A Practical Fuzzy Logic Controller for the Path Tracking of Wheeled Mobile Robots

By T.H. Lee, H.K. Lam, F.H.F. Leung, and P.K.S. Tam

This article tackles the path-tracking problem of wheeled mobile robots (WMRs) that are used in the Micro Robot Soccer Tournament (MiroSot). The basic configuration of MiroSot (Figure 1) comprises a football stadium (ground plane) with two teams [1]-[2]. Each team has three WMRs (Figure 2), a camera for image capture, a host computer, and an RF data transmitter. The camera captures the football stadium's images that are sent to and processed by the host computer. Based on the real-time locations of the robots and the ball and the soccer game strategy, the host computer determines the actions of its team of robots. The objective is to score points by pushing the ball to the opponent's goal as many times as possible and to prevent the opponent team from scoring points. To achieve this objective, a path has to be generated by the game strategy for the home robot to follow—a path-tracking problem. For each robot, the host computer generates the corresponding control signals driving the wheel at each side to ensure good path-tracking capability. The robot will have both linear and angular displacements until it arrives at the target position.

To tackle the path-tracking problem, some other problems have to be solved. First, owing to the nonlinear dynamic and nonholonomic characteristics of the WMRs, the controller design will be a difficult problem, as we do not have a systematic and simple controller design methodology for nonlinear systems. Traditionally, we may derive a mathematical system model from which a suitable controller is designed [3]. However, the model of the WMRs used in MiroSot is very complicated. The WMR dynamic equations are obtained by the well-known Lagrange equations [6]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}} = \boldsymbol{\tau}, \qquad (1)$$

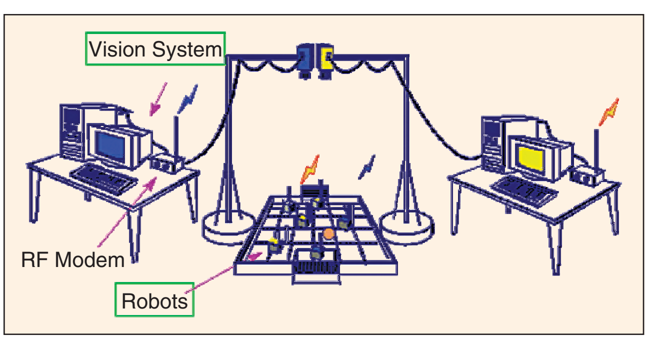

Figure 1. *MiroSot system configuration.*

where $\boldsymbol{\tau}$ represents the torque. The Lagrangian variable L is equal to the difference between the kinetic energy and the potential energy. Based on (1), the dynamic equation can be expressed as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}_m(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{B}(\mathbf{q})\boldsymbol{\tau} - \mathbf{A}^T(\mathbf{q})\lambda, \qquad (2)$$

where $\mathbf{M}(\mathbf{q})$ represents the 3×3 inertia matrix (which is symmetric), $\mathbf{V}_m(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ represents the 3×1 vector of centrifugal and Coriolis torques, $\mathbf{G}(\mathbf{q})$ represents the gravity torques, $\mathbf{A}(\mathbf{q})$ is given by the nonholonomic constraints, λ is a Lagrange multiplier associated with the constraints, and $\mathbf{B}(\mathbf{q})$ is a 3×2 matrix. In the present case, the variables are defined as follows:

$$\mathbf{q} = \begin{bmatrix} x_c \\ y_c \\ \theta \end{bmatrix}, \quad \mathbf{M}(\mathbf{q}) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$\mathbf{V}_m(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{G}(\mathbf{q}) = \mathbf{0}, \quad \mathbf{A}^T(\mathbf{q}) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix},$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}, \quad \mathbf{B}(\mathbf{q}) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix},$$

$\lambda = -m(\dot{x}_c\cos\theta + \dot{y}_c\sin\theta)\dot{\theta}$. x_c and y_c are the x and y coordinates of the WMR in the football stadium, respectively; θ is the heading angle of the WMR; I is the moment of inertia of the WMR about its center (which is difficult to obtain); m is the mass of the robot; τ_r and τ_l are torque control inputs generated by the right and left motors, respectively; and R and r are the distances between the two wheels and the radius of the wheel, respectively. Substituting the above variables into (2) and simplifying the equation, we have

$$\begin{bmatrix} \ddot{x}_c \\ \ddot{y}_c \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -\dot{\theta}\sin\theta\cos\theta & -\dot{\theta}\sin^2\theta & 0 \\ \dot{\theta}\cos^2\theta & \dot{\theta}\sin\theta\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{\cos\theta}{mr} & \frac{\cos\theta}{mr} \\ \frac{\sin\theta}{mr} & \frac{\sin\theta}{mr} \\ \frac{R}{Ir} & \frac{-R}{Ir} \end{bmatrix} \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}. \qquad (3)$$

