

# Harmonic Analysis of EMUs in Railway Systems

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**Abstract--The increasing use of power electronic drives and devices in Electric Multiple Units (EMU) of railway systems has increased the levels of harmonics in the electricity feeding systems of railways. While the harmonic current for a single EMU can be easily measured or simulated, it is not easy to estimate the possible worst case for a number of trains of EMU operating simultaneously on a line according to a time schedule, but with statistical variation. This paper presents a systematic approach to estimate harmonic distortion, which incorporates a novel directional track shift method to deal with the train schedule. The railway simulation is performed in the ATP version of the Electromagnetic Transient Program (EMTP) and the results of the harmonic simulation are compared with measurements made on a railway system. Summaries of the parameters affecting the harmonic performance of the traction system and estimates of the most predominant harmonics in network are also presented.**

*Index terms*—Harmonic, Railway, Train Schedule

## 1. INTRODUCTION

The statistical approach has commonly been applied to industrial loads where the loads are time-varying. In this simulation, the train loads are varying and moving continuously. In this paper a stochastic method is presented which incorporates the statistical nature of the train schedule with the traction system in order to predict the harmonic distortion in the AC-side of a DC railway system. Consequently, the harmonic currents of input feeders and the total

harmonic distortion (THD) are dependent on the passenger loads, train schedule of all trains in up and down tracks and are described by the probability-of-occurrence of the harmonic currents.

The conventional method with time-domain train schedule [1] has been applied for harmonic analysis. However, the conventional method is modelled for the real operations of the trains. For the simulations of all train schedules, the simulation time of the train schedule should be the same as the real operations. In normal train operation, the velocity and acceleration pattern of single typical train more or less repeats. In order to consider all situations of different track schedule, a novel directional track shift method is developed for the various train schedule patterns. This method can overcome the omission of some possible train schedule.

## 2. SIMULATION METHODOLOGY

Time domain multi-train method [1] simulates the train movement in a certain period of time (e.g. 30 minutes) in accordance with train schedule in real operations. However, the simulation of train schedule may omit some possible train schedule, especially the different train schedules in different tracks. Therefore, we develop a novel directional track shift method for harmonic analysis of multi-train railway systems and the details are described in Section 2.1.

Matlab was used to generate the train schedule for the EMTP simulation according to typical train information. This Matlab train schedule was imported into the EMTP simulation and the EMTP determined the targeted current waveforms. In the simulation it was assumed that all voltage sourced from the electric supply

company are constant and free of harmonics. The waveforms determined by EMTP were then transferred to Matlab where Fast Fourier Transform and the Statistical Toolbox were used to create cumulative distribution curves, which were then used to analyse the harmonic situation.

## 2.1. TRAIN SCHEDULE

### 2.1.1. DIRECTIONAL TRACK SHIFT METHOD FOR HARMONIC ANALYSIS

The train schedules of both up track and down track with train time interval are considered in the simulation in order to calculate the multi-train schedule. A typical train schedule for the rush hour is shown in Figure 1, and it assumed that the pattern of the train schedule repeats. Only the period in the window is used in the simulation because the train schedule is the same as other train schedules with the same time interval. In order to cover all cases of train schedule patterns in harmonic analysis, the even distribution of time shift between up track and down track is assumed. Based on this concept, one of the track schedules is kept constant and another track shifts a time interval  $\Delta t_{shift}$  each time in one direction across the time axis. Therefore, the numerical shift of time interval in down track covers one cycle of the time schedule in the simulations until next train is located at the start point of last train. This loop of simulations can cover different locations of trains in up track and down track. If the intervals of up tracks and down tracks are different, the least common multiple (*LCM*) of their headways in terms of time intervals of both tracks is applied to the total simulation interval of the train schedule  $I_{sim}$ , i.e. the repeatable rectangular window in train schedule. This is used to ensure the coverage of all cases during the train operation in both tracks. The total number of simulation time steps in train schedule of both tracks is shown in Equation 1.

$$I_{sim} = LCM(\Delta I_1, \Delta I_2)$$

$$T_{shift} = \frac{I_{sim}}{\Delta t_{shift}} \quad (1)$$

where  $I_{sim}$  is the total simulation interval of the train schedule, i.e. repeatable rectangular window,

$T_{shift}$  is total no. of shifts in train schedule,

*LCM* is the function of least common multiple for the parameters inside the bracket,

$\Delta I_1$  and  $\Delta I_2$  are the headways in terms of train interval between two consecutive trains in track 1 and 2,

$\Delta t_{shift}$  is the time interval of track shift.

