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It's Showtime: Live Streaming E-commerce and Optimal Promotion Insertion Policy

Wenpo Huang
Experimental Center of Data Science and Intelligent Decision Making, Hangzhou Dianzi University, Hangzhou, China
School of Management, Hangzhou Dianzi University, Hangzhou, China
whuang@hdu.edu.cn

Wei Jiang
Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China
jiangwei@sjtu.edu.cn

Xinggang Luo
Experimental Center of Data Science and Intelligent Decision Making, Hangzhou Dianzi University, Hangzhou, China
School of Management, Hangzhou Dianzi University, Hangzhou, China
xgluo@mail.neu.edu.cn

Xiaowei Mei
Department of Management and Marketing, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
michael.mei@polyu.edu.hk

Lai Wei
Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China
laiwei@sjtu.edu.cn

Correspondence:
Xiaowei Mei, Department of Management and Marketing, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
Email: michael.mei@polyu.edu.hk
Lai Wei, Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China
Email: laiwei@sjtu.edu.cn

Production and Operations Management

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Wenpo Huang
Experimental Center of Data Science and Intelligent Decision Making, Hangzhou Dianzi University, Hangzhou, China
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Wei Jiang
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Xinggang Luo
Experimental Center of Data Science and Intelligent Decision Making, Hangzhou Dianzi University, Hangzhou, China
School of Management, Hangzhou Dianzi University, Hangzhou, China

Xiaowei Mei*
Department of Management and Marketing, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

Lai Wei†
Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China

Abstract: Live streaming e-commerce has gained tremendous success as a new form of business model over the past few years. Nevertheless, there has been scarce research examining the effect of exogenous stimulus-driven factors (i.e., social cues such as promotion coupons) on the profit of the live streaming platform. We model the continuous evolvement of the aggregate viewer involvement level with geometric Brownian motion and investigate the optimal timing and depth of promotion insertion policy. Our findings reveal that the optimal promotion insertion policy for the live streaming room is a threshold policy that consists of a start-to-promote threshold and an involvement target when the trend of the aggregate involvement level is not large and the promotion cost is not high. Under some scenarios, the promotion planning process is constrained by various business rules, such as the promotion depth being limited by a discrete set (e.g., integral multiples of D). Although this problem involves more complex constraints, we analytically show that a variation of the abovementioned threshold policy is the optimal strategy among all feasible policies, i.e., when the aggregate viewer involvement drops below certain level (P), the live streaming host promotes with the depth of the least integral multiple of D to raise the viewer involvement level above P. Finally, to examine the effectiveness of our proposed policy, we compare it with prevalent industry practices. The result shows that the proposed promotion insertion policy outperforms prevailing ones significantly in generating profit for the live streaming room.

Key words: Social Technology, Live Streaming E-commerce, Promotion Planning, Geometric Brownian Motion
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* The corresponding author: michael.mei@polyu.edu.hk
† The corresponding author: laiwei@sjtu.edu.cn

1 Introduction

Social network platforms such as Facebook, Pinterest, Twitter, and TikTok these days serve more than a community for users to connect with others. A report by Ryu and Park (2020) shows that 85% of all US shoppers have accessed social network sites to inform their shopping decisions and 30% of them have purchased a product after seeing it on social media. The worldwide revenue of using social media as a tool for e-commerce (*social commerce*) is valued at 724 billion U.S. dollars in 2022 and is forecasted to surpass six trillion by 2023 (Statista 2022). A commonly adopted strategy is to endorse products through social media influencers who are able to affect their numerous followers on social media platforms and turn online traffic into sales, which is referred as *influencer marketing* (Mallipeddi et al. 2022).

When social commerce and video collide, it defines a new type of business model: *live streaming e-commerce*. With the rapid development of telecommunication and internet technology, live streaming broadcast has become increasingly popular in recent years as an interactive form of multimedia entertainment. Live streaming platforms allow hosts to broadcast to their audiences through live videos with various activities, such as dancing, singing, and playing video games. Different from conventional video advertising on social media platforms such as YouTube, live streaming platforms do not provide recorded content. Rather, they serve as a conduit connecting hosts with viewers, thus facilitating better interactions than conventional social media platforms. Video advertising, such as those on YouTube, is one-sided communication from firms or brands to viewers who passively receive information without any interaction. In contrast, live streaming e-commerce is two-way communication. Viewers not only receive information about the products they are interested in, but they can also ask questions and get feedback from the live streaming host in real time. Furthermore, they can ask the host to showcase the endorsed products from different angles or even request the host to try on certain products such as clothing. On the other hand, the host can use reinforcements like promotional coupons to boost the aggregate viewer involvement level in real time when viewers are not well engaged in the live streaming room, which is not feasible for traditional video advertising. This distinctive feature of live streaming broadcast has grabbed the attention of the retail industry, as live streaming platforms integrate commercial activities with streaming hosts who are capable of conducting product endorsement for the vendors through the hosts' live streaming channels.

A report by Bloomberg in 2020 shows that the market size of live streaming e-commerce was US$60 billion globally in 2019 with China being the largest market. In 2020, the global market almost doubled as the coronavirus pandemic accelerated changes in people's shopping behavior (Kharif and Townsend 2020). The closing of retail stores in order to slow down the spreading of the virus has propelled more people online. As a typical example, TalkShopLive, a US live streaming platform that debuted in 2018 has increased its sales by seven-fold during Covid-19.[1] The quick development of the live streaming market incentivizes

[1] See https://wwd.com/beauty-industry-news/beauty-features/brian-moore-talkshop-live-livestream-commerce-1234774942/ (Last accessed December 05, 2022)

retailers and social platforms to capitalize on this new commerce opportunity. For example, Amazon Live hosts online shows on products related to fitness, makeup, and cooking every day.[2] Forrester predicts that live streaming e-commerce in China will reach US$100 billion by 2023, representing a compound annual growth rate of 45.7%.[3]

Compared with traditional e-commerce which adopts celebrity endorsements (such as athletes, supermodels, and actors) to promote products or services, live streaming e-commerce relies more heavily on social media influencers or so-called internet celebrities. By enthusiastically sharing their knowledge about the endorsed products through the medium of live streaming shows, these social media influencers have gained a large follower base (Lin et al. 2021, Mallipeddi et al. 2022). Therefore, compared to the traditional e-commerce logic of "people looking for goods", the innovation of live streaming e-commerce is a recommendation-type logic of "goods looking for people". While the entertaining and immersive nature of live streaming e-commerce keeps viewers watching longer and well engaged, more importantly, it telescopes the customer decision processes from awareness to purchase and improves sales conversion. According to Pana (2021), for traditional e-commerce, 43.8% of the total visits view a product page, only a third of those (14.5%) add a product to the shopping cart, and finally, only 2.12% of the total visits finish their visit with a transaction. In contrast, 70% of the total viewers in a live streaming broadcast view the product page and 10% of total viewers become buyers, a rate which is almost 5 times as much as traditional e-commerce. In many cases, customers may have already made their purchase decision while watching the live streaming broadcast and the product page just serves as confirmation before checking out.

Despite the tremendous success of live streaming e-commerce in boosting sales, one of the biggest challenges in live streaming rooms is to consistently grab viewers' attention and maintain a high involvement level which is highly correlated with sales (Sun et al. 2019). There are two kinds of attention suggested by the selective attention theory: endogenous attention and exogenous attention (Posner 1980, Theeuwes 1991, Yantis 2000, Serences and Yantis 2007). Endogenous attention refers to voluntary, goal-directed attention as a result of subjective effort. Viewers' attention to the endorsed products is evidently endogenous attention since it directly relates to focal targets and is goal driven. Previous literature has found that viewers' attributes, such as curiosity and their match with the endorsed products, contribute significantly to their involvement level in the live streaming room (Yi et al. 2015, Zhou et al. 2019). During an e-commerce live streaming show, the host is the anchor and indispensable source of product information and plays an important role in attracting viewers' attention. Previous literature also found that personal characteristics of live streaming hosts such as their attractiveness and communication style, among others, are important factors in contributing to viewers' involvement level (Lu and Chen 2021).

[2] See https://advertising.amazon.com/solutions/products/amazon-live (Last accessed December 05, 2022)

[3] See https://www.forrester.com/report/Forrester-Analytics-Social-Commerce-Forecast-2018-To-2023-China/RES157177/ (Last accessed December 05, 2022)

In contrast, exogenous attention refers to involuntary, stimulus-driven attention triggered by distracting factors, such as social cues. In the context of live streaming e-commerce, these social cues include interaction texts or promotions which suddenly appear on viewers' screens during the live streaming broadcast. One of the most effective social cues used by live streaming hosts is coupon messages containing promotion information (Fei et al. 2021). More than often, a sense of urgency is also generated by using time-limited tactics such as one-off coupons. Previous literature has examined the effects of social cues, e.g., the number of deals purchased for a daily deal promotion or friends' purchase history, on consumers' decision making (Ye et al. 2013, Kukar-Kinney and Xia 2017, Windels et al. 2018, Qiu et al. 2021). However, there has been a lack of study on the optimal operational policy regarding these social cues, especially in the context of live streaming e-commerce. In this work, we formulate a stochastic model where the aggregate viewer involvement level in a live streaming room evolves constantly over time. The stochastic process of viewers' update of their aggregate involvement level is captured through a drifted geometric Brownian motion. Next, we highlight the key research questions and findings of our study.

