

Twin-Models-Based Controller Output Observer for Vehicle Dynamics Estimation: Concept, Design and Validation^{*}

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Abstract: This paper presents a new concept for vehicle dynamics estimation based on the *twin-models-based controller output observer*. The objective is to estimate lateral speed, considered as an unmeasured state, as well as longitudinal wheel torque and steering torque, which are treated as vehicle unknown inputs (UIs). The central idea is to recast the estimation problem as a reference tracking control scheme composed of two model-based control loops. In the first loop, Takagi-Sugeno (TS) modeling is used to capture time-varying vehicle dynamics, and a state feedback controller is designed for tracking. This loop provides initial estimates of vehicle states and UIs. To address the mismatch between the TS model and the actual vehicle system, we introduce a second control loop featuring a static output feedback (SOF) compensator that offers a correction term based on a nominal vehicle model. We demonstrate that the control inputs and states of the nominal vehicle model in this correction loop can effectively serve as estimates of the vehicle UIs and states. The control design in both loops is formulated as convex optimization problems under linear matrix inequality (LMI) constraints. The proposed vehicle estimation method is validated using experimental data from the INSA autonomous vehicle on a real test track.

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Keywords: Vehicle dynamics estimation, torque estimation, autonomous vehicles, EPS systems, Takagi–Sugeno fuzzy systems, controller output observers.

1. INTRODUCTION

Autonomous vehicles have generated significant interest from both research communities and industries. They have the potential to transform transportation systems by preventing deadly crashes, providing mobility for people with physical disabilities, and reducing environmental pollution (Fagnant and Kockelman, 2015). For safety, it is essential to have real-time and accurate information about vehicle state variables and control actions. However, onboard sensors are not always available due to economic or technical reasons (Tian et al., 2024). As a result, there is growing interest in developing estimation algorithms that reliably reconstruct vehicle dynamics using data from low-cost sensors (Ghorai et al., 2022).

Estimating vehicle lateral speed and sideslip angle is crucial for optimizing vehicle handling and improving lateral stability. However, the high cost of commercial sensors for measuring these vehicle variables presents a significant barrier to their

integration in mass-production vehicles. To address this issue, various estimation methods have been developed, including Kalman filtering techniques (Wang et al., 2022), kinematic-based approaches (Selmanaj et al., 2017), nonlinear observers (Zhang et al., 2016; Pan et al., 2022), and neural network-based observers (Nguyen et al., 2023). More details on the latest developments in this area can be documented in the recent survey paper (Tian et al., 2024). Nevertheless, most model-based estimation methods overlook longitudinal-lateral coupling, primarily due to the challenges in filter and observer design in the presence of nonlinear dynamics and unknown disturbances (Zhang et al., 2016; Nguyen et al., 2021). This oversight can significantly compromise estimation performance, especially in critical driving situations. Furthermore, existing estimation methods often require fine-tuning to achieve satisfactory performance, which can hinder their practical application in industrial contexts (Selmanaj et al., 2017).

Steering torque is crucial for controlling vehicle direction and serves as a fundamental input for path tracking, planning, active safety, and driver failure detection in intelligent vehicles (Rajamani, 2011; Nguyen et al., 2024b). However, low-cost torque sensors can yield inaccurate measurements or faulty steering signals (Wang et al., 2018; Pan et al., 2022). Similarly, longitudinal wheel torque is vital for applications like brake torque control, speed control, and adaptive cruise control (Moon et al.,

^{*} This work is supported by the PROCORE-France/Hong Kong Joint Research Scheme, by the HM-Science ANR-21-CE48-0021, by the RITMEA project, by the Hauts-de-France Region, by the European Community, by the Ministry of Higher Education and Research, by the French National Center for Scientific Research, by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Brasil - Finance Code 001, and by the CAPES-COFECUB program (Grant numbers: 88887.987117/2024-00, Ma1076/25, 52828NB).

2009). Yet, directly measuring this variable in commercial cars poses economic and technical challenges (Nguyen et al., 2021). As a result, estimating steering and longitudinal wheel torques has gained increasing research interest, leveraging techniques based on steady-state look-up tables or vehicle dynamics models (Franco et al., 2008; Alcantar et al., 2018; Nguyen et al., 2021).

