# Image Restoration by Regularization in Uncorrelated Transform Domain

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***Abstract:*** *Conventional spatially-adaptive regularized image restoration schemes weight the amount of regularization according to the spatial content of an image. In this correspondence, we first separately decorrelate the signals under analysis into uncorrelated components and then weight the amount of regularization performed to these components accordingly. The proposed approach works better than conventional schemes especially in edge regions.*

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# Introduction

In image restoration, an image degradation process can be generally formulated by $\boldsymbol{y}=\boldsymbol{Hx}+\boldsymbol{n}$, where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the lexicographically ordered original and degraded images, $\boldsymbol{n}$ is a noise vector and $\boldsymbol{H}$ represents a linear degradation operator[1].

Solving this equation directly to get the solution $\hat{\boldsymbol{x}}$ from the observable $\boldsymbol{y}$ is not feasible as it is basically an ill-posed problem[2]. Restoration methods based on regularization theory[3] are widely used instead as they can successfully replace the ill-posed problem by a well-posed problem. Constrained optimal method[4] is the simplest methodology to realize regularization. In this method, an algebraic objective function of $\hat{\boldsymbol{x}}$ is defined based on different constraint sets. The solution is then obtained by minimizing the objective function with respect to $\hat{\boldsymbol{x}}$.

In general, two constraints are used. One of them tries to keep the solution faithful to the information provided by the observed version $\boldsymbol{y}$, which is usually given as $\|\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}\|^2 < \varepsilon$, while the other one tries to remain the solution faithful to the *a priori* information about the original image, which can be generally given as $\|\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})\|^2 < e$. Here, $\breve{\boldsymbol{x}}$ represents our priors about the solution and $\boldsymbol{L}$ is a linear operator. The bounds $\varepsilon$ and $e$ respectively describe how "faithful" the possible solution to the observed information and the *a priori* information should be. In other words, their values tell the relative significance of the constraints to the solution and hence should be used to weight the contribution of the constraints in constructing the objective function[5]. The objective function derived from this idea is given as $J=\|\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}\|^2+\alpha\|\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})\|^2$, where $\alpha=\varepsilon/e$.

By probing further, one can see that, in fact, different elements of $\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}$ make different amount of contribution to the error function $\|\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}\|^2$ in fulfilling the constraint $\|\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}\|^2 < \varepsilon$. In order to have a good solution $\hat{\boldsymbol{x}}$, their contribution should also be weighted. Similar case happens when we investigate the elements of $\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})$. By taking these factors into account, the objective function should be modified to be

$$J=\|\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}\|_{\boldsymbol{R}}^2+\alpha\|\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})\|_{\boldsymbol{S}}^2 \quad (1).$$

Here $\|.\|_{\boldsymbol{R}}^2$ and $\|.\|_{\boldsymbol{S}}^2$ denote weighted norms. This generalized formulation describes almost all spatially-adaptive regularized restoration methods reported in the literature[6-10]. In general, these methods differ by their ways to evaluate $\boldsymbol{R}$ and $\boldsymbol{S}$. Both $\boldsymbol{R}$ and $\boldsymbol{S}$ are usually oversimplified to be diagonal matrices in these methods.

If introducing weighting factors to weight the contribution of different elements is the right direction to improve the restoration performance, the objective function given as (1) is obviously not the ultimate solution. Consider the objective function (1) again. Since $\boldsymbol{R}$ and $\boldsymbol{S}$ are diagonal matrices, each element of $\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}$ and $\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})$ is weighted separately. This implies the elements are considered to be independent of each other. This is obviously not true as adjacent image pixels in an image are highly correlated. By using such a simplified weighting approach, the weighting effect of different weighting factors may counteract each other and hence not be able to provide a good restoration result effectively.

To solve this problem, we suggest considering $\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}$ and $\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})$ as two different signals and separately decomposing them into a number of uncorrelated channels by transforms for weighting. By doing this, two advantages can be gained. First, it is easier for one to determine the weighting factor for a particular channel as uncorrelated channels do not interact with each other. They are isolated and easier to be handled. Weighting a particular channel will not affect the other channels. The second advantage also comes from the decorrelation property of the image transform. In practical circumstances, one has to estimate the weighting factors from either the distorted image or the *a priori* information, so there must be some estimation errors. The less correlated the channels are, the less sensitive is the restoration result to the estimation errors in the channels.

