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RESEARCH ARTICLE

Heterogeneous early health signals and deferred annuities

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Abstract

According to an influential study on uncertainty resolution, offering deferred annuities to annuitants with identical early health signal eliminates adverse selection by completely crowding out the immediate annuity market. However, identical early health signal may not be an appropriate assumption when deferred annuities are offered to annuitants at their prime working years. We provide a more general specification of health information revelation process, in which annuitants' early health signals are heterogeneous and have some, but not full, predictive power about future survival probabilities. It is shown that both deferred and immediate annuity transactions coexist in this case. Using the general framework which nests identical early signal as a limiting case, we explain and reconcile both sets of results.

Keywords Deferred annuity · Immediate annuity · Adverse selection · Heterogeneous early health signals

JEL Classification G52 · D80

1 Introduction

In recent decades, many countries (especially developing countries) have adopted the defined-contribution retirement income system. While this kind of system may better

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avoid the financial unsustainability problem typically associated with the defined-benefit pay-as-you-go system, one disadvantage is that the burden of longevity risk (that is, the risk of outliving financial resources when a person lives longer than expected) is shifted from the government to the senior citizens. Insuring against longevity risk can in principle be achieved through the financial market (as in Yaari (1965); Davidoff et al. (2005); Jang et al. (2016)), but in practice, few people purchase annuities from the private market.[1]

One reason of low annuity purchase is adverse selection, which can be traced back to the literature of asymmetric information applied to the insurance market (Rothschild and Stiglitz 1976). The impact of adverse selection in the annuity market has been found in many studies (such as Friedman and Warshawsky 1990; Finkelstein and Poterba 2002). To the extent that adverse selection is a major factor of the low demand for annuities (particularly the immediate annuities), offering deferred annuity at an early age has been suggested as a possible solution,[2] because it is generally believed that annuitants do not show big differences in health characteristics when they are young. Brugiavini (1993) is a pioneer study on this issue. In the main model in which there is only survival probability uncertainty, she shows that adverse selection is present when only the immediate annuity is available at a later age corresponding roughly to retirement age, but offering the deferred annuity at an early age when annuitants are identical in health condition eliminates adverse selection. This contribution is widely recognized by many researchers, including Finkelstein and Poterba (2002), Sheshinski (2007), Steinorth (2012), Reichling and Smetters (2015) and d'Albis and Kalk (2021). However, a strong implication of her results is that the immediate annuity market is completely crowded out, which is inconsistent with empirical observation. Brugiavini (1993, p. 45) is aware of this feature, and shows in the second part of the paper that transactions in immediate annuities are present when there is an additional factor of labor income uncertainty.

This paper studies the provision of deferred annuities, with the emphasis on public deferred annuities in a defined-contribution individual-account retirement income system. This question becomes more relevant as many countries adopting this system have shifted to focus on the wealth decumulation phase in recent years, after mainly focusing on the wealth accumulation phase when the retirement income system was first set up. For example, Singapore established the Central Provident Fund system in the 1950s to promoting savings for retirement, but only introduced the Lifelong Income for the Elderly program in 2009 (Chia 2016).

A distinguishing feature in this paper is the use of a more realistic assumption about the heterogeneity of early health signals, which is different from several well-known papers (such as Brugiavini 1993; Sheshinski 2007). With the focus on the

[1] This phenomenon is observed in many countries, and various factors (such as bequest motive, intra-family mortality risk sharing, and high annuity price due to adverse selection) have been suggested to explain this so-called annuity puzzle (Modigliani 1986; Benartzi et al. 2011).

[2] The deferred annuity is a financial product that is "purchased today but does not pay until the annuitant survives to a pre-specified age" (Chen et al. 2020, p. 373). Two major types of deferred annuities, namely the working-age deferred annuity and retirement-age deferred annuity (also known as advanced-age deferred annuity), have been discussed in the literature (Milevsky 2005; Brown 2008; Chen et al. 2020). The relative merits of these two types of deferred annuities are likely to be different. Given our focus on heterogenous early health signals, we only examine the first type in this paper.

effect of uncertainty resolution on the timing of annuity provision, Brugiavini (1993) uses a three-period model and makes the natural assumption of identical early health signal for all annuitants (in the beginning period). Since many working-age deferred annuities are offered when the annuitants are at their prime working years (perhaps around aged 45 to 55), they probably have some private information regarding their health risks, including the survival probability to an advanced age. We think that it is more appropriate to assume annuitants' early health signals are heterogeneous when the working-age deferred annuities are offered. Steinorth (2012) has also mentioned a similar point about heterogeneous early signals. Based on empirical evidence that people learn about their health information gradually over time (Smith et al. 2001; Hurd and McGarry 2002), we propose a more general health information revelation process in which the early health signals are heterogeneous and have predictable power about survival probabilities in a future period.

Using the proposed information revelation process, we show that the annuitants purchase the deferred annuity at an earlier age, and then only those with sufficiently high survival probabilities buy the immediate annuity at a later age. In particular, we prove that the annuitants with the lowest value of early health signal but highest value of subsequent survival probability always have the incentive to buy immediate annuities. Unlike the results in Brugiavini (1993) and Sheshinski (2007), the immediate annuity market is only incompletely crowded out when the annuitants' early health signals are heterogeneous.

A useful by-product of the more general specification of health information revelation in this paper is that it nests the specification in Brugiavini (1993) as a limiting case. Moreover, our specification suggests a useful interpretation regarding the revelation of health information of these two models: the early health signals are partially informative to the annuitants in our model, but totally uninformative in hers. Since the specification in Brugiavini (1993) is a limiting case of our proposed model, it is puzzling why both deferred and immediate annuity transactions coexist in our specification, but the immediate annuity market is completely crowded out in her model. After a careful investigation of the factors causing different results in these two papers, we find that the result of complete crowding out of the immediate annuity market after the introduction of deferred annuities (Brugiavini 1993) disappears even when the annuitants' early health signals are slightly diverse. The crucial factor leading to the complete crowding out of the immediate annuity market in her model with uncertainty in survival probability only is that the annuitants' early health signals are identical.

The remaining sections are organized as follows. In Section 2, we briefly provide the literature review and link our paper to existing studies. In Section 3, we present a simple three-period model with a more general process of health information revelation. In Section 4 we prove the coexistence of deferred and immediate annuity transactions, and compare our results with those in Brugiavini (1993). We conclude in the last section.

2 Related literature

The starting point of our analysis is that in the presence of asymmetric information in the insurance market (such as the immediate annuity market), higher-risk buyers purchase larger amounts of the product, leading to efficiency loss (Rothschild and Stiglitz 1976). The canonical solution to eliminate the inefficiency of adverse selection is "to mandate that everyone purchase insurance" (Einav and Finkelstein 2011, p. 120). However, mandatory policies restricting citizens' choice may not be popular. Moreover, some researchers, such as Feigenbaum et al. (2013) and Heijdra et al. (2014), have suggested that mandatory annuitization policies may also be suboptimal in general equilibrium models incorporating unintended bequests and capital accumulation. Even if a mandatory annuity program is adopted, simply requiring the citizens to buy annuities may not completely eliminate adverse selection. In some cases such as the compulsory section of the UK annuity market before 2015, participation was mandatory but the participants were able to choose among a range of annuity products with different characteristics (Finkelstein et al. 2009). Their choices, though limited, may lead to a partial offsetting effect to the policy of mandatory participation. Lau and Zhang (2023) discuss the diverse practices of public annuity policies in various economies. In particular, choices in annuity product and/or annuitization amount are still present in the mandatory public annuity programs in Lithuania, Singapore and Sweden.

In this context, an alternative policy suggestion in which annuitants are given the choice is to offer annuities at an earlier time. The main reason of this suggestion is that while the private health characteristics of the annuitants may be quite diverse when buying annuities at retirement age (which is the main factor leading to adverse selection), their health characteristics are usually more similar at an early age. To the best of our knowledge, Brugiavini (1993) is the first study to systematically examine these issues. In analyzing deferred and immediate annuities, she assumes that the annuitants have the same health signal at a young age but different health characteristics at an advanced age, and shows that there is no transaction in immediate annuities. Sheshinski (2007) adopts similar assumptions and obtains the result of complete crowding out of the immediate annuity market in a continuous-time model when the utility function is state-invariant.

Besides buying deferred annuities at an earlier age to avoid risk classification, individuals may also consider enhanced annuities, which offer higher payouts to buyers who are able to provide evidence of a reduced life expectancy. Motivated by the observed market share of enhanced annuities, Steinorth (2012) compares them with deferred annuities. Instead of following Brugiavini's (1993) assumption of identical health conditions at a younger age, she argues that "individuals already have an idea of their future risk type when young that becomes clearer over time" (p. 974) and labels this feature as heterogeneous private information. She shows that enhanced annuities crowd out standard deferred annuities offered at an earlier age when the annuitants have incomplete knowledge of their future risk type, but both products coexist (with lower risk annuitants buying enhanced annuities while higher risk annuitants buying deferred annuities) when annuitants are sufficiently risk averse and the proportion of low risk type is sufficiently high.

Unlike Brugiavini (1993) and Steinorth (2012) who focus on risk sharing among the same cohort of annuitants in a defined-contribution individual-account system by using a partial-equilibrium life-cycle model, d'Albis and Kalk (2021) compare deferred annuities with immediate annuities (labelled as long-term and short-term annuities, respectively, in their paper) using an overlapping-generations model with production. In their framework, redistribution between cohorts is present in deferred annuities offered at an early age, but is absent in immediate annuities offered at a later age. They further derive robust results that annuitants buy deferred annuities early in life in a dynamically inefficient economy (with over-accumulation of capital), but postpone the decisions to purchase annuities at a later age if the economy is dynamically efficient. Moreover, in an extended model with the assumption that neither annuity buyers nor providers know with certainty in an early period about future survival probabilities, they show (in Proposition 2 of their paper) that the two annuity markets coexist when the marginal product of capital lies in some intermediate range.