The first research question we want to address is: what is the optimal policy for inserting promotional coupons to boost the aggregate viewer involvement level in the live streaming room? Specifically, we would like to investigate the optimal timing and promotion depth (Shaffer and Zhang 2002, Sinitsyn 2017, Zhang et al. 2020, Calvo et al. 2023). Since the aggregate viewer involvement level continuously evolves stochastically as the live streaming goes on and different promotion depth comes at different costs, the answer to this research question seems complex. Nevertheless, we find that the optimal promotion insertion policy is a (θ, Θ) policy, i.e., the live streaming host should wait until the aggregate viewer involvement level drops to a certain threshold (θ), then release a promotion with appropriate depth to raise aggregate viewer involvement level to a specific target (Θ). This finding sheds light on current practices of live streaming e-commerce, most of which simply release promotional messages with fixed discounts at some specific time point.

Under some scenarios, promotions are released in batches of time-limited one-off coupons taking values of multiples of integers. Thus, the promotion depth is actually constrained by a discrete set of integral multiples. To further examine the optimal promotion insertion policy under this practical restriction, we investigate this research question: What is the optimal promotion strategy if the promotion depth is limited by a discrete set of integral multiples? One may expect the optimal policy to be more complex than the one we find for our first research question because an additional constraint is introduced into the problem. Nevertheless, our analysis demonstrates that a (P, D) policy is optimal in this case, i.e., the live streaming host should delay the promotion until the real-time involvement level in the live streaming room decreases below the threshold P and then release the least number of coupons (each increases the aggregate involvement level by D) to bring the involvement level above P.

Moreover, we also perform a comparative static analysis by examining the effect of different parameters on the optimal promotion policy. We find that fixed promotion cost and unit promotion cost have different effects on the optimal promotion policy. Specifically, as the fixed cost increases, the start-to-promote threshold decreases while the involvement target increases and the optimal promotion depth also increases. In contrast, as the unit cost increases, both the start-to-promote threshold as well as the involvement target decrease and the optimal promotion depth also diminishes. We also have an interesting finding regarding the volatility of aggregate viewer involvement level. Intuitively, as volatility increases, the aggregate viewer involvement level changes more dramatically, and the chance of reaching the start-to-promote threshold becomes high, indicating that the live streaming host might need to adjust the start-to-promote threshold downward to avoid releasing promotions too frequently. However, it is not evident how the optimal promotion depth should evolve with volatility accordingly. Our study reveals that the optimal promotion depth first increases and then decreases in volatility. These findings provide practical guidelines for practitioners to adjust the promotion policy to accommodate different market conditions dynamically.

To examine the effectiveness of our proposed policy, we further examine how it performs in comparison with current industry practices. We compare our proposed policy with four prevalent industry policies where the live streaming host releases promotions at the beginning of the live streaming, during the live streaming, both at the beginning of or during the live streaming or at some random time point. The result reveals that our proposed policy outperforms all these prevalent industry practices significantly. The managerial implication of this finding here is that the timing and depth of the promotion are essential for the live streaming room. Therefore, to maximize the profit, the live streaming hosts should release the right depth of promotions at the right moment instead of doing it at either some predetermined time point or randomly with promotions of fixed depth.

The rest of the paper is organized as follows. We review the literature and discuss the differences and contributions of our work in Section 2. In Section 3, we describe our model setup. We analyze the basic model, present the optimal solution, and perform comparative static analyses in Section 4. In Section 5, we extend the basic model by discussing promotions with fixed depth and a finite horizon setting. In Section 6, we evaluate the effectiveness of our proposed promotion policy for live streaming e-commerce by comparing it with current industry practices. Finally, Section 7 concludes and presents directions for future research.

2 Literature Review

Our study is related to the literature in three streams: (i) the recent development of e-commerce business models and promotion planning, (ii) influencer marketing and live streaming e-commerce, and (iii) the application of stochastic processes such as Brownian motion and Geometric Brownian motion in operations management, information systems, and marketing.

Regarding the first stream of research, a substantial amount of work has studied channel selection and pricing contracts between different parties involved in the e-commerce business. For example, Tian et al. (2018) consider an operation model called ”online marketplace” for the retailer who can either act as a reseller of the suppliers’ products, or as an independent supplier on the intermediary platform. This study shows that the degree of competition intensity and the magnitude of fulfillment costs can affect the retailer’s decision on the operational model. With the emergence of powerful e-commerce platforms, the retailer business model has been widely adopted, wherein the manufacturer needs to pay a slotting fee and share a portion of the revenue with the platform. Shen et al. (2019) study the manufacturer’s optimal channel selection problem among the choices of the platform retailer, the traditional reseller, or both channels. Other works have investigated information incentives of different parties on the online market. Li et al. (2021) consider the information sharing between the online marketplace, the upstream manufacturer, and the reseller, where the reseller and the manufacturer cooperate while the online marketplace has superior demand information. The design of the pricing scheme and compensation contract under asymmetric information can also be found in Chen et al. (2021). Qiu et al. (2021) propose an analytical framework of multidimensional observational learning to characterize the interaction between product differentiation and social ties. Consumers may refer to the information from early adopters’ subjective quality reviews while the seller can strategically choose to withhold or disclose the variously high- or low-quality information to suit their targets.

Our work is also closely related to the literature on promotion planning in operations management and information systems area. Such strategies include but are not limited to social coupons (Kumar and An 2012) and social red packets (Gao et al. 2020). Social coupons use promotional discounts to increase the coverage in consumer’s social network (Kumar and An 2012). Luo et al. (2014) investigate the impact of deal popularity and the social influence on a group-buying platform, which provides discounted products and services posted on the website and consumers are influenced by deal popularity at both the purchase and redemption phases. Subramanian and Rao (2016) analyze strategic implications of displaying deals on some daily deal websites which offer an online promotion to attract more consumers. Gao et al. (2020) investigate the use of social red packets and find that consumers with more social network connections enjoy better promotion rewards. Conversely, consumers are encouraged to enhance their social network connections in order to receive better promotion rewards.

For the second stream of research, influencer marketing and live streaming e-commerce on social media have gained spectacular attention during the past couple of years. Brand owners favor social media celebrities for their influence on their online followers. By monetizing the online influences of social media celebrities, live streaming e-commerce has become a new channel for vendors to boost sales. Ajorlou et al. (2018) study the optimal dynamic pricing for selling products to consumers through social networks, where the diffusion of information about a product is through word-of-mouth communication contributed by both buyers and non-buyers. Cheng et al. (2019) combine econometrics with deep learning to test how live streaming affects the landscape of e-commerce in order to understand the business value created by live streaming. They also suggest that live streaming boosts online sales by reducing product uncertainty and can substitute for other uncertainty-mitigating mechanisms such as online reviews. Schouten et al. (2020) conduct experiments to investigate the endorsement effectiveness of traditional celebrities and social media influencers, and show that social media influencers are closer to their fans and are more trustworthy than traditional celebrities. Mallipeddi et al. (2022) develop a model to facilitate the ability to identify independent influencers from a complex social network so that companies can optimally post their advertisement on social media platforms. Fei et al. (2021) use the stimulus-organism-response (S-O-R) theory to show that social cues like promotion messages and interaction texts can affect the viewer's attention allocation and purchase intention for the live streaming e-commerce. They also carry out a within-subject eye-tracking experiment and propose that anchor characteristics such as host attractiveness may moderate the effects of social cues. Lu et al. (2021) reveal that there exists a positive relationship between audience size and average tip per viewer under the pay-what-you-want pricing strategy where viewers could send the broadcaster voluntary payments in the form of virtual gifts and virtual cash. Exploring another related aspect, Lin et al. (2021) employ a vector autoregression model to illuminate a positive relationship between broadcaster emotion and viewer emotion.

Lastly, our work also enriches the application of stochastic processes such as Brownian motion (BM) and Geometric Brownian motion (GBM) in new business models. Both BM and GBM have been widely used to model the stochastic process in operations management, information systems, and marketing. In the marketing area, Branco et al. (2012, 2016) use BM to characterize the evolution of the product information search process and focus on the study of consumers' optimal stopping rules. Villas-Boas (2018) uses BM to model the evolution of consumers' preferences and determine a company's optimal repositioning strategy. In the information systems area, Benaroch (2018) adopts the call option model to study cybersecurity investment using GBM. Kumar et al. (2020) study the optimal advertisement insertion policy where users' involvement with certain digital content is modeled as the BM. In the operations management area, Kouvelis and Tian (2014) study the decision of flexible capacity investment when the predicted aggregate demand is modeled as a GBM process. Angelus (2021) address the investment in distinct and distributed renewable energy by an individual consumer participating in a regulated electricity market when his/her demand for electricity follows GBM. The main difference between Kouvelis and Tian (2014), Angelus (2021), and our work is that the investment decision in Kouvelis and Tian (2014) is continuous with time while the investment of renewable power generation in Angelus (2021) is determined once at most. In contrast, our decision on the timing and depth of the promotion is related to impulse control problems in which the control makes the underlying process discontinuous and there could be an infinite number of controls.