Despite significant advances in vehicle estimation techniques, the *joint* estimation of the lateral speed (considered an unmeasured vehicle state), steering torque and longitudinal torque (treated as vehicle unknown inputs) has not been addressed. This estimation solution is important because the estimated lateral speed can aid in active vehicle safety control (Rajamani, 2011), while the estimates of steering and longitudinal wheel torques are valuable for fault detection and diagnosis in intelligent automotive systems (Shi and Zhang, 2020). Motivated by these issues, we develop a new estimation method based on the controller output observer concept from (Alcantar et al., 2018). This approach reformulates the estimation problem as a reference tracking control problem, enabling a simple and effective scheme. The proposed *twin-models-based controller output observer* consists of two control loops: the Main Loop (ML) and the Correction Loop (CL). Vehicle measurements from onboard low-cost sensors serve as reference signals for both control loops. In the ML, a state-feedback tracking controller is designed using a Takagi-Sugeno (TS) model of the vehicle system. When the TS model accurately represents the vehicle dynamics, the ML handles most of the estimation task and provides initial estimates of vehicle states and UIs. Inspired by the simple adaptive control scheme (Kaufman et al., 1998, Chapter 2), the CL is integrated to address the inevitable mismatch between vehicle models and the actual system. A static output feedback (SOF) compensator is designed in the CL based on a nominal vehicle model to provide a correction term, enhancing estimation performance against model-to-reality discrepancies. The main contributions of the paper are summarized as follows.

- We propose a cost-effective solution that solely uses on-line data from low-cost sensors with reduced fine-tuning effort to jointly estimate vehicle lateral speed, steering torque, and longitudinal torque.
- Compared to the related results in (Alcantar and Assadian, 2018; Alcantar et al., 2018), the new twin-models-based controller output observer enhances estimation performance in the presence of modeling uncertainties. Additionally, Lyapunov stability theory is employed to ensure robustness performance. The tracking control design for both ML and CL is reformulated as an LMI-based optimization problem, allowing for the convenient obtaining of estimation solutions using standard numerical solvers.
- The proposed estimation scheme is validated using experimental data obtained from real-world driving scenarios with an autonomous vehicle on a test track, see Fig. 1.

Notation. For a matrix A , A^T denotes the transpose of A , $\text{He}(A) = A + A^T$. A positive definite matrix A is denoted by $A > 0$. $\text{diag}(A_1, A_2)$ is the block-diagonal matrix composed of A_1, A_2 . An identity matrix of dimension n is denoted by I_n , and a zero matrix of dimension $m \times n$ is denoted by $0_{m \times n}$. Symbol $*$ represents the terms generated by symmetry in block matrices. Arguments are omitted when there is no confusion.

2. VEHICLE MODELING

This section reviews the modeling of the INSA autonomous vehicle, shown in Fig. 1, for observer design. The vehicle parameters are listed in Table 1, and its nonlinear dynamics can be described as follows (Rajamani, 2011; Nguyen et al., 2023):

$$\begin{aligned} m_v(\dot{v}_x - y_r v_y) &= \frac{T_w}{R_t} - F_{ax} \\ m_v(\dot{v}_y + y_r v_x) &= F_{yf} + F_{yr} \\ I_z \dot{y}_r &= l_f F_{yf} - l_r F_{yr} \end{aligned} \quad (1)$$

where v_x is the vehicle longitudinal speed, v_y is the vehicle lateral speed, y_r is the vehicle yaw rate, δ is the front wheel steering angle, $F_{ax} = 0.5 C_{dx} \rho_a A_{fx} v_x^2$ is the longitudinal aerodynamic drag force. Assuming small angles and no longitudinal slip, the lateral tire forces F_{yf} and F_{yr} on the front and rear tires can be modeled as (Rajamani, 2011)

$$F_{yf} = 2C_f \alpha_f, \quad F_{yr} = 2C_r \alpha_r \quad (2)$$

where the sideslip angles α_f and α_r for the front and rear tires are given by

$$\alpha_f = \delta - \frac{v_y + l_f y_r}{v_x}, \quad \alpha_r = \frac{l_r y_r - v_y}{v_x}. \quad (3)$$

The EPS dynamics is described as (Nguyen et al., 2024b)

$$I_s R_s \ddot{\delta} + R_s B_s \dot{\delta} = T_s - \frac{2K_p C_f \eta_t}{R_s} \left(\delta - \frac{v_y + l_f y_r}{v_x} \right). \quad (4)$$