This paper focuses on a defined-contribution individual-account system and examines issues related to Brugiavini's (1993) result of complete crowding out of the immediate annuity market after deferred annuities are offered. We think that the assumption of identical early health signal for all annuitants is inconsistent with the evidence of heterogeneous health characteristics of citizens in their fifties. Building on the idea of heterogeneous early health information in Steinorth (2012), we propose an alternative specification that early health signals are diverse and have predictive power about future survival probabilities. Unlike the assumption of symmetric information in d'Albis and Kalk (2021), we assume that annuitants' survival probabilities are not known to annuity providers. Both Steinorth (2012) and this paper consider deferred annuities at an early age and assume asymmetric information regarding the annuitant's early health signal. However, Steinorth (2012) focuses on enhanced annuities at a later age and specifies that the annuitant's risk type is observable to the enhanced annuity providers. On the other hand, we focus on immediate annuities at a later age and assume that the annuitant's future survival probability is not known to the immediate annuity providers. Moreover, while early health signal in both papers is modelled as a continuous random variable, our specification of the annuitant's survival probability at a later age as a continuous variable is different from that of the two risk types in Steinorth (2012, Figure 1). With this specification, we are able to derive useful results to understand why the crowding out of immediate annuity market is complete when early health signals are identical but is incomplete in the model with heterogeneous early signals.

3 A model of annuities with heterogeneous early health signals

In this paper we use a simple three-period model of deferred and immediate annuities, which is based on a similar one used in Brugiavini (1993). [3] However, we specify a

[3] To focus on the main issue of adverse selection, the model in Brugiavini (1993) does not address annuitants' liquidity concern. In this paper, we analyze the effects of introducing deferred annuities when the annuitants use their individual-account retirement wealth to purchase this product. In this environment,

more general revelation process of annuitant's health information, which includes the specification used in Brugiavini (1993) as a limiting case.

In this section, we first describe the sequence of events in our model and suggest a general specification of how private health information is revealed. We then derive the annuitants' annuitization choices. We also analyze the outcome without the deferred annuity, which serves as the reference economy when we compare with the outcome after the deferred annuity is introduced.

3.1 Sequence of events

As in Brugiavini (1993), all events happen in three periods of time: Periods 0, 1 and 2. These periods represent the three stages of a typical life-cycle of adulthood: working stage, early and later stages after retirement, respectively. We assume that all citizens survive at least the first two periods, but there is some probability that a person may not survive to Period 2. In this model, the sequence of events happens as follows.

In period 0, all citizens receive health information (z). This information in Period 0 (to be denoted as early health signal) is useful in predicting her health characteristics at a later age, as described in details in Section 3.2. In the same period, all citizens know their wealth level (w). [4] The government offers a deferred annuity product in Period 0.[5] Based on the health signal and given her wealth level, the citizen makes her deferred annuity choice (β_z).

In Period 1, the citizen's health information is further revealed. This piece of information in Period 1 (to be denoted as survival probability) tells her about the probability (θ) of surviving in Period 2. [6] In the same period, annuity providers offer an immediate annuity.[7] Based on the survival probability θ and given the choice of β_z in the

since annuitants cannot use the lock-in retirement wealth for other purchases until the retirement age, it is not misleading in using the simplifying assumption of ignoring liquidity constraints.

4 In Brugiavini's (1993) specification, an annuitant earns labor income in Periods 0 and 1. In particular, she studies the effect of labor income uncertainty on annuity purchase by assuming (in Section 4 of her paper) that labor income in Period 1 is random. In this paper, we focus on the working-age deferred annuities, and use the assumption of heterogeneous early health signals, instead of identical early health condition for all annuitants. To contrast sharply our results under the new assumption with the original one used in Brugiavini (1993), we do not study labor income uncertainty in this paper. Moreover, we simplify the model by assuming that all income arrives in Period 0.

5 As mentioned in the Introduction, we analyze the effects of offering the deferred annuity when the annuitants use their individual-account retirement wealth to purchase this product. In this environment, whether the deferred annuity is supplied by the government or the private sector (as the products in the Compulsory Purchase Annuity market in the UK before 2015, which can only be purchased by using the annuitant's individual-account wealth) is not crucial. For convenience, we assume that the deferred annuity is provided by the government or a statutory body (such as the observed public annuity programs summarized in Table 1 of Lau and Zhang 2023), and label it as the public deferred annuity.

6 Brugiavini (1993) uses two different specifications about the distribution of annuitants' survival probabilities in Period 1: a two-point distribution in one case and a continuous distribution in another. We use the formulation that θ is a continuous random variable.

7 Our major objective is to study two annuities: an immediate annuity offered around retirement age (say, age 65) and a deferred annuity offered during working age (say, age 50) when people are less certain about their health states during retirement. In most existing annuity contracts, annuitants receive survival-contingent payout shortly after purchasing the immediate annuity but they wait for a longer duration (usually up to

Table 1 Sequence of events

	Period		
	0	1	2
Information about survival probability	z	θ	
Wealth	w	0	0
Consumption	0	$c^1_{z\theta}$	$c^2_{z\theta}$
Annuity purchase	β_z	$\alpha_{z\theta}$	

previous period, the citizen with z in Period 0 and θ in Period 1 chooses the amount of immediate annuity purchase ($\alpha_{z\theta}$) and consumption level in Period 1 ($c^1_{z\theta}$).

In Period 2, some citizens die. For those who are still alive, they do not make active choices and simply consume $c^2_{z\theta}$, the amount received from annuity payouts based on their previous annuitization choices.

The events described above are summarized in Table 1.

3.2 Predicting future survival probabilities based on heterogeneous early health signals

Deferred annuities in many societies are offered when the annuitants have already shown some differences in health characteristics. Moreover, empirical evidence (such as Smith et al. 2001; Hurd and McGarry 2002) shows that people learn about their health information gradually. We combine these two features and propose the following health information revelation process.

Denote the probability density function of $z \in \left[\underline{z}, \overline{z}\right]$ as $f_z\left(z\right)$, and denote the probability density function of $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ conditional on health signal z as $f_{\theta|z}\left(\theta|z\right)$. Denote the probability density function of survival probability $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ as $f_\theta\left(\theta\right)$. Given the above specification of the sequence of events, they are related according to

$$f_\theta\left(\theta\right) = \int_{\underline{z}}^{\overline{z}} f_{\theta|z}\left(\theta|z\right) f_z\left(z\right) dz = \int_{\underline{z}}^{\overline{z}} f\left(z, \theta\right) dz, \tag{1}$$

where $f\left(z, \theta\right)$ is the joint probability density function of z and θ. It is assumed that the various parameters satisfy

$$0 \leq \underline{\theta} < \underline{z} < \overline{z} < \overline{\theta} \leq 1. \tag{2}$$

retirement age around 65) if they purchase working-age deferred annuities. Following Brugiavini (1993), we capture the essential elements of these two annuities in a three-period model in which survival uncertainty is present in Period 2 by assuming that the deferred annuity is offered in Period 0 and the immediate annuity is offered in Period 1. For convenience, the latter contract is labelled as “immediate annuity” even though strictly speaking, the survival-contingent payments appear one period afterwards in this model.

We impose discipline on the process of health information revelation by assuming a consistency condition such that the expected value of θ conditional on z equals to z:

$$E\left(\theta|z\right)=\int_{\underline{\theta}}^{\overline{\theta}}\theta f_{\theta|z}\left(\theta|z\right)d\theta=z. \tag{3}$$

Equation (3) is the most crucial element of the proposed health information revelation process. According to (3), a high (resp. low) value of health signal (z) in Period 0 means that survival probability (θ) in Period 1 is also high (resp. low) on average. The early health signal z is useful in predicting future value of θ.[8] Our specification builds on, but is different from, the concepts in Steinorth (2012).

Based on the specification about the conditional mean in (3), it can be shown that[9]

$$E\left(\theta\right)=E\left(z\right). \tag{4}$$

Thus, the unconditional mean of annuitants' survival probabilities and that of their early health signals are equal.

Brugiavini (1993) assumes that all annuitants receive the same health signal in Period 0 about the distribution of survival probability in Period 1. In terms of our model, her specification of equal early health signal is as follows:

$$0\leq\underline{\theta}<\underline{z}=z_0=\overline{z}=E\left(\theta\right)<\overline{\theta}\leq 1. \tag{5}$$

It is easy to see that (5) is a limiting case of (2), when the range of the health signal degenerates to a single point at $z_0=E\left(\theta\right)$.

Besides viewing the difference of Brugiavini (1993) and this paper as whether early health signals are assumed to be homogeneous or heterogeneous, an alternative interpretation is as follows. As seen from (2) and (3), the process of health information revelation is represented by an increase in the mean-preserving spread ($=\left(\overline{\theta}-\underline{\theta}\right)-\left(\overline{z}-\underline{z}\right)$) in Period 1. Viewed from this perspective, we consider an environment in which annuitants' health signals at the time of buying deferred annuities are *partially informative* about future survival probabilities. Brugiavini (1993), on the other hand, considers the limiting case (5) that the early health signals are *totally uninformative*, with the largest increase in the mean-preserving spread ($=\overline{\theta}-\underline{\theta}$).

8 A specific example of $f\left(z,\theta\right)$ that satisfies (2) and (3) is presented in the Appendix.

9 Multiplying the middle and right-hand side terms of (3) by $f_z\left(z\right)$ and integrating with respect to z, we obtain

$$E\left[E\left(\theta|z\right)\right]=\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}}\theta f_{\theta|z}\left(\theta|z\right)f_z\left(z\right)d\theta dz=\int_{\underline{z}}^{\overline{z}}zf_z\left(z\right)dz.$$

Using $f_{\theta|z}\left(\theta|z\right)f_z\left(z\right)=f\left(z,\theta\right)$, the middle term of the above equation becomes $\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}}\theta f\left(z,\theta\right)d\theta dz=E\left(\theta\right)$. (The above result follows directly from the Law of Iterated Expectations.) It is straightforward to see that the right-hand side term of the above equation is $E\left(z\right)$. Combining these results leads to (4).

3.3 Annuitization choices

Except for the process of health information revelation, the remaining parts of the model are very similar to that in Brugiavini (1993). We summarize the main features of the annuity contracts and derive the optimal annuitization choices.