We would like to point out that most applications of BM and GBM focus on the optimal stopping problems (Branco et al. 2012, 2016, Kouvelis and Tian 2014, Zhang and Zhang 2015, Ke et al. 2016, Benaroch 2018, Villas-Boas 2018). Different from the optimal stopping problem, our work is an impulse control problem in which the process might jump because of the control. In the literature of the impulse control problem, Angelus (2021) investigates the optimal investment time and capacity based on the dynamic Stackelberg game. Their modeling of electricity demand is indeed a stochastic process with the restriction of one impulse. Kumar et al. (2020) is more relevant to our work. Both papers study the optimal impulse control problem without the constraint of control times. The differences between their work and ours are 1) Kumar et al. (2020) use the BM process to model the stochastic process while our paper uses the GBM since the aggregate involvement level is positive and follows a skewed distribution; 2) the objective function in their work is the discounted profit when inserting an advertisement while the objective function in our paper comprises of the discounted cumulative sum of revenue and the discounted promotion cost; 3) Our paper further examines the case under which the promotion depth is restricted to be integral multiples of certain values, which is a more complicated and practical problem not studied in Kumar et al. (2020). Compared with prior impulse control literature, our main contributions lie in the following aspects. Our work theoretically solves the impulse control of the GBM problem with discounted cost criterion under two different cases: Case 1 without any restriction on promotion depth, and Case 2 where the promotion depth is restricted to be integral multiples of certain values, which is not studied in prior impulse control literature. In addition, we explore the case where the cost of impulse control can be monetary and non-monetary as well as the case of a finite horizon with expected future profit, and find interesting non-monotone behavior of the optimal promotion policy.

3 Model Setup

In this section, we first introduce our basic model. Then a detailed discussion is provided on how promotions affect the aggregate viewer involvement level in the live streaming room.

3.1 Aggregate Viewer Involvement Process in the Live Streaming room

We consider a live streaming room in which the host (also called streamer) endorses products to potential customers who enter the room randomly. The sales of products depend highly on the aggregate viewer involvement level, which reflects the overall engagement of all viewers with the host in the live streaming room and changes constantly during the live streaming. Once in a while, the host uses social cues, such as discount coupons and interaction texts, to grab viewers' attention and increase their involvement level. Thus, the aggregate viewer involvement level is continuously updated depending on viewers' match with the products as well as their interactions with the host in the live streaming room. Therefore, we model the continuously updating process of the aggregate viewer involvement level by following Branco et al. (2012,

2016) and Kumar et al. (2020). One noteworthy difference between our work and theirs is that we apply GBM instead of BM to model the aggregate viewer involvement level. The main reason for this modeling choice is that BM models the evolution of a variable that spans the entire field of real numbers. However, in our context, the aggregate viewer involvement level never drops below zero, which can be characterized by GBM.

GBM has been widely used for modeling stock prices. In the literature studying options, the pricing behavior of the underlying asset is usually modeled as GBM. Various applications of real option models for investment decisions under demand uncertainty can be found in Dixit et al. (1994). Another work by Kouvelis and Tian (2014) uses GBM to model the aggregate product demand. The process of GBM indicates that the aggregated involvement level exhibits the Markovian property and is log-normally distributed.

The index of the aggregate involvement level is shown on the screens of both viewers and the live streaming host. In Figure **??** in Section EC.10 of the Online Appendix, we display a screenshot of one live streaming room from China's leading live streaming platform *Kuaishou* where the continuously updated index is shown on the right. The details of our model are illustrated as follows.

Let X_t represent the aggregate viewer involvement level at time t, which is determined by two factors. The first factor is the deterministic trend of the viewer involvement level, which could be reflected by the match between the products and viewers' expectations or the popularity of the live streaming host. Secondly, a stochastic component contributes to the aggregate viewer involvement level, such as the interactions between viewers and the host. In practice, customers' involvement level is skewed and long-tailed. Specifically, in our paper, the dynamics of the aggregate viewer involvement level after t time units are modeled as $dX_t = X_t(\mu dt + \sigma dB_t)$ where B_t is the standard Brownian motion. Here the first term μdt is the deterministic part, which evolves with time but is not stochastic. The term μ is also called the drift term, which captures the trend of aggregate viewer involvement level over time. For instance, if the live streaming host does a good job of endorsing the products and interacting with viewers, it is highly possible that more people will join the live streaming room and the overall involvement level in the room is likely to increase over time. In this case, the drift term μ is positive. The term can also be negative under some circumstances, such as when customers may have a high initial level of expectation for the endorsed products, but finally lose interest because the products do not match their expectations, which results in less interaction in the room. The second term σdB_t captures the random evolution of aggregate viewer involvement level, with $\sigma > 0$ being the volatility. Similar to the drift term, the realization of random change in viewer involvement level can be either positive or negative. For example, viewer involvement level may be boosted by promotional coupons the live streaming host releases at a certain time point. On the contrary, it can also be brought down if the host makes some mistake when endorsing the products. We provide the following example explaining how the aggregate viewer involvement level evolves with time in Figure 1.

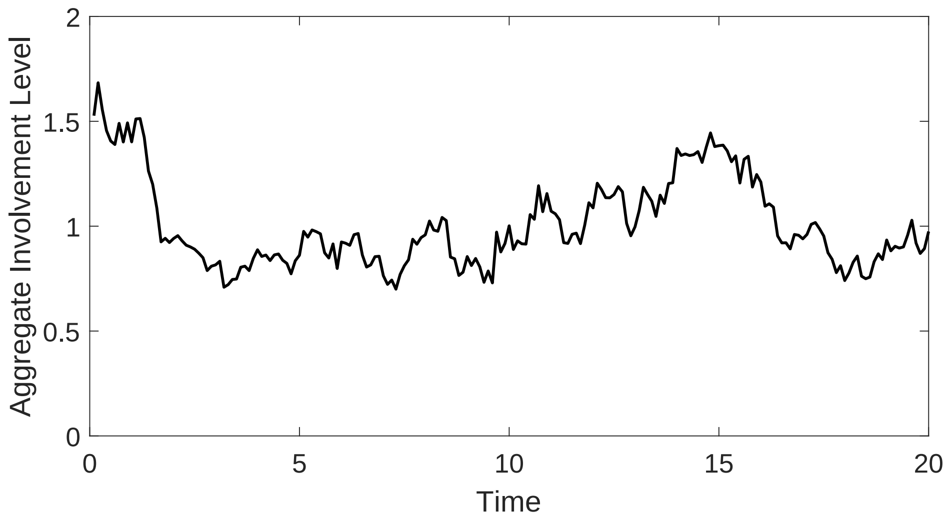

Figure 1 A Sample Path of Viewers' Aggregate Involvement Level in a Live Streaming Room without Promotion

3.2 Impact of Promotion and Host's Decision

Promotion through social cues is a common practice conducted in the live streaming room in order to grab the attention of the audiences and increase the purchase intention of target customers (Park and Lin 2020). In this subsection, we first discuss the impact of promotional activities in a live streaming room, then we investigate the host's optimal decision on the depth and timing of the promotion.

The impact of conducting promotion activities can be two-fold. On the one hand, promotions increase the aggregate viewer involvement level in the live streaming room, boosting product sales. On the other hand, releasing promotions incurs a cost of $c(v)$ where v is the promotion depth and $c(\cdot)$ is an increasing function. We model the increment of the aggregate viewer involvement level with the promotion depth, such that with a promotion depth of v, the aggregate viewer involvement level rises from x to $x+v$.

Let the revenue in the live streaming room at time t be $h(x)$ and is assumed to be concave (Geunes et al. 2006, Demirezen et al. 2020, Qin et al. 2022) and increasing in the aggregate viewer involvement level x, which is further modeled with the power function $h(x) = \beta x^{\alpha}$ with $0 < \alpha < 1$, which is verified by using empirical data from one of the largest live streaming platforms *Kuaishou*. The platform *Kuaishou* uses a composite index "Renqi" which is related to the number of active users, click rate, duration of stay, and so on as the aggregate involvement level. Detailed discussions can be found in Appendix EC.10. Let $H(x)$ be the cumulative revenue in the live streaming room if there is no promotion so that we have

$$H(x) = \mathbb{E}\left[\int_0^{\infty} e^{-rt} h(X_t)dt | X_0 = x\right] = Kx^{\alpha}, \tag{1}$$

where r is the discount factor and $K = (r - (\alpha\mu + (\alpha^2 - \alpha)\sigma^2/2))^{-1}\beta$. The discount factor is widely adopted in the finance literature to capture the present value of future cash flows (Gordon 1963, Campbell and Shiller 1988, Campbell 2000, Bakshi et al. 2021). In other words, it is a conversion factor when computing the time value of money. Therefore, it becomes reasonable and necessary to include a discount factor under the infinite horizon setting of our model. This also makes the technical analyses feasible and tractable. For the finite horizon, we do not consider the discount factor since the duration of the live streaming broadcast is short and the profit of the platform is less sensitive to temporal discounts. We focus on the case that $r > \alpha\mu + (\alpha^2 - \alpha)\sigma^2/2$. The trivial case of $r \leq \alpha\mu + (\alpha^2 - \alpha)\sigma^2/2$ is discussed in Lemma 1 below. The derivation of the above equation is provided in Section **??** of the Online Appendix.

The host's decisions, thus, are a sequence of promotion time $(\tau_1, \tau_2, \cdots)$ and promotion depth $(v_1, v_2, \cdots)$, which is denoted as $w = \{(\tau_1, v_1), (\tau_2, v_2), \cdots\}$ with $\tau_k \leq \tau_{k+1}, v_k \geq 0$. Note that $X_{\tau_i^-}$ and X_{τ_i} are the aggregate involvement level immediately before and after the host releases the promotion at time τ_i. Let W represent the set of all feasible decisions. For a given $w \in W$, the stochastic process of the aggregate involvement level is

$$\begin{cases} dX_t = \mu X_t dt + \sigma X_t dB_t, & \tau_i \leq t < \tau_{i+1}, i \geq 0, \\ X_{\tau_i} = X_{\tau_i^-} + v_i, & i \geq 1, \end{cases} \tag{2}$$

where the first equation models the evolution of the aggregate viewer involvement level in the live streaming room between promotions, and the second equation denotes the jump in the aggregate involvement level after the host releases a promotion.

