

Pricing Strategies for Processing Intermediaries in Agri-chains with Side Markets: Paradox of Ex-Post Pricing Flexibility

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Abstract

We study an agricultural supply chain in an emerging economy, where farmers make land allocation decisions among alternative crops and have the option to sell their harvest either in a side market or directly to a processing plant, and where a government-mandated minimum price guarantee (MPG) is in place. Our study examines a range of pricing strategies that the processing plant can employ to favor a protected crop and the utilization of the processing plant. We begin by considering the Benchmark strategy, in which the plant strictly adheres to the MPG. Subsequently, we investigate two distinct pricing strategies: the Ex-ante strategy, in which the plant commits to an optimal predetermined final price at the start of the growing season; and the Responsive strategy, in which the optimal final price is determined at the end of the season after yield and side-market uncertainties have been resolved. Additionally, we propose the Hybrid strategy, which integrates both ex-ante pricing as a pre-season commitment and ex-post price adjustments in response to realized uncertainties. We develop two-stage stochastic program models to analyze the farmer allocation and plant pricing decisions. Among other results, our study suggests that the ex-post pricing flexibility inherent in the Responsive approach may leave the plant worse off, even though price adjustments are plant-profit optimizing. We refer to this phenomenon as the ex-post pricing flexibility paradox. Due to this phenomenon, the Hybrid strategy, while consistently outperforming the Responsive approach, does not always dominate the Ex-ante approach. We propose execution variants of the Hybrid strategy that can guarantee outperformance on the Ex-ante benchmark. In long-term repeated interactions between the plant and farmers, the Hybrid strategy with historically credible probabilistic after-harvest price adjustments, proves effective. For short-term practices, defining a contingency region of relevant outcomes (e.g., yield realizations, side-market attractiveness, and processing margins) that triggers after-harvest price adjustments enables the Hybrid strategy to outperform. These findings urge caution when the plant considers adjusting after-harvest prices, and highlight the importance of building long-term and stable relationships in agri-chains for effective execution of pricing strategies that align incentives and optimize profits for all involved parties.

Key words: Pricing, agricultural supply chain, land allocation, pricing flexibility, random yield

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1 Introduction

For decades, governments in developing economies have enacted policies designed to bolster agricultural crop production for a variety of purposes, including enhancing the living standards of farmers, ensuring national food security, and boosting exports to global markets. A notable strategy in this regard is the implementation of minimum price guarantees. Examples include the minimum support prices in India

(Aditya et al., 2017) and the minimum guarantee prices in Brazil and Mozambique (Gomes and Teixeira, 2019; The World Bank Report, 2010). The typical supply chain for a protected crop (e.g., commodities such as cotton, corn, soybeans, olives, etc.) by a country government has farmers in the growing regions as the suppliers selling crops to intermediate processing plants (e.g., converting cottonseed, crushing and refining soybeans, corn milling operations, pressing olive into oil, etc.) before they can be distributed within the country or exported to global markets. Before the planting season begins, the government guarantees a price floor that government-registered intermediate processing plants or distributors must offer to farmers for their crops. The primary intention of minimum price guarantees is to incentivize increased protected crop production in an effort to support higher utilization of expensive governmental and/or private investments in intermediate processing capacities, and to support efficiency of the protected crop supply chain in serving the local market and growing exports. However, recent studies have raised doubts about the execution efficacy of these practices, often with the underutilization of intermediate processing capacities as offered evidence (see World Bank Report 2010, Li et al. 2023).

Examining the supply side of the issue reveals that farmers, even when provided with price guarantees at the start of the growing season, have the flexibility to diversify their crop production prior to planting. They can subsequently sell their harvested crops to various downstream outlets (Aditya et al., 2017) and in certain cases divert their protected crop consumption away from the government-intended processing sector. This phenomenon has been observed in several contexts, as illustrated in the following scenarios.

- **Regional monopsony buyer vs. buyers of neighboring regions:** Mozambique operates a national concessionary system for production of cotton, the country's third-largest export crop (The World Bank, 2010). The cotton-growing regions are divided into different concessionaires, with farmers in a concession expected by the government to grow and sell cotton exclusively to the designated ginner. However, if farmers find better prices for their harvest outside their assigned concession, they may opt to sell their cotton in neighboring regions or countries instead.

- **Processing food markets vs. fresh produce markets:** Fruit farmers in emerging economies have the flexibility to market their produce, including oranges, apples, and strawberries, through different channels. They may choose to sell to processors involved in the production of processed foods and beverages, who possess the financial resources and capacity to provide pre-season price commitments and are typically favored by the government due to their scale efficiency and exporting capabilities. Alternatively, farmers can opt to sell their produce as fresh goods to spot market-driven local buyers, who, albeit logistically convenient, are typically more financially constrained and sensitive to demand fluctuations, and who do not typically offer pre-season price commitments.

Although downstream processing plants may make reactive price offers after harvesting in response to market competition (Aditya et al. 2017), some adopt a more nuanced price mechanism with ex-ante and ex-post crop production prices to engage with farmers. For instance, Mozambique's national minimum pricing system for cotton (The World Bank Report, 2010) exemplifies such an approach. Administered by the cotton authorities in Mozambique, this system is implemented by all ginneries who are granted geographical concessions. Within this framework, ginneries and farmers agree on an indicative price as early as seven to eight months before cotton procurement commences. This early pricing information is intended to help farmers make informed decisions regarding their cotton production plans and to ensure an adequate supply of cotton for downstream ginneries. The final price to be paid upon cotton delivery is announced just one to two months before the start of cotton procurement. The latter price reflects yield and market conditions for both cotton and its competing cash crop option, which in Mozambique is typically sesame (The World Bank Report 2011), with price adjustments favoring the

increased distribution of the protected government crop (i.e., cotton). The same report also highlights that despite the existence of this pricing mechanism and concession system, farmers retain the option to sell their produce in alternative markets, such as neighboring regions, and the resulting underutilization of ginning facilities leads to ginner capacities mothballing and/or closing down, i.e., referred to as “instability of ginneries.”

The previous discussion raises several research questions on the strategic interaction between upstream farmers and downstream processing plants in the presence of various pricing strategies. Specifically, we would like to investigate the following set of major questions: What influence can downstream processing plants, through their pricing schemes, exert on the farmers’ decisions? Should these plants adhere solely to the minimum price guarantee and subsequently compete with the side market via an ex-post price, or should they consider a hybrid approach that combines these strategies? Alternatively, should processing plants announce a different ex-ante price than the minimum price guarantee? Should price adjustments after harvesting, and closer to downstream delivery dates, be ex-ante committed or presented as a potential option conditional on observed yield and market condition? Furthermore, how do uncertainties in crop yield and the dynamics of the side market affect pricing strategies? How does the price premium that intermediate processing plants charge to their local or export customers affect the efficacy of different pricing schemes?

To address our research questions, we examine an agricultural supply chain of an MPG-supported crop in an emerging economy. Farmers in the growing region make choices between the protected crop and an alternative (e.g., in Mozambique the competing crops are cotton and sesame). The available arable land is estimated with reasonable accuracy, and the government MPG aims to effect its allocation in a favorable ratio for the protected crop, and in alignment with downstream processing capacities. As observed in current practices and literature (Aditya et al., 2017; and Li et al., 2023), farmers do not make quantity commitments, and have the option to sell their crop harvest either in a side market or to a government-favored processing plant. The downstream processing plant is supported to accept all delivered harvest/output up to its processing capacity, a figure often known from the previous year’s government records. Even in the low-likelihood case of capacitated downstream processing, the government pays farmers for the over-capacity harvested crop. (Hereafter, unless otherwise explicitly stated, all crop references are meant for the MPG-supported crop.) The side-market price decreases linearly with the total harvest that farmers sell to the market, with an uncertain intercept. Before the crop production season begins, the government establishes a guaranteed minimum price, which farmers use to plan their crop production. While the processing plant must develop a pricing strategy to engage farmers, farmers must decide on land allocation for the protected crop at the start of the season and determine how to distribute their crop harvest between the side market and the processor.

Altogether we consider four pricing strategies for the processing plant: (1) the passive Benchmark strategy, in which the processing plant adopts and executes literally the government-specified minimum price guarantee (MPG); (2) the Ex-ante strategy, in which the processing plant commits to an optimally chosen price, at least as high as the MPG, at the start of the season and executes it as the final price at the season end; (3) the Responsive strategy, in which the processing plant adopts the MPG at the season start and chooses a price (no lower than the MPG) ex post to buy at the season end in response to the realized crop yield and market price; and (4) the Hybrid Two-Price (choice) strategy, in which the processing plant commits to an ex-ante price (at least as high as the MPG) at the season start and commits to ex-post price adjustments reflecting yield and market conditions at the season end, with a chosen final price (no less than the committed ex-ante price). We develop stylized two-stage stochastic

models for relevant decisions under different pricing strategies. To ensure analytical tractability and to provide useful managerial insights, we make the simplifying assumption that farmers in the region work as a cooperative and are effectively represented by a single farmer in the models. Hereafter, we refer to members in the cooperative as “farmers” and the farmer cooperative as “the farmer.”

The Benchmark analysis offers insights on the farmer’s trade-off between the side market and the processing plant. When the guaranteed price is low, the farmer may opt for a smaller protected crop production (i.e., favor its alternative crop) and sell their harvest at a higher price in the side market. In this case, the farmer opts first for the side market, and then sells whatever the side market does not take to the processing plant at the guaranteed price. When the guaranteed price is sufficiently high, the farmer allocates all available land to the protected crop. At the end of the season, the realized uncertainties in crop yield and side-market price then determine how the harvest is allocated between the processing plant and the side market. Then, considering all potential ex-ante prices that meet or exceed the guaranteed price, the Ex-ante strategy outperforms the Benchmark strategy on profits for farmers and the processing plant, with favorable land allocation for the protected crop production. By committing to an optimally chosen price at the start of the season, the processing plant can incentivize farmers to increase their crop production, leading to higher profits for both the processing plant and the farmers. This strategy not only safeguards the processing plant against the potential risks of adverse side-market conditions that could impact harvest procurement, but also offers farmers higher profit assurance when planning their crop production.

The insights from the ex-post Responsive strategy are interesting: For a fixed land allocation, both the farmer and the processing plant can be better off. In this strategy, the ex-post price tends to decrease as the total allocated land increases. However, anticipating higher ex-post prices when yield is low, the farmer may intentionally produce less of the crop in order to create conditions of higher prices. While this behavior may benefit the farmer, it leads to higher procurement costs for the processing plant, revealing a potential inefficiency in strategic interactions. Hence, while farmers are better off with the Responsive strategy over the Benchmark, the processing plant may be worse off. That is, the ex-post pricing flexibility may potentially backfire and lead to decreased profit of the processing plant.

Interestingly, we discover that the Hybrid strategy with committed ex-ante price and ex-post price adjustments, which includes both the Ex-ante and Responsive strategies as unique cases, may not consistently yield the best results for the processing plant due to the aforementioned strategic interactions between the plant and farmers. While it outperforms the Responsive strategy and the Benchmark strategy in profitability for both farmers and the processing plant, the Hybrid strategy may have the processing plant worse off as compared to the Ex-ante strategy. We find again that ex-post pricing flexibility does not always guarantee better outcome for all stakeholders when operated as an ex-ante commitment. However, we will argue, the processing plant may want to adopt a plausible randomized version of the Hybrid strategy: The plant commits only to an ex-ante price and leaves it as an uncertain option (i.e., hard to predict based on past outcomes) whether it will adjust the final price ex post. While it might pose challenges in short-term implementation depending on whether the trigger events are internal or external, this randomized strategy is possible in a longer time horizon with repeated interactions between farmers and the processing plant. For example, the processing plant can forgo small profit improvements via price adjustment in a given period in exchange for credibility of the uncertainty in exercising the price adjustment option in the future. If such an implementation is feasible, both farmers and the processing plant can be better off. For short-term practices, we suggest a contingency plan-based execution of the Hybrid strategy. We define a contingency region of relevant outcomes (e.g.,

yield realizations, side-market attractiveness, and processing margins) that triggers after-harvest price adjustments, and if done appropriately the Hybrid strategy outperforms.

Our analysis and numerical studies reveal insights on how the ex-ante price commitment from the processing plant affects the farmer’s land allocation to the protected crop. When the ex-ante committed price exceeds a certain threshold, the farmer opts for the full land allocation to the crop. We refer to this price threshold as the “full land allocation (FLA)” threshold. The FLA threshold is dependent on the pricing strategy used. The FLA for the Ex-ante strategy is higher as compared to the Benchmark strategy, since the processing plant tends to use ex-ante committed prices above the MPG for increased competitiveness with the side market. Our analysis further shows that the FLA threshold under the Responsive strategy is also higher than that under the Benchmark. This is because, in anticipation of significant end-of-season price adjustments when harvests are low, the farmer has a strong incentive to withhold full commitment to land allocation, particularly when the ex-ante committed prices are not sufficiently high. The FLA thresholds are the same under the Responsive and Hybrid strategies, when the Hybrid strategy has a committed price adjustment at the season’s end. While all the above analytical and numerical results are obtained under the proportional yield assumption, we extend our models to the case of heterogeneous land productivity.

The contributions of our research are twofold. First, our study, albeit stylized, provides a novel model of agribusiness practices in emerging economies where farmers make land allocation decisions among competing cash crops and have the option to choose downstream delivery after harvest between a side market and the processing plant. We offer a unified framework incorporating different pricing strategies the processing plant may use to interact with farmers in the presence of a government-specified MPG. Second, we provide analytical and numerical results characterizing decision trade-offs for both farmers and the downstream processing plant in the presence of these strategies. In the presence of the side market vying for harvests at the season’s end, optimal ex-ante pricing adjustment capabilities at the processing plant benefit both the farmer and the plant, as evident in the Ex-ante strategy. However, the processing plant’s ex-post pricing flexibility, responding to realized yields and uncertainties in the side market, may unfavorably influence farmers’ land allocation to the protected crop and inadvertently undermine the plant’s profitability. This represents a counterintuitive result, the paradox of ex-post pricing flexibility, and it is revealed and explained in our analysis of the Responsive and Hybrid Two-Price strategies. Nonetheless, when the processing plant does not make an ex-ante commitment on the ex-post price adjustment at the season’s end (i.e., when executing a randomized version of the Hybrid scheme), both the farmer and the plant may be better off as compared to the Ex-ante strategy. Below, we position our research in the existing literature.