In this correspondence, based on the idea we have mentioned, we make use of the transform theory[11] to decorrelate images into uncorrelated transform components and then weight these components according to their variances. This approach can definitely improve the restoration performance. In particular, it is found in our experiments that the details of the edge regions of restored images can be greatly improved with the proposed approach while the other conventional spatially-adaptive approaches cannot[6-10].

Note there are some reported literatures which consider a distorted image as a multichannel signal and restore it in the frequency domain[12]. However, their motivations and implementations are quite different from ours. Generally speaking, decorrelating signal before weighting is not their basic concern. In these approaches, the discrete Fourier transform (DFT) is typically used to decompose the distorted image into a number of frequency channels for subsequent restoration. Since DFT components are still correlated, these approaches can be considered as simultaneously performing spatial weighting schemes to a number of subband images.

## Algorithm

Suppose $\boldsymbol{b}$ is a vector of random variables. The value of each random variable is of a certain uncertainty but its statistical characteristics are known or can be estimated. Without lose of generality, we assume $\boldsymbol{E}[\boldsymbol{b}]=\vec{0}$, where $\boldsymbol{E}[.]$ is the expectation operator and $\vec{0}$ denotes the zero vector. Note, since its statistical characteristics are known, $\boldsymbol{b}$ can always be zero-meaned. Assume $\boldsymbol{T}$ is the unitary Karhunen-Loeve transform (KLT) of $\boldsymbol{b}$[13]. Then $\boldsymbol{T}$ can completely decorrelate $\boldsymbol{b}$ and $\boldsymbol{E}[\boldsymbol{Tbb^tT^t}]$ is a diagonal matrix. The i$^{th}$ diagonal element of the matrix, denoted as $\langle \boldsymbol{E}[\boldsymbol{Tbb^tT^t}]\rangle_{\boldsymbol{ii}}$, is the variance of $[\boldsymbol{Tb}]_{\boldsymbol{i}}$, where $[\boldsymbol{Tb}]_{\boldsymbol{i}}$ is the i$^{th}$ element of $\boldsymbol{Tb}$. In formulation, we have $\langle \boldsymbol{E}[\boldsymbol{Tbb^tT^t}]\rangle_{\boldsymbol{ii}} = \boldsymbol{E}[[\boldsymbol{Tb}]_{\boldsymbol{i}}^2]$. Obviously, $\boldsymbol{E}[[\boldsymbol{Tb}]_{\boldsymbol{i}}^2]$ indicates the relative degree of uncertainty of $[\boldsymbol{Tb}]_{\boldsymbol{i}}$ with respect to the other elements of $\boldsymbol{Tb}$. This information can hence be used to weight the contribution of each element of $\boldsymbol{Tb}$ to $\|\boldsymbol{Tb}\|^2$. Specifically, the weighting factor should be proportional to $1/\boldsymbol{E}[[\boldsymbol{Tb}]_{\boldsymbol{i}}^2]$. If $\boldsymbol{E}[\boldsymbol{bb^t}]$ is the *a priori* information we know about $\boldsymbol{b}$, then $\boldsymbol{E}[[\boldsymbol{Tb}]_{\boldsymbol{i}}^2]$ can be easily determined as $\boldsymbol{E}[[\boldsymbol{Tb}]_{\boldsymbol{i}}^2] = \langle \boldsymbol{E}[\boldsymbol{Tbb^tT^t}]\rangle_{\boldsymbol{ii}} = \langle \boldsymbol{TE}[\boldsymbol{bb^t}]\boldsymbol{T^t}\rangle_{\boldsymbol{ii}}$.

