fractal and fractional

Editorial

Fractal and Fractional Theories in Advancing Geotechnical Engineering Practices

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Fractal and fractional theories have developed over several decades and have gradually grown in popularity, with significant applications in geotechnical engineering [1,2]. The contours and pore structures of geotechnical materials often exhibit complex morphologies and scales that fractal theory can effectively describe. Experimental techniques such as scanning electron microscopy (SEM), computerized tomography (CT) scanning [3,4], and mercury intrusion porosimetry (MIP) can be used to obtain pore structure data, which can then be quantitatively characterized based on fractal dimensions and other parameters to capture the complexity and connectivity of the pores. For example, because fractal theory can accurately represent the irregular geometry of natural soils and matches well with their complex structures, it has been applied in predictive models of the soil–water characteristic curve (SWCC) for unsaturated soils [5]. Moreover, fractal theory can be applied to study the distribution and complexity of particle contact networks [6], aiding in understanding the interaction mechanisms between particles and their influence on the macroscopic mechanical behavior of geotechnical materials. Furthermore, traditional soil constitutive models often fall short in describing the complex mechanical behaviors of geotechnical materials. In contrast, fractional-order constitutive models can more precisely capture the nonlinear and memory effects inherent in these materials [7]. For instance, fractional constitutive models can effectively describe the mechanical behavior of geotechnical materials under long-term loading [8], thereby providing more reliable tools for long-term stability analysis in geotechnical engineering.

This Special Issue, "Fractal and Fractional in Geotechnical Engineering", focuses on exploring and applying the critical roles of fractal and fractional theories in geotechnical engineering design. This collection features 13 papers addressing the application of fractal and fractional concepts in geotechnical engineering and construction materials, covering topics such as rock fracture behavior, elastic wave propagation, slurry seepage theory, single-particle breakage, artificial frozen soil, and tunnel settlement prediction. An overview of these papers is provided below.

Shi and Xiao [9] investigated how fractal dimensions affect the mechanical properties and fracture behavior of multi-mineral rocks using three numerical models, i.e., digital texture, Voronoi polygon, and Weibull distribution. Particle flow simulations and uniaxial compression tests were performed on 2D and 3D models, respectively. The results indicated that the Weibull model had the highest fractal dimension and complexity, while the Voronoi model had the lowest, as well as a more regular structure. Compressive strength increased roughly linearly with fractal dimension in all models. Additionally, 3D models showed higher strength than 2D models, but the relationship between fractal dimension and strength followed a similar trend in both cases. This highlights the importance of fractal dimensions in describing rock geometry and mechanics.

Wang and Zhang [10] conducted a numerical study on elastic wave propagation in two-dimensional fractional Brownian fields, examining the effects of the Hurst exponent and standard deviation on wave behavior. Their results demonstrate that higher standard deviation and lower Hurst exponent increase wavefront roughness, cause asynchronous arrivals, and amplify energy attenuation due to stronger scattering and greater modulus variability. This study enhances our understanding of wave propagation in fractal heterogeneous media, with significant implications for seismic exploration and subsurface imaging in complex geological formations.

Gong et al. [11] developed a theoretical penetration grouting model for Bingham fluids based on fractal theory, incorporating the influence of pore structure on slurry infiltration. The model was validated experimentally using a custom apparatus simulating constant flow rate penetration. The results showed strong agreement between theoretical predictions and experimental data, demonstrating the model's effectiveness in describing slurry pressure distribution during grouting. This work provides valuable guidance for grouting design and related engineering applications.

Li et al. [12] developed a comprehensive framework integrating experiments and numerical simulations to study single-particle breakage using 3D particle reconstruction based on a vision foundation model. Their calibrated discrete element method simulations accurately replicated particle breakage behavior. The study revealed strong correlations between 3D fractal dimensions and particle size, crushing strength, and morphology, highlighting fractal dimension as a valuable descriptor of particle properties. This framework advances the understanding of particle breakage mechanics and provides a robust tool for future research.

Sun et al. [13] analyzed highway slope surface displacement using multifractal detrended fluctuation analysis (MF-DFA) and developed a particle swarm optimization (PSO)-optimized long short-term memory (LSTM) model for prediction. Their study revealed multifractal characteristics in the displacement data and classified the slope warning levels. The PSO-LSTM model demonstrated high predictive accuracy. These findings support routine monitoring and provide a strong foundation for improved slope safety management.

