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Convex Hull Pricing via An Explicit Formulation for the Lagrangian Dual of the Network-constrained Unit Commitment

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***Abstract*—Convex hull pricing (CHP) is a pivotal approach to enhance market transparency by minimizing uplift costs. This paper revisits the mathematical foundation of CHP and provides an explicit formulation of the Lagrangian dual formulation for network-constrained unit commitment (NCUC), further defining the CHP. Here, a convex hull model for single-unit commitment (1UC) problems is established with ramping constraints and minimum on/off time, making this explicit formulation implementable and further delivering the optimal Lagrangian dual solution via two linear programming (LP) models. The first LP reformulates the NCUC by replacing mixed-integer constraints with convex hull relaxations, while the second, obtained by fixing the inner variables in the Lagrangian dual problem of the NCUC to their optimal values from the first LP, generates the optimal Lagrangian dual solution. Numerical experiments on the IEEE-118 and Polish-2383 systems validate the superiority of CHP in reducing uplift costs and of this proposed pricing method in computational efficiency.**

***Index Terms*—Convex hull pricing, unit commitment, electricity spot market, uplift cost, Lagrangian duality**

NOMENCLATURE

Sets

$\Omega_g^{\text{MILP-1UC}}$	Feasible region of unit g modeled as the mixed-integer linear programming (MILP)
$\boldsymbol{S}_g$	State space of the unit g
$\boldsymbol{A}_g(\bullet)$	State set of the unit g that contains all successor states of the input state
$\boldsymbol{A}_g^-(\bullet)$	State set that contains all former states of the input state
$\boldsymbol{S}_g^+$	State set of the unit g only including all states that this unit is online
$\boldsymbol{S}_g^-$	State set of the unit g only including all states that this unit is offline

Parameters

G	Numbers of units
T	Numbers of scheduling periods
B	Numbers of buses
$P_{b,t}^{\text{load}}$	Load demand of the bus b at the period t
$H_{l,b(g)}$	Power transfer distribution factors from the bus of the unit g to the transmission line l
$H_{l,b}$	Power transfer distribution factors from the bus of the bus b to the transmission line l
F_l	Capacity of the transmission line l
C_g^{u}	Start-up and shut-down costs of the unit g
C_g^{d}	Shut-down costs of the unit g
$K_{g,i}$	Slope of the i-th segment of the cost function for the unit g
$M_{g,i}$	Intercept of the i-th segment of the cost function for the unit g
$P_{g,t}^{\text{seg}}$	Segment point of the i-th segment of the cost function for the unit g
$P_g^{\min}$	Minimum power outputs of unit g
$P_g^{\max}$	Maximum power outputs of unit g
R_g^{regu}	Regular ramp rate of the unit g
R_g^{init}	Start-up/shut-down ramp rate of the unit g
T_g^{on}	Minimum on time of the unit g
T_g^{off}	Minimum off time of the unit g
$\Pi_{b,t}$	Convex hull price of the bus b at the period t

Variables

$c_{g,t}$	Cost of the unit g at the period t
$p_{g,t}$	Power output of the unit g at the period t
$x_{g,t}$	Online state of the unit g at the period t
$u_{g,t}$	Start-up instruction of the unit g at the period t
$d_{g,t}$	Shut-down instruction of the unit g at the period t
s_t	State of a unit at the period t
$\omega_g(s_t, s_{t+1})$	Lagrangian dual variable corresponding to the constraint, which describes the state transition from the state s_t to the state s_{t+1}
λ_t	Lagrangian dual variable corresponding to the power balance at the period t
$\beta_{l,t}$	Lagrangian dual variable corresponding to the reverse capacity of the line l at the period t

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$\gamma_{l,t}$	Lagrangian dual variable corresponding to the forward capacity of the line l at the period t
$f_{l,t}$	Power flow on the line l at the period t

Functions

$f(\bullet,\bullet)$	Fuel cost of the unit with the input power output and the online state
$\varpi_g(\bullet)$	State-value function to the input state
$V_g(\bullet)$	Cost of the input state
$[\bullet]_i$	i-th component of the input state
$\mathrm{ConvH}(\bullet)$	convex hull of the MILP feasible set

I. Introduction

The deregulation of electricity markets has underscored the importance of transparent pricing mechanisms to reflect resource values and guide market equilibrium [1]-[3]. Since the main content of electricity market construction, the electricity price and its pricing mechanism can reflect the resource values and guide the market participants to join the market in a sufficiently competitive manner, attracting wide attention from both academic and industrial fields [4]-[7].

To ensure market transparency, the electricity spot market is expected to be established as a perfectly competitive market in which each participant attempts to maximize its own profit until the market equilibrium is satisfied and finally happens to be the maximum welfare. In this situation, the electricity price will be the marginal cost of the whole power system. Therefore, in past practice, locational marginal prices (LMPs), which represent this marginal cost, have been widely implemented in independent system operators (ISOs), such as PJM, New York ISO, New England ISO, California ISO, Midwest ISO, and several Chinese provincial ISOs [8]-[9].

LMPs have been investigated in [3] and [10]-[15]. Among them, in [3], the authors started utilizing the marginal costs to price the electricity energy production. Later, realizing the necessity of managing network congestion, the authors in [10] took the congestion cost into the electricity price, eventually giving the LMPs. Note that such LMPs, which now can reflect not only the energy cost but also its delivery cost, have been widely applied today. Furthermore, LMPs have been extended in [11] to take into account the incalculable transmission loss in the past, thus making LMPs more accurate. Then, in [12], authors further explored the accuracy of the LMPs considering the transmission loss and analyzed the sensitivity of such LMPs with respect to the system load. Moreover, a solution named continuous LMPs has been proposed to eliminate the inherent step change in LMPs due to the load variation in [13],the loss of the transmission in LMPs then has been embedded in [14], and the authors in [15] introduced uncertainty to LMPs, further accommodating the transition to renewable energy.

Note that the above-mentioned LMPs were derived from the network-constrained economic dispatch (NCED) problem with online states of the unit fixed at the optimal solution of the network-constrained unit commitment (NCUC) problem [16]. In fact, start-up/shut-down and fixed costs determined by these online states are important and should be reflected in electricity prices [17]-[18]. However, the non-convexity brought by such binary decisions makes it hard to establish a uniform electricity price that supports maximum welfare [19], which indicates that the participant units tend to deviate from the scheduling instructions issued by ISOs for more revenue. To incentivize compliance, the uplift cost is needed [20]-[22]. However, the uplift cost was defined in a non-uniform manner, whose non-transparency would undermine the fairness of the electricity spot market, and should be minimized as much as possible [23].

Motivated by this, the authors in [21] then introduced an approach that treated online states as continuous variables to price non-convexities in the NCUC, which reduced uplift costs. In a similar vein, non-convexities have been priced by relaxing integrality constraints in the NCUC problem while incorporating equality constraints that fixed the online state to its optimal value in [17] and [24]. Later, in [22], the authors have introduced the concept of the convex hull pricing, which has since attracted significant attention due to its theoretical potential to minimize the uplift cost. Specifically, the minimum uplift cost has been proved to be equal to the duality gap between the NCUC problem and its Lagrangian dual problem. Convex hull prices, developed by the optimal dual solution, naturally achieve the minimum uplift cost.

However, obtaining the convex hull price is not straightforward, as deriving the exact Lagrangian dual solution poses challenges. Therefore, in [25], a sub-gradient simplex cutting plane method has been proposed to approximate the convex hull price. Further advancements have been made in [26], which incorporated an incentive-compatibility constraint into the pricing model to get closer to the convex hull price. In [27], a pricing scheme then has been designed to reduce price discrimination and align it with the minimum uplift cost. Subsequently, a convex primal formulation for convex hull pricing was introduced in [23], and Benders decomposition was employed in [28] to accelerate this formulation. Note that the key to the convex primal formulation lies in developing the convex hull model for the single-unit commitment (1UC) problem. To make the convex primal formulation implementable, the authors in [29] then proposed a convex hull model for the 1UC without ramping constraints, while the authors in [30] developed a more advanced state-transition formulation. In addition, a network-flow formulation in [31] described the feasible region of each unit, offering an approximate convex hull price. Moreover, several studies also focused on the polyhedral description of 1UC problems. In [32], the authors excluded integers and proposed a polynomial algorithm for the 1UC. In [33], a tight polyhedral approximation has been introduced for quadratic cost functions. Beyond the 1UC problem, the ramping production is described as a polyhedron by a series of inequalities [34], which were applicable to 1UC. Later, the online interval has been investigated and several inequalities also have been introduced for the min-up/down constraints in UCs in [35]. Most notably, the authors in [36] conducted a comprehensive polyhedral study, deriving strong valid inequalities to yield a convex hull model for the fuel-constrained 1UC problem. Next, several studies have investigated solving the Lagrangian dual problem of the NCUC.

