Switching Controller for Fuzzy Systems Subject to Unknown Parameters: Analysis and Design Based on a Linear Matrix Inequality (LMI) Approach [1]

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***Abstract* - This paper presents a switching control scheme for multivariable nonlinear systems. Based on a fuzzy plant model of the multivariable nonlinear system, a switching controller consisting of a number of simple sub-controllers is proposed. The parameters of the switching controller can be obtained by solving some LMIs. A switching algorithm is developed. To illustrate the merits and the design procedures of the proposed switching controller, an application example is given.**

I. INTRODUCTION

Control of nonlinear systems is difficult because we do not have systematic mathematical tools to help finding a necessary and sufficient condition to guarantee the stability and performance. The problem will become more complex if some of the parameters of the plant are unknown. Fuzzy control is one of the control techniques to deal with this class of system. By using a fuzzy plant model [1], a nonlinear system can be expressed as a weighted sum of some simple sub-systems. This model gives a fixed structure to some of the nonlinear systems that facilitates the analysis of the systems. Some authors gave a sufficient stability condition to this class of systems [8, 10] based on Lyapunov stability theory. However, the problem of choosing the parameters of the controllers remains open. Some other control techniques can also be found to tackle this class of fuzzy systems such as adaptive control [3-4, 6, 9, 13-15] and sliding mode control [5, 12]. Robust analysis for this class of system has also been reported [7, 16].

In this paper, we propose a switching controller to tackle the fuzzy control system subject to unknown parameters with given bounds. This switching controller consists of a number of simple sub-controllers. One of the sub-controllers will be chosen to control the plant based on a derived switching scheme. We shall formulate the design problem of the parameters of the switching controller into a linear matrix inequality (LMI) problem [11]. These LMIs can be solved readily by employing existing LMI tools. Stabilization of a cart-pole type inverted pendulum system with unknown parameters will be given as an application example to illustrate the design procedures and the merits of the proposed switching controller.

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This paper is organized as follows. The fuzzy plant model and the fuzzy controller will be briefly introduced in section 2. In section 3, the design method of the membership functions of the fuzzy controller will be provided, and the system stability of the fuzzy control systems will be analyzed. In section 4, an application example, namely a mass-spring-damper system, will be given. A conclusion will be drawn in section 5.

II. FUZZY PLANT MODEL AND SWITCHING CONTROLLER

An uncertain multivariable nonlinear system is to be controlled. In this paper, we consider multivariable nonlinear systems which are in the following form,

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) \tag{1}$$

where $\mathbf{F}(\mathbf{x}(t)) \in \Re^{n\times n}$ is the unknown system matrix, $\mathbf{B}(\mathbf{x}(t)) \in \Re^{n\times m}$ is the known input matrix, $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector. The system of (1) is represented by a fuzzy plant model which expresses the multivariable nonlinear system as a weighted sum of linear systems. A switching controller is to close the feedback loop.

A. Fuzzy Plant Model

Let p be the number of fuzzy rules describing the multivariable nonlinear plant of (1), the i-th rule is of the following format,

$$\text{Rule } i: \text{IF } f_1(\mathbf{x}(t)) \text{ is } \mathrm{M}_1^i \text{ and } \ldots \text{ and } f_\Psi(\mathbf{x}(t)) \text{ is } \mathrm{M}_\Psi^i$$
$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \tag{2}$$

where M_α^i is a fuzzy term of rule i corresponding to the known function $f_\alpha(\mathbf{x}(t))$, $\alpha = 1, \ldots, \psi$, $i = 1, \ldots, p$, Ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are the known system and input matrices respectively of the i-th rule sub-system. The system dynamics are given by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\big), \tag{3}$$

where,

$$\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1,\; w_i(\mathbf{x}(t)) \in [0 \quad 1] \text{ for all } i \tag{4}$$