Undetermined random deviations of train interval in train schedule can also be one of the attributes in the directional track shift method. Since the real situations on the railway operation should have some deviations in the velocity profile, the actual time and location of trains should differ from the train schedule. However, the railway company has not conducted any research on this aspect and no related statistical data is available. As a result, sampling site measurement for checking the time accuracy was used. The purpose is to check all trains to ascertain if they follow the accurate train schedule without deviation, by checking the time display located at the headwall of the train platform. From the sampling site measurement, the mean and standard deviation of train departure time are measured, respectively. In train schedule, the train departure times of trains are generated using the mean and standard deviation. Based on the typical pattern of single train, the train schedule of up track and down track with the deviation of train departure time are calculated by the time interval of two consecutive trains. The parameters used in this directional track shift method are described in the section of “System Parameters”.

### 2.1.2. SIMULATION PROCEDURE

The electrical performance part of the system was modelled using EMTP. However for traction systems, some parameters are difficult to determine. Furthermore, EMTP can only simulate one case at a time. Therefore, Matlab was interfaced with EMTP to run the statistical simulations.

The sequence is as follows:

Step 1. Determine the headways in terms of time interval  $\Delta I$  of two consecutive trains and the starting locations of multi-trains in both up track and down track and calculate the typical velocity profile of single train in both tracks. Set the number of track shift  $N_{shift}$  to be 1. Determine the time interval of track shift  $\Delta t_{shift}$  and calculate the total simulation interval of the train schedule  $I_{sim}$  and the number of track shift  $T_{shift}$  in this traction simulation in equation 1.

Step 2. Calculate a typical train schedule [4] [6] in accordance with the headways in terms of time interval  $\Delta I$ , starting locations and typical velocity profiles of multi-trains in both tracks, as shown in the enlarged repeatable rectangular window  $I_{sim}$  of Figure 2.

Step 3. From the train schedule, the accelerations, velocities, locations and the slope of the track at the corresponding locations of all trains are determined in the current timestep  $t_{schedule}$  in the train schedule. Those data together with the total train mass, which is the sum of empty train mass and total passenger mass in the train, are then imported into the mechanical model and electrical model of traction system within EMTP. EMTP is used to simulate the current waveform of the target points.

Step 4. Continue to next step if the schedule timestep is at the end of the rectangular window of train schedule. Otherwise, repeat from step 2 with next timestep  $t_{schedule+1}$  increased by  $\Delta t$  in the train schedule.

Step 5. Continue to next step if the number of track shift  $N_{shift}$  in this traction simulation equals the total number of track shift  $T_{shift}$ . Otherwise, the schedule of one of the tracks, e.g. up track, is kept stationary and that of the rest of the tracks, e.g. down track, is shifted with a timestep  $\Delta t_{shift}$ . Add 1 to the number of track shift  $N_{shift}$  and redefine starting locations of multi-trains in the stationary track (up track) and the shifted track (down track). Repeat the procedures from step 2 to recalculate the shifted train schedule as shown in Figure 3.

Step 6. Fast Fourier Transform is used to perform the harmonic analysis of the above-mentioned waveforms of all target points.

Step 7. The Statistical Blockset in Matlab is used to create cumulative distribution curves of current harmonics, total harmonic distortion and total demand distortion.

## 2.2. EMTP MODELLING

### 2.2.1. MECHANICAL MODEL

The profile of desired speed against distance is according to the train schedule in Figure 1. From the train schedule, a train's old position and the train acceleration can be determined and the new train's position can be calculated for the current discrete time sample. The total load torque is the summation of the torque from the air resistance, the net weight of the train and the weight of passengers on the train. The total load torque ( $T_l$ ) per one unit train engine transformed to the motor shaft.

### 2.2.2. ELECTRICAL MODEL

The configuration of the motor traction motor unit is shown in Figure 4. The differential equations for the model of a separately excited DC motor are as follows:

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E_{back} \quad \text{for armature circuit}$$

$$T_m = K_t i_a \quad \text{for motor electric torque} \quad (2)$$

Where the voltage constant  $K_e$  is equal to the torque constant  $K_t$ . Overhead lines and rails in this DC traction system are complex components to model because rails are different from conventional transmission lines in shape and in electrical characteristics. Since this study focuses on harmonic analysis, the DC overhead lines and rails shown can be modelled in EMTP as distributed parameter frequency dependent line models. This model is known as the JMARTI overhead line model in EMTP. The frequency dependence and distributed nature of the parameters are well approximated. Both of the DC overhead lines and rails were modelled as untransposed overhead lines.

