Fuzzy Control of DC-DC Switching Converters: Stability and Robustness Analysis[1]

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Abstract **This paper presents the fuzzy control of a PWM (pulse width modulation) boost DC-DC switching converter based on the Takagi-Sugeno (TS) fuzzy modeling approach. Stability and robustness of the fuzzy controlled DC-DC converter will be analyzed. Hardware results on regulating the boost DC-DC converter by a fuzzy controller will be given and compared to those by a traditional PI controller.**

I. INTRODUCTION

It is well known that DC-DC switching power converters are highly nonlinear system with uncertain parameters owing to, e.g., the output load changes during the operation. Consequently, the control (regulation) of DC-DC switching power converters can be a difficult task, especially when the range of operating condition is large and/or the dynamic model has right-half plane zeros (non-minimum phase characteristics). Should this happen, the conventional control method based on averaging and linearization techniques [1-2] will result in poor dynamic performance or system instability. In this paper, the problem is tackled from the nonlinear control point of view. The DC-DC switching converter is to be modeled using the Tagaki-Sugeno (TS) fuzzy modeling approach [3]. The TS-fuzzy model, which consists of some linear sub-systems, is a powerful tool to represent accurately a given nonlinear system. A nonlinear fuzzy controller can then be designed based on this fuzzy model with guaranteed stability and good dynamic behavior [5-6]. Hardware results on regulating a PWM boost converter by the proposed fuzzy controller will be presented. A comparison with a PI-controller in terms of performance will show the merits of the proposed control approach. It should be noted that the fuzzy models of other types of DC-DC switching converters (PWM and quasi-resonant type) can be obtained using the same way discussed in this paper, and the fuzzy controller designs are also similar.

II. TS-FUZZY PLANT MODEL

Fig. 1 shows the circuit of a PWM boost DC-DC switching converter. The parameters of the circuit are chosen as follows: $R \in [R_{\min} \quad R_{\max}] = [15\Omega \quad 51\Omega]$ is the loading of the converter, $C = 47\mu\text{F}$, $L = 0.5\mu\text{H}$, $V_{in} = 5\text{V}$ is the input voltage to the converter, $V_D = 0.7\text{V}$ is the forward voltage of the diode D. From [4], the dynamics of the PWM boost DC-DC switching converter can be described by a TS-fuzzy model with the following rules,

Rule i: IF $v_C(t)$ is M_1^i AND $i_L(t)$ is M_2^i THEN
$$\dot{\mathbf{x}}(t) = (\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i u(t) + \mathbf{E}\ ,\ i = 1, 2, 3, 4 \qquad (1)$$

where $v_C(t)$ is the voltage across the capacitor C, $i_L(t)$ is current flowing through the inductor L, $\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^{\mathrm{T}} = [v_C(t) \quad i_L(t)]^{\mathrm{T}}$, $u(t) \in [0.1 \quad 0.9]$ is the control signal of duty ratio of the PWM switch. The control objective is to regulate the output voltage, $V_{out}(t)$, across the loading resistance R at the reference output voltage $V_{ref} = 12\text{V}$.

$$\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3 = \mathbf{A}_4 = \begin{bmatrix} \dfrac{-1}{R_{mid}C} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix},\ \mathbf{B}_1 = \begin{bmatrix} \dfrac{-i_{L_{\min}}}{C} \\ \dfrac{v_{C_{\min}} + V_D}{L} \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} \dfrac{-i_{L_{\max}}}{C} \\ \dfrac{v_{C_{\min}} + V_D}{L} \end{bmatrix},\qquad \mathbf{B}_3 = \begin{bmatrix} \dfrac{-i_{L_{\min}}}{C} \\ \dfrac{v_{C_{\max}} + V_D}{L} \end{bmatrix},$$

$$\mathbf{B}_4 = \begin{bmatrix} \dfrac{-i_{L_{\max}}}{C} \\ \dfrac{v_{C_{\max}} + V_D}{L} \end{bmatrix},\quad \mathbf{E} = \begin{bmatrix} 0 \\ \dfrac{V_{in} - V_D}{L} \end{bmatrix},\quad \Delta\mathbf{A}_1 = \Delta\mathbf{A}_2 =$$

