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HOW CAN THE COMMISSION MECHANISM OPTIMIZE SUPPLY CHAIN IN E-COMMERCE? EXPLORING THE IMPACT OF QUALITY IMPROVEMENT AND SPILLOVER EFFECT

Li Cui[1], Shuai Du[1], Ke Yan[1,*], T.C.E. Cheng[2] and Jing-Xin Dong[3]

Abstract. Along with the accelerating development of live streaming, the enterprises are increasingly utilizing not only their own online shops, but also cooperate with the celebrities who showcase products in live broadcasts. To explore the impact of quality improvement and the spillover effect of live streaming on the optimal decisions of the enterprise and the celebrity, we construct the Stackelberg game to research decentralized and centralized decision-making structures. Additionally, we consider the role of the commission rate, treated as an exogenous variable, in coordinating the two sales modes (*i.e.*, live streaming and online store). We find that the commission rate has the beneficial impact on the optimal price and profit for supply chain participants, when the spillover effect of live streaming is relatively high under the decentralized decision-making structure. We show that the cost-sharing joint commission mechanism in the quality improvement model of decentralized decision-making will establish the coordination within the two sales modes, leading to the win–win situation for all parties involved, as well as increasing the system profit. Finally, the numerical experiments show that the increase in the commission rate will reduce the celebrity's sharing proportion of quality improvement cost, thereby reducing the sharing proportion of the supply chain system profit.

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1. Introduction

With the rapid development of the Internet, there has been a significant rise in the popularity of live streaming e-commerce. This new online marketing model involves selling products through the dynamics and fashion endorsements of celebrities on social media platforms [33,37]. The distinctive features of products from numerous enterprises make them exceptionally-suited for live streaming e-commerce. Therefore, many enterprises adopt the live streaming mode while operating the online shops.

The celebrities promote products in a vivid and interactive manner, showcasing their style, functionality and experience of use, and offering consultations, thereby stimulating consumer interest in live streaming [21,34,83].

Keywords. Supply chain management, quality improvement, spillover effect, supply chain coordination, game theory.

[1] Business School, Beijing Technology and Business University, No. 11, Fucheng Road, Haidian District, Beijing 100048, P.R. China.

[2] Dept Logist & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, P.R. China.

[3] Business School, Newcastle University, 5 Barrack Road, Newcastle Upon Tyne NE1 4SE, UK.

*Corresponding author: keyan@btbu.edu.cn

The live streaming e-commerce has engendered immense product exposure, a sharp increase in the demand for products, and a large audience base for product promotion [8, 37]. For instances, Nailiang Jia, a popular streamer on the TikTok platform, set the record by selling 50 thousand bags of rice within just half an hour. When the live broadcast ends, numerous consumers will be attracted to the enterprises' own online store to purchase goods due to the spillover effects. This will lead to transform the purchasers in live streaming rooms into loyal customers of online shops, thereby boosting online sales of products [50]. According to the Ministry of Commerce and All China Federation of Supply and Marketing Cooperatives, online sales of products have witnessed the remarkable 166% increase from 2016 to 2021, reaching 422.1 billion RMB and accounting for 15.3% of total product sales.

Considering the spillover effect of live streaming, the enterprises are increasingly engaged in collaborating with celebrities to enhance the sales of their online shops. They employ the two sales strategy, leveraging both their dedicated online shops without celebrity endorsements, and live streaming sales mode collaborating with the celebrities. Simultaneously, the celebrities conduct the quality inspections, showcase the products, provide tests, and offer evaluations from a consumer's perspective in live streaming rooms, influencing and guiding the viewers' purchase intentions [10].

However, there are several serious issues that arise in the collaboration between the enterprises and celebrities. Recently, the frequent quality and safety incidents of products, such as "Saccharin jujube" incident [74], "Problematic cucumbers" incident [36], and Austin Li's "non-stick pan" incident [62] and Luo Yonghao's "5–20 rose gift box" incident [76], having the made consumers increasingly concerned about the quality of the product. However, investing more in enhancing the quality of products for enterprises often leads to increased costs. The decision on whether to improve product quality becomes a major concern for these enterprises. Moreover, the enterprises are obligated to share a portion of their sales revenue with celebrities as a commission, typically fluctuating around 30%, while the net profit margin of products is only around 10% [19]. Given these financial pressures, several scholars have recognized the commission rate as an exogenous variable in supply chain coordination, as it is often determined by external factors such as platform policies or market competition, rather than by the internal decisions of the enterprises involved [15, 23, 73, 77]. Treating the commission rate as exogenous allows for a clearer analysis of its impact on the coordination between manufacturers, retailers, and celebrities, particularly in aligning their incentives and optimizing overall supply chain performance. Additionally, the price of products in live streaming is expected to be the lowest compared to other channels. Consequently, most enterprises find it challenging to offset the high costs associated with celebrity endorsements in live streaming e-commerce. If the actual spillover effect of live streaming falls significantly below the expectations of enterprises in terms of promotion and advertising, they may face substantial losses. Previous literature studies have primarily focused on offline and traditional e-commerce channels in the of dual-channel supply chains, neglecting the field of live-streaming e-commerce [49,60,61,75,81]. Furthermore, few studies have examined the spillover effects of live-streaming and manufacturers' quality improvement strategies in the realm of live-streaming e-commerce [20, 35, 37, 41, 46].

To fill in these gaps, we consider inclusion of the spillover effect of live streaming in the two sales modes which involves enterprise and celebrity. Enterprises determines level of quality improvement effort. Based on the aforementioned analysis, we have identified the following research questions: (i) Do employ quality improvement strategies while considering the spillover effects of live streaming? (ii) How does the commission rate impact the optimal decision-making process of the supply chain? (iii) What constitutes the most effective coordination mechanism for the supply chain?

To address these issues, we propose a decision-making and coordination framework for a dual-channel supply chain combining live streaming and online stores, which includes a comparison between the basic model (*i.e.*, without quality improvement) and the quality improvement model that considers live streaming spillover effects. Additionally, with the consideration of celebrities' sales efforts to promote products, we analyze the optimal decisions of all stakeholders by comparing two models with decentralized and centralized decision-making structures. We also investigate the impact of the commission rate and spillover effect. Finally, we consider the commission rate as an exogenous variable and design a coordination mechanism for the quality improvement model, which includes a cost-sharing joint commission.

Our main contributions can be summarized as follows: (i) Given the limited research on the spillover effect in live streaming, we incorporate this effect into the two sales mode (*i.e.*, live streaming and online store), considering the quality improvement strategy of enterprises. (ii) We apply and extend the coordination mechanism of cost-sharing joint commission to the field of live streaming e-commerce, further exploring the influence of the commission rate on the coordination mechanism. (iii) We aim to provide a theoretical foundation for quality strategies and the coordination mechanism in dual-channel supply chains, considering the spillover effect of live streaming.

We also offer the valuable management insights to supply chain members. Enterprises must prioritize quality-centric operational governance by implementing consumer-centric investments that strategically balance premium positioning with market share objectives, particularly across heterogeneous consumer sensitivity profiles. Simultaneously, it is critical to rigorously evaluate potential anchors that can make authentic connections with viewers and products. For celebrities, dynamically adjusting sales efforts and incorporating interactive elements (*e.g.*, real-time Q&A, personalized recommendations) into live broadcasts can enhance consumer experiences and strengthen audience engagement. Furthermore, both parties should institutionalize collaborative contractual arrangements such as cost-sharing mechanisms, to maximize operational synergies and achieve mutual profit maximization.

The rest of the paper is organized as follows: In Section 2, we review the literature to provide an overview of previous research and establish the direction of our study. In Section 3, we develop the basic model and the quality improvement model under decentralized and centralized decision-making structures. In Section 4, we compare the results of the different models and discuss the influence of the commission rate on the supply chain. In Section 5, we propose a coordination mechanism for the quality improvement model within a decentralized decision-making framework. In Section 6, we conduct numerical analysis to validate our models. In Section 7, we have expanded our model. In Section 8, We offer corresponding management insights. Finally, in Section 9, we summarize our key findings and discuss the managerial implications of our research.

2. Literature review

Our study is located at the intersection of three categories of literature: Spillover effects of dual-channel supply chains, quality management in the supply chain of products, and coordination contract in the supply chain.

2.1. Spillover effects of dual-channel supply chains

Dual-channel supply chains include both traditional offline dual channels and online-offline dual channels [27]. Additionally, there is the integrated online-offline dual-channel model. The evolution of dual-channel supply chain research has reflected technological advancements and shifting market demands, progressing through distinct phases. Initially, studies focused on traditional offline dual-channel systems where manufacturers coordinated with retailers, addressing operational challenges such as channel coordination [6, 69], pricing strategies [17], and channel conflict [42]. Chen *et al.* [6] studied coordination schemes in dual-channel supply chains and found that contracts involving the manufacturer's wholesale price and the direct channel price could effectively coordinate both supply channels, enabling a win–win situation for both the manufacturer and the retailer. Perlman [42] examines the impact of showrooming and identifies specific levels of showrooming at which a retailer may decide to exit the dual-channel system, despite this choice being disadvantageous for both parties involved. With the rise of e-commerce, the integration of online and offline channels has emerged as a pivotal research domain in dual-channel supply chain management. This phase of research has highlighted issues like channel coordination [57, 61, 74, 81], pricing strategies [18, 38, 58] and return policies [18, 48]. Xiao and Shi [58] explored pricing and channel priority strategies in dual-channel supply chains under conditions of supply shortages due to random yield rates, investigating the impact of channel coordination and the sequencing of decisions on channel priority strategies. Similarly, Pal [40] examined the influence of retail services on profits and pricing decisions under different return strategies. Zhang *et al.* [74] considered a multi-manufacturer, multi-retailer dual-channel network, analyzing the influence of key factors on equilibrium states and profits.

As e-commerce has grown, so has the importance of spillover effects within dual-channel supply chains, with research expanding to cover cross-channel impacts and their implications on decision-making. The spillover effect refers to the influence of one channel's actions on the other, shaping decisions across a range of variables, from pricing to inventory management. For instance, Alaei *et al.* [1] investigated the impact of cross-channel spillovers on channel choice and profit sharing when manufacturers adopt online shops, marketplaces, and distributor channels. Zhen *et al.* [80] further examined bidirectional spillover effects (offline-online and online-offline), finding that retailers' third-party platform adoption and selling format choices are influenced by spillover direction and intensity, with usage peaking at moderate levels of spillovers. The spillover effect has been explored in various contexts, though its application in live-streaming environments is still emerging. Fan *et al.* [13] examined how live streaming's spillover effects on purchase intentions for related products are influenced by real-time interactions with influencers, which boost consumer trust and indirectly drive sales in online stores. Yang *et al.* [71] compared spillover impacts under three cooperation scenarios – no Key Opinion Leader (KOL), manufacturer-KOL, and retailer-KOL partnerships – finding that manufacturers consistently prefer direct KOL collaborations to maximize spillover benefits, regardless of consumer acceptance levels. This shift has important implications for the dual-channel supply chain, as KOL collaborations in live streaming not only influence consumer behavior but also reshape supply chain strategies by amplifying spillovers.

Despite these advances, most studies focus primarily on sales or pricing outcomes, often overlooking how spillovers might influence strategic decisions like quality improvement. Ghosh *et al.* [17] incorporated carbon regulations and consumer preferences into dual-channel models but neglected the role of spillovers in driving eco-innovation. Similarly, Zhang *et al.* [75] explored reference price effects on channel performance without linking spillovers to long-term quality investments. This gap underscores the need for future research to explore how spillover effects can be leveraged to enhance quality improvement initiatives, potentially optimizing overall supply chain performance in e-commerce.

2.2. Quality management in the supply chain of products

The quality management of products has become a popular topic in current academic research and has received significant attention from researchers. Existing research have primarily focused on factors that influence the product quality improvement [44,72], strategies for disclosing quality information [22], supply chain decision-making with deteriorating products [4, 45, 59, 68], and product quality strategy under different channel power structures [56, 64], and the effect of a quality label on purchase intention [14].

However, these studies have been based on traditional supply chain and have not taken into account the dual-channel supply chain or discussed the influence of quality improvement strategies on supply chain profits. For instance, Xiao and Xu [59] examine the coordination strategy of a traditional supply chain under vendor-managed inventory (VMI), considering the deterioration of products. Cambra-Fierro *et al.* [5] offer a unifying framework to understand the linkages between customer equity, experience quality and non-transactional behaviors in an e-commerce context. Hong *et al.* [22] explored the quality information disclosure strategies of green manufacturers after acquiring quality information in a closed-loop supply chain. Cai *et al.* [4] coordinated a three-tier fresh produce supply chain, consisting of manufacturers, third-party logistics providers, and distributors, by designing two contracts: Wholesale Market Clearing (WMC) and Wholesale Price Discount Sharing (WDS). These contracts addressed the double marginalization problem and achieved the overall supply chain's optimal decision-making. Fan *et al.* [14] investigate the impact of traceability on quality and price competition in a duopoly. Xu *et al.* [64] investigated optimal supply quality and price subsidy strategies for a traditional supply chain consisting of a producer and a seller under different channel power structures. Ho Yoo and Cheong [72] investigated several incentive mechanisms for collaborative product quality improvement in a buyer-driven supply chain, and the impacts of those mechanisms on supply chain performance. Wang *et al.* [52] considered an outsourcing logistics channel and investigate the impact of logistics service quality on distributors' ordering decisions. Chen *et al.* [7] investigated price and quality decisions in dual-channel supply chains, and emphasized the effects of the quality sensitivity parameters of different channels on price and product quality, as well as profits and consumer surplus.

With the development of the economy and the increasing improvement of people's consumption level, consumers have become widely concerned about the quality and safety of products [64, 68]. As a new type of product sales mode, live streaming e-commerce does not allow consumers to directly interact with products and evaluate product quality as traditional channels do [14]. Therefore, enterprises need to invest in product quality improvement to reduce consumer uncertainty regarding the quality of products.

2.3. Coordination contract in the supply chain

The coordination contract in the supply chain has received considerable attention in the existing research for the purpose of coordinating decentralized supply chain, managing risks, and optimizing performance. The research on coordination contracts mainly focuses on cost-sharing contracts [26, 30, 47, 52], revenue-sharing contracts [9, 28, 43, 61, 66, 67, 70], and two-part tariff contracts [3, 54, 69, 79]. Wang *et al.* [52] design cost-sharing and wholesale premium contracts for the traditional low-carbon supply chain to address carbon emission reduction. Bai *et al.* [3] proposed a revenue and promotional cost-sharing contract, as well as a two-part tariff contract, to coordinate sustainable supply chain systems. In contrast, Lan and Yu [28] utilized the group leader's commission as a coordination tool, and by combining it with the traditional revenue-sharing contract, they designed a revenue-sharing-commission coordination contract to achieve coordinated operation within the community group-buying supply chain. Similarly, Yang and Chen [70] examined the impact of retailers on manufacturers' carbon emission reduction investments and supply chain profits through revenue sharing (RS), cost sharing (CS), double sharing (B), or no sharing (N) mechanisms, considering consumer environmental awareness and carbon tax policies. On the other hand, Yan *et al.* [66] explored how both the manufacturer's and retailer's fairness concerns influence the fresh agricultural product supply chain within the framework of Nash bargaining. Lastly, Liu *et al.* [30] delved into the supply chain coordination challenges arising from the One Belt, One Road (OBOR) initiative, investigating the impact of cost-sharing contracts on key decisions in logistics service supply chains focused on mass customization. However, to the best of our knowledge, the coordination contract for the new sales mode, which includes the online shop channel and the live streaming channel, has been rarely studied by scholars, despite extensive research on the coordination of various supply chain systems. Xu *et al.* [63] proposed a contract the two-way revenue sharing contract and demonstrate by coordinating the dual-channel supply chain with risk-averse, and analyzed that how the risk attitude changes the parameters of the coordinating contract. Chen *et al.* [10] prove the feasibility of sharing the R&D costs of the green degree and sales effort costs of the advertisement contract through bargaining problems achieving a win–win situation. And sharing the R&D costs of the green degree and sales effort costs of the advertisement contract can not only help to improve the green degree and the price of the product, but also improve the profitability of all supply chain members.

