# An Iterative Algorithm for Restorating Color-Quantized Images

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## ABSTRACT

This paper studies the restoration of color-quantized images. Restoration of color-quantized images is rarely addressed in the literature, and direct applications of existing restoration techniques are generally inadequate to deal with this problem. We propose a POCS-based restoration algorithm specific to color-quantized images, which makes a good use of the available color palette to derive useful *a priori* information for restoration. The proposed restoration algorithm is shown to be capable of improving the quality of an color-quantized image to a certain extent.

## 1 Introduction

Color quantization is the process of reducing the number of colors in a digital image by replacing them with a representative color selected from a palette [1]. It is widely used nowadays as it lessens the burden of massive image data on storage and transmission bandwidth in many multimedia applications.

A color-quantized image can be considered as a degraded version of the original full-color image. Accordingly, there should be some image restoration techniques for recovering the original image from its color-quantized version whenever it is necessary. Although there is quite an amount of work in the literature on digital restoration of noisy and blurred color images [2]-[7], by far little effort has been seen in the literature to address the restoration of color-quantized images. Obviously, the degradation models of the two cases are completely different and hence a direct adoption of conventional restoration algorithms may not work effectively. This paper is devoted to formulating the problem of color-quantized image restoration and developing a restoration algorithm specific to restoration of color-quantized images.

## 2 Image Degradation in color quantization

A pixel of a 24-bit full color image generally consists of three color components. The intensity values of these three components form a vector in a 3D space. In color quantization, each pixel vector is compared with a set of representative color vectors, $\hat{\mathbf{v}}_i$, $i = 1, 2, \cdots, N_c$, which are stored in a previously generated *color palette*. The best-matching color is selected based on the minimum Euclidean distance criterion, where the Euclidean distance measure between vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ is defined as $d(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2) \equiv \|\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2\|$. In other words, a pixel vector $\vec{\mathbf{v}}$ is represented by color $\hat{\mathbf{v}}_k$ if and only if $d(\vec{\mathbf{v}}, \hat{\mathbf{v}}_k) \leq d(\vec{\mathbf{v}}, \hat{\mathbf{v}}_j)$ for all $j = 1, 2, \cdots, N_c$. Once the best-matching colors for all pixel vectors of the source image have been selected from the color palette, the indices of the selected colors are transmitted to the receiver with the color palette. At the receiver, with the same color palette, the color quantized image can be reconstructed based on the received indices.

## 3 Restoration of color quantized images

### 3.1 Formulation of A Priori Information

A color image $\mathbf{X}$ generally consists of three color planes, say, $\mathbf{X_r}$, $\mathbf{X_g}$ and $\mathbf{X_b}$, which represents the red, the green and the blue color planes of the image respectively. The $(i,j)^{\text{th}}$ pixel of the image is hence a 3D vector represented by $\vec{\mathbf{X}}_{(i,j)} = (\mathbf{X}_{(i,j)r}, \mathbf{X}_{(i,j)g}, \mathbf{X}_{(i,j)b})$, where $\mathbf{X}_{(i,j)c} \in [0,1]$ is the intensity value of the $c^{\text{th}}$ color component of the $(i,j)^{\text{th}}$ pixel. Here, we assume that the maximum and the minimum intensity values of a pixel are, respectively, 1 and 0. Consider the case that image $\mathbf{X}$ is encoded as $\mathbf{Y}$ by color quantization with a color palette $C$ containing $N_c$ colors:

$$C = \left\{ \hat{\mathbf{v}}_i, i = 1, 2, \cdots, N_c \right\}. \tag{1}$$

According to color quantization theory, the $N_c$ colors in the palette partition the whole color vector space, say $\Gamma$, into $N_c$ non-overlapped Voronoi regions. For a particular Voronoi region $R_k = \{\vec{\mathbf{v}} | \vec{\mathbf{v}} \in \Gamma$ and $d(\vec{\mathbf{v}}, \hat{\mathbf{v}}_k) \leq d(\vec{\mathbf{v}}, \hat{\mathbf{v}}_j)$ for $j = 1, 2, \cdots, N_c\}$, the associated variance vector is given as

$$\vec{\sigma}_k = (\sigma_{k,r}^2, \sigma_{k,g}^2, \sigma_{k,b}^2) \tag{2}$$

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where

$$\sigma_{k,c}^2 = \frac{1}{N_{o,k}} \sum_{\vec{v} \in R_k} \left(v_c - \hat{v}_{k,c}\right)^2 \quad for \;\; c \in \{r, g, b\}. \quad (3)$$