Based on the idea we have mentioned, the objective function can be given as

$$J = \|\boldsymbol{T}_1(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}-\boldsymbol{M})\|_{\boldsymbol{R}}^2 + \alpha\|\boldsymbol{T}_2(\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})-\boldsymbol{M}_{\boldsymbol{f}})\|_{\boldsymbol{S}}^2 \quad (2),$$

where $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ are the KLTs for $\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}-\boldsymbol{M}$ and $\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})-\boldsymbol{M}_{\boldsymbol{f}}$ respectively. Here, $\boldsymbol{M}=\boldsymbol{E}[\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}]$ and $\boldsymbol{M}_{\boldsymbol{f}}=\boldsymbol{E}[\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})]$. The parameter $\alpha$ should be determined as $\alpha=\varepsilon_1/e_1$, where $\varepsilon_1$ and $e_1$ are the bounds of $\|\boldsymbol{T}_1(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}-\boldsymbol{M})\|_{\boldsymbol{R}}^2$ and $\|\boldsymbol{T}_2(\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})-\boldsymbol{M}_{\boldsymbol{f}})\|_{\boldsymbol{S}}^2$ respectively. The weighting matrices $\boldsymbol{R}$ and $\boldsymbol{S}$ are diagonal matrices intrinsically and their i$^{th}$ diagonal elements can be determined as

$$r_i = \frac{1}{\langle \boldsymbol{E}[\boldsymbol{T}_1(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}-\boldsymbol{M})(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}-\boldsymbol{M})^{\boldsymbol{t}}\boldsymbol{T}_1^{\boldsymbol{t}}]\rangle_{\boldsymbol{ii}}} \quad (3)$$

$$s_i = \frac{1}{\langle \boldsymbol{E}[\boldsymbol{T}_2(\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})-\boldsymbol{M}_{\boldsymbol{f}})(\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})-\boldsymbol{M}_{\boldsymbol{f}})^{\boldsymbol{t}}\boldsymbol{T}_2^{\boldsymbol{t}}]\rangle_{\boldsymbol{ii}}} \quad (4)$$

Note eqns. (2)-(4) provide the general formulations for restoring a degraded image.

Now let us consider the case when smoothness constraint is applied. In such a case, we can let $\boldsymbol{L}(\hat{\boldsymbol{x}}-\breve{\boldsymbol{x}})$ be $\boldsymbol{C}\hat{\boldsymbol{x}}$, where $\boldsymbol{C}$ is a spatial 2D highpass Laplacian filter represented in matrix form. As $\boldsymbol{C}\hat{\boldsymbol{x}}$ theoretically contains no low frequency component, we assume $\boldsymbol{M}_{\boldsymbol{f}}=\boldsymbol{E}[\boldsymbol{C}\hat{\boldsymbol{x}}]=\vec{0}$ in order to simply the analysis. In addition, if ***y-Hx*** is a zero-mean white noise of variance $\sigma_n^2$, we can assume $\boldsymbol{E}[(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}})(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}})^{\boldsymbol{t}}]=\sigma_n^2\,\mathbf{I}$ and $\boldsymbol{M}=\boldsymbol{E}[\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}]=\vec{0}$. This implies $\boldsymbol{T}_1=\mathbf{I}$ and $r_i=1/\sigma_n^2$. Hence, the objective function (2) can be simplified as

$$J=\frac{1}{\sigma_{\boldsymbol{n}}^2}\left\|\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}\right\|^2+\alpha\left\|\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}\right\|_{\boldsymbol{S}}^2 \tag{5}$$

The minimization of *J* with respect to $\hat{\boldsymbol{x}}$ results in the normal equation

$$\left(\boldsymbol{H}^{\boldsymbol{t}}\boldsymbol{H}+\alpha\sigma_{\boldsymbol{n}}^2\,\boldsymbol{C}^{\boldsymbol{t}}\boldsymbol{T}_2^{\boldsymbol{t}}\boldsymbol{S}\boldsymbol{T}_2\boldsymbol{C}\right)\hat{\boldsymbol{x}}=\boldsymbol{H}^{\boldsymbol{t}}\boldsymbol{y} \tag{6}$$

