A Chattering Elimination Algorithm for Sliding Mode Control [1]

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Abstract - **It is well known that sliding mode control is capable of tackling systems with uncertainties. However, the discontinuous control signal causes a significant problem of chattering. An algorithm is proposed in this paper to eliminate chattering by removing the discontinuous control signal when the system is operating near the sliding plane. The transient performance as well as the robustness properties will not be affected and zero steady-state error is ensured. Mathematical derivation of the algorithm is to be detailed. This algorithm is applied to a system with parameter uncertainties to show its ability and merits.**

I. INTRODUCTION

Sliding mode control (SMC) [1, 2] is widely accepted as a powerful method of tackling uncertain systems. When the upper bound of the uncertainties is known, a control strategy can be designed to drive the system into a pre-defined sliding plane. Once the sliding plane is hit, the system responses will only be governed by the plane and are insensitive to parameter variations. Therefore SMC offers good robustness against parameter uncertainties for both linear and non-linear systems. However, since the controller will switch between two structures during operation, the system will undergo oscillation near the sliding plane. This phenomenon of chattering [1-5] is a major drawback of SMC. It will excite unmodelled dynamics of the plants and generate system noise. Moreover, the control power will be unnecessarily increased.

Basically, the control signal of SMC can be divided into two parts: a continuous control signal (usually called equivalent control) which controls the system when its states are on the sliding plane, and a discontinuous control signal which handles uncertainties. Chattering is only caused by the discontinuous control signal. The greater the uncertainties, the larger are the discontinuous control signal's amplitude and the chattering. A common method to alleviate chattering is by inserting a boundary layer [1, 2] near the sliding plane such that a continuous control signal replaces the discontinuous one when the system is near the sliding plane. This method can eliminate chattering but a finite steady-state error will result. A trade-off between chattering and tracking accuracy is thus needed. Another method is to minimize the amplitude of the discontinuous control signal during the controller design. However, the robustness properties of the controller are affected and the transient performance of the system will become poor. An alternative approach proposed by Parra-Vega *et. al.* [3] is to reduce the amplitude of the discontinuous input only when the system is near the sliding plane so that the transient responses are not affected. However, small chattering still exists and the problem is not solved yet. Recently, Chang *et. al.* [4] proposed a chattering alleviation control method which switches from two control algorithms: one governs the reaching phase and the other governs the steady-state phase. Although a recursive prediction of the system performance is employed, the chattering problem can only be reduced to a certain extent. Besides, the recursive prediction procedure is problem dependent, making the controller design a complicated task. By applying an integrator before the control signal is fed into the plant, an approach that can eliminate chattering and ensure zero steady-state error was proposed [5]. However, the system order is increased by one and the transient responses must be worsened.

In view of these weaknesses, an algorithm is proposed in this paper which can totally eliminate chattering without greatly affecting the robustness properties and the transient performance. Also zero steady-state error is ensured. It is done by adding an auxiliary continuous control signal to the control input when the system is operating on the sliding plane. However, if the system is far from the sliding plane, the auxiliary control is set small so that the system is mainly controlled by the discontinuous control signal. Therefore the robustness properties and the transient response are not greatly affected. The value of this auxiliary control is essentially the same as the average discontinuous control signal originally required to overcome parameter uncertainty. Hence, zero steady-state error is ensured. At the same time, the discontinuous control signal is replaced by the auxiliary control so that it can be removed. As a result, chattering is eliminated.

In this paper, Section II gives a brief review of the conventional SMC for a general second order uncertain system. The proposed chattering elimination algorithm is derived in section III. Then in section IV, the algorithm is applied to a non-linear system with parameter uncertainty to show its ability. A conclusion is drawn in Section V.

II. SLIDING MODE CONTROLLER DESIGN IN UNCERTAIN SYSTEM

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For ease of presentation, consider a single-input second order system:

$$\ddot{x} = f + u \quad (1)$$

where u is the control input, x is the state and f is a nonlinear function of x which is not exactly known but estimated as f_{eq}. The upper bound F of the uncertainty of f is defined as the smallest real number satisfying

$$|f_{eq} - f| < F \quad (2)$$

The aim of the controller is to drive the state x to a desired state x_d. Define the state error as:

$$e = x_d - x \quad (3)$$

Then, $\dot{e} = \dot{x}_d - \dot{x}$ (4)

and, $\ddot{e} = \ddot{x}_d - \ddot{x}$ (5)