Here, $N_{o,k}$ is the total number of vectors in $R_k$, and, $v_c$ and $\hat{v}_{k,c}$ are, respectively, the $c^{\text{th}}$ color components of $\vec{v}$ and $\hat{\mathbf{v}}_k$. The information carried by $\sigma_{k,c}^2$ provides us an important constraint to seek the original image $\mathbf{X}$. In practice, in order to reduce the realization complexity, one may select some typical images to form a training set $\Omega_t$ and then approximate $\vec{\sigma}_k$ with vectors in $\Omega_t \cap R_k$ instead of $R_k$. Note that all $\vec{\sigma}_k$'s are solely determined by the color palette which is attached with the image or known by default and hence no extra information is required for their estimation.

Let $\vec{\mathbf{X}}_{(i,j)}$ and $\vec{\mathbf{Y}}_{(i,j)}$ be the 3-dimensional vectors representing the $(i,j)^{\text{th}}$ pixels of the original image $\mathbf{X}$ and the encoded image $\mathbf{Y}$, respectively. If $\hat{\mathbf{v}}_k \in C$ is the color used to represent $\vec{\mathbf{X}}_{(i,j)}$, then we will have $\vec{\mathbf{Y}}_{(i,j)} = \hat{\mathbf{v}}_k$, and the distance between $\vec{\mathbf{Y}}_{(i,j)}$ and $\vec{\mathbf{X}}_{(i,j)}$ will be bounded by the boundary of the Voronoi region $R_k$. For any particular color component $\mathbf{Y}_{(i,j)c}$ in $\vec{\mathbf{Y}}_{(i,j)}$, its deviation from the corresponding color component in $\vec{\mathbf{X}}_{(i,j)}$, $\mathbf{X}_{(i,j)c}$, is also bounded. In formulation, we have

$$(\mathbf{Y}_{(i,j)c} - \mathbf{X}_{(i,j)c})^2 \leq \varepsilon_{(i,j)c} \quad (4)$$

where $\varepsilon_{(i,j)c}$ is the corresponding bound. Since we have $\vec{\mathbf{Y}}_{(i,j)} = \hat{\mathbf{v}}_k$, $\mathbf{Y}_{(i,j)c} = \hat{v}_{k,c}$ holds and the bound $\varepsilon_{(i,j)c}$ can be estimated to be a function of $\sigma_{k,c}^2$. By assuming that $\varepsilon_{(i,j)c}$ is proportional to $\sigma_{k,c}^2$, we have $|\mathbf{Y}_{(i,j)c} - \mathbf{X}_{(i,j)c}| \leq \beta\sigma_{k,c}$, where $\beta$ is a scaling parameter. This forms a constraint set $S_1 = \{\mathbf{I} : \vec{\mathbf{I}}_{(i,j)} \in \Gamma$ and $|\mathbf{Y}_{(i,j)c} - \mathbf{I}_{(i,j)c}| \leq \beta\sigma_{k,c}$ for all $i,j\}$, where $k$ is the index of the codeword $\hat{\mathbf{v}}_k = \vec{\mathbf{Y}}_{(i,j)}$, to restore $\mathbf{X}$.

Typical images would generally have weak high frequency components as the intensity of neighboring pixels is highly correlated. This feature can be exploited as an additional *a priori* information in the restoration of color-quantized images. The conventional approach to incorporate this information into restoration is to assume that the energy of the high frequency components of image $\mathbf{X}$ is bounded. In our approach, we assume that the energy of each high frequency component of $\mathbf{X}$ is bounded and the bound of each component is estimated with the low-pass filtered $\mathbf{X}$. In particular, we have $|[\mathcal{T}(\mathbf{X})]_{(i,j)}| \leq |[\mathcal{T}(\mathcal{F}(\mathbf{Y})]_{(i,j)}|$ for $(i,j) \in \Omega_H$, where $\mathcal{F}$ and $\mathcal{T}$ are, respectively, a linear low-pass filtering operator and a 2D DCT operator, $[\bullet]_{(i,j)}$ denotes the $(i,j)^{\text{th}}$ element in the transform domain and $\Omega_H$ defines the set of high frequency components which should be bounded. This forms a smoothness constraint set $S_0 = \{\mathbf{I} : \vec{\mathbf{I}}_{(i,j)} \in \Gamma$ and $|[\mathcal{T}(\mathbf{I})]_{(i,j)}| \leq |[\mathcal{T}(\mathcal{F}(\mathbf{Y})]_{(i,j)}|$ for $(i,j) \in \Omega_H\}$ for restoring image $\mathbf{X}$.