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$$w_i(\mathbf{x}(t))=\frac{\mu_{M_1^i}(f_1(\mathbf{x}(t)))\times\mu_{M_2^i}(f_2(\mathbf{x}(t)))\times\cdots\times\mu_{M_\psi^i}(f_\psi(\mathbf{x}(t)))}{\sum_{k=1}^{p}\left(\mu_{M_1^k}(f_1(\mathbf{x}(t)))\times\mu_{M_2^k}(f_2(\mathbf{x}(t)))\times\cdots\times\mu_{M_\psi^k}(f_\psi(\mathbf{x}(t)))\right)} \quad (5)$$

is an unknown nonlinear function and $\mu_{M_\alpha^i}(f_\alpha(\mathbf{x}(t)))$, $\alpha = 1, 2, \ldots, \psi$, are the known membership function. (Details about fuzzy modeling can be founded [1, 10].).

B. Switching Controller

A switching controller is employed to control the nonlinear plant of (1). The switching controller consists of some simple sub-controllers. These sub-controllers will switch among each other to control the system of (1) according to an appropriate switching scheme. The switching controller is described by,

$$\mathbf{u}(t)=-\sum_{j=1}^{p}m_j(\mathbf{x}(t))\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \quad (6)$$

where $m_j(\mathbf{x}(t))$ takes the value of 0 or 1 according to a switching scheme discussed later, $\mathbf{R}\in\Re^{m\times m}$ and $\mathbf{P}\in\Re^{n\times n}$ are symmetric positive definite matrices to be designed, $(\cdot)^{\mathrm{T}}$ denotes the transpose of a matrix or a vector. It can be seen from (6) that the switching controller consists of 2^p state-feedback controllers which are linear combinations of $-\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)$, $j = 1, 2, \ldots, p$.

III. System Stability, Switching Scheme and Switching Controller Design based on LMI

We shall present the design of the switching controller in this section. A switching scheme for $m_j(\mathbf{x}(t))$, $j = 1, 2, \ldots, p$, will be derived under the consideration of system stability. From (3), (6), and writing $w_i(\mathbf{x}(t))$ as w_i, and $m_j(\mathbf{x}(t))$ as m_j, the closed-loop system is described by,

$$\begin{aligned}\dot{\mathbf{x}}(t)&=\sum_{i=1}^{p}w_i\left(\mathbf{A}_i\mathbf{x}(t)-\mathbf{B}_i\sum_{j=1}^{p}m_j\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)\\
&=\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\right)\mathbf{x}(t)\\
&\quad-\sum_{i=1}^{p}\sum_{j=1}^{p}w_im_j\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)+\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\\
&=\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\right)\mathbf{x}(t)\\
&\quad-\sum_{j=1}^{p}(m_j-w_j)\sum_{i=1}^{p}w_i\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\end{aligned} \quad (7)$$

Comparing (1) and (3), it is obvious that $\mathbf{B}(\mathbf{x}(t))=\sum_{i=1}^{p}w_i\mathbf{B}_i$, so, (7) becomes,

$$\begin{aligned}\dot{\mathbf{x}}(t)&=\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\right)\mathbf{x}(t)\\
&\quad-\sum_{i=1}^{p}(m_i-w_i)\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\end{aligned} \quad (8)$$

To investigate the system stability of the closed-loop system of (8), we consider the following quadratic Lyapunov function,

$$V=\frac{1}{2}\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \quad (9)$$

Differentiating (9), we have,

$$\dot{V}=\frac{1}{2}\left(\dot{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)+\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\dot{\mathbf{x}}(t)\right) \quad (10)$$