We assume that annuitants know more about their own health risks than the annuity providers do, and they make annuitization decisions based on the private health information. As in Abel (1986), Brugiavini (1993) and Hosseini (2015), we assume that annuity providers offer non-exclusive annuity contracts with linear pricing, instead of exclusive contracts with price convexity (such as Eckstein et al. 1985; Eichenbaum and Peled 1987). Under a non-exclusive contract with linear pricing, all annuitants pay the same price for each unit of a particular annuity product. As mentioned in Abel (1986), it is hard to determine whether a buyer also holds annuities issued by other providers. This well-known difficulty affects the effectiveness of exclusive contracts with convex pricing in the annuity market.[10]

Under the assumption that survival is certain in Periods 0 and 1 but is uncertain in Period 2, an annuitant's main decision is how to insure against this longevity risk. In the model, the annuitants can purchase deferred or immediate annuities to achieve this task. The two annuity contracts operate as follows. The deferred annuity is offered in Period 0 and can only be purchased in this period, and the price of one unit of annuity is $\frac{q}{(1+r)^2}$. If an annuitant buys one unit of the deferred annuity, she will receive one dollar in Period 2 if and only if she is alive. The immediate annuity is offered in Period 1, and the price of one unit of annuity is $\frac{p}{1+r}$.[11] If an annuitant buys one unit of the immediate annuity, she will receive one dollar in Period 2 if and only if she is alive. In this environment, the budget constraints are

$$c^1_{z\theta} = (1+r)\left[w - \frac{q}{(1+r)^2}\beta_z\right] - \frac{p}{1+r}\alpha_{z\theta} \tag{6}$$

and

$$c^2_{z\theta} = \beta_z + \alpha_{z\theta}, \tag{7}$$

where r is the risk-free interest rate. Since borrowing against annuity contracts is not allowed in most societies, we impose the restriction of non-negative amounts of annuity purchase: $\beta_z \geq 0$ and $\alpha_{z\theta} \geq 0$. Notice that we express the prices as $\frac{p}{1+r}$ and $\frac{q}{(1+r)^2}$. In subsequent analysis, we compare the "normalized" prices p and q, which remove the effect of interest rate due to the different time periods in purchasing deferred and immediate annuities.

[10] This difficulty has also been mentioned by Chiappori (2000, p. 369): "agents can always "linearize" the schedule by buying a large number of small contracts from different insurers."

[11] We define each of the two annuity contracts in terms of the annuity price ($\frac{p}{1+r}$ or $\frac{q}{(1+r)^2}$), as in Brown (2003). In this case, the annuity payout is fixed at the normalized level of 1. Alternatively, it can be defined in terms of the annuity payout level, as in Abel (1986) and Brugiavini (1993). The two specifications are equivalent, with the annuity price negatively related to the annuity payout.

We use backward induction to solve the annuitant's decision problem. In Period 2, the annuitants simply consume what they have. In Period 1, after purchasing β_z of deferred annuity in Period 0 and knowing the updated information of survival probability θ, an annuitant with $z\theta$ (i.e., with health signal z in Period 0 and survival probability θ in Period 1) chooses $\alpha_{z\theta}$ (the amount of immediate annuity) to maximize

$$U_{z\theta} = u\left(c^1_{z\theta}\right) + \frac{\theta}{1+\rho}u\left(c^2_{z\theta}\right)$$

$$= u\left((1+r)\,w - \frac{q}{1+r}\beta_z - \frac{p}{1+r}\alpha_{z\theta}\right) + \frac{\theta}{1+\rho}u\left(\beta_z + \alpha_{z\theta}\right), \tag{8}$$

where ρ is the subjective discount rate, and the standard assumptions $u'(.) > 0$ and $u''(.) < 0$ hold.

Denote the optimal choice as $\alpha^*_{z\theta}$. It is straightforward to show that if $\theta \geq \theta_\alpha(z)$, where

$$\theta_\alpha(z) = \frac{p\,(1+\rho)\,u'\left((1+r)\,w - \frac{q}{1+r}\beta_z\right)}{(1+r)\,u'(\beta_z)}, \tag{9}$$

then $\alpha^*_{z\theta}$ is an interior solution, which is determined according to

$$\frac{p}{1+r}u'\left((1+r)\,w - \frac{q}{1+r}\beta_z - \frac{p}{1+r}\alpha^*_{z\theta}\right) = \frac{\theta}{1+\rho}u'\left(\beta_z + \alpha^*_{z\theta}\right). \tag{10}$$

On the other hand, if $\theta < \theta_\alpha(z)$, then

$$\frac{p}{1+r}u'\left((1+r)\,w - \frac{q}{1+r}\beta_z\right) > \frac{\theta}{1+\rho}u'(\beta_z), \tag{11}$$

leading to the corner solution of $\alpha^*_{z\theta} = 0$.

The left-hand side of (10) is the marginal cost of purchasing one unit of the immediate annuity by annuitant $z\theta$. It equals to the marginal benefit term on the right-hand side when immediate annuity choice is an optimal interior solution. On the other hand, the marginal cost is higher than marginal benefit in (11) when $\alpha^*_{z\theta} = 0$.

To determine the equilibrium normalized price of the immediate annuity, we assume the zero-profit assumption, as in Abel (1986), Brugiavini (1993) and Hosseini (2015). Annuity providers receive annuity premium $\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}} \frac{p}{1+r}\alpha^*_{z\theta} f(\theta, z)\, d\theta dz$ from all buyers in Period 1. Annuity providers then distribute payments $\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}} \theta\alpha^*_{z\theta} f(\theta, z)\, d\theta dz$ to all surviving buyers in Period 2. The expected present discounted value of total annuity payment equals to the total premium received under the zero-profit condition. As a result, the normalized equilibrium immediate annuity price (denoted by p^*) is determined according to

$$p^* = \frac{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \theta \alpha_{z\theta}^* f(\theta, z)\, d\theta dz}{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha_{z\theta}^* f(\theta, z)\, d\theta dz} = \frac{\int_{\underline{z}}^{\overline{z}} \left[\int_{\theta_\alpha(z)}^{\overline{\theta}} \theta \alpha_{z\theta}^* f_{\theta|z}(\theta|z)\, d\theta \right] f_z(z)\, dz}{\int_{\underline{z}}^{\overline{z}} \left[\int_{\theta_\alpha(z)}^{\overline{\theta}} \alpha_{z\theta}^* f_{\theta|z}(\theta|z)\, d\theta \right] f_z(z)\, dz}, \tag{12}$$

where $\theta_\alpha(z)$ is defined in (9).

We now consider the purchase of deferred annuities in Period 0. Anticipating the choices of $\alpha_{z\theta}^*$ in Period 1, an annuitant who receives signal z in Period 0 chooses β_z to maximize her expected utility

$$E(U_{z\theta}|z) = \int_{\underline{\theta}}^{\overline{\theta}} U_{z\theta} f_{\theta|z}(\theta|z)\, d\theta, \tag{13}$$

where $U_{z\theta}$ is given by (8) with $\alpha_{z\theta}^*$ determined optimally according to (10) if it is an interior solution or $\alpha_{z\theta}^* = 0$ otherwise. Note that we simplify the optimization problem by assuming that consumption choice in Period 0 is irrelevant in the annuitants's objective function, as in Brugiavini (1993). [12]

Denote annuitant z's optimal purchase of the deferred annuity as β_z^*. Differentiating (13) with respect to β_z and applying the envelope theorem, we show that β_z^* is an interior solution if

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{q}{1+r} u'\left((1+r)\, w - \frac{q}{1+r}\beta_z^* - \frac{p}{1+r}\alpha_{z\theta}^* \right) f_{\theta|z}(\theta|z)\, d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{1+\rho} u'\left(\beta_z^* + \alpha_{z\theta}^*\right) f_{\theta|z}(\theta|z)\, d\theta \tag{14}$$

holds, where $\alpha_{z\theta}^*$ is defined in (10) if $\theta \geq \theta_\alpha(z)$ or $\alpha_{z\theta}^* = 0$ otherwise. On the other hand, $\beta_z^* = 0$ if

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{q}{1+r} u'\left((1+r)\, w - \frac{p}{1+r}\alpha_{z\theta}^* \right) f_{\theta|z}(\theta|z)\, d\theta > \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{1+\rho} u'\left(\alpha_{z\theta}^*\right) f_{\theta|z}(\theta|z)\, d\theta. \tag{15}$$

The left-hand side of (14) is annuitant z's marginal cost of purchasing one unit of deferred annuity while the right-hand side is the corresponding marginal benefit. They

[12] As in Brugiavini (1993), there are several simplifying assumptions in the annuitant's objective function. First, there is no bequest motive in this model. As a result, saving in terms of risk-free bond is dominated by any annuity product that offers higher return (e.g., Yaari 1965; Davidoff et al. 2005). Second, we assume that the annuitant's lifetime utility does not depend on consumption level in Period 0 and all annuitants have the same level of wealth. Since our study focuses on the issues related to how the revelation of health information affects the purchase of deferred and immediate annuities, the above factors are not the most important ones. Relaxing these assumptions leads to more complicated analysis, but does not provide additional insight for the issues analyzed in this paper.

are equal for an interior solution of β_z^*. On the other hand, the marginal cost is higher than marginal benefit in (15) when $\beta_z^* = 0$.

As in the private immediate annuity market, we assume that the government manages the deferred annuity plan under zero-profit condition. This assumption is reasonable. The main purpose of the public deferred annuity plan is to encourage citizens to buy annuities at an early stage, and earning profit from this plan is generally not an objective of this policy. Moreover, the assumption of zero-profit condition allows us to focus on the information friction on the demand side, the source of market imperfection in our study. Applying the zero-profit condition, the normalized equilibrium price of deferred annuity (q^*) is given by[13]

$$q^* = \frac{\int_{\underline{z}}^{\overline{z}} \left[\beta_z^* \int_{\underline{\theta}}^{\overline{\theta}} \theta f_{\theta|z}(\theta|z)\, d\theta \right] f_z(z)\, dz}{\int_{\underline{z}}^{\overline{z}} \beta_z^* f_z(z)\, dz} = \frac{\int_{\underline{z}}^{\overline{z}} z \beta_z^* f_z(z)\, dz}{\int_{\underline{z}}^{\overline{z}} \beta_z^* f_z(z)\, dz}, \tag{16}$$

after using $\int_{\underline{\theta}}^{\overline{\theta}} \theta f_{\theta|z}(\theta|z)\, d\theta = z$ according to (3).

3.4 The economy without the deferred annuity

We now study the annuitants' behavior before the deferred annuity is offered. The results will be useful in subsequent analysis when we compare with the economy when both deferred and immediate annuities are available.