In general, $\hat{\boldsymbol{x}}$ cannot be evaluated directly from this equation as it requires the inversion of a huge matrix. An alternative approach is to use a steepest descent algorithm to approximate $\hat{\boldsymbol{x}}$ iteratively[6]. This approach leads to the following iterative equation:

$$\hat{\boldsymbol{x}}_{\boldsymbol{k}+1}=\hat{\boldsymbol{x}}_{\boldsymbol{k}}+\beta\left[\boldsymbol{H}^{\boldsymbol{t}}(\boldsymbol{y}-\boldsymbol{H}\hat{\boldsymbol{x}}_{\boldsymbol{k}})-\alpha\sigma_{\boldsymbol{n}}^2\boldsymbol{C}^{\boldsymbol{t}}\boldsymbol{T}_2^{\boldsymbol{t}}\boldsymbol{S}\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}}\right] \tag{7a}$$

$$\hat{\boldsymbol{x}}_0=\beta\boldsymbol{H}^{\boldsymbol{t}}\boldsymbol{y} \tag{7b}$$

where $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$ is the estimate of $\hat{\boldsymbol{x}}$ at the $k^{th}$ iteration. The iteration converges if $\beta$ satisfies the condition $0<\beta<2/\lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of the matrix $\boldsymbol{H}^{\boldsymbol{t}}\boldsymbol{H}+\alpha\sigma_{\boldsymbol{n}}^2\,\boldsymbol{C}^{\boldsymbol{t}}\boldsymbol{T}_2^{\boldsymbol{t}}\boldsymbol{S}\boldsymbol{T}_2\boldsymbol{C}$.

The weighting matrix ***S*** can be estimated at each iteration based on the available form of the restored image $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$. In particular, by substituting the assumptions mentioned earlier into eqn. (4), we have

$$s_i=\frac{1}{\left\langle \boldsymbol{E}[\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}}(\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}})^{\boldsymbol{t}}]\right\rangle_{\boldsymbol{ii}}} \tag{8}$$

Note that $\left\langle \boldsymbol{E}[\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}}(\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}})^{\boldsymbol{t}}]\right\rangle_{\boldsymbol{ii}}$ is in fact the variance of the $i^{th}$ element of $\boldsymbol{T}_2\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}}$. In practice, it is estimated with the ensemble $\Omega=\{\,\boldsymbol{T}_2(\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}})^{<\boldsymbol{m},\boldsymbol{n}>}:\ |m|,|n|\le d\,\}$, where *d* is an integer parameter which defines the size of the ensemble and $(\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}})^{<\boldsymbol{m},\boldsymbol{n}>}$ denotes the shift version of $\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}}$ obtained by shifting all its elements m steps up and n steps right in the spatial domain. Specifically, its estimated value $\theta_i$ is given as

$$\theta_i=\frac{1}{(2d+1)^2}\sum_{m=-d}^{d}\sum_{n=-d}^{d}\left[\boldsymbol{T}_2\left((\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}})^{<\boldsymbol{m},\boldsymbol{n}>}-\boldsymbol{M}_{\boldsymbol{c}}\right)\right]_{\boldsymbol{i}}^2 \tag{9}$$

where

$$\boldsymbol{M}_{\boldsymbol{c}}=\frac{1}{(2d+1)^2}\sum_{m=-d}^{d}\sum_{n=-d}^{d}\left(\boldsymbol{C}\hat{\boldsymbol{x}}_{\boldsymbol{k}}\right)^{<\boldsymbol{m},\boldsymbol{n}>} \tag{10}$$

In contrast to the approaches which make use of local spatial variance[6-8], the proposed approach approximates weighting factors in the transform domain. As we have mentioned in previous section, it would be helpful to obtain a better restoration result because of the lower sensitivity to approximation error in the transform domain.

The value of $\theta_i$ could fluctuate violently from i to i, so $s_i$ may not be stable if one directly lets $s_i$ be $\theta_i^{-1}$ with eqn. (8). In order to make $s_i$ stable, the equation $s_i = 1/(1+\kappa\theta_i)$ is used instead to confine $s_i$ in the interval (0,1]. The parameter $\kappa$ is a tuning parameter that can be adjusted experimentally to make the weighting effect be able to provide a good restoration result from the human visual point of view.