From (8) and (10),

$$\begin{aligned}\dot{V}&=\frac{1}{2}\left(\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\right)\mathbf{x}(t)\right.\\
&\quad\left.-\sum_{i=1}^{p}(m_i-w_i)\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\\
&\quad+\frac{1}{2}\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\left(\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\right)\mathbf{x}(t)\right.\\
&\quad\left.-\sum_{i=1}^{p}(m_i-w_i)\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)\\
&=\frac{1}{2}\mathbf{x}(t)^{\mathrm{T}}\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\left(\mathbf{A}_i^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}_i-\mathbf{P}\mathbf{B}_j\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}-\mathbf{P}\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P}\right)\mathbf{x}(t)\\
&\quad-\sum_{i=1}^{p}(m_i-w_i)\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\\
&=\frac{1}{2}\mathbf{x}(t)^{\mathrm{T}}\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\mathbf{Q}_{ij}\mathbf{x}(t)\\
&\quad-\sum_{i=1}^{p}(m_i-w_i)\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\end{aligned} \quad (11)$$

where,

$$\mathbf{Q}_{ij}=\mathbf{A}_i^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}_i-\mathbf{P}\mathbf{B}_j\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}-\mathbf{P}\mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\mathbf{P} \quad (12)$$

From (11),

$$\begin{aligned}\dot{V}&=\frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{x}(t)+\frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_{ji}\mathbf{x}(t)\\
&\quad-\sum_{i=1}^{p}(m_i-w_i)\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\\
&=\frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p}w_iw_j\mathbf{x}(t)^{\mathrm{T}}\left(\mathbf{Q}_{ij}+\mathbf{Q}_{ji}\right)\mathbf{x}(t)\\
&\quad-\sum_{i=1}^{p}(m_i-w_i)\mathbf{x}(t)^{\mathrm{T}}\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\end{aligned}$$

$$= \frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \mathbf{x}(t)^{\mathrm{T}} \mathbf{J}_{ij}\mathbf{x}(t)$$
$$- \sum_{i=1}^{p}(m_i - w_i)\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \quad (13)$$

where,

$$\mathbf{J}_{ij} = \mathbf{Q}_{ij} + \mathbf{Q}_{ji} \quad (14)$$

From (14), let,

$$\mathbf{J}_{ij} < 0 \text{ for all } i \text{ and } j \quad (15)$$

$$m_i = \frac{1 + \operatorname{sgn}\left(\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)}{2} \text{ for all } i \quad (16)$$

$$\operatorname{sgn}(z) = \begin{cases} 1 & \text{if } z > 0 \\ -1 & \text{otherwise} \end{cases} \quad (17)$$

From (13) to (16), we have,

$$\begin{aligned}\dot{V} &= \frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \mathbf{x}(t)^{\mathrm{T}} \mathbf{J}_{ij}\mathbf{x}(t) \\ &\quad - \sum_{i=1}^{p}\left[\frac{1 + \operatorname{sgn}\left(\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)}{2} - \frac{1}{2} - \left(w_i - \frac{1}{2}\right)\right]\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \\ &= \frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \mathbf{x}(t)^{\mathrm{T}} \mathbf{J}_{ij}\mathbf{x}(t) \\ &\quad - \sum_{i=1}^{p}\left[\frac{\operatorname{sgn}\left(\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)}{2} - \left(w_i - \frac{1}{2}\right)\right]\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t) \\ &\le \frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \mathbf{x}(t)^{\mathrm{T}} \mathbf{J}_{ij}\mathbf{x}(t) \\ &\quad - \sum_{i=1}^{p}\left(\frac{1}{2} - \left|w_i - \frac{1}{2}\right|\right)\left|\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right|\end{aligned} \quad (18)$$

As $w_i - \frac{1}{2} \in \left[-\frac{1}{2} \;\; \frac{1}{2}\right]$, $i = 1, 2, \ldots, p$, due to the property of the fuzzy plant model, it can be shown that (18) satisfies the following inequality,

$$\dot{V} \le \frac{1}{4}\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \mathbf{x}(t)^{\mathrm{T}} \mathbf{J}_{ij}\mathbf{x}(t) < 0 \quad (19)$$

Hence, we can conclude that the closed-loop system of (7) is asymptotically stable, i.e., $\mathbf{x}(t) \to 0$ as $t \to \infty$, if the stability condition of (15) is satisfied and the switching scheme of (16) is applied.