In this economy, the optimization problem of annuitant $z\theta$ after knowing her private information of survival probability θ in Period 1, is to choose $\widehat{\alpha}_{z\theta} = \widehat{\alpha}_\theta$ to maximize

$$\widehat{U}_{z\theta} = \widehat{U}_\theta = u\left((1+r)\, w - \frac{\widehat{p}}{1+r} \widehat{\alpha}_\theta \right) + \frac{\theta}{1+\rho} u\left(\widehat{\alpha}_\theta\right), \tag{17}$$

which is the utility function (8) in an economy with no deferred annuity ($\beta_z = 0$). It is obvious from (17) that health signal z does not affect the annuitant's choice in Period 1 when only immediate annuities are available. Note that we use the symbol "$\widehat{\ }$" above a variable to represent the corresponding variable before the deferred annuity is introduced.

It is straightforward to show that the optimal immediate annuity choices (denoted by $\widehat{\alpha}_\theta^*$) before the introduction of deferred annuity are given by

$$\frac{\widehat{p}}{1+r} u'\left((1+r)\, w - \frac{\widehat{p}}{1+r} \widehat{\alpha}_\theta^* \right) = \frac{\theta}{1+\rho} u'\left(\widehat{\alpha}_\theta^*\right). \tag{18}$$

[13] The deferred annuity provider receives annuity premium $\int_{\underline{z}}^{\overline{z}} \frac{q}{(1+r)^2} \beta_z^* f_z(z)\, dz$ from all buyers in Period 0, and then distributes payments $\int_{\underline{z}}^{\overline{z}} \left[\int_{\underline{\theta}}^{\overline{\theta}} \theta \beta_z^* f_{\theta|z}(\theta|z)\, d\theta \right] f_z(z)\, dz$ to all surviving buyers in Period 2. Under the zero-profit condition, the present discounted value of total annuity payment equals to total premium received, leading to (16).

Thus, the corresponding normalized equilibrium price ($\widehat{p}^*$) of the immediate annuity is

$$\widehat{p}^* = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \theta \widehat{\alpha}_\theta^* f_\theta(\theta)\, d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \widehat{\alpha}_\theta^* f_\theta(\theta)\, d\theta}. \tag{19}$$

From now on, we just use the simpler term “annuity price” without the adjective “normalized” when referring to the terms in (12), (16) and (19).

4 The outcome after introducing deferred annuities

We use the model with heterogeneous early health signals to analyze the outcome after the government introduces deferred annuities. In particular, we consider an environment with the following restriction on the highest level of early health signals:

$$\overline{z} < \widehat{p}^*, \tag{20}$$

where $\widehat{p}^*$ is defined in (19).

The restriction in (20) puts an upper bound on the range of early health signals (z). Following the interpretation mentioned in Section 3.2, this restriction means that early health signals of the annuitants are *informative but not very informative*. Since we have in mind that Period 0 roughly corresponds to age 50 and Period 1 corresponds to age 65, it is reasonable to assume that the increase in mean-preserving spread is not trivially small. On the other hand, if early health signals are very informative (say, when deferred annuities are offered at an age around 60) in the sense that the increase in mean-preserving spread is very small, then the potential benefit of offering deferred annuities is much reduced. We do not consider this less interesting case and impose the restriction $\overline{z} < \widehat{p}^*$ in (20).

4.1 Coexistence of both types of annuities

The first important result is given in the following proposition.

Proposition 1 *Consider an economy in which the annuitants’ early health signals satisfy restriction (20) and each annuitant’s private health information is revealed according to (2) and (3). After the government introduces the deferred annuity,*

(a) the immediate annuity market is not completely crowded out; and

(b) there are transactions in deferred annuities.

Proof We prove by contradiction. Suppose the immediate annuity market is completely crowded out (i.e., $\alpha_{z\theta}^* = 0$ for all annuitants). Then all annuitants will buy deferred annuities to insure against longevity risk and thus, the first-order condition (14) for an interior solution holds for all annuitants. When $\alpha_{\underline{z}\theta}^* = 0$ for all θ, condition (14) of annuitants with $\underline{z}$ becomes

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{q^*}{1+r} u'\left((1+r)\,w - \frac{q^*}{1+r}\beta^*_{\underline{z}}\right) f_{\theta|z}\left(\theta|\underline{z}\right) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{1+\rho} u'\left(\beta^*_{\underline{z}}\right) f_{\theta|z}\left(\theta|\underline{z}\right) d\theta.$$

Since the integrand on the left-hand side of the above equation is independent of θ and $\int_{\underline{\theta}}^{\overline{\theta}} \theta f_{\theta|z}\left(\theta|\underline{z}\right) d\theta = \underline{z}$ on the right-hand side because of (3), we obtain

$$\frac{q^*}{1+r} u'\left((1+r)\,w - \frac{q^*}{1+r}\beta^*_{\underline{z}}\right) = \frac{\underline{z}}{1+\rho} u'\left(\beta^*_{\underline{z}}\right). \tag{21}$$

Consider annuitants with lowest health signal ($\underline{z}$) in Period 0 but highest survival probability ($\overline{\theta}$) in Period 1. Since $\underline{z}$ is the lowest health signal and $\overline{\theta}$ is the highest survival probability, we obtain

$$p^* = \frac{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \theta \alpha^*_{z\theta} f\left(\theta, z\right) d\theta dz}{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha^*_{z\theta} f\left(\theta, z\right) d\theta dz} \leq \frac{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \overline{\theta} \alpha^*_{z\theta} f\left(\theta, z\right) d\theta dz}{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha^*_{z\theta} f\left(\theta, z\right) d\theta dz} = \overline{\theta}, \tag{22}$$

because $\theta < \overline{\theta}$ for $\theta \in \left[\underline{\theta}, \overline{\theta}\right)$, and

$$q^* = \frac{\int_{\underline{z}}^{\overline{z}} z \beta^*_z f_z\left(z\right) dz}{\int_{\underline{z}}^{\overline{z}} \beta^*_z f_z\left(z\right) dz} > \underline{z}, \tag{23}$$

because $z > \underline{z}$ for $z \in \left(\underline{z}, \overline{z}\right]$.[14] Combining (21) to (23), we obtain

$$\frac{p^*}{1+r} u'\left((1+r)\,w - \frac{q^*}{1+r}\beta^*_{\underline{z}}\right) < \frac{\overline{\theta}}{1+\rho} u'\left(\beta^*_{\underline{z}}\right). \tag{24}$$

Comparing with (10) and (11), we see from (24) that for annuitants with $\underline{z}\overline{\theta}$, the marginal cost of buying the immediate annuity (when $\alpha^*_{\underline{z}\overline{\theta}} = 0$) is lower than the marginal benefit. Thus, condition (24) is inconsistent with the supposition of complete crowding out of the immediate annuity market (i.e., $\alpha^*_{z\theta} = 0$ for all annuitants). This proves part (a).

If no annuitant buys the deferred annuity ($\beta^*_z = 0$ for all z), then $\alpha^*_{z\theta} > 0$ for all z and all θ. In this case, it is easy to show that $p^* = \widehat{p}^*$, where $\widehat{p}^*$ is defined in (19). Combining $\overline{z} < \widehat{p}^*$ according to (20) under the deferred annuity policy with $q^* \leq \overline{z}$,

[14] In this model, a higher value of early health signal (z) indicates that the annuitant is more likely to have higher survival probability (θ) and demand more deferred annuities. A necessary condition for $q^* = \underline{z}$, instead of (23), to hold is that annuitants with the lowest level of early health signal ($\underline{z}$) are indifferent in purchasing the first unit of the deferred annuity and not purchasing, while all other annuitants (with higher level of z) strictly prefer not to purchase. Since this is contradictory to the above property of the model, we rule out the possibility of $q^* = \underline{z}$.

which is implied by the zero-profit condition (16), we obtain $q^* < \widehat{p}^*$. Thus, the possibility of $q^* \geq p^* = \widehat{p}^*$ is ruled out. We thus only need to consider whether the combination of $\beta_z^* = 0$ for all annuitants and

$$q^* < p^* = \widehat{p}^* \tag{25}$$

is possible.

If $\beta_z^* = 0$ for all z, then all $\alpha_{z\theta}^*$ are interior solutions and (10) becomes

$$\frac{p^*}{1+r} u' \left((1+r)\, w - \frac{p^*}{1+r} \alpha_{z\theta}^* \right) = \frac{\theta}{1+\rho} u' \left(\alpha_{z\theta}^* \right).$$

Multiply both sides of this equation by $f_{\theta|z}(\theta|z)$ and integrate with respect to θ. Combining the resulting equation with $q^* < p^* = \widehat{p}^*$ leads to

$$\begin{aligned} \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{1+\rho} u' \left(\alpha_{z\theta}^* \right) f_{\theta|z}(\theta|z)\, d\theta &= \int_{\underline{\theta}}^{\overline{\theta}} \frac{p^*}{1+r} u' \left((1+r)\, w - \frac{p^*}{1+r} \alpha_{z\theta}^* \right) f_{\theta|z}(\theta|z)\, d\theta \\ &> \int_{\underline{\theta}}^{\overline{\theta}} \frac{q^*}{1+r} u' \left((1+r)\, w - \frac{p^*}{1+r} \alpha_{z\theta}^* \right) f_{\theta|z}(\theta|z)\, d\theta. \end{aligned} \tag{26}$$

Comparing with (15), we see from (26) that for annuitants with z, the marginal benefit of buying the deferred annuity (when $\beta_z^* = 0$) is higher than the marginal cost. The above argument holds for any z. Thus, condition (26) is inconsistent with the supposition of no purchase of the deferred annuity (i.e., $\beta_z^* = 0$ for all annuitants). This proves part (b). □

The intuition of the above proof is as follows. Under the supposition of complete crowding out of the immediate annuity market, (21) would hold for annuitants with $\underline{z}$ and $\overline{\theta}$. Together with the two inequalities (22) and (23), condition (21) implies (24) in the proof of part (a) of Proposition 1. Intuitively, annuitants with lowest z but highest θ always have the incentive to buy the first unit of the immediate annuity after they have purchased the smallest amount of the deferred annuity earlier, leading to the impossibility of complete crowding out of the immediate annuity market.

Under the supposition that no annuitant buys the deferred annuity, all annuitants insure against the longevity risk by buying the immediate annuity after observing θ in Period 1. Combining the equality of marginal benefit and marginal cost at every states of θ, together with $q^* < p^*$ according to (25), we obtain (26) in the proof of part (b) of Proposition 1. According to (26), the marginal cost of buying the first unit of the (less expensive) deferred annuity is lower than the marginal benefit. As a result, it is impossible to have no transaction in deferred annuities.

