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Process Flexibility: A Distribution-Free Approach to Long Chain Resilience

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Abstract. Process flexibility has been a well-established supply chain strategy in both theory and practice for managing demand uncertainty. This study extends its application to mitigating supply disruptions by analyzing a *long chain* system. Specifically, we investigate the effectiveness of long chains in the face of random supply disruptions and demand uncertainty. We derive a closed-form, tight bound on the expected sales ratio of a long chain relative to full flexibility under random disruptions, thus providing a service level guarantee. Our analysis shows that when designed capacity equals expected demand, the fraction of benefits a long chain achieves relative to full flexibility increases with disruption probability; however, it decreases when capacity is instead expanded to match expected demand under disruptions. The long chain also demonstrates superior resilience, absorbing a significant portion of unexpected disruptions due to its sparsity.

To generalize our findings, we introduce a *moment decomposition* approach that readily adapts to general piecewise polynomial performance metrics while maintaining tractability through a semidefinite program (SDP). This approach extends the traditional type-II service metric (expected sales) to include type-I metric (probability of meeting full demand) and supports more flexible capacity-demand relationships. Applying this approach to the capacity configuration problem, we find that without disruption, a long chain achieves target service levels with capacity comparable to full flexibility, even with limited demand information. In contrast, disruptions significantly raise capacity requirements, although long chains maintain a substantial advantage over dedicated systems. Our results highlight the resilience of long chains and the critical need to adapt capacity configuration decisions to supply disruption risks.

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Key words: process flexibility, worst-case bound, supply disruption, capacity configuration

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1. Introduction

The vulnerabilities in supply chain systems and production strategies are being exposed everywhere due to the recent supply shocks[1]. The pandemic-induced lockdown measures resulted in a labor shortage and reduced production capacity. Additionally, the conflicts in Ukraine, along with disruptions caused by the pandemic, have destabilized the supply of essential commodities like steel, resins, and oil, leading to increased production costs in recent months. Faced with these unprecedented challenges, companies are dedicating their efforts to overcoming disruptions and enhancing resilience. McKinsey conducted a recent survey encompassing 113 supply chain leaders from various global industries. The survey results shed light on prevailing industry practices[2]. It was found that while the most widely adopted practice (80% of respondents) for enhancing resilience involves maintaining a larger safety stock as a buffer, companies are actively exploring more intelligent approaches to ensure resilience while effectively managing inventory costs. One such approach involves diversifying their supply base, thereby reducing dependence on a single source. Indeed, many firms are adopting longer-term strategies like dual-sourcing (81% of respondents) to enhance their resilience in the face of disruptions.

Over the past two decades, supply chain disruption has garnered significant interest from both academics and practitioners. These disruptions, characterized by their rare yet impactful nature, have highlighted the importance of preparedness. Examples of such disruptions include natural disasters like the COVID-19 pandemic (Shen and Sun 2023) and the Great East Japan Earthquake in 2011 (Hendricks et al. 2020), as well as human-induced events like strikes at General Motors parts plants (Bode and Wagner 1998) and US coal mining disasters Madsen (2009). For this study, we will adopt Bode and Wagner (2015)'s definition of supply chain disruption, which refers to "the combination of an unintended and unexpected triggering event that occurs somewhere in the upstream supply chain (the supply network), the inbound logistics network, or the purchasing (sourcing) environment, and a consequential situation which presents a serious threat to the normal course of business operations of the focal firm."

When addressing supply chain disruptions, inventory management (e.g., Meyer et al. 1979, Moinzadeh and Aggarwal 1997, Gao et al. 2019) and sourcing flexibility (e.g., Anupindi and Akella 1993, Tomlin and Wang 2005) have been recognized as crucial mitigation strategies. Simchi-Levi et al. (2018) further proposes a hybrid strategy, examining the interaction between process flexibility and inventory to enhance resilience.

[1] Global Supply Chains in a Post-Pandemic World (September-October 2020); retrieved from https://hbr.org/2020/09/global-supply-chains-in-a-post-pandemic-world on September 26, 2022.

[2] Taking the pulse of shifting supply chains (August 26, 2022); retrieved from https://www.mckinsey.com/capabilities/operations/our-insights/taking-the-pulse-of-shifting-supply-chains on September 26, 2022.

1.1. Process flexibility

In both theory and practice, the concept of *process flexibility* holds great importance in effectively tackling uncertainties in demand. Consider a production system represented by a bipartite graph, where a set of n product nodes, denoted as $[n] \triangleq \{1, \cdots, n\}$, is connected to a set of m facility/plant nodes, denoted as $[m] \triangleq \{1, \cdots, m\}$, through a unique set of links that define the *configuration*,

$$\mathscr{G} \subseteq [m] \times [n] = \{(j, i), i \in [n], j \in [m]\}. \tag{1}$$

A link connecting product node i with plant node j means that plant j can produce/serve product i. Let $\boldsymbol{D} \triangleq (D_1, ..., D_n) \sim \mathbb{P}$ denote the demand vector for all products with an unknown joint distribution $\mathbb{P}$, where D_i denote the stochastically independent demand for the product i. Similarly, let $\boldsymbol{C} \triangleq (C_1, ..., C_m)$ denote the vector of production capacity over all plants. We assume that the operators of a production system observe demand realizations and aim to maximize total sales (i.e., demand fulfillment). Under a configuration $\mathscr{G}$, the maximum sales problem can be solved as a max-flow problem, as shown in (2) below. The effectiveness of the production system can be assessed based on its *expected sales* $\mathbb{E}[f_{\mathscr{G}}(\boldsymbol{D}, \boldsymbol{C})]$, where

$$\begin{aligned} f_{\mathscr{G}}(\boldsymbol{D}, \boldsymbol{C}) \triangleq \max \; & \sum_{(j,i) \in \mathscr{G}} x_{j,i} \\ \text{s.t.} \; & \sum_{j \in [m]} x_{j,i} \leq D_i \;\; \forall i \in [n] \\ & \sum_{i \in [n]} x_{j,i} \leq C_j \;\; \forall j \in [m] \\ & x_{j,i} \geq 0 \qquad \forall j \in [m], i \in [n] \\ & x_{j,i} = 0 \qquad \forall (j,i) \notin \mathscr{G} \end{aligned} \tag{2}$$

and $x_{j,i}$ represents the resource allocated from plant j to serve the demand of product i.

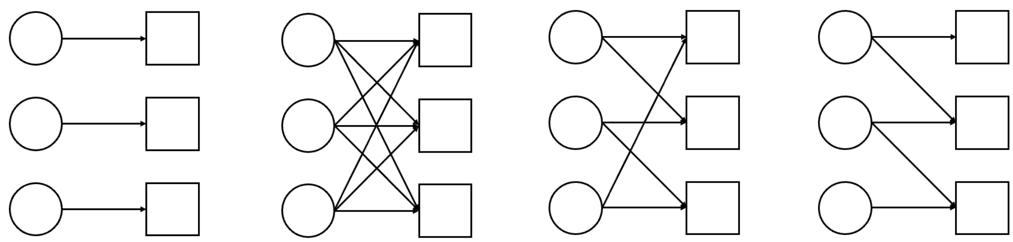
We define a production system or a configuration to be *balanced* if $m = n$, and a production system to be *symmetric* when C_j's have the same value for all $j \in [m]$ and D_i's are independent and identically distributed (i.i.d.) random variables.

Notable configurations studied in the literature include the *dedicated system*, where each plant can only produce a single product and the *fully flexible system*, where each plant is capable of producing all the products. For example, Figure 1-(a) depicts a balanced dedicated system, denoted by $\mathscr{D}_n \triangleq \{(i, i), i \in [n]\}$; and Figure 1-(b) visualizes the corresponding fully flexible system, denoted by $\mathscr{F}_n \triangleq \{(j, i), i \in [n], j \in [n]\}$. Additionally, the "chaining" strategy, widely known as the *long chain*, stands out due to its superior performance achieved through limited flexibility in deploying capacity to satisfy demand (see Figure 1-(c)):

$$\mathscr{C}_n \triangleq \{(i, i), (i, i+1) | i = 1, ..., n-1\} \cup \{(n, n), (n, 1)\}. \tag{3}$$

As shown by Jordan and Graves (1995), in a balanced and symmetric production system, the "chaining"

Figure 1: Configurations with $m = n = 3$



(a) Dedicated system (b) Fully flexible system (c) Long chain (d) Open chain

strategy can result in significant advantages in terms of expected sales (demand fulfillment) and capacity utilization, comparable to the fully flexible system.

The recent supply shocks raise two questions: *Does the remarkable capability of the "chaining" strategy in pooling demand uncertainty remain effective in the presence of supply disruption? Can a chaining system effectively mitigate such disruption?* Appendix A presents a simple illustrative example that affirms both, demonstrating the long chain's ability to hedge against supply risk while pooling demand. While prior literature (to be reviewed in §2) has established that a flexibility configuration, such as a long chain, effectively balance supply and demand under uncertainty, their performance under supply disruption remains less understood. This paper provides analytical insights into the expected sales performance of long chain systems facing both demand uncertainty and supply disruption through asymptotic analysis.

1.2. Contributions

Below, we summarize the contributions of this paper:

- This study advances the process flexibility literature by analyzing the long chain's performance under both demand uncertainty and supply disruptions. We derive a tight closed-form bound on its asymptotic expected sales when designed capacity matches expected demand but is subject to random disruptions, with demand characterized by its mean, support, and mean absolute deviation (MAD). Our results show that the long chain can surpass both dedicated and fully flexible systems in resilience measured by the ratio of expected sales guarantee under disruption to those without. We also connect the asymptotic guarantee to the performance of finite systems.
- To generalize our findings, we relax assumptions on supply disruptions, demand characterization, performance metrics, and capacity levels. We propose a moment decomposition approach, leveraging semidefinite programming (SDP) to analyze long chain performance under general settings. This framework accommodates any piecewise polynomial performance metric and moment-based demand information, systematically generalizing prior work (e.g., Wang and Zhang 2015, Bidkhori et al. 2016) and enabling new analyses, such as the probability of demand fulfillment (type-I service level).

- The proposed method offers a versatile framework for analyzing long chain resilience across broader operational contexts. Using the moment decomposition approach, we underscore the need to adjust capacity based on disruption risk. Our results show that in the absence of disruptions, a long chain can achieve a target type-II service level with capacity similar to a fully flexible system, even with limited demand information. However, disruption significantly raises capacity requirements, highlighting the importance of accounting for such risks in planning. We also quantify the effectiveness of mitigating disruptions through capacity expansion.

Organization. The paper is organized as follows. In §2, we review the relevant literature. §3 provides analytical characterizations of the performance guarantee for long chain systems under random supply disruptions and demand uncertainty. §4 introduces a general framework for analyzing the asymptotic performance of long chains under disruption, extending the analysis in §3 by relaxing its assumptions. We develop a moment decomposition approach and present three applications. In §5, we explore the capacity configuration problem for supply disruption mitigation. Finally, §6 concludes the paper. All proofs are provided in Appendix B.

Notations. We denote by $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{N}$ the set of real numbers, nonnegative real numbers and nonnegative integers. For a positive integer n, we use $[n]$ to denote the index set $\{1, 2, \cdots, n\}$. We denote by $\mathrm{C}_{n,r}$ as the corresponding binomial coefficient given $n, r \in \mathbb{N}$. We denote the closure of the set $\mathcal{Z}$ by $\mathrm{cl}(\mathcal{Z})$. For a logical expression $\mathcal{E}$, we define the indicator function $\mathbb{1}\{\mathcal{E}\}$ equals 1 if $\mathcal{E}$ is true and 0 otherwise. Similarly, we denote $\mathbb{1}_{\mathcal{Z}}$ as the indicator function of the set $\mathcal{Z}$, i.e., $\mathbb{1}_{\mathcal{Z}}(\boldsymbol{x})$ represents $\mathbb{1}\{\boldsymbol{x} \in \mathcal{Z}\}$. We use $\mathbf{0}$ to represent zero vectors or matrices. For a matrix $\boldsymbol{A} \in \mathbb{R}^{m\times n}$, we denote its transpose by $\boldsymbol{A}^\top$. For a symmetry matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, we use $\boldsymbol{A} \succeq \mathbf{0}$ to represent that $\boldsymbol{A}$ is positive semidefinite. We denote by $\mathcal{P}_0(\mathcal{Z})$ the set of all probability distributions over the set $\mathcal{Z}$. We define plus function $[x]^+ \triangleq \max\{x, 0\}$. We use the expectation operator $\mathbb{E}[\cdot]$ and $\mathbb{E}_{\mathbb{P}}[\cdot]$ to signify the expectation taken with respect to a probability distribution $\mathbb{P}$. We also use "w.p." to represent "with probability" and "$\sim$" to represent distribution equivalence of random variables or probability distributions.