Then, from (1)–(4), the nonlinear vehicle dynamics can be rewritten in the form

$$\Sigma: \dot{x} = A(v_x, y_r)x + Bu \quad (5)$$

where $x = [v_y \ y_r \ v_x \ \delta]^T \in \mathbb{R}^{n_x}$ is the vehicle state, $u = [T_w \ T_s]^T \in \mathbb{R}^{n_u}$ is the vehicle input, with $n_x = 5$ and $n_u = 2$. The state-space matrices of system Σ are given by

$$\begin{aligned} A(v_x, y_r) &= \begin{bmatrix} -\frac{a_{11}}{v_x} & \frac{a_{12}}{v_x} & -y_r & 2\frac{C_f}{m_v} & 0 \\ \frac{a_{21}}{v_x} & -\frac{a_{22}}{v_x} & 0 & 2\frac{C_f l_f}{I_z} & 0 \\ y_r & 0 & -C_x v_x & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{a_f}{v_x} & \frac{l_f a_f}{v_x} & 0 & -a_f & -\frac{B_s}{I_s} \end{bmatrix} \\ B^T &= \begin{bmatrix} 0 & 0 & \frac{1}{R_t m_v} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{I_s R_s} & 0 \end{bmatrix}, \quad a_f = \frac{2K_p C_f \eta_t}{I_s R_s^2} \\ a_{11} &= \frac{2(C_f + C_r)}{m_v}, \quad a_{12} = \frac{2(C_r l_r - C_f l_f)}{m_v} \\ a_{21} &= \frac{2(C_r l_r - C_f l_f)}{I_z}, \quad a_{22} = \frac{2(C_f l_f^2 + C_r l_r^2)}{I_z} \\ C_x &= \frac{C_{dx} \rho_a A_{fx}}{2m_v}. \end{aligned}$$

Due to the physical limitations related to the driving conditions performed with the test track, the state variables of the INSA autonomous vehicle are bounded as (Nguyen et al., 2024a)

$$x \in \mathcal{D}_x = \{v_y \in [\underline{v}_y, \bar{v}_y], y_r \in [\underline{y}_r, \bar{y}_r], v_x \in [\underline{v}_x, \bar{v}_x]\} \quad (6)$$

with $\bar{v}_y = -\underline{v}_y = 1.5$ [m/s], $\bar{y}_r = -\underline{y}_r = 1.1$ [rad/s], $\underline{v}_x = 1.5$ [m/s] and $\bar{v}_x = 20$ [m/s].

As in practice, we assume that vehicle speed v_x and yaw rate y_r can be measured cost-effectively. The steering angle δ and

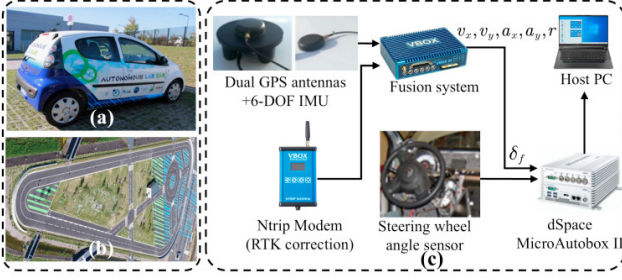


Fig. 1. Experimental facilities. (a) INSA test vehicle, (b) Gyrovia test track, (c) Data acquisition system.

Table 1. Vehicle parameters.

Symbol	Description	Value
m_v	Vehicle mass	1077 [kg]
C_f	Front cornering stiffness	47135 [N/rad]
C_r	Rear cornering stiffness	56636 [N/rad]
l_f	Distance from the CoG to the front axle	1.08 [m]
l_r	Distance from the CoG to the rear axle	1.24 [m]
I_z	Vehicle yaw moment of inertia	1442 [kgm ²]
R_t	Tires effective rolling radius	0.26 [m]
ρ_a	Air density	1.23 [kg/m ³]
C_{dy}	Lateral drag coefficient	0.35 [-]
C_{dx}	Longitudinal drag coefficient	0.32 [-]
A_{fx}	Longitudinal frontal area	1.97 [m ²]
η_t	Tire length contact	0.1 [m]
R_s	Steering gear ratio	10 [-]
K_p	Manual steering column coefficient	0.1 [-]
B_s	Steering system damping	4.03 [-]
I_s	Inertia of the steering system	0.05 [kgm ²]

rate $\dot{\delta}$ are obtained via an onboard optical encoder (Nguyen et al., 2024b). However, lateral speed v_y measurement is often unavailable for commercial vehicles due to high sensor costs (Tian et al., 2024). Thus, the output equation of system (5) is defined as

$$y = Cx, \quad C = [0_{n_y \times 1} | I_{n_x - 1}] \quad (7)$$

with $n_y = 4$. The longitudinal wheel torque T_w and the steering torque T_s are assumed to be immeasurable. Consequently, the vehicle unknown input u must be estimated.