Current research on supply chain coordination contracts primarily focuses on traditional dual-channel models (online-offline). However, the unique dynamics of live streaming e-commerce, particularly the integration of online shops and live streaming sales mode, remain underexplored.

Existing literature has partially explored the use of exogenous commission rates for supply chain coordination. For instance, He *et al.* [20] investigates how different contract types, with an exogenous commission rate, influence the coordination and profit-sharing between manufacturers, retailers, and streamers within a supply chain. Similarly, Zhang and Tang [73] examines the impact of the exogenous commission rate on the manufacturer's decisions regarding live-streaming modes, pricing, and promotional efforts in a dual-channel supply chain. In contrast, Chen *et al.* [11] focuses on the role of the exogenous commission rate in coordinating pricing and sales efforts between manufacturers and internet celebrities, considering various power structures within a live-streaming supply chain.

Moreover, Zhou *et al.* [82] explores how the commission rate of live-streaming platforms influences their decision to promote products exclusively or nonexclusively, showing that a higher commission rate may benefit

Table 1. Related studies and positioning of this paper in the literature. The contribution of our work (✓: covered).

Literature	Supply chain structure	Dual channels	Two sales mode	Quality management	Coordination contract	Spillover effect
[1]	Two manufacturers one e-tailer	✓				✓
[13]	One manufacturer one retailer		✓			✓
[2]	One manufacturer one retailer	✓				✓
[20]	One manufacturer one retailer one streamer		✓		✓	
[44]	One manufacturer one retailer	✓		✓		
[78]	One manufacturer one retailer	✓			✓	
[24]	One manufacturer one retailer		✓			
[65]	One manufacturer one retailer	✓				✓
[72]	One manufacturer one buyer	✓		✓		
[3]	One manufacturer one retailer	✓			✓	
[47]	One manufacturer one retailer	✓			✓	
[80]	One retailer and one third party platform	✓	✓			✓
[31]	One supplier,an online retailer	✓	✓			✓
[82]	live-streaming platforms and traditional retail platforms		✓		✓	
[73]	A manufacturer and a live-streaming selling platform		✓		✓	
Our work	A manufacturer a celebrity		✓	✓	✓	✓

the brand under exclusive promotion, while nonexclusive promotion can favor traditional platforms under certain conditions. In addition, Huang *et al.* [23] examines how commission rates and mismatch costs affect competing retailers' decisions to introduce live-streaming channels, finding that an endogenous commission rate can prevent a prisoner's dilemma and influence the equilibrium strategy between retailers. Furthermore, Jiang *et al.* [25] analyzes the optimal live-streaming choices for online retailers, revealing that influencer commission rates and fixed fees negatively affect the retailer, but a profit-sharing mechanism can improve the outcomes for both the retailer and the influencer. Finally, Zhang *et al.* [77] models the impact of commission rates and fixed signing bonuses on a manufacturer's choice between merchant and influencer live-streaming modes, demonstrating that these rates influence retail prices and promotional efforts, and that the hybrid live-streaming mode is preferable to both the manufacturer and platform under specific conditions.

Addressing these gaps, our study proposes live-streaming spillover effects into a game-theoretic framework to analyze the interaction between live streaming demand and quality improvement incentives, while introducing quality improvement strategies into our model. We further propose an innovative cost-sharing joint mechanism, incorporating the exogenous commission rate, that allocates quality improvement costs between enterprises and celebrities. This framework differentiates short-term operational decisions aiming to optimize channel synergies and maximize mutual profitability. By addressing both the direct and indirect effects of live streaming, this approach not only enhances the operational efficiency of the supply chain but also improves long-term sustainability. This approach advances strategic quality management in digitalized supply chains by integrating promotional spillovers with contractual mechanisms.

To emphasize our contribution, we have summarized the most relevant papers in Table 1.

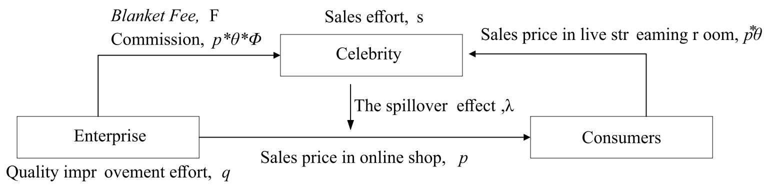

FIGURE 1. Dual-channel supply chain structure diagram.

3. Model description

In this study, we examine the two sales modes (*i.e.*, live streaming and online store) that involves an enterprise and a celebrity, centered on an innovative live streaming-commerce framework [15, 16, 51]. The supply chain facilitates the sale of a specific product through both an online shop and a live streaming room, with the latter serving as a "live streaming + online store" for real-time transactions and immersive brand promotion.

The product is priced at p in the online shop and at $p\theta(0<\theta<1)$ in the live streaming room [32, 77]. The celebrity makes sales effort (s) by actively engaging with viewers in the live streaming room, providing detailed information about the product, and increasing the viewers' purchase intention. The sales effort in turn indirectly boosts the demand for the product in the online shop, creating a spillover effect. The coefficient λ is used to quantify this spillover effect. Under the cooperation arrangement, the enterprise pays a "blanket fee" (F) that can range from a few to hundreds of thousands [20]. Additionally, they share a proportion (ϕ) of the revenue with the celebrity as a commission, a percentage that generally hovers around 30%. Since we focus on analyzing the short-term operational decisions of the supply chain system in this paper, we set the commission rate as an exogenous variable, following the approach adopted by several scholars [23, 73, 82]. We compare and analyze the optimal decisions made under different models of decision-making, including centralized and decentralized approaches. Furthermore, we differentiate between two models: the basic model by subscript N, which does not include any quality improvement efforts, and the quality improvement model denoted by subscript Y, which accounts for initiatives to enhance the product's quality. The enterprise makes quality improvement effort (q) through investing in advanced equipment, refining production technology, ensuring production standardization, and implementing quality monitoring measures. For simplicity, in the remainder of this study, we refer to the enterprise as male (using pronouns *he* and *his*) and the celebrity as female (using pronouns *she* and *her*). Figure 1 illustrates the structure of the supply chain.

According to the research conducted by Yan *et al.* [69], the requirements of the enterprise and celebrity can be represented as linear functions of the product price in the online shop, the enterprise's quality improvement effort, and the celebrity's sales effort. Thus, the demand functions for the online shop channel (D_1) and live streaming channel (D_2) in the basic model and quality improvement model are respectively given as follows:

$$D_1^N = (1-\gamma)\alpha - ep + \lambda s \tag{1}$$
$$D_2^N = \gamma\alpha - ep\theta + s \tag{2}$$
$$D_1^Y = (1-\gamma)\alpha - ep + gq + \lambda s \tag{3}$$
$$D_2^Y = \gamma\alpha - ep\theta + gq + s. \tag{4}$$

In the equations (1)–(4), α represents the potential market demand online. The parameter γ denotes the proportion of products sold in the live streaming, γ $(0<\gamma<1)$ and $1-\gamma$ represents the proportion of products sold through the online shop. The parameter e and g signifies customers' price sensitivity and quality sensitivity.

We define the cost for the enterprise's quality improvement effort as $C(q) = \frac{k_1 q^2}{2}$, and the cost for the celebrity's sales effort as $C(s) = \frac{k_2 s^2}{2}$. Previous studies, such as those conducted by Ofek *et al.* [39], Zhang *et al.* [74] and Liu *et al.* [30], have explored these cost functions. Here, k_1 and k_2 represent the cost coefficients

TABLE 2. Denotations for modeling.

Symbol	Description
Parameter	
θ	Price discount coefficient in the live streaming room ($0 < \theta < 1$)
F	"Blanket fee" paid by an enterprise to a celebrity ($F > 0$)
ϕ	Commission rate ($0 < \phi < 0.3$)
γ	The proportion of products sold in the live streaming room ($0 < \gamma < 1$)
g	Quality sensitivity of customers ($g > 0$)
e	Price sensitivity of customers ($e > 0$)
α	Potential online market demand ($\alpha > 0$)
λ	The coefficient of live streaming's spillover effect ($\lambda > 0$)
η	The celebrity's sharing proportion of quality improvement cost ($0 < \eta < 1$)
Decision variable	
p	The product price in the online shop
q	The level of quality improvement effort
s	The level of sales effort
Derived function	
D_ϵ ($\epsilon = 1, 2$)	Demand for online shop channel ($\epsilon = 1$) and live streaming channel ($\epsilon = 2$)
π_a	Profit of the enterprise
π_o	Profit of the celebrity
π_C	Profit of supply chain system, $\pi_C = \pi_a + \pi_o$
Superscript	
c	The model of centralized decision-making
d	The model of decentralized decision-making
N	The basic model
Y	The quality improvement model
co	The coordination model
$*$	The optimal decision

for the quality improvement effort and sales effort, respectively. Additionally, we assume $k_1 = 1$, $k_2 = 1$ and the unit production cost of products as 0, as suggested by Liang *et al.* [29] and Choi and Fredj [12]. Consequently, the costs of the enterprise's quality improvement effort and the celebrity's sales effort can be expressed as $C(q) = \frac{q^2}{2}$ and $C(s) = \frac{s^2}{2}$, respectively. The profit functions of the enterprise (π_a) and the celebrity (π_o) are the following:

$$\pi_a^Y = (1-\phi)p\theta D_2^Y + pD_1^Y - \frac{q^2}{2} - F \tag{5}$$

$$\pi_o^Y = \phi p\theta D_2^Y - \frac{s^2}{2} + F \tag{6}$$

$$\pi_a^N = (1-\phi)p\theta D_2^N + pD_1^N - F \tag{7}$$

$$\pi_o^N = \phi p\theta D_2^N - \frac{s^2}{2} + F. \tag{8}$$

The notations and subscripts used in the paper are summarized in Table 2.

4. Model solutions and discussions

In this section, we will obtain and compare the optimal decisions of the supply chain members in both the basic model and the quality improvement model . In addition, we will analyze the influence of the commission rate on the supply chain. The proof of the propositions can listed in the Appendix A. And the expressions of parameters are provided in Appendix B.

4.1. The basic model

In the basic model, no efforts are made by the enterprise to improve the quality. The demand for products is mainly influenced by the product price and the sales efforts of the celebrity.

4.1.1. The basic model of centralized decision-making

Under the centralized decision-making structure, the enterprise and the celebrity are vertically integrated as a whole system. they make joint decisions on the product price in the online shop and sales effort in order to maximize the profit. The total profit of the supply chain is shown as follows:

$$\pi_C^{N^c} = p\theta(\gamma\alpha - ep\theta + s) + p\left((1-\gamma)\alpha - ep + \lambda s\right) - \frac{s^2}{2}. \tag{9}$$

According to equation (9), we can obtain the Hessian matrix of $\pi_C^{N^c}(p,s)$

$$H(p,s) = \begin{bmatrix} -2e - 2e\theta^2 & \theta+\lambda \\ \theta+\lambda & -1 \end{bmatrix}.$$

If $e > \frac{(\theta+\lambda)^2}{2(1+\theta^2)}$, $|H(p,s)| = 2e(1+\theta^2) - (\theta+\lambda)^2 > 0$. So the profit function $\pi_C^{N^c}(p,s)$ is concave and there exists an optimal solution with respect to the product price in the online shop and sales effort. Let $\frac{\partial \pi_C^{N^c}}{\partial p} = 0$ and $\frac{\partial \pi_C^{N^c}}{\partial s} = 0$, the optimal decisions are shown as follows:

$$p^{N^{c*}} = \frac{\alpha\left(1+\gamma(-1+\theta)\right)^2}{2e(1+\theta^2) - (\theta+\lambda)^2} \tag{10}$$

$$s^{N^{c*}} = \frac{\alpha\left(1+\gamma(-1+\theta)\right)(\theta+\lambda)}{2e\left(1+\theta^2\right) - (\theta+\lambda)^2} \tag{11}$$

$$\pi_C^{N^{c*}} = \frac{\alpha^2\left(1+\gamma(-1+\theta)\right)^2}{4e(1+\theta^2) - 2(\theta+\lambda)^2}. \tag{12}$$

4.1.2. The basic model of decentralized decision-making

We employ the Stackelberg game to analyze this scenario, where both the enterprise and the celebrity make decisions independently in order to maximize their respective profits. The game unfolds in the following sequence: the enterprise announces the product price in the online shop first, followed by the celebrity determining her sales effort level in response. By substituting equations (1) and (2) into equations (7) and (8), we can express the profit functions of the enterprise and the celebrity as follows.

$$\pi_a^{N^d} = (1-\phi)p\theta(\gamma\alpha - ep\theta + s) + p\big((1-\gamma)\alpha - ep + \lambda s\big) - F \tag{13}$$

$$\pi_o^{N^d} = \phi p\theta(\gamma\alpha - ep\theta + s) - \frac{s^2}{2} + F. \tag{14}$$

According to equation (14), the celebrity's profit function about s is obtained by the backward induction method. As $\frac{\partial^2 \pi_o^{N^d}}{\partial s^2} = -1 < 0$, the celebrity's profit function is a concave function of the sales effort and has a maximum value. Let $\frac{\partial \pi_o^{N^d}}{\partial s} = 0$, we can conclude that

$$s = p\theta\phi. \tag{15}$$

Substituting equation (15) into the enterprise's profit function, *i.e.*, equation (13), we obtain that when $e > \frac{(\theta+\lambda)^2}{2(1+\theta^2)}$, $\frac{\partial^2 \pi_a^{N^d}}{\partial p^2} = 2\big(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\big) < 0$. So the profit function is concave, and there exists an optimal solution of $\pi_a^{N^d}(p)$. Let $\frac{\partial \pi_a^{N^d}}{\partial p} = 0$, the optimal decision is shown as follows:

$$p^{N^{d^*}} = \frac{\alpha\left(\theta\gamma(\phi-1)+\gamma-1\right)}{2\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)}. \tag{16}$$

Substituting equation (16) into the celebrity's profit function, *i.e.*, equation (14), we can get

$$s^{N^{d^*}} = \frac{\alpha\theta\phi\left(\theta\gamma(\phi-1)+\gamma-1\right)}{2\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)}. \tag{17}$$

By substituting the optimal solution into equations (13) and (14), we have the optimal value of the enterprise's and celebrity's profit. The optimal solutions are shown as follows:

$$\pi_a^{N^{d^*}} = -\frac{\alpha^2(\theta\gamma(\phi-1)+\gamma-1)^2}{4\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)} - F \tag{18}$$

$$\begin{aligned}\pi_o^{N^{d^*}} &= \frac{\alpha^2\theta\phi(\theta\gamma(\phi-1)+\gamma-1)\left(\gamma\theta^2(1-\phi)(3\phi-2e)\right)}{8\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)^2}\\ &\quad + \frac{\alpha^2\theta\phi(\theta(1-\gamma)(2e-\phi)+4\gamma(\lambda\theta\phi-e))}{8\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)^2} + F\end{aligned} \tag{19}$$

$$\begin{aligned}\pi_C^{N^{d^*}} &= \frac{\alpha^4\theta(-1+\gamma+\gamma\theta(-1+\phi))^3\phi A_1\phi}{32\left(e\left(-1+\theta^2(-1+\phi)\right)+\theta\phi(\theta+\lambda-\theta\phi)\right)^3}\\ &\quad + \frac{\alpha^4\theta(-1+\gamma+\gamma\theta(-1+\phi))^3\phi 2e\gamma(2+\theta(1+\theta-\theta\phi))}{32\left(e\left(-1+\theta^2(-1+\phi)\right)+\theta\phi(\theta+\lambda-\theta\phi)\right)^3}\end{aligned} \tag{20}$$

$A_1 = (-2e\theta+\theta\phi+\gamma\theta(-1-4\lambda)+\theta(-1+\phi))$.