For a given strategy $w = \{(\tau_1, v_1), (\tau_2, v_2), \cdots\} \in W$, the expected discounted profit in the live streaming room at time t with aggregate viewer involvement level x is

$$J^w(t,x) = \mathbb{E}_x\left[\int_t^\infty e^{-rs} h(X_s) ds - \sum_{i:\tau_i \geq t} e^{-r\tau_i} c(v_i)\right]. \tag{3}$$

The objective of the live streaming host is to maximize his/her total expected profit. To achieve that, he/she needs to find an optimal strategy $w^* \in W$ such that

$$V(t,x) = J^{w^*}(t,x) = \max_{w \in W} J^w(t,x). \tag{4}$$

There are two trade-offs for the host when solving this optimization problem. The first trade-off concerns the promotion depth. Heavy promotions could potentially boost sales more significantly. However, it also comes with high costs. The other trade-off deals with the timing of the promotion. The effect of the promotion relies heavily on the number of viewers in the live streaming room, which constantly evolves over time. The host may want to release promotions at the beginning of a live streaming show. However, this may not be quite effective since the number of viewers in the live streaming room might not be large. Similarly, if the host releases the promotion at a later time, there is a chance that many viewers may have already left the live streaming room and the promotion will not be effective again. The main notations we use are summarized in the following Table 1.

Table 1 Notations

X_t	Aggregate viewer involvement level in the live streaming room at time t
μ	Trend of aggregate viewer involvement level
σ	Volatility of aggregate viewer involvement level
τ_i	Time of the ith promotion
v_i	Depth of the ith promotion
$c(v)$	Total cost of one promotion under the promotion depth v
r	Discount factor
$h(x)$	Host's revenue function with the form of $h(x) = \beta x^\alpha$, $0 < \alpha < 1$ and $\beta > 0$
A, B	Output parameters in the differential equation
D	Depth of increased involvement level under the promotion constraint

4 Optimal Solution of the Basic Model

In the live streaming room, the host aims to increase audiences' involvement level in order to boost sales using promotion tools such as limited-time coupons. In order to derive the closed-form solution for our analytical results, we first investigate the optimal promotion strategy under an infinite horizon. There are two reasons for us to start the analyses of the focal research questions with the infinite horizon. First, as the competition in live streaming e-commerce is becoming more and more fierce, live streaming platforms have been deploying their crew of hosts who work on shifts to provide non-stop live streaming shows online. Second, major Internet giants such as Tencent, NetEase, and ByteDance have moved one step further by developing their own virtual live streaming hosts (similar to virtual YouTubers) trained by large scale AI models (e.g., Large Language Models), that can provide non-stop live streaming broadcast (Xinhua 2020, Gu 2022).

4.1 Optimal Promotion Strategy

In this subsection, we consider a general setting where the live streaming host can arbitrarily choose the promotion depth. The cost of the promotion consists of two parts: a fixed cost and a marginal cost.[4] We adopt the following linear cost function $c(v) = c_0 + c_1 v$ with $v \geq 0$, where c_0 and c_1 represent the fixed and marginal cost of the promotion. We first discuss a special case where the trend of the aggregate viewer involvement level is high.

LEMMA 1. *When $\mu \geq \frac{r}{\alpha} + \frac{1-\alpha}{2}\sigma^2$, the expected discounted profit of the live streaming room becomes infinity if there is no promotion.*

The proof is provided in Section **??** of the Online Appendix. Lemma 1 shows that when the trend of aggregate viewer involvement level is high enough, the live streaming room can generate infinite profit

[4] We refer to this case as monetary promotion. In Section EC.14 of the Online Appendix, we discuss the case of non-monetary promotion.

without promotion, which is a trivial case we do not focus on in our study. Based on the results of Lemma 1, we focus on the case of $\mu < \frac{r}{\alpha} + \frac{1-\alpha}{2}\sigma^2$ in the remainder of this paper.

For notation simplicity, we denote $\gamma_2 := \left[\sqrt{(\sigma^2-2\mu)^2+8r\sigma^2}-(\sigma^2-2\mu)\right]/(2\sigma^2)$ and $\bar{c}_0 := (1-\alpha)\left(\frac{\alpha K}{c_1^\alpha}\right)^{\frac{1}{1-\alpha}}$. We first consider the case of $0 < c_0 \leq \bar{c}_0$. The case of $c_0 > \bar{c}_0$ is discussed in Corollary 1. We derive the following lemma to characterize thresholds and parameters (θ, Θ, A), which are used in Theorem 1 and Proposition 1.

LEMMA 2. *For $0 < c_0 \leq \bar{c}_0$ and $\mu < \frac{r}{\alpha} + \frac{1-\alpha}{2}\sigma^2$, there exists unique (θ, Θ, A) such that*

$$A\theta^{-\gamma_2} + H(\theta) = A\Theta^{-\gamma_2} + H(\Theta) - c_0 - c_1(\Theta - \theta) \tag{5}$$

$$-A\gamma_2\theta^{-\gamma_2-1} + H'(\theta) = c_1 \tag{6}$$

$$-A\gamma_2\Theta^{-\gamma_2-1} + H'(\Theta) = c_1. \tag{7}$$

The proof is provided in Section **??** of the Online Appendix. Based on the results of Lemma 2, we are able to provide the optimal solution for our optimization problem (4) in the following theorem, where (θ) and (Θ) represent the start-to-promote threshold and the aggregate viewer involvement target after the promotion, respectively. Basically, Lemma 2 proves the existence and uniqueness of the thresholds (θ) and (Θ), as well as the output parameter A that are used in Theorem 1. Note that the thresholds (θ, Θ) characterized in Lemma 2 depend on $(\mu, \sigma, r, c_0, c_1)$.

THEOREM 1. *Under the conditions in Lemma 2, the value function is*

$$V(t,x) = e^{-rt}\begin{cases} Ax^{-\gamma_2} + H(x) & \text{for } x > \theta \text{ (waiting region)} \\ A\Theta^{-\gamma_2} + H(\Theta) - c_0 - c_1(\Theta - x) & \text{for } x \leq \theta \text{ (action region)} \end{cases}, \tag{8}$$

where θ, Θ, A are determined in Lemma 2.

The proof of Theorem 1, along with the following proposition (Proposition 1), is provided in Section **??** of the Online Appendix. Based on the structure of the value function in Theorem 1, we elaborate on the characteristics of the optimal promotion strategy in Proposition 1.

PROPOSITION 1. *With the conditions in Lemma 2, under the infinite horizon, the optimal promotion policy is to delay the promotion until the real-time aggregate involvement level (x) decreases below the start-to-promote threshold θ and then release the promotion with the cost of $c_0 + c_1(\Theta - x)$, which increases the involvement level to Θ.*

We refer to the (θ, Θ) policy in Proposition 1 as the *threshold policy*, which we have proved as the optimal policy among all classes of policies. Intuitively, one may expect that the live streaming host would like to promote only when the aggregate involvement level is low and the promotion policy might be quite complex in order to maximize profits. For example, the preroll policy allows the streaming host to release promotions only at the beginning of the broadcast, while the midroll policy enables the host to promote once halfway through the broadcast. In comparison, with a random policy, the host can promote at any nonspecific time point throughout the broadcast. However, we prove mathematically that a (θ, Θ) policy is the optimal among all classes of policies. Essentially, the host needs only to release promotions to lift the real-time aggregate involvement level (x) up to Θ with the cost of $c_0 + c_1(\Theta - x)$ once the aggregate involvement level falls below θ.

The parameters μ, σ, r, c_0, c_1 that are used to characterize the thresholds can be different for different live streaming rooms, depending on factors such as types of products, host's influence, promotion types and broadcast time, etc. The parameters μ, σ, r, which rely on the live streaming room and types of products, can be estimated from historical live streaming data. The parameters c_0 and c_1 are cost-related parameters that depend on the type of promotions provided by the streaming host.

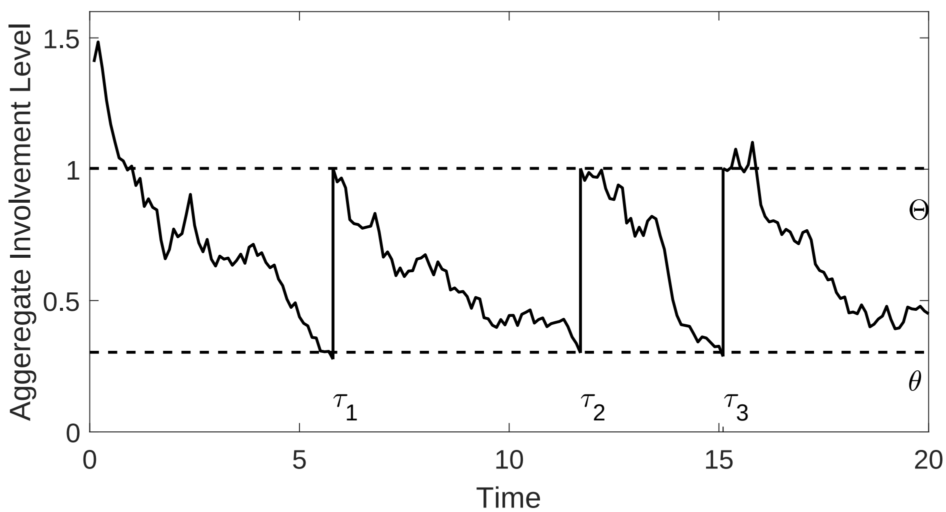

Figure 2 Illustration of the Threshold Policy

We illustrate the optimal *threshold policy* in Figure 2. In this example, viewers have a negative attitude toward the live streaming show which is reflected by a downward trend in viewer involvement level. This could be due to limited interactions in the live streaming room during that time period or a mismatch between the endorsed products and viewer expectations. To maximize the expected profit, the streaming host releases promotions whenever the aggregate involvement level is below the start-to-promote threshold. As shown in Figure 2, three promotions are released to lift the aggregate viewer involvement level in the live streaming room. Note that if the initial aggregate viewer involvement level is sufficiently low (i.e., $X_0 < \theta$), our policy suggests that the host should immediately release the promotion. This recommendation is consistent with the current practice of many live streaming hosts who always try to increase aggregate viewer involvement level at the very beginning of the broadcast. Alternatively, when the initial aggregate involvement level is above θ, the live streaming hosts should choose not to release the promotion until the viewer involvement level drops below θ. Based on these findings, the current practice of live streaming hosts always releasing promotions at specific time points regardless of the aggregate viewer involvement level might be a sub-optimal strategy.