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This plant model is the basis for designing the position controller of the WMR. More details about the modeling of wheeled mobile robots can be found in [6]. The objective is to drive the robot, of output cordinates x_c and y_c, to a desired position (position control) and follow a desired path (path tracking) in the football stadium by feeding an appropriate torque (input of the system), τ, to the robot. Unfortunately, the model of (3) may not be very useful because of the parameter uncertainties in practical robots. This makes the design of the controller difficult to realize. Even if a nonlinear controller can be designed based on the mathematical model, the controller may be too complicated for implementation in software. The fact that three robots are to be controlled further complicates the problem. Moreover, controlling of the robots is just one of the tasks (others include decision making, game strategy, and path planning) performed by the host computer. On the other hand, the position and heading angle of the robot, which are the inputs to the controller, are obtained from the captured image only. Figure 3 shows a functional block diagram of the closed-loop control system. The WMR has no embedded position sensor, and its RF receiver only listens to commands from the host computer. The position information is obtained by recognizing the color mark on top of each WMR. In view of the low resolution of the camera (320×240 pixels), the readings of the robots' positions and heading angles are subject to tolerances. In practice, for a stationary WMR, the tolerance of position will be about three pixels, whereas that of the heading angle will be even greater (sometimes more than 20°). Errors of input signals are therefore inevitable. The situation will be further worsened if the designed controller requires measured linear and angular velocities as inputs. This is because the velocity is estimated by dividing a pixel count by time, which is inherently inaccurate when the pixel reading itself has errors.

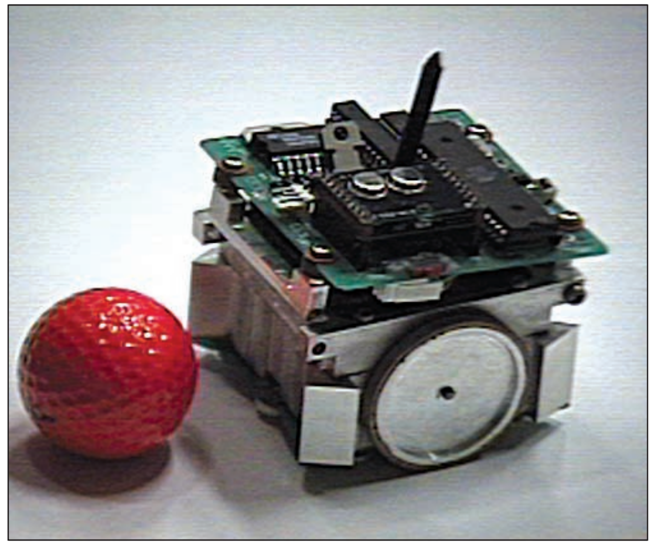

Figure 2. *Wheeled mobile robot.*

Some researchers have proposed the use of sliding mode control for the WMR [6]. Although this controller can provide a faster response, the structure of the sliding controller is quite complex and the computational demand is also high. Moreover, the derivation of the controller in [6] is subject to the following assumptions: 1) the system states used for the controller can be measured exactly, and 2) the heading angle of the robot and the angle coordinate cannot be perpendicular to each other.

In view of the difficulties mentioned above, a fuzzy logic controller (FLC) is proposed in this article. This FLC is designed based on a simple P-controller (proportional