Proposition 1 implies that the only possibility is the coexistence of deferred and immediate annuities. When they coexist, the annuity prices (q^* and p^*) and the annuitants' optimal annuity purchases (β_z^* and $\alpha_{z\theta}^*$) are mutually dependent. According to

(10) or (11), $\alpha^*_{z\theta}$ is a function of p^*, as well as β^*_z chosen earlier and q^*. Similarly, according to (14) or (15), β^*_z is a function of q^*, as well as p^* and the anticipated choices of $\alpha^*_{z\theta}$. Moreover, each of p^* and q^* is determined by the corresponding zero-profit condition: (12) or (16). In this environment, determining the equilibrium outcome involves solving the two fixed points (p^* and q^*) simultaneously.

The following proposition shows that an equilibrium exists when there are transactions in both deferred and immediate annuities.[15]

Proposition 2 *When the annuitants' early health signals satisfy restriction (20) and each annuitant's private health information is revealed according to (2) and (3), introducing the deferred annuity leads to the coexistence of immediate and deferred annuity transactions, with*

$$q^* < p^*. \tag{27}$$

Proof In this model, annuitants' optimal annuitization choices (β^*_z and $\alpha^*_{z\theta}$) and the two annuity prices (q^* and p^*) are mutually dependent. We use a sequential approach based on backward induction to show that an equilibrium exists in this environment.

In Period 1, conditional on an arbitrary value of deferred annuity price q, we look for the equilibrium value of immediate annuity price, denoted by $p^*(q)$, such that $p^*(q)$ determined according to (12) is consistent with the annuitants' optimal choices, given by (10) for $\alpha^*_{z\theta}$ (or $\alpha^*_{z\theta} = 0$ if $\theta < \theta_\alpha(z)$) and (14) for β^*_z (or $\beta^*_z = 0$ if (15) holds).[16] Based on (12), we define the following function:

$$K(p;q) = \frac{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \theta \alpha^*_{z\theta}(p,q) f(\theta,z)\, d\theta dz}{\int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha^*_{z\theta}(p,q) f(\theta,z)\, d\theta dz}, \tag{28}$$

where $\alpha^*_{z\theta}$, which is defined in (10) if $\theta \geq \theta_\alpha(z)$ or $= 0$ otherwise, is rewritten as $\alpha^*_{z\theta}(p,q)$, in order to show its dependence on p and q explicitly. Based on (12) and (28), the equilibrium price $p^*(q)$ is defined by

$$K\left(p^*(q);q\right) = p^*(q). \tag{29}$$

For any given value of $q \in \left[\underline{z}, \overline{z}\right]$, we examine $K(p;q)$ as a function of p and look for the intersection of this function and the 45-degree line when $p \in \left[\underline{\theta}, \overline{\theta}\right]$. Similar

15 On the other hand, we simply assume, but do not attempt to prove, that the equilibrium annuity prices are unique. (We have performed computational analysis based on the probability distributions in the Appendix and the logarithmic utility function. The equilibrium for the economy in this case, or in other cases when we vary the parameter values, is unique.) It is well known that the uniqueness of equilibrium is not guaranteed generally when annuitization decisions and annuity prices are mutually dependent; see, for example, Abel (1986, p. 1086) and Villeneuve (2003, p. 534). Note that even if multiple equilibria are present, it is reasonable to focus on the equilibrium in which (a) the government aims to maximize annuitants' welfare by choosing the deferred annuity with the lowest price, subject to the zero-profit condition, and (b) each of the private annuity providers offers the immediate annuity with the lowest price, otherwise its market share will drop to zero when at least one competitor sets the price at this level.

16 Note that the zero-profit condition (16) for deferred annuities does not necessarily hold in this intermediate step.

to the steps in the proof in Proposition 1, it can be shown that

$$K\left(\underline{\theta};q\right)=\frac{\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}}\theta\alpha_{z\theta}^{*}\left(\underline{\theta},q\right)f\left(\theta,z\right)d\theta dz}{\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}}\alpha_{z\theta}^{*}\left(\underline{\theta},q\right)f\left(\theta,z\right)d\theta dz}>\underline{\theta} \tag{30}$$

at the beginning point $p=\underline{\theta}$, and

$$K\left(\overline{\theta};q\right)=\frac{\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}}\theta\alpha_{z\theta}^{*}\left(\overline{\theta},q\right)f\left(\theta,z\right)d\theta dz}{\int_{\underline{z}}^{\overline{z}}\int_{\underline{\theta}}^{\overline{\theta}}\alpha_{z\theta}^{*}\left(\overline{\theta},q\right)f\left(\theta,z\right)d\theta dz}<\overline{\theta} \tag{31}$$

at the end point $p=\overline{\theta}$.[17] The overall budget of annuity providers is in deficit if the price is set at a low level of $\underline{\theta}$, and in surplus if the price is set at a high level of $\overline{\theta}$.

According to (30) and (31), $K\left(p;q\right)$ is above the 45-degree line at the starting point $\underline{\theta}$ but below it at the end point $\overline{\theta}$. Since the $K\left(p;q\right)$ function is continuous in p, we conclude that $p^{*}\left(q\right)$ defined in (29) exists within the interval $\left[\underline{\theta},\overline{\theta}\right]$, as shown in Panels A and B of Figure 1. The function $p^{*}\left(q\right)$ for $q\in\left[\underline{z},\overline{z}\right]$ is shown in Panel C of Figure 1.[18]

Using the results derived above, we now study the economy in Period 0. We look for the equilibrium value of deferred annuity price q^{*} determined according to (16), such that it is consistent with $p^{*}\left(q^{*}\right)$ determined by (29) and the annuitants' optimal choices, given by (10) for $\alpha_{z\theta}^{*}$ (or $\alpha_{z\theta}^{*}=0$ if $\theta<\theta_{\alpha}\left(z\right)$) and (14) for β_{z}^{*} (or $\beta_{z}^{*}=0$ if (15) holds). Based on (16), we define the following function:

$$L\left(q\right)=\frac{\int_{\underline{z}}^{\overline{z}}z\beta_{z}^{*}\left(p^{*}\left(q\right),q\right)f_{z}\left(z\right)dz}{\int_{\underline{z}}^{\overline{z}}\beta_{z}^{*}\left(p^{*}\left(q\right),q\right)f_{z}\left(z\right)dz}, \tag{32}$$

where β_{z}^{*} is rewritten as $\beta_{z}^{*}\left(p,q\right)$. The equilibrium price q^{*} is defined by

$$L\left(q^{*}\right)=q^{*}. \tag{33}$$

We examine $L\left(q\right)$ as a function of q and look for the intersection of this function and the 45-degree line when $q\in\left[\underline{z},\overline{z}\right]$. Similar to the analysis of the $K\left(p;q\right)$ function, we obtain

$$L\left(\underline{z}\right)=\frac{\int_{\underline{z}}^{\overline{z}}z\beta_{z}^{*}\left(p^{*}\left(\underline{z}\right),\underline{z}\right)f_{z}\left(z\right)dz}{\int_{\underline{z}}^{\overline{z}}\beta_{z}^{*}\left(p^{*}\left(\underline{z}\right),\underline{z}\right)f_{z}\left(z\right)dz}>\underline{z} \tag{34}$$

[17] Note that unlike (22), we have strict inequality in (31) when the transactions in deferred and immediate annuities coexist. In this case, $\alpha_{z\theta}^{*}\left(\overline{\theta},q\right)>0$ holds for annuitants with $\theta=\overline{\theta}$ and some other high values of θ.

[18] The various functions in Figure 1 are generated according to the uniform distributions, logarithmic utility function and parameter values discussed in the Appendix.

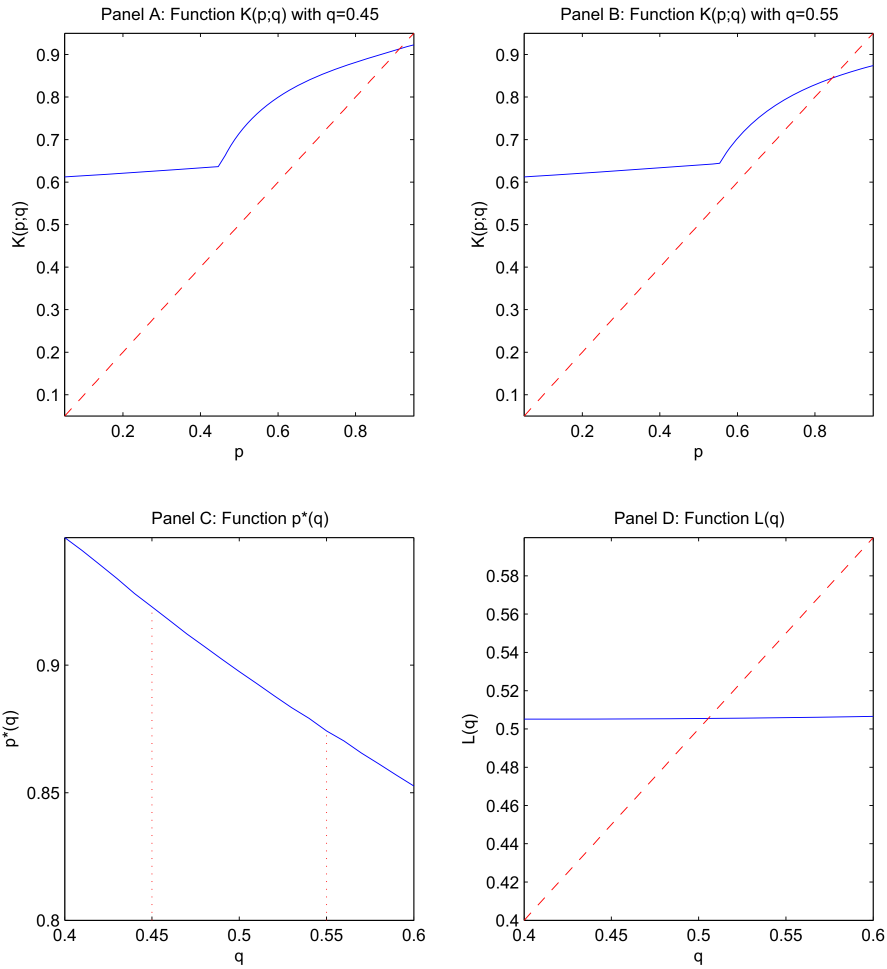

Fig. 1 Existence of equilibrium prices p* and q*

and

$$L\left(\overline{z}\right)=\frac{\int_{\underline{z}}^{\overline{z}} z\beta_z^*\left(p^*\left(\overline{z}\right),\overline{z}\right) f_z\left(z\right) dz}{\int_{\underline{z}}^{\overline{z}} \beta_z^*\left(p^*\left(\overline{z}\right),\overline{z}\right) f_z\left(z\right) dz}<\overline{z}. \tag{35}$$

According to (34) and (35), $L\left(q\right)$ is above the 45-degree line at $q=\underline{z}$ but below it at $q=\overline{z}$. Since the $L\left(q\right)$ function is continuous in q, we conclude that the equilibrium deferred annuity price q^* defined in (33) exists within the interval $\left[\underline{z},\overline{z}\right]$, as shown in Panel D of Figure 1.