The above discussion demonstrates that the (θ, Θ) policy is optimal among all feasible policies under the infinite horizon when the fixed promotion cost is low, i.e., $c_0 < \bar{c}_0$. In order to make our analyses of the model complete, we also investigate the case where the fixed promotion cost is high ($c_0 \geq \bar{c}_0$). The result is presented in the following corollary.

COROLLARY 1. *For $c_0 \geq \bar{c}_0$ and $\mu < \frac{r}{\alpha} + \frac{1-\alpha}{2}\sigma^2$, it is optimal for the host not to release any promotions under the infinite horizon.*

The proof is provided in Section **??** of the Online Appendix. Corollary 1 indicates that when the fixed cost is high ($c_0 \geq \bar{c}_0$), it is not worth increasing the viewer involvement level through promotions. The optimal strategy for the live streaming host is actually not to release any promotion.

4.2 Comparative Static Analysis

In this subsection, we provide several managerial implications by investigating the impact of different parameters on the optimal promotion policy of the live streaming room. Specifically, we derive the impact of the fixed promotion cost (i.e., c_0) and the marginal promotion cost (i.e., c_1) analytically as well as that of the drift (i.e., μ) and volatility (i.e., σ) numerically. The results are presented in the following subsections.

4.2.1 Fixed Promotion Cost

The parameter c_0 represents the fixed promotion cost. Our analysis demonstrates how this fixed cost affects the optimal promotion policy. We find that as the fixed cost c_0 increases, the start-to-promote threshold θ decreases while the involvement target Θ increases. Hence the promotion depth $(\Theta - \theta)$ increases. We summarize this finding in Proposition 2 and further illustrate that in Figure 3.

PROPOSITION 2. *As the fixed cost c_0 increases, the start-to-promote threshold θ decreases while the involvement level Θ increases, and the optimal promotion depth increases.*

The proof is provided in Section **??** of the Online Appendix. This result is in line with our initial hypothesis. When the fixed promotion cost increases, the live streaming host bides her time when the aggregate viewer involvement level in the live streaming room decreases and releases a heavy promotion only when the involvement level drops to a certain level. The reason is that it becomes costly for the live streaming host to release promotions of any depth when the fixed promotion cost c_0 increases. Therefore, rather than releasing a few promotions of medium depth, which comes with a fixed cost for every promotion, the live streaming host would wait until the involvement level in the live streaming room becomes low enough, then release a heavy promotion to boost the level of aggregate viewer involvement to a high target.

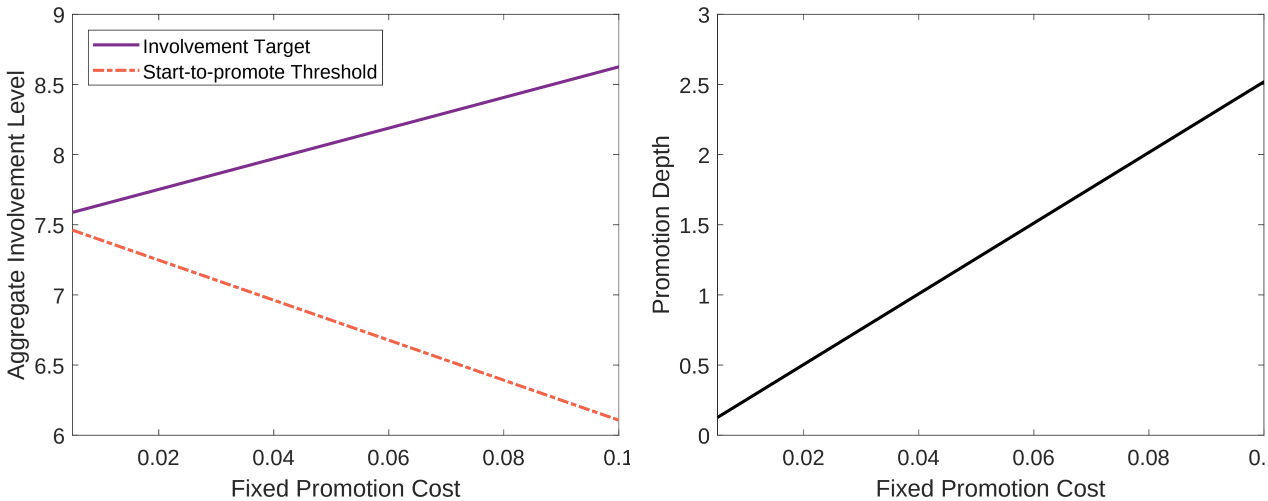

Figure 3 Comparative Statics of Fixed Cost under the Threshold Policy

4.2.2 Unit Promotion Cost

The parameter c_1 represents the unit promotion cost. Our analysis also demonstrates how the unit promotion cost affects the optimal promotion policy. Similar to our previous discussion with the fixed cost c_0, we find that as the unit promotion cost c_1 increases, the start-to-promote threshold θ decreases. However, the involvement target Θ decreases as well under this circumstance. We summarize the impact of the unit promotion cost c_1 in Proposition 3.

PROPOSITION 3. *As the unit promotion cost c_1 increases, both the start-to-promote threshold θ and the involvement target Θ decrease, and the optimal promotion level decreases as well.*

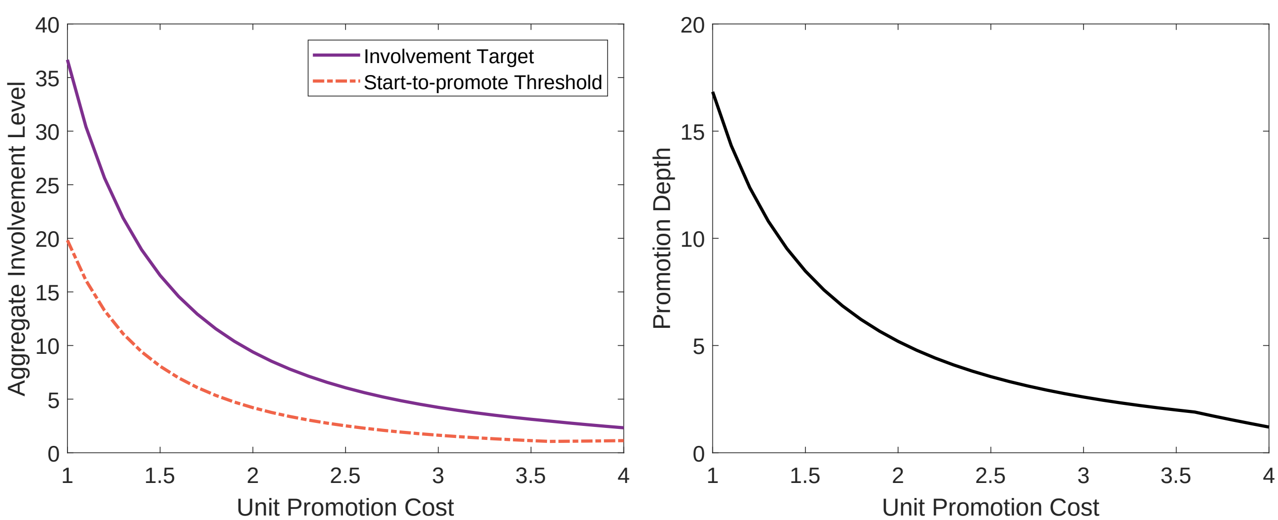

Figure 4 Comparative Statics of the Unit Promotion Cost under the Threshold Policy

The proof of Proposition 3 is provided in Section **??** of the Online Appendix. Figure 4 below demonstrates how the unit cost c_1 affects the optimal promotion insertion policy. As the unit cost c_1 increases, it becomes more expensive for the live streaming host to release promotions. Thus, the live streaming host exhibits more patience with the decrease of the aggregate viewer involvement level in the live streaming room and releases promotions when viewer involvement drops to a lower level. Since the unit promotion cost is high, it would be expensive to release heavy promotions. Rather, the optimal strategy for the live streaming host is to target a lower aggregate viewer involvement level with promotions of a lower depth. Propositions 2 and 3 provide practical managerial insights for the live streaming industry. Due to the fact that the fixed and unit cost of promotions for different live streaming rooms vary a lot, live streaming managers should prepare promotion coupons with different nominal values so that the streaming host can use accordingly.

4.2.3 Drift

The parameter μ represents the expected trend of the aggregate viewer involvement level in the live streaming room. In this subsection, we analyze the comparative statics of the start-to-promote threshold and involvement target with respect to both negative and positive drift terms. This suggests that the overall trend of the aggregate viewer involvement level in the live streaming room can either decrease or increase over time without any intervention. Our numerical analyses provide the following result.

Observation 1: As the drift increases, the start-to-promote threshold, the aggregate viewer involvement target, and the optimal promotion depth increase.