3. ESTIMATION PROBLEM FORMULATION

This section introduces the proposed scheme for estimating the unknown input and unmeasurable state of the vehicle system. The core idea is to frame the estimation problem as a reference tracking control problem, utilizing two loops: the Main Loop and the Correction Loop, which are detailed below. Then, the estimation problem is formulated.

3.1 Main Loop

The ML consists of a state feedback tracking controller, designed using a TS model of the vehicle system (5). To build the TS model while reducing numerical complexity and conservatism, we employ the same variable change and first-order Taylor approximation as in (Nguyen et al., 2024a):

$$\begin{aligned} \frac{1}{v_x} &= \frac{1}{v_0} + \frac{1}{v_1} \xi, & v_x &\simeq v_0 \left(1 - \frac{v_0}{v_1} \xi \right) \\ v_0 &= \frac{2\bar{v}_x v_x}{v_x + \bar{v}_x}, & v_1 &= \frac{2\bar{v}_x v_x}{v_x - \bar{v}_x}. \end{aligned} \quad (8)$$

The new variable $\xi \in [-1, 1]$ is used to characterize the variations of v_x within its lower and upper limits. Note that using (8), the matrix $A(v_x, y_r)$ in (5) now linearly depends on

each element of the vector of premise variables $z = [z_1 \ z_2]^T$, with $z_1 = \xi$ and $z_2 = y_r$. Note that each premise variable can be equivalently rewritten as $z_i = \phi_1^i(z_i)\bar{z}_i + \phi_2^i(z_i)\underline{z}_i$, where

$$\phi_1^i(z_i) = \frac{z_i - \underline{z}_i}{\bar{z}_i - \underline{z}_i}, \quad \phi_2^i(z_i) = 1 - \phi_1^i(z_i), \quad i \in \{1, 2\} \quad (9)$$

where $\underline{z}_i$ and $\bar{z}_i$ are respectively the lower and upper bounds of z_i , for $i = 1, 2$. Consequently, using the sector nonlinearity approach (Tanaka and Wang, 2001, Chapter 2), system (5) together with the change of variable (8) can be modeled by the following TS form:

$$\Sigma: \begin{cases} \dot{x}(t) = A(h)x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (10)$$

with $A(h) = \sum_{i=1}^4 h_i(z)A_i$, where the constant matrices A_i , for $i \in \{1, \dots, 4\}$, are obtained from $A(\xi, y_r)$ with the corresponding upper and lower bounds of ξ and y_r . The membership function, defined as $h_i(z) = \prod_{j=1}^2 \phi_{i_j}^j(z_j)$, $i_j \in \{1, 2\}$, satisfies the convex sum property $\sum_{i=1}^4 h_i(z) = 1$, $h_i(z) \geq 0$. The detailed expressions of A_i and h_i , for $i \in \{1, \dots, 4\}$, are omitted for brevity. Motivated by (Alcantar and Assadian, 2018), the controller output observer corresponding to system Σ in (10) is defined as

$$\hat{\Sigma}: \begin{cases} \dot{\hat{x}}(t) = A(h)\hat{x}(t) + B\hat{u}(t) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (11)$$

where $\hat{x}$, $\hat{y}$ and $\hat{u}$ are, respectively, estimates of x , y and u . The goal of the estimation here is to design $\hat{u}$ such that $\hat{x}(t) \rightarrow x(t)$ and $\hat{u}(t) \rightarrow u(t)$, when $t \rightarrow \infty$. However, as mentioned previously, v_y is not available for the state tracking control purposes. To avoid this issue, the idea of “vehicle inverse model” is used. To this end, an “open-loop” estimate $\tilde{v}_y$ of v_y can be obtained using the nonlinear dynamic equation of v_y in (5) as

$$\dot{\tilde{v}}_y = -\frac{a_{11}}{v_x} \tilde{v}_y + \frac{a_{12}}{v_x} y_r - y_r v_x + 2 \frac{C_f}{m_v} \delta. \quad (12)$$

Then, the reference vector to be tracked in the main loop is defined as $\tilde{x}_r = [\tilde{v}_y \ y_r \ v_x \ \delta \ \dot{\delta}]^T$. As a result, the goal is to design a control law $\hat{u}(t)$ such that

$$\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} (\hat{x}(t) - \tilde{x}_r(t)) \rightarrow 0.$$