4.2. The quality improvement model

With the rapid growth of the economy, consumers' demand for products has shifted from simply satisfying quantity to seeking better quality [72]. In response, the enterprise have taken on the responsibility of enhancing product quality by implementing standardized production process, providing technology training, and minimizing the excessive use of pesticides and fertilizers to the best of their ability. In this study, we specifically examine quality improvement models that adopt both centralized and decentralized decision-making structures.

4.2.1. The quality improvement model of centralized decision-making

Similarly, the enterprise and celebrity are closely connected as part of a unified system. Their collaboration determines the product price, the efforts put into quality improvement, and the sales strategy, all with the ultimate goal of maximizing profit.

The total profit of supply chain can be expressed as follows:

$$\pi_C^{Y^c} = p\theta(\gamma\alpha - ep\theta + q + s) + p\big((1-\gamma)\alpha - ep + q + \lambda s\big) - \frac{q^2}{2} - \frac{s^2}{2}. \tag{21}$$

According to equation (21), we can obtain the Hessian matrix of $\pi_C^{Y^c}(p,q,s)$

$$H(p,q,s) = \begin{bmatrix} -2e(1+\theta^2) & g(1+\theta) & \theta+\lambda \\ g(1+\theta) & -1 & 0 \\ \theta+\lambda & 0 & -1 \end{bmatrix}.$$

$\frac{\partial^2 \pi_C^{Y^c}}{\partial p^2} = -2e(1+\theta^2) < 0$ can be derived. If $e > \frac{g^2(\theta+1)^2+(\lambda+\theta)^2}{2(\theta^2+1)}$, we can get that $\frac{\partial^2 \pi_C^{Y^c}}{\partial p^2} * \frac{\partial^2 \pi_C^{Y^c}}{\partial p \partial q} - \frac{\partial^2 \pi_C^{Y^c}}{\partial q \partial p} * \frac{\partial^2 \pi_C^{Y^c}}{\partial q^2} = 2e(1+\theta^2) - g^2(1+\theta)^2 > 0$ and $|H(p,q,s)| = (\lambda+\theta)^2 - 2e(1+\theta^2) - g^2(1+\theta)^2 < 0$ So the profit function is concave, indicating that $\pi_C^{Y^c}(p,q,s)$ has a unique maximum. Let $\frac{\partial \pi_C^{Y^c}}{\partial p} = 0$, $\frac{\partial \pi_C^{Y^c}}{\partial q} = 0$, and $\frac{\partial \pi_C^{Y^c}}{\partial s} = 0$, the optimal decisions are shown as follows.

$$p^{Y^{c^*}} = \frac{a(-1+\gamma-\gamma\theta)}{g^2(1+\theta)^2 - 2e(1+\theta^2) + (\theta+\lambda)^2} \tag{22}$$

$$q^{Y^{c^*}} = -\frac{g(a+\alpha\gamma(-1+\theta))(1+\theta)}{g^2(1+\theta)^2 - 2e(1+\theta^2) + (\theta+\lambda)^2} \tag{23}$$

$$s^{Y^{c^*}} = -\frac{(a+\alpha\gamma(-1+\theta))(\theta+\lambda)}{g^2(1+\theta)^2 - 2e(1+\theta^2) + (\theta+\lambda)^2} \tag{24}$$

$$\pi_C^{Y^{c^*}} = \frac{(a+\alpha\gamma(-1+\theta))^2}{4e(1+\theta^2) - 2(g^2(1+\theta)^2 + (\theta+\lambda)^2)}. \tag{25}$$

4.2.2. The quality improvement model of decentralized decision-making

In a similar manner, the Stackelberg game is employed to analyze the decentralized decision-making structure. The game proceeds in the following order: the enterprise initiates by announcing his efforts towards improving the quality of their products and setting the prices in the online shop. Next, the celebrity responds by determining the level of sales efforts they will put in. By substituting equations (3) and (4) into equations (5) and (6), the enterprise's profit function and celebrity's profit function can be expressed as follows.

$$\pi a^{Y^d} = (1-\phi)p\theta(\gamma\alpha - ep\theta + gq + s) + p((1-\gamma)\alpha - ep + gq + \lambda s) - \frac{q^2}{2} - F \tag{26}$$

$$\pi o^{Y^d} = \phi p\theta(\gamma\alpha - ep\theta + gq + s) - \frac{s^2}{2} + F. \tag{27}$$

According to equation (27), the celebrity's profit function about s is obtained by the backward induction method. As $\frac{\partial^2 \pi_o^{Y^d}}{\partial s^2} = -1 < 0$, the celebrity's profit function is a concave function related to the sales effort and has a maximum value. To obtain the maximum sales effort, let $\frac{\partial \pi_o^{Y^d}}{\partial s} = 0$, we can get that

$$s = p\theta\phi. \tag{28}$$

Substituting equation (28) into the enterprise's profit function in equation (26), we obtain the Hessian matrix

$$H(q,p) = \begin{bmatrix} -1 & g + g\theta(1-\phi) \\ g + g\theta(1-\phi) & -2e + 2\theta^2(e-\phi)(-1+\phi) + 2\theta\lambda\phi \end{bmatrix}.$$

If $e > \frac{g^2(\theta+1)^2+(\lambda+\theta)^2}{2(\theta^2+1)}$, we can get that $|H(q,p)| = -1 + 2e(1-\theta^2(-1+\phi)) + \theta(2g(-1+\phi) - g^2\theta(-1+\phi)^2 - 2\phi(\theta+\lambda-\theta\phi)) > 0$. Therefore, the profit function is concave and there has a unique maximum of $\pi_a^{Y^d}(q,p)$. Let $\frac{\partial \pi_a^{Y^d}}{\partial q} = 0$ and $\frac{\partial \pi_a^{Y^d}}{\partial p} = 0$, we have:

$$p^{Y^{d^*}} = \frac{a(-1+\gamma+\gamma\theta(-1+\phi))}{2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)} \tag{29}$$

$$q^{Y^{d^*}} = -\frac{ag(-1+\theta(-1+\phi))(-1+\gamma+\gamma\theta(-1+\phi))}{2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)}. \tag{30}$$

Substituting equations (29) and (30) into the celebrity's profit function in equation (27), we have

$$s^{Y^{d*}} = \frac{a\theta\phi(-1+\gamma+\gamma\theta(-1+\phi))}{2e\left(-1+\theta^2(-1+\phi)\right)+g^2(1+\theta-\theta\phi)^2+2\theta\phi(\theta+\lambda-\theta\phi)}. \tag{31}$$

By substituting the optimal solution into equations (26) and (27), we can conclude the enterprise's and celebrity's profits as follows:

$$\pi_a^{Y^{d*}} = -\frac{a^2\left(-1+\gamma+\gamma\theta(-1+\phi)\right)^2}{4e\left(-1+\theta^2(-1+\phi)\right)+2g^2\left(1+\theta-\theta\phi\right)^2+4\theta\phi(\theta+\lambda-\theta\phi)} - F \tag{32}$$

$$\begin{aligned}\pi_o^{Y^{d*}} &= -\frac{\theta A_2\phi\left(-2e\theta+2g^2\left(-1+2\gamma\right)\left(-1+\theta(-1+\phi)\right)\right)}{A_1} \\ &\quad - \frac{\theta A_2\phi\left(\gamma\theta\left(-1-4\lambda+3\theta(-1+\phi)\right)\phi\right)}{A_1} \\ &\quad - \frac{\theta A_2\phi\left(2e\gamma\left(2+\theta(1+\theta-\theta\phi)\right)\right)}{A_1} + F\end{aligned} \tag{33}$$

$$\begin{aligned}\pi_C^{Y^{d*}} &= \frac{A_2\left(-2e(1+\gamma(-1+\theta))(1+\theta^2)2e\theta A_3\right)}{A_1} \\ &\quad + \frac{A_2\left(\theta\phi\left(2(1+\gamma(-1+\theta))(\theta+\lambda)+\theta A_4-\gamma\theta^2\phi^2\right)\right)}{A_1} \\ &\quad - \frac{A_2\left(g^2\left(-1+\theta(-1+\phi)\right)\left(1+\theta-3\theta\phi+\gamma A_5\right)\right)}{A_1}\end{aligned} \tag{34}$$

$A_1 = 2\left(2e\left(-1+\theta^2(-1+\phi)\right)+g^2\left(1+\theta-\theta\phi\right)^2+2\theta\phi(\theta+\lambda-\theta\phi)\right)^2$,
$A_2 = a^2\left(-1+\gamma+\gamma\theta(-1+\phi)\right)$, $A_3 = (2\theta+\gamma(-1+(-2+\theta)\theta))\phi$, $A_4 = (-3+\gamma(3-\theta+2\lambda))\phi$, $A_5 = \left(-1+\theta\left(\theta(-1+\phi)^2+4\phi\right)\right)$.

4.3. Analysis and comparison

Proposition 1. *With respect to the decentralized decision-making structure for the basic model and quality improvement model, the impact of the commission rate on the optimal decision variables is shown as follows:*

(1) *When* $\lambda > \lambda_1$, $\frac{\partial p^{Y^{d*}}}{\partial\phi} > 0$, *otherwise,* $\frac{\partial p^{Y^{d*}}}{\partial\phi} < 0$; *When* $\lambda > \lambda_2$, $\frac{\partial p^{N^{d*}}}{\partial\phi} > 0$, *otherwise,* $\frac{\partial p^{N^{d*}}}{\partial\phi} < 0$;
(2) $\frac{\partial q^{Y^{d*}}}{\partial\phi} < 0$, $\frac{\partial s^{Y^{d*}}}{\partial\phi} > 0$, $\frac{\partial s^{N^{d*}}}{\partial\phi} > 0$.

As Proposition 1 shown, the spillover effect of live streaming has a threshold that impacts the relationship between the product price in the online shop and the commission rate, regardless of whether there is a quality improvement. When the spillover effect of live streaming is relatively low, the increase in sales from the live streaming room to the online shop is minimal. An increase in commission rates reduces corporate profits. Consequently, enterprises lower prices in online stores to attract consumer attention, indirectly decreasing product prices in live-streaming channels to stimulate sales volume. However, when the spillover effect of live streaming exceeds the threshold, customers who repurchase products from the online shop after watching a live streaming will increase and recommend products to others, resulting in a significant influx of new customers to the online shop. In this case, as the commission rate increases, the enterprise will increase the product price of the online shop to maximize profits.

Moreover, an increase in the commission rate incentivizes the celebrity to enhance sales efforts, as commissions constitute their primary source of income. A higher commission rate translates to a greater revenue share, motivating the celebrity to intensify promotional activities and stimulate consumer purchasing intentions. For instance, the celebrity may highlight after-sales services to alleviate consumer concerns or engage with fans by offering product samples and other interactive experiences during live streaming. These strategies not only extend viewership duration but also reinforce consumer engagement and purchasing intent.

Proposition 2. *The influence of commission rate on the profits of supply chain under the basic model and quality improvement model of decentralized decision-making is as follows:*

(1) *When* $0 < \lambda < \lambda_3$, $\frac{\partial \pi_a^{Y^{d^*}}}{\partial \phi} < 0$, *otherwise,* $\frac{\partial \pi_a^{Y^{d^*}}}{\partial \phi} > 0$;

(2) *When* $0 < \lambda < \lambda_4$, $\frac{\partial \pi_a^{N^{d^*}}}{\partial \phi} < 0$, *otherwise,* $\frac{\partial \pi_a^{N^{d^*}}}{\partial \phi} > 0$;

(3) *When* $\lambda > \lambda_5$ *and* $\gamma_1 < \gamma < 1$, $\frac{\partial \pi_o^{Y^{d^*}}}{\partial \phi} > 0$, *otherwise,* $\frac{\partial \pi_o^{Y^{d^*}}}{\partial \phi} < 0$;

(4) $\frac{\partial \pi_o^{N^{d^*}}}{\partial \phi} > 0$.

In Proposition 2, it is demonstrated that when the spillover effect of live streaming is below the threshold, the profit of the enterprise declines as the commission rate increases, regardless of the presence of quality improvement. When the spillover effect of live streaming is relatively low, the increase in product demand in the online shop resulting from live streaming is minimal. As a result, the profits of the enterprise cannot compensate for the increased costs caused by the higher commission rate, leading to a reduction in profit. However, when the spillover effect of live streaming exceeds the threshold, the popularity and sales volume of the online shop experience significant growth. The increased demand for dual-channel products brings more profits to the enterprise compared to the increased costs caused by the commission rate. Therefore, even with an increase in the commission rate, the profit of the enterprise will continue to rise.

In the quality improvement model, when both the spillover effect of live streaming and the proportion of products sold in the live streaming room surpass the threshold, the commission rate has a positive impact on the celebrity's profit. In the basic model, the profit of the celebrity always increases with a higher commission rate. This phenomenon can be attributed to the substantial sales volume generated from the live streaming room under the quality improvement model. As the revenue share for the celebrity increases, the growth in profits resulting from increased demand outweighs the cost of sales efforts for live streaming, leading to an improvement in the celebrity's profit.

Proposition 3. *The influence of live-streaming spillover effects on the profits and decision-making of supply chain under the quality improvement model of decentralized decision-making is as follows:*

(1) $\frac{\partial^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial^{Y^{d^*}}}{\partial \lambda} > 0$;

(2) $\frac{\partial \pi_a^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial \pi_o^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial \pi_C^{Y^{d^*}}}{\partial \lambda} > 0$.

In Proposition 2, it is demonstrated that when the spillover effect of live streaming is below the threshold, the profit of the enterprise declines as the commission rate increases, regardless of the presence of quality improvement. When the spillover effect of live streaming is relatively low, the increase in product demand in the online shop resulting from live streaming is minimal. As a result, the profits of the enterprise cannot compensate for the increased costs caused by the higher commission rate, leading to a reduction in profit. However, when the spillover effect of live streaming exceeds the threshold, the popularity and sales volume of the online shop experience significant growth. The increased demand for dual-channel products brings more profits to the enterprise compared to the increased costs caused by the commission rate. Therefore, even with an increase in the commission rate, the profit of the enterprise will continue to rise.