### 2.2.3 HARMONIC CALCULATION

Once the electrical network built, the instantaneous dc load current of EMUs and the incoming feeders to overhead line for trains can be calculated using EMTP. Passing through the converters and incoming transformers, the AC current waveforms from the power supplies can be calculated. These waveforms are then transferred to Matlab and calculate the harmonics using FFT. Repeat the above procedures with different parameters in train schedule, passenger loads, unbalanced voltage supplies and background loads of the EMUs and obtain different harmonics in various situations. Once all electrical simulations with various situations are finished, the characteristic harmonics such as 11<sup>th</sup>, 13<sup>th</sup>, 23<sup>rd</sup> and 25<sup>th</sup> harmonics are selected and calculate the cumulative probability curve of the harmonics.

### 3. SIMULATION AND MEASUREMENT

In this study, the stochastic behaviour of the harmonic distortion under different train scheduling and passenger loadings was determined both by simulation and measurement. The measurements were performed during rush hours from 8:30 am to 9:00 am for one week in summer, and the current harmonics on the AC feeders were measured with 1800 samples using a data-logging instrument. A stochastic simulation model with 800 samples was created in the EMTP environment where models of the converters, motors, overhead lines and the rails were developed.

Since the EMUs include the smoothing chokes for the chopper circuits, the output current of the EMUs are smoothed which do not cause the harmonic problems. Therefore, the major harmonics in AC side of the DC railway are generated from 12-pulse converters. The simulation and measurement results of fundamental, 11th, 13th, 23rd and 25th order of current harmonics (i.e. characteristic harmonics of a 12-pulse converter) at point T11P (in Figure 5) are presented below. Since the trains present random fluctuating loads, probability-of-

occurrence curves are used to present probability distributions of the presence in their harmonic components. Figures 6 and 8 are simulation results of probability-of-occurrence of the fundamental, 11th, 13th, 23rd and 25th orders of harmonic currents at point T11P in rush hour. Figures 12 and 14 are the corresponding total demand distortion (TDD) and total harmonic distortion (THD) curves. Figures 7 and 9 are the corresponding measurement results.

### 4. CONCLUSIONS

Railway traction systems present randomly varying loads to the electrical supply system. Consequently, the prediction of harmonic contents is especially difficult, and requires stochastic techniques. Statistically varying parameters include number of passengers on trains, velocity profiles, train schedule, train locations, as well as variation in the loading of air conditioning and lighting.

This paper presents a study of the current distortions in an electrical distribution system feeding a DC traction system. The study involves both field measurements and computer simulation. A directional track shift method was used to account for the train schedule, and stochastic variables, such as the probability density functions of the train schedule deviations were included in the harmonic simulation study. Parameters which were not deterministic, such as passenger numbers, passenger weight on trains and typical velocity profiles of trains were represented by mean values, determined from historical and design parameters supplied by the railway company.

The major contribution in this paper is the simulation technique: the comparisons of the statistical distributions of harmonic content determined by simulation from field measurements are quite close, and indicate that the simulation methods developed in this study can be used to predict harmonic contents of new traction systems that are being designed. Additionally, the directional track shift method can be used for load flow studies in traction systems.

## 5. REFERENCES

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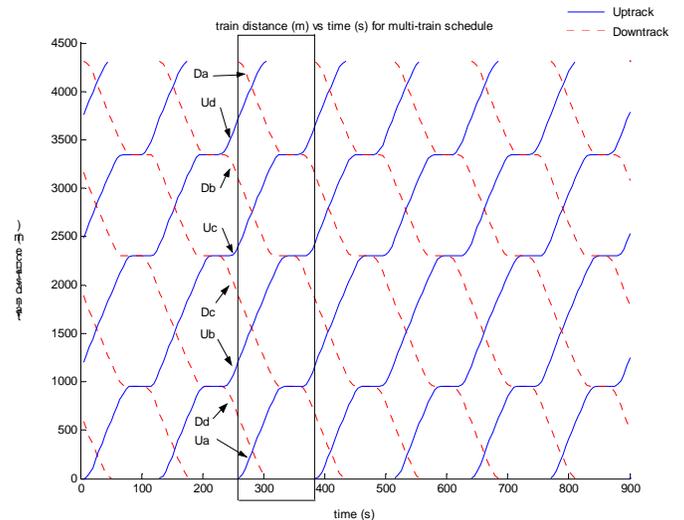