The error dynamics $\dot{e}(t) = \dot{\hat{x}}(t) - \dot{\tilde{x}}_r(t)$ can be defined from (11) as

$$\dot{e}(t) = A(h)\hat{x}(t) + B\hat{u}(t) - \dot{\tilde{x}}_r(t). \quad (13)$$

Now, adding and subtracting the quantity $A(h)\tilde{x}_r(t)$ to the expression $\dot{e}(t)$ in (13), it follows that

$$\dot{e}(t) = A(h)e(t) + B\hat{u}(t) - \dot{\tilde{x}}_r(t) + A(h)\tilde{x}_r(t). \quad (14)$$

Note that we have the access of all states of system $\hat{\Sigma}$ for state tracking control. Let us define

$$\hat{u}(t) = \hat{u}_{fb}(t) + \hat{u}_{ff}(t) \quad (15)$$

where the feedback control $\hat{u}_{fb}(t)$ and the feedforward control $\hat{u}_{ff}$ are given by

$$\hat{u}_{fb}(t) = K(h)e(t), \quad \hat{u}_{ff}(t) = B^\dagger(\dot{\hat{x}}(t) - A(h)\tilde{x}_r(t)) \quad (16)$$

where $K(h) = \sum_{i=1}^4 h_i(z)K_i$. Note that the term $\dot{\hat{x}}(t) - A(h)\tilde{x}_r(t)$ in (14) can be seen as a *known* disturbance, and the feedforward control part $\hat{u}_{ff}(t)$ is used to mitigate its effects on the tracking error dynamics. It follows from (14)–(16) that

$$\dot{e}(t) = (A(h) + BK(h))e(t). \quad (17)$$

Remark 1. Note that if $K(h)$ is designed to stabilize the error system (17), and if there are no modeling errors, then

$\hat{x}(t) \rightarrow \tilde{x}_r(t) \rightarrow x(t)$ as $t \rightarrow \infty$. Based on the idea of a controller output observer, we have $\Sigma \rightarrow \hat{\Sigma}$, which implies that $\hat{u}(t) \rightarrow u(t)$. Similar to the twin-in-the-loop concept in (Dettù et al., 2024), if model Σ in (10) accurately represents the vehicle system, the main loop handles most of the estimation. In this case, the compensator in the correction loop functions primarily as a tracking controller for a linearized system near the operating point.

3.2 Correction Loop

Modeling errors are unavoidable in practice, such as parametric uncertainties and external disturbances. Therefore, the correction loop of the proposed twin-model-based estimation scheme is implemented to mitigate their adverse effects on estimation performance. The tracking control design in this loop is based on the following nominal vehicle model Σ_0 , derived from Jacobian linearization of system Σ around the operating point $x_0 = [0 \ 0 \ 10 \ 0 \ 0]^T$:

$$\Sigma_0 : \begin{cases} \dot{x}_0(t) = A_0 x_0(t) + B u_0(t) \\ y_0(t) = C x_0(t) \end{cases} \quad (18)$$

The control input $u_0(t)$ applied to model Σ_0 is defined as

$$u_0(t) = \hat{u}(t) + s(t) \quad (19)$$

where the SOF compensation law $s(t)$ is designed such that

$$\lim_{t \rightarrow \infty} e_0(t) \rightarrow 0$$

with $e_0(t) = x_0(t) - x(t)$. Note that

$$\dot{e}_0(t) = A_0 x_0(t) + B u_0(t) - \dot{x}(t). \quad (20)$$

Adding and subtracting the quantity $A_0 x(t)$ and replacing $u_0(t)$ by (19) in (20), the error dynamics becomes

$$\dot{e}_0(t) = A_0 e_0(t) + B \hat{u}(t) + B s(t) - \dot{x}(t) + A_0 x(t). \quad (21)$$

Replacing $\hat{u}(t)$ in (21) by its expression (15), it follows that

$$\dot{e}_0(t) = A_0 e_0(t) + B K(h) e(t) + B s(t) + w(t) \quad (22)$$

where $w(t) = -\dot{x}(t) + A_0 x(t) + B \hat{u}(t)$ is a disturbance, which represents the modeling errors around the operating point. We define the SOF compensation signal $s(t)$ as

$$s(t) = L e_y(t) \quad (23)$$

where $e_y(t) = y_0(t) - y(t)$, and the feedback gain $L \in \mathbb{R}^{n_u \times n_y}$ is to be determined. Note that the error $e_y(t)$ is used to “measure” online the modeling mismatch between the model Σ_0 and the real vehicle system.