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Proposition 4. *The optimal decision variables in the basic model and quality improvement model of decentralized and centralized decision-making structures are in the following order:* $p^{Y^{c^*}} > p^{Y^{d^*}} > p^{N^{d^*}}$, $p^{Y^{c^*}} > p^{N^{c^*}} > p^{N^{d^*}}$, $q^{Y^{c^*}} > q^{Y^{d^*}}$, $s^{Y^{c^*}} > s^{Y^{d^*}} > s^{N^{d^*}}$, $s^{Y^{c^*}} > s^{N^{c^*}} > s^{N^{d^*}}$.

Proposition 4 demonstrates that in both the centralized or decentralized decision-making structures, the online shop's product price and the enterprise's sales effort in the quality improvement model are consistently higher than those in the basic model. The higher price in the online shop is primarily driven by the enterprise's desire for increased profit resulting from his investment in quality improvement. Meanwhile, the celebrity further enhance sales effort level ensuring comprehensive comprehension of the improved quality through detailed product information explanation.

In addition, in both decision-making structures, the product price, sales effort, and quality improvement effort reach their peak in the centralized structure. This is due to ,the enterprise and the celebrity collaborate to make optimal decisions, setting the highest possible product price and providing the highest levels of quality improvement effort and sales effort in this structure.

Proposition 5. *Comparing the profits of the supply chain between the basic model and quality improvement model with decentralized and centralized decision-making structures, we can conclude that:* $\pi_a^{Y^{c^*}} > \pi_a^{N^{c^*}}, \pi_a^{Y^{d^*}} > \pi_a^{N^{d^*}}, \pi_o^{Y^{c^*}} > \pi_o^{N^{c^*}}, \pi_o^{Y^{d^*}} > \pi_o^{N^{d^*}}, \pi_C^{Y^{c^*}} > \pi_C^{Y^{d^*}}, \pi_C^{N^{c^*}} > \pi_C^{N^{d^*}}$.

Proposition 5 reveals that the quality improvement of products not only stimulates consumers' intention to purchase but also generates quality premiums, leading to increased profits for both supply chain members under the quality improvement model. The efforts made by the enterprise to improve the quality of products present a promising method for achieving significant development in product e-commerce. The centralized decision-making structure stands as the ideal method for maximizing profits for each member and the whole system. This illustrates that the centralized decision-making structure can avoid the systemic efficiency loss caused by the double marginalization of the decentralized decision-making structure. Therefore, while the quality improvement model is superior, the degree of cooperation between the enterprise and the celebrity is insufficient in the decentralized decision-making structure. The establishment of a coordinating mechanism is essential to achieving system coordination for the quality improvement model within the decentralized decision-making structure.

5. Coordination mechanism

In this section, we propose a cost-sharing joint commission mechanism between the supply chain members for the quality improvement model with a decentralized decision-making structure. According to [53], a contract can achieve supply chain coordination if $\pi_a = \delta\pi + A$, where A is a constant and $0 < \delta < 1$. In other words, if the enterprise's profit is an affine function of the supply chain system's profit, coordination is achievable.

In the two sales mode (*i.e.*, live streaming and online store), the enterprise can increase demand for its products by enhancing its quality improvement efforts, thereby ensuring quality assurance for consumers. As a result, the celebrity is incentivized to encourage the enterprise to elevate its quality improvement efforts and share a portion of the associated costs. In this coordination mechanism, the enterprise actively increases its quality improvement efforts to align with the requirements of a centralized decision-making structure. In this model, we derive the expression for the commission rate, treated as an exogenous variable, that facilitates coordination. It is assumed that the commission rate is denoted by $\overline{\phi}$, and the celebrity's sharing proportion of the quality improvement cost is $\eta(0 < \eta < 1)$. Notably, the model focuses on deriving this expression for coordination, without making any decisions regarding the commission rate itself.

Thus, the enterprise's profit is

$$\pi_a^{Yco} = \left(1 - \overline{\phi}\right) p\theta(\gamma\alpha - ep\theta + gq + s) + p((1-\gamma)\alpha - ep + gq + \lambda s) - \frac{(1-\eta)q^2}{2} - F. \tag{35}$$

The celebrity's profit is

$$\pi_o^Y co = \overline{\phi} p\theta \left(\gamma a - e^{p\theta} + gq + s\right) - \frac{s^2}{2} - \frac{\eta q^2}{2} + F. \tag{36}$$

Proposition 6. *The cost-sharing joint commission mechanism can achieve system coordination, if* $(\bar{\phi}, \eta)$ *satisfies* $\begin{cases} \overline{\phi} & = \phi_1 \\ \eta & = \delta \end{cases}$ *, where* δ *represents the celebrity's sharing proportion of the system profit and*

$$\begin{aligned} \phi_1 = & \frac{q^2 + 2gpq\eta - s^2\eta + 2ap(-1+\eta)(1+\gamma(-1+\theta))}{2\theta p(a\gamma - e\theta p + gq + s)} \\ & + \frac{2gpq\eta\theta + 2ps\eta\theta - 2ep^2(-1+\eta)(1+\theta^2) + 2ps(-1+\eta)\lambda}{2\theta p(a\gamma - e\theta p + gq + s)} \\ & + \frac{2F - 2gpq - 2q^2\eta - 2gpq\theta - 2ps\theta}{2\theta p(a\gamma - e\theta p + gq + s)}. \end{aligned} \tag{37}$$

We can conclude that when the cost-sharing joint commission mechanism is implemented, the profit functions of the enterprise and the celebrity are affine functions of the system profit, which can achieve the system coordination. At this point, the decision variables satisfy that

$$p = p^{Y^{c^*}} = \frac{a(-1+\gamma-\gamma\theta)}{(g^2(1+\theta)^2 - 2e\left(1+\theta^2\right) + (\theta+\lambda)^2)} \tag{38}$$

$$q = q^{Y^{c^*}} = -\frac{g(a + a\gamma(-1+\theta))(1+\theta)}{g^2(1+\theta)^2 - 2e\left(1+\theta^2\right) + (\theta+\lambda)^2} \tag{39}$$

$$s = s^{Y^{c^*}} = -\frac{(a + a\gamma(-1+\theta))(\theta+\lambda)}{g^2(1+\theta)^2 - 2e\left(1+\theta^2\right) + (\theta+\lambda)^2}. \tag{40}$$

Substituting the optimal solution into equation (37), we get the commission rate as follows:

$$\overline{\phi} = -\frac{-2FA_1^2 - (aA_3)^2\left(-g^2(-1+2\eta)(1+\theta)^2 + A_2\right)}{2a^2A_3\theta\left(g^2(-1+2\gamma)(1+\theta) + A_4 + (\theta+\lambda)(-1+\gamma+\gamma\lambda)\right)} \tag{41}$$

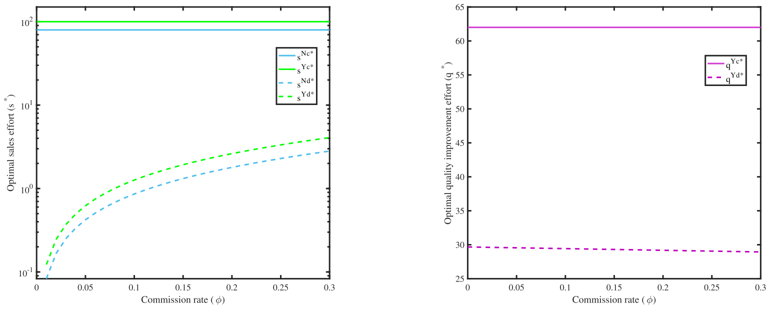
$A_1 = g^2(1+\theta)^2 - 2e(1+\theta^2) + (\theta+\lambda)^2$, $A_2 = 2e(-1+\eta)(1+\theta^2) - \eta(\theta+\lambda)^2$, $A_3 = 1 + \gamma(-1+\theta)$, $A_4 = e(\theta - \gamma(2+\theta+\theta^2))$.

In the quality improvement model of decentralized decision-making, the precondition for supply chain members to accept the coordination mechanism is that the profits of each member after coordination are not lower than those under the decentralized decision-making structure, so it is necessary to meet

$$\begin{cases} \eta\pi_C^{Y^{c^*}} & \geq \pi_a^{Y^{d^*}} \\ (1-\eta)\pi_C^{YC^*} & \geq \pi_o^{Y^{d^*}} \end{cases}$$

therefore the sharing cost proportion meets

$$\frac{\pi_a^{Y^{d^*}}}{\pi_C^{Y^{c^*}}} \leq \eta \leq 1 - \frac{\pi_o^{Y^{d^*}}}{\pi_C^{Y^{c^*}}}.$$



(a) (b)

FIGURE 2. Optimal sales effort and quality improvement effort. (a) Optimal sales effort. (b) Optimal quality improvement effort.

6. Numerical analysis

In this section, we begin by analyzing the impact of the commission rate on the optimal outcomes of the basic model and the quality improvement model in both decentralized and centralized decision-making. Through numerical analysis, we also compare different models. Finally, we demonstrate the effectiveness of the coordination mechanism.

6.1. The effect of commission rate

This section examines how the commission rate affects the product price in the online shop, the level of quality improvement effort and sales effort, and the profit of supply chain members in both the basic model and quality improvement model. Empirical data from Taobao Live (Alibaba) and Douyin (TikTok) indicate that commission rates for live streaming influencers vary between 10% and 30%. By rationally setting parameters, some researchers have validated their findings through numerical experiments [32,51].Other scholars have derived significant managerial implications from sensitivity analyses [15,55]. Based on this evidence,we assume values of $\alpha = 120, F = 80, \theta = 0.7, \gamma = 0.4, e = 3, g = 1, \lambda = 1.7, 0 < \phi < 0.3$ to verify the effect of commission rate on optimal sales effort, quality improvement effort, and supply chain systems' profits.

Then, we assume $\lambda = 1.7$ and 0.1 to verify the effect of the commission rate on sales price in the online shop and the profits of supply chain members.

Figures 2 and 3 show a positive correlation between the commission rate and sales effort, and a negative correlation with quality improvement effort in both the basic model and quality improvement model of decentralized decision-making. From Figures 4 and 5, we observe that when the spillover effect of live streaming is minimal, the commission rate positively influences the product price in the online shop and negatively impacts the profit of the enterprise under the decentralized decision-making structure, regardless of whether quality improvement efforts are made or not. However, when the spillover effect of live streaming is relatively high, the product price in the online shop increase with the commission rate, leading to boosted profit for the enterprise. Figure 5 demonstrates that with an increasing commission rate, the celebrity's profit always increase in the basic model. However, under the quality improvement model of decentralized decision-making, the celebrity's profits increase with the commission rate only when the spillover effect of live streaming and the products sold from the live streaming room meet the threshold.

The figures above illustrate that the optimal decisions and profit under the centralized decision-making structure are always superior to those under the decentralized decision-making structure. In addition, regardless of decision-making structure, the optimal decisions and profits under the quality improvement model consistently outperform those under the basic model. This aligns with the conclusion of Propositions 1–5.

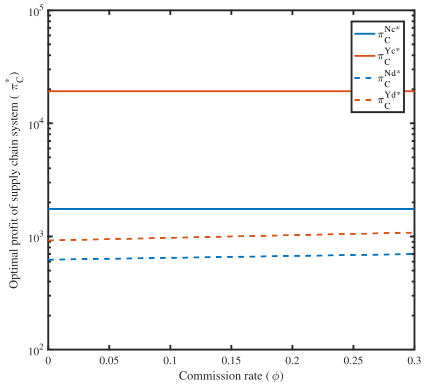
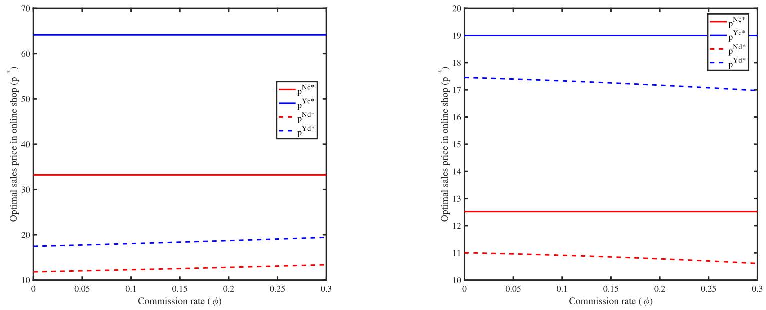
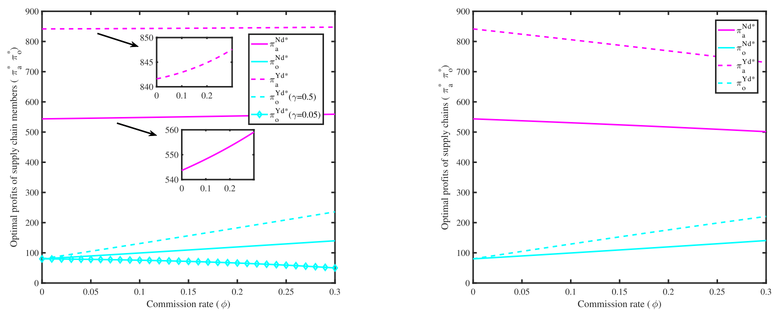


FIGURE 3. Optimal profit of the supply chain system.



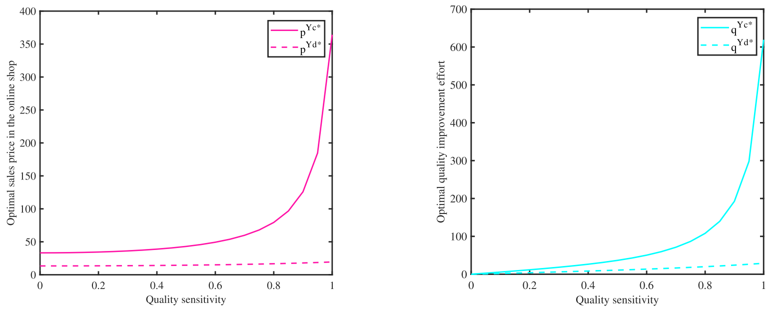
(a) (b)

FIGURE 4. Optimal sales price in the online shop. (a) $\lambda = 1.7$. (b) $\lambda = 0.1$.



(a) (b)

FIGURE 5. The profits of supply chain member. (a) $\lambda = 1.7$. (b) $\lambda = 0.1$.



(a) (b)

FIGURE 6. Impact of quality sensitivity on sales prices and quality improvement efforts. (a) Sales prices. (b) Quality improvement efforts.

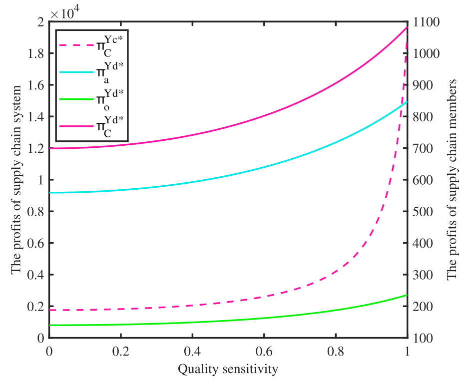


FIGURE 7. Optimal profit of the supply chain system.