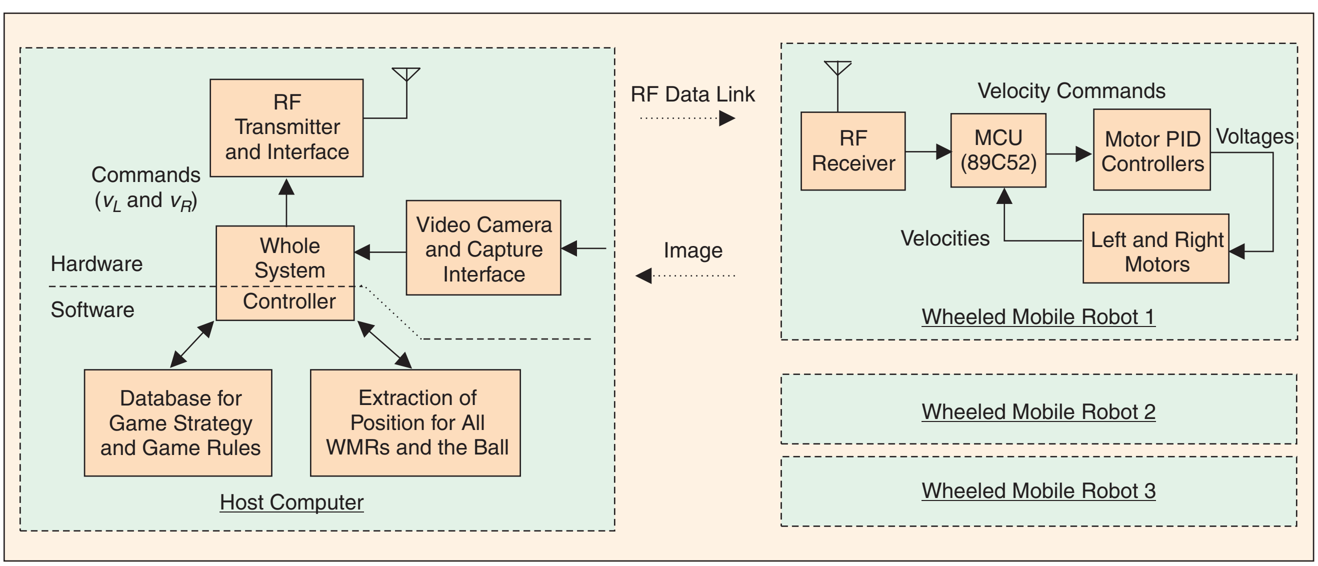

Figure 3. *Block diagram of closed-loop control system.*

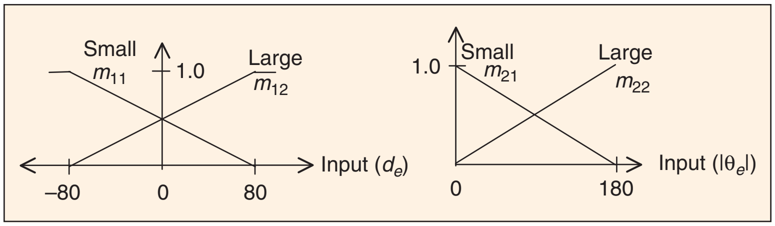

Figure 4. *Input membership functions.*

controller) that is able to control the robot practically. The P-control law is given by

$$\mathbf{u} = \begin{bmatrix} v_l \\ v_r \end{bmatrix} = \begin{bmatrix} k_d d_e + k_\theta \theta_e \\ k_d d_e - k_\theta \theta_e \end{bmatrix}, \tag{4}$$

$$d_e = \sqrt{(x_r - x_c)^2 + (y_r - y_c)^2},$$

$$\theta_e = \theta_r - \theta,$$

where v_l and v_r are voltages applied to the left-wheel and right-wheel motors, respectively; k_d is the gain for the error distance d_e between the current and the desired positions; k_θ is the gain for the error angle θ_e between the current and desired orientations; and x_r, y_r, and θ_r are the reference horizontal position, vertical position, and heading angle, respectively. Using the control law of (4), the P-controller controls the translational and rotational motions of the robot. In particular, when $\theta_e = 0$, the P-control law becomes $v_l = v_r = k_d d_e$, which results in translational motion only. On the other hand, when $d_e = 0$, we have $v_l = k_\theta \theta_e$ and $v_r = -k_\theta \theta_e$, which results in a rotational motion only. The advantages of this P-controller include: 1) the control law is simple, 2) the effect of the input error tolerance is reduced as only two system states are used, and 3) the plant model need not be known. However, the values of k_d and k_θ have to be determined based on trial and error. They need not be optimal but are obtained as a tradeoff between speed and stability.

To improve the performance, the proposed FLC [4]-[5] is used. It incorporates expert knowledge into the controller design process using some linguistic rules. Such an FLC is a nonlinear controller that retains the advantages of the P-controller but with an adaptive gain for each state variable so that a quick response can be achieved. Since the velocity and acceleration are not used as inputs, error accumulation is not a problem.