Figure 1. Typical Multi-train Schedule in Rush Hour

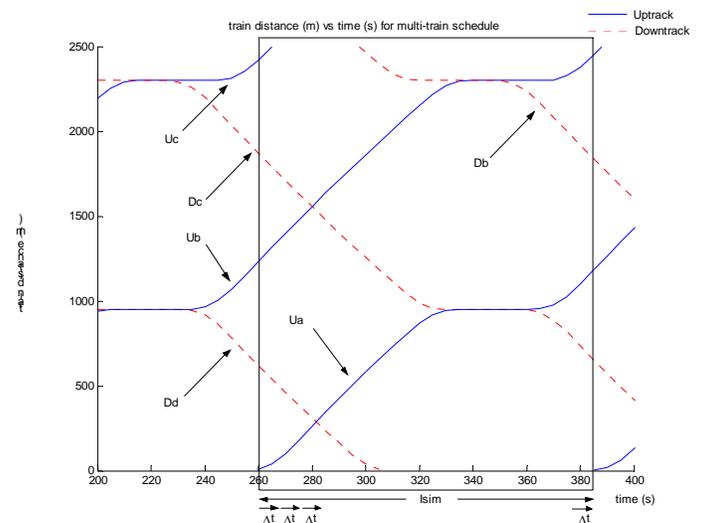


Figure 2. Enlarged Simulation Interval in Train Schedule

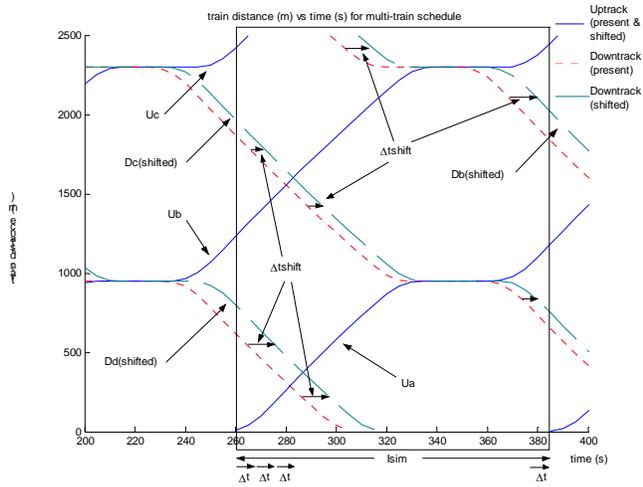


Figure 3. Enlarged Simulation Interval in Shifted Train Schedule

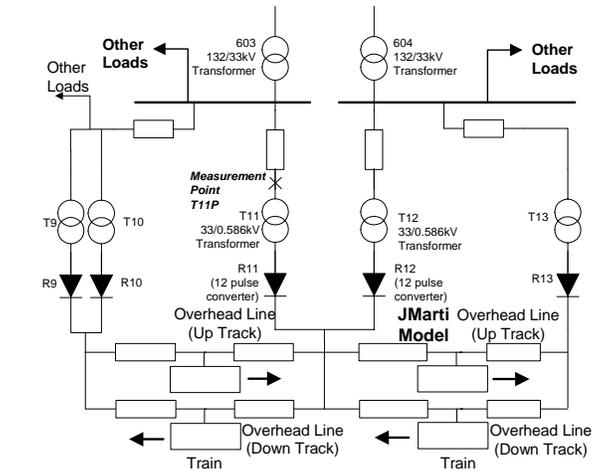


Figure 5. Schematic diagram of the traction network

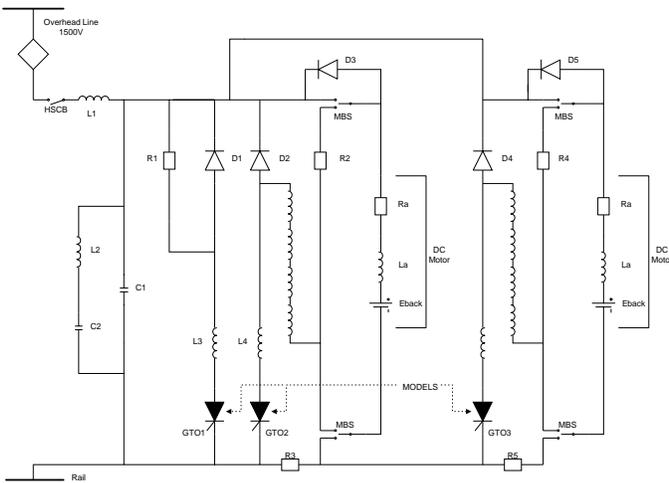


Figure 4. Configuration of one motor car

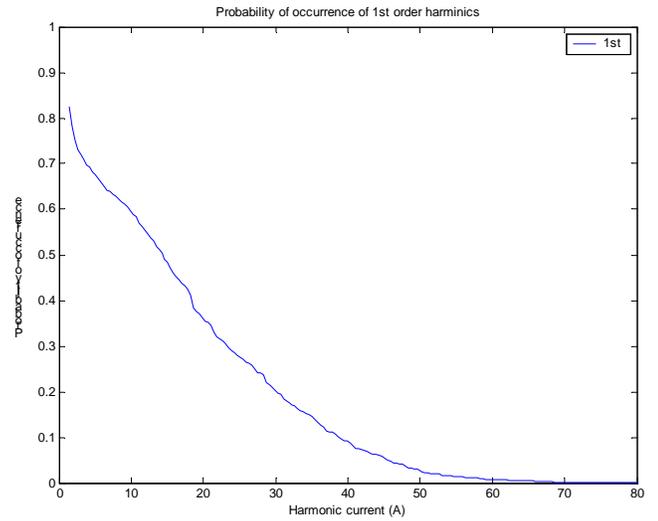


