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Command Filtered Adaptive Tracking Consensus of Random Nonlinear Multi-Agent Systems

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***Abstract*—This article investigates the topic of adaptive tracking consensus for a family of nonlinear multi-agent systems modeled by random differential equations, which are different from well-known stochastic differential equations. This paper provides some primitive results on random nonlinear multi-agent systems. An improved backstepping method named command filtered control is adopted to derive the adaptive control law, where the convergence of the filtering error is guaranteed. To tackle the serious nonlinearities and uncertainties, a series of dynamic gains are introduced in the design process. The tracking errors for each follower concerning the output of the leader can be regulated arbitrarily to a small enough value by choosing appropriate tuning parameters. All signals of the closed-loop system are analyzed to have bounds almost surely. Moreover, the feasibility of the theory developed in this paper is validated by an example of numerical simulation.**

Note to Practitioners: Inspired by all kinds of biological clustering or grouping behaviors in our natural world, the research on multi-agent systems has long been popular in both theoretical and engineering scenarios. In addition, colored noises are ubiquitous and negligible when dealing with control problems of different engineering plants. Based on the above observation, this paper was motivated to realize the tracking consensus control of a class of nonlinear multi-agent systems disturbed by colored noise, which is also called random nonlinear systems, with the help of an enhanced adaptive backstepping approach called command filtered control. Although there exist nonlinearities and random noises in each follower agent, the objective of output consensus still could be achieved with the combination of the methods of dynamic gains and command filtering. The application potential of this research is considerable, such as drone formation performance and drone cruise. Our future research will further investigate control problems of random nonlinear multi-agent systems under some practical obstacles such as actuator failures.

***Index Terms*—Random differential equations (RDEs), nonlinear multi-agent systems (NMASs), command filtered control (CFC), tracking consensus**

I. Introduction

In recent decades, multi-agent systems (MASs) have gained great popularity from extensive researchers [1]-[8]. While, due to their complexity compared with single systems, the research on MASs becomes much harder. As known to all, nonlinearities and uncertainties are ubiquitous in the natural world [9], thus the study on nonlinear MASs (NMASs) is quite meaningful and necessary. Nevertheless, not like that the results on linear MASs are abundant and well developed [2]-[4], the outcomes of NMASs are somewhat inadequate and their theoretical framework is incomplete. It is gratifying that there have emerged a lot of excellent papers working on NMASs so far, such as [10]-[12]. Consensus is a fundamental issue of MASs, whose control purpose is to drive the output or state of each agent to arrive at a consistent value when stability is achieved. Thus it can be classified into state consensus [1] and output (tracking) consensus [10]-[12]. From another perspective, consensus problems are cast into leader-following consensus [10], [12], and leaderless consensus [11]. The function of a leader is to provide followers with a reference signal. Following these existing results, this article further concentrates on tracking consensus issues of a category of leader-following NMASs using adaptive techniques.

As known to all control scientists and engineers, backstepping is a widespread and effective tool to deal with nonlinear adaptive control problems [13]. However, the cumbersome derivation process always causes trouble when designing controllers for high-dimensional systems. To surmount the dilemma, the method of dynamic surface control (DSC) was put forward in [14], where a series of filters are used to estimate derivatives of virtual control laws. DSC can be regarded as an improved version of backstepping control and it effectively decreases the computation burden. Further, a novel enhanced method named command filtered control (CFC) was established in [15], where the effect of the command filtering (CF) is analyzed and a compensated tracking error is introduced. CFC retains the intrinsic design philosophy of the backstepping approach and it significantly simplifies the backstepping implementation [16]-[19]. CFC is also adopted in this paper to achieve the aim of adaptive tracking.

In applications, stochastic characteristics of control systems can not be overlooked [20]-[22]. Thus the study on stochastic nonlinear MASs (SNMASs) has arisen in recent years [6], [23]. The model of a single agent in SNMASs is described by a stochastic differential equation (SDE): $dx = r(x,t)dt + s(x,t)dW_t$, where r, s are nonlinear functions, and W_t is a *Winner process*, whose formal derivative is deemed to be white noise. While the strict derivative of a Brownian motion does not exist. Besides, viewing that the real existing noise is always colored noise, SDEs may fail to be the most suitable model for practical significance. A few years ago, a different model characterized by the following random differential equation (RDE) was developed in [24]: $\dot{x}(t) = r(x,t) + s(x,t)\xi(t)$, where $\xi(t)$ is a wide stationary process, representing the

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colored noise in most cases. Some primitive theories of RDEs can be referred to [25] and [26]. Recently, the investigation of random nonlinear systems (RNSs) has grown prominently [27]-[32]. The criterion of noise-to-state stability for RNSs was established in [27] and [28]. The stability results for a category of switched RNSs were analyzed in [29]. For RNSs with time delay and impulse, their stability criteria were developed in [30] and [31], respectively. Beyond practical stability and asymptotic stability, [32] further achieved finite-time stabilization of random nonlinear systems. [33] and [34] investigated the event-triggered control subject for RNSs via the LMI technique and backstepping method, respectively. [35] investigated the issue of tracking control for a family of RNSs with techniques of high-gain observer and neural approximation. Besides, [36], [37], and [38] concentrated on adaptive command filtered control design of RNSs with different dynamics. To get a more exhaustive observation of stability analysis and control design of RNSs, readers may refer to the survey article [39] for details. Nevertheless, there are only a few academic papers concentrating on control design for random nonlinear MASs (RNMASs). The recently published work [40] investigated the tracking consensus problem of networked RNMASs with intermittent communications, where an intermittent communication mechanism was proposed by introducing a series of distributed auxiliary linear systems. Compared with [40], the highlights of this paper are twofold. Firstly, CFC method was introduced as an improvement of the normal backstepping method adopted in [40], aiming to reduce the computation and design burden. Secondly, the systems considered in this paper involve severe nonlinearities that cannot be transformed into the strict feedback form as studied in [40], which greatly increases the design difficulty.

The contributions of this paper are concluded as below:

(1) Viewing that control problems of RNMASs have gotten rare attention so far, this paper aims to provide some related results and try to enrich this theoretical realm. Compared with the well-researched deterministic NMASs, RNMASs are more challenging in control design and stability analysis.

(2) To avoid the overwhelming computation burden caused by “the curse of dimension”, a series of delicately designed command filters with compensation mechanisms are introduced in this paper. This work provides the possibility and feasibility of applying the control scheme developed in this paper to high-dimensional RNMASs even with complex dynamics.

(3) Although nonlinear systems in strict feedback form are extensively considered in all kinds of control problems, sometimes it becomes extremely difficult or even impossible to transform an engineering model into this form. So, it is imperative to develop different control schemes to tackle this situation. This paper gives a solution for tracking control of this kind of system with severe nonlinearities that cannot presented as strict feedback. Inspired by [41], some dynamic gains are introduced to offset the undesired influence of the nonlinearities and uncertainties.

The remnant of this article is constructed as below. In Section II, some preliminaries about RNMASs are given. In Section III, the main body is developed, where the CFC technique is utilized to derive the adaptive controller, and the tracking performance is evaluated. In the simulation Section IV, a numerical example validates the effectiveness of the main results. The conclusions are drawn in Section V.

Notations: In this article, let $\mathbb{R}^n$ be the Euclidean space with $n-$dimension; Let $\mathbb{R}^+$ be the set of all positive real numbers. Denote $\|v\|$ to be the Euclidean norm of vector v. For a list of variables $x_1, x_2, ..., x_i$, the vector $\bar{x}_i$ is the abbreviation for $[x_1, x_2, ..., x_i]^T$. $\vee$ is the disjunctive operator. And $E\{\cdot\}$ represents the expectation operator concerning its corresponding probability measure P. The term *almost surely* is abbreviated as *a.s.*. The operator $\overline{\lim}$ means taking limit superior. $sgn(\cdot)$ denotes the sign function, that is, $sgn(x) = 1$, if $x > 0$; $sgn(x) = 0$, if $x = 0$; $sgn(x) < 0$, and if $x < 0$. $diag\{\cdot\}$ presents a diagonal matrix with $\cdot$ being its diagonal elements.