3.3 Problem Formulation

We define $\tilde{e}(t) = [e_0^T(t) \ e^T(t)]^T$. Then, the dynamics of the augmented tracking error can be obtained from (17), (22) and (23) as

$$\Sigma_e : \dot{\tilde{e}}(t) = \tilde{A}(h) \tilde{e}(t) + \tilde{B} w(t) \quad (24)$$

with

$$\tilde{A}(h) = \begin{bmatrix} A_0 + BLC & BK(h) \\ 0_{n_x} & A(h) + BK(h) \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} I_{n_x} \\ 0_{n_x} \end{bmatrix}.$$

To minimize the disturbance effects on tracking control performance, we consider a performance vector associated with system Σ_e of the form

$$\tilde{z} = \tilde{C} \tilde{e}, \quad \tilde{C} = [C_z \ 0_{n_x}] \quad (25)$$

where C_z is to be specified for tracking control design. We are now ready to formalize the estimation problem.

Problem 1. Consider the extended tracking error system Σ_e , defined in (24). Design the state feedback control gain $K(h)$ at the ML and the SOF compensator gain L at the CL such that Σ_e verifies the following closed-loop specifications.

- If $w(t) \neq 0$, for $t \geq 0$, the augmented tracking error $\tilde{e}(t)$ converges exponentially towards the origin with a decay rate less than a predefined $\alpha > 0$.
- If $w(t) \neq 0$, for $t \geq 0$, there exists a scalar $\gamma > 0$ such that system Σ_e is stable. Moreover, the augmented tracking error satisfies $\mathcal{L}_2$ performance $\|\tilde{z}(t)\|_2 \leq \gamma \|w(t)\|_2$.

Remark 2. It is important to note that the proposed scheme reformulates the simultaneous estimation of vehicle and UIs as a tracking control problem. We will show that the state x_0 and the input u_0 of system Σ_0 can effectively serve as estimates of the vehicle state and UIs, respectively.

4. TWIN-MODELS-BASED CONTROLLER OUTPUT OBSERVER DESIGN

The following theorem provides LMI conditions to simultaneously design the parameter-dependent state-feedback control gain $K(h)$ in the ML and the static feedback compensator gain L in the CL, ensuring that the tracking errors dynamics (24) satisfy the properties defined in Problem 1.

Theorem 1. Consider the augmented error system Σ_e , defined in (24). If there exist positive scalars α and γ , symmetric positive definite matrices $W_1 \in \mathbb{R}^{n_x \times n_x}$ and $W_2 \in \mathbb{R}^{n_x \times n_x}$, matrices $G_i \in \mathbb{R}^{n_u \times n_x}$, $U \in \mathbb{R}^{n_u \times n_y}$ and $V_1 \in \mathbb{R}^{n_y \times n_y}$, such that the following LMIs hold:

$$V_1 C = C W_1 \quad (26)$$

$$\begin{bmatrix} \Psi_{11} & W_1 C_z^T & B G_i & I_{n_x} \\ * & -I_{n_x} & 0_{n_x} & 0_{n_x} \\ * & * & \Psi_{33i} & 0_{n_x} \\ * & * & * & -\gamma^2 I_{n_x} \end{bmatrix} < 0 \quad (27)$$

for $i \in \{1, \dots, 4\}$, with

$$\Psi_{11} = \text{He}(A_0 W_1 + B U C + \alpha W_1)$$

$$\Psi_{33i} = \text{He}(A_i W_2 + B G_i + \alpha W_2).$$

Then, the state-feedback gain $K(h) = G(h) W_2^{-1}$ and the SOF gain $L = U V_1^{-1}$ ensure that the augmented error system (24) satisfies the closed-loop properties defined in Problem 1. Consequently, $u_0(t)$ estimates the unknown input $u(t)$ of the vehicle system, while $x_0(t)$ estimates the vehicle state $x(t)$.

Proof. The proof of Theorem 1 is based on the convexification procedure in (Crusius and Trofino, 1999) and the well-known bounded real lemma (Boyd et al., 1994), which is omitted here for brevity.