2 Relevant Literature

This paper contributes to three streams of literature: supply risk management, operational flexibility via price postponement, and agricultural supply chain management.

First, our work is related to the broad literature on supply risk management, which concerns unreliable supply in production systems (see, e.g., Feng 2010, Feng et al. 2019, 2021, etc.). One stream focuses on stochastic proportional yields. The classic work by Henig and Gerchak (1990) characterizes the optimal sole sourcing inventory policy as a threshold policy under which the firm places an order when the inventory falls below the threshold, and both the order quantity and the threshold are functions of the yield distribution. In more recent work, Kouvelis et al. (2018) provide important

distributional conditions to study the production inflation issue in a decentralized supply chain under yield uncertainty. Besides inventory, supply diversification is another common proactive strategy to hedge against supply shortage, and is widely studied in the literature (see, e.g., Dada et al., 2007, Dong et al., 2022, etc.). Mitigation strategies to fight against potential supply yield shortages are studied in the literature, such as emergency production (Kouvelis and Li, 2013), reliability improvement (Lee and Lu, 2015), and backup supply (Pan et al., 2022). Our work differs from these previous studies as we investigate how to design effective pricing mechanisms for an agricultural supply chain of farmers and intermediate processing plants interacting in the presence of production yield and an uncertain side market. Specifically, we examine how the downstream processing plant can use pricing to incentivize upstream farmers to grow and sell more of a protected crop harvest.

Second, our work contributes to the broad literature on operational flexibility, and particularly research on pricing flexibility. This body of literature recognizes that pricing can play a powerful role in mitigating the mismatch of supply and demand when price commitment is postponed until after all uncertainties are resolved. Van Mieghem and Dada (1999) are the first to study the impact of the pricing time on the inventory decision when demand is uncertain. Granot and Yin (2008) investigate price and order postponement in a bilateral supply chain. In the context of uncertain yield, Tang and Yin (2007) and Kouvelis et al. (2021) analyze the impact of responsive pricing for risk-neutral and risk-averse firms, respectively. Kazaz and Webster (2011) study the impact of yield-dependent trading cost on the firm's pricing and quantity decisions under responsive pricing. Li et al. (2017) and Dong et al. (2023) evaluate the impact of price postponement on the adoption of supply diversification under random capacity and random yield models, respectively. Geng et al. (2023) investigate the interaction between price postponement and payment postponement in mitigating supply yield risk in a bilateral supply chain. In contrast to the aforementioned studies, which do not consider farmers' choice in land allocation among competing crops and after-harvest distribution choice in the presence of an uncertain side market, we explore a variety of (ex-ante and ex-post) pricing mechanisms that a downstream processing plant can employ to better manage the strategic behavior of farmers. Unlike Granot and Yin (2008) and Geng et al. (2023), who find that operational flexibility may backfire for the Stackelberg follower, our study shows that ex-post pricing flexibility does not necessarily benefit the downstream processing plant as the Stackelberg leader. This is because farmers may strategically adjust their land allocation for the protected crop in response. The identification of this ex-post pricing flexibility paradox in the agri-food supply chain is a key contribution of our work.

Third, our work contributes to the emerging agricultural supply chain literature. Here, we focus only on closely related literature that investigates farmers' land allocation decisions in the presence of government supporting policies. Alizamir et al. (2019) study the impacts of two government subsidies (agriculture risk coverage and price loss coverage) in the agriculture industry on consumers, farmers, and the government. Akayya et al. (2021) study the impact of taxes and subsidies on innovations, including sustainable practices. Chintapalli and Tang (2021) study the impact of minimum price support on farmer surplus, consumer surplus, expenditure, and net welfare with a risk-averse constraint, while Guda et al. (2021) study guaranteed support price by analyzing a Stackelberg game between a social planner and a group of homogeneous farmers. Relatedly, while Tang et al. (2023) examine and compare the effects of input and output subsidy programs on farmer welfare, income disparity, and consumer surplus, Fan et al. (2023) compare a set of subsidy programs to evaluate the efficiency of resource usage and the distribution among the farmers. In contrast to these papers, our work examines the land allocation decision in a supply chain setting with an aggregate farmer and a downstream processing

plant, and the interactions between the aggregate farmer and the processing plant. Recently, Li et al. (2023) study competition and coordination issues in an agricultural supply chain that comprises farmers and downstream processors, in the presence of MPG. Emphasis is placed on characterizing the second-price auction that allocates harvest among competing processors in the chain. Unlike Li et al. (2023), we study various pricing schemes the downstream processing plant uses to offer the right incentives to farmers in land allocation and harvest distribution. Our emphasis is on revealing the potentially detrimental effect of pricing flexibility in the agri-chain context.

Finally, we note that a substantial body of literature has explored agricultural supply chains in emerging economies without government subsidies, albeit focusing on a variety of other issues. Huh and Lall (2013) investigate optimal crop choices, water capacity reservation, and irrigation allocation from a farmer’s perspective. Tang et al. (2014) analyze whether farmers should use market information to improve their production plans or adopt agricultural advice to optimize their operations. Behzadi et al. (2018) explore the role of allocation flexibility and the effectiveness of various risk management strategies in mitigating demand disruption risks within agricultural supply chains. Yi et al. (2021) examine how different financing formats influence smallholder farmers’ production decisions. Dong et al. (2024) and Zheng et al. (2024) respectively assess the impact of precision agriculture (PA) technology on farmer welfare and environmental sustainability. For a broader review, with the challenges and opportunities in transitioning to a resilience-focused, eco-friendly agricultural value chain, see Dong (2021).

The remainder of the paper is organized as follows. In Section 3, we present the modeling framework for the agricultural supply chain and introduce the four pricing mechanisms. Section 4 formulates and analyzes the Benchmark and Ex-ante strategies, and this is followed by the formulation and analysis of the Responsive and Hybrid strategies in Section 5. Subsequently in Section 6, we study numerically the impact of various modeling parameters on the profitability of these strategies. In Section 7, we discuss randomized variants of the Hybrid strategy, in which the processing plant commits only to the ex-ante price, while leaving uncertain the option to adjust the final price ex post. Section 8 briefly extends the models to account for heterogeneous land productivity. Finally, we conclude the paper in Section 9.

3 The Framework

We consider an agricultural supply chain of a protected crop by the government in an emerging economy, whose production is supported by minimum price guarantee (MPG). Farmers cultivating the protected crop are not required to commit ex ante to supplying a specific quantity to any downstream buyer (Aditya et al., 2017; The World Bank Report, 2010 and 2011)¹. Instead, they have the flexibility to sell their harvest either in a side market or directly to a processing plant. Based on estimates of the arable land in the growing region and the anticipated response of farmers to given incentives, the government estimates the quantity of planted crop and ensures that it aligns with downstream processing capacities. The side market is characterized by a linear price model: $r = A - bq$ (see, e.g., Tang and Yin, 2007); that is, for a given total quantity q farmers sell in the side market, its price is $A - bq$. We assume that A is a random variable with a probability density function (PDF) $\psi(\cdot)$ and a cumulative distribution function (CDF) $\Psi(\cdot)$. A may represent the maximum willingness to pay for the crop on the side market, and is referred to as (uncertain) price intercept. The processing plant, which the government approves as the

¹In situations where farmers have to fulfill explicit legal or financial obligations, committing to a specific quantity is often more appropriate. One such example is when farmers use their harvest as collateral for loans (Yi et al., 2021).

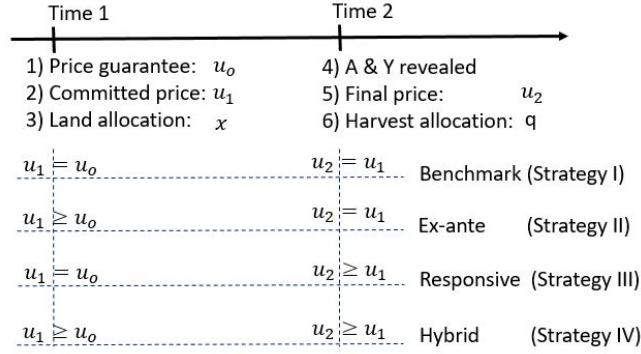


Figure 1: Sequence of events, and strategies

primary processor for the crop in its region, pays a procurement price that meets or exceeds the MPG². We assume that the processing plant is un-capacitated in that it has a capacity to process a historically peak-high crop harvest from the region. The region's land size is L units. The government sets the MPG in a way that offers incentives to increase the utilization of often underused downstream assets. We will discuss for the purpose of theoretical completeness an extension with a restrictive processing capacity in Appendix V.

A general sequence of events in the supply chain is illustrated in Figure 1 for a crop growing season. Prior to the start of the growing season, the government announces a MPG u_o , which requires the processing plant to commit to a price u_1 that is no less than u_o (i.e., $u_1 \geq u_o$) at the beginning of the season (Time 1). With committed price u_1 from the processing plant, farmers decide the amount of land x allocated out of L available arable units to the protected crop. We assume that all farmers in the region have the same proportional crop yield; this is appropriate when similar weather conditions and other unpredictable factors affect the crop production and growth in the whole region (see, e.g., Alizamir et al., 2019). The proportional crop yield per unit of land is random and denoted as Y , with a PDF $\phi(\cdot)$ and CDF $\Phi(\cdot)$. After uncertainties for both Y and A are realized at the end of the season (Time 2), the processing plant³ offers a final procurement price u_2 (which is greater than or equal to u_1), and farmers allocate their harvest between the side market and the processing plant. We denote q as the amount of crop harvest sold to the side market, and accordingly $xY - q$ is the amount of crop harvest sold to the processing plant. Each unit of land allocated to the crop incurs a production cost c_f , and each unit of the remaining land $L - x$ generates a leftover profit c_l . c_l represents the average profit margin for farmers from growing a diversification crop or other use of the land. We assume c_l is constant; for example, this may be the result of a MPG for growing another government-supported crop (adjusted for average yield), contractual revenue from leasing the land for another use, or the mean of an exogenously varying profit from other land use. We let m be the processing margin for harvest units that go through the processing plant's output, which is the difference between the selling price and processing cost. Downstream crop processing plants sell their output (e.g., lint, edible oils, etc.) to global markets or market-controlling consumer packaged good companies at established prices known at the start of the season. Similarly, processing costs (e.g., milling, refining, etc.) are contractually set

²We implicitly assume that the processing plant is committed to meeting this government-mandated MPG standard, and that its implementation can be verified through an enforceable agreement between the plant and the government.

³The processing plant is typically located near or within the growing region of the protected crop. It employs field specialists and regional staff, often recruited from local growers or elected representatives of the farming community. This organizational structure provides the plant with direct access to realizations of A and Y .

for both exporters and downstream buyers at the beginning of the season. The resulting margin (e.g., price minus processing costs) is known and fixed at the beginning of the time period of our decisions. We assume $m > u_o$ holds, i.e., the processing plant has the price latitude for adjustment consideration.

The processing plant must determine the optimal approach for engaging with farmers regarding their crop production (land allocation) and harvest allocation decisions, using the pricing options available at Times 1 and 2. Within the framework depicted in Figure 1, we consider four pricing strategies for the processing plant. The first strategy, introduced below in Section 4.1, is a passive Benchmark approach in which the processing plant sets u_o at both Times 1 and 2; this is followed by an Ex-ante strategy in which the processing plant optimally chooses $u_1 (\geq u_o)$ at Time 1 and implements it at Time 2. In Section 5 we consider a Responsive strategy in which the processing plant adopts u_o at Time 1, and after observing Y and A , optimally chooses a final price $u_2 (\geq u_o)$ at Time 2. Finally, we examine a Hybrid strategy in which the processing plant optimally commits to a price $u_1 (\geq u_o)$ at Time 1 and also announces a committed price adjustment later. The plant then optimally, given realized conditions, sets a final price $u_2 (\geq u_1)$ at Time 2. For expositional brevity, we may also refer to these strategies as Strategy *I*, *II*, *III*, and *IV* in their above-mentioned order. A table summarizing the characteristics of each studied strategy is available in the Appendix IV (see Table 2). Our study of these strategies is from an aggregate farming level; that is, we assume that farmers act as a group (like a farmer cooperative or co-op) and thus they are represented by one farmer (pronoun: her). Farmer co-ops are an observed farming organizational practice, and there are a variety of benefits for farmers to form co-ops (see, e.g., An et al., 2015). We assume all information is public and all decision-makers are risk-neutral.

We formulate the models for these four pricing strategies and their corresponding decisions at Times 1 and 2 in two-stage stochastic programs, and solve them backward starting from the decisions at Time 2. A brief summary of the notations is given in Table 1 of Appendix *II*. In Section 4, we formulate and analyze single-choice ex-ante strategies, including Strategies *I* and *II*. Subsequently, Section 5 introduces the formulation and analysis of ex-post pricing flexibility strategies, specifically Strategies *III* and *IV*.

4 Single-Price Choice Ex-ante Strategies

We begin our analysis with the passive Benchmark strategy in Section 4.1, in which we derive the optimal land and harvest allocation rules and conduct sensitivity analysis on the land allocation and expected profit. In Section 4.2, we analyze the Ex-ante strategy and compare expected profits between the Benchmark and Ex-ante strategies.