II. Problem formulation and preliminaries

To begin with, some preliminaries about graph theory need to be reviewed. The topology of communication among followers is supposed to be undirected. The topology graph is described by a triple as $G = (V, E, \mathscr{A})$, where $V = \{v_1, ..., v_N\}$ is the nonempty set of vertices, $E \subseteq \{(v_i, v_j)|v_i, v_j \in V, i \neq j\}$ is the set of edges, and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix, where $a_{ij} \in \mathbb{R}^+$. If there exists an edge between vertices i and j, then $(v_i, v_j), (v_j, v_i) \in E$ and $a_{ij} = a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0$. We define that $a_{ii} = 0$ for $i = 1, ..., N$. Define the degree matrix as $\mathscr{D} = diag\{d_1, ..., d_N\} \in \mathbb{R}^{N\times N}$, where $d_i = \sum_{j=1}^{N} a_{ij}$, $i = 1, ..., N$ is the degree of the ith follower. The Laplacian matrix of the graph G is defined as $\mathscr{L} = [l_{ij}] = \mathscr{D} - \mathscr{A} \in \mathbb{R}^{N\times N}$. Let $\mathscr{B} = diag\{b_1, ..., b_N\} \in \mathbb{R}^{N\times N}$ be the pinning matrix of the MASs, where $b_i \in \mathbb{R}^+$ for $i = 1, ..., N$ is the pinning gain of the leader to the ith follower. $b_i = 1$ if the leader can contact the ith follower directly, otherwise it is zero.

Consider the following RNMAS which includes N follower agents and one leader agent, the dynamics of the lth $(1 \le l \le N)$ follower is described as:

$$\begin{cases} \dot{x}_{l,i} = \delta_{l,i}(\bar{x}_{l,i}, t)x_{l,i+1} + f_{l,i}(x_l, u, t) + g_{l,i}(\bar{x}_{l,i}, t)\xi_{l,i}(t), \\ \qquad\qquad\qquad\qquad\qquad\qquad i = 1, 2, ..., n_l - 1, \\ \dot{x}_{l,n_l} = \delta_{l,n_l}(x_l, t)u_l + f_{l,n_l}(x_l, u, t) + g_{l,n_l}(x_l, t)\xi_{l,n_l}(t), \\ y_l = x_{l,1}, \end{cases} \tag{1}$$

where $x_l = \bar{x}_{l,n_l} \in \mathbb{R}^{n_l}$ is the state of the lth follower; and $u_l, y_l \in \mathbb{R}$ denote the control input and the output of the system, respectively; $\delta_{l,i} \neq 0$ are known (virtual) control coefficients; $f_{l,i}$ represent system nonlinearities and $g_{l,i}$ are known intensity coefficients of the colored noise $\xi_{l,i}$, which both satisfy local Lipschitz condition and $|f_{l,i}(0, 0, t)| \vee |g_{l,i}(0, t)| < d_0$ with d_0 being a positive constant, $1 \le l \le N$ and $1 \le i \le n_l$. The leader dynamics are presented as:

$$\begin{cases} \dot{x}_{0,i} = x_{0,i+1}, \quad i = 1, 2, ..., n_0 - 1, \\ \dot{x}_{0,n_0} = f_0(x_0, t), \quad y_0 = x_{0,1}, \end{cases} \tag{2}$$

where $x_0 = \bar{x}_{0,n_0} \in \mathbb{R}^{n_0}$ is the state of the leader; and $y_0 \in \mathbb{R}$ is the output of the leader; f_0 is a Lipschitz continuous nonlinear function satisfying $f_0(0, t) = 0$.

To ensure controllability, the following assumptions need to be clarified:

***Assumption** 1:* The control coefficients are bounded, that is to say, for $l = 1, ..., N$ and $i = 1, ..., n_l$, there exists a series of constants $\bar{\delta}_{l,i} > \underline{\delta}_{l,i} > 0$ such that

$$\underline{\delta}_{l,i} \leq \delta_{l,i} \leq \bar{\delta}_{l,i}.$$

***Assumption** 2:* The vectors of random processes $\xi_l(t) = [\xi_{l,1}, \xi_{l,2}, ..., \xi_{l,n_1}], 1 \leq l \leq N$ are $\mathcal{F}_t-$adapted and piecewise continuous, and there is a constants $M > 0$ such that their 2-nd moments satisfy

$$\sup_{t_0 \leq s \leq t, 1 \leq l \leq N} E\{\|\xi_l(s)\|^2\} < M.$$

***Remark** 1:* According to the Lemma 4 in [24], the unique existence of closed-loop solution on the maximal existence interval $[0, t_m)$ follows straightforwardly from the local Lipschitz properties of $f_{l,i}$ and $|f_{l,i}(0,0,t)| \vee |g_{l,i}(0,t)| < d_0$, and Assumption 2.

***Assumption** 3:* It is assumed that the output of the leader and its derivative are bounded, that is

$$\sup_{t \geq 0}(|y_0(t)| + |\dot{y}_0(t)|) \leq \hbar,$$

where $\hbar > 0$ is an unknown constant.

***Assumption** 4:* The nonlinear functions $f_{l,i}$ satisfy

$$|f_{l,i}(x_l, u, t)| \leq \theta_{l,i}\|x_l\| + \epsilon_{l,i},$$

for $1 \leq l \leq N$ and $1 \leq i \leq n_l$, where $\theta_{l,i}$ and $\epsilon_{l,i}$ are positive unknown constants.

***Remark** 2:* It is the growth condition of nonlinearities that should be especially noticed. The domination takes a form which is affine of $\|x_l\|$ instead of the simple $\|\bar{x}_{l,i}\|$ in strict-feedback form. Moreover, an unknown parameter $\theta_{l,i}$ is multiplied ahead of $\|x_l\|$. These mean that $f_{l,i}$ may not dominated by an upper bound which is precisely known. This feature increases the difficulty of dealing with the nonlinearities, which are more severe than other nonlinearities appearing in most existing results.

***Lemma** 1:* [36] Let $V(t)$ be a positive function that has a continuous derivative. If $V(t)$ satisfies the following inequality:

$$E\{\dot{V}(t)\} \leq -aE\{V(t)\} + b,$$

where $a, b > 0$ are two terms, then

$$E\{V(t)\} \leq V(t_0) + \frac{b}{a}, \quad \overline{\lim_{t \to +\infty}} E\{V(t)\} \leq \frac{b}{a}.$$

***Assumption** 5:* The topology graph G of the RNMAS (1) is fixed and connected, and it has a spanning tree with the leader as its root.

The tracking consensus error for the lth ($1 \leq l \leq N$) follower is defined as $\rho_l = y_l - y_0$ for $1 \leq l \leq N$, and the corresponding relative tracking error is defined as

$$e_l = \sum_{j \in N_l} a_{lj}(\rho_l - \rho_j) + b_l \rho_l. \tag{3}$$

Then it is easy to have $e = (\mathscr{L} + \mathscr{B})\rho$ in compact form, where $e = [e_1, ..., e_N]^T$ and $\rho = [\rho_1, ..., \rho_N]^T$.

The control objective in this paper is to design a series of command filters (CFs) and further design the corresponding adaptive control protocols for each follower agent, so that outputs of the followers can track the output of the leader as precisely as possible and other closed-loop signals of the RNMAS (1) are all bounded almost surely.