An extreme-point subdifferential method was introduced in [37] and [38], focusing on solving the Lagrangian dual problem of the NCUC. It ensured a better convergence performance, eventually yielding a more effective convex hull price. In addition, the authors in [39] has employed the level method, which can converge within fewer iterations to a certain gap, yielding a price that outperformed many of the traditional approaches. Moreover, the authors in [40] conducted a comprehensive survey, which reviewed and compared the performance across many existing approaches. In general, the authors in [41] described convex hull pricing by Dantzig-Wolfe decomposition and Column Generation, achieving convergence. Its innovation lay in that it analyzed the essence of the convex primal formulation and cut along the edge of this formulation. In theory, this can lead to the exact Lagrangian dual solution, thus yielding the convex hull price. In recent research, more researchers in [42]-[43] announced the benefits of convex hull pricing in transparency, and there have researchers compared the convex hull pricing with several prices that have been applied in practice, showing its excellent performance in [44].

Although the convex hull pricing has gained attention due to its excellent performance, challenges still exist in its applications, either due to the fact that obtaining the exact optimal Lagrangian dual solution of the NCUC is hard [45], or requiring massive iterations. To address this issue, this paper redefines the formulation for the optimal Lagrangian dual solution of the NCUC, and the main contributions of this paper are as follows:

i) An explicit Lagrangian dual formulation of the NCUC is derived, where the convex hull model for 1UC problems is employed with ramping constraints and minimum on/off time, enabling the optimal Lagrangian dual solution via two linear programming (LP) models. The first LP reformulates the NCUC by replacing mixed-integer constraints with convex hull relaxations, while the second, which is obtained by fixing the inner variables in the Lagrangian dual problem to their optimal values from the first LP, gives the optimal Lagrangian dual solution.

ii) A convex hull pricing method is proposed utilizing the above proposed explicit Lagrangian dual formulation of the NCUC. Compared with the traditional iteration methods, the proposed method only needs to solve the LP model for the optimal Lagrangian dual solution of the original NCUC. Therefore, the convex hull pricing model is also explicit and does not require any iteration, definitely avoiding the convergence issue and huge computational burden, compared with the previous iterative methods when addressing pricing problems.

The remainder of this paper is organized as: Section II illustrates the explicit Lagrangian dual formulation of NCUC, Section III shows the proposed convex hull pricing, Section IV does several experiments, and Section V concludes this paper.

II. Explicit Formulation for the Lagrangian Dual of the NCUC

A. Basis of the NCUC Problem

At the beginning, the NCUC problem aims to minimize the total operation costs, including fuel cost and unit start-up/shut-down costs, while satisfying power balance and transmission capacity constraints. Generally speaking, the NCUC problem can be denoted as the following MILP formulation:

P1
$$\min \sum\nolimits_{g=1}^{G}\sum\nolimits_{t=1}^{T} c_{g,t} \tag{1a}$$

s.t.
$$\sum\nolimits_{g=1}^{G} p_{g,t} = \sum\nolimits_{b=1}^{B} P_{b,t}^{\text{load}}, \forall t \tag{1b}$$

$$\left| \sum\nolimits_{g=1}^{G} H_{l,b(g)} p_{g,t} - \sum\nolimits_{b=1}^{B} H_{l,b} P_{b,t}^{\text{load}} \right| \le F_l, \forall l, \forall t \tag{1c}$$

$$(c_{g,t}, p_{g,t}, x_{g,t}, u_{g,t}, d_{g,t}) \in \Omega_g^{\text{MILP-1UC}}, \forall g \tag{1d}$$

where G, T, and B represent the numbers of units, scheduling periods, and buses, respectively; $c_{g,t}$, $p_{g,t}$, $x_{g,t}$, $u_{g,t}$, and $d_{g,t}$ are the cost, power output, online state, start-up instruction, and shut-down instruction of the unit g at the period t, respectively; $P_{b,t}^{\text{load}}$ denotes the load demand of the bus b at the period t; $H_{l,b(g)}$ and $H_{l,b}$ are the power transfer distribution factors from the bus of the unit g and bus b to the transmission line l; and F_l is the capacity of the transmission line l; $\Omega_g^{\text{MILP-1UC}}$ denotes the feasible region of unit g modeled as the mixed-integer linear programming (MILP). In this NCUC model, (1a) denotes the objective function, representing the total cost; (1b)-(1c) represent these system-wide constraints, which include both power balance and transmission limits; and (1d) denotes the feasible region of the 1UC problems. Specifically, the feasible region $\Omega_g^{\text{MILP-1UC}}$ is:

$$c_{g,t} = C_g^{\text{u}} u_{g,t} + C_g^{\text{d}} d_{g,t} + f_g(p_{g,t}, x_{g,t}), \forall g, \forall t \tag{2a}$$

$$f_g(p_{g,t}, x_{g,t}) \ge K_{g,i}(p_{g,t} - P_{g,i}^{\text{seg}} x_{g,t}) + M_i x_t, \forall g, \forall t, \forall i \tag{2b}$$

$$u_{g,t} + d_{g,t} \le 1, \forall g, \forall t \tag{2c}$$

$$u_{g,t} - d_{g,t} = x_{g,t} - x_{g,t-1}, \forall g, \forall t \tag{2d}$$

$$x_{g,t} P_g^{\min} \le p_{g,t} \le x_{g,t} P_g^{\max}, \forall g, \forall t \tag{2e}$$

$$p_{g,t} - p_{g,t-1} \le R_g^{\text{regu}} + u_{g,t}(R_g^{\text{init}} - R_g^{\text{regu}}), \forall g, \forall t \tag{2f}$$

$$p_{g,t-1} - p_{g,t} \le R_g^{\text{regu}} + d_{g,t}(R_g^{\text{init}} - R_g^{\text{regu}}), \forall g, \forall t \tag{2g}$$

$$\sum\nolimits_{\tau=t}^{t+T_g^{\text{on}}-1} x_{g,\tau} \ge u_{g,t} T_g^{\text{on}}, \forall g, \forall t \tag{2h}$$

$$\sum\nolimits_{\tau=t}^{T} x_{g,\tau} \ge u_{g,t}(T - t + 1), \forall g, \forall t \tag{2i}$$

$$\sum\nolimits_{\tau=t}^{t+T_g^{\text{off}}-1} (1 - x_{g,\tau}) \ge d_{g,t} T_g^{\text{off}}, \forall g, \forall t \tag{2j}$$

$$\sum\nolimits_{\tau=t}^{T} (1 - x_{g,\tau}) \ge d_{g,t}(T - t + 1), \forall g, \forall t \tag{2k}$$

$$u_{g,t}, d_{g,t}, x_{g,t} \in \{0,1\}, \forall g, \forall t \tag{2l}$$

where C_g^{u} and C_g^{d} are start-up and shut-down costs of the unit g; $f(p_{g,t}, x_{g,t})$ is the fuel cost of the unit g operating at period t when its power output is $p_{g,t}$ and its online state is $x_{g,t}$; $K_{g,i}$, $M_{g,i}$, and $P_{g,t}^{\text{seg}}$ are slope, intercept, and segment point of the i-th segment of the cost function for the unit g; $P_g^{\min}$ and $P_g^{\max}$ are minimum and maximum power outputs of unit g; R_g^{regu} is the regular ramp rate of the unit g; R_g^{init} is the start-up/shut-down ramp rate of the unit g; T_g^{on} and T_g^{off} are minimum on and off time of the unit g; $x_{g,\tau}$ is auxiliary variable for constructing minimum on/off constraints, which denotes the online state of the unit g at the period τ. In this region, (2a)-(2b) is the cost; (2c)-(2d) define the logical relationship among the online status, start-up instruction, and shut-down instruction; (2e) is the power output range; (2f)

-(2g) limit the maximum ramp rate of the unit; and (2h)-(2k) are the minimum on/off time limits; (2l) are binary variables.