4.1 The Passive Benchmark

The Benchmark strategy involves the processing plant setting both the early committed price u_1 and the final price u_2 to the MPG u_o . The decisions in the strategy reduce to the farmer's land and harvest allocations only. Below, we first characterize the farmer's trade-off at Time 2 between the side market and the processing plant. Then we determine the optimal land allocation at Time 1 and analyze the impact of uncertainty on the final equilibrium outcomes. At Time 2, after the relevant uncertainties have been revealed and with a total harvest of $x \cdot Y$, the farmer determines the optimal amount q to sell in the side market and $xY - q$ to sell to the processing plant in order to maximize the total revenue.

The optimal value of q is obtained by using the following optimization model:

$$\begin{aligned} \max_q \quad & \{V_f(q, x, u_o, Y, A) := (A - bq)q + u_o(xY - q)\} \\ \text{s.t.} \quad & 0 \leq q \leq xY, \end{aligned}$$

where $V_f(q, x, u_o, Y, A)$ is the farmer's total revenue for given q sold in the side market, land allocation x , final price u_o , as well as realizations of Y and A . The first term of $V_f(q, x, u_o, Y, A)$ is the revenue from selling in the side market, and the second term is the revenue from selling to the processing plant. It can be seen that from the first order condition (FOC) and concavity of $V_f(q, x, u_o, Y, A)$ w.r.t. q , the optimal harvest allocation q^* to the side market is determined as follows:

$$q^* = \begin{cases} 0 & \text{if } A \leq u_o \\ \frac{A - u_o}{2b} & \text{if } A > u_o \text{ \& } \frac{A - u_o}{2b} \leq xY \\ xY & \text{if } A > u_o \text{ \& } \frac{A - u_o}{2b} > xY. \end{cases} \quad (1)$$

The harvest allocation rule for the farmer is straightforward. When the MPG from the processing plant is higher than the realized side-market price intercept A (i.e., $A \leq u_o$), it is more profitable to sell the entire harvest to the processing plant. Conversely, when the side-market price is higher than the MPG (i.e., $u_o < A$), the side market is the first choice, and the processing plant receives a portion of the harvest only if the harvest xY exceeds the threshold $\frac{A - u_o}{2b}$. Specifically, when the harvest xY is sufficiently high, or equivalently when u_o is large enough (i.e., $\frac{A - u_o}{2b} \leq xY$), $\frac{A - u_o}{2b}$ is sold to the side market and the processing plant receives the remainder $xY - \frac{A - u_o}{2b}$. On the other hand, when the harvest xY is not high enough, or equivalently when u_o is small (i.e., $xY < \frac{A - u_o}{2b}$), the entire harvest xY is sold to the side market and the processing plant receives nothing. The corresponding farmer's crop revenue at q^* , denoted as $V_f^*(x, u_o, Y, A) := V_f(q^*, x, u_o, Y, A)$, is calculated for each combination of Y and A and folds back to Time 1 as follows:

$$V_f^*(x, u_o, Y, A) = \begin{cases} u_o xY & \text{if } A \leq u_o \\ \frac{(A - u_o)^2}{4b} + u_o xY & \text{if } A > u_o \text{ \& } \frac{A - u_o}{2b} \leq xY \\ (A - bxY)xY & \text{if } A > u_o \text{ \& } \frac{A - u_o}{2b} > xY. \end{cases} \quad (2)$$

At Time 1 the choice of the crop land amount x is to maximize the farmer's expected profit below:

$$\max_x \{ \mathcal{V}_{f,\xi}(x, u_o) := \mathbb{E}_\xi [V_f^*(x, u_o, Y, A)] - c_f x + c_l(L - x) \}, \quad (3)$$

where $\mathcal{V}_{f,\xi}(x, u_o)$ represents the expected profit for the farmer, given x and committed price u_o at Time 1. We denote $x^*(u_o, \xi)$ as the optimal value to (3). For notational convenience, we refer to it as x^* or $x^*(u_o)$ by suppressing its dependency on u_o , and ξ when the context is clear. The first term of $\mathcal{V}_{f,\xi}(x, u_o)$ in (3) is the expected revenue from Time 2 over all possible Y and A , the second term is the incurred production cost for x , and the third term is the profit from the remaining land $L - x$.

Lemma 1 *The optimal land allocation x^* is uniquely determined in the first-order condition below:*

$$u_o \mathbb{E} [Y \cdot \mathbf{1}_{\{A \leq u_o\}}] + u_o \mathbb{E} \left[Y \cdot \mathbf{1}_{\{A > u_o \text{ \& } \frac{A - u_o}{2b} \leq xY\}} \right] + \mathbb{E} \left[(A - 2bxY)Y \cdot \mathbf{1}_{\{A > u_o \text{ \& } xY < \frac{A - u_o}{2b}\}} \right] = c_f + c_l. \quad (4)$$

where $\mathbf{1}_{\{\text{condition}\}}$ is an indicator function with a value of 1 when "condition" is true, and a value of 0 otherwise.

All proofs can be found in Appendix I. The right-hand side of (4) represents the cost of allocating one unit of land to the protected crop, which includes both the production cost c_f and the opportunity cost c_l . The left-hand side terms fully capture the trade-off faced by the farmer in harvest allocation, as reflected in (1), between the side market and the processing plant. It is worth noting that on one hand, x^* can be greater than zero even when $u_o < c_f + c_l$, and that on the other hand, x^* can be smaller than L even when $u_o > c_f + c_l$. The former occurs because the farmer may choose to sell at a price of $A - 2bxY$, which might be greater than $c_f + c_l$, resulting in a positive land allocation to the crop. The latter occurs when the expected crop yield is small and u_o is only slightly greater than $c_f + c_l$. Additionally, it is evident that when u_o is large enough, $x^* = L$, as seen in Corollary 1 below.

Corollary 1 *If $u_o \geq \frac{c_f + c_l}{\mathbb{E}[Y]}$, then $x^* = L$.*

We refer to $\frac{c_f + c_l}{\mathbb{E}[Y]}$ as the u_o threshold for full land allocation (for brevity, the “full allocation threshold” or FLA, and denoted as u_{FL}^I) in the Benchmark strategy. With the above characterization of the optimal land allocation x^* , now we are ready to analyze how the side market and the MPG affect x^* and the associated profit $\mathcal{V}_{f,\xi}^*(u_o) := \mathcal{V}_{f,\xi}(x^*, u_o)$. To this end, we observe that both x^* and \mathcal{V}_f^* depend on the MPG u_o as well as on the distribution information for Y and A . Accordingly, we express them explicitly as functions of u_o , Y and A : $x^*(u_o, \xi)$ and $\mathcal{V}_{f,\xi}^*(u_o)$, respectively. We will use the concepts of first-order stochastic dominance (denoted as \leq_{st} or \geq_{st}) and convex-order stochastic dominance (denoted as \leq_{cx} or \geq_{cx}) between random variables. The formal definitions of these concepts can be found in Shaked and Shanthikumar (2006). Corollary 2 below summarizes properties of x^* .

Corollary 2 *1) If $A^1 \leq_{st} A^2$, then $x^*(u_o, \xi^1) \leq x^*(u_o, \xi^2)$, where $\xi^i = (Y, A^i)$, $i \in \{1, 2\}$.
2) $x^*(u_o, \xi)$ is convex-increasing in u_o .*

The price of the side market is determined by A , and a higher price motivates the farmer to grow more of the crop. The MPG u_o from the processing plant is also an important factor, as a higher u_o means that the farmer gets a higher price and is motivated to grow more of the crop. The corollary below summarizes properties of the farmer’s optimal profit $\mathcal{V}_{f,\xi}^*(u_o) := \mathcal{V}_{f,\xi}(x^*, u_o)$.

Corollary 3 *1) If $A^1 \leq_{st} A^2$, then $\mathcal{V}_{f,\xi^1}^*(u_o) \leq \mathcal{V}_{f,\xi^2}^*(u_o)$, where $\mathcal{V}_{f,\xi^i}^*(u_o)$ is the farmer’s optimal profit with committed price u_o at Time 1 when the side-market linear price is $A^i - bq$.*

- 2) If $A^1 \leq_{cx} A^2$, then $\mathcal{V}_{f,\xi^1}^*(u_o) \leq \mathcal{V}_{f,\xi^2}^*(u_o)$.
3) $\mathcal{V}_{f,\xi}^*(u_o)$ increases in u_o .*

Whenever the realization (or mean) of A or the value of u_o is higher, the farmer’s profit at Time 2 improves, resulting in a larger expected farmer profit at Time 1. A more variable A allows the farmer to benefit from larger realizations of A while reducing losses from low realizations of A through u_o . Therefore, a higher variance in A leads to greater profits for the farmer.

Having established the farmer’s decisions and profits at Time 1 and Time 2 as described above, we now shift our focus to the processing plant. For notational simplicity, we express x explicitly as a function of u_o , while suppressing its dependence on b , Y and A . Denote $\mathcal{V}_{p,\xi}(x, u_o)$ as the profit of the processing plant at Time 1, given land allocation x and committed price u_o , as expressed below.

$$\mathcal{V}_{p,\xi}(x, u_o) = \mathbb{E}_\xi [V_p(x, u_o, Y, A) := (m - u_o)(xY - q^*)], \quad (5)$$

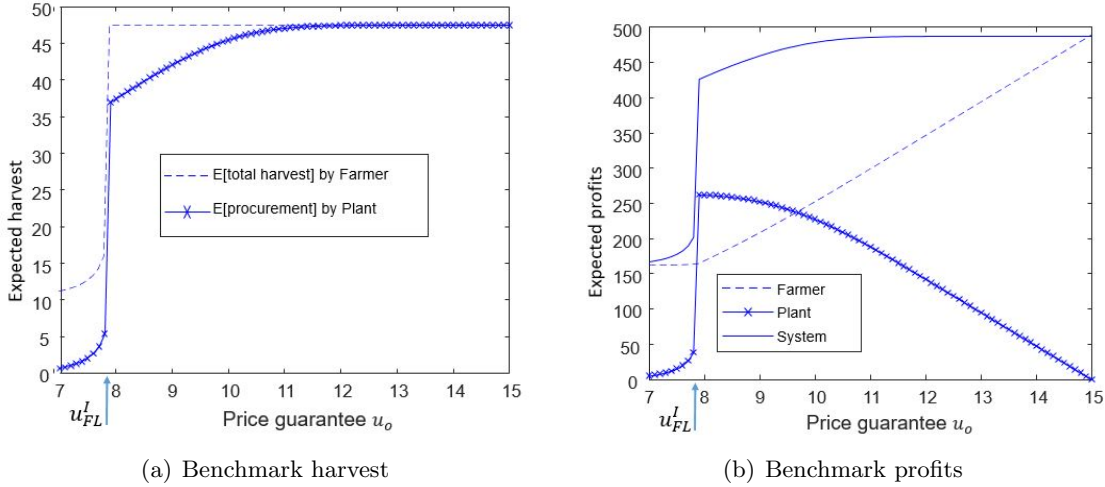


Figure 2: Performance under Benchmark strategy

where $V_p(x, u_o, Y, A)$ represents the processing plant's profit viewed at Time 2. The first factor of $V_p(x, u_o, Y, A)$ is the profit margin for the plant, while the second factor indicates the allocated amount of crop harvest. It will be seen later that $\mathcal{V}_{p,\xi}(x^*(u_o), u_o)$ is not necessarily concave. We next present a sufficient condition under which $\mathcal{V}_{p,\xi}(x^*(u_o), u_o)$ is unimodal.

Proposition 1 *Under full land allocation to the crop, i.e., $u_o \geq u_{FL}^I$, if A has a PF_2 probability density function, then $\mathcal{V}_{p,\xi}(x^*(u_o), u_o)$ is unimodal in u_o .*

The above proposition effectively indicates the unimodality of $\mathcal{V}_{p,\xi}(x^*(u_o), u_o)$ when u_o is above the FLA threshold u_{FL}^I and A has a PF_2 probability density. Note that PF_2 densities (Karlin, 1968) include the classes of truncated normal and gamma families. Next, we examine through numerical studies the impact of u_o on the processing plant's expected harvest procurement and expected profit as well as the farmer's expected profit. The numerical examples hereto assume the following parameters (unless otherwise specified): $A \sim Normal(\text{mean} = 10, \text{std} = 1)$, $b = 0.1$, $Y \sim Normal(\text{mean} = 0.95, \text{std} = 0.1)$, $c_f = 4.5$, $c_l = 3$, $m = 15$, and $L = 50$.

We begin by examining the impact of u_o on the expected harvests for the farmer and the plant. For values of u_o below the FLA u_{FL}^I , e.g., 7.9 in Figure 2(a), x^* increases but remains less than L . Because of small u_o values, the plant receives a small percentage of the harvest, e.g., ranging from about 5.96% to 33.76% in Figure 2(a). Once u_o surpasses the FLA threshold, where $u_{FL}^I = 7.9$, the percentage of harvests received by the plant substantially increases when x^* becomes L , e.g., it is 77.8% at $u_o = u_{FL}^I$ in Figure 2(a), and it keeps increasing as u_o increases until it reaches 100%.

Next, we examine the impact of u_o on the expected profits for the farmer and the plant, as well as their total (i.e., the system's profit). As u_o increases, the farmer gets a higher price for her harvest and is willing to grow more of the crop. Thus the farmer's profit increases in u_o . As u_o increases, the plant receives higher harvest amounts, since the price is more competitive and the farmer grows more of the crop. Thus it is observed that the plant's profit increases as long as $u_o < u_{FL}^I$, see Figure 2(b). When u_o exceeds the FLA threshold u_{FL}^I , the plant's expected profit becomes unimodal (i.e., first increasing and then decreasing). It is noteworthy that while meeting the u_o threshold induces the farmer to fully allocate the land to the protected crop, the plant may find it beneficial to operate at a higher u_o (over u_{FL}^I) in order to be competitive in the side market. However, raising the u_o value leads to a reduced

profit margin, and thus the plant's profit may exhibit only a decreasing trend over $[u_{FL}^I, m]$, as observed in Figure 2(b), when the processing margin m is not sufficiently high.⁴ The higher u_o is, the less of the harvest is sold in the side market, leading to a higher profit for the system.