6.2. Sensitivity analysis

6.2.1. Impact of quality sensitivity

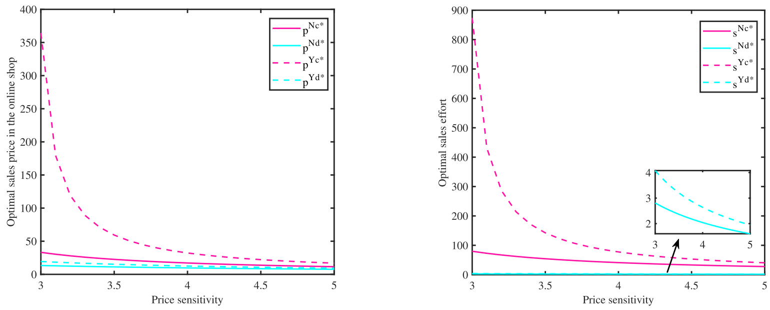
In this section, we present a sensitivity analysis of consumers' price and quality sensitivities. We assume values of $a = 120, F = 80, \theta = 0.7, \gamma = 0.4, \lambda = 1.7$ and $\phi = 0.3$. We first analyze quality sensitivity and we assume that $e = 3$.

Figures 6 and 7 demonstrate that both product pricing and quality improvement efforts exhibit positive correlations with consumer quality sensitivity, thereby enhancing profitability across the supply chain system. Specifically, heightened consumer quality sensitivity incentivizes enterprise to intensify quality investments and implement premium pricing strategies. High-quality products consequently attract greater consumer demand, generating increased profits for the supply chain system.

6.2.2. Impact of price sensitivity

In addition, we analyze price sensitivity and we assume that $g = 1$.

Figures 8 and 9 demonstrate that both product pricing and celebrities' sales efforts exhibit negative correlations with increasing price sensitivity, reducing overall supply chain profitability. Specifically, price-sensitive consumers compel enterprise to adopt downward pricing adjustments to retain market share. Concurrently, higher consumer price sensitivity leads celebrity to reduce their sales effort investments. However, excessively low selling prices may ultimately compromise the profitability of the entire supply chain system.



(a) (b)

FIGURE 8. Impact of price sensitivity on sales prices and sales efforts. (a) Sales price. (b) Sales efforts.

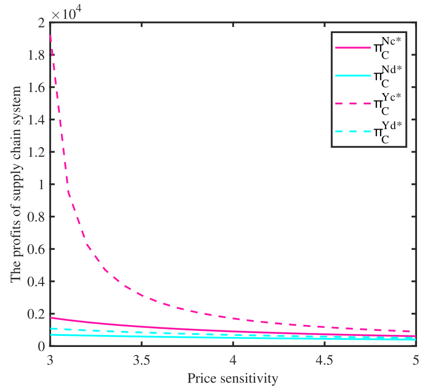


FIGURE 9. Optimal profit of the supply chain system.

6.2.3. Impact of spillover effect

Then, we systematically examine the impact of live streaming spillover effects on supply chain members' operational decisions and bilateral profit allocation.

Figure 10 illustrates the impact of live-streaming spillover effects on key supply chain decision variables. All three metrics-product sales price, quality improvement efforts, and streamers' sales efforts-exhibit positive correlations with live-streaming spillover effects, confirming cross-platform synergy amplification. This suggests that stronger spillover effects incentivize both enterprises and celebrities to enhance product value propositions. Enterprises increase quality investments while cautiously adjusting prices to balance premium positioning and demand retention. Simultaneously, streamers intensify sales efforts to attract more consumers. This tripartite escalation reflects strategic alignment among supply chain members. However, the celebrity's sales efforts exhibit a slower growth rate, likely because increased efforts yield diminishing marginal returns, thereby limiting the incentive for further intensification.

Figures 10 and 11 prove Proposition 3. Figure 11 illustrates the relationship between live streaming spillover effects and profit distribution among supply chain stakeholders. Both enterprise profits and celebrity profits increase with spillover effect intensity. Enterprises leverage spillover-driven demand expansion to optimize pricing strategies while maintaining cost efficiency, whereas celebrity enhance sales conversion rates through extended audience reach. It is noteworthy that corporate profits are growing at a significantly higher rate compared to the celebrity's profit growth. This is because the growth of spillover effects primarily increases demand for online

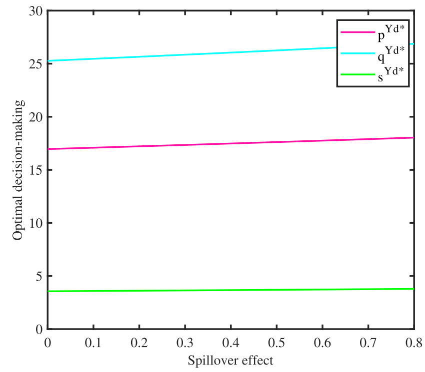


FIGURE 10. Impact of spillover effect on optimal decision-making.

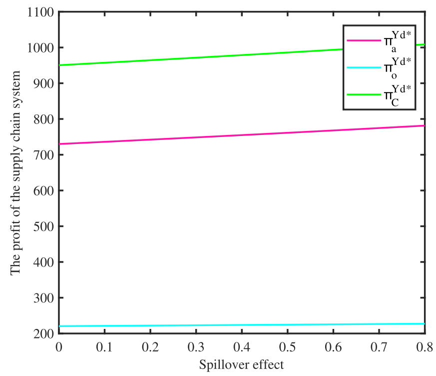


FIGURE 11. Impact of spillover effect on the supply chain system's profits.

stores, yet the celebrity cannot share in the profits generated by these stores, resulting in a comparatively low increase in their profit growth. As shown in Figure 10, despite celebrities' increased sales efforts, they achieve relatively lower profit growth. Thus, a cost-and-profit sharing mechanism must be designed to equitably allocate operational burdens and benefits, ultimately achieving Pareto-optimal profit distribution across the supply chain ecosystem.

6.3. Effectiveness of coordination mechanism

In this section, we analyze the impact of the cost-sharing joint commission mechanism under the quality improvement model of decentralized decision-making. We set the initial commission rate as $\phi = 0.2$. Parameter values assumed in the numerical analysis are listed as follows: $\alpha = 200, \lambda = 0.6, \theta = 0.8, e = 9, g = 1, \gamma = 0.6, F = 100$.

Based on these parameter values, we obtain the following optimal decision and profit calculations: under the centralized decision-making structure, $p^{Y^{c^*}} = 7.24, q^{Y^{c^*}} = 13.03, s^{Y^{c^*}} = 10.13, \pi_C^{Y^{c^*}} = 636.842$; under the decentralized decision-making structure, $\pi_a^{Y^{d^*}} = 409.462, \pi_o^{Y^{d^*}} = 187.741, \pi_C^{Y^{d^*}} = 597.203$. By utilizing the cost-sharing joint commission mechanism, we obtain the results shown in Table 3.

The results in Table 3 demonstrate that the cost-sharing joint commission mechanism effectively coordinates the system, leading to a "win–win" situation for supply chain members and the system. Furthermore, as the sharing proportion of quality improvement cost increases, the celebrity's profit shows an upward trend. There-

TABLE 3. Values of parameters and change rate before and after coordination.

		The profit of the enterprise, the celebrity, and the supply chain system	Profit comparison
Before coordination		$\pi_a^{Y^{d^*}} = 409.462$, $\pi_o^{Y^{d^*}} = 187.741$, $\pi_C^{Y^{d^*}} = 597.203$	After coordination, the overall profit of the system is optimized, and $\pi_a^{Y^{d^*}} < \pi_a^{Ydco^*} \pi_o^{Y^{d^*}} < \pi_o^{Ydco^*}$
After coordination	$\eta = 0.66$	$\pi_a^{Ydco^*} = 419.039(\uparrow 2.3\%)$, $\pi_o^{Ydco^*} = 217.803(\uparrow 16.0\%)$, $\pi_C^{Y^{c^*}} = 636.842$	
	$\eta = 0.66$	$\pi_a^{Ydco^*} = 425.424(\uparrow 3.9\%)$, $\pi_o^{Ydco^*} = 211.418(\uparrow 12.6\%)$, $\pi_C^{Y^{c^*}} = 636.842$	
	$\eta = 0.68$	$\pi_a^{Ydco^*} = 431.809(\uparrow 5.5\%)$, $\pi_o^{Ydco^*} = 205.033(\uparrow 9.2\%)$, $\pi_C^{Y^{c^*}} = 636.842$	
	$\eta = 0.69$	$\pi_a^{Ydco^*} = 438.195(\uparrow 7.0\%)$, $\pi_o^{Ydco^*} = 198.647(\uparrow 5.8\%)$, $\pi_C^{Y^{c^*}} = 636.842$	
	$\eta = 0.70$	$\pi_a^{Ydco^*} = 444.580(\uparrow 8.6\%)$, $\pi_o^{Ydco^*} = 192.262(\uparrow 2.4\%)$, $\pi_C^{Y^{c^*}} = 636.842$	

Notes. When $\overline{\phi} = 1.12 - 1.05\eta$, the system reaches coordination, where $0.64 \leq \eta \leq 0.71$.

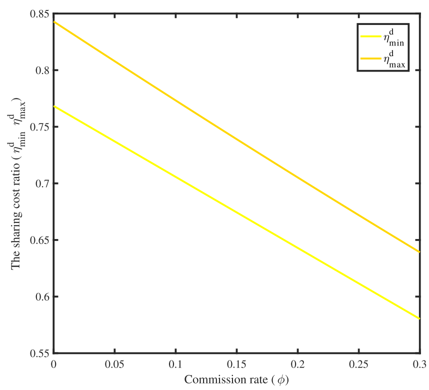


FIGURE 12. The effect of commission rate on the sharing proportion.

fore, by increasing the sharing proportion of quality improvement cost, the celebrity can significantly enhance profitability during the coordination process.

Figure 12 confirms that as the commission rate increases, the sharing proportion of quality improvement cost decrease, and the applicable conditions for the coordination mechanism become narrower. This finding indicates that raising the commission rate reduces the celebrity's share of the system's profit, thereby increasing the profit of the enterprise. conclusion may be surprising since, in general, increasing the commission rate would be expected to increase the profit of the celebrity. However, when employing the coordination mechanism, an increase in the commission rate reduce the celebrity's share of quality improvement cost and, consequently, their share of the supply chain system profit.

Additionally, as the commission rate increases, the change in the maximum value of the sharing proportion of quality improvement cost is greater than the change in the minimum value. This results in a smaller scope of feasible conditions for the coordination mechanism. Consequently, the negotiation space decreases and the difficulty of cooperation and coordination between the enterprise and the celebrity increases. This situation is not conducive to designing an effective coordination mechanism.

7. The extended model

7.1. Model 1: Considering $\theta = 1$

In this section, we extend the basic model by analyzing the scenario where the price of the product in the live streaming room ($p\theta$) is equal to the price in the online shop (p), *i.e.*, $\theta = 1$. This assumption reflects a situation where there is no price differentiation between the two sales modes – the live streaming room and the online shop. The consideration of $\theta = 1$ is particularly important to address concerns raised by reviewers regarding whether products sold through both sales modes should be priced the same. By considering this case, we explore the implications of having identical pricing in both sales modes on the supply chain members' decisions.

7.1.1. Models under centralized decision-making

In the extended model, we continue to focus on the supply chain involving an enterprise and a celebrity in a live-streaming commerce framework. The supply chain is designed to sell a specific product both through an online shop and a live streaming room. The product is priced identically across both sales modes, with the price set at p in both the online shop and the live streaming room.

$$\pi_C^{E_2c} = p(\gamma a - ep + gq + s) + p((1-\gamma)a - ep + gq + \lambda s) - \frac{q^2}{2} - \frac{s^2}{2}. \tag{42}$$

According to equation (42), we can obtain the Hessian matrix of $\pi_C^{E_2c}(p,q,s)$:

$$H(p,q,s) = \begin{bmatrix} -4e & 2g & 1+\lambda \\ 2g & -1 & 0 \\ 1+\lambda & 0 & -1 \end{bmatrix}$$

$\frac{\partial^2 \pi_C^{E_2c}}{\partial p^2} = -4e < 0$ can be derived. If $e > \frac{4g^2+(\lambda+1)^2}{4}$, we can get that $\frac{\partial^2 \pi_C^{E_2c}}{\partial p^2} * \frac{\partial^2 \pi_C^{E_2c}}{\partial p \partial q} - \frac{\partial^2 \pi_C^{E_2c}}{\partial q \partial p} * \frac{\partial^2 \pi_C^{E_2c}}{\partial q^2} = 4e - 4g^2 > 0$ and $|H(p,q,s)| = (\lambda+1)^2 - 4e - 4g^2 < 0$.

So the profit function is concave, indicating that $\pi_C^{Y_C c}(p,q,s)$ has a unique maximum. Let $\frac{\partial \pi_C^{E_2c}}{\partial p} = 0$, $\frac{\partial \pi_C^{E_2c}}{\partial q} = 0$, and $\frac{\partial \pi_C^{E_2c}}{\partial s} = 0$, the optimal decisions are shown as follows.

$$p^{E_2c^*} = \frac{a}{4e - 4g^2 - (\lambda+1)^2} \tag{43}$$

$$q^{E_2c^*} = \frac{-2ag}{-4e + 4g^2 + (\lambda+1)^2} \tag{44}$$

$$s^{E_2c^*} = \frac{-(a(\lambda+1))}{-4e + 4g^2 + (\lambda+1)^2} \tag{45}$$

$$\pi_C^{E_2c^*} = \frac{a^2}{8e - 2(4g^2 + (\lambda+1)^2)}. \tag{46}$$

7.1.2. Models under decentralized decision-making

In a similar manner, the Stackelberg game is employed to analyze the decentralized decision-making structure. The game proceeds in the following order: the enterprise initiates by announcing his efforts towards improving the quality of their products and setting the prices in the online shop. Next, the celebrity responds by determining the level of sales efforts they will put in. The enterprise's profit function and celebrity's profit function can be expressed as follows.

$$\pi_a^{E_2d} = (1-\phi)p(\gamma a - ep + gq + s) + p((1-\gamma)a - ep + gq + \lambda s) - \frac{q^2}{2} - F \tag{47}$$

$$\pi_o^{E_2d} = \phi p(\gamma a - ep + gq + s) - \frac{s^2}{2} + F. \tag{48}$$

According to equation (48), the celebrity's profit function about s is obtained by the backward induction method. As $\frac{\partial^2 \pi_o^{E_2d}}{\partial s^2} = -1 < 0$, the celebrity's profit function is a concave function related to the sales effort and has a maximum value. To obtain the maximum sales effort, let $\frac{\partial \pi_o^{E_2d}}{\partial s} = 0$, we can get that

$$s = p\phi. \tag{49}$$

Substituting equation (49) into the enterprise's profit function in equation (47), we obtain the Hessian matrix

$$H(q,p) = \begin{bmatrix} -1 & g + g(1-\phi) \\ g + g(1-\phi) & -2e + 2(e-\phi)(-1+\phi) + 2\lambda\phi \end{bmatrix}.$$