B. Convex Hull Model for the 1UC Problem

Now, this subsection develops a convex hull model for the 1UC problem. At the beginning, the overview of constructing this convex hull model can be visualized as Fig. 1 shows:

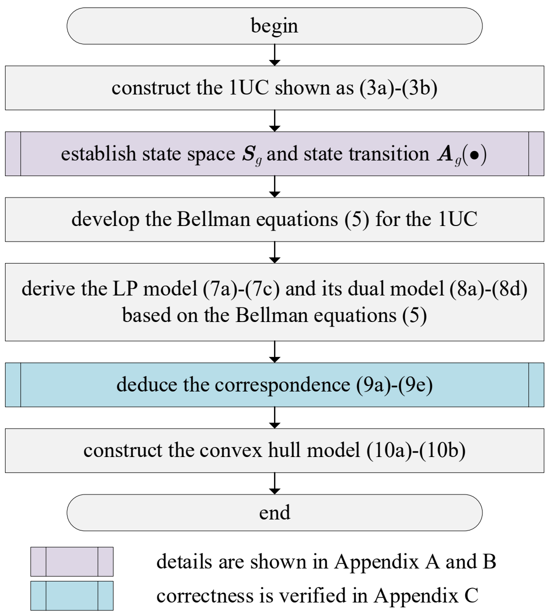

Fig. 1. Flowchart of Constructing the Convex Hull Model for the 1UC.

As shown in Section II-A, the 1UC problem could be:

$$\min \quad \sum\nolimits_{t=1}^{T} c_{g,t} \tag{3a}$$

s.t.
$$(c_{g,t}, p_{g,t}, x_{g,t}, u_{g,t}, d_{g,t}) \in \Omega_g^{\mathrm{MILP-1UC}} \tag{3b}$$

Note that the 1UC problem has been formulated as a sequential decision-making problem. As a result, this 1UC problem can be addressed through the dynamic programming (DP) technique. Here, let $\boldsymbol{S}_g$ be the state space of the unit g, whose components are five-dimensional vectors (see Appendix A). Moreover, let $\boldsymbol{A}_g(\bullet)$ be the state set of the unit g that contains all successor states of the input state based on the feasible region $\Omega_g^{\text{MILP-1UC}}$ (see Appendix B). Now, the above 1UC problem can be described by the following Bellman equations:

$$\sum\nolimits_{t=1}^{T} c_{g,t}^{*} := \min \quad \{\varpi_g(s_1) \mid s_1 \in \boldsymbol{S}_g\} \tag{4}$$

where $c_{g,t}^{*}$ is the optimal value of the $c_{g,t}$, s_t denotes the state at the period t, and $\varpi_g(s_t)$ denotes the state-value function corresponding to the state s_t, which can be represented as follows:

$$\varpi_g(s_t) = \min \quad \{V_g(s_t) + \varpi_g(s_{t+1}) \mid s_{t+1} \in \boldsymbol{A}_g(s_t)\}, \forall s_t \in \boldsymbol{S}_g \tag{5}$$

and $V_g(\bullet)$ represents the cost of the input state, i.e.,

$$V_g(s_t) = C_g^{\mathrm{u}}[s_t]_3 + C_g^{\mathrm{d}}[s_t]_4 + f_g([s_t]_1, [s_t]_2), s_t \in \boldsymbol{S}_g \tag{6}$$

In (6), $[s_t]_i$ is the i-th component of the state s_t. Then, such above Bellman equations can be solved as an LP, i.e.,

$$\max \quad \varpi_g \tag{7a}$$

s.t.
$$\varpi_g \le \varpi_g(s_1), \forall s_1 \in \boldsymbol{S}_g \tag{7b}$$

$$\varpi_g(s_t) \le V_g(s_t) + \varpi_g(s_{t+1}), \forall s_{t+1} \in \boldsymbol{A}_g(s_t), \forall s_t \in \boldsymbol{S}_g \tag{7c}$$

However, only the optimal objective of the 1UC problem can be obtained from this LP model. Therefore, to further deduce the optimal scheduling decision for the 1UC problem, the optimal decision of the Bellman equations is necessary, which can be derived from the more informative dual model of the above LP model (7a)-(7c). This dual model can be denoted as follows:

$$\min \quad \sum\nolimits_{t=1}^{T} \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) V_g(s_t) \tag{8a}$$

s.t.
$$\sum\nolimits_{s_1 \in \boldsymbol{S}_g} \sum\nolimits_{s_2 \in \boldsymbol{A}_g(s_1)} \omega_g(s_1, s_2) = 1 \tag{8b}$$

$$\sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) = \sum\nolimits_{s_{t-1} \in \boldsymbol{A}_g^{-}(s_t)} \omega_g(s_{t-1}, s_t), s_t \in \boldsymbol{S}_g \tag{8c}$$

$$\omega_g(s_t, s_{t+1}) \ge 0, s_{t+1} \in \boldsymbol{A}_g(s_t), s_t \in \boldsymbol{S}_g \tag{8d}$$

where $\omega_g(s_t, s_{t+1})$ is the Lagrangian dual variable corresponding to the associated constraint, which describes the state transition from the state s_t to the state s_{t+1} in (7c). $\boldsymbol{A}_g^{-}(\bullet)$ denotes a set that contains all former states of the input state based on $\Omega_g^{\text{MILP-1UC}}$. According to the complementary slackness condition, once an optimal Lagrangian dual solution is nonzero, its corresponding constraint is binding. Since each constraint in the LP model (7a)-(7c) represents a state transition, the binding one has a specific practical meaning to the optimal Lagrangian dual solution. Specifically, if $\omega_g(s_t, s_{t+1})=1$, $\varpi_g(s_t) \le V_g(s_t) + \varpi_g(s_{t+1})$ is binding, which indicates that the optimal state of unit g is the state s_t at the period t and should transfer to the state s_{t+1} at the period $t+1$. After drawing the correspondence, the scheduling decision can be derived through the following equations:

$$c_{g,t} = \sum\nolimits_{t=1}^{T} \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) V_g(s_t) \tag{9a}$$

$$p_{g,t} = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) [s_t]_1 \tag{9b}$$

$$x_{g,t} = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) [s_t]_2 \tag{9c}$$

$$u_{g,t} = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) [s_t]_3 \tag{9d}$$

$$d_{g,t} = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g(s_t, s_{t+1}) [s_t]_4 \tag{9e}$$

Now, the optimal scheduling decision of the 1UC problem (3a)-(3b) can be obtained from the LP model (8a)-(8d), along with (9a)-(9e). In addition, this LP model, along with (9a)-(9e) could be recognized as the convex hull model of the 1UC problem (3a)-(3b) (see Appendix C). Now, this convex hull model can be represented as the following formulation:

$$\min \quad \sum\nolimits_{t=1}^{T} c_{g,t} \tag{10a}$$

$$(c_{g,t}, p_{g,t}, x_{g,t}, u_{g,t}, d_{g,t}) \in \mathrm{ConvH}(\Omega_g^{\mathrm{MILP-1UC}}) \tag{10b}$$

where $\mathrm{ConvH}(\Omega_g^{\text{MILP-1UC}})$, which is (8b)-(8d) and (9a)-(9e), denotes the convex hull of the original MILP feasible set.