2. Literature review

In this section, we delve into the literature that directly addresses process flexibility and its effectiveness in the face of supply chain disruption.

The conventional literature on process flexibility has primarily emphasized the remarkable capability of the long chain in meeting demand. The influential study by Jordan and Graves (1995) demonstrates, through extensive simulation studies, that limited flexibility, when appropriately configured using a "chaining" strategy, can yield most of the benefits of total flexibility in terms of expected sales and capacity utilization. This pioneering work has been expanded upon in various contexts and has inspired valuable insights in manufacturing and service systems, such as worker cross-training (Hopp et al. 2004), capacity portfolio investment (Bassamboo et al. 2010), production postponement (Chou et al. 2014), workforce deployment (Yan et al. 2018), and vehicle routing (Ledvina et al. 2022), among others. For a comprehensive review of the development of process flexibility over the past decades, please refer to Chou et al. (2008) and Wang et al. (2021).

Meanwhile, efforts have been expanded to theoretically quantify the effectiveness of a flexible system through the metric of expected sales (see, e.g., Chou et al. 2010, Simchi-Levi and Wei 2012, Chen et al. 2015 and Désir et al. 2016, among others) or worst-case demand fulfillment (see, e.g., Simchi-Levi and Wei 2015, Wang et al. 2022). Among this literature, a critical question is to evaluate the expected sales under a long chain structure in a balanced and symmetric system. Chou et al. (2010) develops a method to compute the expected sales of long chain in asymptotic regime given probability distribution with finite support. Simchi-Levi and Wei (2012) analyze the performance of a long chain with finite size ($n < \infty$) under stochastic demand by exploiting supermodularity. To assess the robustness of the long chain's performance under ambiguous demand distribution, Wang and Zhang (2015) study an *infinite system* (i.e., when $n \to \infty$) under ambiguous demand. Specifically, they derive a closed-form lower bound for the ratio of the expected sales of a k-chain relative to that of a fully flexible system given the mean and variance of demand. Similarly, Bidkhori et al. (2016) derives the closed-form lower bound on the expected sales given the mean, support and partial expectation of demand. Their approaches are related to the class of distributionally robust optimization (DRO) models, which seek decisions that hedge against the worst-case distribution within the ambiguity set of probability distributions (see, e.g., Delage and Ye 2010). Pioneered by Scarf (1958), DRO has been applied to derive upper and lower bounds on the expected value of various stochastic optimization problems (e.g., Birge and Louveaux 2011, Kall et al. 1994), such as inventory management (e.g., Gallego 1998, Das et al. 2021) and queuing theory (e.g., Kingman 1962, Bertsimas and Natarajan 2007).

The value of flexibility in systems with disruption, on the other hand, is not well-understood. The initial attempt was carried out by Lim et al. (2011) that studies a finite system with known demand and supply distribution through simulations and numerical experiments. The main observation is that a single long chain is preferable to multiple short ones when the system is subject to node-based failures and the other way round under arc-based disruption. Mehmanchi et al. (2020) further advances such findings with the help of the the plant cover indices introduced by Simchi-Levi and Wei (2015). Mehmanchi et al. (2020) study the worse-case effectiveness of a flexible configuration with the supply and demand uncertainties captured by (budget) uncertain sets (see Bertsimas and Sim 2004). Rujeerapaiboon et al. (2023) extends Simchi-Levi and Wei (2012) to consider random supply disruption and focuses on computing the expected sales of long chain under i.i.d. disruptions and i.i.d. known discrete demand distributions. Another recent study by DeValve et al. (2023) examines the network design problem in flexible configurations under stochastic supply capacity and stochastic demand (i.e., with known distribution). The study proposes and utilizes a structural property called γ-cover modularity to derive the approximation guarantee for greedy methods.

These works do not provide theoretical guarantees for the effectiveness of a long chain in mitigating disruption under ambiguous demand. In contrast, we study the asymptotic performance of a long chain under ambiguous demand and random supply disruption. This problem setting admits closed-form solutions in some cases and allows for a clear interpretation of findings, making it easier to draw managerial insights and capture the interplay among the various parameters. Further, our results are more robust than those in the existing literature, which can be sensitive to the specification of demand distribution and choice of parameters in the experiments.

3. Analysis of long chain under disruption

In this section, we examine balanced and symmetric configurations under random supply disruptions and demand uncertainty. The system comprises of n plants and n products, where all plants have a *designed capacity* C but are subject to random disruptions independently with probability $\epsilon \in [0,1)$. We model the *effective capacity* of each plant as $C_i = C \cdot \xi_i$, where $\xi_i \sim \text{Bernoulli}(1-\epsilon)$ for each $i \in [n]$. This two-state event modeling of disruption is common in the OR/MS literature, where one state corresponds to normal functioning (or "up", "on", etc), and the other corresponds to disruption (or "down", "off", "fail", etc), see Snyder et al. (2016). Such "all-or-nothing" disruption assumption will be relaxed as in §4. Suppose the uncertain demands are i.i.d. and each follows $D_i \sim \mathbb{P} \in \mathcal{P}_0(\mathbb{R})$. We also assume the random vector $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_n)$, representing the binary state vector for all plants, is independent of the demand vector $\boldsymbol{D} = (D_1, \ldots, D_n)$. This implies that the random effective capacity vector $\boldsymbol{C} = (C_1, \ldots, C_n)$ are independent of the demand vector $\boldsymbol{D}$.

We define the performance measure as the expected sales under random disruptions and any demand distribution $\mathbb{P}$, i.e.,

$$MF(\epsilon, \mathbb{P}, \mathscr{G}) \triangleq \mathbb{E}_{\xi}\left[\mathbb{E}_{\mathbb{P}}\left[f_{\mathscr{G}}(\boldsymbol{D}, \boldsymbol{C})\right]\right].$$

The outer expectation $\mathbb{E}_{\xi}[\cdot]$ and inner expectation $\mathbb{E}_{\mathbb{P}}[\cdot]$ account for the randomness of disruptions and demand, respectively. For a dedicated system and a fully flexible system, we have

$$MF(\epsilon, \mathbb{P}, \mathscr{D}_n) = \mathbb{E}_{\xi}\left[\mathbb{E}_{\mathbb{P}}\left[\sum_{i\in[n]} \min\{C \cdot \xi_i, D_i\}\right]\right] = n(1-\epsilon)\mathbb{E}_{\mathbb{P}}\left[\min\{C, D_1\}\right] \tag{4}$$

and

$$MF(\epsilon, \mathbb{P}, \mathscr{F}_n) = \mathbb{E}_{\xi}\left[\mathbb{E}_{\mathbb{P}}\left[\min\left\{\sum_{i\in[n]} C \cdot \xi_i, \sum_{i\in[n]} D_i\right\}\right]\right]. \tag{5}$$

By Jensen's inequality, the expected sales per node under a fully flexible system is bounded as

$$\frac{1}{n}\mathbb{E}\left[\min\left\{C \cdot \sum_{i\in[n]} \xi_i, \sum_{i\in[n]} D_i\right\}\right] \leq \frac{1}{n}\min\left\{\mathbb{E}\left[C \cdot \sum_{i\in[n]} \xi_i\right], \mathbb{E}\left[\sum_{i\in[n]} D_i\right]\right\} = \min\{(1-\epsilon)C, \mu\}, \tag{6}$$

with the equality holding by the law of large numbers as $n \to \infty$.

To investigate whether the effectiveness of flexible configurations in responding to demand uncertainty remains valid under supply disruption, we consider the *relative efficiency (RE)*, which is defined as the ratio of the expected sales attainable by any system $\mathscr{C}$ to that achieved by the fully flexible system $\mathscr{F}_n$, considering the same level of randomness, i.e.,

$$RE(\epsilon, \mathbb{P}, \mathscr{G}) \triangleq \frac{MF(\epsilon, \mathbb{P}, \mathscr{G})}{MF(\epsilon, \mathbb{P}, \mathscr{F}_n)}.$$

The RE of a flexible configuration is inherently influenced by the demand distribution $\mathbb{P}$, which may or may not be known. To show that the expected sales in a configuration $\mathscr{G}$ are comparable to those of a fully flexible system, it is natural and sufficient to provide a guarantee for the expected sales over a set of possible distributions, characterized by an ambiguity set of probability distributions, denoted by $\mathcal{F}$. For example, we consider in this section the ambiguity set:

$$\mathcal{F}_0 \triangleq \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}) \;\middle|\; \begin{array}{l} D \sim \mathbb{P} \\ \mathbb{E}\left[D\right] = \mu \\ \mathbb{E}\left[|D - \mu|\right] = b \\ \mathbb{P}\left[D \in [0, 2\mu]\right] = 1 \end{array} \right\}.$$

The ambiguity set $\mathcal{F}_0$ encompasses all distributions that satisfy the conditions on the mean, support, and MAD. Specifically, we consider a mean μ and support $[0, 2\mu]$, as proposed by Chou et al. (2010). Additionally, MAD is a widely-used dispersion measure of random variable (see, e.g., Postek et al. 2018, Elmachtoub et al. 2021), similar to standard deviation. Nonetheless, in applications demonstrated by van Eekelen et al. (2022), it has been found that MAD can facilitate the derivation of tight analytical bounds, offering an advantage over other measures. We also consider more versatile demand characteristics besides MAD, such as higher-order moments, in §4.

Inspired by Kingman (1962) and Wang and Zhang (2015), among others, we define the *expected sales guarantee* as

$$MFG(\epsilon, \mathcal{F}, \mathscr{G}) \triangleq \inf_{\mathbb{P} \in \mathcal{F}} MF(\epsilon, \mathbb{P}, \mathscr{G}).$$

This definition guarantees that for a configuration $\mathscr{G}$, as long as the demand distribution $\mathbb{P}$ falls within the ambiguity set $\mathcal{F}$, the expected sales under random disruption with probability ϵ will achieve at least the level $MFG(\epsilon, \mathcal{F}, \mathscr{G})$. Moreover, since the expected sales in a fully flexible system cannot exceed a level given by Equation (6), this expected sales guarantee also provides a lower bound for the RE for a configuration $\mathscr{G}$. If such a lower bound is close to one, we can conclude that the configuration $\mathscr{G}$ performs comparably to a fully flexible system, regardless of the distribution.

In a balanced and symmetric dedicated system without supply disruption ($\epsilon = 0$), Chou et al. (2010) observes that $\lim_{n \to \infty} RE(0, \mathbb{P}, \mathscr{D}_n) \geq 1/2$ under any demand distribution $\mathbb{P}$ with mean μ and support

$[0, 2\mu]$. This observation can be supported by the following proposition, which provides a tight lower bound for $MF(\epsilon, \mathbb{P}, \mathscr{D}_n)$.