$$\Delta\mathbf{A}_3 = \Delta\mathbf{A}_4 = \begin{bmatrix} \dfrac{-1}{RC} + \dfrac{1}{R_{mid}C} & 0 \\ 0 & 0 \end{bmatrix}$$ which have uncertain values; $v_{C_{\min}} = 5.5556\text{V}$, $v_{C_{\max}} = 25\text{V}$, $i_{L_{\min}} = 0.16\text{A}$, $i_{L_{\max}} = 2\text{A}$, $R_{mid} = 23\Omega$ is the nominal value of the resistor R. The dynamics of the boost converter is given by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{4} w_i(\mathbf{x}(t))\big((\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i u(t) + \mathbf{E}\big) \qquad (2)$$

where

$$\sum_{i=1}^{4} w_i(\mathbf{x}(t)) = 1,\ w_i(\mathbf{x}(t)) \in [0,\ 1] \text{ for all } i; \qquad (3)$$

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$$w_i(\mathbf{x}(t)) = \frac{\mu_{\mathrm{M}_1^i}(v_C(t)) \times \mu_{\mathrm{M}_2^i}(i_L(t))}{\sum_{i=1}^{4}(\mu_{\mathrm{M}_1^i}(v_C(t)) \times \mu_{\mathrm{M}_2^i}(i_L(t)))} \text{ for all } i \tag{4}$$

which is a nonlinear function of $\mathbf{x}(t)$, $\mu_{\mathrm{M}_\alpha^i}(x_\alpha(t))$, $\alpha = 1$, 2, are the grades of membership of which the value depends on the operating conditions. The membership functions of the fuzzy term M_i^j, $i = 1, 2, j = 1, 2, 3, 4$ are defined as follows.

$$\mu_{\mathrm{M}_1^1}(v_C(t)) = \mu_{\mathrm{M}_1^2}(v_C(t)) = \frac{v_C(t) - v_{C_{\max}}}{v_{C_{\min}} - v_{C_{\max}}},$$

$$\mu_{\mathrm{M}_1^3}(v_C(t)) = \mu_{\mathrm{M}_1^4}(v_C(t)) = \frac{-v_C(t) + v_{C_{\min}}}{v_{C_{\min}} - v_{C_{\max}}},$$

$$\mu_{\mathrm{M}_2^1}(i_L(t)) = \mu_{\mathrm{M}_2^3}(i_L(t)) = \frac{i_L(t) - i_{L_{\max}}}{i_{L_{\min}} - i_{L_{\max}}},$$

$$\mu_{\mathrm{M}_2^2}(i_L(t)) = \mu_{\mathrm{M}_2^4}(i_L(t)) = \frac{-i_L(t) + i_{L_{\min}}}{i_{L_{\min}} - i_{L_{\max}}}.$$

It should be noted that (2) is equivalent to the state-space average model of the boost converter [1-2]. In order to regulate the output voltage of the boost converter, a fuzzy controller with integral control will be employed. The TS-fuzzy plant model of (2) is augmented as,

$$\dot{\overline{\mathbf{x}}}(t) = \sum_{i=1}^{4} w_i(\mathbf{x}(t))\left((\overline{\mathbf{A}}_i + \Delta\overline{\mathbf{A}}_i)\overline{\mathbf{x}}(t) + \overline{\mathbf{B}}_i u(t) + \overline{\mathbf{E}}\right) \tag{5}$$

where $\overline{\mathbf{x}}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}}$, $x_3(t) = x_3(t_o) + \int_o^\infty \left(V_{ref} - x_1(t)\right) \mathrm{d}t$, $x_3(t_o)$ is the initial value of $x_3(t)$ at an arbitrary time t_o.