C. Explicit Formulation Construction

Since the convex hull of the 1UC problem is developed in an explicit manner, an LP model can be deduced, which is:

P2
$$\min \quad \sum\nolimits_{g=1}^{G} \sum\nolimits_{t=1}^{T} c_{g,t} \tag{11a}$$

s.t.
$$\sum\nolimits_{g=1}^{G} p_{g,t} = \sum\nolimits_{b=1}^{B} P_{b,t}^{\mathrm{load}}, \forall t \tag{11b}$$

$$|\sum_{g=1}^{G} H_{l,b(g)} p_{g,t} - \sum_{b=1}^{B} H_{l,b} P_{b,t}^{\text{load}}| \le F_l, \forall l, \forall t \quad (11c)$$

$$(c_{g,t}, p_{g,t}, x_{g,t}, u_{g,t}, d_{g,t}) \in \text{ConvH}(\Omega_g^{\text{MILP-1UC}}), \forall g \quad (11d)$$

Note that, formally, **P2** is the same as **P1**, with the only difference being that each unit in **P2** is replaced with its convex hull. Now, based on the well-known result in Lagrangian relaxation world (see e.g., Theorem 5.1 in [46]), the following **Proposition 1** extends this result to the NCUC, which illustrates that **P2** can be used to derive the optimal dual solution of **P1**.

Proposition 1: Note that the optimal dual objective of **P1** is equivalent to the optimal objective of **P2**.

Proof: Here, let **P1** be denoted as its compact formulation, i.e.,

C1
$$\min \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} \quad (12a)$$

s.t.
$$\boldsymbol{D}\boldsymbol{y} \ge \boldsymbol{d} : (\boldsymbol{\pi}) \quad (12b)$$

$$\boldsymbol{y} \in \mathbf{Y} \quad (12c)$$

where (12b) represents (1b)-(1c), (12c) represents (1d), and $\boldsymbol{\pi}$ is the Lagrangian dual variable of the corresponding constraint (1b)-(1c). Now, the associated Lagrangian dual problem is:

D-C1
$$\max_{\boldsymbol{\pi} \in \mathbf{R}_+} \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}) = \max_{\boldsymbol{\pi} \in \mathbf{R}_+} \min_{\boldsymbol{y} \in \mathbf{Y}} \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{\pi}(\boldsymbol{D}\boldsymbol{y} - \boldsymbol{d}) \quad (13)$$

Notice that the dual function $\mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi})$ is an MILP optimizing a linear function within the MILP set **Y**. Given that the optimal solution of a linear function should be the vertex of its feasible region, the value of the Lagrangian dual function $\mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi})$ can be obtained through the following LP model, i.e.,

$$\max \quad \varphi \quad (14a)$$

s.t.
$$\varphi \le \boldsymbol{q}^{\mathrm{T}} - \boldsymbol{\pi}(\boldsymbol{D}\boldsymbol{y}_i^{\circ} - \boldsymbol{d}), i = 1, \ldots, |\operatorname{vert}(\mathbf{Y})| \quad (14b)$$

where $\boldsymbol{y}_i^{\circ}$ is the i-th vertex of the set **Y,** and $\operatorname{vert}(\bullet)$ is an operator used to generate all vertices of the input set. Then, by substituting (14a)-(14b) into (13), $\boldsymbol{\pi}$ becomes the decision variable and the Lagrangian dual problem (13) could be reformulated as an LP model, which is represented as follows:

$$\max \quad \varphi \quad (15a)$$

s.t.
$$\varphi \le \boldsymbol{q}^{\mathrm{T}} - \boldsymbol{\pi}(\boldsymbol{D}\boldsymbol{y}_i^{\circ} - \boldsymbol{d}) : (\delta_i), i = 1, \ldots, |\operatorname{vert}(\mathbf{Y})| \quad (15b)$$

$$\boldsymbol{\pi} \in \mathbf{R}_+ \quad (15c)$$

where δ_i is the dual variable associated with its constraint.

Note that an LP model will satisfy the strong duality if it is feasible. Hence, the optimal objective of the above LP model could be obtained from its dual problem, i.e.,

$$\min \quad \sum_{i=1}^{|\operatorname{vert}(\mathbf{Y})|} \delta_i \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}_i^{\circ} \quad (16a)$$

s.t.
$$\sum_{i=1}^{|\operatorname{vert}(\mathbf{Y})|} \delta_i (\boldsymbol{D}\boldsymbol{y}_i^{\circ} - \boldsymbol{d}) \ge 0 \quad (16b)$$

$$\sum_{i=1}^{|\operatorname{vert}(\mathbf{Y})|} \delta_i = 1 \quad (16c)$$

$$\delta_i \ge 0, \boldsymbol{y}_i^{\circ} \in \operatorname{vert}(\mathbf{Y}), \forall i \quad (16d)$$

Note that all vertices of the set **Y** take the convex combination by Lagrangian dual variables, represented as:

$$\sum_{i=1}^{|\operatorname{vert}(\mathbf{Y})|} \delta_i \boldsymbol{y}_i^{\circ}, \sum_{i=1}^{|\operatorname{vert}(\mathbf{Y})|} \delta_i = 1 \quad (17a)$$

$$\delta_i \ge 0, \boldsymbol{y}_i^{\circ} \in \operatorname{vert}(\mathbf{Y}), \forall i \quad (17b)$$

which defines a feasible region that coincides with the convex hull of the MILP set **Y**, enabling (16a)-(16d) to be denoted as:

C2
$$\min \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} \quad (18a)$$

s.t.
$$\boldsymbol{D}\boldsymbol{y} \ge \boldsymbol{d} : (\boldsymbol{\pi}) \quad (18b)$$

$$\boldsymbol{y} \in \text{ConvH}(\mathbf{Y}) \quad (18c)$$

Through the above transformations, the optimal dual objective of **C2** is verified to be the optimal objective of **C2**, yielding:

$$\max_{\boldsymbol{\pi} \in \mathbf{R}_+} \quad \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}) = \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}^{\mathbf{C2}(*)} \quad (19)$$

where $\boldsymbol{y}^{\mathbf{C2}(*)}$ is the optimal solution of (18a)-(18c). Note that **C2** can be recognized as the compact formulation of **P2**, further showing that the optimal dual objective of **P1** is equivalent to the optimal objective of **P2**. **Q.E.D.**

Moreover, this LP, i.e., **P2**, can be used to derive the optimal dual solution of **P1**, as demonstrated in **Proposition 2**.

Proposition 2: Optimal Lagrangian dual solutions **P1** and **P2** coincide with each other.

Proof: let the compact formulations **C1** and **C2** represent **P1** and **P2**. Here, it begins with the Lagrangian dual of **C2**, i.e.,

D-C2
$$\max_{\boldsymbol{\pi} \in \mathbf{R}_+} \mathcal{D}^{\mathbf{C2}}(\boldsymbol{\pi}) = \max_{\boldsymbol{\pi} \in \mathbf{R}_+} \min_{\boldsymbol{y} \in \text{ConvH}(\mathbf{Y})} \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{\pi}(\boldsymbol{D}\boldsymbol{y} - \boldsymbol{d}) \quad (20)$$

Based on Strong Duality, no duality gap will exist between **C2** and its dual problem **D-C2**, resulting in an equivalence:

$$\mathcal{D}^{\mathbf{C2}}(\boldsymbol{\pi}^{\mathbf{C2}(*)}) = \min_{\boldsymbol{y} \in \text{ConvH}(\mathbf{Y})} \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{\pi}^{\mathbf{C2}(*)}(\boldsymbol{D}\boldsymbol{y} - \boldsymbol{d}) = \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}^{\mathbf{C2}(*)} \quad (21)$$

where $\boldsymbol{\pi}^{\mathbf{C2}(*)}$ represents the optimal dual solution of **C2**, and it also can be a feasible dual solution of **C1**, yielding:

$$\mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}^{\mathbf{C2}(*)}) = \min_{\boldsymbol{y} \in \mathbf{Y}} \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{\pi}^{\mathbf{C2}(*)}(\boldsymbol{D}\boldsymbol{y} - \boldsymbol{d}) \quad (22)$$

Generally speaking, optimizing a linear function within its feasible region is equivalent to doing so over the convex hull of that region. Therefore, the following derivation can be made:

$$\mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}^{\mathbf{C2}(*)}) = \min_{\boldsymbol{y} \in \text{ConvH}(\mathbf{Y})} \quad \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{\pi}^{\mathbf{C2}(*)}(\boldsymbol{D}\boldsymbol{y} - \boldsymbol{d}) \quad (23)$$

Note that the right-hand side of (23) is $\mathcal{D}^{\mathbf{C2}}(\boldsymbol{\pi}^{\mathbf{C2}(*)})$, which is equal to $\boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}^{\mathbf{C2}(*)}$. Now, according to (19), the relationship has:

$$\boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}^{\mathbf{C2}(*)} = \max_{\boldsymbol{\pi} \in \mathbf{R}_+} \quad \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}) \ge \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}^{\mathbf{C2}(*)}) = \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}^{\mathbf{C2}(*)} \quad (24)$$

which, based on the Squeeze Theorem, suggests that:

$$\max_{\boldsymbol{\pi} \in \mathbf{R}_+} \quad \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}) = \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}^{\mathbf{C2}(*)}) \Rightarrow \boldsymbol{\pi}^{\mathbf{C2}(*)} \in \arg\max_{\boldsymbol{\pi} \in \mathbf{R}_+} \quad \mathcal{D}^{\mathbf{C1}}(\boldsymbol{\pi}) \quad (25)$$

Here, the optimal dual solution of **C2** is equivalent to that of **C1**, verifying that the optimal Lagrangian dual solutions **P1** and **P2** have coincided with each other. **Q.E.D.**

Now, according to **Proposition 1** and **Proposition 2**, an alternative LP formulation, i.e., **P2**, has been illustrated that it can be employed to address the Lagrangian dual problem of **P1**.