PROPOSITION 1. *Suppose $C \geq \mu$. For the dedicated system $\mathscr{D}_n$, the expected sales guarantee under demand ambiguity $\mathcal{F}_0$ is*

$$MFG(\epsilon, \mathcal{F}_0, \mathscr{D}_n) = n(1-\epsilon)\left(\mu - \left[1 - \frac{C}{2\mu}\right]^+ b\right).$$

We remark that when $C = \mu$ and for any possible MAD ($b \in [0, \mu]$), the lower bound becomes $n(1-\epsilon)\mu\,(1 - b/(2\mu))$ and, regardless the disruption probability ϵ, we have

$$RE(\epsilon, \mathbb{P}, \mathscr{D}_\infty) \triangleq \lim_{n\to\infty} RE(\epsilon, \mathbb{P}, \mathscr{D}_n) \geq 1 - \frac{b}{2\mu}. \tag{7}$$

3.1. The relative efficiency of a long chain

We next focus on relative efficiency of a long chain under random disruption,

$$RE(\epsilon, \mathbb{P}, \mathscr{C}_\infty) \triangleq \lim_{n\to\infty} RE(\epsilon, \mathbb{P}, \mathscr{C}_n).$$

Note that the computation of expected sales involves solving the max-flow problem (2) for all possible demand and disruption scenarios, which lacks an analytical solution and presents a challenge in general. However, in the asymptotic analysis of a long chain, it is known that evaluating the effectiveness of a long chain is equivalent to evaluating that of an open chain (as discussed in Chou et al. 2010 and Wang et al. 2021), where an open chain is defined as $\mathscr{O}_n \triangleq \{(i,i),(i,i+1) | i = 1,\ldots,n-1\} \cup \{(n,n)\} = \mathscr{C}_n \backslash \{(n,1)\}$ (see, e.g., Figure 1-(d)), and a greedy allocation is optimal for an open chain. We next extend such analysis to the cases with random disruption. For any given disruption and demand vector $(\boldsymbol{\xi}, \boldsymbol{D})$, it proceeds as follows: first use plant 1 to satisfy the demand of product 1; if there is remaining capacity in plant 1, use it to satisfy the demand of product 2; then use plant 2 to satisfy any excess demand of product 2; if there is any remaining capacity in plant 2, use it to satisfy the demand of product 3; and so on. We formally summarize this greedy approach as follows: (i) let $W_1 \leftarrow 0$ and W_i be the amount of remaining capacity at the plant $i-1$ that is allocated to the product i for $i \in [n]\backslash\{1\}$; (ii) for each $i \in [n-1]$, recursively set $W_{i+1} \leftarrow C \cdot \xi_i - \min\{C \cdot \xi_i, [D_i - W_i]^+\}$. Clearly W_{i+1} is continuous and monotone in W_i for any fixed ξ_i and D_i, hence there is a stationary distribution for this stochastic process. Since each W_i is bounded by C, it converges in distribution to a random variable W as $n \to \infty$. Then the steady state is characterized by

$$W \sim \left[C \cdot \xi - [D - W]^+\right]^+ \tag{8}$$

where "$\sim$" means equivalent in distribution. Because W_{i+1}, D_{i+1} and ξ_{i+1} are independent by construction, we know that W, D and ξ are independent at the steady state.

In the absence of supply disruption, Simchi-Levi and Wei (2012) shows that the performance of long chain $MF(0,\mathbb{P},\mathscr{C}_n)$ can be characterized by the demand fulfillment of product n in the open chain $\mathscr{O}_n$ under i.i.d. demand. This result has been extended to account for random disruption (Rujeerapaiboon et al. 2023). By integrating stochastic orders with their results, we show that asymptotic analysis offers insights into the expected sales of long chains.

PROPOSITION 2. *The expected sales of a long chain $\mathscr{C}_n$ are bounded by:*

$$(n-1)\mathbb{E}\left[\min\left\{C\cdot\xi+W,D\right\}\right]\leq MF(\epsilon,\mathbb{P},\mathscr{C}_n)\leq n\mathbb{E}\left[\min\left\{C\cdot\xi+W,D\right\}\right],$$

Moreover, the asymptotic RE of long chain provides a lower bound for the RE of long chains:

$$RE(\epsilon,\mathbb{P},\mathscr{C}_\infty)=\frac{\mathbb{E}\left[\min\left\{C\cdot\xi+W,D\right\}\right]}{\min\left\{(1-\epsilon)C,\mu\right\}}\leq\frac{n}{n-1}RE(\epsilon,\mathbb{P},\mathscr{C}_n).$$

Proposition 2 implies that, to obtain a performance guarantee for $\mathscr{C}_\infty$ for all distributions within the set $\mathcal{F}_0$, it is sufficient to solve the following model:

$$\begin{aligned}\inf_{\mathbb{P}\in\mathcal{F}_0}\ & \mathbb{E}_{\mathbb{P}}\left[\mathbb{E}\left[\min\{C\cdot\xi+W,D\}\right]\right]\\ \text{s.t. }& W\sim\left[C\cdot\xi-[D-W]^+\right]^+\\ & D,W,\xi\text{ are independent}\end{aligned}\tag{9}$$

where the inner expectation $\mathbb{E}[\cdot]$ is taken with respect to the randomness of (W,ξ) induced by ξ and D at steady state (8). The two constraints capture distribution equivalence (8) and the independence, respectively.

We analytically solve the Problem (9) when $C=\mu$.

THEOREM 1. *Suppose $C=\mu$, then for any demand distribution $\mathbb{P}\in\mathcal{F}_0$, we have*

$$\mathbb{E}\left[\min\{C\cdot\xi+W,D\}\right]\geq(1-\epsilon)\mu\left[1-\frac{(b/\mu)^2}{4(\epsilon+(1-\epsilon)b/\mu)}\right].$$

It follows that

$$RE(\epsilon,\mathbb{P},\mathscr{C}_\infty)\geq\rho(\epsilon,b)\triangleq1-\frac{(b/\mu)^2}{4(\epsilon+(1-\epsilon)b/\mu)}.$$

Moreover, the above bounds are tight.

Theorem 1 provides a tight closed-form bound on the asymptotic performance of long chain under disruption. We observe that when the expected demand equals the designed capacity, the asymptotic RE of long chain is at least $\rho(\epsilon,b)$ for any demand distribution with support set $[0,2\mu]$ and MAD value of b under disruption probability ϵ. In the absence of supply disruption ($\epsilon=0$), this bound coincides with the asymptotic bound in Bidkhori et al. (2016). Table 1 summarizes how this analytical bound responds to the disruption probability (ϵ) and the demand dispersion (measured by b/μ), compared with the analytical bound of a dedicated system.

Table 1: Exact analytical bounds on asymptotic RE of long chain (Theorem 1) and dedicated system (Equation (7)) under disruption probability ϵ when $C=\mu$.

		The ratio of MAD to mean (b/μ)									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mathscr{C}_\infty$	$\epsilon=0$	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7750	0.7500
	$\epsilon=0.1$	0.9868	0.9643	0.9392	0.9130	0.8864	0.8594	0.8322	0.8049	0.7775	0.7500
	$\epsilon=0.2$	0.9911	0.9722	0.9489	0.9231	0.8958	0.8676	0.8388	0.8095	0.7799	0.7500
	$\epsilon=0.3$	0.9932	0.9773	0.9559	0.9310	0.9038	0.8750	0.8449	0.8140	0.7823	0.7500
	$\epsilon=0.4$	0.9946	0.9808	0.9612	0.9375	0.9107	0.8816	0.8506	0.8182	0.7846	0.7500
	$\epsilon=0.5$	0.9955	0.9833	0.9654	0.9429	0.9167	0.8875	0.8559	0.8222	0.7868	0.7500
$\mathscr{D}_\infty$	$\epsilon\in[0,1)$	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000

We have shown that a long chain retains much of the flexibility benefits under supply disruptions in terms of expected sale guarantees, extending its known demand pooling advantages to settings with supply risk. By pooling risks in a sparse manner, the long chain strikes a balance between two extremes. At one extreme, a fully flexible system allows each plant to produce all products, and vice versa. This flexibility facilitates risk pooling by aggregating both random product demand and the random effective capacity of the plants. However, it is more susceptible to disruptions: if any plant is disrupted, all the demand fulfillment it supports can be interrupted, leading to greater losses compared to a sparse system when disruptions occur. Conversely, at the other extreme, a dedicated system is sparse: a disruption to a plant affects only the demand fulfillment of one product. However, the dedicated system cannot effectively pool risks.

Theorem 1 demonstrates that, when $C=\mu$, the long chain retains a higher fraction of its benefits compared to a fully flexible system as disruption probability ϵ increases. While both experience declining expected sales under rising disruption risk, the fully flexible system suffers a greater relative loss. This is because its advantage in reallocating capacity diminishes when capacity becomes more constrained. In §5, we show that this pattern changes when designed capacity is adjusted to $C=\mu/(1-\epsilon)$ so that the expected effective capacity matches the expected demand.

COROLLARY 1 **(Comparative statics).** *Suppose $C=\mu$, then for $\epsilon\in[0,1)$, there exists $b_0(\epsilon)\triangleq\frac{\sqrt{\epsilon(8+\epsilon)}-3\epsilon}{2(1-\epsilon)}\in[0,1)$, such that if $b/\mu\le b_0(\epsilon)$, then $\frac{\partial^2\rho}{\partial\epsilon\partial b}\ge 0$; otherwise $\frac{\partial^2\rho}{\partial\epsilon\partial b}<0$.*

While a long chain is less effective than a fully flexible system in pooling demand variations, as evidenced by a decrease in RE as demand variation increases ($\partial\rho/\partial b<0$), it gains resilience from its sparse structure, particularly when disruptive risks are high ($\partial\rho/\partial\epsilon\ge 0$). Corollary 1 characterizes the joint effect of demand variation and supply disruption on RE, identifying a threshold $b_0(\cdot)$ dependent on ϵ. Below this threshold, the marginal loss from increased variation decreases with disruption probability ($-\partial^2\rho/\partial b\partial\epsilon < 0$); above it, disruption amplifies the loss. Since the threshold increases with ϵ, complementarity between disruption and variation is more likely when demand variability is low, and becomes substitutive as demand variability grows.

Long chain resilience

To assess a system's ability to resist and recover from supply risks and demand uncertainty, we introduce the notion of *system resilience (SR)*, defined as a ratio of the system's expected sales guarantee under disruption to its disruption-free counterpart for any given system $\mathscr{G}$,

$$SR(\epsilon,\mathcal{F},\mathscr{G}) \triangleq \frac{MFG(\epsilon,\mathcal{F},\mathscr{G})}{MFG(0,\mathcal{F},\mathscr{G})}.$$

For example, Proposition 1 clearly demonstrates that a dedicated system exhibits a system resilience of

$$SR(\epsilon,\mathcal{F}_0,\mathscr{D}_n) = 1-\epsilon,$$

under demand ambiguity $\mathcal{F}_0$. By the law of large numbers, the fully flexible system demonstrates a system resilience asymptotically as

$$SR(\epsilon,\mathcal{F}_0,\mathscr{F}_\infty) = \frac{\min\{\mu(1-\epsilon),\mu\}}{\mu} = 1-\epsilon.$$

Recall that $MFG(\epsilon,\mathcal{F}_0,\mathscr{C}_\infty) = (1-\epsilon)\mu\cdot\rho(\epsilon,b)$ in Theorem 1, which implies the following result on the system resilience of an infinite long chain system.

COROLLARY 2. *Suppose $C=\mu$. The system resilience of an infinite long chain under demand ambiguity $\mathcal{F}_0$ is*

$$SR(\epsilon,\mathcal{F}_0,\mathscr{C}_\infty) = (1-\epsilon)\left(1+\eta(\epsilon,b)\right)$$

where

$$\eta(\epsilon,b) \triangleq \frac{1}{4-b/\mu}\cdot\left(\frac{1}{\epsilon(1-b/\mu)}+\frac{\mu}{b}\right)^{-1} = \frac{1-b/\mu}{4-b/\mu}\epsilon + o(\epsilon).$$

Corollary 2 extends the result of Theorem 1 to analyze the resilience of the long chain. In the case of an infinite system ($n\to\infty$), both the fully flexible system and the dedicated system exhibit a resilience of $1-\epsilon$, directly translating the disruption probability into discounted expected sales. In contrast, the long chain can withstand a non-negligible fraction of the disruption, characterized by an additional term, $\eta(\epsilon,b)$, that recovers the discounted expected sales a little bit. This term depends on the disruption probability ϵ and the dispersion of demand b/μ. Additionally, the higher the disruption probability, the greater the resilience of the long chain system, as evidenced by $\partial\eta(\epsilon,b)/\partial\epsilon > 0$. Our analysis expands the understanding of the long chain by demonstrating that its benefits extend beyond efficiency, highlighting its resilience to disruptions.

Up to this point, our analysis has focused on a specific scenario in which the expected demand equals the designed capacity ($C=\mu$) and the uncertainty in demand is captured by the MAD-based ambiguity set $\mathcal{F}_0$. The analysis thus far highlights the significance of addressing Problem (9), which motivates us to refine our methodology and generalize it to accommodate more general cases.

4. A general framework for long chain analysis

We now present a comprehensive approach to analyzing performance guarantees of long chains under supply disruption, extending previous section's analysis in four aspects by (i) supporting performance metrics beyond expected sales, (ii) incorporating broader demand characterization beyond the mean-MAD specification, (iii) accommodating flexible disruption modeling beyond the traditional "all-or-nothing" scenario, and (iv) relaxing the assumption that designed capacity must equal the expected demand. The proposed approach simplifies and generalizes the asymptotic analysis in the existing literature, providing new insights for more diverse scenarios.