For ease of reference, we denote \mathcal{V}_f^I and \mathcal{V}_p^I as the maximum expected profits for the farmer and the processing plant, respectively, in the Benchmark strategy for a given committed price u_1 at Time 1. Additionally, we let x^I be the optimal land allocation to the crop for given u_1 . Note that $u_1 = u_o$ in the Benchmark strategy, where u_o is exogenously given and is (assumed to be) determined by government officials. The external determination of u_o may result from trade-offs that encompass various factors, including the government's total budget available for crop support policies, the crops eligible for such support, world market prices for these crops, living expenses, and the standard of living within the relevant farming community, as well as the population size and dynamics of the community, etc. (see, e.g., The World Bank Report, 2010). While the overarching goal is to support the livelihood and safeguard the living standards of the farming society, these considerations may not align seamlessly with downstream links, such as the processing plant, within the agricultural supply chain. With the above-defined profit functions for the farmer and the processing plant in the Benchmark, below we consider the Ex-ante strategy, which allows the plant to make an ex-ante price commitment at Time 1.

4.2 The Ex-ante Strategy

In the Ex-ante strategy, instead of passively adopting the MPG u_o , the processing plant commits to an optimally chosen value of $u_1 (\geq u_o)$ at Time 1 and pays for the delivered harvest at Time 2 at this price. Since otherwise the timeline of events in the Ex-ante strategy is the same as that in the Benchmark, we can conveniently use the dynamics and results in the Benchmark strategy to study and present the Ex-ante strategy. Following the harvest allocation rule described in (1) at Time 2, the farmer allocates the harvest between the side market and the processing plant based on the observed Y and A , and the pre-committed price u_1 . The corresponding optimal farmer revenue at Time 2 is then folded back to calculate the expected farmer profit in (3) at Time 1 with a pre-committed price u_1 and to find its optimal land allocation $x(u_1)$. Knowing the land allocation $x(u_1)$ from the farmer, the processing plant's choice of the pre-committed price u_1 is formulated into the following optimization model:

$$\max_{u_1 \geq u_o} \mathcal{V}_{p,\xi}(x, u_1). \quad (6)$$

Let u_1^{II} be the u_1 value that maximizes $\mathcal{V}_{p,\xi}(x, u_1)$ in (6) over all $u_1 \geq u_o$, and u_{FL}^{II} be the smallest u_1 value such that $x^{II}(u_1) = L$ (i.e., the FLA threshold for the Ex-ante strategy).

Proposition 2 *Let us consider the full land allocation region to the crop (i.e., $u_o \geq u_{FL}^I$), and that A has a PF_2 probability density function. If $m > \tilde{m}$, then $u_{FL}^{II} > u_{FL}^I$; and if $m \leq \tilde{m}$, then $u_{FL}^{II} = u_{FL}^I$, where \tilde{m} is defined as follows:*

$$\tilde{m} = \left(L\mu_Y\Psi(u_1) + \int_{a>u} \int_{y>\frac{a-u_1}{2bL}} (Ly - \frac{a-u_1}{2b})d\Phi(y)d\Psi(a) + \frac{u_1}{2b} \int_{a>u} (1 - \Phi(\frac{a-u_1}{2bx}))d\Psi(a) \right) \Bigg|_{u_1 = \frac{c_f+c_l}{\mu_Y}}. \quad (7)$$

⁴We note that in our example the maximum plant profit occurs when $u_o = u_{FL}^I$, but this is not necessarily the case when m is larger, as, for example, an illustrating supportive example can be easily constructed when $m = 20$.

Proposition 2 establishes a critical link between the u_o full land threshold in the Benchmark strategy and the plant-preferred full land u_1 , denoted as u_{FL}^{II} , in the Ex-ante strategy. Assuming u_o is sufficiently large to incentivize a full land allocation for the farmer, this proposition identifies sufficient conditions under which $u_{FL}^{II} > u_{FL}^I$ and $u_{FL}^{II} = u_{FL}^I$ hold true, respectively. The conditions underscore the pivotal role of the plant's processing margin m in deciding whether $u_{FL}^{II} > u_{FL}^I$ or $u_{FL}^{II} = u_{FL}^I$ occurs. When m is sufficiently large, i.e., $m > \tilde{m}$, there is ample space in the plant's margin to adopt more aggressive ex-ante pricing, resulting in a larger u_{FL}^{II} ; this way, not only can the plant get a full land allocation, it also is more competitive against the side market. Conversely, for any smaller m values, $u_{FL}^{II} = u_{FL}^I$.

For ease of reference, we denote \mathcal{V}_f^{II} and \mathcal{V}_p^{II} as the maximum expected profit for the farmer and the processing plant, respectively, in the Ex-ante strategy over all committed price u_1 at Time 1. The following proposition compares the performance of the Benchmark and Ex-ante strategies.

Proposition 3 *For both the farmer and the processing plant profits, $\mathcal{V}_f^I \leq \mathcal{V}_f^{II}$ and $\mathcal{V}_p^I \leq \mathcal{V}_p^{II}$ hold.*

Using insights from Propositions 2 and 3, we can outline relevant trade-offs the government may face in choosing the appropriate MPG. While setting the MPG close to u_{FL}^I will lead to full land allocation under a simple Benchmark strategy, it will not come close to driving the high utilization of the processing plant capacity. This second goal might be achieved for an MPG closer to u_{FL}^{II} , as demonstrated by a higher ex-ante price under the Ex-ante strategy. However, the government might hesitate to assume such aggressive MPGs for agri-chains with high processing margins. For these cases, the government expects the intermediate processing plant to assume the extra cost (i.e., sacrificing some of its margins) in providing the needed incentives to farmers for a full-land allocation and receiving a higher percentage of the harvests over the side market. An optimized ex-ante price leads to closer achievement of both efficiency measures in the agri-chain: full-land allocation to the crop, and higher utilization of intermediate processing capacity. Furthermore, it is Pareto optimal relative to the Benchmark strategy, as Proposition 3 shows. The Ex-ante strategy leads to a win-win outcome for both the processing plant and the farmer. That is, while for low-profit-margin agri-chains the government MPG is necessary for reasonable performance of the chain, in high-margin environments the intermediate plant has the motivation to offer prices that lead to improved profit performance for all parties in the chain.

While the Ex-ante strategy enables the processing plant to incentivize farmers by committing to pay a price that may exceed the government's announced MPG, other pricing strategies use ex-post pricing flexibility to account for yield and side market conditions and, if needed, offer higher procurement prices at Time 2 (the end of the season). We discuss them next in Section 5.

5 Ex-Post Pricing Flexibility: Responsive and Hybrid Strategies

In this section, we first analyze a pure ex-post Responsive strategy in Section 5.1. Then we examine a Hybrid Two-Price choice strategy that incorporates both an ex-ante price pre-commitment and an ex-post price adjustment in Section 5.2.

5.1 The Responsive Strategy

By allowing the processing plant to adjust the final price at the end of the season, the Responsive strategy offers a mechanism to account for yield realizations in the crop production or side-market conditions. In what follows, we derive the optimal responsive price for the processing plant at Time 2,

followed by the expected profits for the farmer and the processing plant at Time 1 for a given x and $u_1 = u_o$, and then compare the Responsive strategy with the Benchmark strategy.

Following the optimal harvest allocation rule in Equation (1) but substituting u_2 for u_o , the farmer allocates the harvest to the side market, denoted explicitly as $q^*(u_2)$ for its dependency on u_2 . The optimization model for the processing plant's choice of u_2 in the Responsive strategy is as follows:

$$\max_{u_2 \geq u_1} \{V_p(x, u_2, Y, A) := (m - u_2)(xY - q^*(u_2))\}. \quad (8)$$

where $V_p(x, u_2, Y, A)$ represents the profit for the processing plant at Time 2. The following lemma characterizes the optimal u_2 .

Lemma 2 *The optimal price u_2 , denoted as u_2^* for a given committed price u_1 at Time 1, is given below:*

$$u_2^*(x) = \max \left(u_1, \min \left(\frac{m + A - 2bxY}{2}, A, m \right) \right). \quad (9)$$

As shown in the proof, $V_p(x, u_2, Y, A)$ is zero at the beginning of the domain (i.e., in the interval $u_2 \in [0, A - 2bxY]$), and concave on $[A - 2bxY, \min(A, m)]$, and decreasing on $[\min(A, m), \max(A, m)]$. Consequently, the optimal u_2 for the processing plant falls in $[A - 2bxY, \min(A, m)]$, since $V_p(x, u_2, Y, A)$ is also continuous in u_2 . The unconstrained optimal u_2 , determined via FOC for $V_p(x, u_2, Y, A)$, is $\frac{m+A-2bxY}{2}$. Taking into account the boundary condition $u_2 \geq u_1$ and the magnitude relationship between A and m , we can obtain (9). The formula shows that the final price is higher than the originally announced u_1 (which is set to u_o for the Responsive strategy), except when u_1 is large enough (i.e., $A \leq u_1 \leq m$) or when the harvest is large enough (i.e., $\frac{m+A-2u_1}{2b} < xY$). The ex-post price reaches its maximum when the harvest is very small (i.e., $xY \leq \frac{A-m}{2b}$). This finding suggests that compared to the Ex-ante strategy, the Responsive strategy provides the processing plant with an opportunity to compete for harvest even when the harvested amount is small. Furthermore, it can be seen that u_2^* increases in A , u_1 , and m , and decreases in x and Y . With the expression for u_2^* in (9), the corresponding optimal profit of the processing plant at Time 2 for given committed price u_1 can be calculated; we denote this profit as $V_p(x, u_2^*, Y, A)$. It can be seen $V_p(x, u_2^*, Y, A)$ that is continuous in u_1 .

To derive the optimal profit from the crop at Time 2 for the farmer, conditioned on realizations of Y and A , we need only to apply u_2^* to (2) by substituting u_o with u_2^* , and after algebraic manipulation, can obtain $V_f^*(x, u_2^*, Y, A)$ below:

$$V_f^*(x, u_2^*, Y, A) = \begin{cases} u_1 x Y & \text{if } A \leq u_1 \leq m \\ \frac{1}{4b} (A - u_1)^2 + u_1 x Y & \text{if } u_1 < A < m \ \& \ \frac{m+A-2u_1}{2b} < xY \\ \frac{1}{4b} \left(\frac{A-m+2bxY}{2} \right)^2 + \left(\frac{m+A-2bxY}{2} \right) xY & \text{if } u_1 < A < m \ \& \ \frac{m-A}{2b} < xY \leq \frac{m+A-2u_1}{2b} \\ AxY & \text{if } u_1 < A < m \ \& \ xY \leq \frac{m-A}{2b} \\ \frac{1}{4b} (A - u_1)^2 + u_1 x Y & \text{if } u_1 \leq m \leq A \ \& \ \frac{m+A-2u_1}{2b} < xY \\ \frac{1}{4b} \left(\frac{A-m+2bxY}{2} \right)^2 + \left(\frac{m+A-2bxY}{2} \right) xY & \text{if } u_1 \leq m \leq A \ \& \ \frac{A-m}{2b} < xY \leq \frac{m+A-2u_1}{2b} \\ \frac{1}{4b} (A - m)^2 + mxY & \text{if } u_1 \leq m \leq A \ \& \ xY \leq \frac{A-m}{2b}. \end{cases} \quad (10)$$

The optimal choice of the crop land amount x is to maximize the expected profit for the farmer at Time 1 as given below:

$$\max_x \{ \mathcal{V}_{f,\xi}(x, u_1) = \mathbb{E}_\xi [V_f^*(x, u_2^*, Y, A)] - c_f x + c_l (L - x) \}. \quad (11)$$

The lemma below summarizes properties of $\mathcal{V}_{f,\xi}(x, u_1)$.

Lemma 3 1) $\mathcal{V}_{f,\xi}(x, u_1)$ convexly increases in u_1 .

2) If $A^1 \leq_{st} A^2$, then $\mathcal{V}_{f,\xi^1}(x, u_1) \leq \mathcal{V}_{f,\xi^2}(x, u_1)$, where $\xi^i = (Y, A^i)$ for $i \in \{1, 2\}$.

Part 1 of Lemma 3 highlights the effect of the processing plant's ex-ante price commitment u_1 at Time 1: the farmer's expected profit increases with a higher price commitment u_1 . Part 2 implies that a stochastically higher side-market willingness-to-pay A leads to a higher overall expected profit for the farmer after taking into account all possible x values. Since this result aligns with Part 3 of Corollary 3, it suggests that the responsive pricing strategy does not change the monotonicity of the farmer's expected profit with respect to A .

While it can be shown that $V_f^*(x, u_2^*, Y, A)$ is piecewise concave in x for each of three-piece domains – i.e., $xY \in [0, \frac{m-A}{2b}]$, $[\frac{m-A}{2b}, \frac{m+A-2u_1}{2b}]$, $[\frac{m+A-2u_1}{2b}, \infty)$, it should be noted that the function is not necessarily concave for the entire domain $[0, +\infty)$. This is due to the fact that the function starts to tilt up at a higher linear rate at $xY = \frac{m+A-2u_1}{2b}$ in the end tail for both cases $A \geq m$ and $A < m$. As $V_f^*(x, u_2^*, Y, A)$ is not necessarily concave in x for $xY \in [0, +\infty)$, we do not anticipate concavity of the function $\mathcal{V}_{f,\xi}(x, u_1)$. Proposition 4 presents sufficient conditions to develop properties of $\frac{\partial \mathcal{V}_{f,\xi}(x, u_1)}{\partial x}$.

Proposition 4 1) If the optimal x^* for (11) is an interior point in $(0, L)$, then x^* is the solution of $\frac{\partial \mathcal{V}_{f,\xi}(x, u_1=u_o)}{\partial x} = 0$.