III. Exact Convex Hull Pricing Based on The Proposed Explicit Formulation

Generally speaking, the convex hull price is described by the marginal cost of each bus based on the NCUC in a direct manner. To obtain the convex hull, it could begin by defining the functional relationship between the minimum cost and the load demand of the bus b at the period t as follows:

$$\phi(P_{b,t}^{\text{load}}) := \begin{array}{ll} \min & \sum_{g=1}^{G}\sum_{t=1}^{T} c_{g,t} \\ \textbf{s.t.} & \text{(1b) - (1d)} \end{array} \tag{26}$$

where $\phi(P_{b,t}^{\text{load}})$, indeed, is the functional relationship between the optimal objective of **P1** and the associated load demand.

Of course, the associated marginal cost can be obtained by differentiating $\phi(P_{b,t}^{\text{load}})$ on $P_{b,t}^{\text{load}}$ according to the Equilibrium Theory [21]-[24]. However, identifying the implicit function $\phi(P_{b,t}^{\text{load}})$ is obviously difficult. Therefore, the Envelope Theorem is taken, providing the equivalence to differential $\phi(P_{b,t}^{\text{load}})$ over $P_{b,t}^{\text{load}}$, which can be denoted as the following formulation:

$$\frac{\mathrm{d}\phi(P_{b,t}^{\text{load}})}{\mathrm{d}P_{b,t}^{\text{load}}} = \left.\frac{\partial \mathcal{L}^{\mathbf{P1}}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma})}{\partial P_{b,t}^{\text{load}}}\right|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}^{\mathbf{P1}(*)},\boldsymbol{\beta}=\boldsymbol{\beta}^{\mathbf{P1}(*)},\boldsymbol{\gamma}=\boldsymbol{\gamma}^{\mathbf{P1}(*)}} \tag{27}$$

where $\boldsymbol{\lambda}^{\mathbf{P1}(*)}$, $\boldsymbol{\beta}^{\mathbf{P1}(*)}$, and $\boldsymbol{\gamma}^{\mathbf{P1}(*)}$ represent associated dual solutions while $\mathcal{L}^{\mathbf{P1}}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma})$ is the Lagrangian function of **P1**, i.e.,

$$\begin{aligned} \mathcal{L}^{\mathbf{P1}}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma}) = & \sum_{g=1}^{G}\sum_{t=1}^{T} c_{g,t} \\ & -\sum_{t=1}^{T} \lambda_t (\sum_{g=1}^{G} p_{g,t} - \sum_{b=1}^{B} P_{b,t}^{\text{load}}) \\ & -\sum_{l=1}^{L}\sum_{t=1}^{T} \beta_{l,t}(F_l + f_{l,t}) \\ & -\sum_{l=1}^{L}\sum_{t=1}^{T} \gamma_{l,t}(F_l - f_{l,t}) \end{aligned} \tag{28}$$

with λ_t, $\beta_{l,t}$, and $\gamma_{l,t}$ are Lagrangian dual variables corresponding to the power balance at the period t, the reverse transmission limit of the line l at the period t, and forward transmission limit of the line l at the period t. In addition, $f_{l,t}$ represents the power flow on the line l, at the period t, whose detail is as follows:

$$f_{l,t} = \sum_{g=1}^{G} H_{l,b(g)} p_{g,t} - \sum_{b=1}^{B} H_{l,b} P_{b,t}^{\text{load}}, \forall l, \forall t \tag{29}$$

Now, based on the equivalence denoted by (27), the convex hull price can be generated as the following formulation, i.e.,

$$\begin{aligned} \Pi_{b,t} &= \left.\frac{\partial \mathcal{L}^{\mathbf{P1}}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma})}{\partial P_{b,t}^{\text{load}}}\right|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}^{\mathbf{P1}(*)},\boldsymbol{\beta}=\boldsymbol{\beta}^{\mathbf{P1}(*)},\boldsymbol{\gamma}=\boldsymbol{\gamma}^{\mathbf{P1}(*)}} \\ &= \lambda_t^{\mathbf{P1}(*)} + \sum_{l=1}^{L} H_{l,b}(\beta_{l,t}^{\mathbf{P1}(*)} - \gamma_{l,t}^{\mathbf{P1}(*)}) \end{aligned} \tag{30}$$

where $\Pi_{b,t}$ denotes the convex hull price of the bus b at the period t. In fact, the convex hull price is obtained from the exact optimal Lagrangian dual solution of **P1**. However, only approximate dual solutions are achieved in most implementations.

As shown in Section II, an explicit LP formulation, denoted by **P2**, is introduced, whose Lagrangian dual solution is equivalent to that of **P1**. Therefore, the convex hull price can be obtained through **P2**. Specifically, solving the dual problem of **P2** can be solved through two steps involving two LPs:

Step 1: Solve the LP model **P2**, and subsequently provide its optimal solutions $c_{g,t}^{\mathbf{P2}(*)}$, $p_{g,t}^{\mathbf{P2}(*)}$, $x_{g,t}^{\mathbf{P2}(*)}$, $u_{g,t}^{\mathbf{P2}(*)}$, and $d_{g,t}^{\mathbf{P2}(*)}$.

Step 2: Solve the following LP model, i.e.,

$$\max_{\boldsymbol{\lambda}\in\mathbf{R},\boldsymbol{\beta}\in\mathbf{R}_+,\boldsymbol{\gamma}\in\mathbf{R}_+} \mathcal{D}^{\mathbf{P2}(*)}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma}) \tag{31}$$

where $\mathcal{D}^{\mathbf{P2}(*)}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma})$ is the Lagrangian dual function of **P2** with $c_{g,t}$, $p_{g,t}$, $x_{g,t}$, $u_{g,t}$, and $d_{g,t}$ fixed at their respective optimal values obtained from **P2**, i.e., $c_{g,t}^{\mathbf{P2}(*)}$, $p_{g,t}^{\mathbf{P2}(*)}$, $x_{g,t}^{\mathbf{P2}(*)}$, $u_{g,t}^{\mathbf{P2}(*)}$, and $d_{g,t}^{\mathbf{P2}(*)}$. Specifically, $\mathcal{D}^{\mathbf{P2}(*)}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma})$ is represented as follows:

$$\begin{aligned} \mathcal{D}^{\mathbf{P2}(*)}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma}) = & \sum_{g=1}^{G}\sum_{t=1}^{T} c_{g,t}^{\mathbf{P2}(*)} \\ & -\sum_{t=1}^{T} \lambda_t (\sum_{g=1}^{G} p_{g,t}^{\mathbf{P2}(*)} - \sum_{b=1}^{B} P_{b,t}^{\text{load}}) \\ & -\sum_{l=1}^{L}\sum_{t=1}^{T} \beta_{l,t}(F_l + f_{l,t}^{\mathbf{P2}(*)}) \\ & -\sum_{l=1}^{L}\sum_{t=1}^{T} \gamma_{l,t}(F_l - f_{l,t}^{\mathbf{P2}(*)}) \end{aligned} \tag{32}$$

and $f_{l,t}^{\mathbf{P2}(*)}$ is the value of $f_{l,t}$ as $p_{g,t}$ fixed at $p_{g,t}^{\mathbf{P2}(*)}$. Next, solving (31) can generate the dual solution of **P2** (i.e., $\boldsymbol{\lambda}^{\mathbf{P2}(*)}$, $\boldsymbol{\beta}^{\mathbf{P2}(*)}$, and $\boldsymbol{\gamma}^{\mathbf{P2}(*)}$), coinciding with **P1**. Hence, the convex hull price, denoted by (30), could be determined as follows:

$$\Pi_{b,t} = \lambda_t^{\mathbf{P2}(*)} + \sum_{l=1}^{L} H_{l,b}(\beta_{l,t}^{\mathbf{P2}(*)} - \gamma_{l,t}^{\mathbf{P2}(*)}) \tag{33}$$

Now, through **P2**, the exact dual solution of the NCUC is obtained by solving two LPs, yielding the exact convex hull price.