III. Main results

Firstly, a series of dynamic gains $H_l(t)$ are introduced to repress the poor effect of system uncertainties and nonlinearities:

$$\dot{H}_l(t) = \max\{|e_l| - \omega_l, 0\}, \quad H_l(0) = 1, \tag{4}$$

where $\omega_l := \kappa_l + \bar{\varrho}_{l,1}$, κ_l and $\bar{\varrho}_{l,1}$ are positive constants to be specified later. It is apparent that

$$H_l(t) \geq 1, \quad \forall t \geq 0.$$

Inspired by [16] and [19], the following CFs are introduced to estimate the derivatives of virtual controllers:

$$\sigma_l \dot{\hat{\alpha}}_{l,i} = -\varpi \varepsilon_{l,i} - sgn(\varepsilon_{l,i}\dot{\alpha}_{l,i})\dot{\alpha}_{l,i}, \tag{5}$$

where $\sigma_l, \varpi > 0$ are design constants and

$$\varepsilon_{l,i} = \hat{\alpha}_{l,i} - \alpha_{l,i}$$

for $1 \leq i \leq n_l - 1$ are filtering errors with $\alpha_{l,0} := y_0$ and $\hat{\alpha}_{l,n_l} = \alpha_{l,n_l} = u_l$.

***Remark** 3:* In (5), $\alpha_{l,i}$, $\hat{\alpha}_{l,i}$ and $\dot{\hat{\alpha}}_{l,i}$ represent the input, the state and the output of the filter, respectively. Via Laplace transformation, (5) in time domain is identical to

$$\hat{A}(s) = \frac{\varpi + sgn(\varepsilon_{l,i}\dot{\alpha}_{l,i})\varpi/\sigma_l}{\sigma_l s + \varpi}A(s) - \frac{sgn(\varepsilon_{l,i}\dot{\alpha}_{l,i})}{\sigma_l}A(s)$$

in the form of frequency domain, which is a proportional segment plus an integral segment (inertia segment) concerning the input $A(s)$. Thus $\hat{\alpha}_{l,i}$ can be interpreted as a filtered version of $\alpha_{l,i}$.

Perform the following coordinate transformation to the system (1):

$$\zeta_{l,1} = e_l, \quad \zeta_{l,i} = x_{l,i} - \hat{\alpha}_{l,i-1}, \quad 2 \leq i \leq n_l. \tag{6}$$

***Remark** 4:* It should be noted that in the coordinate transformation (6), estimated values of virtual control laws are used. In most existing cases, original values are used directly, which causes great effort in the design process because of the tedious derivation process. The CFC method used in this paper helpfully provides these estimated values, which alleviates lots of computation burden.

The following error-compensating signals are introduced to compensate for the influence of filtering errors:

$$\begin{cases} \dot{\varrho}_{l,1} = -\bar{\tau}_{l,1}\varrho_{l,1} + h_l\delta_{l,1}(\varrho_{l,2} + \varepsilon_{l,1}) + m_{l,1}sgn(\varrho_{l,1}), \\ \dot{\varrho}_{l,2} = -\bar{\tau}_{l,2}\varrho_{l,2} + \delta_{l,2}(\varrho_{l,3} + \varepsilon_{l,2}) - h_l\delta_{l,1}\varrho_{l,1} + m_{l,i}sgn(\varrho_{l,2}), \\ \dot{\varrho}_{l,i} = -\bar{\tau}_{l,i}\varrho_{l,i} + \delta_{l,i}(\varrho_{l,i+1} + \varepsilon_{l,i}) - \delta_{l,i-1}\varrho_{l,i-1} + m_{l,i}sgn(\varrho_{l,i}), \\ \qquad 3 \leq i \leq n_l - 1, \\ \dot{\varrho}_{l,n_l} = -\bar{\tau}_{l,n_l}\varrho_{l,n_l} - \delta_{l,n_l}\varrho_{l,n_l-1} + m_{l,n_l}sgn(\varrho_{l,n_l}), \end{cases} \tag{7}$$

where $\bar{\tau}_{l,i} > \frac{1}{2}, i = 1, ..., n_l$ are design constant parameters. They further lead to the compensated tracking errors as below:

$$\eta_{l,i} = \zeta_{l,i} - \varrho_{l,i}, \quad 1 \le i \le n_l. \tag{8}$$

Define positive functions as

$$V_{l,\eta_i} = \frac{\eta_{l,i}^2}{2}, \quad V_{l,\alpha_i} = \frac{(\hat{\alpha}_{l,i} - \alpha_{l,i})^2}{2},$$

for $1 \le i \le n_l$, and

$$V_\Theta = \frac{\tilde{\Theta}_l^2}{2\gamma},$$

where $\gamma > 0$ is a design constant and define $\tilde{\Theta}_l$ will be specified later. With (5) it is not hard to obtain

$$\dot{V}_{l,\alpha_i} \le -\varpi(\hat{\alpha}_{l,i} - \alpha_{l,i})^2 \le 0. \tag{9}$$

***Remark** 5:* According to the Lyapunov stability theory, inequality (9) implies that the filtering errors $\varepsilon_{l,i}$ converge and they are bounded almost surely.

A. Design Process

For the lth $(1 \le l \le N)$ follower, we will adopt backstepping technique to design adaptive control protocols for the tracking consensus purpose.

Step 1: From (1), (3), (6) and (8) it holds that

$$\begin{aligned}\dot{V}_{l,\eta_1} =& \eta_{l,1}(\dot{e}_l - \dot{\varrho}_{l,1}) = -\eta_{l,1}[\sum_{j\in N_l} a_{lj}(\delta_{j,1}x_{j,2} + f_{j,1} + g_{j,1}\xi_{j,1}) \\ &+ b_l\dot{y}_0 - (d_l + b_l)(\delta_{l,1}x_{l,2} + f_{l,1} + g_{l,1}\xi_{l,1}) + \dot{\varrho}_{l,1}] \\ =& -\eta_{l,1}\{\sum_{j\in N_l} a_{lj}\delta_{j,1}x_{j,2} + b_l\dot{y}_0 + \dot{\varrho}_{l,1} - (d_l + b_l)\cdot \\ &[\delta_{l,1}(\eta_{l,2} + \varrho_{l,2} + \alpha_{l,1} + \varepsilon_{l,1}) + F_{l,1} + \widehat{G\xi}_{l,1}]\}, \end{aligned} \tag{10}$$

where $F_{l,1} := f_{l,1} - (\sum_{j\in N_l} a_{lj}f_{j,1})/(d_l + b_l)$, and $\widehat{G\xi}_{l,1} := g_{l,1}\xi_{l,1} - (\sum_{j\in N_l} a_{lj}g_{j,1}\xi_{j,1})/(d_l + b_l)$.

By Assumptions 3 and 4 and using Young inequality, it is easy to see

$$-\eta_{l,1}b_l\dot{y}_0 \le \frac{H_l b_l^2\eta_{l,1}^2}{2} + \frac{\hbar^2}{2H_l},$$

$$\begin{aligned}-(d_l + b_l)\eta_{l,1}\delta_{l,1}\eta_{l,2} \le& -\eta_{l,1}\sum_{j\in N_l} a_{lj}\delta_{j,1}x_{j,2} \\ \le& \frac{H_l}{2}\sum_{j\in N_l} a_{lj}\eta_{l,1}^2\delta_{j,1}^2x_{j,2}^2 + \frac{d_l}{2H_l},\end{aligned}$$

and

$$\begin{aligned}&\eta_{l,1}(d_l + b_l)F_{l,1} \le (d_l + b_l)|\eta_{l,1}F_{l,1}| \le \\ &(d_l+b_l)|\eta_{l,1}|(\theta_{l,1}\|x_l\|+\epsilon_{l,1})+|\eta_{l,1}|\sum_{j\in N_l} a_{lj}(\theta_{j,1}\|x_j\|+\epsilon_{j,1}) \\ \le& \frac{H_l\eta_{l,1}^2}{2}[(d_l+b_l)^2+d_l]\Theta + \frac{\|x_l\|^2+\sum_{j\in N_l} a_{lj}\|x_j\|^2+d_l+1}{2H_l}\end{aligned}$$