$$\overline{\mathbf{A}}_1 = \overline{\mathbf{A}}_2 = \overline{\mathbf{A}}_3 = \overline{\mathbf{A}}_4 = \begin{bmatrix} -1/R_{mid}C & 1/C & 0 \\ -1/L & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix},$$

$$\overline{\mathbf{B}}_1 = \begin{bmatrix} -i_{L_{\min}}/C \\ (v_{C_{\min}} + V_D)/L \\ 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_2 = \begin{bmatrix} -i_{L_{\max}}/C \\ (v_{C_{\min}} + V_D)/L \\ 0 \end{bmatrix},$$

$$\overline{\mathbf{B}}_3 = \begin{bmatrix} -i_{L_{\min}}/C \\ (v_{C_{\max}} + V_D)/L \\ 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_4 = \begin{bmatrix} -i_{L_{\max}}/C \\ (v_{C_{\max}} + V_D)/L \\ 0 \end{bmatrix},$$

$$\Delta\overline{\mathbf{A}}_1 = \Delta\overline{\mathbf{A}}_2 = \Delta\overline{\mathbf{A}}_3 = \Delta\overline{\mathbf{A}}_4 = \begin{bmatrix} -1/RC + 1/R_{mid}C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

which have uncertain values; $\overline{\mathbf{E}} = \begin{bmatrix} 0 \\ \dfrac{V_{in} - V_D}{L} \\ V_{ref} \end{bmatrix}$. The ranges of the parameter uncertainties are given by:

$$\Delta\overline{\mathbf{A}}_1 = \Delta\overline{\mathbf{A}}_2 = \Delta\overline{\mathbf{A}}_3 = \Delta\overline{\mathbf{A}}_4$$

$$\in \left(\begin{bmatrix} -1/R_{\min}C + 1/R_{mid}C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -1/R_{\max}C + 1/R_{mid}C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} -493.3703 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 507.8812 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right).$$

III. FUZZY CONTROLLER

A four-rule fuzzy controller employing the same membership functions of the TS-fuzzy plant model of (5) will be used to control the converter. The rules of the fuzzy controller are as follows,

Rule j: IF $v_C(t)$ is M_1^j AND $i_L(t)$ is M_2^j THEN $u(t) = \mathbf{G}_j\overline{\mathbf{x}}(t), j = 1, 2, 3, 4$ (6)

The feedback gains $\mathbf{G}_j$ are designed as $\mathbf{G}_1 = \mathbf{G}_3 = [-0.6811 \quad -4.5874 \quad 4695.8259]$ and $\mathbf{G}_2 = \mathbf{G}_4 = [-0.1868 \quad -1.0838 \quad 1142.9961]$. The output of the fuzzy controller is,

$$u(t) = \sum_{j=1}^{4} w_j(\mathbf{x}(t))\mathbf{G}_j\overline{\mathbf{x}}(t) \tag{7}$$

IV. STABILITY AND ROBUSTNESS ANALYSIS

In the previous section, a fuzzy controller of (7) is designed based on the TS-fuzzy plant model of (5) for the boost converter. The closed-loop system is guaranteed to be input-to-state stable by the following Lemma.

Lemma 1 [5-7]: *The fuzzy control boost converter is stable if* $\mathbf{TH}_{ii}\mathbf{T}^{-1}$ *and* $\mathbf{TJ}_{ij}\mathbf{T}^{-1}$ *are designed such that,*

$$\begin{cases} \mu\left[\mathbf{TH}_{ii}\mathbf{T}^{-1}\right] + \left\|\mathbf{T}\Delta\mathbf{H}_i\mathbf{T}^{-1}\right\|_{\max} \le -\varepsilon \text{ for all } i \\ \mu\left[\mathbf{TJ}_{ij}\mathbf{T}^{-1}\right] + \left\|\mathbf{T}\Delta\mathbf{J}_i\mathbf{T}^{-1}\right\|_{\max} \le -\varepsilon \text{ for all } i<j \end{cases}$$