Discussion: *Employing this explicit formulation, the Lagrangian dual solution of NCUC, i.e.,* ***P1****, can be obtained through (11a)-(11d), i.e.,* ***P2****, which only needs to solve two LP models with well-established LP algorithms. In comparison, the traditional implicit Lagrangian dual formulation relies on iterative techniques, leading to a huge computational burden and endless oscillations. Meanwhile, the oscillations might result in a low-quality solution. In contrast, by using this proposed explicit formulation, obtaining the Lagrangian dual solution of the NCUC does not rely on iterations, improving the computational efficiency and simultaneously enhancing the solution quality.*

On the basis of the mathematical foundation of the convex hull price[17], [22] and [27], the following **Proposition 3** has been employed to demonstrate that the uplift cost yielded by the above proposed convex hull price (33) is equivalent to the duality gap between **P1** and its associated Lagrangian dual problem, which achieves the uplift minimization in theory.

Proposition 3: The uplift cost yielded by the convex hull price (33) equals the duality gap between **P1** and its dual problem.

Proof: Now, the uplift cost to (33) is the fee, including the opportunity costs and the financial transmission rights [22], i.e.,

$$\underbrace{\left\{\begin{array}{ll} \max & \sum_{g=1}^{G}\sum_{t=1}^{T}[(\Pi_{b(g),t}p_{g,t} - c_{g,t}) - (\Pi_{b(g),t}p_{g,t}^{\mathbf{P1}(*)} - c_{g,t}^{\mathbf{P1}(*)})] \\ \textbf{s.t.} & \text{(1d)} \end{array}\right\}}_{\text{Opportunity Costs}} \\ + \underbrace{\sum_{l=1}^{L}\sum_{t=1}^{T}[\beta_{l,t}^{\mathbf{P2}(*)}(F_l + f_{l,t}^{\mathbf{P1}(*)}) + \gamma_{l,t}^{\mathbf{P2}(*)}(F_l - f_{l,t}^{\mathbf{P1}(*)})]}_{\text{Financial Transmission Rights}} \tag{34}$$

where $c_{g,t}^{\mathbf{P1}(*)}$, $p_{g,t}^{\mathbf{P1}(*)}$, and $f_{l,t}^{\mathbf{P1}(*)}$ are optimal values of $c_{g,t}$, $p_{g,t}$, and $f_{l,t}$ derived from the **P1**. In (34), $\Pi_{b(g),t}$ corresponds to (33), i.e.,

$$\Pi_{b(g),t} = \lambda_t^{\mathbf{P2}(*)} + \sum_{l=1}^{L} H_{l,b(g)}(\beta_{l,t}^{\mathbf{P2}(*)} - \gamma_{l,t}^{\mathbf{P2}(*)}) \tag{35}$$

Here, by performing the transformations (details in Appendix D) on (34), this uplift cost (34) will become:

$$\sum_{g=1}^{G}\sum_{t=1}^{T} c_{g,t}^{\mathbf{P1}(*)} - \mathcal{D}^{\mathbf{P1}}(\boldsymbol{\lambda}^{\mathbf{P2}(*)},\boldsymbol{\beta}^{\mathbf{P2}(*)},\boldsymbol{\gamma}^{\mathbf{P2}(*)}) \tag{36}$$

Since $\boldsymbol{\lambda}^{\mathbf{P2}(*)}$, $\boldsymbol{\beta}^{\mathbf{P2}(*)}$, and $\boldsymbol{\gamma}^{\mathbf{P2}(*)}$ coincide with $\boldsymbol{\lambda}^{\mathbf{P1}(*)}$, $\boldsymbol{\beta}^{\mathbf{P1}(*)}$, and $\boldsymbol{\gamma}^{\mathbf{P1}(*)}$, then the term after the minus sign could be:

$$\max_{\boldsymbol{\lambda}\in\mathbf{R},\boldsymbol{\beta}\in\mathbf{R}_+,\boldsymbol{\gamma}\in\mathbf{R}_+} \mathcal{D}^{\mathbf{P1}(*)}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma}) \tag{37}$$

which further yields the following formulation, i.e.,

$$\sum\nolimits_{g=1}^{G}\sum\nolimits_{t=1}^{T} c_{g,t}^{\mathbf{P1}(*)} - \max_{\boldsymbol{\lambda}\in\mathbf{R},\boldsymbol{\beta}\in\mathbf{R}_{+},\boldsymbol{\gamma}\in\mathbf{R}_{+}} \mathcal{D}^{\mathbf{P1}(*)}(\boldsymbol{\lambda},\boldsymbol{\beta},\boldsymbol{\gamma}) \quad (38)$$

As (38) demonstrates, this uplift cost is equivalent to the optimal objective of **P1** minus its optimal dual objective, i.e., the duality gap, thereby verifying **Proposition 3**. **Q.E.D.**

IV. Case Study

To demonstrate the effectiveness of the proposed convex hull pricing approach, this paper then investigates and compares the locational marginal prices (i.e., LMPs), Lagrangian relaxation prices (i.e., LRPs), integer relaxation prices (i.e., IRPs), the convex hull prices based on Dantzig-Wolfe (D-W) decomposition approach (i.e., DWPs) [41], and the proposed convex hull prices (i.e., CHPs) on the IEEE-30 and the Polish-2383 systems. More details of these mentioned approaches are as follows:

M-LMP: Locational marginal prices derived from the optimal Lagrangian dual solution of the SCED with integer fixed;

M-LRP: Approximate convex hull prices derived from the sub-optimal Lagrangian dual solution of the NCUC provided by the sub-gradient algorithm;

M-IRP: Approximate convex hull prices derived from the optimal Lagrangian dual solution of the NCUC, with its integer variables relaxed to continuous variables varying from 0 to 1;

M-DWP: Convex hull prices obtained from the Dantzig-Wolfe decomposition proposed in [41];

M-CHP: Convex hull prices from the optimal Lagrangian dual solution of NCUC provided by the proposed approach.

All these numerical experiments are conducted on a personal computer equipped with an Intel (R) Core (TM) i7-13700K and a 32.0 GB RAM and solved using the solver COPT 7.2.8.

A. IEEE-30 System

Here, this applied IEEE-30 system, containing 6 units, 30 buses, and 41 branches, is tested with the five prices [47]. Generally, ISOs issue the scheduling instructions based on the optimal solution of NCUC and request all units to follow the instructions. Then, ISOs should pay units according to electricity prices. Here, LMPs, LRPs, IRPs, DWPs, and CHPs associated with each unit are depicted in Fig. 2 with issued instructions.