If $e > \frac{4g^2+(\lambda+1)^2}{4}$, we can get that $|H(q,p)| = -1+2e(1-(-1+\phi))+(2g(-1+\phi)-g^2(-1+\phi)^2-2\phi(1+\lambda-\phi)) > 0$. Therefore, the profit function is concave and there has a unique maximum of $\pi_a^{Y^d}(q,p)$. Let $\frac{\partial \pi_a^{E_2d}}{\partial q} = 0$ and $\frac{\partial \pi_a^{E_2d}}{\partial p} = 0$, we have:

$$p^{E_2d^*} = \frac{a(-1+\gamma\phi)}{1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi + (-2+g^2)\,\phi^2} \tag{50}$$

$$q^{E_2d^*} = \frac{-a(-1+g(-1+\phi))(-1+\gamma\phi)}{1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi + (-2+g^2)\,\phi^2}. \tag{51}$$

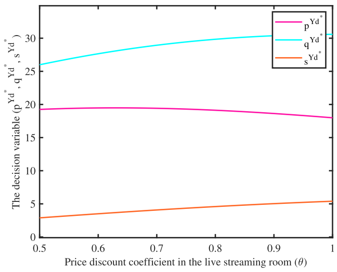
Substituting equations (50) and (51) into the celebrity's profit function in equation (49), we have

$$s^{E_2d^*} = \frac{a\phi(-1+\gamma\phi)}{1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi + (-2+g^2)\phi^2}. \tag{52}$$

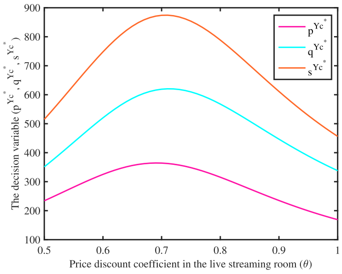
By substituting the optimal solution into equations (47) and (48), we can conclude the enterprise's and celebrity's profits as follows:

$$\begin{aligned}\pi_a^{E_2d^*} = &-\frac{a^2(-1+\gamma\phi)^2((1+g)^2+2e(-2+\phi)+6\phi)}{2(1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi+(-2+g^2)\phi^2)^2} \\ &-\frac{a^2(-1+\gamma\phi)^2(\phi(-2g(1+g)+6\lambda+(-6+g^2)\phi))}{2(1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi+(-2+g^2)\phi^2)^2} - F\end{aligned} \tag{53}$$

$$\pi_o^{E_2d^*} = -\frac{\phi a(-1+\gamma\phi)\begin{pmatrix} a\,(2e+2g(-1+g(-1+\phi))+3\phi)\,(-1+\gamma\phi) \\ -2\gamma a \begin{pmatrix} (1-4e+2g+g^2+(-2+g^2)\phi^2 \\ +2(1+e-g(1+g)+\lambda)\phi \end{pmatrix} \end{pmatrix}}{2(1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi+(-2+g^2)\phi^2)^2} + F \tag{54}$$



(a)



(b)

FIGURE 13. Impact of price discount on decision variable. (a) Decentralized model. (b) Centralized model.

$$\pi_C^{E_2d^*} = -\frac{a(-1+\gamma\phi)\begin{pmatrix} a(-1+\gamma\phi)\begin{pmatrix}((1+g)^2+4e(-1+\phi)-4g(1+g)\phi \\ +6(1+\lambda)\phi+3(-1+g^2)\phi^2)\end{pmatrix} \\ +2\gamma a\phi\left(-1-2e(-2+\phi)+2g(-1+\phi)\right) \\ +2\gamma a\phi\left(-g^2(-1+\phi)^2+2\phi(-1-\lambda+\phi)\right)\end{pmatrix}}{2(1-4e+2g+g^2+2(1+e-g(1+g)+\lambda)\phi+(-2+g^2)\phi^2)^2}. \tag{55}$$

7.1.3. Analysis and comparison

In this extended model with $\theta = 1$, we compare the optimal decisions and profits under two scenarios: one where $\theta = 1$ and another where θ is a free variable (*i.e.*, when the prices in the online shop and live streaming room are equal). By reviewing existing literature and incorporating real-world cases, we propose the following hypotheses, with the following assumptions: $a = 120, F = 80, \gamma = 0.4, e = 3, g = 1, \lambda = 1.7, \phi = 0.3$ (Fig. 13).

In the extended model with $\theta = 1$, we find that under the decentralized model, the sales price p is lower compared to when θ is a free variable, while both quality improvement effort q and sales effort s are higher. We suggests that when the price is uniform across both sales modes, supply chain members are incentivized to invest more in quality improvements and sales efforts to drive demand. In contrast, under the centralized model with $\theta = 1$, all three decision variables – price, quality improvement effort, and sales effort – are lower than when θ is a free variable. We indicates that when θ is a free variable, there is greater flexibility in setting prices, which allows for a more efficient allocation of resources and leads to higher profitability with lower effort. These results emphasize the importance of pricing flexibility and coordination strategies. Managers should recognize that in a decentralized model, higher investments in quality and sales efforts are essential to offset lower pricing, while in a centralized model, offering flexible pricing and optimizing resource allocation can lead to better profitability with reduced effort. Overall, We highlight the strategic advantage of having θ as a free variable, as it allows for more tailored pricing strategies that enhance supply chain performance and maximize profits.

Figure 14 reveals that in the decentralized model, when $\theta = 1$, both the profits of the supply chain members and the overall system profits are higher compared to when θ is a free variable. We suggests that, under a decentralized decision-making structure, a uniform pricing strategy across both sales modes (*i.e.*, $\theta = 1$) encourages higher investments in quality improvement and sales efforts, leading to greater overall profits. On the other hand, in the centralized model, $\theta = 1$ results in lower profits compared to when θ is a free variable. It indicates that, in a centralized system, more flexible pricing (*i.e.*, allowing θ to vary) leads to better resource allocation and higher profitability. The findings suggest that when supply chain members adopt a centralized decision-making approach, setting θ as a free variable can be advantageous. It allows for more optimal pricing strategies that can drive higher profits with less effort. However, when the supply chain operates under a decentralized structure, fixing $\theta = 1$ may lead to higher profits for the system as a whole, as both parties invest more in quality improvements and sales efforts to offset lower prices. These insights underline the importance of

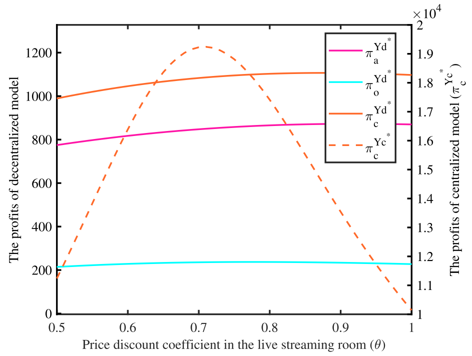


FIGURE 14. Impact of price discount on profits of supply chain system.

decision-making structure and pricing flexibility in determining supply chain success. Managers should recognize that in decentralized systems, setting $\theta = 1$ can be a tool for aligning incentives and driving investment in quality and sales efforts, ultimately benefiting the entire supply chain. Conversely, in centralized systems, allowing θ to be a free variable provides pricing flexibility, leading to better resource allocation and higher profitability. Therefore, selecting the appropriate decision-making structure and managing pricing strategies effectively can significantly improve both individual and overall supply chain profits.

7.2. Model 2: Considering no spillover effects

In this study, we also explore the extended model that does not incorporate live-streaming spillover effects, which serves as a benchmark for understanding the influence of spillover on supply chain dynamics. In the absence of spillover effects, the interactions between supply chain members – such as enterprises and celebrities – are treated independently, with no external influence from the actions of one actor directly affecting others. The demand functions for the online shop and live streaming sales modes are respectively given as follows:

$$D_1^{E_3} = (1-\gamma)a - ep + gq \tag{56}$$

$$D_2^{E_3} = \gamma a - ep\theta + gq + s. \tag{57}$$

The profit functions of the enterprise (π_a) and the celebrity (π_o) are the following:

$$\pi_a^{E_3} = (1-\phi)p\theta D_2^{E_3} + pD_1^{E_3} - \frac{1}{2}q^2 - F \tag{58}$$

$$\pi_o^{E_3} = \phi p\theta D_2^{E_3} - \frac{1}{2}s^2 + F. \tag{59}$$

7.2.1. Models under centralized decision-making

$$\pi_C^{E_2 c} = p(\gamma a - ep + gq + s) + p\theta((1-\gamma)a - ep\theta + gq + \lambda s) - \frac{1}{2}q^2 - \frac{1}{2}s^2. \tag{60}$$

According to equation (60), we can obtain the Hessian matrix of $\pi_C^{E_3 c}(p,q,s)$:

$$H(p,q,s) = \begin{bmatrix} -2e(1+\theta^2) & 1+g\theta & \theta \\ 1+g\theta & -1 & 0 \\ \theta & 0 & -1 \end{bmatrix}$$

$\frac{\partial^2 \pi_C^{E_3 c}}{\partial p^2} = -2e(1+\theta^2) < 0$ can be derived. If $e > \frac{(1+g\theta)^2+\theta^2}{2(1+\theta^2)}$, we can get that $\frac{\partial^2 \pi_C^{E_3 c}}{\partial p^2} * \frac{\partial^2 \pi_C^{E_3 c}}{\partial p \partial q} - \frac{\partial^2 \pi_C^{E_3 c}}{\partial q \partial p} * \frac{\partial^2 \pi_C^{E_3 c}}{\partial q^2} = -(1+g\theta)^2 + 2e(1+\theta^2) > 0$ and $|H(p,q,s)| = 1 - 2e + 2g\theta + \theta^2 - 2e\theta^2 + g^2\theta^2 < 0$. So the profit function is concave, indicating that $\pi_C^{E_3c}(p,q,s)$ has a unique maximum. Let $\frac{\partial \pi_C^{E_3c}}{\partial p}=0$, $\frac{\partial \pi_C^{E_3c}}{\partial q}=0$, $\frac{\partial \pi_C^{E_3c}}{\partial s}=0$, the optimal decisions are shown as follows:

$$p^{E_3c^*}=\frac{a(-1+\gamma-\gamma\theta)}{1-2e(1+\theta^2)+\theta(\theta+g(2+g\theta))} \tag{61}$$

$$q^{E_3c^*}=\frac{-a(1+\gamma(-1+\theta))(1+g\theta)}{1-2e(1+\theta^2)+\theta(\theta+g(2+g\theta))} \tag{62}$$

$$s^{E_3c^*}=\frac{a\theta(-1+\gamma-\gamma\theta)}{1-2e(1+\theta^2)+\theta(\theta+g(2+g\theta))} \tag{63}$$

$$\pi_C^{E_3c^*}=\frac{(a+a\gamma(-1+\theta))^2}{-2+4e(1+\theta^2)-2\theta(\theta+g(2+g\theta))}. \tag{64}$$

7.2.2. Models under centralized decision-making

$$\pi_a^{E_3d}=(1-\phi)\,p\theta(\gamma a-ep\theta+gq+s)+p\left((1-\gamma)a-ep+gq+\lambda s\right)-\frac{1}{2}q^2-F \tag{65}$$

$$\pi_o^{E_3d}=\phi\,p\theta(\gamma a-ep\theta+gq+s)-\frac{1}{2}s^2+F. \tag{66}$$

According to equation (66), the celebrity's profit function with respect to s is obtained by the backward induction method. Since $\frac{\partial^2\pi_o^{E_3d}}{\partial s^2}=-1<0$, the celebrity's profit function is concave with respect to the sales effort and has a maximum value. To obtain the maximum sales effort, let $\frac{\partial \pi_o^{E_3d}}{\partial s}=0$, we get that

$$s=p\phi\theta. \tag{67}$$

Substituting equation (67) into the enterprise's profit function in equation (65), we obtain the Hessian matrix:

$$H(q,p)=\begin{bmatrix}-1 & 1-g\theta(-1+\phi)\\ 1-g\theta(-1+\phi) & -2e-2\theta^2(-1+\phi)(-e+\phi)\end{bmatrix}.$$

If $e>\frac{(1+g\theta)^2+\theta^2}{2(1+\theta^2)}$, we get that $|H(q,p)|=-1+2e\left(1-\theta^2(-1+\phi)\right)-\theta(-1+\phi)\left(-2g+g^2\theta(-1+\phi)-2\theta\phi\right)>0$. Therefore, the profit function is concave, and there is a unique maximum of $\pi_a^{E_3d}(q,p)$. Let $\frac{\partial \pi_a^{E_3d}}{\partial q}=0$ *and* $\frac{\partial \pi_a^{E_3d}}{\partial p}=0$, we have:

$$p^{E_3d^*}=\frac{-a(-1+\gamma+\gamma\theta(-1+\phi))}{2e+2\theta^2(-1+\phi)(-e+\phi)-(1+g(\theta-\theta\phi))^2} \tag{68}$$

$$q^{E_3d^*}=\frac{-a(-1+g\theta(-1+\phi))(-1+\gamma+\gamma\theta(-1+\phi))}{1+2e(-1+\theta^2(-1+\phi))+\theta(-1+\phi)\left(-2g+g^2\theta(-1+\phi)-2\theta\phi\right)}. \tag{69}$$

Substituting equations (68) and (69) into the celebrity's profit function in equation (67), we have

$$s^{E_3d^*}=\frac{a\theta(-1+\gamma+\gamma\theta(-1+\phi))\phi}{2e+2\theta^2(-1+\phi)(-e+\phi)-(1+g(\theta-\theta\phi))^2}. \tag{70}$$

By substituting the optimal solution into equations (65) and (66), we can conclude the enterprise's and celebrity's profits as follows:

$$\pi_a^{E_3d^*} = \frac{-a^2(-1+\gamma+\gamma\theta(-1+\phi))^2}{2A_1} - F \tag{71}$$

$$\pi_o^{E_3d^*} = F - \frac{A_2\phi\begin{pmatrix} 2g(1+\gamma(-1+\theta(-1+\phi))) \\ +2g^2(-1+\gamma)\theta(-1+\phi)+\theta(-2e+\phi)+A_3 \end{pmatrix}}{2A_1^2} \tag{72}$$

$$\pi_C^{E_3d^*} = \frac{-(A_4+A_5)\phi + (A_6+\gamma(-3+\theta))\theta^2\phi^2 + (1+g^2)\gamma\theta^3\phi^3}{2A_1^2} \tag{73}$$

$$A_1 = 1 + 2e(-1+\theta^2(-1+\phi)) + \theta(-1+\phi)(-2g+g^2\theta(-1+\phi)-2\theta\phi)$$
$$A_2 = a^2\theta(-1+\gamma+\gamma\theta(-1+\phi))$$
$$A_3 = \gamma\left(-2+\theta(-1+3\theta(-1+\phi))\phi + 2e(2+\theta(1+\theta-\theta\phi))\right)$$
$$A_4 = a^2(-1+\gamma+\gamma\theta(-1+\phi))\left((1+\gamma(-1+\theta))\left(-(1+g\theta)^2+2e(1+\theta^2)\right)\right)$$
$$A_5 = \theta\begin{pmatrix} (-1+2e)\gamma + 2g(2+\gamma(-2+\theta)) + 2(1+2e)(-1+\gamma)\theta \\ -2(1+e)\gamma\theta^2 + g^2\theta(4+\gamma(-4+3\theta)) \end{pmatrix}$$
$$A_6 = 3 - 3g^2(1+\gamma(-1+\theta)).$$

7.2.3. Analysis and comparison

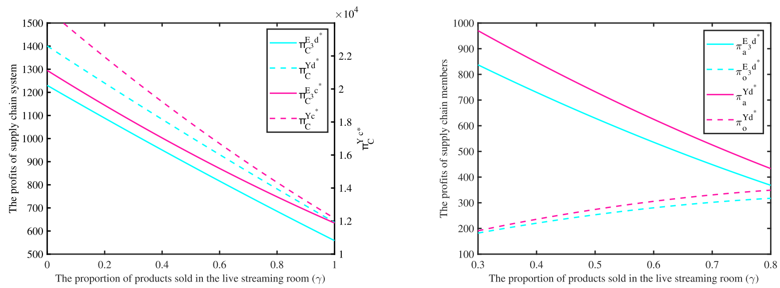
Proposition 7. *By comparing the models with and without spillover effects, we get the following conclusion:*

(1) $p^{E_3c^*} < p^{Y^{c^*}}$, $q^{E_3c^*} < q^{Y^{c^*}}$, $s^{E_3c^*} < s^{Y^{c^*}}$;
(2) $p^{E_3d^*} < p^{Y^{d^*}}$, $q^{E_3d^*} < q^{dc^*}$, $s^{E_3d^*} < s^{Y^{d^*}}$.