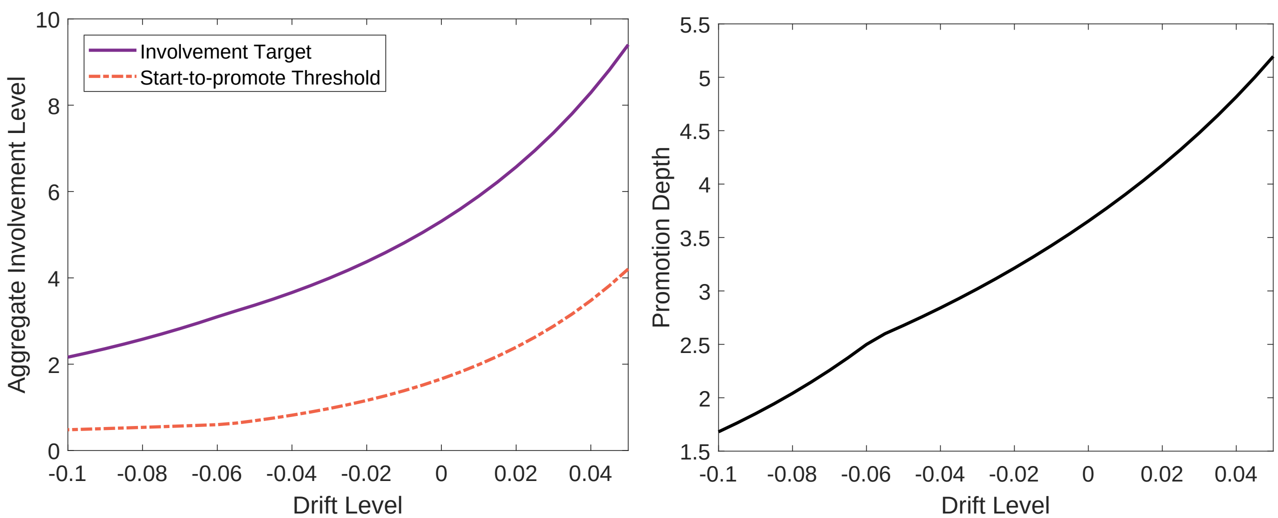

Figure 5 Comparative Statics of Drift under the Threshold Policy

We show the effect of the drift term on the optimal promotion strategy in Figure 5. As the drift term increases, the trend of the aggregate viewer involvement level in the live streaming room evolves upwards, approaching a flat line when the drift term gets closer to 0. Thus, it becomes less likely for the aggregate viewer involvement level to hit a low start-to-promote threshold. In order to maintain the aggregate viewer involvement on a high level, it is optimal for the live streaming host to set a high start-to-promote threshold and release heavier promotions when the drift gets larger.

4.2.4 Volatility

Recall that we use the parameter σ to denote the volatility of the aggregate viewer involvement level in a live streaming room. By conducting extensive numerical analyses, we have the following observation.

Observation 2: When the volatility increases, both the start-to-promote threshold and the involvement target decrease. Correspondingly, the optimal promotion depth first increases and then decreases in volatility.

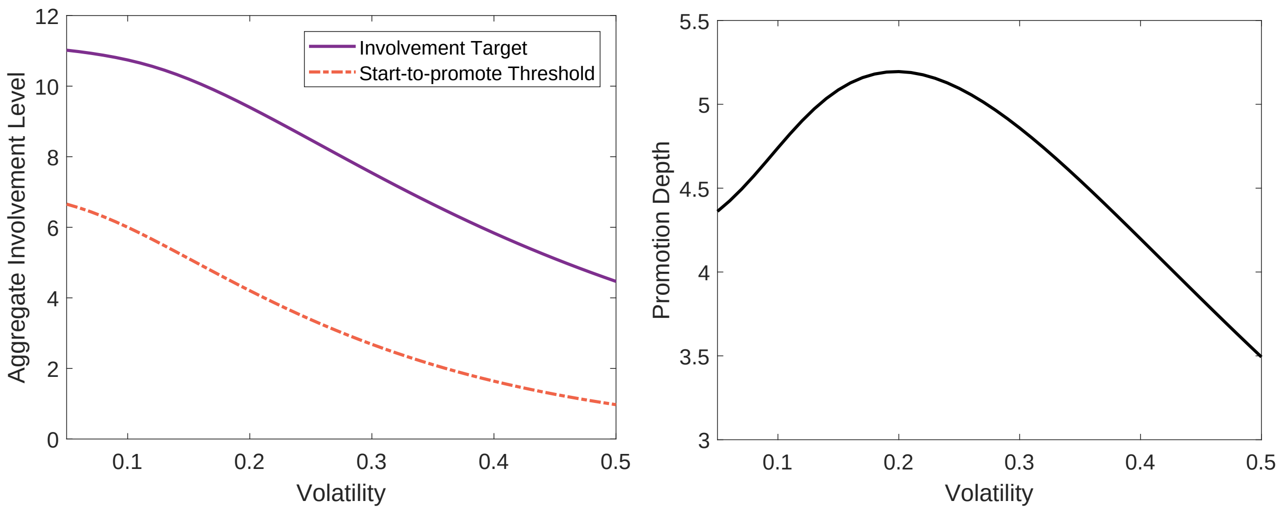

Figure 6 Comparative Statics of Volatility under the Threshold Policy

We illustrate how the volatility affects the optimal promotion policy in Figure 6. The left figure shows that the optimal start-to-promote threshold decreases in volatility. The intuition is that with larger volatility, the aggregate viewer involvement level fluctuates more violently. The chance of reaching the start-to-promote threshold becomes larger, thus, the live streaming host is more likely to release a promotion. In order to circumvent the high cost of releasing promotions with excessive frequency, the live streaming host chooses to adjust the start-to-promote threshold downward. The righthand figure suggests that the optimal promotion depth first increases and then decreases in volatility. The reason is that when the volatility is low, the aggregate viewer involvement level evolves less violently. Thus, the chance of reaching the start-to-promote threshold and releasing promotions is relatively lower. The live streaming host can actually afford promotions with higher depth to boost the aggregate viewer involvement to a higher level and generate more revenue. Yet, when the volatility is high, the chance of reaching the start-to-promote threshold and releasing promotions becomes high. Even with a reduced start-to-promote threshold, the live streaming host could only afford promotions with lower depth. Our Observation 2 suggests that as the volatility increases, the optimal choice for the host is a trade-off between the start-to-promote threshold and the promotion depth.

5 Extensions

In this section, we extend the discussion of our basic model in two directions. In Section 5.1, we consider the case where the promotion depth is integral multiples of certain value rather than continuous real numbers, which is the prevailing industry practice. In Section 5.2, we study two scenarios where the planning horizon is treated as finite. We first investigate the scenario in which viewers leave the live streaming room and cannot make any purchases once the broadcast ends. Following that, we then study the scenario where viewers can still stay online and purchase products after the broadcast is over.

5.1 Optimal Policy with Promotion Constraint

Our basic model focuses on the case where nominal values of coupons are arbitrarily chosen. In other words, the live streaming host could release promotions of any depth. However, under some circumstances, the promotion planning process is challenging and constrained by many practical business rules. For example, in business practice, a substantial number of promotions are in the form of time-limited one-off coupons taking values of integral multiples of certain value. Therefore, the promotion depth is chosen from a discrete set, which makes the promotion planning problem more complex and motivates us to investigate the same problem with the constraint of this business rule. In this subsection, we assume that the streaming host can only release promotions with a depth of integral multiples of R/c, where c is the marginal cost of increasing the aggregate involvement level. For notation brevity, we denote $D := R/c$. Following our settings in the basic model, the aggregate viewer involvement level is increased by kD if k such coupons are released. Let

$$\rho(x,y) := \min\{k \in \mathbb{Z}^+ : x + kD > y\} = \left\lfloor \frac{y-x}{D} \right\rfloor + 1,$$

which is the least number of coupons needed to increase the real-time involvement level from x to above y. Therefore, the corresponding promotion cost is $\rho(x,y)cD$. We have the following result to characterize the parameter P, which is further used to illustrate the optimal policy.

LEMMA 3. *For $0 < c < KD^{\alpha-1}$ and $\mu < \frac{r}{\alpha} + \frac{1-\alpha}{2}\sigma^2$, there exists a unique pair of (B,P) such that*

$$BP^{-\gamma_2} + H(P) = B(P+D)^{-\gamma_2} + H(P+D) - cD$$
$$-B\gamma_2 P^{-\gamma_2-1} + H'(P) = -B\gamma_2 (P+D)^{-\gamma_2-1} + H'(P+D).$$

The proof is provided in EC.7 of the Online Appendix. Lemma 3 characterizes the parameters (B,P) which are used in Theorem 2 and Proposition 4. It is worth mentioning that the parameters (B,P) in Lemma 3 are uniquely determined and are functions of the parameters (μ,σ,r,c,D). In the following theorem, we provide the optimal solution under the constraint of the promotion depth being integral multiples of certain value in the infinite planning horizon.

THEOREM 2. *Under the conditions in Lemma 3, the optimal profit function of the live streaming platform is*

$$V(t,x) = e^{-rt} \begin{cases} Bx^{-\gamma_2} + H(x) & \text{for } x > P \text{ (waiting region)} \\ B(x+\rho(x,P)D)^{-\gamma_2} + H(x+\rho(x,P)D) - c\rho(x,P)D & \text{for } x \leq P \text{ (action region),} \end{cases} \tag{9}$$

where B and P are defined in Lemma 3.