2) If the optimal x^* for (11) is an interior point in $(0, L)$, then x^* is increasing in u_1 .

3) If $\frac{\partial \mathcal{V}_{f,\xi}(x, u_1=u_o)}{\partial x} > 0$ for arbitrary non-negative x , then $\frac{\partial \mathcal{V}_{f,\xi}(x, u_1)}{\partial x} \Big|_{u_1 > u_o} > 0$ and x^* for (11) is equal to L .

Part 1 of Proposition 4 prescribes the farmer's optimal land allocation for (11) if the solution is an interior point in $(0, L)$. Provided that this optimal land allocation is an interior point, Part 2 says that a higher MPG u_o leads to a higher land allocation to the crop. Part 3 says that if the profit margin of the crop is positive for a given MPG u_o , then for a higher guaranteed price the profit margin is still positive and the optimal solution for (11) is full land allocation to the crop. Let u_{FL}^{III} be the FLA threshold for the Responsive strategy, i.e., it is the u_o threshold value such that $x^*(u_o) = L$ for any $u_o \geq u_{FL}^{III}$.

The processing plant's expected profit, given land allocation x and committed price u_1 at Time 1, can be computed as follows:

$$\mathcal{V}_{p,\xi}(x, u_1) = \mathbb{E}_\xi [V_p(x, u_2^*, Y, A)]. \quad (12)$$

The corollary below summarizes properties of $\mathcal{V}_{p,\xi}(x, u_1)$.

Corollary 4 1) $\mathcal{V}_{p,\xi}(x, u_1)$ convexly increases in x and m , and decreases in u_1 .

2) If $Y^1 \leq_{st} Y^2$, then $\mathcal{V}_{p,\xi^1}(x, u_1) \leq \mathcal{V}_{p,\xi^2}(x, u_1)$, where $\xi^i = (Y^i, A)$ for $i \in \{1, 2\}$.

3) If $A^1 \leq_{st} A^2$, then $\mathcal{V}_{p,\xi^1}(x, u_1) \geq \mathcal{V}_{p,\xi^2}(x, u_1)$, where $\xi^i = (Y, A^i)$ for $i \in \{1, 2\}$.

Higher values of x lead to an increased available harvest, and larger values of m result in a greater margin, both of which increase the profit for the processing plant. However, a higher committed price u_1 hurts the processing plant's profit when the land allocation to the protected crop x is given. Additionally, a stochastically higher yield also results in an increased available harvest and increased profit for the processing plant. On the other hand, a stochastically higher value of A results in an increase in the price of the side market, which is adversarial to the process plant profit by reducing either the margin

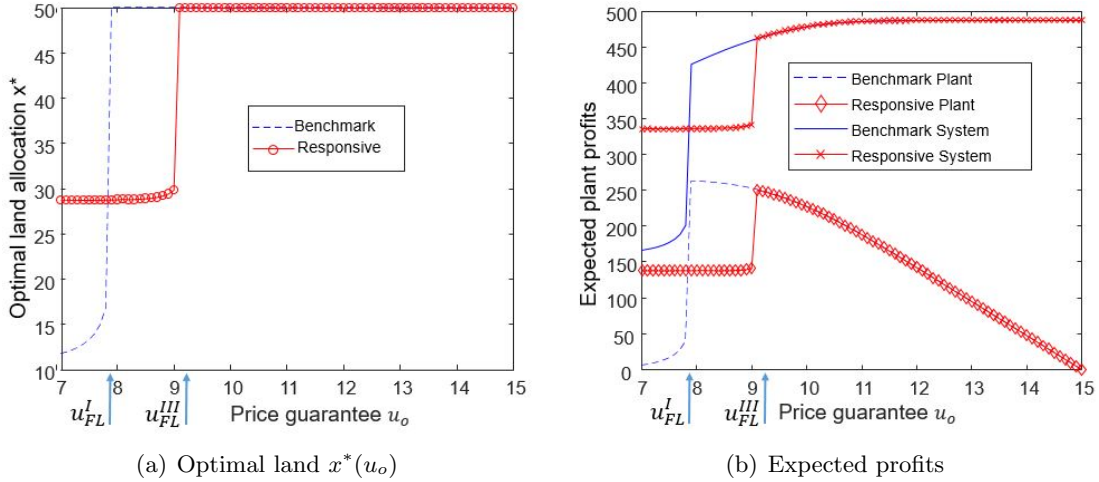


Figure 3: Comparison between Strategies I and III

(i.e., $m - u_2$) or the processed amount (i.e., $xY - q^*$) for the plant. For ease of reference, we denote \mathcal{V}_f^{III} and \mathcal{V}_p^{III} as the maximum expected profit for the farmer and the processing plant in the Responsive strategy, respectively, for a committed price $u_1 = u_o$ at Time 1. Additionally, we let x^{III} be the optimal land allocation to the crop for given $u_1 = u_o$. The following proposition compares the performance between the Benchmark and Responsive strategies.

Proposition 5 *For the farmer profit it always holds that $\mathcal{V}_f^I \leq \mathcal{V}_f^{III}$. However, there exists an ex-ante set price $u_1 = u_o$, as seen in Figure 3(b), at which the processing plant profits $\mathcal{V}_p^I > \mathcal{V}_p^{III}$.*

Below, we conduct numerical studies to explore how the ex-ante committed price $u_1 = u_o$ and the margin for the processing plant m affect the optimal land allocations and the optimal expected profits for shareholders under the Responsive strategy.

We examine the impact of u_o on the optimal land allocation by contrasting the optimal land allocations for the Responsive and Benchmark as u_o increases. The optimal land allocation to the crop in the Benchmark strategy, denoted as $x^I(u_o)$, is convexly increasing in u_o (Corollary 2), and is sensitive to changes in u_o , e.g., Figure 3(a), when u_o is below the FLA threshold u_{FL}^I . $x^I(u_o)$ is just the full land L once u_o surpasses the FLA threshold. In contrast to the Benchmark strategy, the optimal land allocation in the Responsive strategy $x^{III}(u_o)$ may be less sensitive to changes in u_o for low values of u_o . This occurs because, when the processing margin m is high, the final price at Time 2 is more likely to be either $\frac{m+A-2bxY}{2}$ or $\min(A, m)$, which is higher than and independent of u_o . As a result, for low values of u_o , the farmer allocates to the crop a much greater amount of land in the Responsive strategy than in the Benchmark to take advantage of higher final prices at Time 2. As u_o increases moderately, the farmer's allocation to the crop under the Responsive strategy remains relatively stable, as long as $\frac{m+A-2bxY}{2}$ or $\min(A, m)$ remains above the FLA threshold. When u_o gets sufficiently large, the optimal allocated land $x^{III}(u_o)$ leaps to the total available land as depicted in Figure 3(a), which also demonstrates that the FLA threshold u_{FL}^{III} in the Responsive strategy is larger than that in the Benchmark u_{FL}^I (in our examples, $u_{FL}^I = 7.9$, $u_{FL}^{III} = 9.2$).

Next, we explore the impact of u_o on the processing plant's expected profits in the Responsive and Benchmark strategies. For low values of u_o , the prospect of obtaining a higher final price at Time 2 in the Responsive strategy motivates the farmer to grow and sell more of the harvest to the processing

plant. As a result, the expected profits for the processing plant are higher in the Responsive strategy, as are the expected profits for the system, see Figure 3(b). That is, the Responsive strategy is apparently effective in improving the processing plant's and the system's profits when the committed price u_o at Time 1 is lower than the FLA threshold u_{FL}^I . Once u_o surpasses u_{FL}^I , the expected profits for the processing plant and the system are higher in the Benchmark than in the Responsive, and this relationship continues until u_o reaches the FLA threshold u_{FL}^{III} in the Responsive strategy. As seen in Figure 3(b), for this particular example, $\mathcal{V}_{p,\xi}^I(x, u_o)$ is maximized at $u_o = 7.9$ with $\mathcal{V}_p^I = 262.47$, while $\mathcal{V}_{p,\xi}^{III}(x, u_o) |_{u_o=7.9} = 137.98$. This suggests that for u_o values between u_{FL}^I and u_{FL}^{III} , $\mathcal{V}_p^{III} < \mathcal{V}_p^I \leq \mathcal{V}_p^{II}$.

That is to say, the processing plant may benefit more from an ex-ante price commitment when the government price guarantee u_o lies in an intermediate range $[u_{FL}^I, u_{FL}^{III}]$. These intermediate values of u_o , while reasonably competitive against the side market, may not necessarily align with the plant's best interest when we account for the farmer's response to the expected adjustment for the ex-post price at the season end. The farmer, expecting an upward price adjustment, strategically adjusts the land allocation to the crop (i.e., reduces it) in order to more likely receive higher prices on the allocated amount. This leads to a paradox: An ex-post price adjustment, optimized to increase processing plant profit by incorporating recent information on yield and side market condition, ultimately results in inferior performance for both the processing plant and the system compared to the ex-ante-set Benchmark strategy. For expositional convenience, we refer to this observation as "the paradox of ex-post pricing flexibility", and we will see it recurring in the analyzed two-price strategy below.

5.2 The Hybrid Two-Price Strategy

In the Hybrid strategy, the processing plant commits to an ex-ante price u_1 at Time 1 and concurrently announces its intention to revise the final price $u_2 \geq u_1$ based on the realized values of Y and A . Note that another plausible alternative strategy is to announce an ex-ante committed price at Time 1 (which may exceed the minimum guaranteed price u_o , MPG, set at Time 0), while allowing for a downward/upward adjustment at Time 2, provided that the final price still meets or exceeds the MPG. Under this approach, the processor's decision on the final price u_2 at Time 2 is effectively constrained only by $u_2 \geq u_o$, rather than by $u_2 \geq u_1$. Recognizing this, the farmer interprets the processing plant's commitments as simply $u_2 \geq u_o$ at Time 2, with u_2 unknown at Time 1. As a result, this alternative strategy is effectively reduced to, and equivalent to, the Responsive strategy. Our analysis hereafter assumes only the upward price adjustment, i.e., $u_2 \geq u_1$, in the Hybrid strategy.

Our results in the previous three strategies serve conveniently as building blocks for analysis of the Hybrid strategy. Specifically, at Time 2 the farmer responds to a final price u_2 with her harvest allocation in Equation (1) as in the Benchmark strategy, and the processing plant chooses the best ex-post price in Equation (9) as in the Responsive strategy. At Time 1, the farmer chooses the best crop land allocation x that maximizes the profit $\mathcal{V}_{f,\xi}(x, u_1)$ in Equation (11) as in the Responsive strategy and, finally knowing the optimal land allocation x from the farmer, the processing plant chooses a committed price $u_1 (\geq u_o)$, maximizing the profit $\mathcal{V}_{p,\xi}(x, u_1)$ given in Equation (12) as in the Ex-ante strategy. Let u_{FL}^{IV} be the FLA threshold for the Hybrid Two-Price strategy, i.e., the u_1 threshold value such that $x^*(u_1) = L$ for any $u_1 \geq u_{FL}^{IV}$. For ease of reference, we denote \mathcal{V}_f^{IV} and \mathcal{V}_p^{IV} as the maximum expected profit for the farmer and the processing plant, respectively, in the Hybrid strategy over all committed prices u_1 at Time 1. The proposition below compares the FLA threshold value and the expected profits between the Responsive and Hybrid Two-Price strategies.⁵

⁵We would like to remind the reader that the FLA thresholds of the two strategies are based on the ex-ante committed

Proposition 6 1) *The FLA values of the Responsive and Hybrid Two-Price strategies are the same: $u_{FL}^{III} = u_{FL}^{IV}$.*

2) *For both the farmer and plant profits the following hold: $\mathcal{V}_f^{III} \leq \mathcal{V}_f^{IV}$, and $\mathcal{V}_p^{III} \leq \mathcal{V}_p^{IV}$.*

Part 1 of Proposition 6 states that the FLA threshold value is the same for both the Responsive and the Hybrid Two-Price strategies. This occurs because the maximum profit function for the processing plant at Time 2 is decreasing in u_1 for a given $x = L$. Part 2 states that both the farmer and the processing plant favor the Hybrid strategy over the Responsive strategy. The farmer is better off in the Hybrid strategy because the pre-committed price at Time 1 is either equal to or higher than that in the Responsive strategy, and consequentially her profit will never be worse in the Hybrid strategy for each land allocation x . Likewise, the processing plant is better off in the Hybrid strategy because it allows the plant to choose the optimal pre-committed price at Time 1, avoid the troublesome region $[u_{FL}^I, u_{FL}^{III}]$ and consequentially improve its profitability in comparison to the Responsive strategy. Similarly, it can be easily shown that $\mathcal{V}_f^I \leq \mathcal{V}_f^{IV}$, and $\mathcal{V}_p^I \leq \mathcal{V}_p^{IV}$.

While there is a clear-cut ranking relationship among the Hybrid, Responsive, and Benchmark strategies, there is no clearly dominating relationship between the Ex-ante and Hybrid strategies. This occurs because when the MPG u_o is between two allocation thresholds u_{FL}^I and u_{FL}^{III} , i.e., in $[u_{FL}^I, u_{FL}^{III}]$, the processing plant is better off in the Ex-ante strategy than in the Responsive strategy. It has to do with farmers' land allocation for MPGs in that range, which is smaller under Hybrid as compared to the Ex-ante strategy (see Section 5). As a result, if the processing plant's maximum expected profit for the Ex-ante strategy in the u_o range is higher than the plant's maximum profit in the Responsive strategy over all $u_o (\geq u_{FL}^{III})$ values, then we have $\mathcal{V}_p^{II} > \mathcal{V}_p^{IV}$. The proposition below presents some sufficient conditions for profit comparison between the Ex-ante and Hybrid strategies.