Two more constraint sets can be used to restore $\mathbf{X}$. One is that confines the intensity value of a particular pixel to be valid. In formulation, we have $S_2 = \{\mathbf{I} : \vec{\mathbf{I}}_{(i,j)} \in \Gamma$ and $0 \leq \mathbf{I}_{(i,j)c} \leq 1$ for all $i,j\}$. The other is $S_3 = \{\mathbf{I} : \vec{\mathbf{I}}_{(i,j)} \in \Gamma$ and $\mathcal{Q}(\vec{\mathbf{I}}_{(i,j)}) = \vec{\mathbf{Y}}_{(i,j)}$ for all $i,j\}$, where $\mathcal{Q}$ is the color quantization operator. This set makes sure that the color quantized output of the restored $\mathbf{X}$ is $\mathbf{Y}$.

### 3.2 Formulation of Restoration Algorithm

Based on the *a priori* information concerning the color quantization process and the original image, four constraint sets are defined in the previous subsection. All these constraint sets are convex sets. Based on the theory of POCS [8], an iterative algorithm can hence be defined as

$$\mathbf{X}^{(k+1)} = P_3 P_2 P_1 P_0 \mathbf{X}^{(k)}, \quad (5)$$

where $\mathbf{X}^{(k)}$ is the estimate of $\mathbf{X}$ at iteration $k$ and $P_i$ is a projection operator projecting a given image $\mathbf{I}$ onto $S_i$, where $i \in \{0,1,2,3\}$. In particular, they are defined as

$$P_0 \;:\; [\mathcal{T}(\mathbf{I})]_{(i,j)} = [\mathcal{T}(\mathcal{F}(\mathbf{Y})]_{(i,j)} \quad if \;\; |[\mathcal{T}(\mathbf{I})]_{(i,j)}| > |[\mathcal{T}(\mathcal{F}(\mathbf{Y})]_{(i,j)}|, \quad for \; (i,j) \in \Omega_H \quad (6)$$

$$P_1 \;:\; \mathbf{I}_{(i,j)c} = \begin{cases} \mathbf{Y}_{(i,j)c} + \beta\sigma_{k,c} & if \; \mathbf{I}_{(i,j)c} > \mathbf{Y}_{(i,j)c} + \beta\sigma_{k,c} \\ \mathbf{Y}_{(i,j)c} - \beta\sigma_{k,c} & if \; \mathbf{I}_{(i,j)c} < \mathbf{Y}_{(i,j)c} - \beta\sigma_{k,c} \end{cases} \quad (7)$$

$$P_2 \;:\; \mathbf{I}_{(i,j)c} = \begin{cases} 1 & if \; \mathbf{I}_{(i,j)c} > 1 \\ 0 & if \; \mathbf{I}_{(i,j)c} < 0 \end{cases} \quad (8)$$

$$P_3 \;:\; \vec{\mathbf{I}}_{(i,j)} = \vec{\mathbf{Y}}_{(i,j)} \quad if \; \mathcal{Q}(\vec{\mathbf{I}}_{(i,j)}) \neq \vec{\mathbf{Y}}_{(i,j)} \quad (9)$$

The initial estimate $\mathbf{X}^{(0)}$ is set to be $\mathcal{F}(\mathbf{Y})$, although theoretically no restriction are imposed on the initial estimate.

Based on a similar idea presented in [9], it is desirable to keep $S_1$ to be a subset of $S_3$. In general, the size of $S_1$ should be as big as possible under the constraint so as not to exclude the original $\mathbf{X}$ in the set of potential candidates. In our proposed algorithm, in order to achieve this objective, we first select an appropriate $\beta$ such that $S_1$ and $S_3$ are of similar size at the beginning and then reduce the size of $S_1$ by adjusting the value of $\beta$ at each iteration until $S_1 \subset S_3$ is achieved. Specifically, projection $P_1$ is modified as

$$P_1 \;:\; \mathbf{I}_{(i,j)c} = \begin{cases} \mathbf{Y}_{(i,j)c} + \beta_k\sigma_{k,c} & if \; \mathbf{I}_{(i,j)c} > \mathbf{Y}_{(i,j)c} + \beta_k\sigma_{k,c} \\ \mathbf{Y}_{(i,j)c} - \beta_k\sigma_{k,c} & if \; \mathbf{I}_{(i,j)c} < \mathbf{Y}_{(i,j)c} - \beta_k\sigma_{k,c} \end{cases} \quad (10)$$