where $\mathbf{J}_{ij} = \dfrac{\mathbf{H}_{ij} + \mathbf{H}_{ji}}{2}$, $\Delta\mathbf{J}_i = \Delta\mathbf{H}_i$, $\mathbf{H}_{ij} = \overline{\mathbf{A}}_i + \overline{\mathbf{B}}_i\mathbf{G}_j$, $\Delta\mathbf{H}_i = \Delta\overline{\mathbf{A}}_i$; $\mathbf{T} \in \Re^{3\times 3}$ *is an arbitrary invertible matrix;* $\left\|\mathbf{T}\Delta\mathbf{H}_i\mathbf{T}^{-1}\right\|_{\max}$ *and* $\left\|\mathbf{T}\Delta\mathbf{J}_i\mathbf{T}^{-1}\right\|_{\max}$ *are the maximum possible values of* $\left\|\mathbf{T}\Delta\mathbf{H}_i\mathbf{T}^{-1}\right\|$ *and* $\left\|\mathbf{T}\Delta\mathbf{J}_i\mathbf{T}^{-1}\right\|$ *respectively;* ε *is a non-zero positive scalar;* $\|\cdot\|$ *denotes the* l_2 *norm for vectors and* l_2 *induced norm for matrices;* $\mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right] = \lim\limits_{\Delta t \to 0^+} \dfrac{\left\|\mathbf{I} + \mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right\| - 1}{\Delta t} = \lambda_{max}\left(\dfrac{\mathbf{TH}_{ij}\mathbf{T}^{-1} + (\mathbf{TH}_{ij}\mathbf{T}^{-1})^*}{2}\right)$ *is the corresponding matrix measure of the induced matrix norm of* $\left\|\mathbf{TH}_{ij}\mathbf{T}^{-1}\right\|$ *(or the logarithmic derivative of* $\left\|\mathbf{TH}_{ij}\mathbf{T}^{-1}\right\|$*);* $\lambda_{max}(\cdot)$ *denotes the largest eigenvalue, * denotes the conjugate* *transpose.*

Table 1 and Table 2 summarize the stability and robustness results. From the numerical values listed in the tables, and by Lemma 1 with

$$\mathbf{T}=\begin{bmatrix} 1.0095\times10^{-5} & 3.5013\times10^{-6} & -1.4428\times10^{-5} \\ 3.5013\times10^{-6} & 4.2026\times10^{-5} & -6.8181\times10^{-5} \\ -1.4428\times10^{-5} & -6.8181\times10^{-5} & 0.09837 \end{bmatrix},$$

the closed-loop system is guaranteed to be input-to-state stable. This is because the fourth rows of these two tables all contain only negative values.

V. EXPERIMENTAL RESULTS

The PWM boost DC-DC switching converter and the fuzzy controller have been realized in hardware. The part numbers of the MOSFET, the diode and the PWM controller used in the boost converter are IRF540, 1N5822 and TL494 respectively. To implement the proposed fuzzy controller, analog multipliers (AD633), operational amplifier (LM324) and some passive components have been used. Fig. 2 shows the output response of $v_C(t)$ controlled by the fuzzy controller when the boost converter is subject to load changes. To illustrate the merits of the proposed fuzzy controller, a traditional PI controller is employed to control the same plant for comparison. The output of the PI controller is given by $u(t)=K_p e(t)+K_I\int_0^\infty e(t)\mathrm{d}t+0.5833$ where $e(t)=V_{ref}-v_C(t)$, $K_p=0.017$ is the proportional gain and $K_I=1000$ is the integral gain. Fig. 3 shows the response of $v_C(t)$ controlled by the PI controller when the boost converter is subject to load changes. In both responses, the load R is changing from $51\Omega\rightarrow15\Omega\rightarrow51\Omega$ in 25Hz. $R=51\Omega$ during the on period of the square wave (lower line) in Fig. 2 and Fig. 3 while $R=15\Omega$ during the off period. It can be seen from these two figures that the fuzzy controller gives a better performance than the PI controller.

VI. CONCLUSION

A fuzzy controller has been designed for regulating a switching PWM boost DC-DC converter subject to parameter changes based on the TS-fuzzy model approach. Stability and robustness of the closed-loop system have been analyzed. Experimental results given by the fuzzy controller and the traditional PI controller have been presented.