with $\Theta := \max_{1\le i\le n_l, 1\le l\le N}\{\theta_{l,i}^2 + \epsilon_{l,i}^2\}$ being an unknown parameter, and

$$\begin{aligned}&\eta_{l,1}(d_l + b_l)\widehat{G\xi}_{l,1} \le (d_l + b_l)|\eta_{l,1}\widehat{G\xi}_{l,1}| \le \\ &\eta_{l,1}^2[\frac{(d_l + b_l)^2g_{l,1}^2}{4c_{l,1}} + \sum_{j\in N_l}\frac{a_{lj}g_{j,1}^2}{4c_{j,1}}] + c_{l,1}\xi_{l,1}^2 + \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2\end{aligned}$$

with $c_{l,1}$ and $c_{j,1}$ being positive design constants.
Therefore, (10) can be further arranged as

$$\begin{aligned}\dot{V}_{l,\eta_1} \le& \eta_{l,1}[h_l\delta_{l,1}(\varrho_{l,2}+\alpha_{l,1}+\varepsilon_{l,1})-\dot{\varrho}_{l,1}]+\eta_{l,1}^2\{\frac{H_l}{2}[b_l^2+ \\ &\sum_{j\in N_l} a_{lj}\delta_{j,1}^2x_{j,2}^2+(h_l^2 + d_l)\hat{\Theta}_l]+\frac{h_l^2g_{l,1}^2}{4c_{l,1}}+\sum_{j\in N_l}\frac{a_{lj}g_{j,1}^2}{4c_{j,1}}\} \\ &+\frac{H_l\eta_{l,1}^2}{2}(h_l^2 + d_l)\tilde{\Theta}_l + h_l\delta_{l,1}\eta_{l,1}\eta_{l,2}+\sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 \\ &+ c_{l,1}\xi_{l,1}^2 + \frac{\|x_l\|^2+\sum_{j\in N_l} a_{lj}\|x_j\|^2 + 2d_l + 1 + \hbar^2}{2H_l},\end{aligned} \tag{11}$$

where $h_l := d_l + b_l$, and $\hat{\Theta}_l$ is the estimate of the unknown parameter Θ produced by lth agent with $\tilde{\Theta}_l = \Theta - \hat{\Theta}_l$ being the estimation error.

Choose the Lyapunov function candidate as

$$V_{l,1} = V_{l,\eta_1} + V_{l,\alpha_1} + V_\Theta,$$

then we have

$$\begin{aligned}\dot{V}_{l,1} \le& \eta_{l,1}[h_l\delta_{l,1}(\varrho_{l,2}+\alpha_{l,1}+\varepsilon_{l,1})-\dot{\varrho}_{l,1}]+\eta_{l,1}^2\{\frac{H_l}{2}[b_l^2+ \\ &\sum_{j\in N_l} a_{lj}\delta_{j,1}^2x_{j,2}^2+(h_l^2 + d_l)\hat{\Theta}_l]+\frac{h_l^2g_{l,1}^2}{4c_{l,1}}+\sum_{j\in N_l}\frac{a_{lj}g_{j,1}^2}{4c_{j,1}}\} \\ &+ c_{l,1}\xi_{l,1}^2+\sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 + \frac{\tilde{\Theta}_l}{\gamma}[\frac{H_l\gamma\eta_{l,1}^2}{2}(h_l^2 + d_l) - \dot{\hat{\Theta}}_l] \\ &+\frac{\|x_l\|^2+\sum_{j\in N_l} a_{lj}\|x_j\|^2+2d_l+1+\hbar^2}{2H_l} + h_l\delta_{l,1}\eta_{l,1}\eta_{l,2}.\end{aligned} \tag{12}$$

Take the first virtual control law for the lth follower agent as

$$\begin{aligned}\alpha_{l,1} =& -\frac{\eta_{l,1}}{h_l\delta_{l,1}}\{\frac{H_l}{2}[b_l^2+\sum_{j\in N_l} a_{lj}\delta_{j,1}^2x_{j,2}^2+(h_l^2+d_l)\hat{\Theta}_l] \\ &+\frac{h_l^2g_{l,1}^2}{4c_{l,1}}+\sum_{j\in N_l}\frac{a_{lj}g_{j,1}^2}{4c_{j,1}} +\varphi_l(\eta_{l,1})\} - \frac{\bar{\tau}_{l,1}\zeta_{l,1}}{h_l\delta_{l,1}},\end{aligned} \tag{13}$$

where φ_l is a bounded continuous positive function to be used in the last step. Bringing (7) and (13) into (12), we obtain

$$\begin{aligned}\dot{V}_{l,1} \le& -\bar{\tau}_{l,1}\eta_{l,1}^2 + c_{l,1}\xi_{l,1}^2+\sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 + h_l\delta_{l,1}\eta_{l,1}\eta_{l,2} \\ &+\frac{\tilde{\Theta}_l}{\gamma}[\frac{H_l\gamma\eta_{l,1}^2}{2}(h_l^2 + d_l) - \dot{\hat{\Theta}}_l] - m_{l,1}\eta_{l,1}sgn(\varrho_{l,1}) \\ &+\frac{\|x_l\|^2+\sum_{j\in N_l} a_{lj}\|x_j\|^2+2d_l+1+\hbar^2}{2H_l} - \eta_{l,1}^2\varphi_l.\end{aligned} \tag{14}$$

Step 2: From (1), (6) and (8), we know that

$$\begin{aligned}\dot{\eta}_{l,2} =&\dot{x}_{l,2} - \dot{\hat{\alpha}}_{l,1} - \dot{\varrho}_{l,2} = \delta_{l,2}(\eta_{l,3} + \varrho_{l,3} \\ &+ \alpha_{l,2} + \varepsilon_{l,2}) + f_{l,2} + g_{l,2}\xi_{l,2} - \dot{\hat{\alpha}}_{l,1} - \dot{\varrho}_{l,2}. \end{aligned} \quad (15)$$

Define $V_{l,\eta_2} = \frac{\eta_{l,2}^2}{2}$, then it follows from (15) that

$$\begin{aligned}\dot{V}_{l,\eta_{l,2}} =&\eta_{l,2}[\delta_{l,2}(\eta_{l,3}+\varrho_{l,3}+\alpha_{l,2}+\varepsilon_{l,2}) \\ &+ f_{l,2} + g_{l,2}\xi_{l,2} - \dot{\hat{\alpha}}_{l,1} - \dot{\varrho}_{l,2}] \\ \leq&\eta_{l,2}\delta_{l,2}(\eta_{l,3}+\varrho_{l,3}+\alpha_{l,2}+\varepsilon_{l,2})-\eta_{l,2}(\dot{\hat{\alpha}}_{l,1}+\dot{\varrho}_{l,2}) \\ &+ \frac{H_l\eta_{l,2}^2}{2}\Theta + \frac{\|x_l\|^2+1}{2H_l} + \frac{\eta_{l,2}^2 g_{l,2}^2}{4c_{l,2}} + c_{l,2}\xi_{l,2}^2, \end{aligned} \quad (16)$$

where the last inequality above is derived by using Assumption 4 and Young inequality.