Figure 6. Fundamental current (Rush Hour Simulation)

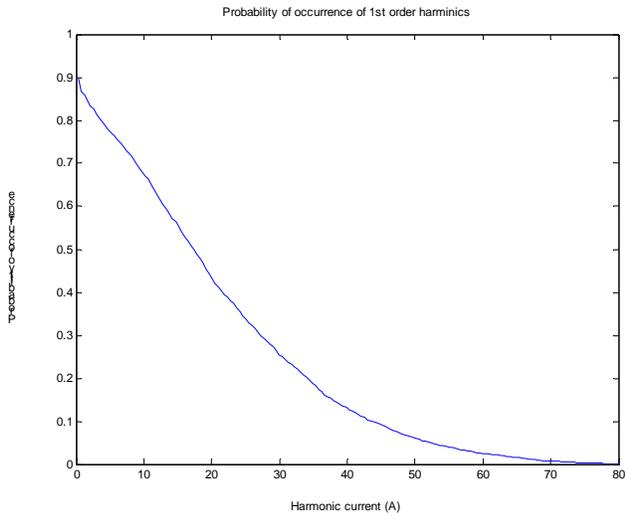


Figure 7. Fundamental current (Rush Hour Measurement)

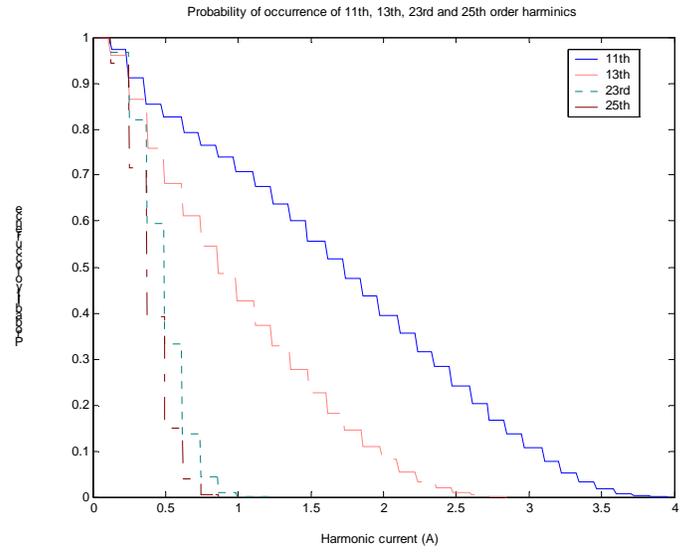


Figure 9. 11<sup>th</sup>, 13<sup>th</sup>, 23<sup>rd</sup> and 25<sup>th</sup> order harmonic current (Rush Hour Measurement)

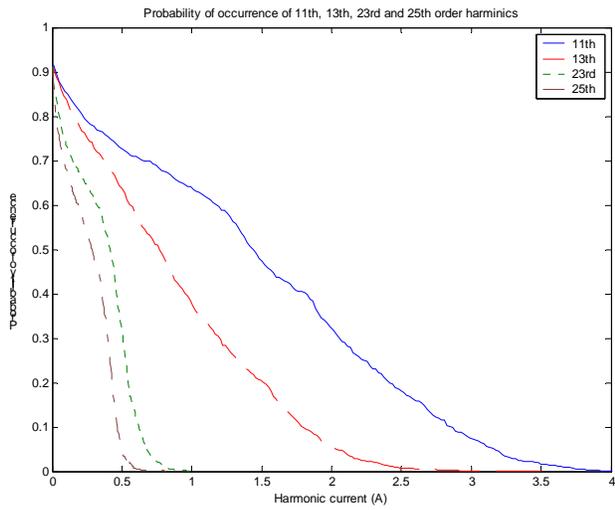


Figure 8. 11th, 13th, 23rd and 25th order harmonic current (Rush Hour Simulation)