The system equation becomes

$$\ddot{e} = \ddot{x}_d - f - u \quad (6)$$

A sliding plane is defined as:

$$\sigma = \dot{e} + \lambda e \quad (7)$$

where λ is a positive constant which determines the convergence rate of the system when the sliding plane is hit. To design the sliding mode controller, the methodology in [1] can be followed. Consider

$$\begin{aligned} \dot{\sigma} &= \ddot{e} + \lambda\dot{e} \\ &= \ddot{x}_d - f - u + \lambda\dot{e} \end{aligned} \quad (8)$$

When the system is staying on the sliding plane, it is controlled by the continuous control signal $u = u_{eq}$ which is called equivalent control. Also, (8) equals to zero and $f = f_{eq}$. Hence,

$$u_{eq} = \ddot{x}_d - f_{eq} + \lambda\dot{e} \quad (9)$$

To tackle the uncertainty of f, a discontinuous control signal u_d is added to the control input.

$$u = u_{eq} + u_d \quad (10)$$

$$u_d = k\,\mathrm{sgn}(\sigma) \quad (11)$$

where sgn is the sign function. k is a positive constant that describes the amplitude of the discontinuous control signal. It should be large enough to overcome the uncertainty of f and will be derived later. To ensure the system stability, the existence and the reachability of the sliding plane, the following condition should be satisfied:

$$\sigma\dot{\sigma} < -\eta|\sigma| \quad (12)$$

where η is a positive constant that governs the reaching time, i.e. the time taken to hit the sliding plane if the initial state is not on the plane. Then, from (8) to (12),

$$\begin{aligned} \sigma\dot{\sigma} &< \sigma(\ddot{x}_d - f - u + \lambda\dot{e}) \\ &= \sigma(\ddot{x}_d - f + \lambda\dot{e} - u_{eq} - u_d) \\ &= \sigma(\ddot{x}_d - f + \lambda\dot{e} - \ddot{x}_d + f_{eq} - \lambda\dot{e} - k\,\mathrm{sgn}(\sigma)) \\ &= \sigma(f_{eq} - f) - k|\sigma| \\ &< -\eta|\sigma| \end{aligned}$$

A sufficient condition for k is

$$k > F + \eta \quad (13)$$

The above derivation is a complete design of a sliding mode controller. It is seen that the discontinuous control signal u_d will either take the value k or $-k$. Then, on the average, u_d is equivalent to a continuous signal u_{ed} where

$$u_{ed} = \alpha k - (1 - \alpha) k \qquad 0 \le \alpha \le 1 \quad (14)$$

We call u_{ed} the equivalent discontinuous control and this term will be used in the next section. Note that u_{ed} can take any value between k and $-k$ but the magnitude is bounded by k. Conversely, if u_{ed} is small, the value of k can be set small. This relationship between u_{ed} and k can be stated as follows:

$$u_{ed} = \beta \qquad \Rightarrow \qquad k \ge |\beta| \quad (15)$$

III. CHATTERING ELIMINATION ALGORITHM

It is shown in section II that the control signal can be divided into a continuous control signal u_{eq} and a discontinuous control signal u_d. Chattering is only caused by u_d, or equivalently caused by k. The proposed chattering elimination algorithm will eliminate k on approaching steady-state such that chattering is eliminated, but it will maintain u_d during the transient responses in order to preserve a good transient behavior and robustness of the controller. In practice, chattering during the transient response is not a significant problem. In this paper, it is assumed that the uncertainty F is constant. Then by (13) and (14), the equivalent discontinuous control signal u_{ed} is also constant in steady-state. The proposed algorithm can be explained by dividing the control action into two phases. First, an auxiliary continuous control signal u_x is added to u. It is used to replace u_{ed} for compensating the uncertainty. As a result, u_{ed} will finally become zero. Second, since u_x completely replaces u_d and u_{ed} is zero, from (15), k can be reduced to zero. The chattering is thus eliminated.

In practice, the two phases of the algorithm works together. Therefore one should be careful that k cannot be reduced too fast. Otherwise, (12) cannot be satisfied and the controller cannot tackle the system uncertainties. The mathematical derivation of the algorithm is shown below.