Choose the Lyapunov function candidate as

$$V_{l,2} = V_{l,1} + V_{l,\eta_2} + V_{l,\alpha_2},$$

then from (9), (14) and (16) we have

$$\begin{aligned}\dot{V}_{l,2} \leq& -\bar{\tau}_{l,1}\eta_{l,1}^2 - \eta_{l,2}(\dot{\hat{\alpha}}_{l,1}+\dot{\varrho}_{l,2}) - m_{l,1}\eta_{l,1}sgn(\varrho_{l,1}) - \eta_{l,1}^2\varphi_l \\ &+\eta_{l,2}\delta_{l,2}(\eta_{l,3}+\varrho_{l,3}+\alpha_{l,2}+\varepsilon_{l,2}) + \sum_{k=1}^{2} c_{l,k}\xi_{l,k}^2 + \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 \\ &+ \frac{2\|x_l\|^2 + \sum_{j\in N_l} a_{lj}\|x_j\|^2 + 2d_l + 2 + \hbar^2}{2H_l} + \frac{\eta_{l,2}^2 g_{l,2}^2}{4c_{l,2}} + \frac{H_l\eta_{l,2}^2}{2}\hat{\Theta}_l \\ &+ \frac{\tilde{\Theta}_l}{\gamma}\{\frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2+d_l) + \eta_{l,2}^2] - \dot{\hat{\Theta}}_l\} + h_l\delta_{l,1}\eta_{l,1}\eta_{l,2}. \end{aligned} \quad (17)$$

Take the second virtual control law for the lth follower agent as

$$\alpha_{l,2} = -\frac{1}{\delta_{l,2}}[\bar{\tau}_{l,2}\zeta_{l,2} + \eta_{l,2}(\frac{g_{l,2}^2}{4c_{l,2}} + \frac{H_l}{2}\hat{\Theta}_l) - \dot{\hat{\alpha}}_{l,1} + h_l\delta_{l,1}\zeta_{l,1}], \quad (18)$$

Bringing (7) and (18) into (17), we obtain

$$\begin{aligned}\dot{V}_{l,2} \leq& -\sum_{k=1}^{2}\bar{\tau}_{l,k}\eta_{l,k}^2 - \sum_{k=1}^{2} m_{l,k}\eta_{l,k}sgn(\varrho_{l,k}) + \sum_{k=1}^{2} c_{l,k}\xi_{l,k}^2 \\ &+ \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 + \frac{2\|x_l\|^2 + \sum_{j\in N_l} a_{lj}\|x_j\|^2 + 2d_l + 2 + \hbar^2}{2H_l} \\ &+ \frac{\tilde{\Theta}_l}{\gamma}\{\frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2+d_l) + \eta_{l,2}^2] - \dot{\hat{\Theta}}_l\} + \delta_{l,2}\eta_{l,2}\eta_{l,3} - \eta_{l,1}^2\varphi_l. \end{aligned} \quad (19)$$

Step i** ($2 \leq i \leq n_l - 1$)**: From (1), (5), (6) and (8), we have

$$\begin{aligned}\dot{\eta}_{l,i} =&\dot{x}_{l,i} - \dot{\hat{\alpha}}_{l,i-1} - \dot{\varrho}_{l,i} = \delta_{l,i}(\eta_{l,i+1} + \varrho_{l,i+1} \\ &+ \alpha_{l,i} + \varepsilon_{l,i}) + f_{l,i} + g_{l,i}\xi_{l,i} - \dot{\hat{\alpha}}_{l,i-1} - \dot{\varrho}_{l,i}. \end{aligned} \quad (20)$$

Define $V_{l,\eta_i} = \frac{\eta_{l,i}^2}{2}$, then it follows from (20) that

$$\begin{aligned}\dot{V}_{l,\eta_{l,i}} =& \eta_{l,i}[\delta_{l,i}(\eta_{l,i+1}+\varrho_{l,i+1}+\alpha_{l,i}+\varepsilon_{l,i}) \\ &+ f_{l,i} + g_{l,i}\xi_{l,i} - \dot{\hat{\alpha}}_{l,i-1} - \dot{\varrho}_{l,i}] \\ \leq&\eta_{l,i}\delta_{l,i}(\eta_{l,i+1}+\varrho_{l,i+1}+\alpha_{l,i}+\varepsilon_{l,i}) - \eta_{l,i}(\dot{\hat{\alpha}}_{l,i-1}+\dot{\varrho}_{l,i}) \\ &+ \frac{H_l\eta_{l,i}^2}{2}\Theta + \frac{\|x_l\|^2+1}{2H_l} + \frac{\eta_{l,i}^2 g_{l,i}^2}{4c_{l,i}} + c_{l,i}\xi_{l,i}^2, \end{aligned} \quad (21)$$

where the last inequality above is derived by using Young inequality and Assumption 4.

Choose the Lyapunov function candidate as

$$V_{l,i} = V_{l,i-1} + V_{l,\eta_i} + V_{l,\alpha_i},$$

and assume that

$$\begin{aligned}\dot{V}_{l,i-1} \leq& -\sum_{k=1}^{i-1}\bar{\tau}_{l,k}\eta_{l,k}^2 - \sum_{k=1}^{i-1} m_k\eta_{l,k}sgn(\varrho_{l,k}) \\ &+ \sum_{k=1}^{i-1} c_{l,k}\xi_{l,k}^2 + \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 + \delta_{l,i-1}\eta_{l,i-1}\eta_{l,i} \\ &+ \frac{(i-1)\|x_l\|^2 + \sum_{j\in N_l} a_{lj}\|x_j\|^2 + 2d_l + i - 1 + \hbar^2}{2H_l} \\ &+ \frac{\tilde{\Theta}_l}{\gamma}\{\frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2+d_l) + \sum_{k=2}^{i-1}\eta_{l,k}^2] - \dot{\hat{\Theta}}_l\} - \eta_{l,1}^2\varphi_l, \end{aligned} \quad (22)$$

then from (9), (21), (22) and by using Young inequality, we have

$$\begin{aligned}\dot{V}_{l,i} \leq& -\sum_{k=1}^{i-1}\bar{\tau}_{l,k}\eta_{l,k}^2 - \eta_{l,1}^2\varphi_l + \delta_{l,i-1}\eta_{l,i-1}\eta_{l,i} \\ &+\eta_{l,i}\delta_{l,i}(\eta_{l,i+1}+\varrho_{l,i+1}+\alpha_{l,i}+\varepsilon_{l,i}) - \eta_{l,i}(\dot{\hat{\alpha}}_{l,i-1}+\dot{\varrho}_{l,i}) \\ &+\sum_{k=1}^{i} c_{l,k}\xi_{l,k}^2 + \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 - \sum_{k=1}^{i-1} m_k\eta_{l,k}sgn(\varrho_{l,k}) \\ &+ \frac{i\|x_l\|^2 + \sum_{j\in N_l} a_{lj}\|x_j\|^2 + 2d_l + i + \hbar^2}{2H_l} + \frac{\eta_{l,i}^2 g_{l,i}^2}{4c_{l,i}} \\ &+ \frac{\tilde{\Theta}_l}{\gamma}\{\frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2+d_l) + \sum_{k=2}^{i}\eta_{l,k}^2] - \dot{\hat{\Theta}}_l\} + \frac{H_l\eta_{l,i}^2}{2}\hat{\Theta}_l. \end{aligned} \quad (23)$$

Take the ith virtual control law for the lth follower agent as

$$\alpha_{l,i} = -\frac{1}{\delta_{l,i}}[\bar{\tau}_{l,i}\zeta_{l,i} + \eta_{l,i}(\frac{g_{l,i}^2}{4c_{l,i}} + \frac{H_l}{2}\hat{\Theta}_l) - \dot{\hat{\alpha}}_{l,i-1} + \delta_{l,i-1}\zeta_{l,i-1}]. \quad (24)$$

Bringing (7) and (24) into (23), we obtain

$$\begin{aligned}\dot{V}_{l,i} \leq& -\sum_{k=1}^{i}\bar{\tau}_{l,k}\eta_{l,k}^2 - \sum_{k=1}^{i}\eta_{l,k}m_{l,k}sgn(\varrho_{l,k}) + \delta_{l,i}\eta_{l,i}\eta_{l,i+1} \\ &+ \frac{i\|x_l\|^2 + \sum_{j\in N_l} a_{lj}\|x_j\|^2 + 2d_l + i + \hbar^2}{2H_l} + \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 \\ &+\sum_{k=1}^{i} c_{l,k}\xi_{l,k}^2 + \frac{\tilde{\Theta}_l}{\gamma}\{\frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2+d_l) + \sum_{k=2}^{i}\eta_{l,k}^2] - \dot{\hat{\Theta}}_l\} - \eta_{l,1}^2\varphi_l. \end{aligned} \quad (25)$$