4.1. A general asymptotic bound

We begin with a generalized version of the Problem (9) given by:

$$\begin{aligned} \inf_{\mathbb{P}\in\mathcal{F}} \ & \mathbb{E}_{\mathbb{P}}\left[\mathbb{E}\left[\kappa(C\cdot\xi, W, D)\right]\right] \\ \text{s.t. } & W \sim \left[C\cdot\xi - [D-W]^+\right]^+ \\ & D, W, \xi \text{ are independent} \end{aligned} \tag{10}$$

The expectation here has the same meaning as in Problem (9). Note that we replace the ambiguity set $\mathcal{F}_0$ with a more general ambiguity set $\mathcal{F}$ and denote the performance metric by a function $\kappa(\cdot)$.

ASSUMPTION 1. *We make the following assumptions on Problem* (10)*.*

(a) The performance metric function $\kappa(\cdot)$ *is piecewise polynomial in* $C\cdot\xi, W, D$.

(b) The random demand $D \sim \mathbb{P}$ *is characterized by a moment-based ambiguity set*

$$\mathcal{F} = \left\{ \mathbb{P}\in\mathcal{P}_0(\mathbb{R}_+) \;\middle|\; \begin{array}{l} \mathbb{E}\left[g_j(D)\right] = \delta_j \quad \forall j\in[J] \\ \mathbb{P}\left[D\in\mathcal{Z}_D\right] = 1 \end{array} \right\}$$

where each function g_j *is a piecewise polynomial of degree* $q_j \in \mathbb{N}$.

(c) The random disruption ξ *characterized by a discrete distribution with* Ω *scenarios so that* $\mathbb{P}\left[\xi = \lambda_\omega\right] = \epsilon_\omega$ *for each* $\omega\in[\Omega]$ *where* $\lambda_\omega \in [0,1]$, $\epsilon_\omega \geq 0$ *and* $\sum_{\omega\in[\Omega]}\epsilon_\omega = 1$.

Assumption 1 has strong modeling power in practice. For example, we can consider the *type-I service level* as the expectation of an indicator function

$$\kappa_1(C\cdot\xi, W, D) \triangleq \mathbb{1}\left\{C\cdot\xi + W \geq D\right\} \tag{11}$$

and the *type-II service level* in terms of demand fulfillment

$$\kappa_2(C\cdot\xi, W, D) \triangleq \min\{C\cdot\xi + W, D\}. \tag{12}$$

Problem (10) involves constraints on distribution matching and stochastic independence among random variables. We relax them by moment matching constraints, proposed below.

PROPOSITION 3. *Under Assumption 1, for all integer $d \geq \max_{j\in[J]}\{q_j\}/2$, the optimal value of Problem (10) is bounded from below by the optimal value of the following problem,*

$$\min_{\mathbb{P}} \ \mathbb{E}\left[\sum_{\omega\in[\Omega]} \epsilon_\omega \kappa(C\cdot\lambda_\omega, W, D)\right] \tag{13a}$$

$$\text{s.t.} \ \mathbb{E}\left[W^p - \sum_{\omega\in[\Omega]} \epsilon_\omega \left(\left[C\cdot\lambda_\omega - [D-W]^+\right]^+\right)^p\right] = 0 \qquad \forall p\in[2d] \tag{13b}$$

$$\mathbb{E}\left[g_j(D)W^p - \delta_j W^p\right] = 0 \qquad \forall j\in[J], p\in[2d-q_j] \tag{13c}$$

$$\mathbb{E}\left[g_j(D)\right] = \delta_j \qquad \forall j\in[J] \tag{13d}$$

$$\mathbb{P}\left[(D,W)\in\mathcal{Z}_D\times[0,C]\right] = 1. \tag{13e}$$

Proposition 3 provides a lower bound for the asymptotic performance of a long chain under the metric $\kappa(\cdot)$ by solving Problem (13). As a special case, the proof of Theorem 1 analytically solves the Problem (13), under the Bernoulli disruption where $\xi \sim \text{Bernoulli}(1-\epsilon)$, with type-II service level performance metric $\kappa_2(\cdot)$, the MAD ambiguity set $\mathcal{F}_0$ and when $C=\mu$. However, solving Problem (13) analytically as a generalized moment problem (see, e.g., Lasserre 2009) is generally intractable. To address this challenge, we next introduce a moment decomposition approach that leverages semidefinite programming to address generalized moment problems effectively.

REMARK 1. We focus on the long chain analysis in this paper, although it is possible to extend the analysis to k-chain in certain cases. Without disruption, Wang and Zhang (2015) show that k-chains can be analyzed using two random variables: demand and total remaining capacity. However, extending the analysis to k-chains under disruption is far more challenging. Even for $k=3$, the steady state involves dependent random variables, making the analysis significantly more complex. For $k\geq 4$, the dependencies among remaining capacities at all prior nodes and demand further complicate the analysis. In Appendix C, we show that even characterizing the expected sales of a 3-chain in the steady state is substantially harder than for long chains due to these dependencies.

4.2. Analysis via a moment decomposition approach

We now present the moment decomposition approach for deriving an SDP lower bound for generalized moment problems. To facilitate understanding, we briefly introduce to the generalized moment problem below:

$$\begin{aligned} \min_{\mathbb{P}} \ & \mathbb{E}_{\mathbb{P}}\left[\psi_0(\boldsymbol{z})\right] \\ \text{s.t.} \ & \mathbb{E}_{\mathbb{P}}\left[\psi_j(\boldsymbol{z})\right] = \delta_j \ \forall j\in[J] \\ & \mathbb{P}\left[\boldsymbol{z}\in\mathcal{Z}\right] = 1. \end{aligned} \tag{14}$$

Here, $\boldsymbol{z} \sim \mathbb{P}$ is a random vector with a support set $\mathcal{Z} \subseteq \mathbb{R}^n$. For each $j \in \{0\} \cup [J]$, ψ_j is a piecewise polynomial, i.e., $\psi_j(\boldsymbol{z}) \triangleq \sum_{\ell \in [L]} \psi_{j,\ell}(\boldsymbol{z}) \cdot \mathbb{1}_{\mathcal{Z}_\ell}(\boldsymbol{z})$. Here, we assume the support set $\mathcal{Z}$ can be decomposed into L scenarios, denoted by $\mathcal{Z} = \cup_{\ell=1}^L \mathcal{Z}_\ell$, $\cap_{\ell=1}^L \mathcal{Z}_\ell = \varnothing$, and each $\psi_{j,\ell}(\cdot)$ is a polynomial of degree at most $2d$ in the components of $\boldsymbol{z}$, i.e.,

$$\psi_{j,\ell}(\boldsymbol{z}) \triangleq \sum_{\boldsymbol{\alpha} \in \mathbb{N}_{2d}^n} \psi_{j,\ell,\boldsymbol{\alpha}} \boldsymbol{z}^{\boldsymbol{\alpha}} = \sum_{\boldsymbol{\alpha} \in \mathbb{N}_{2d}^n} \psi_{j,\ell,\boldsymbol{\alpha}} z_1^{\alpha_1} \cdots z_n^{\alpha_n} \tag{15}$$

where $\mathbb{N}_r^n \triangleq \{\boldsymbol{\alpha} \in \mathbb{N}^n : \sum_{i=1}^n \alpha_i \leq r\}$ is the index set of multi-indices with sum of entries at most r, and $\psi_{j,\ell,\boldsymbol{\alpha}} \in \mathbb{R}$ is the corresponding coefficient of the monomial $\boldsymbol{z}^{\boldsymbol{\alpha}} \triangleq z_1^{\alpha_1} \cdots z_n^{\alpha_n}$. Note that the cardinality of $\mathbb{N}_r^n$ is the binomial coefficient $\mathrm{C}_{n+r,n}$.

Problem (14) involves optimization over probability measures, making it inherently an infinite-dimensional problem when the support set $\mathcal{Z}$ has infinite cardinality. The moment decomposition approach addresses this challenge by approximating the optimization problem over probability distributions through the moments of the random variables within each scenario. This reformulation enables a finite-dimensional approximation, making it computationally efficient and tractable.

To implement this approach, we introduce the concepts of moment vectors and moment matrices.

DEFINITION 1 (MOMENT VECTOR AND MOMENT MATRIX). For each $\ell \in [L]$, we define the *moment vector* $\boldsymbol{m}_\ell \in \mathbb{R}^{\mathrm{C}_{n+2d,n}}$, indexed by $\boldsymbol{\alpha} \in \mathbb{N}_{2d}^n$, with each component given by

$$m_{\ell,\boldsymbol{\alpha}} \triangleq \mathbb{E}\left[\boldsymbol{z}^{\boldsymbol{\alpha}} \mathbb{1}_{\mathcal{Z}_\ell}(\boldsymbol{z})\right]. \tag{16}$$

For each $r \in \{0\} \cup [d]$, we define the corresponding *moment matrix* mapping

$$\boldsymbol{M}_r : \mathbb{R}^{\mathrm{C}_{n+2d,n}} \to \mathbb{R}^{\mathrm{C}_{n+r,n} \times \mathrm{C}_{n+r,n}}$$

where the entry of $\boldsymbol{M}_r(\boldsymbol{m}_\ell)$ indexed by $(\boldsymbol{\alpha}, \boldsymbol{\alpha}') \in \mathbb{N}_r^n \times \mathbb{N}_r^n$ is $m_{\ell,\boldsymbol{\alpha}+\boldsymbol{\alpha}'} = \mathbb{E}\left[\boldsymbol{z}^{\boldsymbol{\alpha}+\boldsymbol{\alpha}'} \mathbb{1}_{\mathcal{Z}_\ell}(\boldsymbol{z})\right]$.

In short, the moment vector and the matrix are used to summarize key information about a probability distribution in terms of its moments. Intuitively, the moment vector collects all the moments of the random vector $\boldsymbol{z}$ up to a certain degree (e.g., $2d$) within a specific region or "scenario" $\mathcal{Z}_\ell$. The moment matrix organizes the relationships between moments into a structured, matrix form, reflecting how different powers of $\boldsymbol{z}$ interact. To relate the support information to the moment vectors, we also introduce the concept of localizing moment matrices.

DEFINITION 2 (LOCALIZING MOMENT MATRIX). Given a polynomial $\phi(\boldsymbol{z}) = \sum_{\boldsymbol{\gamma} \in \mathbb{N}_{2(d-r)}^n} \phi_{\boldsymbol{\gamma}} \boldsymbol{z}^{\boldsymbol{\gamma}}$ and $r \in \{0\} \cup [d]$, we define the associated *localizing moment matrix* mapping $\boldsymbol{M}_{r,\phi} : \mathbb{R}^{\mathrm{C}_{n+2d,n}} \to \mathbb{R}^{\mathrm{C}_{n+r,n} \times \mathrm{C}_{n+r,n}}$ such that its $(\boldsymbol{\alpha}, \boldsymbol{\alpha}')$ entry is $\sum_{\boldsymbol{\gamma} \in \mathbb{N}_{2(d-r)}^n} \phi_{\boldsymbol{\gamma}} m_{\ell,\boldsymbol{\gamma}+\boldsymbol{\alpha}+\boldsymbol{\alpha}'}$, i.e., $\mathbb{E}\left[\phi(\boldsymbol{z}) \boldsymbol{z}^{\boldsymbol{\alpha}+\boldsymbol{\alpha}'} \mathbb{1}_{\mathcal{Z}_\ell}(\boldsymbol{z})\right]$.

The localizing moment matrix generalizes the moment matrix, as $\boldsymbol{M}_{r,\phi} = \boldsymbol{M}_r$ when $\phi(\boldsymbol{z}) = 1$ for all $\boldsymbol{z} \in \mathcal{Z}$. For a fixed ϕ, $\boldsymbol{M}_{r,\phi}$ defines as a linear map. The term "localizing" reflects the matrix's ability to "focus" on the interaction between the probability distribution and the specified polynomial $\phi(\boldsymbol{z})$. The entries of the localizing moment matrix are weighted by $\phi(\boldsymbol{z})$, enabling it to capture the influence of ϕ on the random vector $\boldsymbol{z}$ within each scenario $\mathcal{Z}_\ell$. In the following proposition, we leverage the localizing moment matrix to incorporate support information into the moment vectors by focusing on polynomials derived from the support constraints.

With these definitions in place, we are now ready to present a tractable SDP lower bound for the generalized moment problem (14).