In the following, we shall formulate the design of the switching controller (6) and the stability condition of (15) into an LMI problem. In order words, we formulate the finding of **R** and **P** into an LMI problem. Considering (12) and (16) and restate as follows,

$$\begin{aligned}&\mathbf{A}_i^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_i + \mathbf{A}_j^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_j - 2\mathbf{P}\left(\mathbf{B}_j\mathbf{R}\mathbf{B}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\right)\mathbf{P} < 0 \\ &\Rightarrow \mathbf{P}^{-1}[\mathbf{A}_i^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_i + \mathbf{A}_j^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_j \\ &\quad - 2\mathbf{P}\left(\mathbf{B}_j\mathbf{R}\mathbf{B}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\right)\mathbf{P}]\mathbf{P}^{-1} < 0 \\ &\Rightarrow \mathbf{P}^{-1}\mathbf{A}_i^{\mathrm{T}} + \mathbf{A}_i\mathbf{P}^{-1} + \mathbf{P}^{-1}\mathbf{A}_j^{\mathrm{T}} + \mathbf{A}_j\mathbf{P}^{-1} \\ &\quad - 2\left(\mathbf{B}_j\mathbf{R}\mathbf{B}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\right) < 0\end{aligned} \quad (20)$$

It can be seen that (20) is an LMI. **R** and **P** can be solved readily by employing some LMI tools. The analysis results, switching scheme and design of the switching controller are summarized by the following lemma.

Lemma 1: *The closed-loop control system of* (6) *is guaranteed to be asymptotically stable if the parameters of the switching controller,* **R** *and* **P** *are symmetric positive definite and satisfy the following LMIs,*

$$\mathbf{P}^{-1}\mathbf{A}_i^{\mathrm{T}} + \mathbf{A}_i\mathbf{P}^{-1} + \mathbf{P}^{-1}\mathbf{A}_j^{\mathrm{T}} + \mathbf{A}_j\mathbf{P}^{-1} - 2\left(\mathbf{B}_j\mathbf{R}\mathbf{B}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{R}\mathbf{B}_j^{\mathrm{T}}\right) < 0$$

for all i and j,

and choose $m_i(\mathbf{x}(t)) = \dfrac{1 + \operatorname{sgn}\left(\mathbf{x}(t)^{\mathrm{T}} \mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^{\mathrm{T}}\mathbf{P}\mathbf{x}(t)\right)}{2}$ *for all i*

The design procedures of the switching controller are summarized by the following steps.

Step I). Obtain the fuzzy plant model of a nonlinear plant, e.g., by means of fuzzy modeling methods [1, 10] or other means.

Step II). Design the parameters of the switching controller by solving the LMIs in Lemma 1

Step III). Design $m_i(\mathbf{x}(t))$ according to Lemma 1.

IV. Application Example

An application example will be given in this section to show the design procedures and the merits of the proposed switching controller. A cart-pole type inverted pendulum system [8] is shown in Fig. 2. A switching controller will be designed for it by following the design procedures given in previous section.

Step I). The dynamic equation of the cart-pole type inverted pendulum system is given by,

$$\ddot{\theta}(t) = \frac{g\sin(\theta(t)) - aml\dot{\theta}(t)^2\sin(2\theta(t))/2 - a\cos(\theta(t))u(t)}{4l/3 - aml\cos^2(\theta(t))} \quad (21)$$

where θ is the angular displacement of the pendulum, g = 9.8m/s^2 is the acceleration due to gravity, m = 0.1kg is the mass of the pendulum, $a = 1/(m + M)$, $M \in [0.5 \quad 1]kg$ is the mass of the cart, $2l$ = 1m is the length of the pendulum, and u is the force applied to the cart. The objective of this application example is to design a fuzzy controller to close the feedback loop of (21) such that $\theta = 0$ at steady state. The inverted pendulum of (21) can be modeled by a fuzzy plant model having four rules. The i-th rule can be written as follows,