The proof of Theorem 2 is provided in EC.8 of the Online Appendix. Based on Theorem 2, we have the optimal policy under the constraint of the promotion depth being integral multiples of certain value in the following proposition.

PROPOSITION 4. *Under the conditions in Lemma 3, when promotion depth is integral multiples of value D, the streaming host's optimal policy under the infinite horizon is to delay the promotion until the real-time aggregate viewer involvement level in the live streaming room decreases below the start-to-promote threshold P and then release the least number of coupons to bring the viewer involvement level above P.*

The proof of Proposition 4 is provided in EC.8 of the Online Appendix. Proposition 4 shows that the (P,D) policy is still optimal among all feasible policies under the constraint of promotions being integral multiples of certain value. It is optimal for the streaming host to release the least number of coupons to increase the aggregate viewer involvement level above the threshold of P when the involvement level decreases below P. Particularly, when the real-time involvement level x is below P, the optimal promotion depth is $\rho(x,P)D$. Our analyses focus on the case that the nominal values of coupons are fixed, which is similar to the inventory management problem where the ordering quantity can only be placed by a fixed amount. In this case, the streaming host can only decide when to release promotions, which makes our study of this case pivotal. Our optimal promotion policy provides direct guidelines for the live streaming host's decision-making regarding the promotion timing. In order to make our analyses of the model complete, we also investigate the case where the unit promotion cost is high ($c \geq KD^{\alpha-1}$). The result is presented in the following corollary.

COROLLARY 2. *For $c \geq KD^{\alpha-1}$ and $\mu < \frac{r}{\alpha} + \frac{1-\alpha}{2}\sigma^2$, it is optimal for the platform not to release any promotion under the infinite horizon.*

The proof of Corollary 2 is provided in EC.9 of the Online Appendix. Corollary 2 shows that when the unit promotion cost is high, the optimal strategy for the streaming host is not to release any promotion.

5.2 Finite Horizon

Section 4 discusses the infinite horizon case in order to derive analytical solutions and generate relevant managerial insights, which can be extended to more general settings. However, the non-stop live streaming shows provided by virtual live streaming hosts are still in a nascent stage. There are still many live streaming broadcasts that are hosted within a limited duration of time. Thus, in this subsection, we investigate this scenario where the planning horizon is treated as finite. We present the discussion of the finite horizon in two cases. First, we consider the scenario where no further revenue is generated after the broadcast ends (Case 1). In the business practice of live streaming e-commerce, the platform usually provides consumers with the option of purchasing the endorsed products even after the broadcast is over. For example, the host can retain the purchase links on the platform such that viewers can click on and place their orders even after the live streaming show. Therefore, we further examine the second case where the platform can gain profit from viewers' orders placed after the broadcast is over (Case 2). We consider the case that the profit of the live streaming room after the broadcast ends is proportional to the power form of its involvement level at time T, that is λX_T^{δ} for $\delta > 0$. Thus, the expected discounted profit can be written as

$$\mathbb{E}_x\left[\int_0^T e^{-rs}h(X_s)ds - \sum_{i:\tau_i \leq T} e^{-r\tau_i}c(v_i) + e^{-rT}\lambda X_T^{\delta}\right], \tag{10}$$

where the first two terms are similar to those in Equation (3).

Both the start-to-promote threshold and involvement target under the finite horizon is time-varying. They converge to those under the infinite horizon after sufficiently long live streaming time. Since the closed-form solution to impulse control problems under the finite horizon can be hardly obtained (Elliott and Filinkov 2007, Björk 2009, Feng and Muthuraman 2010), we employ numerical analyses to further investigate the focal research questions under this setting and we have the following two observations.

Observation 3: In the finite horizon, when the purchase links are not accessible after the broadcast ends, both the optimal start-to-promote threshold and the involvement target decrease as time goes by and finally approach zero.

Note that under the infinite horizon, the start-to-promote threshold and the involvement target do not depend on the time. However, both change with time under a finite horizon. The left panel of Figure 7 illustrates how the start-to-promote threshold $\theta(t)$ and the involvement target $\Theta(t)$ evolve with time when the purchase links are not accessible after the broadcast ends. We can observe that both the optimal start-to-promote threshold and the involvement target decrease in time and become zero when it gets close to the end of the broadcast. This observation is robust under different parameter settings and the finding is in line with our expectations. At the beginning of the planning horizon, the live streaming host's optimal policy is similar to that under the infinite horizon since there is plenty of time left. However, under the focal scenario of a finite planning horizon, no more profits could be gained once the live streaming room is closed. Therefore, when approaching the end of the live streaming show, the live streaming host's incentive to increase viewers' involvement level gradually decreases. Consequently, the start-to-promote threshold and the involvement target diminish to zero when it gets close to the end of the broadcast.

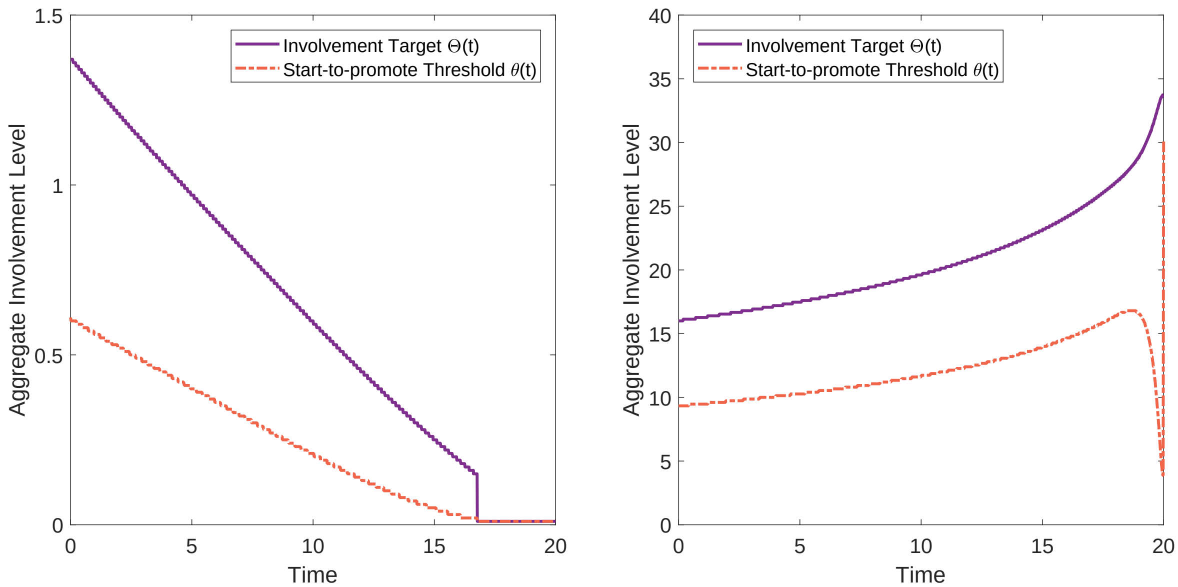

Figure 7 Optimal Thresholds and Targets in the Finite Horizon under Case 1 (Left) and Case 2 (Right)

Observation 4: In the finite horizon, when the purchase links are accessible after the broadcast ends, the optimal start-to-promote threshold first increases, then decreases, and finally increases as the live streaming show approaches the end.

The right panel of Figure 7 illustrates how the start-to-promote threshold $\theta(t)$ and the involvement target $\Theta(t)$ evolve when the purchase links are accessible after the broadcast ends. As it gets close to the end of the broadcast, one might expect that the host may become more eager to release promotions (i.e., increased optimal start-to-promote threshold) because a high viewer involvement level at the end of the broadcast can generate high post live streaming revenue and the time left for releasing any further promotions is diminishing. However, our findings reveal that the start-to-promote threshold actually first decreases and then increases when it approaches the end of the broadcast, suggesting that the host should consider the trade-off between promotion time and depth throughout the broadcast. At the beginning of the planning horizon, there is plenty of time for the host to release multiple promotions of modest depth to increase the aggregate viewer involvement level to a specific target. However, when it approaches the end of the live streaming show, the host would be concerned that less time is left to release promotions. Therefore, the host would prefer to release promotions with larger depth, depicted by the increase in the difference between the optimal involvement target and start-to-promote threshold. When the broadcast gets to the end, the start-to-promote threshold is increased again because the host would like to boost viewers' involvement level by releasing promotions at the last minute to earn the post live streaming profit.