Step** n_l**: Define $V_{l,\eta_{n_l}} = \frac{\eta_{l,n_l}^2}{2}$, and

$$V_l = V_{l,n_l-1} + V_{l,\eta_{n_l}} + V_{l,\alpha_{n_l}}.$$

The actual control input for the lth follower agent is designed as

$$u_l = \alpha_{l,n_l} = -\frac{1}{\delta_{l,n_l}} \cdot [\bar{\tau}_{l,n_l}\zeta_{l,n_l} + \eta_{l,n_l}(\frac{g_{l,n_l}^2}{4c_{l,n_l}} + \frac{H_l}{2}\hat{\Theta}_l) - \dot{\hat{\alpha}}_{l,n_l-1} + \delta_{l,n_l-1}\zeta_{l,n_l-1}]. \quad (26)$$

Let the function φ_l introduced in *Step 1* satisfy the following inequality:
$-\eta_{l,1}^2\varphi_l + (n_l\|x_l\|^2 + \sum_{j\in N_l} a_{lj}\|x_j\|^2)/(2H_l) \le -\sum_{k=1}^{n_l-1}\frac{\varepsilon_{l,k}^2}{2}$.

Following the same idea of *Step* i, define $\eta_{l,n_l+1} := 0$ and we can finally obtain

$$\dot{V}_l \le -\sum_{k=1}^{n_l}\bar{\tau}_{l,k}\eta_{l,k}^2 - \sum_{k=1}^{n_l-1}\frac{\varepsilon_{l,k}^2}{2} + \frac{2d_l + n_l + \hbar^2}{2H_l} + \sum_{k=1}^{n_l} c_{l,k}\xi_{l,k}^2 + \sum_{j\in N_l} a_{lj}c_{j,1}\xi_{j,1}^2 - \sum_{k=1}^{n_l}\eta_{l,k}m_{l,k}sgn(\varrho_{l,k}) + \frac{\tilde{\Theta}_l}{\gamma}\{\frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2 + d_l) + \sum_{k=2}^{n_l}\eta_{l,k}^2] - \dot{\hat{\Theta}}_l\}. \quad (27)$$

Consider the following adaptive updating law for $\hat{\Theta}_l$:

$$\dot{\hat{\Theta}}_l = \frac{H_l\gamma}{2}[\eta_{l,1}^2(h_l^2 + d_l) + \sum_{k=2}^{n_l}\eta_{l,k}^2] - \gamma l_0(\hat{\Theta}_l - \Theta_0). \quad (28)$$

Taking (28) into (27), also noting that $H_l \ge 1$, with the help of Young inequality we have

$$l_0\tilde{\Theta}_l(\hat{\Theta}_l - \Theta_0) \le l_0[-\frac{\tilde{\Theta}_l^2}{2} + \frac{(\Theta - \Theta_0)^2}{2H_l}],$$

and

$$-\sum_{k=1}^{n_l}\eta_{l,k}m_{l,k}sgn(\varrho_{l,k}) \le \sum_{k=1}^{n_l}(\frac{\eta_{l,k}^2}{2} + \frac{m_{l,k}^2}{2}).$$

Based above observation, we thus can obtain

$$\dot{V}_l \le -\sum_{k=1}^{n_l}\tau_{l,k}\eta_{l,k}^2 + \sum_{k=1}^{n_l}c_{l,k}\xi_{l,k}^2 + \sum_{j\in N_l}a_{lj}c_{j,1}\xi_{j,1}^2 + \sum_{k=1}^{n_l}\frac{m_{l,k}^2}{2} + \frac{2d_l + n_l + \hbar^2 + l_0(\Theta - \Theta_0)^2}{2H_l} - \frac{l_0\tilde{\Theta}_l^2}{2} - \sum_{k=1}^{n_l-1}\frac{\varepsilon_{l,k}^2}{2}, \quad (29)$$

where $\tau_{l,k} := \bar{\tau}_{l,k} - \frac{1}{2}$ for $1 \le k \le n_l$.

Define $\beta_l := \min_{1\le k\le n_l}\{2\tau_{l,k}, \frac{\gamma l_0}{2}, \frac{1}{2}\}$. Denote $\mathscr{V}_l := E\{V_l(t)\}$ and take expectation on both side of (29), then by the definition of V_l we can finally acquire

$$E\{\dot{V}_l\} \le -\beta_l\mathscr{V}_l + \frac{\Delta_l}{H_l} + \Omega_l, \quad (30)$$

where $\Delta_l := d_l + [n_l + \hbar^2 + l_0(\Theta - \Theta_0)^2]/2$, $\Omega_l := (1 + d_l)\bar{c}M + \sum_{k=1}^{n_l}\frac{m_{l,k}^2}{2}$, and $\bar{c} := \max_{1\le k\le n_l, 1\le l\le N}\{c_{l,k}\}$.

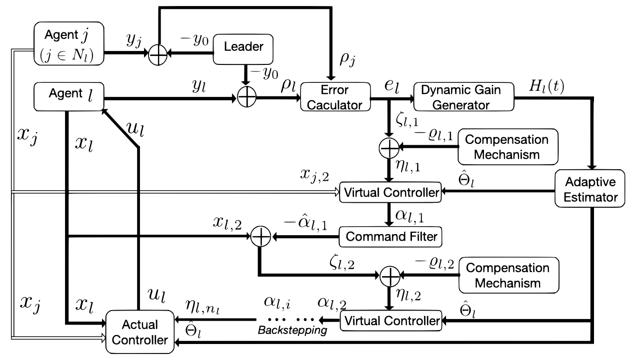

Fig. 1. Algorithmic diagram of control design

B. Performance Analysis

Based on the laborious design derived above, we are ready to present the main body of this paper which is condensed to a theorem as below.

Theorem 1: Under the adaptive control protocols (26) and (28), the following conclusions hold:

a. The RNMAS (1) has a unique solution globally;

b. The outputs of N agents in the form of (1) can follow the output of the leader agent (2), and the tracking errors can be made arbitrarily tiny by regulating involved tuning parameters;

$$\lim_{t\to+\infty} E\{|e_l|\} \le \sqrt{\frac{2\Xi_l}{\beta_l}} + \bar{\varrho}_{l,1}, \quad (31)$$

where $\Xi_l := \Delta_l + \Omega_l$ are defined in (30) and $\bar{\varrho}_{l,1}$ is the upper bound for $E\{\varrho_{l,1}\}$;

c. All the closed-loop signals are bounded a.s. on $[0, +\infty)$.

Proof. Because of the Lipschitz property of $H(t)$ and following the progressive analysis similar with [41], it is concluded that there exists a unique closed-loop solution for each follower system, that is $(x_l(t), H_l(t)), 1 \le l \le N$ on $[0, +\infty)$, with its initial value $x_l(0) \in \mathbb{R}^n$. This proves the part ***a.*** in Theorem.