$$\text{Rule } i\text{: IF } f_1(\mathbf{x}(t)) \text{ is } \mathrm{M}_1^i \text{ AND } f_2(\mathbf{x}(t)) \text{ is } \mathrm{M}_2^i$$
$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t) \text{ for } i = 1, 2, 3, 4 \quad (22)$$

so that the system dynamics is described by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{4} w_i\left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)\right) \quad (23)$$

where $\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^T = [\theta(t) \quad \dot{\theta}(t)]^T$, $\theta(t) \in [\theta_{\min} \quad \theta_{\max}] = \left[-\frac{7\pi}{18} \quad \frac{7\pi}{18}\right]$ and $\dot{\theta}(t) \in [\dot{\theta}_{\min} \quad \dot{\theta}_{\max}] = [-5 \quad 5]$;

$f_1(\mathbf{x}(t)) = \frac{g - amlx_2(t)^2\cos(x_1(t))}{4l/3 - aml\cos^2(x_1(t))}\left(\frac{\sin(x_1(t))}{x_1(t)}\right)$ and

$f_2(\mathbf{x}(t)) = -\frac{a\cos(x_1(t))}{4l/3 - aml\cos^2(x_1(t))}$; $\mathbf{A}_1 = \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ f_{1_{\min}} & 0 \end{bmatrix}$

and $\mathbf{A}_3 = \mathbf{A}_4 = \begin{bmatrix} 0 & 1 \\ f_{2_{\max}} & 0 \end{bmatrix}$; $\mathbf{B}_1 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ f_{2_{\min}} \end{bmatrix}$ and

$\mathbf{B}_2 = \mathbf{B}_4 = \begin{bmatrix} 0 \\ f_{2_{\max}} \end{bmatrix}$; $f_{1_{\min}} = 9$ and $f_{1_{\max}} = 20$, $f_{2_{\min}} = -2.8571$

and $f_{2_{\max}} = -0.8677$; $w_i = \frac{\mu_{\mathrm{M}_1^i}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^i}(f_2(\mathbf{x}(t)))}{\sum_{i=1}^{4}\left(\mu_{\mathrm{M}_1^i}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^i}(f_2(\mathbf{x}(t)))\right)}$;

$\mu_{\mathrm{M}_1^\beta}(f_1(\mathbf{x}(t))) = \frac{-f_1(\mathbf{x}(t)) + f_{1_{\max}}}{f_{1_{\max}} - f_{1_{\min}}}$ for β = 1, 2 and $\mu_{\mathrm{M}_1^\delta}(f_1(\mathbf{x}(t))) = 1 - \mu_{\mathrm{M}_1^1}(f_1(\mathbf{x}(t)))$ for δ = 3, 4; $\mu_{\mathrm{M}_2^\varepsilon}(f_2(\mathbf{x}(t))) = \frac{-f_2(\mathbf{x}(t)) + f_{2_{\max}}}{f_{2_{\max}} - f_{2_{\min}}}$ for ε = 1, 3 and $\mu_{\mathrm{M}_2^\phi}(f_2(\mathbf{x}(t))) = 1 - \mu_{\mathrm{M}_2^1}(f_2(\mathbf{x}(t)))$ for ϕ = 2, 4 are the membership functions.

Step II) A switching controller is designed for the plant of (21) and is as follows,

$$\mathbf{u}(t) = -\sum_{j=1}^{4} m_j(\mathbf{x}(t))\mathbf{R}\mathbf{B}_j^T\mathbf{P}\mathbf{x}(t) \quad (24)$$

It can be seen that we have 16 simple controllers. By solving the LMIs given in the Lemma 1, we have $\mathbf{R}$ = 6.3450 and $\mathbf{P} = \begin{bmatrix} 0.4134 & -1.5489 \\ -1.5489 & 6.0771 \end{bmatrix}$.

Step III) According to Lemma 1, we have $m_i(\mathbf{x}(t)) = \frac{1 + \operatorname{sgn}\left(\mathbf{x}(t)^T\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{R}\mathbf{B}_i^T\mathbf{P}\mathbf{x}(t)\right)}{2}$ for i = 1, 2, 3, 4.