By comparing the models with and without spillover effects, several key observations emerge regarding the impact of spillover on supply chain optimization in e-commerce. In the centralized model, the absence of spillover effects results in lower values for pricing, quality improvement efforts, and sales efforts compared to the model with spillover effects. Specifically, the pricing in the no-spillover model is lower than that in the spillover model, indicating that spillover effects facilitate higher pricing by enhancing consumer engagement and perceived value. Similarly, quality improvement efforts and sales efforts in the no-spillover model are lower compared to their counterparts in the spillover model, suggesting that spillover effects incentivize greater investment in both product quality and sales activities. This demonstrates how spillover effects, particularly in a live-streaming or influencer-driven context, can drive more proactive involvement from supply chain participants. These trends also hold true in the decentralized model, with pricing, quality improvement, and sales efforts all being lower in the no-spillover model compared to the spillover model. This further emphasizes the positive role that spillover effects play, not only in centralized decision-making structures but also in decentralized settings where individual players act independently. Importantly, the presence of spillover effects appears to significantly enhance the incentive structures within the supply chain, promoting higher levels of quality improvement, pricing optimization, and sales efforts in both centralized and decentralized contexts.

In this extended model, we explore how the proportion of live-streaming sales with and without spillover effects affects the profits of supply chain members. By reviewing existing literature and incorporating real-world cases, we propose the following hypotheses. Assumptions $a = 120, F = 80, \theta = 0.7, e = 3, g = 1, \lambda = 1.7, \gamma = 0.4$ are made to verify the impact of spillovers on the profits of the supply chain system (Fig. 15).

In the model with live-streaming spillover effects, both the profits of the enterprise and the celebrities are higher compared to the model without spillover effects. This can be attributed to the increased consumer engagement and sales driven by spillover effects, where positive word-of-mouth and celebrities-driven promotion significantly amplify demand. The presence of spillover effects enhances product visibility and consumer trust,



(a) (b)

FIGURE 15. The impact of live streaming spillover effects on supply chain members and systems. (a) The profits of supply chain system. (b) The profits of supply chain member.

thereby increasing the willingness to pay and encouraging higher purchase frequencies. As a result, the enterprise benefits from higher sales volumes and is able to command higher prices, while the celebrities, motivated by performance-based incentives, also experience greater commission-based earnings. Furthermore, the overall profit of the supply chain system is higher in the spillover model than in the no-spillover model. This suggests that spillover effects not only benefit individual stakeholders within the supply chain but also generate synergies that improve the collective profitability of the entire system. By fostering a more interconnected and mutually reinforcing environment among supply chain members, spillover effects promote a more efficient and profitable supply chain ecosystem, ultimately enhancing performance and profit distribution across all parties involved.

8. Managerial insights

This paper provides valuable insights for the management of two sales modes (*i.e.*, live streaming and online store), focusing on the collaboration between enterprises and celebrities, considering the live broadcasting spillover effect and quality improvement decisions of enterprises. The management implications for agribusiness and celebrities are discussed below:

8.1. Management implications for the enterprise

Maintaining high product quality is critical in today's competitive e-commerce environment, as discerning consumers are increasingly willing to pay a premium for quality goods. To meet this demand, companies must not only prioritize quality in their supply chain operations, but also strategically invest in high-quality raw materials, state-of-the-art machinery and ongoing training for their employees. Establishing effective customer feedback mechanisms can provide valuable insights that enable companies to proactively anticipate and address quality issues. In addition, using data analytics for demand forecasting can optimize inventory management and ensure consistent availability of high-quality products. This quality focus is further supported by strategic partnerships with reliable suppliers, fostering collaboration on innovative product development.

As live sales become a significant revenue stream, it is critical to rigorously evaluate potential anchors that can make authentic connections with viewers and products. Collaborative cost-sharing agreements with anchors can align incentives to improve sales performance. Finally, adopting a multi-channel approach in sales strategy can diversify revenue streams, reduce dependence on a single platform, and ultimately strengthen a company's market position and consumer trust. Through these integrated strategies, companies can effectively thrive in the complex e-commerce environment.

8.2. Management implications for celebrity

To further enhance the effectiveness of celebrity partnerships in e-commerce supply chains, it is crucial to focus on integrated relationship management and value co-creation with enterprises. Celebrities should engage in strategic collaborations with enterprises to develop co-branded marketing campaigns that emphasize both product quality and shared brand values. This not only elevates the brand image but also creates a more cohesive consumer experience. Moreover, engaging in joint innovation with enterprises can lead to the development of exclusive product lines or unique offerings that cater specifically to the preferences of the celebrity's audience, thereby driving brand loyalty and increased sales. Additionally, celebrities should leverage data analytics to gain insights into consumer behavior patterns and preferences. By understanding these dynamics, they can tailor their marketing strategies and broadcast content to better align with consumer expectations, enhancing engagement and conversion rates. Further, incorporating interactive elements during live broadcasts, such as real-time Q&A sessions or personalized recommendations based on viewer interaction, can elevate the consumer experience and foster a stronger connection with the audience. It is also essential to prioritize transparency and authenticity in all marketing efforts. Celebrities should clearly communicate product attributes and their genuine opinions to build trust with consumers, avoiding overstated claims that could harm their credibility and reputation. Finally, establishing robust feedback loops and continuously refining sales tactics based on consumer responses will ensure responsiveness to market changes and help maintain a competitive edge. Through these comprehensive strategies, celebrities and enterprises can collaboratively optimize their e-commerce presence, driving sustainable growth and mutual benefits.

9. Conclusion

Taking into account the spillover effect of live streaming and the potential impact of quality improvement, we have developed a "live streaming + online store" sales model for products. This model consists of an online and live streaming sales modes, with centralized and decentralized decision-making structures. Through the construction of a game-theoretic model, we aim to determine the influence of the commission rates and the quality improvement on the optimal decisions and profits for the enterprises and celebrity involved in the supply chain. To achieve supply chain coordination, we propose a cost-sharing joint commission mechanism, incorporating the exogenous commission rate, to facilitate coordination between the enterprise and celebrity. This mechanism ensures that both parties align their incentives and share the costs associated with quality improvement, ultimately optimizing the overall supply chain performance. Our analysis has yielded several key insights:

Firstly, when the spillover effect of the live streaming is relatively high, increasing the commission rate positively affects the optimal price and the profit of the supply chain members under the decentralized decision-making structure. In addition, a higher commission rate leads to an increase sales effort in both the basic model and the quality improvement model, but decreased efforts in quality improvement in the latter case. Secondly, the adoption of quality improvement strategies is beneficial for supply chain members. Thirdly, employing the cost-sharing joint commission mechanism allows for the coordination of the two sales modes (*i.e.*, live streaming and online store), thereby enabling a win–win situation for all involved members and increasing the overall system's profit. Lastly, our results indicate that in the coordination mechanism, the celebrity's sharing proportion of quality improvement costs aligns with their sharing proportion of the system's profit, while the commission rate has a negative impact on their individual profit.

It's important to acknowledge the limitations of our study. We have only considered a two-echelon dual-channel supply chain with a single enterprise and celebrity. Furthermore, our analysis focuses solely on the commission charged by celebrities to the enterprise, not accounting for the transaction fees that enterprise also pay to the e-commerce platform. Therefore, further research can explore a three-stage dual-channel supply chain model, involving the enterprise, the celebrity, and the e-commerce platform.

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Appendix A.

A.1. Proof of Proposition 1

$$\frac{\partial p^{Y^{d^*}}}{\partial \phi} = \frac{a\theta\left(a\theta\left(-g^2(-2+\gamma(3+\theta(-1+\phi)))(-1+\theta(-1+\phi))+\lambda-\gamma\lambda\right)\right)}{\left(2e(\theta^2(\phi-1)-1)-\theta\phi(\theta\phi-2\lambda+2)+(\theta+1)^2\right)^2} + \frac{2a\theta(\theta-e(\gamma+(-1+\gamma)\theta)-2\theta\phi+\gamma\theta(-1+\lambda+\theta(-1+\phi)^2+2\phi))}{\left(2e(\theta^2(\phi-1)-1)-\theta\phi(\theta\phi-2\lambda+2)+(\theta+1)^2\right)^2}.$$

The denominator is greater than 0.

When $\lambda > \frac{2e\gamma-2(1+e)\theta+g^2\left(-2+\gamma\left(3+\theta\left(-1+\phi\right)\right)\right)\left(-1+\theta\left(-1+\phi\right)\right)+2\theta\left(\gamma\left(1+e-\theta\left(-1+\phi\right)^2-2\phi\right)+2\phi\right)}{2+2\gamma(-1+\theta)}$, the molecule is greater than 0. Hence, $\frac{\partial p^{Y^{d^*}}}{\partial \phi} > 0$.

$$\frac{\partial p^{N^{d^*}}}{\partial \phi} = \frac{\alpha\theta\left(\theta(\gamma-1)(2\phi-e)+(\gamma(\theta-1)+1)(\theta+\lambda)+\gamma\theta^2(\phi(\phi-2)-e\gamma)\right)}{2\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)^2}.$$

The denominator is greater than 0 and the molecule is expressed as $\alpha\theta$ times the first-order function of γ. First, we make a monotonicity judgment. It can be expressed as $f(e)$. It is monotonically decreasing. Let $e = e_1$ into $f(e)$, we get

$$\frac{-\lambda(2+\lambda)-\theta(2+\lambda^2-4\phi)+2\theta^4(-1+\phi)^2+\theta^3(-3+2\lambda+4\phi)}{2(1+\theta^2)} + \frac{\theta^2\left(1-4\lambda+2(-2+\phi)\phi\right)}{2(1+\theta^2)}$$

it can be expressed as the quadratic function $f(\lambda)$, which goes downwards. $\Delta_{f(\lambda)} = \frac{1+\theta\left(-2+4\phi+\theta\left(2+(-2+\theta)\theta-4\theta\phi+2(1+\theta)\phi^2\right)\right)}{\left(1+\theta^2\right)} > 0$, $f(\lambda = 0) < 0$, so $f(\lambda)$ always less than 0 and $f(e)$ less than 0. Hence, the function of γ is monotonically decreasing.

$$f(\gamma = 0) = \theta + e\theta + \lambda - 2\theta\phi > 0$$

when $\lambda > \frac{\gamma\left(e-(1-\phi)^2\theta^2\right)+\theta(1-\gamma)(2\phi-1-e)}{1+\gamma(-1+\theta)}$, $f(\gamma = 1) = -e + \theta\left(\lambda + \theta(-1+\phi)^2\right)$ is greater than 0. Hence, when $\lambda > \frac{\gamma\left(e-(1-\phi)^2\theta^2\right)+\theta(1-\gamma)(2\phi-1-e)}{1+\gamma(-1+\theta)}$, $\frac{N^{d^*}}{\partial\phi} > 0$.

$$\frac{\partial q^{Y^{d^*}}}{\partial\phi} = \frac{ag\theta\begin{pmatrix} 2(1+\gamma(-1+\theta))(1+\theta)(\theta+\lambda)+g^2(-1+2\gamma)\left(1+\theta-\theta\phi\right)^2 \\ -4(1+\gamma(-1+\theta))\theta(1+\theta)\phi+2\theta^2(1+\gamma\theta-\gamma\lambda)\phi^2 \\ -2e\left(1+\theta\left(-1+\gamma\left(3+\theta^2(-1+\phi)^2-2\phi\right)\right)\right)\end{pmatrix}}{\left(2e\left(-1+\theta^2(-1+\phi)\right)+g^2\left(1+\theta-\theta\phi\right)^2+2\theta\phi\left(\theta+\lambda-\theta\phi\right)\right)^2}$$

$$\frac{\partial s^{Y^{d^*}}}{\partial\phi} = \frac{a\theta\begin{pmatrix} 2e\left(1+\theta^2+\gamma\left(-1+\theta\left(1+\theta\left(-1+\theta\left(-1+\phi\right)^2\right)-2\phi\right)\right)\right) \\ +2\theta^2\left(-1+\gamma+\gamma\lambda\right)\phi^2-g^2\left(-1+\theta\left(-1+\phi\right)\right)\cdot \\ \left(-1+\gamma-\gamma\theta^2+\gamma\theta\left(3+\theta\right)\phi-\theta\left(1+\phi\right)\right)\end{pmatrix}}{\left(2e\left(-1+\theta^2\left(-1+\phi\right)\right)+g^2\left(1+\theta-\theta\phi\right)^2+2\theta\phi\left(\theta+\lambda-\theta\phi\right)\right)^2}$$

$$\frac{\partial s^{N^{d^*}}}{\partial\phi} = \frac{a\begin{pmatrix} e(1+\gamma(-1+\theta))(\theta+\theta^3)-2e\gamma\theta^2(1+\theta^2)\phi \\ +\theta^3\left(-1+\gamma(1+e\theta+\lambda)\right)\phi^2\end{pmatrix}}{\left(2\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)^2\right)}.$$

The proof of $\frac{\partial q^{Y^{d^*}}}{\partial\phi} < 0$, $\frac{\partial s^{Y^{d^*}}}{\partial\phi} > 0$ and $\frac{\partial q^{N^{d^*}}}{\partial\phi} > 0$ is similar to the above.

A.2. Proof of Proposition 2

$$\frac{\partial\pi_{a^{Y^{d*}}}}{\partial\phi} = -\frac{\alpha^2\theta A_1\left(A_2+\gamma(2-e(2+\theta+\theta^2)-\lambda+e\theta^2\phi+A_3)\right)}{((1+\theta)^2+2e\left(-1+\theta^2(-1+\phi)\right)-\theta\phi(2-2\lambda+\theta\phi))^2}$$

where $A_1 = -1+\gamma+\gamma\theta(-1+\phi)$, $A_2 = -1+e\theta+\lambda-\theta\phi$, $A_3 = \theta(1+\theta+\lambda-\theta\phi+\lambda\phi)$.

The denominator is greater than 0.