Since an auxiliary control u_x is added, (10) becomes

$$u = u_{eq} + u_d + u_x \tag{16}$$

Since k is going to be reduced according to u_{ed}, it should be a function of u_{ed}. Moreover, in order to recover k in case a sudden disturbance is exerted during the steady-state, driving the states away from the sliding plane σ, k should also be a function of σ. Hence, set

$$k = k_{ed} \, | \, u_{ed} \, | + k_\sigma |\sigma| \qquad 0 \le k \le k_{max} \tag{17}$$

where k_{ed} and k_σ are constants to be determined. Note that when the system is operating on the sliding plane, the second term of the right hand side of (17) is zero. On the other hand, since u_{ed} will be replaced by u_x, the first term of (17) will also be zero. Hence k_{ed} and k_σ govern the decrease and increase rate of k. Since the final value of k is zero, chattering is totally eliminated.

u_{ed} and u_x can be derived by the following equations:

$$u_{ed} = u_d - \dot{u}_{ed} \tag{18}$$

$$u_x = \frac{1}{1+|\sigma|} \int u_{ed} \, dt, \qquad \text{subject to } | \, u_x \, | \le F \tag{19}$$

It can be seen that u_{ed} is equivalent to u_d passing through a low-pass filter. u_x is the integration of u_{ed} with respect to time. The gain term in (19) is added to minimize the effect of u_x during the transient responses. u_x is intentionally bounded by F. The reason is given as follows. Consider (12) again,

$$\begin{aligned} \sigma\dot{\sigma} &< \sigma(\ddot{x}_d - f + \lambda\dot{e} - u_{eq} - u_d - u_x) \\ &= \sigma(f_{eq} - f - u_x) - k|\sigma| \\ &< -\eta|\sigma| \end{aligned}$$

A sufficient condition for k is

$$k > \eta + | \, f_{eq} - f - u_x \, | \tag{20}$$

Under the worst case condition, (20) holds if

$$k_{max} = \eta + 2F \tag{21}$$

During the steady-state, $|\sigma|$ is zero. Also the reaching time is not important during the steady-state so that η can be set to 0. From (17) and (20),

$$k = k_{ed} \, | \, u_{ed} \, | > | \, f_{eq} - f - u_x \, | \tag{22}$$

Note that $f_{eq} - f - u_x$ is the uncertainty of the system and will be compensated by the discontinuous control. Thus

$$f_{eq} - f - u_x = u_{ed} \tag{23}$$

$$\Rightarrow k_{ed} > 1 \tag{24}$$

Another parameter k_σ in (17) determines the incremental rate of k. When the system suffers from a sudden disturbance and/or parameter change and the states are driven away from the sliding plane, the discontinuous control signal's amplitude should be increased again. k_σ defines the sensitivity of k under this sudden change of states. The larger the value of k_σ, the greater the value of k increase according to $|\sigma|$. If one wants to recover k to k_{max} when the distance from the sliding plane is larger than γ, k_σ should be given by:

$$k_\sigma = \frac{k_{\max}}{\gamma} \tag{25}$$

IV. RESULTS

The proposed chattering elimination algorithm is applied to the following non-linear system

$$\ddot{x} = -2\dot{x} - ax + \dot{x}^2 + u \tag{26}$$

where a is fixed but unknown. The estimated value of a is 2. Now, let the actual value of a be 5. The parameters k_{ed}, k_σ and k_{max} are selected to be 2, 10 and 10 respectively. The constant λ in (7) is set to be 1. In this simulation, the discontinuous switching is nearly ideal. The time delay of the switch is so small that chattering cannot be seen in the output. Fig. 1 and Fig. 3 show the closed-loop transient responses of the system with respect to a step input without and with applying the chattering elimination algorithm respectively. It can be seen that the transient responses are similar. This means that the proposed algorithm does not affect the transient response very much. Fig. 2 and Fig. 4 show the discontinuous control input u_d of the system without and with applying the proposed algorithm respectively. It is clearly shown that the amplitude of u_d reduces to zero on applying the proposed algorithm. Chattering is successfully eliminated.

V. CONCLUSION

The cause of chattering in SMC due to the presence of a discontinuous control signal has been addressed. A chattering elimination algorithm has been proposed to tackle the problem. By inserting an auxiliary control input, the discontinuous control input can be finally replaced. This algorithm has applied to a non-linear system and chattering elimination has successfully been achieved. The transient response is nearly not affected. Moreover, zero steady-state error is ensured since the auxiliary control input can completely tackle the uncertainties. The control power is saved due to the removal of switching action of the control signal. The design and implementation of the algorithm is also simple.