After q^* is determined as above, we use the $p^*(q)$ function defined in (29) to obtain the equilibrium immediate annuity price (p^*) according to

$$p^* = p^*\left(q^*\right). \tag{36}$$

The above results show the existence of equilibrium annuity prices p^* and q^* when transactions in deferred and immediate annuities coexist.

We now examine whether (27) or $q^* \geq p^*$ holds. In the above model, both deferred and immediate annuities perform the same function in hedging the longevity risk, but the immediate annuities are offered at a later time when more private health information is revealed. It is obvious that if the equilibrium price of the deferred annuity is equal to or higher than that of the immediate annuity (i.e., $q^* \geq p^*$), the annuitants would prefer to wait to receive further health information in Period 1 and then buy immediate annuities, implying that there is no purchase of the deferred annuity in Period 0. Thus, it is impossible to have the coexistence of deferred and immediate annuity transactions combined with $q^* \geq p^*$. We conclude that (27) holds.

This proves Proposition 2.

In the above proof, we apply the idea that the annuity providers' budget will be in surplus if the annuity price is set too high, but in deficit when it is set too low. Together with the continuity of the $K(p; q)$ function in (28) and the $L(q)$ function in (32), we show that the two zero-profit conditions hold and the equilibrium annuity prices (p^* and q^*) exist when early health signals are heterogeneous.

4.2 A limiting case: Annuitants have identical early health signal

The result that deferred and immediate annuity transactions coexist in our framework contrasts sharply to Brugiavini (1993) who predicts a complete crowding out of immediate annuities in the model with only survival probability uncertainty when annuitants' health signals are the same, as in (5). In this case, the outcome of the economy is summarized as:

Proposition 3 *(Brugiavini 1993). When each annuitant's private health information is revealed according to (5), introducing the deferred annuity leads to a complete crowding out of the immediate annuity market.*

The above result has been shown in Brugiavini (1993). Its importance has been recognized and discussed in many papers, such as Sheshinski (2007), Steinorth (2012), Reichling and Smetters (2015) and d'Albis and Kalk (2021).

Since the specification of homogeneous early health signal in Brugiavini (1993) is a limiting case of (2) and (3) in this paper, it is puzzling that the outcomes in these two papers are so different. One objective of this paper is to link our results with hers, and to explain why the coexistence of deferred and immediate annuities in the general environment (2) and (3) becomes a complete crowding out of immediate annuities in the limiting case (5).

For the model with identical early health signal given by (5), Brugiavini (1993) shows that there is no immediate annuity transaction ($\alpha^*_{z_0\theta} = 0, \forall\theta$) after the introduction of deferred annuities. At the equilibrium, the deferred annuity price in (16) with $\underline{z} = \overline{z} = z_0$ becomes

$$q^* = \frac{\int_{\underline{z}}^{\overline{z}} z\beta_z^* f_z(z)\, dz}{\int_{\underline{z}}^{\overline{z}} \beta_z^* f_z(z)\, dz} = z_0 = E(\theta), \tag{37}$$

and the deferred annuity purchase in (14) is determined according to

$$\frac{q^*}{1+r} u'\left((1+r)\, w - \frac{q^*}{1+r}\beta_{z_0}^*\right) = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{1+\rho} u'\left(\beta_{z_0}^*\right) f_{\theta|z_0}(\theta|z_0)\, d\theta$$

$$= \frac{E(\theta)}{1+\rho} u'\left(\beta_{z_0}^*\right). \tag{38}$$

We now examine the threshold level in (9) when all annuitants' early health signals are identical. First, no purchase of the immediate annuity by any annuitant implies that

$$\theta_\alpha(z_0) \geq \overline{\theta}. \tag{39}$$

Second, using (37) and (38), we conclude that (9) becomes

$$\theta_\alpha(z_0) = \frac{p^*(1+\rho)\, u'\left((1+r)\, w - \frac{q^*}{1+r}\beta_{z_0}^*\right)}{(1+r)\, u'\left(\beta_{z_0}^*\right)} = p^*. \tag{40}$$

Third, it is straightforward to see from the definition of the equilibrium immediate annuity price in (12) that p^* must satisfy

$$\underline{\theta} \leq p^* \leq \overline{\theta}. \tag{41}$$

Conditions (39) to (41) imply that the equilibrium immediate annuity price is given by

$$p^* = \overline{\theta}. \tag{42}$$

Using (37), (38) and (42), we conclude that

$$\frac{p^*}{1+r} u'\left((1+r)\, w - \frac{q^*}{1+r}\beta_{z_0}^*\right) = \frac{\overline{\theta}}{1+\rho} u'\left(\beta_{z_0}^*\right). \tag{43}$$

Compared with (10), we conclude from (43) that the healthiest annuitants are indifferent between purchasing the first unit of immediate annuity and not purchasing. Since other annuitants have less annuitization demand when compared with the healthiest ones, we conclude that the immediate annuity market is completely crowded out by the introduction of deferred annuities.[19]

The above results for the model with identical early health signal are different to those regarding the incomplete crowding out of the immediate annuity market when early health signals are heterogeneous ($\overline{z} > \underline{z}$). The underlying reason of this difference can be traced to (24) and (43). In each case, we focus on annuitants receiving the *largest revision of health information* ($\theta - z$) between the two periods, because they have the highest demand for immediate annuities in Period 1, after having purchased the smallest amount of deferred annuities in Period 0. When early health signals are identical at $z_0 = E(\theta)$ and carry no useful information, as in (5), the annuitants receiving the largest revision of health information are those with the highest survival probability ($\overline{\theta}$). It is shown in (43) that only annuitants with $\overline{\theta}$ satisfy (10) with $\alpha^*_{z_0\overline{\theta}} = 0$ and other annuitants satisfy (11). As a result, there is a complete crowding out of immediate annuities, as shown in Brugiavini (1993). Adverse selection does not appear in the deferred annuity market when early health signals are identical. The first-best outcome is achieved by buying deferred annuities only. On the other hand, under the general health information revelation process (2) and (3), we show in (24) that for the annuitants with lowest early health signal ($\underline{z}$) and highest survival probability ($\overline{\theta}$), the marginal benefit of buying the first unit of immediate annuity is higher than the marginal cost. Essentially, two factors cause them to buy less deferred annuities in Period 0: (a) their low level of early health signal ($\underline{z}$) and (b) the deferred annuity price (q^*) is higher than $E(\theta)$ due to adverse selection, even though q^* is still lower than p^*. As a result, annuitants with the lowest early health signal ($\underline{z}$) will buy a low level of deferred annuities ($\beta^*_{\underline{z}}$, which is smaller than $\beta^*_{z_0}$) and thus have residual demand for immediate annuities when the value of θ turns out to be very high in Period 1, leading to an incomplete crowding out of the immediate annuity market.[20]

To summarize, the result of complete crowding out of the immediate annuity market in Brugiavini (1993) breaks down even when the early health signals are slightly heterogeneous, in the sense that $\overline{z} - \underline{z}$ is small but still strictly positive. In this environment, some annuitants with bad health signals in Period 0 but high survival probabilities in Period 1 will buy immediate annuities.[21]

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19 Using (43), it is straightforward to see that

$$\frac{p^*}{1+r} u' \left((1+r)\, w - \frac{q^*}{1+r} \beta^*_{z_0} \right) > \frac{\theta}{1+\rho} u' \left(\beta^*_{z_0} \right)$$

when $\theta < \overline{\theta}$, confirming that $\alpha^*_{z_0\theta} = 0$ for $\theta \in \left[\underline{\theta}, \overline{\theta}\right)$ also.

20 Following this reasoning, it is shown in an Online Appendix that the result of incomplete crowding out of the immediate annuity market is robust when the annuitants at an early age are pessimistic about future survival probabilities (according to the empirical evidence in O'Dea and Sturrock 2023). The intuition is that the amount of deferred annuities purchased by pessimistic annuitants with the lowest early health signal ($\underline{z}$) will be even lower than $\beta^*_{\underline{z}}$ in (24). As a result, they will buy immediate annuities when the value of θ turns out to be very high in Period 1.

21 Computational analysis similar to that in the Appendix shows that when $\overline{z} - \underline{z}$ becomes smaller (but is still strictly positive), the number of annuitants buying immediate annuities decreases but is still positive.

5 Conclusion

In societies that have adopted the defined-contribution retirement income system in recent decades, one important task of the senior citizen's post-retirement financial plan is to effectively insure against longevity risk. However, the level of participation in private annuity market is low in many societies, perhaps because of the high annuity price caused by adverse selection. In this environment, one policy suggestion is to offer deferred annuities at an early stage at which the annuitants are similar in health characteristics. This paper studies the introduction of working-age deferred annuity when annuitants' early health signals may be heterogeneous.

We show that when deferred annuities are offered at a time such that heterogeneous early health signals have predictive power about future survival probabilities, annuitants buy deferred annuities at an early age and those who receive substantially positive revisions of health information also buy immediate annuities at retirement. The coexistence of deferred and immediate annuity transactions is different from that of complete crowding out of the immediate annuity market in the main model of the influential study of Brugiavini (1993). Since the more general specification of heterogeneous early health signals in this paper incorporates the specification in her model as a limiting case, we are able to compare the two sets of results in a single framework. We show that the crucial factor leading to complete crowding out of the immediate annuity market in Brugiavini's (1993) model with uncertainty in survival probability only is identical early health signal for all annuitants; as a result, the complete crowding out result disappears even if the annuitants' early health signals are slightly diverse. Brugiavini (1993, Section 4) extends the model to include uncertainty in labor income and shows that both immediate and deferred annuity transactions coexist. This paper, on the other hand, shows that in an environment with longevity uncertainty only, the presence of heterogeneous early health signals is also able to generate the coexistence result.[22]

Besides showing the coexistence of deferred and immediate annuity transactions and clarifying the roles of homogeneous versus heterogeneous early health signals, this paper also contributes by proposing a tractable health information revelation process in which the heterogeneous early signals have some, but not full, predictive power about future survival probabilities. This flexible framework can also be applied to more general economic environment. As an example, one may extend the current model to a multi-period environment, with the adjustment of health information between any two adjacent periods given by equations similar to (2) and (3).[23] The dynamic health information revelation process in a multi-period model will be useful in the analysis of questions such as at what age of the annuitants shall deferred annuities be offered,

22 We also show that even though adverse selection is still present in deferred annuity purchase when early health signals are heterogeneous, introducing deferred annuities will lead to higher level of annuity purchase and an improvement in the annuitants' welfare. Since these results are based on computational methods and differ from the analytical focus in earlier sections, they are presented in the Appendix.