PROPOSITION 4. *Suppose that the closure* $\mathrm{cl}(\mathcal{Z}_\ell) = \left\{\boldsymbol{z} \in \mathbb{R}^n : \boldsymbol{a}_{\ell,k}^\top \boldsymbol{z} + b_{\ell,k} \geq 0,\ \forall k \in [K_\ell]\right\}$ *is a polyhedron for each* $\ell \in [L]$*. Given* $\boldsymbol{\gamma} \in \mathbb{N}_{2d}^{K_\ell}$*, we define the polynomial*

$$\phi_{\ell,\boldsymbol{\gamma}}(\boldsymbol{z}) \triangleq \prod_{k\in[K_\ell]} \left(\boldsymbol{a}_{\ell,k}^\top \boldsymbol{z} + b_{\ell,k}\right)^{\gamma_k}.$$

Then the optimal value of the SDP

$$\begin{aligned}
\min_{\boldsymbol{m}_\ell, \ell\in[L]} \quad & \sum_{\ell\in[L]} \sum_{\boldsymbol{\alpha}\in\mathbb{N}_{2d}^n} \psi_{0,\ell,\boldsymbol{\alpha}} m_{\ell,\boldsymbol{\alpha}} && && \text{(17a)}\\
\text{s.t.} \quad & \sum_{\ell\in[L]} \sum_{\boldsymbol{\alpha}\in\mathbb{N}_{2d}^n} \psi_{j,\ell,\boldsymbol{\alpha}} m_{\ell,\boldsymbol{\alpha}} = \delta_j && \forall j \in [J] && \text{(17b)}\\
& \sum_{\ell\in[L]} m_{\ell,\boldsymbol{0}} = 1 && && \text{(17c)}\\
& \boldsymbol{M}_{r,\phi_{\ell,\boldsymbol{\gamma}}}(\boldsymbol{m}_\ell) \succeq \boldsymbol{0} && \forall \ell \in [L], r \in \{0\} \cup [d-1], \boldsymbol{\gamma} \in \mathbb{N}_{2(d-r)}^{K_\ell} \setminus \mathbb{N}_{2(d-1-r)}^{K_\ell} && \text{(17d)}\\
& \boldsymbol{M}_d(\boldsymbol{m}_\ell) \succeq \boldsymbol{0} && \forall \ell \in [L] && \text{(17e)}\\
& \boldsymbol{m}_\ell \in \mathbb{R}^{\mathrm{C}_{n+2d,n}} && \forall \ell \in [L] && \text{(17f)}
\end{aligned}$$

provides a lower bound of Problem (14).

The objective function (17a) and the constraints (17b) are derived directly from the generalized moment problem, utilizing the definitions of the function $\psi_j(\cdot)$, moment vector, and moment matrix. Additionally, by Definition 1, $m_{\ell,\boldsymbol{0}}$ can be interpreted as $\mathbb{P}[\boldsymbol{z} \in \mathcal{Z}_\ell]$, thereby constraint (17c) ensures that the probabilities across all scenarios sum to one. Constraints (17d) to (17e) are semidefinite constraints that establish the connection between moment vectors and the support information of the probability distribution via localizing moment matrix mappings associated with polynomials derived from the support constraints. As such, they can be interpreted as relaxations of the support constraints in the generalized moment problem (14).

We demonstrate applications of the moment decomposition approach to extend the analysis in §3 by: (i) characterizing demand distribution beyond $\mathcal{F}_0$, (ii) broadening long chain effectiveness to other performance metrics, (iii) relaxing the "all-or-nothing" disruption assumption, and (iv) generalizing to cases where $C \neq \mu$. The first three perspectives are addressed in this section, while the last is illustrated through a capacity configuration problem in the next section.

4.3. Application 1: analysis with demand characteristics up to the fourth moment

We first provide an illustrative example to demonstrate the application of the moment decomposition approach in accommodating demand characterization through higher moment information. In particular, we derive the SDP lower bound for a generalized moment problem of Problem (13) when the demand is characterized by its mean (μ), standard deviation (σ), skewness (γ_1), kurtosis (γ_2), and support, which define the ambiguity set below.

$$\mathcal{F}_4 \triangleq \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}) \;\middle|\; \begin{array}{l} D \sim \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}[D] = \mu \\ \mathbb{E}_{\mathbb{P}}[(D-\mu)^2] = \sigma^2 \\ \mathbb{E}_{\mathbb{P}}[(D-\mu)^3] = \gamma_1 \sigma^3 \\ \mathbb{E}_{\mathbb{P}}[(D-\mu)^4] = \gamma_2 \sigma^4 \\ \mathbb{P}[D \in [0, 2\mu]] = 1 \end{array} \right\}$$

To obtain a tractable SDP lower bound for Problem (13) under ambiguity set $\mathcal{F}_4$, specifically with $J = 4$ and $\max_{j \in [J]}\{q_j\} = 4$, we begin by considering the corresponding generalized moment problem in the form of Problem (14). To achieve this, we decompose the support set

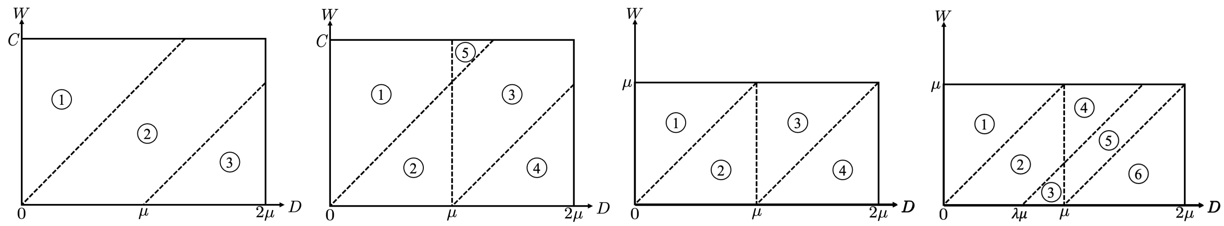
$$\mathcal{Z} = \{(D, W) : 2\mu \geq D \geq 0, C \geq W \geq 0\}$$

into L scenarios such that $\mathcal{Z} = \cup_{\ell \in [L]} \mathcal{Z}_\ell$ and $\cap_{\ell \in [L]} \mathcal{Z}_\ell = \varnothing$. Such decomposition of the support set is contingent upon the availability of information and the performance metrics used. Here, in evaluating the expected sales under the ambiguity set of $\mathcal{F}_4$, the support set can be partitioned into three distinct scenarios ($L = 3$), as depicted in Figure 2-(a), given by

$$\begin{aligned} \mathcal{Z}_1 &= \{(D, W) | 0 \leq D \leq W \leq C\} \\ \mathcal{Z}_2 &= \{(D, W) | 0 \leq W < D \leq 2\mu, D - C \leq W < C\} \,. \\ \mathcal{Z}_3 &= \{(D, W) | 0 \leq W < D - C, D \leq 2\mu\} \end{aligned}$$

Note that if MAD information is accessible, the support set can be subdivided into five scenarios ($L = 5$), as shown in Figure 2-(b). In a special case where the designed capacity equals the expected demand, such as in Theorem 1, this decomposition reduces to four scenarios ($L = 4$), as illustrated in Figure 2-(c). In another case of partial disruption (to be discussed in §4.5), the support set can be partitioned into six scenarios ($L = 6$) as shown in Figure 2-(d).

Figure 2: Illustrating examples of decomposing $\mathcal{Z}$ under different settings



(a) $\mathcal{F}=\mathcal{F}_4$; $C=\mu$; Bernoulli disruption (b) $\mathcal{F}=\mathcal{F}_0$; $C>\mu$; Bernoulli disruption (c) $\mathcal{F}=\mathcal{F}_0$; $C=\mu$; Bernoulli disruption (d) $\mathcal{F}=\mathcal{F}_0$; $C=\mu$; partial disruption

For further analysis, we transform the constraints and objective into piecewise polynomials:

$$\begin{aligned}
\left[C-[D-W]^+\right]^+ &= \mathbb{1}_{\mathcal{Z}_2}\cdot(C+W-D)+\mathbb{1}_{\mathcal{Z}_1}\cdot C\\
\kappa_2(0,W,D) &= \mathbb{1}_{\mathcal{Z}_1}\cdot D+(\mathbb{1}_{\mathcal{Z}_2}+\mathbb{1}_{\mathcal{Z}_3})\cdot W\\
\kappa_2(C,W,D) &= (\mathbb{1}_{\mathcal{Z}_1}+\mathbb{1}_{\mathcal{Z}_2})\cdot D+\mathbb{1}_{\mathcal{Z}_3}\cdot(C+W).
\end{aligned}$$

With moment vectors (see Definition 1) consisting of monomials up to degree four (i.e., $d=2$), we can express the Problem (14) explicitly as follows:

$$\begin{aligned}
\min\ & \mathbb{E}\left[(1-\epsilon)\mathbb{1}_{\mathcal{Z}_3}\cdot(C+W-D)+\epsilon\left(\mathbb{1}_{\mathcal{Z}_2}+\mathbb{1}_{\mathcal{Z}_3}\right)\cdot(W-D)\right]+\mu\\
\text{s.t.}\ & \mathbb{E}\left[W^p\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=(1-\epsilon)\mathbb{E}\left[(\mathbb{1}_{\mathcal{Z}_2}\cdot(C+W-D)+\mathbb{1}_{\mathcal{Z}_1}\cdot C))^p\right] && \forall p\in[4]\\
& \mathbb{E}\left[(DW^p-\mu W^p)\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=0 && \forall p\in[3]\\
& \mathbb{E}\left[(D^2W^p-(\sigma^2+\mu^2)W^p)\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=0 && \forall p\in[2]\\
& \mathbb{E}\left[(D^3W^p-\mathbb{E}[D^3]W^p)\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=0 && \forall p\in[1]\\
& \mathbb{E}\left[D\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=\mu\\
& \mathbb{E}\left[D^2\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=\sigma^2+\mu^2\\
& \mathbb{E}\left[D^3\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=\gamma_1\sigma^3+3\mu\sigma^2+\mu^3\\
& \mathbb{E}\left[D^4\left(\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right)\right]=\gamma_2\sigma^4+4\gamma_1\mu\sigma^3+6\mu^2\sigma^2+\mu^4\\
& \mathbb{E}\left[\textstyle\sum_{\ell\in[L]}\mathbb{1}_{\mathcal{Z}_\ell}\right]=1.
\end{aligned}\tag{18}$$

Guided by Proposition 4, we can obtain the lower bound of Problem (18) through an SDP. The decision variables $\boldsymbol{m}_\ell\in\mathbb{R}^{\mathrm{C}_{6,2}}$ for $\ell\in[3]$ take the form of

$$\boldsymbol{m}_\ell=\mathbb{E}\left[\mathbb{1}_{\mathcal{Z}_\ell}\cdot\begin{pmatrix}1 & D & W & D^2 & DW & W^2 & D^3 & D^2W & DW^2 & W^3 & D^4 & D^3W & D^2W^2 & DW^3 & W^4\end{pmatrix}\right],$$

where each component of $\boldsymbol{m}_\ell$ represents $\mathbb{E}\left[D^{\alpha_1}W^{\alpha_2}\mathbb{1}_{\mathcal{Z}_\ell}\right]$ for certain $(\alpha_1,\alpha_2)\in\mathbb{N}_4^2$. It should be noted that the objective function and all moment matching constraints in Problem (18) can be represented using affine functions of $\boldsymbol{m}_\ell$, corresponding to Equations (17a)-(17c). By the definition of moment matrix, we have the positive semidefinite constraints, corresponding to Equation (17e):

$$\boldsymbol{M}_2(\boldsymbol{m}_\ell) = \mathbb{E}\left[\mathbb{1}_{\mathcal{Z}_\ell} \cdot \begin{pmatrix} 1 & D & W & D^2 & DW & W^2 \\ D & D^2 & DW & D^3 & D^2W & DW^2 \\ W & DW & W^2 & D^2W & DW^2 & W^3 \\ D^2 & D^3 & D^2W & D^4 & D^3W & D^2W^2 \\ DW & D^2W & DW^2 & D^3W & D^2W^2 & DW^3 \\ W^2 & DW^2 & W^3 & D^2W^2 & DW^3 & W^4 \end{pmatrix}\right] \succeq \mathbf{0},\ \forall \ell \in [3].$$