As a final remark, we note here that sometimes an approximation of the KLTs involved in the proposed scheme is required due to two practical reasons. First of all, it is computationally very difficult to determine a KLT kernel of large size. Secondly, even though the KLT kernels are given, the computational complexity of performing them to the images is very high[13].

In practice, the discrete cosine transform (DCT) is used instead to decorrelate the unstacked image $\boldsymbol{C}\hat{\boldsymbol{x}}$. This is because, according to the image transform theory, an image can typically modeled as a highly correlated 2D Markov-I signal and the DCT is asymptotically equivalent to the KLT in decorrelating such a kind of signals[13]. Other reasons for using the DCT are that there are a number of fast algorithms for its realization and its realization complexity is much lower than that of the KLT.

## Simulation Studies

An experiment was first carried out to verify that the DCT was a good approximation to the KLT in decorrelating $\boldsymbol{Cx}$. In this experiment, 4 standard images, namely, "lenna", "cameraman", "house" and "germany", were filtered with operator $\boldsymbol{C}$ and then partitioned into a number of subimages of size $8\times 8$ to form a set $\Gamma_{\boldsymbol{Cx}}$. Then, based on this set of subimages, the correlation coefficients between any two different pixels of $\boldsymbol{Cx}$ were computed and plotted against their magnitude order. Similarly, by using the set $\Gamma_{\boldsymbol{TCx}} = \{\boldsymbol{Tz}: \boldsymbol{z} \in \Gamma_{\boldsymbol{Cx}}\}$, where $\boldsymbol{T}$ is any particular transform operator, we obtained the curve reflecting the correlation among the elements of $\boldsymbol{TCx}$. Fig. 1 shows our experimental result. One can easily observe that performing a transform can definitely decorrelate $\boldsymbol{Cx}$ and the decorrelation performance of the DCT is the best among the transforms.

Simulations were then performed to evaluate the performance of the proposed restoration scheme on a set of 256 level gray-scale digital images of size $256 \times 256$ each. In particular, we would like to see if weighting decorrelated components is more effective than weighting correlated components in providing a good image restoration performance. In order to achieve this, we realized the scheme proposed in [6] as well for comparison. These two schemes are more or less the same except that the proposed scheme weights the components after decorrelating them while the one in [6] does not. Hereafter, we refer to them as non-spatially adaptive weighting (NAW) and spatially adaptive weighting (SAW) schemes respectively.

In the realization of the proposed scheme, three subschemes were simulated for comparison. In the first subscheme, the decorrelation transform was approximated with periodic $8 \times 8$ two-dimensional DCT transform kernels. Specifically, to decorrelate the unstacked image $\boldsymbol{C\hat{x}}$, we first partitioned it into a number of non-overlapped subimages of size $8 \times 8$ and then performed an $8 \times 8$ DCT to each of them. In the second and the third subschemes, $256 \times 256$ DCT and DFT were respectively exploited to decorrelate the unstacked image $\boldsymbol{C\hat{x}}$.

As for the realization of the SAW scheme[6], the solution was obtained by the following iterative equations:

$$\boldsymbol{\hat{x}_{k+1}} = \boldsymbol{\hat{x}_k} + \beta_{\boldsymbol{o}}\left[\boldsymbol{H^t}(\boldsymbol{y} - \boldsymbol{H\hat{x}_k}) - \alpha_{\boldsymbol{o}}\boldsymbol{C^t S' C\hat{x}_k}\right] \tag{11a}$$

$$\boldsymbol{\hat{x}}_0 = \beta_{\boldsymbol{o}}\boldsymbol{H^t y} \tag{11b}$$

Here, $\boldsymbol{S'}$ is a diagonal matrix whose $i^{th}$ diagonal element is given as $s'_i = 1/(1+\kappa\,\theta_i')$, where $\theta_i'$ is the local spatial variance of the corresponding pixel. In particular, we have $\theta_i' = \frac{1}{(2d+1)^2}\sum_{m=-d}^{d}\sum_{n=-d}^{d}\left[(\boldsymbol{\hat{x}_k})^{<\boldsymbol{m,n}>} - \boldsymbol{M}_s\right]_{\boldsymbol{i}}^2$ and $\boldsymbol{M_s} = \frac{1}{(2d+1)^2}\sum_{m=-d}^{d}\sum_{n=-d}^{d}(\boldsymbol{\hat{x}_k})^{<\boldsymbol{m,n}>}$.