During the realization of projection $P_3$, if $\mathcal{Q}(\vec{\mathbf{I}}_{(i,j)}) \neq \vec{\mathbf{Y}}_{(i,j)}$ happens, the $\beta_k$ associated with codeword $\hat{\mathbf{v}}_k = \vec{\mathbf{Y}}_{(i,j)}$ is adjusted by $\beta_k = \alpha\beta_k$, where $\alpha(< 1)$ is a scaling parameter which controls the rate of adjustment. Note this adjustment does not affects the convergence. The adjustment reduces the size of $S_1$. The adjusted $S_1$ is a convex set and hence, at any time, all constraint sets are convex sets. Eventually, $S_1$ becomes a subset of $S_3$ and there will be no more adjustment. When this point is reached, it becomes a typical POCS algorithm and the convergence can be guaranteed.

## 4 Simulation and Comparative Study

Simulation has been carried out to evaluate the performance of the proposed algorithm on a set of color-quantized images. In our simulation, a number of 24-bit full color test images were first color-quantized with a color palette of 256 colors. In our study, two color palettes were generated with median cut algorithm [10] and octree algorithm [11] respectively and used to investigate if the algorithm works with different color palettes. No halftoning is performed during the quantization. The test images applied are a set of *de facto* standard 24-bit full color images of size 256×256 each.

The proposed restoration algorithm was then applied to restore the quantized images. In the realization of the proposed algorithm, A set of 50000 uniformly-distributed 3D random vectors were generated and used as the training set $\Omega_t$ to estimate $\vec{\sigma}_k$. The 3×3 Gaussian filter was used as filter $\mathcal{F}$ and $\Omega_H$ was defined as $\{(i,j) : (256-i)^2+(256-j)^2 < 80000 \text{ and } 0 \leq i,j < 256\}$. The initial value of $\beta_k$ was assigned to be $\beta_k = 0.05/\sigma$ for all $k$, where $\sigma$ is the maximum of $\sigma_{k,c}$'s for all $k$ and $c$, and $\alpha$ was assigned to be 0.8. By considering that the extreme case happens when the whole color space is uniformly partitioned into slices of equal width and that colors are uniformly distributed in the color space, the lower bound of $\sigma^2_{k,c}$ is set to be $\frac{1}{12}(\frac{1}{N_c})^2$, where $N_c$ is the total number of colors in the palette.

Some other restoration algorithms which were originally proposed for restoring noisy and blurred color images were also evaluated for comparison. They were simulated here for comparative study as few schemes had been proposed for restoring color-quantized images and they are typical examples of the type ([2]-[7]). In particular, Galatsanos's algorithm [4] is based on the constrained least square approach and Hunt's algorithm [5] is based on Wiener filtering.

Table 1 shows the SNR improvement (SNRI) achieved by different algorithms. Mathematically, the SNRI is defined as $\text{SNRI} = 10\log\frac{\sum_{(i,j)}||\vec{\mathbf{X}}_{(i,j)}-\vec{\mathbf{Y}}_{(i,j)}||^2}{\sum_{(i,j)}||\vec{\mathbf{X}}_{(i,j)}-\vec{\mathbf{X}}'_{(i,j)}||^2}$ where $\vec{\mathbf{X}}_{(i,j)}$, $\vec{\mathbf{Y}}_{(i,j)}$ and $\vec{\mathbf{X}}'_{(i,j)}$ are, respectively, the $(i,j)^{\text{th}}$ pixels of the original, the color-quantized and the restored images.

In realizing Galatsanos's algorithm [4], the noise power of each channel was estimated with the original full-color image. In realizing Hunt's algorithm [5], three separate Wiener filters were used in three different channels and, during the design of the filters, the noise spectrum of each channel was estimated with the original full-color image. Note that, in practical situation, these parameters must be estimated from the degraded image and hence the restoration results would not be as good as those shown in the table.

From Table 1, one can see that the performance of the proposed algorithm is better as compared with the others and it is consistent even though the input is color-quantized with different color palettes. On average, by applying the proposed algorithm to restore the color-quantized images, a SNRI of 1 dB in image quality was achieved. For visual evaluation of the restoration results, some color-quantized outputs of image "fruits" and the restoration results obtained with various restoration algorithms are provided in Figure 1 for comparison.

The noise introduced by color quantization is basically signal dependent and is not white, which violates the assumptions adopted in most current multichannel restoration algorithms. A better restoration performance can be expected if an algorithm takes this into account. Another study reported in [12] also reveals this fact.