23 Following the idea in the Appendix, we may consider the following $(n+1)$-period model, with $\underline{z}_0 = \overline{z}_0 = E(\theta)$ in the beginning period and $\underline{\theta}_n$ $(< \underline{z}_0)$ and $\overline{\theta}_n$ $(> \overline{z}_0)$ in the last period. In between these two periods, a tractable specification is to use $\underline{z}_{i+1} = \underline{z}_i - g_{i+1}$ and $\overline{z}_{i+1} = \overline{z}_i + g_{i+1}$ until $\underline{\theta}_n = \underline{z}_{n-1} - g_n$ and $\overline{\theta}_n = \overline{z}_{n-1} + g_n$, where $g_i > 0$ for $i = 0, 1, ..., n-1$.

or shall deferred annuities be offered once or several times to any cohort of annuitants in light of various changes in life-cycle events. We leave this investigation to future research work.

Appendix A Effects of offering deferred annuities when early health signals are heterogeneous

Brugiavini (1993) shows that adverse selection is eliminated if deferred annuities are offered at an early period when annuitants are identical in terms of health condition. It turns out that immediate annuities are completely crowded out in her model. In Section 4, we consider a more general health information revelation process and show that immediate annuities are incompletely crowded out. In this environment, however, adverse selection caused by the heterogeneity of early health signals (z) in Period 0 is still present in deferred annuity purchase, even though the degree of heterogeneity in z is less than that of survival probabilities (θ) in Period 1.

Since adverse selection is present in deferred annuity purchase when early signals are heterogeneous, one may be interested to know whether the amount of annuity purchase will be substantially affected or not. A related question is about the impact on annuitants' welfare. We examine these two issues in this Appendix.

In the above model, annuitants' optimal annuitization choices and the equilibrium annuity prices are mutually dependent. Given the interdependence, analytical results about the annuitants' annuitization amounts and utility levels are difficult to obtain. Instead, we adopt the computational approach in the following analysis.

A.1 Functional forms and parameter values

A key feature of our model is that early health signals are heterogeneous and have predictive power about future survival probabilities. In the computational analysis, we adopt the information revelation process with (2) and (3) such that both z and θ are uniformly distributed. In particular, we assume

$$f_z(z) = \begin{cases} \frac{1}{2g}, & z \in [0.5-g, 0.5+g] \\ 0, & z \notin [0.5-g, 0.5+g] \end{cases}, \tag{A1}$$

where $g < 0.5$.[24] Since the probability density function of θ is related to $f_z(z)$ and $f_{\theta|z}(\theta|z)$ according to (1), our task is to find $f_{\theta|z}(\theta|z)$ such that

$$f_\theta(\theta) = \int_{0.5-g}^{0.5+g} f_{\theta|z}(\theta|z)\, f_z(z)\, dz = \frac{1}{\overline{\theta} - \underline{\theta}}.$$

[24] We also impose $g < \widehat{p}^* - 0.5$, so that $\overline{z} < \widehat{p}^*$ in (20) holds.

Focusing on the class of linear conditional probability density functions, we choose[25]

$$f_{\theta|z}(\theta|z) = \frac{1}{\overline{\theta} - \underline{\theta}} + \frac{\left(z - \frac{\underline{\theta}+\overline{\theta}}{2}\right)}{\frac{1}{3}\left(\overline{\theta}^3 - \underline{\theta}^3\right) - \frac{1}{4}\left(\overline{\theta} - \underline{\theta}\right)\left(\underline{\theta} + \overline{\theta}\right)^2}\left(\theta - \frac{\underline{\theta}+\overline{\theta}}{2}\right). \quad \text{(A4)}$$

Therefore, the distributions of $f_z(z)$ and $f_{\theta|z}(\theta|z)$ based on (A1) and (A4) satisfy conditions (2) and (3),[26] and they are used in the computational analysis.

For the within-period utility function in (8), we use the logarithmic form:

$$u(c) = \ln c. \quad \text{(A5)}$$

In the baseline case, the values of exogenous parameters are set as follows: $\underline{\theta} = 0.05$, $\overline{\theta} = 0.95$, $\underline{z} = 0.4$, $\overline{z} = 0.6$, $w = 50$, $r = 0.3$ and $\rho = 0.28$.

A.2 Annuitization decisions

Using the above parameter values in the baseline exercise, we find that the equilibrium deferred annuity price (q^*) is 0.5055 and the equilibrium immediate annuity price (p^*) is 0.8742. The ranking of these two equilibrium prices is consistent with Proposition 2. These values, together with the equilibrium immediate annuity price before the introduction of deferred annuities ($\widehat{p}^* = 0.6073$), are given in Table 2.

The individuals' annuitization choices are helpful in understanding the relative magnitude of these two annuity prices. In Panel A of Figure 2, we plot the optimal deferred

25 Specifically, we consider linear conditional probability density function

$$f_{\theta|z}(\theta|z) = a\theta + b \quad \text{(A2)}$$

that satisfies two properties, where a and b may depend on z. The first property is that for any value of z, $f_{\theta|z}(\theta|z)$ passes through the point $\left(\frac{\underline{\theta}+\overline{\theta}}{2}, \frac{1}{\overline{\theta}-\underline{\theta}}\right)$. According to (A2),

$$\frac{1}{\overline{\theta} - \underline{\theta}} = a\left(\frac{\underline{\theta}+\overline{\theta}}{2}\right) + b. \quad \text{(A3)}$$

The second property is that $a > 0$ when $z > \frac{\underline{\theta}+\overline{\theta}}{2}$, $a < 0$ when $z < \frac{\underline{\theta}+\overline{\theta}}{2}$, and $a = 0$ when $z = \frac{\underline{\theta}+\overline{\theta}}{2}$. These two features guarantee that it satisfies $\int_{0.5-g}^{0.5+g}(a\theta + b)\,dz = 1$ and is a probability density function. After solving a and b from (A3) and $\int_{\underline{\theta}}^{\overline{\theta}}\theta(a\theta + b)\,d\theta = z$, which arises from (3), we obtain (A4).

26 Equivalently, we can generate z and θ by the joint probability density function

$$f(z,\theta) = f_{\theta|z}(\theta|z)\, f_z(z) = \frac{1}{2g}\left[\frac{1}{\overline{\theta} - \underline{\theta}} + \frac{\left(z - \frac{\underline{\theta}+\overline{\theta}}{2}\right)}{\frac{1}{3}\left(\overline{\theta}^3 - \underline{\theta}^3\right) - \frac{1}{4}\left(\overline{\theta} - \underline{\theta}\right)\left(\underline{\theta} + \overline{\theta}\right)^2}\left(\theta - \frac{\underline{\theta}+\overline{\theta}}{2}\right)\right].$$

Table 2 Effects of introducing deferred annuities

Degree of heterogeneity of early health signals (z)	q^*	p^*	$E\left(\beta_z^*\right)$	$E(\alpha_{z\theta}^*)$	$\frac{AEW-w}{w}$
Baseline case: ($\bar{z}-\underline{z} = 0.2$)	0.5055	0.8742	46.04	0.26	1.75%
Lower degree of heterogeneity: ($\bar{z}-\underline{z} = 0.1$)	0.5013	0.9156	47.25	0.06	1.84%
No heterogeneity: ($\bar{z}-\underline{z} = 0$)	0.5	0.95	47.50	0	1.87%

Notes: For the reference economy, $\hat{p}^* = 0.6073$ and $E\left(\hat{\alpha}_\theta^*\right) = 36.86$ in all three cases

annuity choice (β_z^*) as a function of z. This function is upward-sloping because annuitants with good health signals are more likely to have higher survival probabilities and purchase larger amounts of deferred annuities. We also plot the optimal immediate annuity choice ($\alpha_{z\theta}^*$) as a function of survival probability (θ) for three groups of annuitants: with $z = 0.4$, $z = 0.45$ and $z = 0.5$, respectively.[27] Consistent with the results in Section 3, the optimal immediate annuity choice is 0 when θ is low, but is increasing in θ once it passes the threshold given by (9). The upward-sloping aspect arises because of adverse selection: the annuitants with high survival probability (θ) are high-risk buyers and purchase larger amounts of annuities. Moreover, the threshold $\theta_\alpha(z)$ is increasing in z, because annuitants with higher levels of health signal (z) have already purchased larger amounts of deferred annuities and therefore only annuitants with very high survival probabilities need to buy immediate annuities.

Compared with the case of only immediate annuities are available, the amount of annuities purchased increases quite substantially. Adding up the amounts purchased by all the annuitants, the total amount of deferred annuities purchased is $E\left(\beta_z^*\right) = 46.04$ and the total amount of immediate annuities purchased is $E\left(\alpha_{z\theta}^*\right) = 0.26$. The sum of these two terms, which is dominated by $E\left(\beta_z^*\right)$, is much higher than $E\left(\widehat{\alpha}_\theta^*\right)$ when only immediate annuities are available: $E\left(\beta_z^*\right) + E\left(\alpha_{z\theta}^*\right) = 46.30 > 36.86 = E\left(\widehat{\alpha}_\theta^*\right)$.

The reason of the above results can be traced back to the reduction of the severity of adverse selection when deferred annuities are introduced, as in the model by Brugiavini (1993), when early health signals are homogeneous. Our computational analysis shows that this result continues to hold (but with slightly weaker magnitude) when early signals are heterogeneous. In this case, even though there is still some adverse selection in the deferred annuity purchase (since $q^* > E(\theta) = 0.5$), it can be seen that $q^* = 0.5055$ is much lower than $\widehat{p}^* = 0.6073$ (before deferred annuities are introduced). As a result, the amount of $E\left(\beta_z^*\right) = 46.04$ is much larger than $E\left(\widehat{\alpha}_\theta^*\right) = 36.86$. In addition, a small proportion of annuitants also purchase immediate annuities.