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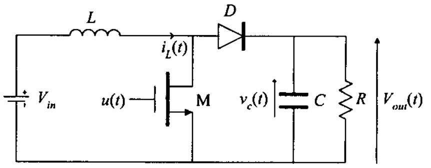

Fig. 1. Circuit diagram of the PWM boost DC-DC converter.

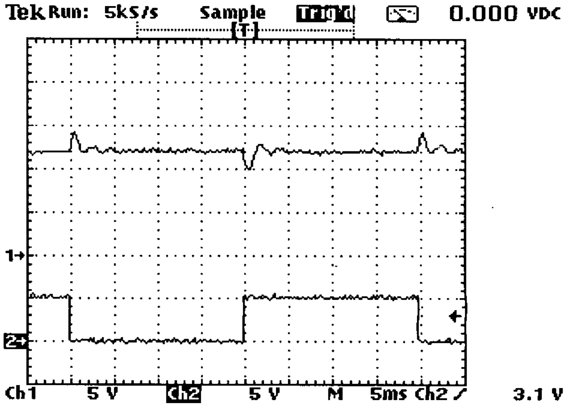

Fig. 2. Response of $v_C(t)$ (upper line) of the PWM boost DC-DC switching converter controlled by the fuzzy controller when $R=51\Omega\rightarrow15\Omega\rightarrow51\Omega$ in 25Hz.

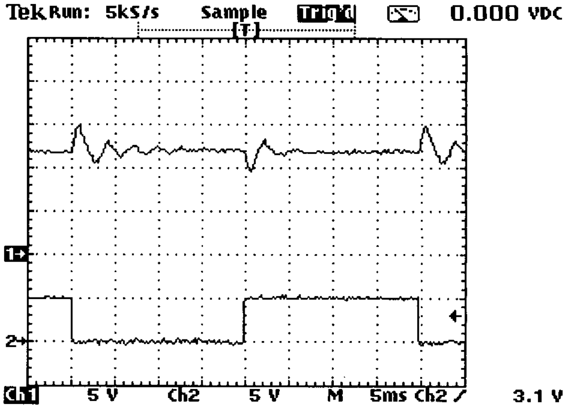

Fig. 3. Response of $v_C(t)$ (upper line) of the PWM boost DC-DC switching converter controlled by the PI controller when $R=51\Omega\rightarrow15\Omega\rightarrow51\Omega$ in 25Hz.

i	1	2	3	4
$\mu[\mathbf{TH}_{ii}\mathbf{T}^{-1}]$	−1620.2266	−1497.1725	−1581.6311	−1581.6311
$\left\Vert\mathbf{T}\Delta\mathbf{H}_i\mathbf{T}^{-1}\right\Vert_{\max}$	933.4205	933.4205	933.4205	933.4205
$\mu[\mathbf{TH}_{ii}\mathbf{T}^{-1}]+\left\Vert\mathbf{T}\Delta\mathbf{H}_i\mathbf{T}^{-1}\right\Vert_{\max}$	−686.8062	−563.7520	−648.2107	−620.7217

Table 1. Stability and robustness analysis result of the regulated boost DC-DC switching converter for $i = j$.

i, j	1, 2	1, 3	1, 4	2, 3	2, 4	3, 4
$\mu[\mathbf{TJ}_{ij}\mathbf{T}^{-1}]$	3269.4959	−3275.3105	−3250.3988	−3265.3988	−3064.8933	−3243.2064
$\left\Vert\mathbf{T}\Delta\mathbf{J}_i\mathbf{T}^{-1}\right\Vert_{\max}$	933.4205	933.4205	933.4205	933.4205	933.4205	933.4205
$\mu[\mathbf{TJ}_{ij}\mathbf{T}^{-1}]+\left\Vert\mathbf{T}\Delta\mathbf{J}_i\mathbf{T}^{-1}\right\Vert_{\max}$	2336.0754	−2341.8900	−2316.9783	−2331.9783	−2131.4727	−2309.7859

Table 2. Stability and robustness analysis result of the regulated boost DC-DC switching converter for $i \neq j$.