Proposition 7 1) *If $x^{II}(u_1) \leq x^{III}(u_1)$ for any $u_1 \geq u_o$, then $\mathcal{V}_p^{II} \leq \mathcal{V}_p^{IV}$, where $x^i(u_1)$ is the optimal x in Strategy $i \in \{II, III\}$.*

2) *If $u^{II}(u_o) \leq u^{IV}(u_o)$ for given u_o , then $\mathcal{V}_f^{II} \leq \mathcal{V}_f^{IV}$.*

3) *There exists some u_o such that $\mathcal{V}_p^{II} > \mathcal{V}_p^{IV}$, i.e., the processing plant is worse off in the Hybrid Two-Price strategy than in the Ex-ante strategy.*

It is worth noting that there exists a value of u_o for which $\mathcal{V}_p^{II} > \mathcal{V}_p^{IV}$ (Part 3 of Proposition 7) and another value for which $\mathcal{V}_f^{II} = \mathcal{V}_f^{IV}$. It can be seen from Figure 3(b) that, for our example, when $u_o = 7.6$ and $u_1 (\geq 7.6)$ is optimally chosen as in the Ex-ante strategy, the processing plant can achieve a maximum expected profit of 262.47 (at $u_1 = 7.9$) using the Ex-ante strategy as compared to a maximum of only 260.15 (at $u_1 = 8.4$) using the Hybrid strategy. There are also numerical examples, e.g., Figure 4(a) suggesting that neither the processing plant nor the farmer is better off under the Hybrid strategy as compared to the Ex-ante strategy, and this may occur more likely in settings when m is low. This observation substantiates the paradox of ex-post price adjustments under either Responsive or Hybrid strategies failing to increase profit performance for either the processing plant or the system for a range of MPG values. However, the detrimental effect may disappear when the processing margin is sufficiently high that the processing plant can preempt farmers' land allocation retreat.

In Figure 4(b), we present a summary of the analytical results regarding the ranking relationships among the four pricing strategies. A connecting segment with highlighted comments (in red) between

prices of each strategy that achieve full land allocation to the crop. However, the ex-ante price for the Responsive strategy is the same as the MPG, while for the Hybrid Two-Price strategy it is optimized and potentially higher than the MPG.

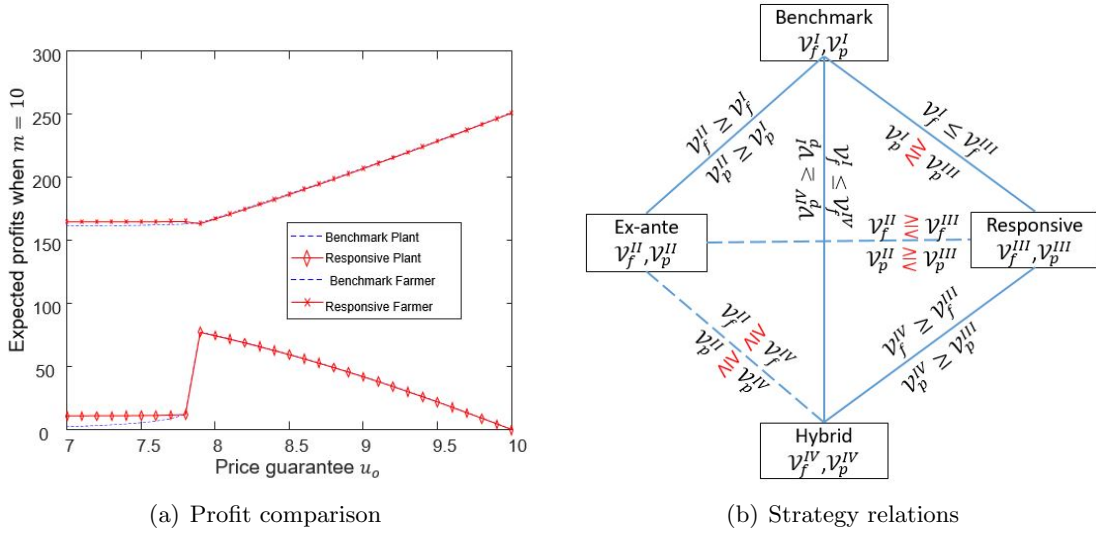


Figure 4: Relations between pricing strategies

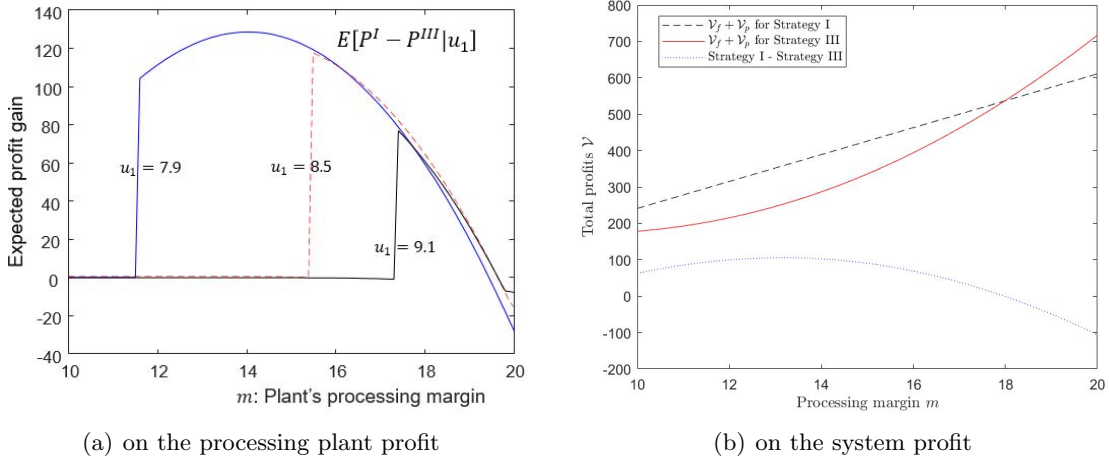
two strategies indicates that their ranking relations can go either way. Now that the analysis above has highlighted the potential advantages and disadvantages of the Responsive and Hybrid strategies in comparison to the Benchmark and Ex-ante strategies, our study below will examine the robustness of these findings. Section 5 focuses on assessing the impact of a few critical modeling parameters through numerical studies. In Section 6, we propose two different execution variants of the Hybrid strategy that circumvent the ex-post pricing flexibility paradox. In long-term repeated interactions between the plant and farmers we can implement a randomized version of the Hybrid strategy. For short-term practices, we suggest a contingency plan-based execution of the Hybrid strategy.

6 Numerical Studies and Discussions

In this section, we numerically investigate how the processing margin (m) and uncertainties in the side market and crop yield impact the expected profits of the processing plant and the system. Specifically, we begin by studying the impact of the processing margin m on the processing plant's profit using the Benchmark and Responsive strategies in Section 6.1. Next, we examine the impact of uncertainties between the Ex-ante and Hybrid strategies, and between the Ex-ante and Responsive strategies in Section 6.2. Section 6.3 assesses how processing margins and uncertainties affect total system profit.

6.1 The Impact of Processing Margin m

The impact of m on $[\mathcal{V}_p^I - \mathcal{V}_p^{III} \mid u_1]$, when interacting with u_1 , is worth noting, where $[\mathcal{V}_p^I - \mathcal{V}_p^{III} \mid u_1]$ is the profit difference between \mathcal{V}_p^I and \mathcal{V}_p^{III} for given u_1 . When m is low and close to u_1 from above, the plant's profit in the Responsive strategy, albeit better, is very close to that in the Benchmark. This is because a low m provides little room for improvement and responsiveness. As m increases to a medium range (with a smaller lower boundary of the range for smaller u_1), the ex-post pricing flexibility paradox occurs and the plant performs better in the Benchmark strategy due to the farmer delaying the full land allocation in the Responsive strategy, see Figure 5(a). When m becomes sufficiently large, the plant performs better in the Responsive strategy. This is because effective responsive pricing at

Figure 5: The impact of processing margin m

Time 2 completely dominates the side market, resulting in a negative $[\mathcal{V}_p^I - \mathcal{V}_p^{III} | u_1]$ as in Figure 5(a). A similar pattern is also observed in the total system profit $\mathcal{V}_f + \mathcal{V}_p$: The system profit difference, $(\mathcal{V}_f^I + \mathcal{V}_p^I) - (\mathcal{V}_f^{III} + \mathcal{V}_p^{III})$, is positive for small to moderate m values, and becomes negative when m is sufficiently large, see Figure 5(b).

We now comment on the effects of m on the profit comparison between \mathcal{V}_p^{II} and \mathcal{V}_p^{IV} . Recall that the profits \mathcal{V}_p^{II} and \mathcal{V}_p^{IV} represent the maximum of (the profit functions of) \mathcal{V}_p^I and \mathcal{V}_p^{III} over all values of the committed price u_1 at Time 1, respectively. It is observed that $\mathcal{V}_p^{II} > \mathcal{V}_p^{IV}$ for moderate m values, with again the paradox of ex-post pricing flexibility kicking in, and that $\mathcal{V}_p^{II} < \mathcal{V}_p^{IV}$ for large m values as shown in Figure 5(a), since the plant is more competitive against the side market with higher m values. Furthermore, this observation is robust when there are changes in the uncertainties, as our discussion in the subsection below will confirm.

6.2 The Impact of Uncertainties

We now examine the impact of the uncertainty from the side market (i.e., market price uncertainty A) and the crop yield uncertainty Y . The first set of examples (see Figure 6(a)) studies the impact on the processing plant profit. To isolate the effect of uncertainty and exclude the arbitrary impact of the MPG at Time 1, our study focuses on the Ex-ante and Hybrid strategies. The effect of the variance in A is clearly observed. As the variance of A increases, it is more likely to observe high and low realizations of A . Higher realizations of A result in increased cost for the processing plant to obtain the harvest, while lower realizations of A are protected by the MPG. Therefore, we observe a decrease in the profit for the processing plant in both the Ex-ante and Hybrid strategies as the variance of A increases. The effect of the variance of Y is more complex, as it is intertwined with that of m values. A higher yield variance increases the likelihood of observing both large and small realizations of Y . Large yields lead to higher allocated crops to the process plant, while small yields lead to lower amounts. For the (symmetric) normal distribution that we use in the numerical studies, the increased harvest obtained from high yields can be balanced out by the decreased amount obtained from low yields. Thus, it is observed that the optimal committed price and the expected profit for the plant in the Ex-ante strategy are less sensitive to changes in the yield variance when the side-market price variance is low. In the Hybrid strategy, only small yield realizations require higher final price adjustments. Consequently,

Ex-ante	m=15							m=20						
	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4
$\sigma_Y = 0.1$	262.71	262.71	262.71	262.67	262.48	262.04	261.26	473.55	471.50	469.05	466.32	463.31	460.12	456.77
0.11	262.71	262.71	262.71	262.67	262.48	262.04	261.26	473.55	471.50	469.05	466.32	463.31	460.12	456.77
0.12	262.71	262.71	262.71	262.67	262.48	262.04	261.26	473.55	471.50	469.05	466.32	463.31	460.12	456.77
0.13	262.71	262.71	262.71	262.67	262.48	262.04	261.26	473.55	471.50	469.05	466.32	463.31	460.12	456.77
0.14	262.71	262.71	262.71	262.67	262.48	262.04	261.26	473.55	471.50	469.05	466.32	463.31	460.12	456.77
0.15	262.71	262.71	262.71	262.67	262.48	262.04	261.27	473.55	471.50	469.05	466.30	463.31	460.12	456.77

Hybrid	m=15							m=20						
	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4
$\sigma_Y = 0.1$	261.53	261.46	261.35	261.08	260.45	259.44	258.10	475.28	475.33	475.41	475.52	475.59	474.17	472.46
0.11	261.45	261.40	261.30	261.05	260.44	259.44	258.13	475.36	475.41	475.49	475.12	474.03	472.47	470.70
0.12	261.39	261.35	261.27	261.03	260.44	259.47	258.17	475.45	475.50	475.36	474.67	473.39	471.73	469.89
0.13	261.36	261.32	261.26	261.03	260.46	259.50	258.23	475.55	475.58	475.34	474.49	473.10	471.37	469.47
0.14	261.34	261.31	261.26	261.05	260.49	259.56	258.31	475.66	475.68	475.38	474.46	473.00	471.22	469.26
0.15	261.34	261.32	261.29	261.09	260.55	259.64	258.41	475.77	475.80	475.47	474.50	473.02	471.20	469.21

Figure 6(a): Impact of variances of Y and A as well as m on profits

the processing plant may commit to a lower price at Time 1, to strike a balance between the ex-ante commitment and a needed price adjustment at Time 2 for a higher profit. In settings with moderate m values, the plant's profit is decreasing in the yield variance, particularly when the side-market price variances are small. However, this diminishing trend gradually shifts to an increasing one when the side-market price variance keeps increasing. Conversely, in settings with large m values, the plant's profit is increasing in the yield variance, when the side-market price variances are small. Nevertheless, this increasing trend gradually shifts to a decreasing one when the price variance increases.

6.3 The Impact on System Profits

The second group of examples (see Figure 5(b) and 6(b)) investigates how the processing margin m as well as yield and side-market uncertainties affect total system profit, defined as $\mathcal{V}_f + \mathcal{V}_p$. Our examples are conducted such that the land allocation to the protected crop reaches its full capacity, allowing for a direct comparison between Strategy I (resp. II) and Strategy III (resp. IV) under maximum land commitment. (It is worth noting that similar patterns have been observed in cases where the farmer allocates only a portion of land to the protected crop, suggesting the robustness of these insights across different allocation regimes.)