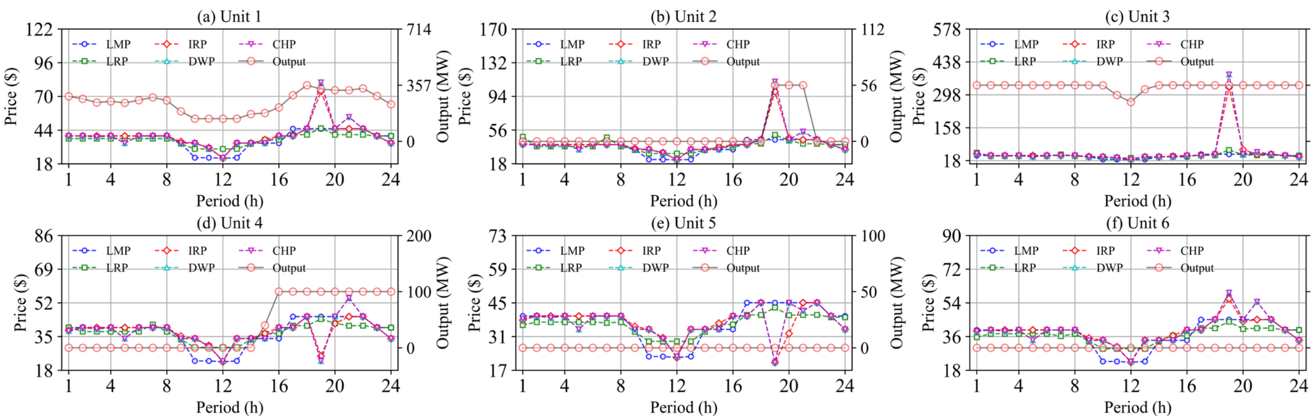

Fig. 2. Prices and Scheduling Instructions.

Note that the M-LMP only considers energy costs, but start-up, shut-down, and fixed costs are included in M-LRP, M-IRP, M-DWP, and M-CHP. If the marginal unit does not tend to start up or shut down, its marginal cost will be determined solely by energy costs, indicating that all of these prices are the same, such as prices during the periods from 0:00 to 14:00 at Unit 5 (Differences might result from the cost generated by financial transmission rights). However, if the marginal unit is about to start up or shut down, its marginal cost will change, causing LMPs to notably differ from LRPs, IRPs, DWPs, and CHPs. Moreover, the changes in the marginal cost of the marginal unit would even alter which unit is considered marginal in the system, finally leading to an elevation in LRPs, IRPs, DWPs, and CHPs compared to LMPs. Specifically, taking the period 11:00 as an example, the LMP is 22.7\$/MW, while LRP, IRP, DWP, and CHP are rising to 29.5\$/MW, 30.5\$/MW, and 30.1\$/MW. Before taking the start-up, shut-down, and fixed costs into account, Unit 1 is online with its minimum output (144MW) and avoids Unit 3 from reaching its maximum output while having the lowest marginal price (22.7\$/MW) to be the marginal unit of the system. Hence, the LMP is 22.7\$/MW. However, once start-up, shut-down, and fixed costs are included, Unit 1 tends to stay offline, causing Unit 3 to reach its limit. To achieve the power balance, Unit 1 is required to keep the online state and will become the marginal unit of the system, thereby leading to an elevation in LRP, IRP, DWP, and CHP. Also, as prices during periods from 0:00 to 14:00 demonstrate, marginal prices at Unit 5 only rely on the energy costs without any intention to start up or shut down, suggesting that LRPs, IRPs, DWPs, and CHPs should be the same as LMPs.

However, LRPs, DWPs, and IRPs mismatch LMPs during these periods, indeed showing that LRPs and IRPs are worse than CHPs. Now, to illustrate the performance of all prices, actual incomes, optimal incomes, and opportunity costs of each unit according to these prices are illustrated in Table I. Actual incomes in the third column of Table I denote the revenues when units settle with LMPs, LRPs, IRPs, DWPs, and CHPs while following the scheduling instructions. Here, these actual incomes show that revenues settled utilizing IRPs (\$55650.5), DWPs (\$64142.6), and CHPs (\$64142.6) exceed those settled utilizing LMPs (\$8480.0), except for LRPs (\$2746.1). Its essence is that by comparing with LMPs, start-up, shut-down, and fixed costs are considered when determining IRPs, DWPs, and CHPs, thereby providing better cost coverage.

Moreover, the optimal incomes are listed in the fourth column, illustrating revenues from self-scheduling. All five prices can guide units to achieve profits. However, there is a difference between the actual and optimal incomes, indeed indicating that these units tend to deviate from the scheduling instructions for more revenue. In the common perception, a smaller difference means less incentive for units to deviate from instructions. To further illustrate the incentives associated with different prices, the opportunity cost denoting this difference is employed and listed in the fifth column. In comparison, the total opportunity cost from LMPs is the highest $18002.9 (3.81), followed by LRPs $9812.5 (2.08), then by IRPs $6137.3 (1.30), and finally with DWPs and CHPs having the lowest $4719.8 (1.00), which indicates that units are the least willing to deviate from the scheduling instructions when settled using DWPs and CHPs.

Table I Cost Comparisons of Each Unit

Appro.	ID	Act. Inc. ($)	Opt. Inc. ($)	Oppt. Costs ($)
	1	15887.1	20511.4	4624.3
	2	-7060.0	0.0	7060.0
	3	-2202.1	1418.3	3620.4
M-LMP	**4**	1855	2188.3	333.3
	5	0.0	2364.9	2364.9
	6	0.0	0.0	0.0
	Total	**8480.0**	**26482.9**	**18002.9**
	1	9064.3	11229.7	2165.4
	2	-7059.0	0.0	7059.0
	3	822.8	1328.9	506.1
M-LRP	**4**	-82.0	0.0	82.0
	5	0.0	0.0	0.0
	6	0.0	0.0	0.0
	Total	**2746.1**	**12558.6**	**9812.5**
	1	28854.4	29919.1	1064.7
	2	-3913.4	0.0	3913.4
	3	31643.9	31868.7	224.8
M-IRP	**4**	-934.4	0.0	934.4
	5	0.0	0.0	0.0
	6	0.0	0.0	0.0
	Total	**55650.5**	**61787.8**	**6137.3**
	1	31818	33445.3	1627.3
	2	-2870.5	0.0	2870.5
	3	35196.4	35417.1	220.7
M-DWP	**4**	-1.3	0.0	1.3
	5	0.0	0.0	0.0
	6	0.0	0.0	0.0
	Total	**64142.6**	**68862.4**	**4719.8**
	1	31818	33445.3	1627.3
	2	-2870.5	0.0	2870.5
	3	35196.4	35417.1	220.7
M-CHP	**4**	-1.3	0.0	1.3
	5	0.0	0.0	0.0
	6	0.0	0.0	0.0
	Total	**64142.6**	**68862.4**	**4719.8**

* "Appro.", "Act.", "Inc.", "Opt.", and "Oppt." denote approach, actual, incomes, and opportunity, respectively.

Then, Table II compares the uplift costs among LMPs, LRPs, IRPs, DWPs, and CHPs. To be more specific, obtained results are $18002.9 (4.62), $13297.0 (1.42), $10077.3 (1.05), $8959.3 (1.00), and $8959.3 (1.00), showing that both DWPs and CHPs achieve the minimum in the uplift costs. In fact, both LRPs and IRPs are also aiming to obtain the DWPs or CHPs, but they fail to achieve this as they do not provide the optimal Lagrangian dual solution for the NCUC, which demonstrates that the DWPs and CHPs are more effective in minimizing uplift costs.

Table II Comparisons of Uplift Costs on IEEE-30

Appro.	Uplift Costs	Percentage
M-LMP	18002.9	200.9%
M-LRP	13297.0	148.4%
M-IRP	10077.3	112.5%
M-DWP	**8959.3**	**100.0%**
M-CHP	**8959.3**	**100.0%**

As seen in Fig. 3, a notable observation from the comparison of results is that the electricity price generated by the proposed M-CHP method is identical to the price obtained from the M-DWP [41]. Since the price derived from M-DWP is recognized as the exact convex hull price, this confirms the effectiveness of our M-CHP method in producing the exact convex hull price, rather than an approximation. Then, for delivering exact convex hull prices, the M-CHP method has outperformed many other approaches, such as traditional sub-gradient and integer relaxation pricing methods, as it provides the exact convex hull price.