Fig. 3 and 4 show the responses of the system states with M = 0.5kg (solid line) and M = 1kg (dotted line) under the initial condition of $\mathbf{x}(0) = [1 \quad 0]^T$.

V. Conclusion

A design of switching controller for multivariable nonlinear plants with unknown parameters based on LMI approach has been given. The parameters of the switching controller can be obtained by solving some LMIs. Switching scheme has been developed under the consideration of system stability. An application example on stabilizing a cart-pole type inverted pendulum system has been give to illustrate the design procedure and the merits of the proposed switching controller.

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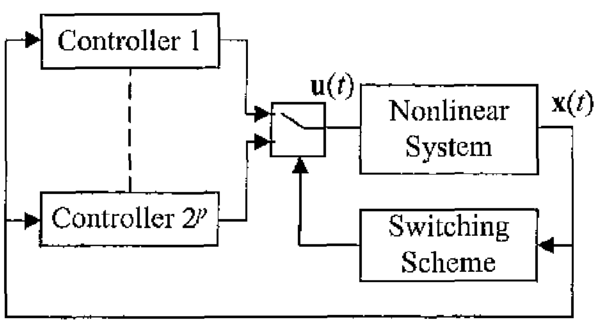

Fig. 1. Block diagram of the switching control system

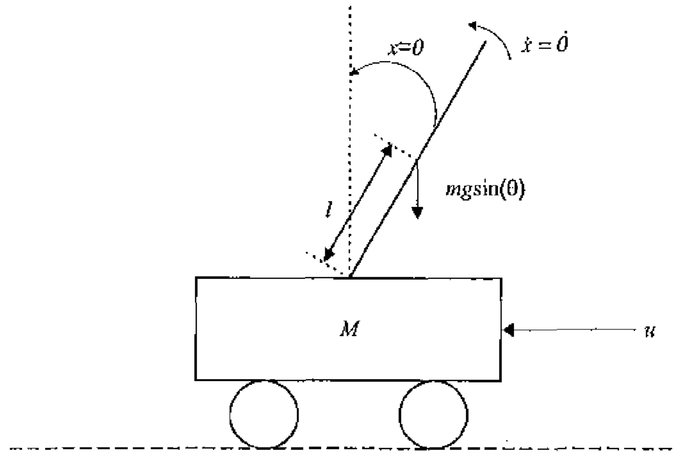

Fig. 2 . A cart-pole type inverted pendulum system.

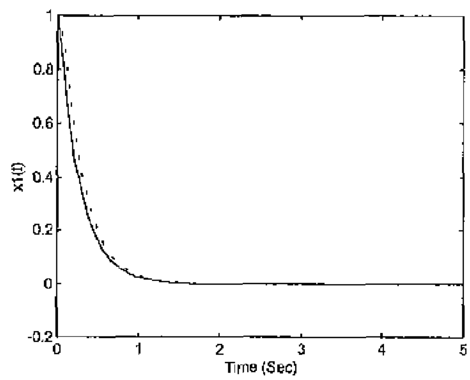

Fig. 3. Responses of $x_1(t)$ under M = 0.5kg (solid line) and M = 1kg (dotted line)

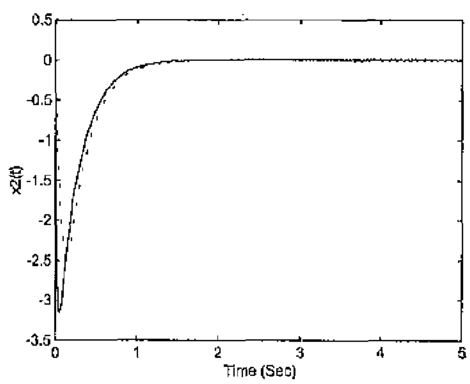

Fig. 3. Responses of $x_2(t)$ under M = 0.5kg (solid line) and M = 1kg (dotted line)