Remark 3. The matrix equality (26) may cause numerical issues when solving the LMI design conditions in Theorem 1. To address this, we can replace (26) with the following strict LMI:

$$\begin{bmatrix} \beta I_{n_y} & * \\ V_1 C - C W_1 & I_{n_y} \end{bmatrix} > 0. \quad (28)$$

where β is an arbitrarily small positive scalar. Indeed, applying Schur complement lemma to (28), it follows that

$$(V_1 C - C W_1)^T (V_1 C - C W_1) < \beta I_{n_y} \quad (29)$$

or

$$\|V_1 C - C W_1\| < \beta. \quad (30)$$

With a sufficiently small β , it follows directly from condition (30) that $V_1 C \approx C W_1$.

5. VALIDATION AND DISCUSSION

This section presents validation using real vehicle data collected from the test facilities in Fig. 1 under two driving scenarios, demonstrating the effectiveness of the proposed twin-model-based controller output observer. Solving LMI conditions (27) and (28) with $\alpha = 0.1$, $\beta = 10^{-5}$, $C_z = \text{diag}(1, 0, 1, 0, 0)$ and $\gamma = 5$, we can then obtain the feedback gains $K(h)$ and L to jointly estimate the unknown inputs T_w and T_s and the unmeasured lateral speed v_y .

5.1 Scenario 1: Normal driving test

For this test, the vehicle travels on a section of the Gyrovia test track under normal conditions, following a trajectory with smooth turns, as shown in Fig. 2(a). The corresponding vehicle longitudinal speed for this scenario is depicted in Fig. 2(b).

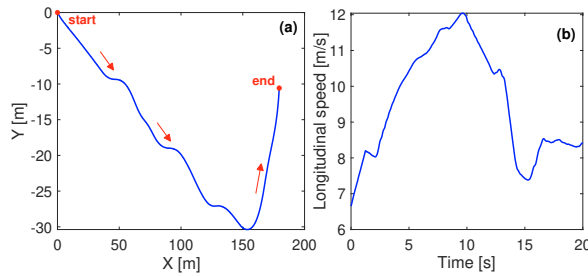


Fig. 2. Scenario 1. (a) Vehicle trajectory, (b) vehicle speed.

The estimation results of the lateral speed v_y , and the wheel torque T_w and the steering torque T_s are shown in Figs. 3(a)–3(c). We can see that the proposed method provides a satisfactory estimation performance for both the unmeasured state and the UIs. Observe also that, as expected, the estimation performance of the ML can be further improved with the CL. It is important to note that using only the ML for estimation is analogous to the classical controller output observer concept in (Alcantar and Assadian, 2018; Alcantar et al., 2018). Table 2 summarizes the error indices obtained with the proposed estimation scheme to provide a quantitative performance analysis. Note that MAE is the mean absolute error and RMSE is the root mean squared error, while NMAE and NRMSE are their normalized values calculated based on the difference between the maximum and minimum values of the respective variable. The quantitative analysis confirms that the proposed method can accurately estimate both the unmeasured state and the UIs of the vehicle, as well as the improvement in estimation performance provided by the correction loop.

Table 2. Performance metrics for Scenario 1.

Loop	Error index	MAE	NMAE [%]	RMSE	NRMSE [%]
ML	v_y [m/s]	0.0555	5.5235	0.0757	7.5282
	T_w [Nm]	103.2667	8.0620	145.4161	11.3527
	T_s [Nm]	1.0086	1.8758	1.3957	2.5958
CL	v_y [m/s]	0.0501	4.9877	0.0676	6.7227
	T_w [Nm]	80.7669	6.3055	123.3704	9.6316
	T_s [Nm]	1.0641	1.9790	1.3924	2.5896

5.2 Scenario 2: Driving with various sharp turns

For this test, the vehicle is driven on road sections featuring various sharp turns. The corresponding vehicle trajectory and

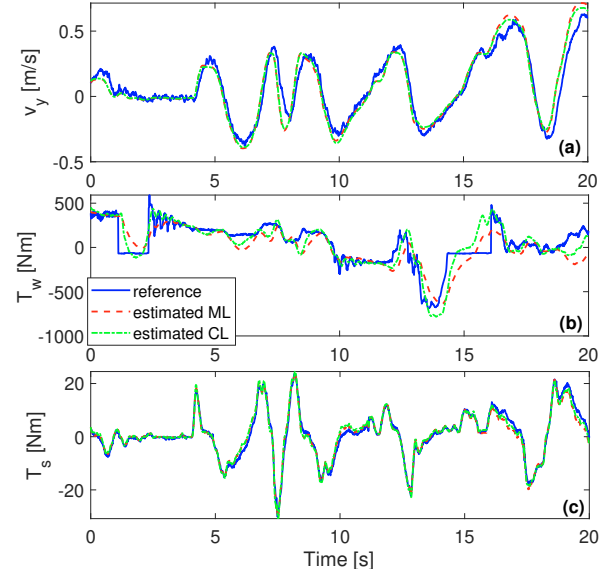


Fig. 3. Estimation performance obtained with Scenario 1. (a) Lateral speed, (b) wheel torque, (c) steering torque.