Fuzzy controllers [7]-[8] have been developed for WMR path tracking. The main difference between the previous work and ours is that in the former, consequent parts of the fuzzy rules are fuzzy terms with triangular-shaped membership functions, whereas rule consequents in our case are P-controllers. In [8], simulation results but no experimental results are provided. Shooting action and obstacle avoidance were also achieved in [7]-[8]. These two actions can also be achieved by our proposed fuzzy controller with a planned path. For instance, the shooting action can be achieved if a path that passes through the ball and avoids all the obstacles is planned for the robot. The target point of the path is at the position of the ball.

Fuzzy Logic Controller

A fuzzy controller having the following four rules, which are designed based on human knowledge, is proposed to control the WMR.

Rule 1: IF d_e is small and $|\theta_e|$ is small, THEN

$$\mathbf{u} = \begin{bmatrix} v_l \\ v_r \end{bmatrix} = \mathbf{u}_1 = \begin{bmatrix} 0.25 d_e + 0.12\theta_e \\ 0.25 d_e - 0.12\theta_e \end{bmatrix}.$$

Rule 2: IF d_e is small and $|\theta_e|$ is large, THEN

$$\mathbf{u} = \begin{bmatrix} v_l \\ v_r \end{bmatrix} = \mathbf{u}_2 = \begin{bmatrix} 0.25 d_e + 0.25\theta_e \\ 0.25 d_e - 0.25\theta_e \end{bmatrix}.$$

Rule 3: IF d_e is large and $|\theta_e|$ is small, THEN

$$\mathbf{u} = \begin{bmatrix} v_l \\ v_r \end{bmatrix} = \mathbf{u}_3 = \begin{bmatrix} 0.8 d_e + 0.12\theta_e \\ 0.8 d_e - 0.12\theta_e \end{bmatrix}.$$

Rule 4: IF d_e is large and $|\theta_e|$ is large, THEN

$$\mathbf{u} = \begin{bmatrix} v_l \\ v_r \end{bmatrix} = \mathbf{u}_4 = \begin{bmatrix} 0.8 d_e + 0.25\theta_e \\ 0.8 d_e - 0.25\theta_e \end{bmatrix}. \tag{5}$$

The gains of this fuzzy controller are manually fine-tuned based on performance with the real system. The input membership functions are defined in Figure 4. They are defined as

$$m_{11}(d_e) = \min\left(\max\left(0, \frac{d_e - 80}{-160}\right), 1\right), \tag{6}$$

$$m_{12}(d_e) = 1 - m_{11}(d_e), \tag{7}$$

$$m_{21}(|\theta_e|) = \frac{-|\theta_e| + 180}{180}, \tag{8}$$

$$m_{22}(|\theta_e|) = 1 - m_{21}(\theta_e). \tag{9}$$

The $\max(\cdot)$ and $\min(\cdot)$ functions in (6) are to restrict the value of $m_{11}(d_e)$ to lie between zero and one even when the value of d_e is larger than 80. As the maximum value of $|\theta_e|$ is 180, the value of $m_{21}(\theta_e)$ will lie between zero and one, and the max(·) and min(·) functions are not necessary. The grades of memberships, w_1 to w_4, of rule 1 to rule 4, respectively, are defined as follows:

$$w_1 = m_{11} \times m_{21}, \quad (10)$$

$$w_2 = m_{11} \times m_{22}, \quad (11)$$

$$w_3 = m_{12} \times m_{21}, \quad (12)$$

$$w_4 = m_{12} \times m_{22}. \quad (13)$$

The output of the FLC is given by

$$\mathbf{u} = \frac{\sum_{i=1}^{4} w_i \mathbf{u}_i}{\sum_{i=1}^{4} w_i}, \quad (14)$$

which shows that the controller has adaptive gains with respect to different operational regions.