There are some open questions in the controller design. First the design of the parameters k_{ed} and k_σ is arbitrary. Only the necessary condition of k_{ed} is derived. The optimal values of the two parameters are not determined. Second, there is a slow dynamic in (18) and (19). Their time constants should be set large comparing with the system time constants. But the optimal values are also not determined.

Actually, the implementation of the proposed algorithm through (18) and (19) is not a unique method. Other methods to determine u_x can also be applied without affecting the algorithm. Further research can be directed to the derivation of other implementation methods like those applying fuzzy logic and neural network.

VI. References

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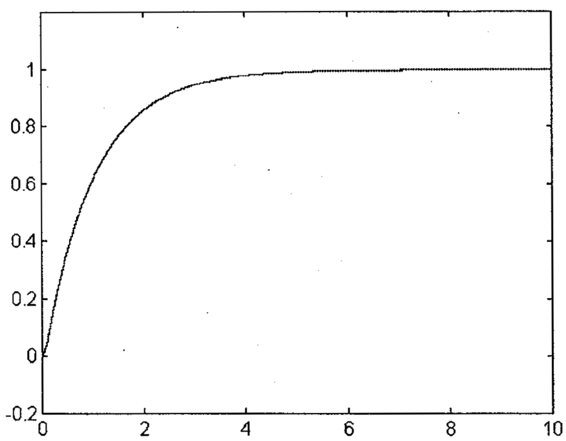

Fig. 1. Step response of the system without applying the proposed algorithm

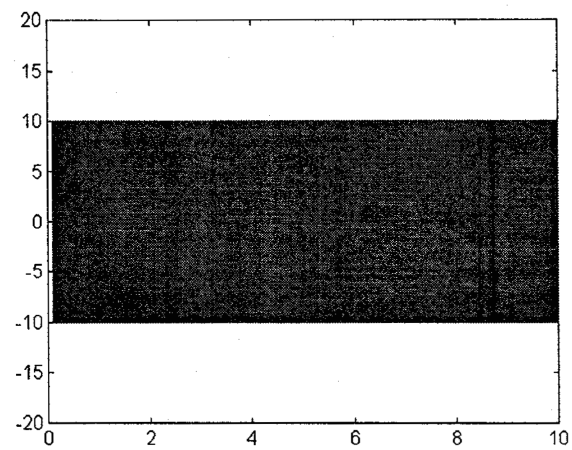

Fig. 2. Discontinuous control input without applying the proposed algorithm

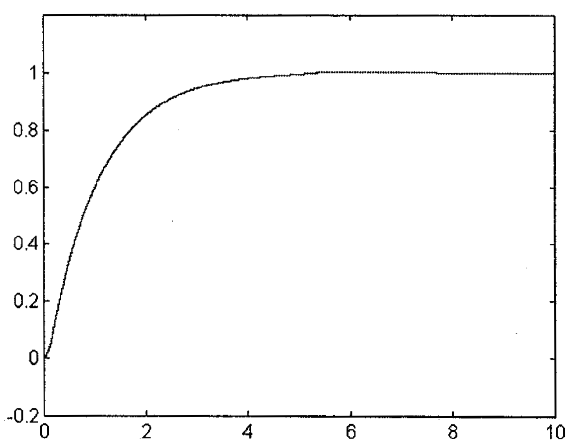

Fig. 3. Step response of the system on applying the proposed algorithm

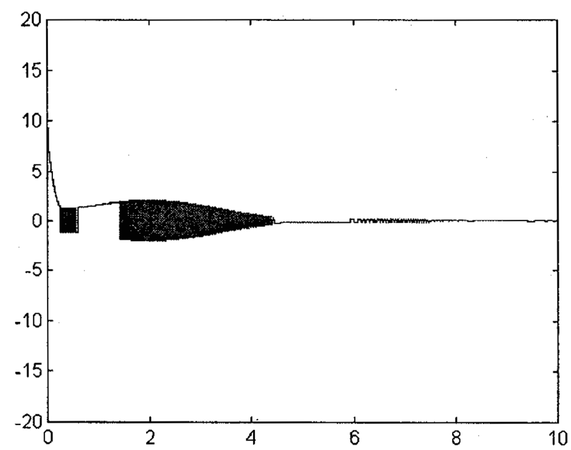

Fig. 4. Discontinuous control input on applying the proposed algorithm