Define $\bar{V}_l = \sum_{i=1}^{n_l}\frac{\varrho_{l,i}^2}{2}$, then by (7) and $h_l \ge 1$ we have

$$\dot{\bar{V}}_l = \sum_{i=1}^{n_l}\varrho_{l,i}\dot{\varrho}_{l,i} = \sum_{i=1}^{n_l}(-\bar{\tau}_{l,i}\varrho_{l,i}^2 + \varrho_{l,i}\delta_{l,i}\varepsilon_{l,i} + m_{l,i}|\varrho_{l,i}|) + (h_l - 1)\varrho_{l,1}\delta_{l,1}\varepsilon_{l,1} \le \sum_{i=1}^{n_l}[-\bar{\tau}_{l,i}\varrho_{l,i}^2 + (h_l\delta_{l,i}\bar{\varepsilon}_{l,i} + m_{l,i})|\varrho_{l,i}|] \le \sum_{i=1}^{n_l}[-\tau_{l,i}\varrho_{l,i}^2 + \frac{(h_l\delta_{l,i}\bar{\varepsilon}_{l,i} + m_{l,i})^2}{2}] \le -\tilde{\tau}_l\bar{V}_l + \Pi_l, \quad (32)$$

where $\tilde{\tau}_l := \min_{1\le i\le n_l}\{\tau_{l,i}\}$ and $\Pi_l := \frac{1}{2}\sum_{i=1}^{n_l}(h_l\delta_{l,i}\bar{\varepsilon}_{l,i} + m_{l,i})^2$. By lemma 1,

$$\overline{\lim_{t\to+\infty}} E\{\bar{V}_l(t)\} \le \frac{\Pi_l}{\tilde{\tau}_l} \quad a.s.,$$

which implies that $\bar{V}_l$ is uniformly bounded *a.s.*, Then according to the definition of $\bar{V}_l$ it can be concluded that $E\{\varrho_{l,i}\}$ are bounded *a.s.* on $[0, +\infty)$.

Define $V=\sum\limits_{l=1}^{N}V_l$ and denote $\mathscr{V}:=E\{V(t)\}$, then from (30) and $H_l\geq 1$ it holds that

$$E\{\dot{V}\}=E\{\sum_{l=1}^{N}\dot{V}_l\}\leq -\beta\mathscr{V}+\Xi, \tag{33}$$

where $\beta:=\min\limits_{1\leq l\leq N}\{\beta_l\}$, $\Xi:=\Delta+\Omega$, $\Delta:=\sum\limits_{l=1}^{N}\Delta_l$, and $\Omega:=\sum\limits_{l=1}^{N}\Omega_l$.

Recalling V is continuously differentiable, by using Fubini's theorem and Dynkin's formula, it then becomes that

$$\int_{t_1}^{t}E\{\dot{V}(s)\}ds=E\{\int_{t_1}^{t}\dot{V}ds\}=\mathscr{V}(t)-\mathscr{V}(t_1).$$

Let $t-t_1\to 0$, and by mean value theorem of integrals we obtain

$$\mathscr{V}(t)-\mathscr{V}(t_1)=\int_{t_1}^{t}E\{\dot{V}(s)\}ds\approx E\{\dot{V}(t)\}(t-t_1),$$

thus we get

$$E\{\dot{V}(t)\}=\lim_{t\to t_1}\frac{\mathscr{V}(t)-\mathscr{V}(t_1)}{(t-t_1)}=\dot{\mathscr{V}}.$$

Then (33) becomes

$$\dot{\mathscr{V}}\leq -\beta\mathscr{V}+\Xi.$$

From lemma 1, we have

$$\mathscr{V}\leq V(0)+\frac{\Xi}{\beta}\quad and \quad \overline{\lim_{t\to+\infty}}\mathscr{V}(t)\leq\frac{\Xi}{\beta}\quad a.s., \tag{34}$$

which implies that V is uniformly bounded *a.s.*. According to the definition of V we can conclude that $\eta_{l,i}(t)$, $\varepsilon_{l,i}(t)$ for $1\leq l\leq N, 1\leq i\leq n_l$ and $\tilde{\Theta}_l(t)$ are all bounded *a.s.* on $[0,+\infty)$. Therefore, $\hat{\Theta}_l(t)$ is also bounded. According to (30) and following the same idea from (33) to (34), we have

$$\overline{\lim_{t\to+\infty}}\mathscr{V}_l(t)\leq\frac{\Xi_l}{\beta_l},$$

where $\Xi_l:=\Delta_l+\Omega_l$. Then by (6), (8) and the definition of V_l, we have

$$|e_l|-|\varrho_{l,1}|\leq|e_l-\varrho_{l,1}|=|\eta_{l,1}|\leq\sqrt{\frac{2\Xi_l}{\beta_l}}.$$

Further, it obtains that

$$E\{|e_l\}|\leq\sqrt{\frac{2\Xi_l}{\beta_l}}+|\varrho_{l,1}|\leq\sqrt{\frac{2\Xi_l}{\beta_l}}+\bar{\varrho}_{l,1},$$

where $\bar{\varrho}_{l,1}$ is the upper bound for $E\{|\varrho_{l,1}|\}$. which proves the part ***b.*** in Theorem 1.

Next, we prove the dynamic gains $H_l(t)$ are bounded using reduction to absurdity. Suppose $H_l(t)$ are unbounded on $[0,+\infty)$. From (6), we know $H_l(t)$ are non-decreasing, then $H_l(t)\to+\infty$ as $t\to+\infty$. Therefore, there exists a finite time T such that

$$H_l(t)\geq\frac{4\Delta_l}{\beta_l\kappa_l^2-4\Omega_l},\forall t>T,$$

which together with (30) yields

$$\dot{\mathscr{V}}_l\leq-\beta\mathscr{V}_l+\frac{\beta_l\kappa_l^2-4\Omega_l}{4}+\Omega_l=-\beta\mathscr{V}_l+\frac{\beta_l\kappa_l^2}{4},\quad\forall t>T.$$

According to the *comparison lemma* in [43] it follows that

$$\mathscr{V}_l\leq\frac{\kappa_l^2}{4}+[V_l(T)-\frac{\kappa_l^2}{4}]e^{-\beta_l(t-T)},\forall t>T,$$

which indicates that for a sufficiently large $T^{'}>T$,

$$\mathscr{V}_l\leq\frac{\kappa_l^2}{2},\quad\forall t>T^{'}.$$

Based on the above analysis, recalling (4) and the definition of V, we obtain

$$\begin{aligned}E\{|e_l|\}=E\{|\zeta_{l,1}|\}\leq&E\{|\eta_{l,1}|\}+E\{|\varrho_{l,1}|\}\\ \leq&\sqrt{2\mathscr{V}_l}+\bar{\varrho}_{l,1}\leq\omega_l,\quad\forall t>T^{'}.\end{aligned}$$

Therefore, from (4) we get $\dot{H}_l(t)=0$, for $\forall\ t>T^{'}$, which implies $\sup\limits_{t\geq0}H_l(t)=H_l(T^{'})<+\infty$, a contradiction. Thus, $H_l(t)$ is actually bounded on $[0,+\infty)$.

From (8), since $\eta_{l,i}$ and $\varrho_{l,i}$ are bounded, then $\zeta_{l,i}$ are bounded. From (5) we know that when $\hat{\alpha}_{l,i}$ are extremely large, its derivative becomes negative enough to decrease the value of $\hat{\alpha}_{l,i}$, so $\hat{\alpha}_{l,i}$ are bounded, then from (6) and the boundedness of $\zeta_{l,i}$ we know that $x_{l,i}$ are bounded. Since $\alpha_{l,i}$ and u_l are continuous functions about $x_{l,i}$, $\zeta_{l,i}$ and $\hat{\Theta}_l$ which are all bounded, $\alpha_{l,i}$ and u_l are also bounded. Until now, the part ***c.*** of Theorem 1 has been proved. The proof is completed.

Remark 6: From the expression given in (31), the upper bounds of the tracking errors are mainly dominated by Ξ_l and β_l. Recalling $\Xi_l=\Delta_l+\Omega_l$ and the definitions of β_l, Δ_l and Ω_l specified around formula (30), the parameter tuning methodology to obtain a better control performance is summarized as follows. If we want smaller upper bounds for the tracking errors, smaller values of β_l, Δ_l, and larger values of Ω_l are preferred. To be precise, we can achieve this by increasing the values of $\tau_{l,k},\gamma,l_0$, or decreasing the values of $d_l,\hbar,l_0,c_{l,k},m_{l,k}$. Of course, parameter tuning should be done carefully, without destroying the stability of the system.