Experimental Results

We developed a fuzzy control program using Visual C++ version 1.52 and implemented it on a Pentium II 450-MHz computer. The dimensions of each robot are 7.5 cm × 7.5 cm × 7.5 cm. It has two driving wheels on two sides and an omnidirectional passive wheel (rolling ball) at the bottom. The CCD camera has a resolution of 320 × 240 pixels and a rate of 30 frames per second. Hence, the sampling period for the robot to get the position and heading angle information is 33 ms. The parameters of the WMR used in this article are as follows: m = 0.45 kg, r = 0.02 m, R = 0.07 m. Because the proposed FLC is designed based on a model-free approach, the value of I is not needed. The maximum linear velocity of the WMR is 1.48 m/s. The maximum torque is 2.1 N-m. The dimensions of the football stadium are 150 cm × 130 cm.

The experiment has two parts. The first part is position control, and we compare the P-controller of (4) with the proposed fuzzy controller of (14) for controlling the same WMR. The second part is path-tracking control, and we compare the performance of the proposed FLC with that of a published sliding mode controller [6], which used a robot of the same specification.

Position Control

To test the position control, the initial position of the WMR is (260, 200) (indicated by the white mark in Figure 5), and we move it to the target position (x_r, y_r) = (40, 40). (Positions are given in units of pixels throughout this article.) The initial heading angle of the WMR is about −90° (i.e., it faces the bottom side of the platform of Figure 5(a)). The fuzzy controller of (14) is employed to perform the position control task. The error distance d_e used by the fuzzy controller is the distance between the current position of the WMR and the target position, and the error angle θ_e is the angle of the center of the WMR toward the target point measured horizontally. For comparison purposes, a P-controller is employed to perform the same task. The gain for the error distance k_d is 0.25, and the gain for the error angle k_θ is 0.12 for the traditional P-controller of (4). The gains of the traditional P-controller are obtained by trial and error. If the gains are set too large for the P-controller, the system will be unstable.

The trajectories of the WMR are captured. Figure 5(a) and (b) shows the actual images of the WMR under the control of the traditional P-controller and the proposed fuzzy controller, respectively. The trajectories are plotted in Figure 6. The solid line corresponds to the fuzzy-logic-controlled WMR, while the dotted line is from the P-controlled WMR. The plotted data are extracted from the image frames. The action

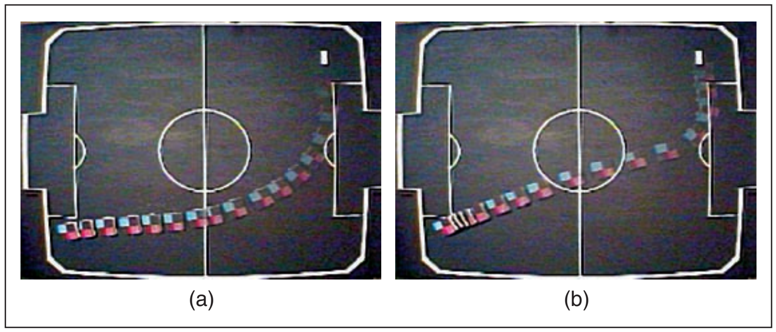

Figure 5. *Position control of the WMR: (a) trajectory of P-controlled WMR and (b) trajectory of fuzzy-logic-controlled WMR.*

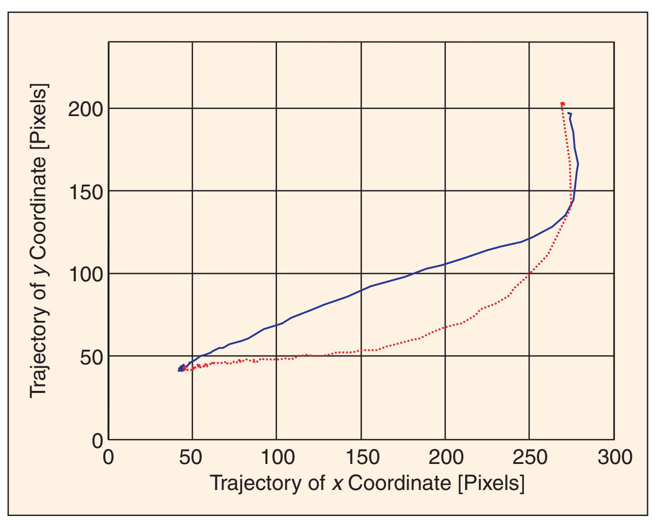

Figure 6. *Trajectory plot of the WMR controlled by the FLC (solid line) and the P-controller (dotted line).*