Next, we illustrate the positive semidefinite constraints of Equation (17d). For example, when $\ell = 1$, we have $K_\ell = 3$ because $\mathcal{Z}_1$ is defined by three linear constraints. Furthermore, for $r \in \{0, 1\}$ and $\boldsymbol{\gamma} \in \mathbb{N}^3_{4-2r} \backslash \mathbb{N}^3_{2-2r}$, we have constraints $\boldsymbol{M}_{r,\phi_{\ell,\boldsymbol{\gamma}}}(\boldsymbol{m}_1) \succeq \mathbf{0}$. Taking $r = 1$ and $\boldsymbol{\gamma} = (1, 1, 0)$ for an example, we have $\phi_{\ell,\boldsymbol{\gamma}}(D, W) = D(W - D)$, because the first two constraints in $\mathcal{Z}_1$ are $D \geq 0$ and $W - D \geq 0$. Therefore, for $\ell = 1$, $r = 1$, $\boldsymbol{\gamma} = (1, 1, 0)$, we have

$$\boldsymbol{M}_{r,\phi_{\ell,\boldsymbol{\gamma}}}(\boldsymbol{m}_1) = \mathbb{E}\left[\mathbb{1}_{\mathcal{Z}_1} \cdot \begin{pmatrix} DW - D^2 & D^2W - D^3 & DW^2 - D^2W \\ D^2W - D^3 & D^3W - D^4 & D^2W^2 - D^3W \\ DW^2 - D^2W & D^2W^2 - D^3W & DW^3 - D^2W^2 \end{pmatrix}\right] \succeq \mathbf{0}.$$

Take another example of $\ell = 1$, $r = 0$, $\boldsymbol{\gamma} = (1, 1, 2)$, we have $\phi_{\ell,\boldsymbol{\gamma}}(D, W) = D(W - D)(C - W)^2$ and

$$\boldsymbol{M}_{r,\phi_{\ell,\boldsymbol{\gamma}}}(\boldsymbol{m}_1) = \mathbb{E}\left[\mathbb{1}_{\mathcal{Z}_1} \cdot \left(-C^2D^2 + C^2DW + 2CD^2W - 2CDW^2 - D^2W^2 + DW^3\right)\right] \geq 0,$$

where the positive semidefinite constraint degenerates to a linear constraint of $\boldsymbol{m}_1$.

Leveraging the SDP formulation, we study a generalized moment problem by extending the problem setting proposed by Wang and Zhang (2015). In particular, they examine the potential benefits of incorporating higher moment information by discretizing the joint distribution of D and W as an approximation. Here, we adopt the problem setting but without discretization. We compute the lower bound in exact, owing to excellent modelling power of our moment decomposition approach which allows us to incorporate more information with ease. We denote the ambiguity set $\mathcal{F}_4$ with skewness $\gamma_1 = 0$ and kurtosis $\gamma_{2,a} = 1$ as $\mathcal{F}_{4,a}$. Similarly, we define $\mathcal{F}_{4,b}$ by replacing $\gamma_{2,a}$ with $\gamma_{2,b} = \mu/\sigma$. We discuss the results against a benchmark where demand is characterized by its mean (μ), variance (σ^2), and support ($[0, 2\mu]$), i.e., $\mathcal{F}_2 \triangleq \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}) \,\middle|\, D \sim \mathbb{P}, \mathbb{E}_{\mathbb{P}}[D] = \mu, \mathbb{E}_{\mathbb{P}}[(D - \mu)^2] = \sigma^2, \mathbb{P}[D \in [0, 2\mu]] = 1 \right\}$. By setting $d = 2$ in Problem (13) and applying the moment decomposition approach, we obtain the results in Table 2.

Table 2 displays the computational results on distribution-free lower bounds of the asymptotic RE of the long chain. We evaluate such bounds for various levels of supply disruption (ϵ) and coefficient of variation (σ/μ), subject to different availability of moment information, either up to the second moment (i.e., $\mathcal{F}_2$) or

Table 2: Lower bounds on asymptotic RE of long chain under disruption probability ϵ when $C=\mu$.

			The coefficient of variation (σ/μ)									
			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mathscr{C}_\infty$	$\epsilon=0.00$	$\mathcal{F}_2$	0.9950	0.9808	0.9587	0.9310	0.9000	0.8676	0.8356	0.8049	0.7762	0.7500
		$\mathcal{F}_{4,a}$	0.9952	0.9821	0.9630	0.9394	0.9120	0.8805	0.8463	0.8139	0.7816	0.7500
		$\mathcal{F}_{4,b}$	0.9952	0.9822	0.9634	0.9398	0.9121	0.8811	0.8475	0.8137	0.7808	0.7500
	$\epsilon=0.05$	$\mathcal{F}_2$	0.9970	0.9849	0.9646	0.9379	0.9064	0.8730	0.8396	0.8075	0.7775	0.7500
		$\mathcal{F}_{4,a}$	0.9987	0.9885	0.9701	0.9464	0.9184	0.8858	0.8502	0.8165	0.7829	0.7500
		$\mathcal{F}_{4,b}$	0.9985	0.9881	0.9697	0.9458	0.9175	0.8856	0.8512	0.8164	0.7822	0.7500
	$\epsilon=0.10$	$\mathcal{F}_2$	0.9977	0.9871	0.9680	0.9422	0.9114	0.8776	0.8433	0.8100	0.7788	0.7500
		$\mathcal{F}_{4,a}$	0.9994	0.9923	0.9755	0.9522	0.9241	0.8907	0.8539	0.8190	0.7841	0.7500
		$\mathcal{F}_{4,b}$	0.9992	0.9915	0.9745	0.9508	0.9220	0.8894	0.8545	0.8188	0.7836	0.7500

up to the fourth moment under different kurtosis (e.g., $\mathcal{F}_{4,a}$ and $\mathcal{F}_{4,b}$). Our numerical experiments complement the analytical findings in §3.1. Fixing the demand characterization, we observe that the asymptotic RE slightly increases with disruption probability, reinforcing the finding from Table 1 that a long chain is more resilient to supply disruptions than a fully flexible system when $C=\mu$.

Our study also examines the benefits of using higher moment information to tighten the lower bound of the asymptotic RE. We find that the extent of improvement is limited and varies based on the coefficient of variation (σ/μ) and the specific higher moment information considered, regardless of the disruption probability (ϵ). When the coefficient of variation is small, incorporating higher moment information has little effect on the lower bound. However, as demand variation increases, incorporating moment information up to the fourth moment can lead to more significant improvements, though the actual impact depends on the specific moment information used.

4.4. Application 2: type-I service level analysis

Next, we evaluate long chains under the type-I service level $\kappa_1(\cdot)$. We demonstrate the essentiality of incorporating higher-moment information under this performance metric. Table 3 compares the lower bounds of type-I service level with and without higher moment information.

Our findings suggest that including higher-moment information beyond mean, variance, and support can result in a significant increase in the lower bound of the type-I service level, particularly when the coefficient of variation σ/μ is low. Specifically, we observe that when $\sigma/\mu=0.1$ and there is no disruption, the lower bound under $\mathcal{F}_{4,a}$ is more than three times higher than that under $\mathcal{F}_2$. This observation regarding the type-I service level contrasts with its type-II counterpart in Table 2. In particular, for the type-II

Table 3: Lower bounds on the type-I service level of long chain in steady-state under disruption probability ϵ when $C = \mu$.

			The coefficient of variation (σ/μ)									
			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mathscr{C}_\infty$	$\epsilon = 0.00$	$\mathcal{F}_2$	0.2490	0.2871	0.3257	0.3603	0.3848	0.4049	0.4271	0.4507	0.4752	0.7500
		$\mathcal{F}_{4,a}$	0.8229	0.7778	0.7376	0.7003	0.6605	0.6495	0.6453	0.6370	0.6392	0.7500
		$\mathcal{F}_{4,b}$	0.5400	0.5069	0.4939	0.4852	0.4792	0.4859	0.4955	0.5013	0.5035	0.7500
	$\epsilon = 0.05$	$\mathcal{F}_2$	0.0733	0.1753	0.2501	0.3059	0.3444	0.3754	0.4043	0.4324	0.4599	0.7125
		$\mathcal{F}_{4,a}$	0.6344	0.6639	0.6592	0.6420	0.6191	0.6134	0.6107	0.6034	0.6054	0.7125
		$\mathcal{F}_{4,b}$	0.2655	0.3667	0.4093	0.4301	0.4416	0.4563	0.4705	0.4808	0.4865	0.7125
	$\epsilon = 0.10$	$\mathcal{F}_2$	0.0405	0.1215	0.1988	0.2617	0.3096	0.3478	0.3818	0.4135	0.4436	0.6750
		$\mathcal{F}_{4,a}$	0.5552	0.5855	0.5941	0.5891	0.5770	0.5786	0.5773	0.5712	0.5724	0.6750
		$\mathcal{F}_{4,b}$	0.1872	0.2863	0.3457	0.3820	0.4057	0.4277	0.4457	0.4595	0.4686	0.6750

service level, with information up to the second moment ($\mathcal{F}_2$), the bounds are already quite tight, and the additional gains from incorporating higher moments are limited. In contrast, for the type-I service level, the bounds under $\mathcal{F}_2$ are much looser. This can be attributed to the discrete nature of the performance metric $\kappa_1(\cdot)$, where the term within the expectation operator is binary, depending on whether the demand is fulfilled or not. In deriving lower bounds, we seek to identify a specific distribution within the ambiguity set that yields the expected sales guarantee. However, deriving the type-I service level guarantee often involves adversarial scenarios (e.g., a demand level that is arbitrarily close to, but exceeds, the available capacity). An intuitive understanding can be drawn from a dedicated system, where we can theoretically demonstrate (proof provided in Appendix B) that when $\epsilon = 0$ the type-I service level guarantee under $\mathcal{F}_2$ is $\inf_{\mathbb{P}\in\mathcal{F}_2} \mathbb{E}\left[\min\{\mu, D\}\right] = \sigma^2/(2\mu^2)$, achieved by a sequence of distributions

$$D_k = \begin{cases} 0 & \text{w.p. } \sigma^2/2 + p_k \\ \mu(1+\epsilon_k) & \text{w.p. } 1-\sigma^2-p_k\,, \\ \mu(2-\delta_k) & \text{w.p. } \sigma^2/2 \end{cases} \qquad k \in \mathbb{N},$$

for some positive numbers ϵ_k, δ_k and p_k such that $\mathbb{E}\left[D_k\right] = \mu$, $\mathbb{E}\left[(D_k-\mu)^2\right] = \sigma^2$, and $\epsilon_k \to 0$, $\delta_k \to 0$, $p_k \to 0$ as $k \to \infty$. Incorporating higher-moment information helps to eliminate such extreme cases from the ambiguity set, thus significantly improving the type-I service level.

Another interesting observation is that the bound on the type-I service level increases with the coefficient of variation in our experiments; see the first row of Table 3 for an example, in contrast to the results presented in Table 2. This trend can also be attributed to the discrete nature of the performance metric $\kappa_1(\cdot)$.

Drawing an analogy from the dedicated system, we note that the expected sales in the absence of disruption, $\sigma^2/(2\mu^2)$, also increases with the coefficient of variation.

4.5. Application 3: analysis under partial supply disruption

Another practically relevant extension to §3 is to generalize the setting to accommodate cases where plants are partially disrupted, discounting their effective capacity by a factor λ, where $\lambda \in [0,1)$. Specifically, $\mathbb{P}[\xi=\lambda]=\epsilon=1-\mathbb{P}[\xi=1]$. We refer to this scenario as *partial supply disruption*. When $\lambda=0$, the problem degenerates to the case discussed in §3. An immediate extension of Proposition 1 provides the expected sales guarantee for a dedicated system as $n[(\mu-b/2)\cdot(1-\epsilon+\epsilon\lambda))]$ when $C=\mu$; and thus the RE of a dedicated system is lower bounded by a ratio that depends solely on the demand distribution, expressed as $1-b/(2\mu)$. For the long chain, a partial supply disruption setting of Problem (13) is as follows:

$$\begin{aligned} &\min\ \mathbb{E}\left[\epsilon\kappa(\lambda C,W,D)+(1-\epsilon)\kappa(C,W,D)\right] \\ &\text{s.t.}\ \ \mathbb{E}\left[W^p-\epsilon\left(\left[\lambda C-[D-W]^+\right]^+\right)^p-(1-\epsilon)\left(\left[C-[D-W]^+\right]^+\right)^p\right]=0\ \forall p\in[2d]\,. \\ &\qquad \text{Constraints (13c), (13d), (13e).} \end{aligned} \tag{19}$$

Decomposing the support set $\mathcal{Z}$ into six scenarios ($L=6$), as illustrated in Figure 2-(d), the resulting problem can be explicitly reformulated as Problem (27) in Appendix B. Table 4 presents the corresponding SDP lower bound on the asymptotic performance of the long chain.