Fig. 2a shows the original image of "Cameraman". In our experiment, it was first defocused with circle of confusion equal to 5 pixels. White noise was then added resulting in a SNR of 20 dB, where SNR was defined as SNR=10log(variance of signal/variance of noise). Fig. 2b shows this distorted image. Fig. 2c and 2d-2f are respectively the restoration results of the SAW scheme and the NAW schemes.

In general, the SAW scheme provides a poorer restoration result around the edge regions compared with all NAW schemes. This is because the local spatial variance in a edge region is typically larger than that in a smooth region. A SAW scheme will adapt to it and try not to suppress the noise in the region accordingly. This results in a noisy edge region.

Among the three NAW subschemes, the one using DFT for decorrelation provided the poorest result as the decorrelation performance of DFT is poorer than that of DCT. One can easily observe the pattern noise in Fig. 2d. As for the other two subschemes, the one using block-based DCT provided a better restoration result in terms of the SNR improvement. Images are actually not stationary signal. Using block-based transform enables the weighting matrix to adapt to the local characteristics of an image. Using a single DCT to decorrelate an image is impossible to achieve this. However, there may be some visible blocking effect in the restored image if block-based DCT is used.

Similar findings can be obtained in restoring motion-blurred images. Fig. 3b shows several magnified portions of a noisy and motion-blurred version of "Lenna". Its original was first blurred by horizontal motion blur over 9 pixels. Noise was then added to achieve a SNR of 20 dB. Figures 3c-3f are the corresponding restoration results obtained with the 4 schemes. One can see that the NAW schemes, especially the one using block-based DCT for decorrelation, can provide better restoration results than the SAW scheme. Table 1 summarizes the objective results of the experiments for comparison.

## Conclusions

Conventional spatially-adaptive regularized image restoration schemes weight the amount of regularization performed to different pixels of the solution according to the spatial content of an image. In this correspondence, we suggest, instead of doing so, we should first separately decorrelate the signals under analysis into a number of uncorrelated components by making use of the image transform theory and then weight these components accordingly. Based on this idea, an effective adaptive iterative restoration algorithm is also proposed. The advantages of the proposed approach over the conventional approaches have been discussed and simulation results have been shown. Simulations verified that weighting decorrelated components could provide a better restoration result than weighting highly correlated image pixels especially around the edge regions.

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| Testing image : | Cameraman | | Lenna | |
|---|---|---|---|---|
| Distortion : | Defocus Blur $5\times5$, 20 dB SNR | | Motion Blur $9\times1$, 20 dB SNR | |
| Parameters : | $\beta=\beta_o=0.67$, $\alpha_{\boldsymbol{o}}=\sigma_{\boldsymbol{n}}^2/10\|\boldsymbol{Cy}\|_{S'}^2$, $\alpha=1/10\|\boldsymbol{TCy}\|_{S}^2$, $\kappa=0.05$, $d=1$ | | | |
| Termination rule: | $\|\hat{\boldsymbol{x}}_{\boldsymbol{k}+1}-\hat{\boldsymbol{x}}_{\boldsymbol{k}}\|/\|\hat{\boldsymbol{x}}_{\boldsymbol{k}}\|<10^{-6}$ | | | |
| | SNR Improvement | Number of iteration | SNR Improvement | Number of iteration |
| SAW | 1.67 dB | 73 | 3.19 dB | 51 |
| NAW | | | | |
| DCT8 (BDCT) | 2.06 dB | 39 | 3.35 dB | 40 |
| DCT256 | 1.84 dB | 34 | 3.15 dB | 50 |
| DFT256 | 1.83 dB | 31 | 3.14 dB | 47 |