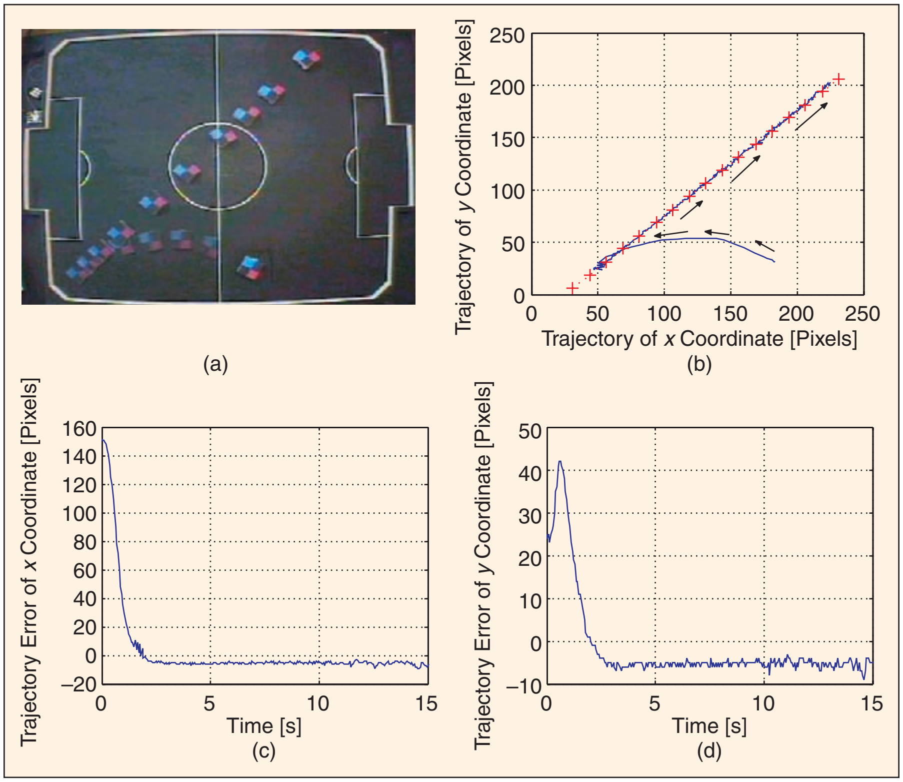

Figure 7. *(a) Straight-line path tracking; (b) actual (solid line) and reference (dotted line with "+") paths (unit of x-axis and y-axis are in pixel); (c) trajectory error of x-coordinate against time; (d) trajectory error of y-coordinate against time.*

time of the fuzzy-logic-controlled WMR from starting to send commands to arriving at the desired point is about 2 s. It is twice as fast as the P-controlled WMR.

Tracking Control

To test the path-tracking control, we use the reference paths stated in [6], which employed a sliding mode controller to realize path tracking. First, we test the tracking performance for a straight-line reference path at an angle of about 45°. The initial position of the WMR is (183, 31), and the initial reference point is (30, 6). The reference path is a straight line governed by $y_r(t) = x_r(t) - 24$. In practice, the reference path is generated as discrete points depending on t. In this example, we put $x_r(t) = t$, and the reference path is a sequence of 201 points of $(x_r(t), y_r(t)) = [(30, 6), (31, 7), \ldots (230, 206)]$ that are generated one by one at each sampling period (33 ms). The initial heading angle is 135° (i.e., the WMR is perpendicular to the reference path). The fuzzy controller (14) is employed to perform the path-tracking task. Effectively, every generated point is a new destination of the WMR after a sampling period. Figure 7(a) shows the image of the trajectory. Figure 7(b) shows the plot of the reference (dotted line with "+") and the actual (solid line) trajectories. Figure 7(c) and (d) shows the trajectory errors of the WMR's x-coordinate and y-coordinate with respect to time, respectively. Arrows in Figure 7(b) indicate the movement direction of the WMR. Figure 7(b) shows that the WMR can reach and follow the reference path even if the initial position of the WMR is far away. It can be seen that the tracking time is about 3 s, whereas the published controller's tracking time is about 8 s.