Table 4: Lower bounds on asymptotic RE of long chain and dedicated system under partial supply disruption with capacity possibly discounted by λ with probability ϵ when $C=\mu$.

		$b/\mu=0.1$			$b/\mu=0.3$			$b/\mu=0.5$		
		$\epsilon=0$	$\epsilon=0.2$	$\epsilon=0.4$	$\epsilon=0$	$\epsilon=0.2$	$\epsilon=0.4$	$\epsilon=0$	$\epsilon=0.2$	$\epsilon=0.4$
$\mathscr{C}_\infty$	$\lambda=0.0$	0.9750	0.9911	0.9946	0.9250	0.9489	0.9612	0.8750	0.8958	0.9107
	$\lambda=0.2$	0.9750	0.9898	0.9936	0.9250	0.9454	0.9571	0.8750	0.8922	0.9053
	$\lambda=0.4$	0.9750	0.9880	0.9924	0.9250	0.9414	0.9519	0.8750	0.8884	0.8992
	$\lambda=0.6$	0.9750	0.9855	0.9904	0.9250	0.9368	0.9454	0.8750	0.8843	0.8922
	$\lambda=0.8$	0.9750	0.9816	0.9855	0.9250	0.9314	0.9368	0.8750	0.8798	0.8843
$\mathscr{D}_\infty$	$\lambda\in[0,1)$	0.9500	0.9500	0.9500	0.8500	0.8500	0.8500	0.7500	0.7500	0.7500

We first observe that in the degenerate case when $\lambda=0$, the results from the proposed moment decomposition approach are consistent with the exact analytical bounds from Theorem 1. For instance, with $b/\mu=0.1$ and $\epsilon=0.2$, the long chain achieves 99.11% of the expected sales guarantee of a fully flexible system according to both methods.

Beyond this consistency, the moment decomposition approach allows us to extend the insights from analytical bounds, showing that a long chain can capture a substantial portion of the benefits of a fully flexible system regardless of the extent of the possible supply disruption. When designed capacity matches expected demand, the long chain exhibits greater resilience to disruption compared to dedicated and fully flexible systems. This resilience is robust to demand variations, whether the disruption is partial or total (i.e., $\lambda = 0$). Another noteworthy observation is that the long chain retains a higher fraction of its benefits not only under increasing disruption probabilities (higher ϵ) but also when disruption extents are greater (lower λ). The intuition is consistent with the case of total disruption, where a fully flexible system becomes more deprived of effective capacity under disruption compared to a long chain.

5. Capacity configuration under disruption

The previous analysis has focused exclusively on the case where designed production capacity is equal to the expected demand, i.e., $C = \mu$. The moment decomposition approach allows for a more general designed capacity. In this section, we will investigate the impact of capacity expansion ($C \geq \mu$) on the asymptotic RE of a long chain compared to a dedicated system under ambiguity set $\mathcal{F}_0$. To this end, we partition the support set $\mathcal{Z}$ into five distinct scenarios, as illustrated in Figure 2-(b). We will further investigate the capacity required to attain a specific service level.

We start with examining a balanced system where the central planner adjusts the designed capacity according to the disruption probability, ensuring that the expected effective capacity equals the expected demand, i.e., $C = \mu/(1-\epsilon)$. This contrasts with the results presented Table 1, where $C = \mu$. According to Equation (6), the expected sales of the fully flexible system is upper bounded by $n\mu$. We therefore study the asymptotic RE of a graph $\mathscr{G}$ as the limit of $MF(\epsilon, \mathbb{P}, \mathscr{G})/(n\mu)$ as n tends to infinity. Table 5 presents the lower bounds on the corresponding asymptotic RE values for both the long chain and the dedicated system.

Table 5 shows that expanding designed capacity to mitigate supply disruptions yields greater benefits for a fully flexible system, as compared with a long chain or dedicated system. While the long chain still exhibits considerably greater resilience to supply disruptions than a dedicated system, our observations regarding the effects of disruption probability on the fraction of benefits captured by the long chain differ from those presented in §3.1. These findings underscore the high sensitivity of the effectiveness of a chaining system to designed capacity, particularly in the presence of supply disruption, raising the question: *how much capacity a long chain requires to achieve a given service level target?* Lyu et al. (2019) and Jiang et al. (2023) shed light on the capacity requirements of a long chain under a known demand distribution without considering the impact of supply disruptions. Their findings indicate that a long chain requires a capacity level that is almost equivalent to that of a fully flexible system to achieve a service level target. In contrast, a dedicated system requires substantially larger capacity. In the rest of this section, we extend the scope of their findings by investigating the capacity requirements in a distribution-free setting and incorporating the impact of supply disruptions.

Table 5: Lower bounds on the asymptotic RE of long chain and dedicated system under disruption ϵ when $C = \mu/(1-\epsilon)$.

		The ratio of mean absolute deviation (MAD) to mean (b/μ)									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mathscr{C}_\infty$	$\epsilon = 0.00$	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7750	0.7500
	$\epsilon = 0.10$	0.9363	0.9116	0.8874	0.8639	0.8407	0.8178	0.7951	0.7725	0.7500	0.7275
	$\epsilon = 0.20$	0.8948	0.8730	0.8514	0.8301	0.8091	0.7886	0.7685	0.7487	0.7292	0.7100
	$\epsilon = 0.30$	0.8472	0.8286	0.8102	0.7921	0.7744	0.7570	0.7400	0.7234	0.7071	0.6910
$\mathscr{D}_\infty$	$\epsilon = 0.00$	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
	$\epsilon = 0.10$	0.8600	0.8200	0.7800	0.7400	0.7000	0.6600	0.6200	0.5800	0.5400	0.5000
	$\epsilon = 0.20$	0.7700	0.7400	0.7100	0.6800	0.6500	0.6200	0.5900	0.5600	0.5300	0.5000
	$\epsilon = 0.30$	0.6800	0.6600	0.6400	0.6200	0.6000	0.5800	0.5600	0.5400	0.5200	0.5000

Notes. The asymptotic RE bounds for dedicated system $\mathscr{D}_\infty$ are tight, see Proposition 1.

5.1. A service level perspective

In the presence of supply disruption, we apply the moment decomposition approach to address the research question under a given performance metric κ (see Assumption 1) with a prespecified target β. Our analysis focuses on the capacity required $C^*_{\mathcal{F}}(\cdot)$, which is defined below

$$C^*_{\mathcal{F}}(\epsilon, \beta) \triangleq \min \left\{ C \mid \mathbb{E}_\xi \left[\mathbb{E}_{\mathbb{P}} \left[\kappa(C \cdot \xi, W, D) \right] \right] \geq \beta,\ \forall \mathbb{P} \in \mathcal{F} \right\}, \tag{20}$$

and solved using a bisection method over the interval $[\mu, 2\mu]$.

As outlined in Proposition 4, our SDP model provides a lower bound for the left-hand side of the constraint in Problem (20), thereby allowing us to derive an upper bound for $C^*_{\mathcal{F}}$. This upper bound, which we will refer to as the *capacity bound* in this study, signifies the capacity level that is sufficient to ensure a specified target β.

5.1.1. Infinite system. We begin our analysis by considering the capacity bound for achieving sales target β under $\kappa_2(\cdot)$, or equivalently a type-II service level of β/μ. For a fully flexible system, we need a capacity of $\max\{\beta/(1-\epsilon), \mu\}$ to achieve a sales target β. As for the dedicated system, Proposition 1 provides a tight bound on the minimum capacity requirement. Specifically, to achieve a sales target β where $\beta \leq (1-\epsilon)\mu$, the required capacity is

$$\max \left\{ 2\mu \cdot \left(1 - \frac{\mu - \beta/(1-\epsilon)}{b} \right), \mu \right\} \tag{21}$$

while achieving a sales target greater than $(1-\epsilon)\mu$ is impossible with arbitrarily large capacity.

Table 6 compares the capacity bounds of long chain with the tight bounds on the minimum capacity requirement in dedicated system under disruption (ϵ) and type-II service level target (β/μ). Clearly, both bounds increase with the service level target, disruption probability, and demand dispersion. In contrast, a fully flexible system's capacity requirement is only affected by the service level target and disruption probability.

Table 6: Capacity bounds (in units of μ) of long chain and dedicated system under disruption ϵ with type-II service level target β/μ.

			The ratio of mean absolute deviation (MAD) to mean (b/μ)									
	β/μ	ϵ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\mathscr{C}_\infty$	0.85	0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0588	1.1111	1.1580	1.2000
		0.05	1.0000	1.0000	1.0000	1.0000	1.0342	1.0945	1.1492	1.1992	1.2439	1.2825
		0.10	1.0000	1.0000	1.0122	1.0740	1.1355	1.1941	1.2483	1.2981	1.3447	1.3892
	0.90	0.00	1.0000	1.0000	1.0000	1.0000	1.0669	1.1255	1.1780	1.2249	1.2666	1.3035
		0.05	1.0000	1.0000	1.0381	1.1047	1.1675	1.2258	1.2783	1.3264	1.3713	1.4141
		0.10	1.0234	1.0791	1.1477	1.2188	1.2861	1.3477	1.4038	1.4563	1.5054	1.5508
	0.95	0.00	1.0000	1.0000	1.0784	1.1489	1.2124	1.2686	1.3181	1.3633	1.4053	1.4448
		0.05	1.0391	1.1177	1.2021	1.2808	1.3491	1.4087	1.4629	1.5127	1.5591	1.6011
		0.10	1.1685	1.2793	1.3799	1.4658	1.5430	1.6113	1.6729	1.7236	1.7646	1.7983
$\mathscr{D}_\infty$	0.85	0.00	1.0000	1.0000	1.0000	1.2500	1.4000	1.5000	1.5714	1.6250	1.6667	1.7000
		0.05	1.0000	1.0000	1.2982	1.4737	1.5789	1.6491	1.6992	1.7368	1.7661	1.7895
		0.10	1.0000	1.4444	1.6296	1.7222	1.7778	1.8148	1.8413	1.8611	1.8765	1.8889
	0.90	0.00	1.0000	1.0000	1.3333	1.5000	1.6000	1.6667	1.7143	1.7500	1.7778	1.8000
		0.05	1.0000	1.4737	1.6491	1.7368	1.7895	1.8246	1.8496	1.8684	1.8830	1.8947
		0.10	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
	0.95	0.00	1.0000	1.5000	1.6667	1.7500	1.8000	1.8333	1.8571	1.8750	1.8889	1.9000
		0.05	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
		0.10	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

Notes. The capacity bound for dedicated system is tight, see Equation (21); The notion "$*$" means 2μ, and "$\otimes$" means target unachievable.

We observe that long chain significantly outperforms dedicated system in terms of the minimum capacity required to achieve a specific service level target. Moreover, the capacity bound of a long chain approaches that of a fully flexible system under the same service level target when the demand dispersion is small and the disruption probability is relatively low. For example, if the ratio of the demand's MAD to its expected value is 0.2 and the service level is 0.9, a dedicated system needs 1.4737 and 2 times the expected demand, respectively, to hedge against supply disruptions of 0.05 and 0.1. In contrast, a long chain requires only 1 and 1.0791 times the expected demand, respectively, which is very close to the fully flexible system's requirement of μ. However, as these parameters (i.e., b/μ and ϵ) increase, a long chain requires additional capacity to buffer the uncertainties from the supply and demand sides. Nonetheless, even with higher levels of demand dispersion and disruption probability, a long chain can still achieve substantial capacity savings compared to a dedicated system. Consider $b/\mu = 1$ and $\beta/\mu = 0.9$, a long chain requires capacity of 1.4141μ and 1.5508μ to mitigate supply disruption with probability of 0.05 and 0.1, under which a dedicated system needs 1.8947μ and 2μ, respectively.