Table 1. Summary of the simulation results

List of Captions

# Decorrelation Performance of Different Transforms

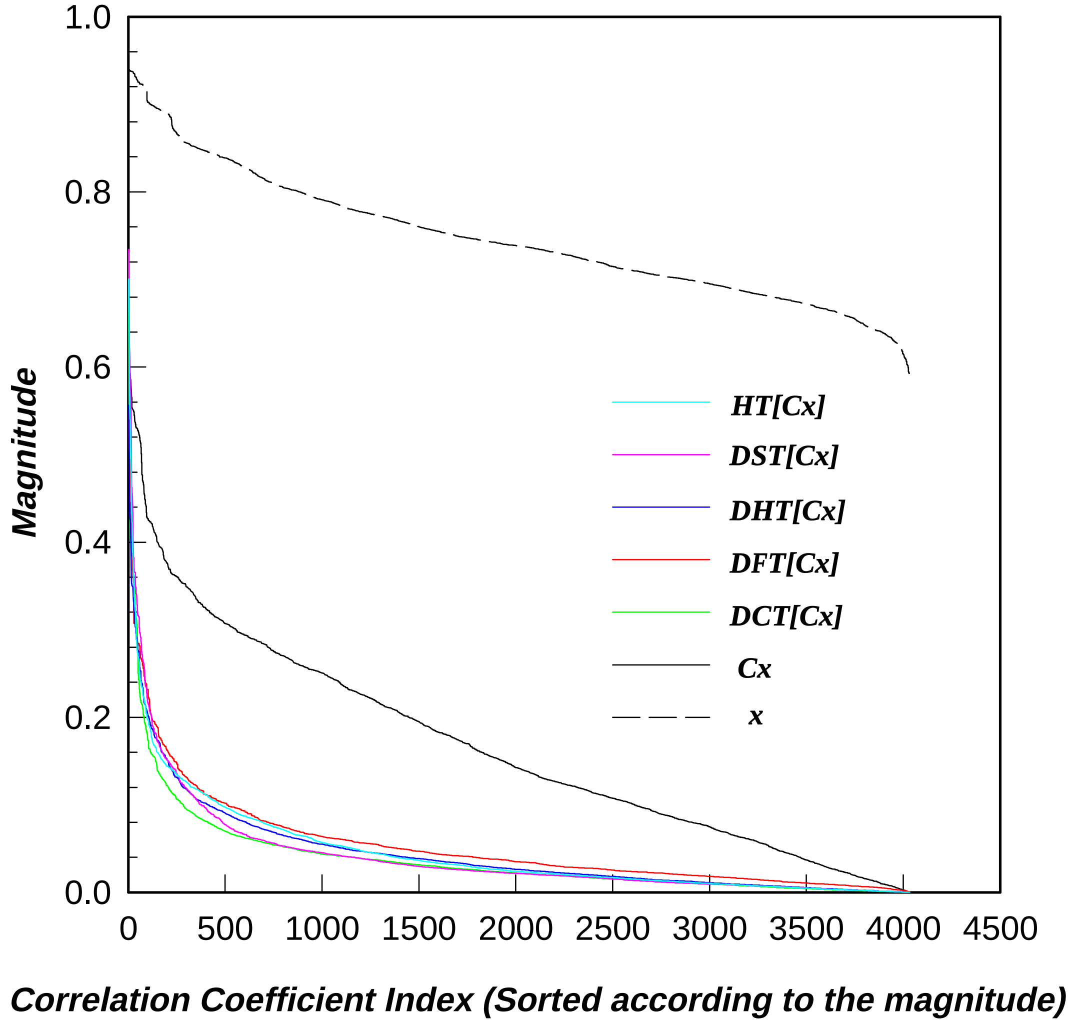


Figure 1. Decorrelation performance of various transforms

| | | |
|---|---|---|
| HT | - | Hadamard Transform |
| DST | - | Discrete Sine Transform |
| DHT | - | Discrete Hartley Transform |
| DFT | - | Discrete Fourier Transform |
| DCT | - | Discrete Cosine Transform |

(a) Original

(b) Degraded

(c) SAW

(d) NAW-DFT256

(e) NAW-DCT256

(f) NAW-BDCT

Figure 2. Restoration performance of various schemes on noisy defocus-blurred image