Figure 5(b) illustrates the evolution of system profits under Strategy I and Strategy III, along with their difference, as the processing margin (m) varies. For both strategies, total profit increases with m . However, the growth under Strategy III is notably more rapid and convex, eventually surpassing the profit under Strategy I when $m > 18$. This pattern can be attributed to the nature of pricing flexibility. At low to moderate margins, responsive pricing at Time 2 offers limited competitiveness in the side market, making ex-ante pricing (as in Strategy II) more effective for securing committed land allocations from the farmer. Conversely, as the processing margin becomes large, the ex-post pricing in Strategy III becomes significantly more appealing, as the processor can respond competitively on the side market against the uncertain price intercept. As a result, the difference in system profits between the two strategies is initially positive favoring Strategy I, peaks at intermediate values of m , and then declines, turning negative once m exceeds 18. These observations suggest that: When processing margins are

Ex-ante	m=15							m=20						
	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4
$\sigma_Y = 0.1$	423.889	424.19	424.69	425.35	426.05	426.68	427.13	711.59	701.49	692.75	685.28	678.45	672.05	666.95
0.11	423.889	424.19	424.69	425.35	426.05	426.68	427.13	711.59	701.49	692.71	685.33	678.82	672.05	666.95
0.12	423.889	424.19	424.69	425.35	426.05	426.68	427.13	711.58	701.49	692.74	685.33	678.45	672.05	666.95
0.13	423.889	424.19	424.69	425.35	426.05	426.68	427.13	711.58	701.54	692.71	685.33	678.45	672.05	666.95
0.14	423.889	424.19	424.69	425.35	426.05	426.68	427.13	711.49	701.5	692.71	685.28	678.45	672.05	666.95
0.15	423.889	424.19	424.69	425.35	426.05	426.68	427.13	711.49	701.5	692.69	685.33	678.45	672.06	666.95
Hybrid	m=15							m=20						
	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4	$\sigma_A = 0.2$	0.4	0.6	0.8	1	1.2	1.4
$\sigma_Y = 0.1$	464.614	464.3	463.58	462.48	461.15	459.72	458.28	719.77	719.16	718.66	717.81	716.79	715.48	713.7
0.11	464.138	463.81	463.11	462.04	460.76	459.38	457.99	718.87	718.46	717.88	716.96	716.2	715.16	713.35
0.12	463.625	463.3	462.62	461.6	460.37	459.05	457.71	718.24	717.72	717.18	716.35	715.58	714.63	713.43
0.13	463.091	462.78	462.13	461.16	459.99	458.72	457.43	717.43	717.1	716.6	715.81	714.97	714.05	712.41
0.14	462.548	462.25	461.64	460.73	459.62	458.4	457.17	716.56	716.31	716.15	715.24	714.4	713.51	712.51
0.15	462.009	461.73	461.17	460.31	459.26	458.11	456.93	715.99	716.04	715.21	714.72	713.89	712.97	711.99

Figure 6(b): System profits $\mathcal{V}_f + \mathcal{V}_p$

low, ex-ante commitments are more advantageous; as margins increase, however, their relative benefit diminishes, and responsive strategies, such as Strategy III or IV, become more profitable due to their built-in pricing flexibility.

Figure 6(b) illustrates how system profit is influenced by uncertainties in crop yield and side-market price, captured by the standard deviations σ_Y and σ_A , under two processing margins ($m = 15$ and $m = 20$). The top half of the table presents results for the Ex-ante strategy, while the bottom half corresponds to the Hybrid strategy. Across all combinations of σ_Y and σ_A , the Hybrid strategy consistently outperforms the Ex-ante strategy in terms of total system profit. This superiority stems from the Hybrid strategy's use of responsive or contingent pricing, which allows it to better capitalize on realized market conditions, especially under uncertainty. In the Ex-ante strategy, system profit remains largely constant as σ_Y or σ_A increases, a pattern particularly evident when $m = 15$. This reflects the static nature of ex-ante pricing, which cannot adjust to volatility or realized conditions. In contrast, profits under the Hybrid strategy exhibit a slight decline as uncertainty rises. This indicates that while responsive pricing is advantageous, greater volatility can erode its effectiveness. Moreover, increasing the processing margin from $m = 15$ to $m = 20$ yields significantly higher system profits for both strategies. However, the Hybrid strategy benefits more substantially from this increase, thanks to its ability to dynamically adjust pricing in response to favorable market margins.

In summary, these results highlight the robust profitability of hybrid strategies, particularly when either crop yield or market demand is uncertain, or when processing margins are high enough to enable responsive pricing to fully realize its value.

7 Discussion on Addressing the Ex-Post Pricing Flexibility Paradox

Our Hybrid Two-Price Strategy of Section 5.2 assumes that the processing plant makes an ex-ante commitment to adjust prices at Time 2 in response to the realized yield Y and side-market intercept A . As a result, a strategic farmer accounting for the plant's responsive price at Time 2 is more likely to not fully allocate the land to the protected crop. As observed, this strategic response of the farmer leads

to the paradox of ex-post pricing flexibility, potentially detrimental to the profits for the plant and the overall system. This paradox behavior explains the issue of “instability of ginneries” in the Mozambique cotton supply chain (The World Bank, 2010): The government’s implementation of a national pricing system with two-price commitments often resulted in less allocated land to the protected crop within a concession, leading to underutilization of the downstream ginning capacity.

Such observations provoke thoughts of rectification via “strategically smart” execution of the Hybrid strategy. It naturally leads to considering a “random” variant of the strategy, in which the processing plant commits to an ex-ante price at Time 1, while preserving flexibility for an uncommitted price adjustment at Time 2. Conceptually, this season-end price adjustment can be viewed as a *randomized option*. Importantly, this option is not exercised arbitrarily. Its execution is either selectively invoked and credibly sustained through repeated interactions with the farmer over multiple years, or triggered by observable regional conditions (e.g., unusually low local yields or a spike in side-market prices). These approaches, which we refer to as the long-term Randomized strategy and the Contingency-Region Hybrid strategy, respectively, are discussed below.

7.1 Long-Term Credible Randomized Strategy

To investigate the effectiveness of this strategy, we use a Bernoulli random variable to represent the processing plant’s decision-making behavior at Time 2: with probability δ the plant reviews the uncertainty realization on yield and side-market conditions and makes the final price adjustment accordingly. While δ could well represent the likelihood of an exogenously given outside event, the processing plant can always choose a value lower than that probability; accordingly, we assume that the processing plant controls the choice of δ . Essentially via the randomized final price adjustment, the processing plant sends no credible ex-ante signals that the prices ex-post will be optimized. We assume that farmers know the selected value of δ ; in a multi-year interaction between farmers and the processing plant, we can interpret δ as the observed frequency of adjusting the season-end price relative to the announced ex-ante price. For notational convenience, we refer to the externally triggered Hybrid strategy as Strategy V , and provide its analysis in this subsection.

We start by noting that this strategy reduces to the Ex-ante strategy (Strategy II) with $\delta=0$, and reduces to the Two-Price Hybrid strategy (Strategy IV) with $\delta=1$. We also note that the decisions for the processing plant at Time 2 for given x , u_1 , and realizations of Y and A can be conveniently characterized with already-developed results. More specifically, if the processing plant decides to respond to the realized Y and A , we can utilize the optimization problem (8) with its optimal final prices characterized in Equation (9) and the corresponding processing plant profits expressed in Equation (20). As a result, considering two possibilities on whether to adjust the final price, the expected profit for the plant after observing the realizations of Y and A , denoted as $V_p^V(x, u_1, Y, A)$, is given below:

$$V_p^V(x, u_1, Y, A) = (1 - \delta)V_p^{II}(x, u_1, Y, A) + \delta V_p^{III}(x, u_2^*, Y, A),$$

where $V_p^{II}(x, u_1, Y, A)$ and $V_p^{III}(x, u_2^*, Y, A)$ are obtained following Equations (5) (for the case of no responsive price adjustment) and (20) (for the case of responsive price adjustments), respectively, with u_1 substituting for u_o . The plant’s expected profit at Time 1 for given x , u_1 , and δ can be written as $\mathcal{V}_{p,\xi}(x, u_1) = (1 - \delta)\mathcal{V}_{p,\xi}^{II}(x, u_1) + \delta\mathcal{V}_{p,\xi}^{III}(x, u_1)$, where $\mathcal{V}_{p,\xi}^i(u_1)$ represents the plant’s maximum profit under Strategy i , $i \in \{II, III\}$.

The expected profits for the farmer at Times 1 and 2 can be calculated as follows. With guaranteed

price u_o and possible ex-ante price commitment u_1 , the farmer profit after observing the realizations of Y and A at Time 2 can be calculated accordingly as follows:

$$V_f^V(x, u_1, Y, A) = (1 - \delta)V_f^{II}(x, u_1, Y, A) + \delta V_f^{III}(x, u_2^*, Y, A), \quad (13)$$

where $V_f^{II}(x, u_1, Y, A)$ and $V_f^{III}(x, u_2^*, Y, A)$ are obtained following Equation (2) (for the case of non-responsive price adjustments) and Equation (10) (for the case of responsive price adjustments). The expected profit for the farmer at Time 1 can be calculated using (2), with $V_f^V(x, u_1, Y, A)$ substituted with (13). For ease of reference, we denote \mathcal{V}_f^V and \mathcal{V}_p^V as the maximum expected profit for the farmer and the processing plant, respectively, in the long-term credible Randomized strategy over all possible values of the committed price u_1 at Time 1. The proposition below summarizes the farmer's full land allocation decision, and compares the profits between the Ex-ante strategy and the Randomized strategy.

Proposition 8 1) *There exists a threshold $\delta_o > 0$, such that for any $\delta < \delta_o$ if $u_1 \geq u_{FL}^I$, then $x(u_1) = L$.*
 2) *Assume $\delta < \delta_o$. Under farmer full land allocation (i.e., $x(u_1) = L$), $\mathcal{V}_f^{II} \leq \mathcal{V}_f^V$ and $\mathcal{V}_p^{II} \leq \mathcal{V}_p^V$.*

For a sufficiently small δ , the (positive) sign of the first order derivative for the farmer's profit function $\mathcal{V}_{f,\xi}(\cdot)$ in Strategy V is not affected in case the processing plant adjusts its final price. As a result, the farmer proceeds with the full land allocation for any $u_o \geq u_{FL}^I$, exactly as in the Ex-ante strategy. With a full land allocation, neither the farmer nor the processing plant gets worse off from the ex-post price adjustment, and the paradox of ex-post pricing flexibility has been avoided. Furthermore, under full land allocation, it can be seen that $V_f^{II}(L, u_1, Y, A) \leq V_f^{III}(L, u_2^*, Y, A)$ and $V_p^{II}(L, u_1, Y, A) \leq V_p^{III}(L, u_2^*, Y, A)$ hold true, leading to \mathcal{V}_f^V and \mathcal{V}_p^V being maximized at δ_o over all $\delta \in [0, \delta_o]$. That is, $\mathcal{V}_f^V = (1 - \delta_o)\mathcal{V}_{f,\xi}^{II}(L, u_1) + \delta_o\mathcal{V}_{f,\xi}^{III}(L, u_1)$ and $\mathcal{V}_p^V = (1 - \delta_o)\mathcal{V}_{p,\xi}^{II}(L, u_1) + \delta_o\mathcal{V}_{p,\xi}^{III}(L, u_1)$. It should also be noted that δ_o could be negligibly small as u_1 approaches u_{FL}^I from the right.

In the Randomized strategy, with sufficiently small δ values, the processing plant may capitalize on responsive pricing in the few instances of severely low yield realizations, avoiding strategic under-allocation of the land to the crop by the farmer. The trade-off in the short term is that in a few moderate yield realizations and aggressive side-market pricing the plant might have to forgo beneficial responsive pricing, and some associated profits, to make the strategy credible. Consequently, the processing plant able to adopt a randomized Hybrid strategy would sustain better long-term profitability and a better-aligned and stable supply chain with higher utilization of all assets used by the protected crop. However, we can also foresee difficulties of executing the strategy when the plant management is evaluated on short-term objectives and/or the tenure of decision-makers in the chain prevents the execution of such long-term strategy. This naturally leads to a contingency-based hybrid strategy below that responds directly to low yield realizations and adverse side-market pricing conditions.

7.2 Contingency Region Hybrid Strategy

We now consider a randomized strategy based instead on the realizations of Y and A within the region. Intuitively, when the crop yield Y is low and the willingness to buy from the side market A is high, the ex-post contingency response is designed to achieve two goals: It enhances the competitiveness of the processing plant's price in the side market and, at the same time, avoids creating a strategic vulnerability that the farmer could exploit ex-ante. Formally, we define a contingency region, denoted

as R_c , as follows.

$$R_c = \left\{ (Y, A) \mid \text{(i) } A - u_1 > 0; \text{(ii) } m - A > 0; \text{(iii) } xY \leq \frac{m - A}{2b} \right\}. \quad (14)$$

The R_c is characterized by three ex-post conditions. Condition (i) ensures that a contingency response is only considered when the side-market willingness to buy exceeds the committed price u_1 ; otherwise, no adjustment is needed. Condition (ii) requires that the selling margin (m) of the protected crop is sufficiently high relative to the realized side-market price intercept A for the processing plant to compete effectively in that market. Finally, condition (iii) indicates that total production for the crop is low, specifically, below the threshold $\frac{m-A}{2b}$, which further justifies invoking the contingency response. Also for notational convenience, we refer to this strategy as Strategy VI. Strategy VI is a randomized hybrid strategy that while ex-ante committing to a price u_1 at Time 1, the processing plant makes ex-post price adjustments whenever realized (Y, A) falls in R_c . Properties of this strategy are summarized below.