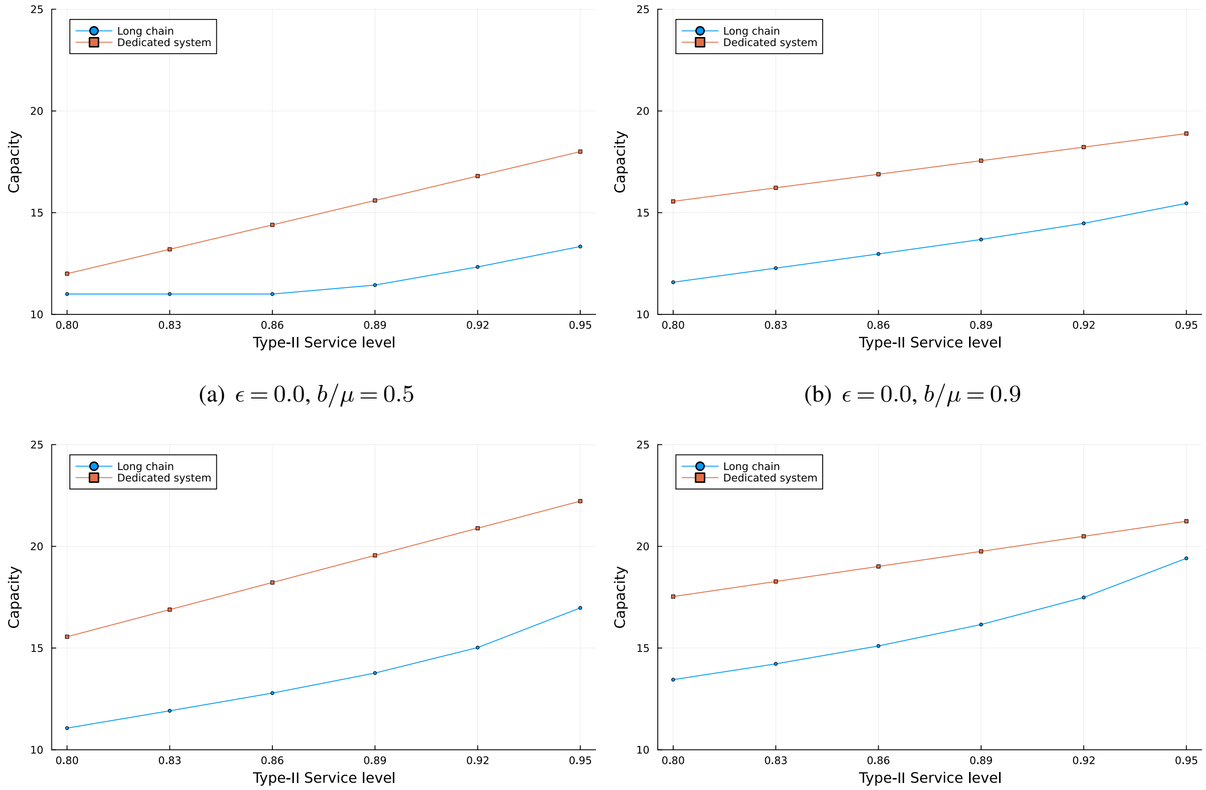
Notably, in a dedicated system, a stringent service level requirement may prevent the system from managing supply disruptions through capacity adjustments, whereas a long chain may still be able to do so. For instance, when $b/\mu = 0.1$ and $\epsilon = 0.1$, a dedicated system is unable to achieve a service level of 0.95 even with arbitrarily large capacity, and it requires a capacity twice the expected demand to achieve a service level of 0.9. We also observe that when $\epsilon = 0.1$, a long chain may require substantial capacity expansion to meet high service levels. In Appendix C, we analyze 3-chain performance under disruption and show that they can achieve similar service levels with lower capacity, consistent with the insight from Simchi-Levi et al. (2018) that 3-chains may be more effective than long chains in high disruption risk settings.

5.1.2. Finite system. While our focus remains to be steady state analysis, we note that an infinite system can guide the capacity configuration into a finite system. Specifically, we can allocate the additional capacity buffer of C to the first product node and reserve a capacity of C for each of the n plants in the long chain, ensuring that each product i receives a total of $W + C \cdot \xi$ units of supply. The following proposition presents a rigorous formulation of this rationale.

PROPOSITION 5. *Consider a balanced and symmetric long chain of size n. Suppose the uncertain demand D has mean μ and each plant has disruption probability ϵ, i.e., $\xi \sim \text{Bernoulli}(1-\epsilon)$. Suppose an infinite system ($n \to \infty$) with designed capacity $C \geq \mu$ can achieve a type-II service level β, then the long chain with an aggregated designed capacity of $(n+1)C$ can also achieve such type-II service level β.*

Figure 3 depicts the capacity bounds obtained from a balanced finite system with $m = n = 10$. This example demonstrates the applicability of Proposition 5 in extending the insights gained from an infinite system, as elucidated in the discussion following Table 6, to a finite system.

Figure 3: Total capacity bound of long chain and dedicated system for $m = n = 10$



(a) $\epsilon = 0.0$, $b/\mu = 0.5$

(b) $\epsilon = 0.0$, $b/\mu = 0.9$

(c) $\epsilon = 0.1$, $b/\mu = 0.5$

(d) $\epsilon = 0.1$, $b/\mu = 0.9$

5.2. Mitigating supply disruption with capacity expansion

The previous subsections also motivate us to consider how to mitigate supply disruption by expanding the designed capacity. Towards this end, we specify the target (β) according to the expected sales attainable by a configuration without disruptions (i.e., $\epsilon = 0$ and $C = \mu$), referred to as *disruption-free sales*. Particularly, for a long chain, the corresponding disruption-free sales is $\min_{\mathbb{P}\in\mathcal{F}_0} \mathbb{E}_{\mathbb{P}}\left[\kappa_2(\mu, W, D)\right]$, which admits an exact analytical form according to the Theorem 1. In the same vein, for a dedicated system, the disruption-free sales is $\mu - b/2$ according to Equation (7). We investigate the capacity expansion requirement that maintains these sales targets when a system is vulnerable to supply disruption.

We begin by examining the case of a dedicated system. Under supply disruption with probability ϵ, to achieve a sales target equal to the disruption-free sales, it is necessary to set the designed capacity to be $\bar{C}(\epsilon, b) \triangleq \mu \cdot (2\epsilon(\mu/b - 1) + 1)/(1 - \epsilon)$, provided that $\epsilon \leq b/2$, see Proposition 1. We highlight that, if ϵ exceeds $b/2$, the target cannot be achieved even with arbitrarily large capacity. Such capacity level $\bar{C}(\epsilon, b)$ is clearly increasing in ϵ, which is intuitive since a higher probability of disruption requires more capacity buffer to achieve the target. One notable observation is that $\bar{C}(\epsilon, b)$ exhibits a decreasing trend with respect to b, indicating that a higher capacity is needed to achieve the disruption-free sales when demand variation is low. This is because, in this case, the disruption-free sales is already very high, making it more challenging to maintain the service level by increasing the capacity. Thus, it is crucial to take into account the effect of demand variability on capacity requirements when designing supply chain systems. Furthermore, the relationship between ϵ and b in terms of their joint impact on capacity configuration (i.e., $\partial^2\bar{C}/\partial\epsilon\partial b \leq 0$) suggests that as demand variation decreases, the additional capacity required to maintain the sales target increases. Table 7 summarizes capacity bounds for dedicated system and long chain under supply disruption.

Table 7: Capacity bounds (in units of μ) of long chain and dedicated system under disruption ϵ with sales target β equals to the disruption-free sales.

		The ratio of mean absolute deviation (MAD) to mean (b/μ)									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	β/μ	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7750	0.7500
	$\epsilon=0.02$	1.0488	1.0469	1.0447	1.0422	1.0398	1.0376	1.0354	1.0337	1.0319	1.0303
	$\epsilon=0.04$	1.1060	1.0928	1.0881	1.0837	1.0796	1.0754	1.0714	1.0677	1.0645	1.0613
$\mathscr{C}_n$	$\epsilon=0.06$	1.1816	1.1445	1.1328	1.1260	1.1196	1.1138	1.1079	1.1025	1.0977	1.0930
	$\epsilon=0.08$	1.2842	1.2051	1.1824	1.1702	1.1609	1.1528	1.1453	1.1382	1.1316	1.1255
	$\epsilon=0.10$	1.4238	1.2793	1.2388	1.2188	1.2051	1.1941	1.1841	1.1750	1.1665	1.1587
	β/μ	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000
	$\epsilon=0.02$	1.3878	1.1837	1.1156	1.0816	1.0612	1.0476	1.0379	1.0306	1.0249	1.0204
	$\epsilon=0.04$	1.7917	1.3750	1.2361	1.1667	1.1250	1.0972	1.0774	1.0625	1.0509	1.0417
$\mathscr{D}_n$	$\epsilon=0.06$	$\otimes$	1.5745	1.3617	1.2553	1.1915	1.1489	1.1185	1.0957	1.0780	1.0638
	$\epsilon=0.08$	$\otimes$	1.7826	1.4928	1.3478	1.2609	1.2029	1.1615	1.1304	1.1063	1.0870
	$\epsilon=0.10$	$\otimes$	$*$	1.6296	1.4444	1.3333	1.2593	1.2063	1.1667	1.1358	1.1111

Notes. The capacity bound for dedicated system is tight, see $\bar{C}(\epsilon, b)$; The notion "$*$" means 2μ, and "$\otimes$" means target unachievable.

We observe a similar trend for the long chain. However, the additional capacity required to recover disruption-free sales is much lower than that of a dedicated system under the same disruption probability when demand variation is low. As an example, consider the case of $\epsilon = 0.02$ and $b/\mu = 0.1$. In this scenario, a long chain requires an additional 4.88% capacity to achieve a 97.50% service level, while a dedicated system requires a much higher increase of 38.78% capacity to guarantee a 95.00% service level. Although the long chain requires slightly more capacity to ensure the disruption-free sales than the dedicated system when the demand variation is high, we observe that the disruption-free sales for the long chain is significantly higher. For instance, when $\epsilon = 0.02$ and $b/\mu = 1$, a long chain requires an additional 3.03% capacity to achieve a 75% service level, while a dedicated system requires an additional 2.04% capacity but can only guarantee a 50% service level. In fact, to achieve a 75% service level, a dedicated system needs a much larger increase of 66.67% capacity under this disruption probability and demand variation.

6. Concluding remarks

This paper advances the process flexibility literature by analyzing the long chain's performance under both demand uncertainty and supply disruptions. We establish performance guarantees for long chains under disruption, showing that when designed capacity matches expected demand and demand lies within a MAD-based ambiguity set, expected sales guarantees can be derived in closed form. To generalize the results, we propose a moment decomposition approach and demonstrated its tractability through an SDP reformulation, enabling new applications and insights. In particular, explore richer ambiguity sets with higher-order moment information, derive type-I service level guarantees, and introduce a partial disruption model. We also apply the method to capacity configuration, quantifying how capacity expansion mitigates disruption under demand uncertainty.

Our findings shed new light on the benefits of process flexibility, especially in managing supply disruptions. When the designed capacity equals expected demand, long chains outperforms both dedicated and fully flexible systems in terms of system resilience. In disruption-free settings, long chains require capacity comparable to fully flexible systems to meet service levels, but disruption significantly increases this requirement. These results highlight the importance of accounting for disruption risk in capacity planning and the value of capacity expansion as a mitigation strategy.

These additional aspects of long chain analysis have been underexplored in the literature and are not fully addressed by existing frameworks for analyzing long chains (e.g., Wang and Zhang (2015)). This gap motivated our investigation, albeit within a stylized setting. The success of the moment decomposition approach relies on the asymptotic analysis of long chains in an infinite homogeneous system. In non-homogeneous systems with unequal capacities and varying demands, numerous factors, such as plant capacities and demand distributions, complicate the problem, making it difficult to derive a steady-state characterization. Moreover, the proposed moment decomposition approach involves solving an SDP relaxation, whose size grows exponentially with the number of random variables (see Equation (17)). The introduction of multiple random variables renders the SDP too large for current solvers. Consequently, we focused on the more tractable homogeneous system setting, which, while extensively studied in the literature, remains challenging to analyze.

We acknowledge, however, that the proposed moment decomposition approach may not be the most suitable starting point for extending the analysis to non-homogeneous systems. Future research could develop new methodologies to address systems with unequal plant capacities and varying product demands, paving the way for more comprehensive frameworks to understand the role of process flexibility in mitigating supply disruptions.

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