To verify our hunch, we further investigate the case where the host is restricted to releasing no more than n promotions when the live streaming time is finite. The optimal start-to-promote thresholds $\theta_n(t)$ when $n = 1$ and 2 are shown in Figure 8. We can observe that the start-to-promote threshold always increases

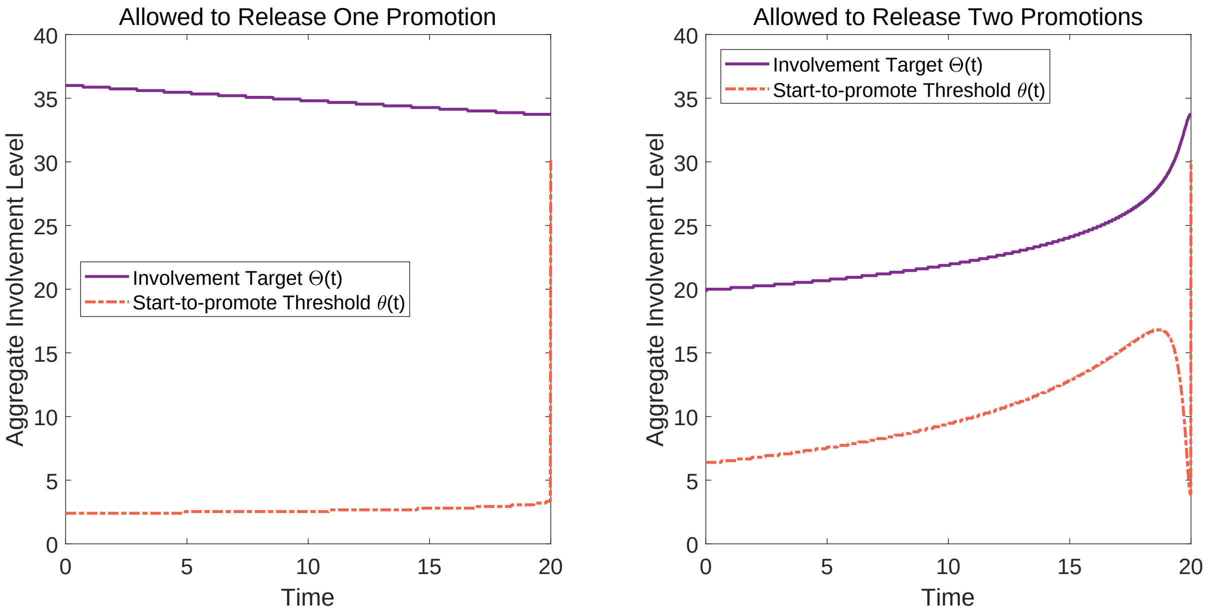

Figure 8 Optimal Thresholds and Targets in the Finite Horizon with Restriction on the Number of Promotions

when the host is constrained to promote only once during the broadcast (the left panel of Figure 8). This result is consistent with our intuition that the host becomes less patient when the broadcast gets close to the end and the time left to release this one promotion rapidly decreases. When the host could make more than one promotion ($n = 2$ in the right panel of Figure 8), we can observe that the optimal start-to-promote threshold shows a similar pattern as Figure 7. Specifically, the optimal start-to-promote threshold first increases, then decreases, and lastly increases. This interesting observation yields practical insights for live streaming platforms with respect to the optimal quantity and depth of promotions under the finite horizon. The optimal start-to-promote threshold is actually affected by the trade-off between the quantity and depth of promotions.

6 Comparison with Other Strategies

In this section, we discuss the effectiveness of our optimal promotion strategy by comparing it with other strategies adopted on the market through numerical analyses. Specifically, we compare our proposed strategy with four industry standard practices.

In practice, the live streaming host usually adopts the following four industry-prevailing promotion strategies. Firstly, the host only releases the promotion at the beginning of the broadcast (the preroll strategy). Secondly, the host promotes once halfway through the broadcast (the midroll strategy). Thirdly, the host promotes once at a random time point during the broadcast (the random strategy). Lastly, the host mixes the preroll and midroll strategies and promotes twice throughout the broadcast (the mix strategy). Here, we consider an alternative strategy where we utilize the start-to-promote threshold θ and the involvement target Θ obtained in the infinite planning horizon as approximations of those under the finite planning horizon, which facilitates the use of the closed-form solution shown in Lemma 2. We refer to this strategy as the steady state strategy. This strategy might not be equally optimal when compared with the dynamic thresholds under a finite horizon. However, the computation workload is largely reduced since the thresholds need to be calculated only once using results from Lemma 2. Next, we would like to compare the performance of this heuristic strategy with other prevailing strategies.

In the setting of our simulation study, the initial involvement level X_0 is assumed to follow a uniform distribution within the interval $[0,4]$, i.e., $X_0 \sim U(0,4)$. We present one illustrative case with the setting of the following parameters: $\{r = 0., \mu = 0.01, \sigma = 0.2, \alpha = 0.2, \beta = 1, c_0 = 0.5, c_1 = 3\}$. We use 500 replications with each simulated for 10 hours and the involvement level is updated three times every minute. We calculate the profitability for each policy from Equation (3) and summarize the results in Table 2.

Table 2 Simulation Results of Profitability for Different Promotion Strategies

Promotion Depth	Preroll	Midroll	Random	Mix2	Steady State	Optimal
1.0	74.45% (1)	74.87% (1)	74.60% (1)	78.53% (2)	95.29% (1.97)	100% (3.31)
1.5	76.54% (1)	76.28% (1)	76.75% (1)	80.77% (2)	95.10% (2.11)	100% (3.45)
2.0	78.41% (1)	78.29% (1)	78.26% (1)	83.12% (2)	95.13% (2.06)	100% (3.38)
2.5	79.32% (1)	79.17% (1)	79.38% (1)	85.32% (2)	95.38% (2.04)	100% (3.39)
3.0	81.37% (1)	81.26% (1)	80.75% (1)	87.41% (2)	95.69% (2.01)	100% (3.24)
3.5	81.80% (1)	81.81% (1)	81.81% (1)	88.21% (2)	95.46% (1.95)	100% (3.23)
4.0	82.07% (1)	82.93% (1)	83.33% (1)	88.39% (2)	95.04% (1.99)	100% (3.32)
4.5	83.59% (1)	83.81% (1)	84.14% (1)	88.88% (2)	95.33% (2.07)	100% (3.39)

From Table 2 we can observe that our proposed policy produces the highest profit compared with other industry policies. For example, when the promotion depth is 4.0, the Mix2 promotion strategy outperforms other industry standard strategies. However, the profit is only 88.39% of what is yielded by our optimal policy. This result is robust under a wide range of parameter values. It demonstrates that our proposed policy with appropriate promotion timing and depth significantly increases the expected profit of the live streaming room.

The start-to-promote threshold and the involvement target under infinite horizon are both fixed and can be derived from Lemma 2. Hence, the steady state strategy is computationally efficient compared with the optimal strategy under a finite horizon. From Table 2 we can observe that the steady state strategy outperforms other industry strategies, attaining a profit of nearly 95% of the optimal strategy. Therefore, our steady state strategy can serve as an effective approximation for the optimal promotion strategy in practice.

7 Conclusions

Live streaming e-commerce as a new type of e-commerce business model has been growing vigorously and is extremely successful at boosting online sales. In this study, we investigate the optimal promotion insertion policy to improve viewer involvement level in the live streaming room, which is one of the most significant challenges for live streaming e-commerce. A geometric Brownian motion is used to model the evolvement of the aggregate viewer involvement level. We start our analyses with an infinite horizon model to capture the structure of the optimal solution and demonstrate that a threshold strategy is optimal among all feasible strategies. This strategy requires the live streaming host to wait until the aggregate viewer involvement level drops to a start-to-promote threshold and then release the promotion to lift it up to a specific involvement target. We further demonstrate that this threshold strategy is still optimal under the constraint of prevailing business rules, which requires the promotion depth to be integral multiples of a certain value. We also conduct comparative statics analysis to investigate how this optimal policy is affected by different parameters, such as the fixed and unit cost of promotions, as well as the volatility and drift of the aggregate viewer involvement level. We then examine the finite horizon problem and find that it is still optimal to have a $(\theta(t), \Theta(t))$ policy with the start-to-promote threshold $(\theta(t))$ and the involvement target $(\Theta(t))$ changing over time. Lastly, we compare the performance of the optimal strategy we propose with current industry practices through extensive numerical analyses.

Our study provides several important theoretical findings. First, we analytically show that a (θ, Θ) strategy is optimal among all feasible strategies although the problem involves complex stochastic processes. The simple structure of this optimal solution provides great convenience when implementing the policy. Additionally, regarding the current business practice of releasing promotions with the depth of integral multiples of certain value, we show that our (P, D) threshold policy is still optimal under this case.

Our work also provides relevant managerial insights for prevalent business practitioners of live streaming e-commerce. First, the promotion coupons should be released when the viewer involvement level drops to some threshold value rather than at some predetermined time points. Current industry practice is to release promotion coupons at some specific time point of the broadcast or randomly, while our analyses show that the proposed policy outperforms all prevalent ones significantly. Furthermore, when promotions are released in batches of time-limited one-off coupons taking values of multiples of integers, our proposed promotion policy is still optimal, i.e., the live streaming host should delay the promotion until the real-time involvement level in the live streaming room decreases below a threshold and then release the least number of coupons to bring the involvement level above another threshold. Our results also indicate that the optimal promotion policy should be adjusted according to different market conditions, such as the fixed and unit cost of promotion, as well as the drift and volatility of viewer involvement level. Finally, under the finite horizon, the optimal promotion policy is actually affected by the trade-off between the quantity and depth of promotions and should be adjusted accordingly.

We briefly discuss the limitations of our work and provide several ideas for future work. First, we use the power function to model the revenue in the live streaming room, which is a rational assumption that facilitates the derivation of the host's optimal promotion policy. This assumption can be further verified when empirical data is available. Second, we assume that the promotion cost is a linear function of promotion depth, which simplifies the model to derive an analytical solution. Future research could investigate the same problem with nonlinear functions for promotion cost. Third, we assume away the fixed cost in our analysis of the optimal policy with promotion constraint in the extension due to the complexity of the problem. Future research could relax this assumption using numerical analysis and investigate the host's optimal decision under this setting. Furthermore, the effects of promotions are modeled in a highly stylized manner, which assumes away the difference between viewers attracted by promotions and organic traffic, i.e., those who join the live streaming organically. This paves an avenue for future research to investigate the behavior of these two types of traffic and provide a more nuanced exploration of promotional strategies. Lastly, future research can also account for unexpected shocks (jumps) during the live streaming process, which we do not consider in this work. Notwithstanding these limitations, our study presents an antecedent attempt to examine the optimal promotion insertion policy (including the promotion timing and depth) for live streaming e-commerce platforms.

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