We conclude that introducing deferred annuities leads to a higher level of overall annuity purchase even if early health signals are heterogeneous.

[27] Note that $\alpha_{z\theta}^* = 0$ for all values of θ when $z = \bar{z} = 0.6$ in this baseline exercise.

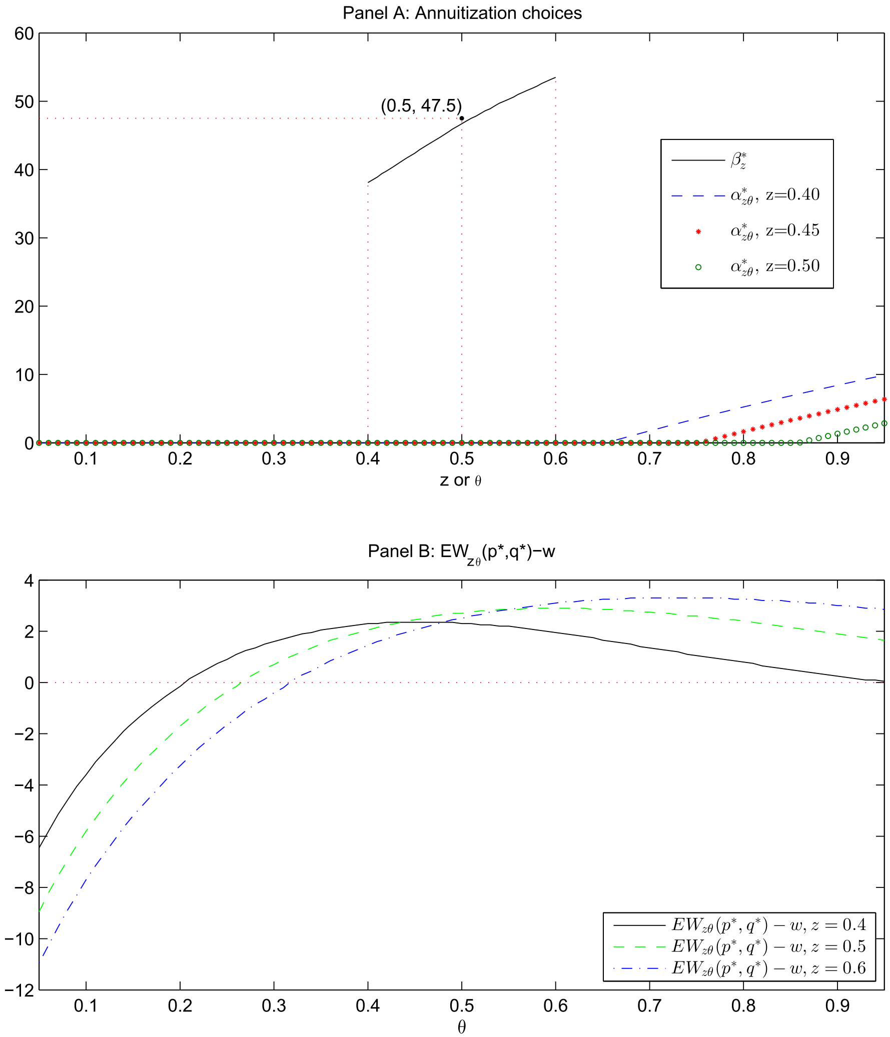

Fig. 2 Annuitization choices and welfare

A.3 Annuitants' welfare

We now examine whether the annuitants gain or lose after the deferred annuity plan is introduced, and we focus on the annuitant's utility level.

A typical way to examine the effect of economic policy intervention (such as the introduction of deferred annuities) is to apply the concept of equivalent wealth based on utility comparison. In the literature about annuity purchases, various versions of equivalent wealth have been used in Mitchell et al. (1999), Brown (2001) and Lau and Ying (2024).

We first focus on the annuitant's utility level in Period 1, after both early health signal (z) and survival probability (θ) are known. Define $EW_{z\theta}\left(p^*, q^*\right)$ as the hypothetical level of wealth of annuitant $z\theta$ such that[28]

$$\widehat{U}^*_{z\theta}\left(EW_{z\theta}\left(p^*, q^*\right), \widehat{p}^*\right) = U^*_{z\theta}(w, p^*, q^*). \tag{A6}$$

The left-hand side term of (A6) is the maximized utility level with wealth $EW_{z\theta}\left(p^*, q^*\right)$ and immediate annuity price $\widehat{p}^*$ according to (17) in the reference economy (with immediate annuities only), and the right-hand side term is the maximized utility level according to (8) after deferred annuities are introduced and the two annuity prices are p^* and q^*.

The definition of (A6) is based on the hypothetical policy of directly transferring cash to (or withdrawing cash from) the annuitants, instead of actually introducing deferred annuities. It has the interpretation that the policy intervention of introducing deferred annuities on the welfare of annuitant $z\theta$ is equivalent to that of cash transfer/withdrawal at the level of $EW_{z\theta}\left(p^*, q^*\right) - w$ if there is no policy change. When $EW_{z\theta}\left(p^*, q^*\right) - w$ is positive, the government has to transfer wealth to annuitants to make them indifferent to the policy change. This implies that introducing deferred annuities benefits annuitants $z\theta$. On the other hand, a negative value of $EW_{z\theta}\left(p^*, q^*\right) - w$ implies that this policy intervention adversely affects annuitants $z\theta$.

For three selected levels of early health signals ($z = 0.4$, 0.5 and 0.6), we plot the $EW_{z\theta}\left(p^*, q^*\right) - w$ (i.e., the difference of equivalent wealth and actual wealth of an annuitant with z and θ) against θ in Panel B of Figure 2. It can be seen that for a particular value of z, $EW_{z\theta}\left(p^*, q^*\right) - w$ is negative for low values of θ, but it increases to become positive after passing some threshold level of θ.[29] We also observe that the function $EW_{z\theta}\left(p^*, q^*\right) - w$ shifts to the right when z is higher, but all three functions show a similar pattern of starting from a negative value when θ is low and then increasing to become positive after passing some threshold levels of θ.

We extend the above concept to the whole group by defining aggregate equivalent wealth for the annuitants as

$$AEW\left(p^*, q^*\right) = \int_{\underline{z}}^{\overline{z}} \int_{\underline{\theta}}^{\overline{\theta}} EW_{z\theta}\left(p^*, q^*\right) f\left(\theta, z\right) dz d\theta. \tag{A7}$$

Based on (A7), we calculate the percentage change of aggregate equivalent wealth (when compared with the original wealth level) as

$$\frac{AEW\left(p^*, q^*\right) - w}{w}. \tag{A8}$$

28 The definition of equivalent wealth involves both initial and final states. The price $\widehat{p}^*$ on the left-hand side of (A6) indicates that the initial state is "immediate annuity only," and the two prices (p^* and q^*) in the equivalent wealth term $EW_{z\theta}\left(p^*, q^*\right)$ indicate the final state after the introduction of the deferred annuity.

29 Annuitants who receive a low value of θ (and thus a downward revision of $\theta - z$, given the value of z) will be adversely affected because the level of deferred annuities purchased in Period 0 turns out to be too high in light of the value of θ revealed in Period 1. A similar pattern is found in Lau and Ying (2024, Figure 3).

As shown in Table 2, the percentage change in aggregate equivalent wealth is $\frac{AEW(p^*,q^*)-w}{w} = 1.75\%$, indicating that offering deferred annuities is welfare improving for the annuitants in the model with heterogeneous early signals.

Once again, the reason behind the welfare improvement is traced to the benefit of offering deferred annuities at an early age. Although adverse selection still appears in deferred annuity purchase (with $q^* > E(\theta)$) when early health signals are heterogeneous, the severity of adverse selection is quite small, as indicated by the difference between $q^* = 0.5055$ and $E(\theta) = 0.5$. Compared with the relatively high value of $\widehat{p}^* = 0.6073$ of the reference economy, the low deferred annuity price q^* leads to beneficial effects to the annuitants.

A.4 Different degrees of heterogeneity of early signals

In the baseline exercise presented above, the range of early health signals (z) is [0.4, 0.6] and that of survival probabilities (θ) is [0.05, 0.95], representing that private health information is gradually revealed over time, with a partial revelation at an early age and a full revelation at an advanced age. We now examine the effects of different degrees of heterogeneity of early health signals on annuitants' behavior and utility levels. Specifically, we keep the range of survival probabilities at [0.05, 0.95], but change the spread of early health signals. We present two cases in addition to the baseline exercise. They are (a) lower degree of heterogeneity with $\underline{z} = 0.45$ and $\overline{z} = 0.55$, and (b) homogeneous early signals (as in Brugiavini 1993) with $\underline{z} = \overline{z} = 0.5$.

In Table 2, we present the numerical results for different degrees of heterogeneity of early signals. For the reference economy, $\widehat{p}^* = 0.6073$ and $E\left(\widehat{\alpha}_\theta^*\right) = 36.86$ are the same for each of the three cases.[30]

In all three cases, we observe similar patterns regarding the ranking of annuity prices, increased annuitization, and improved welfare. When the spread of early signals is [0.45, 0.55], the equilibrium price of the deferred annuity is $q^* = 0.5013$, and the equilibrium price of the immediate annuity is $p^* = 0.9156$. The aggregate annuitization level is $E\left(\beta_z^*\right) + E\left(\alpha_{z\theta}^*\right) = 47.31$. Aggregate equivalent wealth is improved by 1.84%. In the homogeneous early signal case, the corresponding price levels are $q^* = 0.5$ and $p^* = 0.95$. The total annuitization level is $E\left(\beta_z^*\right) + E\left(\alpha_{z\theta}^*\right) = 47.50$. Aggregate equivalent wealth is improved by 1.87%. For all three cases in Table 2, we observe that the relationship $q^* < \widehat{p}^* < p^*$ holds, total annuitization level is increased, and aggregate equivalent wealth is improved. The benefits of offering deferred annuities continue to hold when heterogeneities in early health signals are present.

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30 In each case, changing the spread of health signals in Period 0 does not change the distribution of survival probability revealed in Period 1. As a result, $\widehat{p}^*$ and $E\left(\widehat{\alpha}_\theta^*\right)$ remain unchanged.

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