When $0 < \lambda < \frac{1-e\theta+\theta\phi+(-1+e)\gamma\left(2+\theta(1+\theta-\theta\phi)\right)}{(1+\gamma(-1+\theta+\theta\phi))}$, the molecule is less than 0. Hence, $\frac{\partial\pi_a^{Y^{d^*}}}{\partial\phi} < 0$.

$$\frac{\partial\pi_o^{Y^{d^*}}}{\partial\phi} = -\frac{-A_1\left(A_4+\phi(3\theta^2\phi-A_5)\right)+\phi\left(-2A_2(\theta^2\phi^2+A_4)\right)}{2\left(A_4+\phi(-\theta^2\phi+A_5)\right)^3} - \frac{+\phi^2\left(A_6(2A_4+\phi A_5)-A_3\left(3A_4+\phi(\theta^2\phi+A_5)\right)\right)}{2\left(A_4+\phi(-\theta^2\phi+A_5)\right)^3}$$

where $A_1 = 2\alpha^2\left(1+\gamma(-1+\theta)\right)\theta\left(1+\theta-e\theta-2\gamma(1+\theta)+e\gamma(2+\theta+\theta^2)\right)$, $A_2 = \alpha^2\theta^2\left(-1+\gamma\left(2-6\theta-4\lambda+\gamma\left(1-4e\left(1+\theta^2\right)+\theta(10-3\theta-4\lambda)+4\lambda\right)\right)\right)$, $A_3 = 2\alpha^2\gamma\theta^3\left(2+\gamma\left(-3+(3+e)\theta+2\lambda\right)\right)$, $A_4 = (1+\theta)^2-2e\left(1+\theta^2\right)$, $A_5 = 2\theta(-1+e\theta+\lambda)$, $A_6 = 6\alpha^2\gamma^2\theta^4\phi$.

$$\frac{\partial\pi_a^{Nd}}{\partial\phi} = -\frac{\alpha^2\theta A_1\left(\theta+\lambda+\gamma(-1+\theta)(\theta+\lambda)+\theta\left(-2+\gamma(2-\theta+\lambda)\right)\phi\right)}{4\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)^2} + \frac{\alpha^2\theta A_1 e\left(\theta-\gamma\left(2+\theta+\theta^2\right)+\gamma\theta^2\phi\right)}{4\left(\theta^2(\phi-e)(1-\phi)+\theta\phi\lambda-e\right)^2}$$

where $A_1 = -1 + \gamma + \gamma\theta(-1 + \phi)$.

$$\frac{\partial \pi_o^{N^{d^*}}}{\partial \phi} = \frac{a^2\theta(A_1 + e^2\left(\gamma^2 A_2 - 2\gamma A_3 + \theta^3(\phi+1) + \theta\right) + e\theta\phi(A_4 + A_5 + A_6)}{4\left(e\left(\theta^2(\phi-1)-1\right) + \theta\phi\left(\theta(-\phi) + \theta + \lambda\right)\right)^3}$$

where $A_1 = \theta^3\phi^3(\gamma\lambda + \gamma - 1)(\gamma(\theta(-\phi) + \theta + 2\lambda + 1) - 1)$,
$A_2 = \theta\left(\theta^4(\phi-1)^3 + \theta^2((7-3\phi)\phi - 2) + 2\theta(\phi+1) + 4\phi - 1\right) + 2)$,
$A_3 = (\theta+1)\left(\theta^2(\phi+1)+1\right)$
$A_4 = \left(-2\gamma^2\theta^4\phi^3 + (\gamma(\theta-1)+1)\left(\gamma\left(2\theta^3 + (\theta(3\theta-1)+2)\lambda + \theta + 1\right) + \theta\lambda - 1\right)\right)$,
$A_5 = \gamma\theta^2\phi^2(\gamma(\theta(6\theta+3\lambda-1)+2)+\theta) - 3\theta\phi(\gamma(\gamma\theta(\theta(2\theta+2\lambda-1)+2))+\theta)$,
$A_6 = -3\theta\phi\gamma(+2\gamma\lambda + \gamma + (\theta-2)\theta - 1) - 3\theta^2\phi\gamma$.

The proof of $\frac{\partial \pi_o^{Y^{d^*}}}{\partial \phi}$, $\frac{\partial \pi_a^{Y^{d^*}}}{\partial \phi}$ and $\frac{\partial \pi_a^{N^{d^*}}}{\partial \phi}$ is similar to the above.

A.3. Proof of Proposition 3

$$\frac{\partial p^{Y^{d^*}}}{\partial \lambda} = -\frac{2\phi A_1}{\left(2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)\right)^2}$$

$$\frac{\partial q^{Y^{d^*}}}{\partial \lambda} = \frac{2g\phi A_1(-1+\theta(-1+\phi))}{\left(2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)\right)^2}$$

$$\frac{\partial s^{Y^{d^*}}}{\partial \lambda} = -\frac{2\theta A_1\phi^2}{\left(2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)\right)^2}$$

$$\frac{\partial \pi_a^{Y^{d^*}}}{\partial \lambda} = \frac{4aA_1^2\phi}{\left(4e\left(-1+\theta^2(-1+\phi)\right) + 2g^2(1+\theta-\theta\phi)^2 + 4\theta\phi(\theta+\lambda-\theta\phi)\right)^2}$$

$$\frac{\partial \pi_o^{Y^{d^*}}}{\partial \lambda} = \frac{\begin{array}{c}2a\theta A_1\phi^2\left(2e(\gamma+(-1+\gamma)\theta) + \theta\phi(1-\gamma(1+\theta+2\lambda)+\gamma\theta\phi)\right)\\ + g^2(-2+\gamma(3+\theta(-1+\phi)))(-1+\theta(-1+\phi))\end{array}}{\left(2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)\right)^3}$$

$$\frac{\partial \pi_C^{Y^{d^*}}}{\partial \lambda} = \frac{aA_1\phi\begin{pmatrix}2\theta\phi\left(-((1+\gamma(-1+\theta))(\theta+\lambda)) + \theta(2+\gamma(-2+\theta-\lambda))\phi\right)\\ + g^2(-1+\theta(-1+\phi))(1+\theta-5\theta\phi+\gamma A_2)\\ + 2e\left(1+\theta^2-3\theta^2\phi+\gamma(-1+\theta(1+\phi+\theta A_3))\right)\end{pmatrix}}{\left(2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)\right)^3}$$

where $A_1 = a\theta(-1+\gamma+\gamma\theta(-1+\phi))$, $A_2 = \left(-1+\theta\left(\theta+6\phi-4\theta\phi+3\theta\phi^2\right)\right)$, $A_3 = \left(-1+\theta(-1+\phi)^2+3\phi\right)$.

When $e > e_1$, we can get $2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi) < 0$. Thus, we have $\frac{\partial p^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial q^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial s^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial \pi_a^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial \pi_o^{Y^{d^*}}}{\partial \lambda} > 0$, $\frac{\partial \pi_C^{Y^{d^*}}}{\partial \lambda} > 0$.

A.4. Proof of Proposition 4

$$\frac{p^{Y^{d^*}}}{p^{Y^{c^*}}} = \frac{(-1+\gamma-\gamma\theta)\left(2e\left(-1+\theta^2(-1+\phi)\right) + g^2(1+\theta-\theta\phi)^2 + 2\theta\phi(\theta+\lambda-\theta\phi)\right)}{\left(g^2(1+\theta)^2 - 2e(1+\theta^2) + (\theta+\lambda)^2\right)(-1+\gamma+\gamma\theta(-1+\phi))}.$$

Subtract the denominator from its numerator to get the first-order function about γ. First, we make a monotonicity judgment. It can be expressed as a primary function of ϕ. The graph of $f_1(\phi)$ opens upward and $\Delta_{\left(f_1(\phi)\right)} > 0$. We can know that the function of γ is monotonically increased.

$$f(\gamma=1)=\theta(2\theta\phi\lambda-2\theta(\theta\phi-\lambda)(-1+\phi)$$
$$+\lambda^2(-1+\phi)+(-2e+g^2)\phi+\theta 2(-1+\phi)(1+g2\phi))<0.$$

We can indicate that the $f(\gamma)$ is less than 0, so $\frac{p^{Y^{d^*}}}{p^{Y^{c^*}}}<0$ and $p^{Y^{d^*}}-p^{Y^{c^*}}<0$.

When $\phi=0$, we get $q^{Y^{d^{**}}}=\frac{\alpha(-1-\theta)(-1+\gamma-\theta\gamma)}{\left(-(1+\theta)^2+2e(1+\theta^2)\right)}$. It is minus the quality improvement level under the centralized decision $q^{Y^{c^*}}$ as follows:

$$q^{Y^{c^*}}=\frac{ag(1+\theta)(\theta+\lambda)^2\left(1+(-1+\theta)\gamma\right)}{\left(-g^2(1+\theta)^2+2e(1+\theta^2)\right)\left(g^2(1+\theta)^2-2e(1+\theta^2)+(\theta+\lambda)^2\right)}.$$

The numerator is greater than 0. The $\left(-g^2(1+\theta)^2+2e\left(1+\theta^2\right)\right)$ is a function of e. Let $e=e_1$, we get $(\theta+\lambda)^2>0$. So $\left(-g^2(1+\theta)^2+2e\left(1+\theta^2\right)\right)>0$. $f(e)=\left(g^2(1+\theta)^2-2e\left(1+\theta^2\right)+(\theta+\lambda)^2\right)$ is a function of e, which decreases monotonically concerning e. Let $e=e_1$, we obtain $f(e)=0$, so within the definition field of e, $f(e)$ is less than 0. Therefore, equation (24) is less than 0 and $q^{Y^{d^{**}}}-q^{Y^{c^*}}<0$. Since the $q^{Y^{d^*}}$ decreases with the increases of ϕ, when $\phi>0$, $q^{Y^{d^*}}<q^{Y^{d^{**}}}$. So we can get $q^{Y^{d^*}}-q^{Y^{c^*}}<0$.

The proof of other conclusions is similar to the proof of $p^{d^*}-p^{c^*}<0$, so the paper omits it.

A.5. Proof of Proposition 5

$$\frac{\pi_C^{N^{c^*}}}{\pi_C^{N^{d^*}}}=\frac{4\left(1+\gamma(-1+\theta)\right)^2\left(e\left(-1+\theta^2(-1+\phi)\right)+\theta\phi(\theta+\lambda-\theta\phi)\right)^2}{A_1A_2\left(-2e\left(1+\gamma(-1+\theta)\right)(1+\theta^2)+2\theta A_3\phi+A_4-\gamma\theta^3\phi^3\right)}$$

where $A_1=(2e(1+\theta^2)-(\theta+\lambda)^2)$, $A_2=(-1+\gamma+\gamma\theta(-1+\phi))$, $A_3=(\theta+2e\theta+e\gamma(-1+(-2+\theta)\theta)+\lambda+\gamma(-1+\theta)(\theta+\lambda))$, $A_4=\theta^2(-3+\gamma(3-\theta+2\lambda))\phi^2$.

Subtract the denominator from its numerator to get the quadratic function about γ and the graph of $f(\gamma)$ opens upward. $f(\gamma=0)$ and $f(\gamma=1)$ are greater than 0, and the symmetric axis of the graph of $f(\gamma)$ is less than 0. Hence, $f(\gamma)>0$ and $\pi_C^{N^{c^*}}>\pi_C^{N^{d^*}}$.

The proof of other conclusions is similar to the proof above, so the paper omits it.

A.6. Proof of Proposition 6

Bring $\bar{\phi}=\frac{\begin{array}{c}-2F-2gpq+q^2+2gpq\eta-2q^2\eta-s^2\eta+2ap(-1+\eta)(1+\gamma(-1+\theta))\\-2gpq\theta-2ps\theta+2gpq\eta\theta+2ps\eta\theta-2ep^2(-1+\eta)(1+\theta^2)+2ps(-1+\eta)\lambda\end{array}}{2\theta p(a\gamma-e\theta p+gq+s)}$ into the function of π_a^{Yco} and π_o^{Yco}, we get that $\pi_a^{Yco}=\delta\pi_C^C$ and $\pi_o^{Yco}=(1-\delta)\pi_C^C$.

A.7. Proof of Proposition 7

When $e>\frac{(1+g\theta)^2+\theta^2}{2(1+\theta^2)}$, $(2e-(1-g\theta(-1+\phi))^2+2\theta^2(-1+\phi)(-e+\phi))>0$.

When $e>\frac{1+2g\theta+\theta^2+g^2\theta^2}{2(1+\theta^2)}$, $(1-2e(1+\theta^2)+\theta(\theta+g(2+g\theta)))<0$.

Thus, we can get: $p^{E_3c^*}<p^{Y^{c^*}}$, $q^{E_3c^*}<q^{Y^{c^*}}$, $s^{E_3c^*}<p^{Y^{c^*}}$. $p^{E_3d^*}<p^{Y^{d^*}}$, $q^{E_3d^*}<q^{dc^*}$, $s^{E_3d^*}<p^{Y^{d^*}}$.

Appendix B.

$$\lambda_1 = \frac{\begin{array}{l}2e\gamma - 2(1+e)\theta + g^2\left(-2+\gamma\left(3+\theta\left(-1+\phi\right)\right)\right)\left(-1+\theta\left(-1+\phi\right)\right)\\ +2\theta\left(\gamma\left(1+e-\theta\left(-1+\phi\right)^2-2\phi\right)+2\phi\right)\end{array}}{2+2\gamma(-1+\theta)}$$

$$\lambda_2 = \frac{\gamma\left(e-(1-\phi)^2\theta^2\right)+\theta(1-\gamma)(2\phi-1-e)}{1+\gamma(-1+\theta)}$$

$$\lambda_3 = \frac{1-e^{\theta}+\theta\phi+(-1+e)\gamma\left(2+\theta(1+\theta-\theta\phi)\right)}{1+\gamma(-1+\theta+\theta\phi)}$$

$$\lambda_4 = \frac{\theta(\gamma-1)\left(1+e^{-2\phi}\right)+\gamma\theta^2(\phi-1)(1-e)+2e\gamma}{1+\gamma(-1+\theta+\theta\phi)}$$

$$\lambda_5 = \frac{\sqrt{\theta(2\left(1+\theta^2\right)^2\left(2+\theta+\theta^3\right)+2\theta A_1\phi+\theta^3(A_3+A_4)-\theta^2 A_2}}{2\left(\theta+\theta^3(1+\phi)\right)}$$

where $A_1 = \left(-2+\theta\left(6+\theta(-3+6\theta+\theta^4)\right)\right)$, $A_2 = \left(2+\phi+\theta^2(2+3\phi)\right)$, $A_3 = \left(3+\theta\left(8+10\theta+7\theta^3\right)\right)\phi^2$, $A_4 = \left(6\theta^5(1+\theta^2)\phi^3\right)$

$$\begin{aligned}\gamma_1 =& 2\left((1+\theta)^3-\theta(1+\theta)(-1+\theta+2\lambda)\phi+3\theta^2(1+\theta)\phi^2-\theta^3\phi^3\right)\\ &+6e^2\theta^2\left(\theta+\theta^3(1+\phi)\right)\\ &+2e\left(-2-\theta\left(3-2\phi+\theta\left(4+4\phi-2\lambda\phi+3\theta(1+\phi^2)\right)\right)\right).\end{aligned}$$