Next, we implement path tracking for a curved trajectory. The initial position of the WMR is (39, 89), and the initial reference point is (30, 28). The initial heading angle is –45°. The reference path is approximately governed by the following equations:

$$
\begin{aligned}
y_r(t) = {} & 223.947 \exp(-0.0208 x_r(t)) \\
& -31.7517 \exp(0.0218 x_r(t)) \\
& +32.1982 \exp(0.0222 x_r(t)) \\
& -203.9485 \exp(-0.0259 x_r(t)) \\
& -0.4451 \exp(-67.8449 x_r(t))
\end{aligned}
$$

and

$$x_r(t) = t.$$

This equation is derived from a curve-fitting function based on the data published in [6]. The fuzzy controller (14) is again employed to perform the path-tracking task. Experimental results are shown in Figure 8. The tracking time is about 3 s, whereas the published controller's tracking time is about 10 s. It can been seen from the results of the straight and curved path tracking that steady-state errors are present. This is because no integral action is taken in the proposed FLC for simplicity of its structure and low computational demand. Referring to Figures 7 and 8, the steady-state errors of the x and y trajectories are about five pixels, respectively. As the resolution of the camera is 320×240 pixels and the dimension of the football stadium is 150 cm × 130 cm, the steady-state errors of the x and y trajectories are about $(5 \times 150/320) = 2.3$ cm and $(5 \times 130/240) = 2.7$ cm, respectively. These steady-state errors are acceptable in a soccer game. The trajectories correspond to the x-y position of the center of the WMR, which has a dimension of 7.5 cm × 7.5 cm × 7.5 cm. The steady-state errors will not cause the WMR to miss the ball, as its side dimension is large enough.

Conclusion

A fuzzy logic controller has been proposed to control WMRs in a robot soccer game. A heuristic fuzzy logic controller has been designed based on a model-free approach. Hardware experimental results have been presented to verify that the FLC

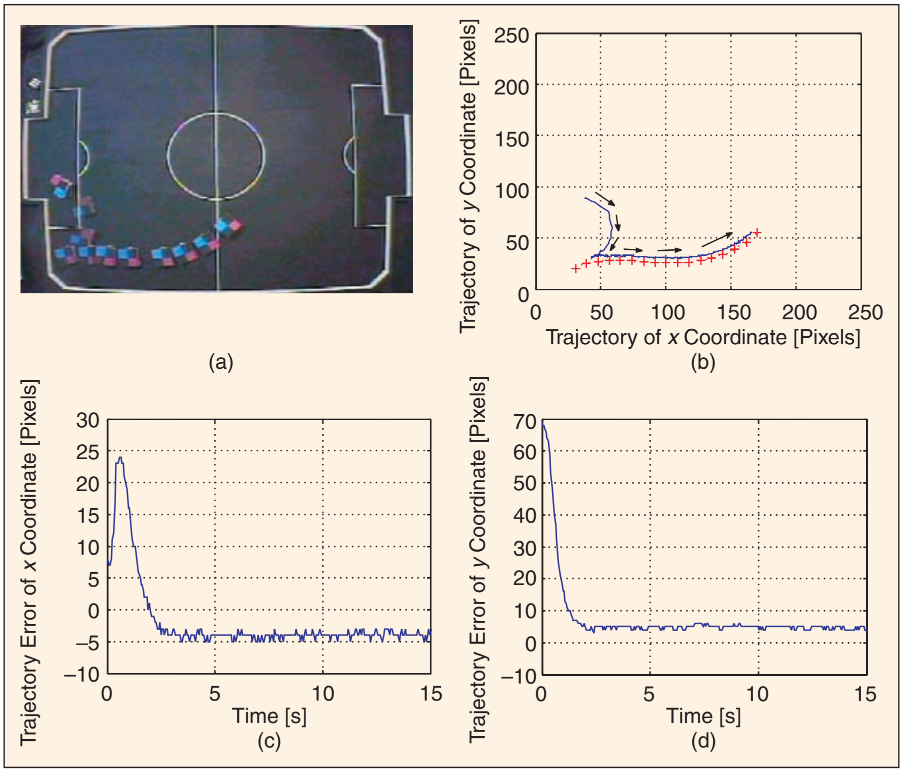

Figure 8. *(a) Path tracking for a curved trajectory; (b) actual (solid line) and reference (dotted line with "+") paths (units of x-axis and y-axis are pixels); (c) trajectory error of x-coordinate against time; (d) trajectory error of y-coordinate against time.*

can control a WMR. The performance of a fine-tuned P-controller has been compared with that of the proposed fuzzy logic controller. The response time of the fuzzy-logic-controlled WMR is two times faster. Good tracking control performance was also obtained from the proposed controller.

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