IV. Simulation

Consider an RNMAS in the form of (1) which includes a leader agent and three follower agents and each follower has second-order dynamics. The configuration and parameters of the system are given in the following.

The topology of the RNMAS is depicted in figure 2, from which we know that $a_{12}=a_{21}=0, a_{23}=a_{32}=a_{13}=a_{31}=1$ and $b_1=b_2=1, b_3=0$.

The nonlinear functions and system parameters involved in (1) are chosen as follows. Nonlinearities are presented as

$$\begin{cases}f_{11}=1.2x_{11}+\sin x_{11},\\ f_{12}=-2.4\cos 0.5x_{12}-0.9x_{11},\\ f_{21}=-2.9\cos x_{21}-0.9x_{21}-1.4,\\ f_{22}=-3x_{22}e^{-0.5x_{21}^2}-x_{22},\\ f_{31}=-3e^{-0.5x_{31}^2},\\ f_{32}=-1.2x_{32}-\sin x_{32}.\end{cases}$$

Noise intensities are given as

$$\begin{cases} g_{11} = -1.1x_{11} - 1.5, \\ g_{12} = 3.8x_{11}x_{12} - 1.3, \\ g_{21} = -1.3x_{21}, \\ g_{22} = -\sin x_{21} - x_{22}, \\ g_{31} = -\sin x_{31}, \\ g_{32} = -1.1x_{31} + 1.8. \end{cases}$$

Control coefficients are

$$\begin{cases} \delta_{11} = 0.62, \\ \delta_{12} = 1, \\ \delta_{21} = 0.52, \\ \delta_{22} = 0.9, \\ \delta_{31} = -0.8, \\ \delta_{32} = 1.2. \end{cases}$$

Other main design parameters are chosen as follows. Parameters in CFs (5) are

$$\begin{cases} \sigma_1 = \sigma_2 = \sigma_3 = 0.5, \\ \varpi = 0.5. \end{cases}$$

Parameters in the filtering error compensating system (7) are

$$\begin{cases} \bar{\tau}_{11} = \bar{\tau}_{12} = \bar{\tau}_{31} = \bar{\tau}_{32} = 1, \\ \bar{\tau}_{21} = 0.1, \quad \bar{\tau}_{22} = 0.9, \\ m_{11} = m_{21} = 0.2, \\ m_{12} = m_{22} = m_{31} = 0.1. \end{cases}$$

The tuning parameter in the Lyapunov function is $\gamma = 0.5$; tuning parameters in the adaptive law are $l_0 = \Theta_0 = 1$; and tuning parameters involved in the backstepping design are

$$c_{11} = c_{12} = c_{21} = c_{22} = c_{31} = c_{32} = 1.$$

Random noises are produced by the dynamics of

$$\begin{cases} \dot{\xi}_{i1}(t) = 0.3sin(t + \chi), \\ \dot{\xi}_{i2}(t) = 0.05cos(t + \chi), \end{cases}$$

where $i = 1, 2, 3$, χ is a random variable uniformly distributed on the region $[0, 2\pi]$, which could be easily generated via instructions of Matlab. The referenced tracking signal outputted by the leader is

$$y_0(t) = -0.4 \sin 500t.$$

And the initial states of the system are set as

$$[x_1(0); x_2(0); x_3(0)]^T = [0.5, -0.3; 1.3, 0.5; 1.3, 0.2]^T.$$

Within the design frame of this article, the response curves of the closed-loop signals in the simulation are shown in figures 3-6. It is apparently observed in the results of simulation that the object of command filtered adaptive tracking consensus for the RNMAS described above has been well achieved.

***Remark** 7:* It can be drawn from the figures of simulation results that all closed-loop signals are bounded and the system is stable. Nevertheless, irregular jumping appears in the state response curves, which can be observed in the first subfigure of Figure 5. This phenomenon is not strange under the influence of random noises. Within the control framework proposed in this paper, the system states can still flow stably regardless of some small abrupt jumping caused by random disturbances.

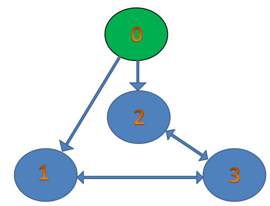

Fig. 2. The topological sketch of the RNMAS (0 denotes the leader and 1-3 denote followers; Arrows indicate the direction of information transmission)

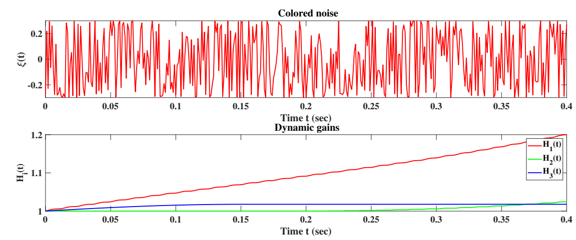

Fig. 3. Response curves of the random noise and dynamic gains

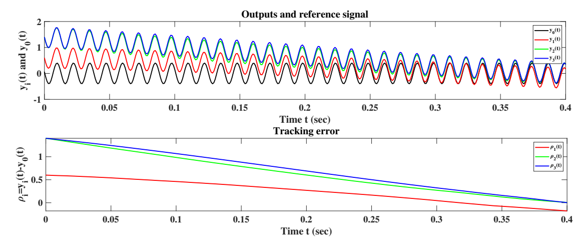

Fig. 4. Response curves of outputs and tracking errors

V. Conclusion

This research solves the problem of the adaptive tracking consensus for leader-following RNMASs. This paper serves as a pioneering result of the control design of RNMASs. The CFC method is adopted to reduce the computation burden, where the convergence of filtering errors is guaranteed. With the help of a series of dynamic gains, tracking errors for each follower could be regulated to an arbitrarily small value by appropriately choosing design parameters. And all the closed-loop signals are guaranteed to be bounded almost surely. Moreover, the feasibility of the theory developed in this paper is validated by a simulation example. Our future research direction will focus on control problems of RNMASs

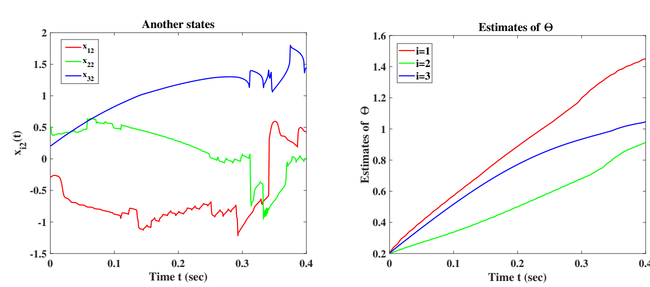

Fig. 5. The response curves of other states and estimates of the unknown parameter

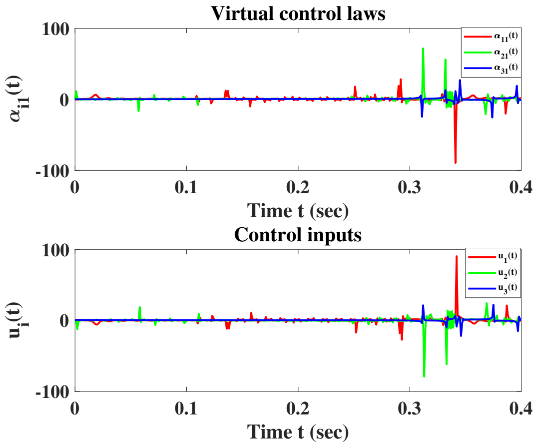

Fig. 6. Response curves of virtual and actual control inputs

with uncertainties, such as unknown control parameters or parametric uncertainties. Moreover, we will try to extend the proposed control scheme to RNMASs within the environment of a more general directed graph.

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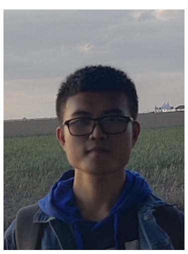

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