Kong et al. [14] investigated the microstructural and fractal characteristics of frozen–thawed sandy soft soil using nuclear magnetic resonance and uniaxial compression tests. Their study demonstrated that freezing temperature and sand content significantly affect pore structure complexity and soil strength, with the fractal dimension serving as a key indicator. Lower freezing temperatures increase fractal dimension and soil strength, while higher sand content enlarges pores and reduces strength. These findings offer valuable insights into freeze–thaw effects on soil stability in soft soil regions.

Liang et al. [15] employed DEM numerical simulations to investigate soil arching mechanisms in granular materials with varying relative densities. They identified three zones based on particle displacement and found that soil arching strength and force chain development are strongly influenced by particle assembly porosity. Denser samples exhibited stronger arching and more pronounced force chain evolution, particularly in the intermediate displacement zone. After reaching peak arching, force chains degraded, leading to a reduction in arching. Ultimately, soil arching at the limit state was found to be independent of relative density. This study highlights the crucial role of contact force chains in soil arching behavior.

Yu and Yin [16] investigated the convergence of stiffness operators and viscoelastic properties in fractal ladder and tree structures. They demonstrated that sequences of stiffness operators converge and that finite-level fractal structures beyond the third hierarchy exhibit behavior similar to infinite-level fractals, simplifying analysis. The study revealed characteristic ultra-long creep times and relaxation tailing effects, underscoring the effectiveness of fractal models in capturing complex viscoelastic behavior. Their work bridges continuous and fractal models, providing valuable insights for materials science and mechanical engineering.

Wang and Wang [17] conducted in situ industrial CT scans to study crack evolution in coal under varying confining pressures, using fractal theory to quantitatively describe crack development. They identified consistent crack evolution stages across pressures and found that fractal dimension changes correlate well with crack dynamics, showing an initial slight decrease followed by significant growth. Higher confining pressures limit crack development and enhance the mechanical strength of coal, primarily leading to shear failure. This study offers a reliable fractal-based method of assessing coal fracture evolution and predicting instability.

Yang et al. [18] applied multifractal theory and multifractal detrended fluctuation analysis (MF-DFA) to characterize tunnel deformation data. They integrated Mann–Kendall analysis to establish dual early warning criteria and employed a particle swarm optimization–long short-term memory (PSO-LSTM) model to predict tunnel settlement. The results showed consistent Class II warning levels across sections and demonstrated that the PSO-LSTM model delivers accurate and stable settlement predictions. This study supports enhanced monitoring and disaster preparedness through quantitative analysis.

Zhang et al. [19] investigated the optimization of peripheral hole charging structures and blasting parameters for extra-long hard rock tunnels to enhance the effectiveness of smooth blasting. By employing laser profiling and multifractal detrended fluctuation analysis (MF-DFA), they compared bidirectional shaped charge blasting with spaced decoupled charge blasting. The results demonstrated that bidirectional shaped charge blasting significantly improved smooth blasting performance, yielding flatter tunnel contours and more uniform overbreak and underbreak. Both blasting methods exhibited multifractal characteristics in overbreak data, with quantitative analyses closely aligning with actual conditions.

Zhang et al. [20] developed an innovative energy calculation method to identify the key factors contributing to rock bursts by comparing elastoplastic and purely elastic models. Their study reveals that a deviatoric stress field, which induces a butterfly-shaped plastic zone, is the primary driver of significant energy release and rock burst occurrence, with trigger stress playing a secondary role. Laboratory experiments employing acoustic emission monitoring validate the butterfly-shaped failure pattern and reveal fractal characteristics in the spatial distribution of events prior to failure. These findings offer a clearer and more quantifiable understanding of the mechanisms underlying rock bursts.

Li et al. [21] developed a dilatancy equation that accounts for the non-coaxiality between the stress and strain rate in granular soils by introducing a novel non-coaxial coefficient grounded in potential theory and material fabric characteristics. This approach establishes a link between plastic strain and the microstructural fabric, enabling the equation to reduce to the classical critical state theory under isotropic conditions while effectively capturing non-coaxial effects under anisotropic conditions. Validation through simple shear tests demonstrates that the equation reliably predicts dilatancy behavior, particularly under principal stress rotation and varying stress ratios.

This collection of 13 papers is expected to inspire new ideas in advancing research on fractal and fractional theories, as well as promoting their broader application in geotechnical engineering. This will facilitate a better characterization of the complex microstructures and mechanical properties of soils, rocks, and structures such as tunnels, ultimately supporting the development of improved design methods and practical tools.

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