Proposition 9 *For given ex-ante price commitment u_1 , the following holds true.*

- 1) *The optimal land allocation for Strategy VI, denoted as $x^{VI}(u_1)$, is no less than that for Strategy II, $x^{II}(u_1)$; that is, $x^{VI}(u_1) \geq x^{II}(u_1)$.*
- 2) *$\mathcal{V}_{f,\xi}^{II}(x, u_1) \leq \mathcal{V}_{f,\xi}^{VI}(x, u_1)$ and $\mathcal{V}_{p,\xi}^{II}(x, u_1) \leq \mathcal{V}_{p,\xi}^{VI}(x, u_1)$.*

Part 1 of Proposition 9 establishes that the contingency region, R_c , provides a set of sufficient conditions on (Y, A) under which ex-post pricing at Time 2, when done on top of an ex-ante commitment, serves as an effective incentive for farmers to allocate more land at Time 1, when compared to Strategy II that has only ex-ante commitment. By ensuring a larger land allocation, Strategy VI mitigates the strategic weaknesses associated with the pricing flexibility of Strategies III and IV. Consequently, starting from the same committed price u_1 at Time 1, both farmers and the processing plant are warranted for higher profits in Strategy V than in Strategy II (as seen in Part 2). Furthermore, Part 1 implies that for any $u_1 \geq u_{FL}^{II}$, both Strategy II and VI induce full-land allocation to the protected crop. Lastly, we note that like Strategy V, Strategy VI may also forgo profit gains that could have been obtained for moderate Y and A realizations since R_c represents extreme cases over all possible outcomes. Having examined ex-ante and ex-post pricing strategies and their variants in the above sections, we now briefly discuss how our modeling framework can be extended to accommodate heterogeneous land productivity, as presented in the section below.

8 Modeling Heterogeneous Land Productivity

In our previous study, we assumed proportional crop yield; that is, each unit of planted crop land x produces the same yield Y , resulting in a total production of xY . While this assumption is reasonable in many practical settings, there are regions where soil conditions vary across parcels, leading to non-uniform productivity. For instances, in the U.S., farmland is often classified into categories based on yield potential, soil quality, and other productivity-related characteristics (Bowman and Pauls, 2006). To capture this heterogeneity, we now denote $Y(x)$ as the total production obtained from planting x units of land with the protected crop. By ranking parcels in decreasing order of average yield rate, it is reasonable to assume that $E[Y(x)]$ is a concave and increasing function of x , i.e., marginally decreasing per unit yield. Furthermore, for analytical simplicity, we assume $Y(x)$ is increasing in x in the first-order

stochastic dominance (e.g., $Y(x)$ is the sum of a (deterministic) concave increasing function, denoted as $\tilde{y}(x)$, and a noise represented by a Normal random variable, denoted as \tilde{Y}). Below, we briefly describe how this nonlinear crop yield function can be incorporated into the four pricing mechanisms.

In the Passive Benchmark strategy, the optimal value of q is obtained by solving the following optimization problem:

$$\begin{aligned} \max_q \{ & V_f(q, x, u_o, Y, A) = (A - bq)q + u_o(Y(x) - q) \} \\ \text{s.t. } & 0 \leq q \leq Y(x). \end{aligned}$$

The concavity of $V_f(q, x, u_o, Y, A)$ w.r.t. q implies that the optimal harvest allocation to the side market q^* is determined by:

$$q^* = \min \left(\frac{(A - u_o)^+}{2b}, Y(x) \right), \quad (15)$$

where $(\cdot)^+ = \max(\cdot, 0)$. The corresponding crop revenue for the farmer at q^* , denoted as $V_f^*(x, u_o, Y, A) = V_f(q^*, x, u_o, Y, A)$, can be computed. Consequently, the farmer's optimal land allocation decision can be found by solving the optimization problem (3). Some properties of $V_f^*(x, u_o, Y, A)$ are summarized in the lemma below.

Proposition 10 1) $V_f^*(x, u_o, Y, A)$ is concave and increasing in $Y(x)$.

2) $V_f^*(x, u_o, Y, A)$ is concave in x .

3) $\mathcal{V}_{f,\xi}(x, u_o)$ is concave in x .

4) If $Y^1(x) \leq_{st} Y^2(x)$, then $\mathcal{V}_{f,\xi^1}(x, u_o) \leq \mathcal{V}_{f,\xi^2}(x, u_o)$, where $\xi^i = (Y^i(x), A)$ for $i \in \{1, 2\}$.

5) If $A^1 \leq_{st} A^2$, then $\mathcal{V}_{f,\xi^1}(x, u_o) \leq \mathcal{V}_{f,\xi^2}(x, u_o)$, where $\xi^i = (Y(x), A^i)$ for $i \in \{1, 2\}$.

6) If $\xi^1 \leq_{supm} \xi^2$, then $\mathcal{V}_{f,\xi^1}(x, u_o) \leq \mathcal{V}_{f,\xi^2}(x, u_o)$, where $\xi^i = (Y^i(x), A^i)$ for $i \in \{1, 2\}$.

Part 1 establishes that the farmer's revenue from the protected crop at Time 2 is both concave and increasing in the total realized production $Y(x)$. Given the assumption that $Y(x)$ is concave and increasing with respect to x , Part 2 confirms that $V_f^*(x, u_o, Y, A)$ is also concave and increasing in x . Consequently, as shown in Part 3, the farmer's expected profit function $\mathcal{V}_{f,\xi}(x, u_o)$, which integrates over all realizations of $Y(x)$ and A , inherits this concavity. This implies that the farmer's optimal land allocation to the protected crop can be uniquely determined, consistent with the decision structure examined in previous sections.

The influence of uncertainty in $Y(x)$ and A on expected farmer profit is further explored in Parts 4, 5, and 6. Notably, Part 6 addresses the case where $Y(x)$ and A are correlated, a relaxation of the independence assumption adopted earlier for analytical tractability. Despite this relaxation, some robustness results can still be established. Specifically, when $\xi = (Y(x), A)$ is jointly normally distributed, the relation $\xi^1 \leq_{supm} \xi^2$ (in a supermodular order; see Shaked and Shanthikumar, 2006) implies a higher correlation between $Y(x)$ and A in ξ^2 . It follows that increased negative correlation between these variables leads to lower expected farmer profit, as higher side-market price intercept tends to coincide with lower yields, reducing effective revenue.

Given the farmer's land allocation decision $x(u_1)$ in response to the committed price u_1 , the processing plant's expected profit and optimal choice of the price commitment u_1 in the Ex Ante Strategy can be formulated through the following optimization problem:

$$\max_{u_1 \geq u_o} \{ \mathcal{V}_{p,\xi}(x, u_1) = \mathbb{E}_\xi [(m - u_1)(Y(x) - q^*)] \}. \quad (16)$$

This formulation is analogous to the model in (6).

Similar to the formulation in (8), we now present the optimization model for the choice of u_2 in the Responsive strategy.

$$\max_{u_2 \geq u_1} \{V_p(x, u_2, Y, A) = (m - u_2)(Y(x) - q^*(u_2))\}, \quad (17)$$

where $q^*(u_2)$ is determined according to the optimal harvest allocation rule in Equation (17), with u_2 substituting for u_o . Consequently, we can characterize the optimal price u_2^* at Time 2, analogously to (9), with xY replaced by $Y(x)$.

Based on properties of $V_p(x, u_2, Y, A)$, the proposition below characterizes the optimal responsive price at Time 2.

Lemma 4 *The optimal price u_2 , denoted as u_2^* for a given committed price u_1 at Time 1, is given below.*

$$u_2^*(x) = \max \left(u_1, \min \left(\frac{m + A - 2bY(x)}{2}, A, m \right) \right). \quad (18)$$

Using the optimal value u_2^* , we can calculate the farmer's expected profit at Time 2, $V_f^*(x, u_2^*, Y, A)$. By integrating over all realizations of $Y(x)$ and A , we obtain the farmer's expected profit at Time 1 for given x , $\mathcal{V}_{f,\xi}(x, u_1)$, using Equation (11). Similarly, we can compute the processing plant's expected profit at Time 1, $\mathcal{V}_{p,\xi}(x, u_1)$, in a spirit similar to (12), which facilitates the search for the optimal pre-committed price u_1 . However, these calculations are rather convoluted due to the nested structure of expectations and optimization, making it challenging to derive analytical results. Nevertheless, some sensitive analysis results analogous to Corollary 4 hold true and are summarized in the corollary below.

Corollary 5 *1) If $Y^1 \leq_{st} Y^2$, then $\mathcal{V}_{p,\xi^1}(x, u_1) \leq \mathcal{V}_{p,\xi^2}(x, u_1)$, where $\xi^i = (Y^i, A)$ for $i \in \{1, 2\}$.*

2) If $A^1 \leq_{st} A^2$, then $\mathcal{V}_{p,\xi^1}(x, u_1) \geq \mathcal{V}_{p,\xi^2}(x, u_1)$, where $\xi^i = (Y, A^i)$ for $i \in \{1, 2\}$.

3) If $\xi^1 \leq_{supm} \xi^2$, then $\mathcal{V}_{p,\xi^1}(x, u_1) \geq \mathcal{V}_{p,\xi^2}(x, u_1)$, where $\xi^i = (Y^i, A^i)$ for $i \in \{1, 2\}$.

Similarly to Parts 4, 5, and 6 in Proposition 10, Corollary 5 reveals how uncertainties in crop yield $Y(x)$ and side-market price intercept A influence the processing plant's expected profit for given x and u_1 . Note that this corollary also allows for interdependence between $Y(x)$ and A , thereby extending the robustness of our insights beyond the assumption of independence.

9 Concluding Remarks

We have considered an agribusiness supply chain for a particular crop that comprises a region of farmers and a processing plant that collects harvest from farmers, in the presence of a government-supported MPG and a potentially lucrative side market for farmers. Specifically, we have examined the land allocation and harvest allocation decisions of farmers in such chains in the presence of competing crops for their land and alternative side markets for their harvest. The pricing decisions of the processing plant heavily influence the farmers' behavior and efficiency of the overall chain. We examine the best processor pricing strategies for these environments.

Our modeling framework provides a unified platform to study the decisions of farmers and the processing plant operating under a range of pricing strategies. These strategies enable the processing plant to interact with farmers, either at the start of the planting season concerning their planting decisions or at the end of the season regarding their harvest selling choices, or potentially at both times.

The strategies include a passive Benchmark, where the plant simply accepts and executes the MPG, as well as three processor-led pricing approaches: the one-price Ex-ante strategy, the one-price Responsive strategy, and the two-price, ex-ante and ex-post, Hybrid strategy.

We examine the profit impact of these pricing strategies, and show that committing to a price at the start of the season (i.e., ex-ante pricing) benefits both the farmers and the processing plant when compared to the passive Benchmark, which includes only the MPG. On the other hand, Responsive pricing, which responds to uncertain crop yield and side market conditions with ex-post price adjustments, benefits farmers but may harm the processing plant depending on the ex-ante announced value of the guaranteed price (MPG or optimized ex-ante price at Time 1). In what we refer to as the paradox of ex-post pricing flexibility, there exists a range of MPGs for which the farmer's land allocation becomes detrimental to processor profits, despite ex-post plant profit optimizing price adjustments.

The Hybrid Two-Price strategy, which combines an ex-ante price commitment at Time 1 with an ex-post price adjustment at Time 2, Pareto dominates the Responsive strategy by offering strictly better outcomes for both the processor and the farmer. However, it does not always outperform the Ex-ante (one-price) strategy from the perspective of the processing plant, thereby illustrating yet another manifestation of the ex-post pricing flexibility paradox. Since the processing plant acts as the leader in a two-stage Stackelberg game, this counterintuitive outcome reflects the paradoxical value of commitment, a phenomenon previously studied in other decision-making contexts within the game theory literature (see Fudenberg and Tirole, 1991). Nevertheless, our observation of this paradox arises in a novel supply chain setting, i.e., the flexibility endowed by the Stackelberg leader, rather than the follower, could potentially backfire.

Moreover, we propose execution variants of the Hybrid strategy that ensure outperformance relative to the Ex-ante approach. In long-term repeated interactions between the processing plant and farmers, a randomized version of the Hybrid strategy, featuring historically credible probabilistic after-harvest pricing adjustments, can be particularly effective. For short-term practices, a well-defined contingency region of relevant outcomes (yield, side market attractiveness, and processing margins) that triggers pricing adjustments after harvest enables the Hybrid strategy to dominate. We recommend building long-term and stable relationships in these agri-chains for effective execution of pricing strategies that align incentives and optimize profits for all involved parties. For the convenience of our reader, the characteristics of each studied strategy in our work appears in the Appendix IV (see Table 2). We also point out that the analysis of our pricing strategies can be readily extended from proportional yield assumptions to accommodate heterogeneous land productivity in more general settings (as in Section 8). On top of analytical results, we conduct numerical examples to offer insights on factors affecting the effectiveness of pricing strategies. The processing margin m affects the plant's profit performance rankings between the Benchmark and Responsive strategies, and between the Ex-ante and Hybrid strategies. Sufficiently large m values offers greater latitude for the plant to respond to the realized yield and market uncertainties, leading to higher plant profits in the Responsive and Hybrid strategies. However, for moderate and small m values, the plant performs better in the Benchmark and Ex-ante strategies. The impact of uncertain side-market price intercept A is also studied: The profits for the processing plant in both the Ex-ante and Hybrid strategies decrease as the variance of side-market prices increases. However, the impact of uncertain crop yield is more nuanced (see Section 5.2).

To conclude, we point to a few directions for future research. Firstly, we have assumed that the quality of the harvest flowing to the two market channels is homogeneous. Investigating the effects of quality differentiation on different pricing strategies would be an interesting area to explore. Secondly, while the

MPG acts as a farmer insurance against extremely low profits under unfavorable conditions, incorporating explicit risk attitudes for farmers and processing plants might affect pricing strategy effectiveness. Thirdly, our study has analyzed the region at an aggregate level, where all farmers act as a cooperative. The dynamics of farmer competition (Zhou et al., 2020) and how they affect different pricing strategies might be worthwhile to investigate. Lastly, one of our research premises assumes that farmers' harvest supply is non-committal, often manifesting through side-markets. Designing enforceable contracts that incentivize protected-crop production among resource-constrained smallholder farmers would therefore be highly desirable. While addressing these issues is outside the scope of our current study, we would suggest further research to enhance understanding of the functioning of agribusiness supply chains in challenging emerging market environments.

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