## 5 Conclusions

By far very little research has been carried out to address the restoration of color-quantized images. Besides, direct applications of existing restoration techniques are generally inadequate to deal with the problem. This paper has proposed a restoration algorithm specific to color-quantized images based on the POCS approach. This algorithm makes good use of the color palette to derive useful *a priori* information for restoration. It has been demonstrated by simulation results and comparative study that the proposed restoration algorithm is capable of improving the quality of a color-quantized image, as compared with other existing restoration approaches such as [4] and [5].

The proposed restoration algorithm requires no extra information other than the color palette to carry out the restoration. This makes it be always able to provide a reasonable restoration performance whatever color quantization scheme with which it works.

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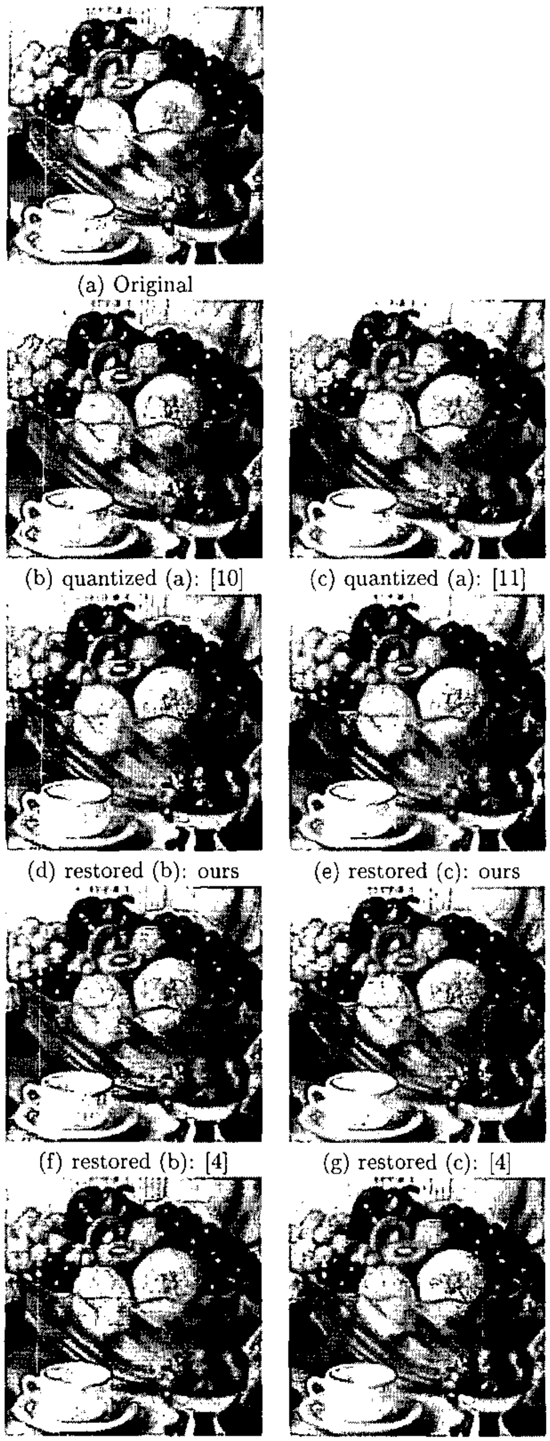

(a) Original

(b) quantized (a): [10] (c) quantized (a): [11]

(d) restored (b): ours (e) restored (c): ours

(f) restored (b): [4] (g) restored (c): [4]

(h) restored (b): [5] (i) restored (c): [5]

Figure 1: Comparison of the restoration performance of various approaches in restoring color-quantized "fruits"

| | SNR Improvement (dB) | | |
|---|---|---|---|
| | Proposed | [4] | [5] |
| Lenna | 0.9288 | 0.3881 | 1.4077 |
| Baboon | 0.6866 | 0.2082 | 0.2381 |
| Boat | 0.9994 | 0.1985 | 0.3990 |
| Peppers | 1.6065 | 0.1881 | 1.5510 |
| **Average** | 1.0553 | 0.2457 | 0.8990 |

(a)

| | SNR Improvement (dB) | | |
|---|---|---|---|
| | Proposed | [4] | [5] |
| Lenna | 0.9983 | 0.3493 | 0.8224 |
| Baboon | 0.8100 | 0.1592 | 0.6033 |
| Boat | 0.3730 | 0.2956 | 0.1443 |
| Peppers | 1.5113 | 0.3457 | 1.4042 |
| **Average** | 0.9232 | 0.2874 | 0.7435 |

(b)

Table 1: SNR Improvements of various algorithms in restoring images color-quantized with (a) median cut algorithm [10] and (b) octree algorithm [11]


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