Table 3. Performance metrics for Scenario 2.

Loop	Error index	MAE	NMAE [%]	RMSE	NRMSE [%]
ML	v_y [m/s]	0.0521	3.4598	0.0678	4.5065
	T_w [Nm]	92.8806	8.6183	132.0285	12.2508
	T_s [Nm]	1.0142	1.4763	1.3646	1.9863
CL	v_y [m/s]	0.0512	3.4054	0.0675	4.4835
	T_w [Nm]	57.2315	5.3104	75.1404	6.9722
	T_s [Nm]	1.1721	1.7061	1.6832	2.4500

speed are shown in Figs. 4(a)–4(b). The estimation results for v_y , T_w , and T_s are respectively presented in Figs. 5(a)–5(c). As in the previous test, we observe that the vehicle state and unknown inputs can be accurately reconstructed using the proposed method, even in the presence of several sharp turns. The error indices obtained from this driving scenario are summarized in Table 3, which quantitatively confirm once again the strong estimation performance of the proposed method, as well as the performance improvement offered by the correction loop.

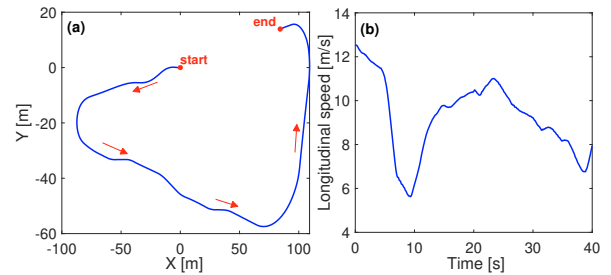


Fig. 4. Scenario 2. (a) Vehicle trajectory, (b) vehicle speed.

6. CONCLUSION

A twin-models-based controller output observer is presented to accurately estimate the unmeasured lateral speed and unknown steering and wheel torques of autonomous vehicles. We reformulate the estimation problem as a tracking control problem, utilizing two control loops. In the ML, a TS state feedback controller performs the primary estimation task. To

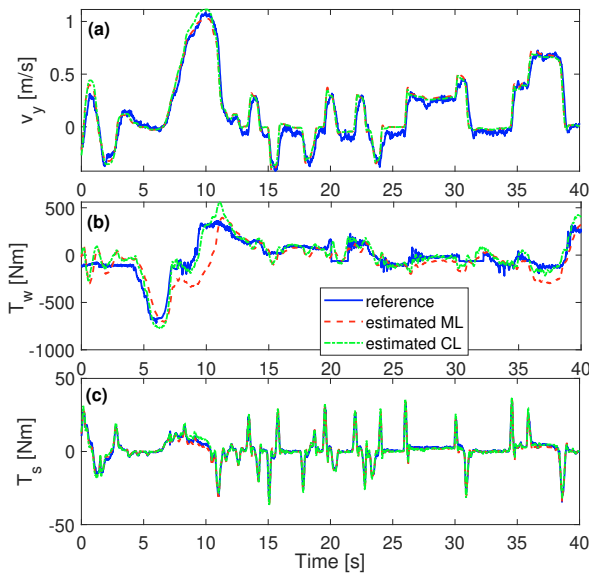


Fig. 5. Estimation performance obtained with Scenario 2. (a) Lateral speed, (b) wheel torque, (c) steering torque.

enhance robustness, a SOF compensator is implemented in the CL to address mismatches between the vehicle models and the actual system. Both controllers are simultaneously designed using Lyapunov stability theory and LMI techniques, which can be solved by common numerical methods. The proposed estimation method is validated with real vehicle measurements under various test scenarios, demonstrating satisfactory performance in real-world conditions. In particular, we show that the correction loop can significantly enhance the estimation accuracy. For future work, we will provide an SOF control solution while avoiding the numerical approximation issue mentioned in Remark 3, for instance, by using the convexification technique from (Nguyen et al., 2018). Additionally, thorough experimental validation and comparative studies with related estimation results in the literature will be conducted to comprehensively demonstrate the effectiveness of the proposed estimation concept across various realistic driving scenarios.

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