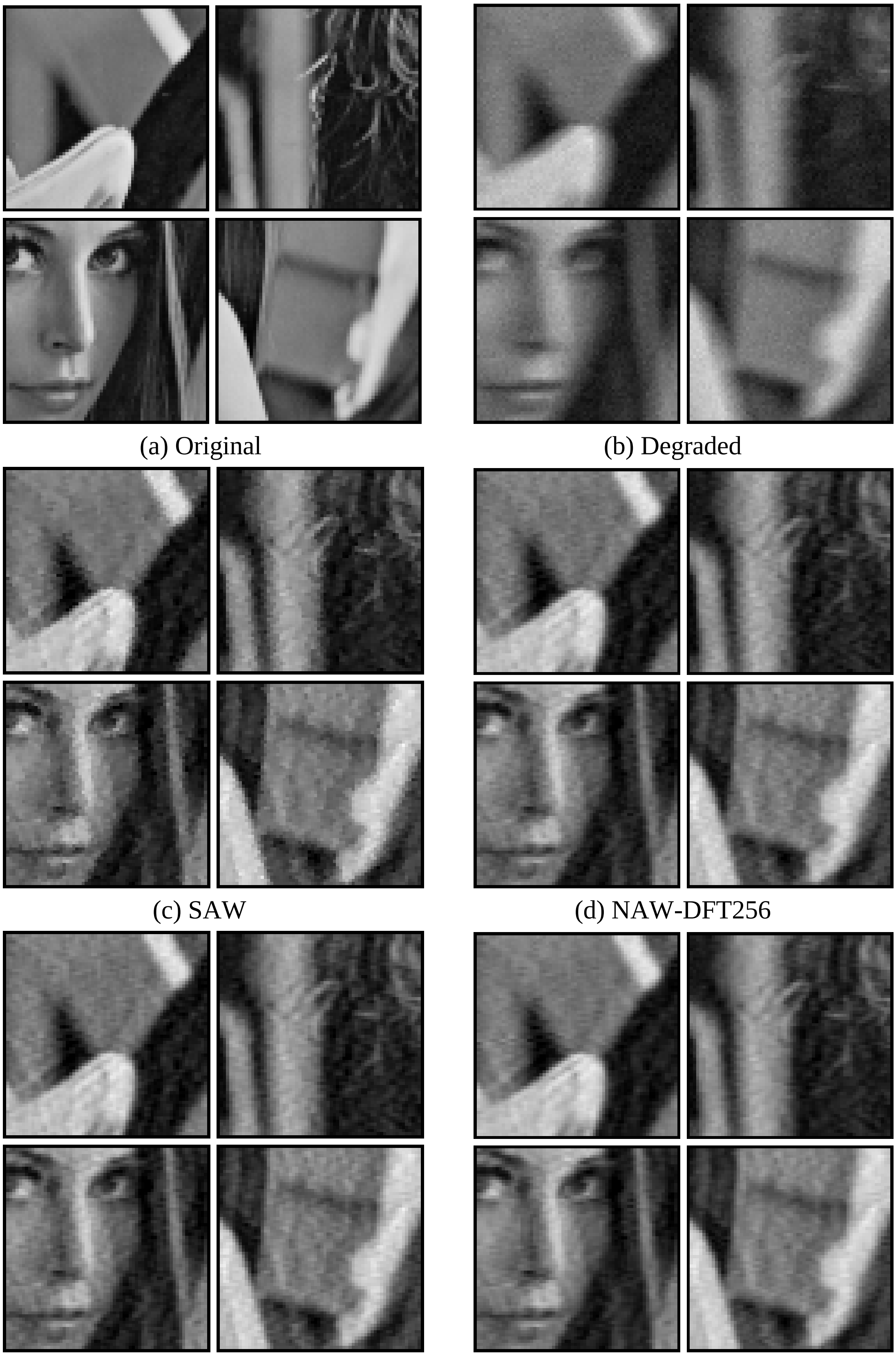


Figure 3. Restoration performance of various schemes on noisy motion-blurred image