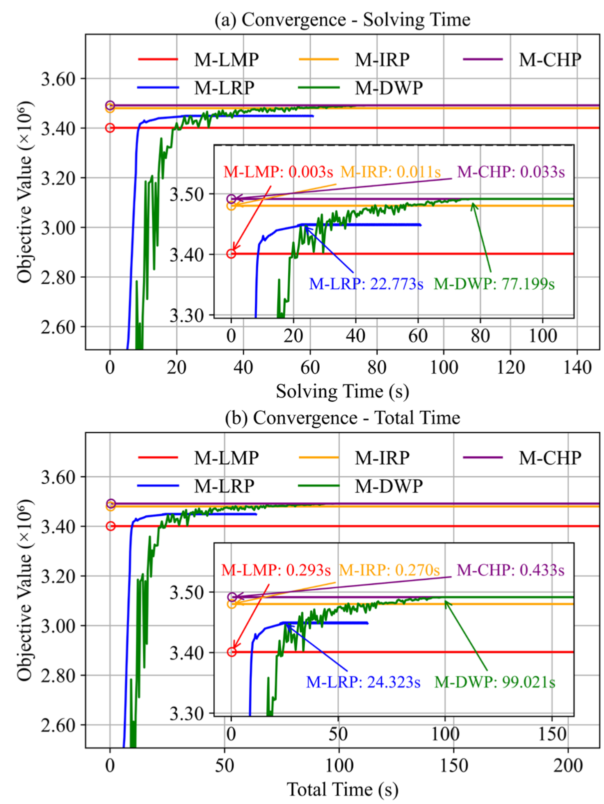

Fig. 3. Convergence Comparisons.

Last but not least, since the M-CHP avoids iterations, another significance of it is its excellent convergence. Here, this is evident from the convergence comparison shown in Fig. 3, where the convergence characteristics of these five methods are evaluated. Sub-plot (a) has demonstrated the solving time and the corresponding Lagrangian dual value, while sub-plot (b) represents the total time, which includes both solving and modeling, along with the Lagrangian dual value. In Fig. 3, the M-LMP, M-IRP, and M-CHP are represented by rays, with the vertical axis indicating the Lagrangian dual value and the horizontal axis indicating the solving time (or total time). Moreover, the curves show the relationship between the Lagrangian dual value and the solving or total time. As plots show, it is clear that the M-CHP is better than M-LMP, M-LRP, and M-IRP, while it is matched by the M-DWP when the latter converges. Although both the M-CHP and the M-DWP will reach the same Lagrangian dual value (the same electricity price), M-DWP requires 77.20s for solving and 99.02s for total time, whereas the M-CHP takes only 0.03s for solving and 0.43s for total time to achieve the exact convex hull price. In conclusion, the M-CHP not only can yield the exact convex hull price but also maintains excellent convergence.

B. Polish-2383 System

A large-scale system, i.e., the Polish-2383 system, involving 323 units, 2383 buses, and 2896 branches, is used to show the effectiveness of the proposed approach in ensuring revenue adequacy and minimizing uplift costs [47].

Table III Comparisons of Total Costs on Polish-2383

Scen.	Act. Inc. ($\times 10^7$\$)					Opt. Inc. ($\times 10^7$\$)					Oppt. Costs ($\times 10^5$\$)					Uplift Costs ($\times 10^5$\$)				
	LMP	**LRP**	**IRP**	**DWP**	**CHP**	**LMP**	**LRP**	**IRP**	**DWP**	**CHP**	**LMP**	**LRP**	**IRP**	**DWP**	**CHP**	**LMP**	**LRP**	**IRP**	**DWP**	**CHP**
#1	0.608	1.055	1.469	1.585	1.585	0.669	1.083	1.470	1.586	1.586	6.109	2.824	0.103	0.089	0.089	6.109	2.881	0.103	0.089	0.089
#2	0.790	1.390	1.723	1.821	1.821	0.846	1.412	1.725	1.822	1.822	5.590	2.133	0.183	0.093	0.093	5.590	2.198	0.205	0.116	0.116
#3	1.026	0.885	0.819	0.927	0.927	1.120	0.904	0.841	0.929	0.929	9.400	1.891	2.220	0.178	0.178	9.400	1.891	2.220	0.180	0.180
#4	0.312	1.010	2.016	2.098	2.098	0.490	1.105	2.020	2.100	2.100	17.79	9.534	0.342	0.188	0.188	17.79	9.668	0.363	0.201	0.201
#5	0.454	0.551	0.577	0.603	0.603	0.460	0.553	0.578	0.603	0.603	0.602	0.167	0.099	0.031	0.031	0.602	0.180	0.099	0.031	0.031
#6	0.323	0.381	0.357	0.377	0.377	0.327	0.383	0.359	0.377	0.377	0.420	0.166	0.195	0.026	0.026	0.420	0.166	0.195	0.026	0.026
Aver.	**0.586**	**0.879**	**1.160**	**1.235**	**1.235**	**0.652**	**0.907**	**1.166**	**1.236**	**1.236**	**6.652**	**2.786**	**0.524**	**0.101**	**0.101**	**6.652**	**2.831**	**0.531**	**0.107**	**0.107**

* "Scen.", "Act.", "Inc.", "Opt.", and "Oppt." denote scenarios, actual incomes, and opportunity, respectively; and the LMP, LRP, IRP, DWP, and CHP represent these methods, i.e., M-LMP, M-LMP, M-LRP, M-IRP, M-DWP, and M-CHP, respectively.

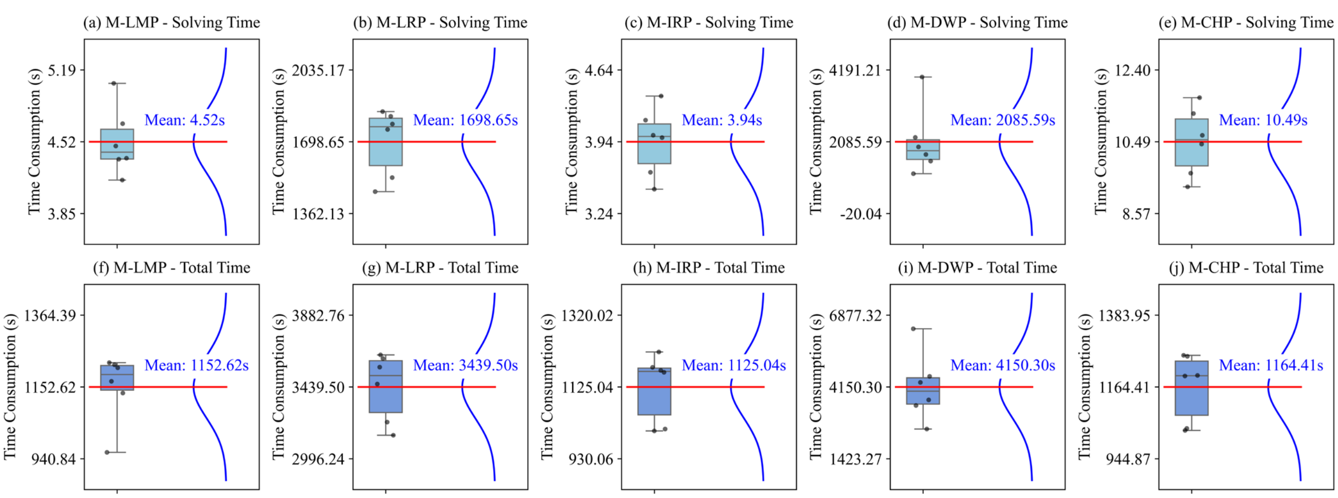

Fig. 4. Comparison of the Time Consumption.

Table III compares the actual income, optimal income, and opportunity cost obtained from these four prices under different scenarios. As actual incomes illustrate, all these prices meet revenue adequacy, where the exact convex hull price obtained from M-DWP and M-CHPs achieves \$$1.235\times10^7$, which is better than LMPs (\$$0.586\times10^7$), LRPs (\$$0.879\times10^7$), and IRPs (\$$1.160\times10^7$). Then, optimal incomes from the self-scheduling with LMPs, LRPs, IRPs, and the exact convex hull price (DWPs and CHPs) are \$$0.652\times10^7$, \$$0.907\times10^7$, \$$1.166\times10^7$, and \$$1.236\times10^7$, all of which are higher than the corresponding actual incomes. Higher than actual incomes further illustrate that units tend to deviate from the instructions of ISOs. This intention demonstrates that the opportunity cost yielded from LMPs (\$$6.652\times10^5$) is higher than LRPs (\$$2.786\times10^5$), IRPs (\$$0.524\times10^5$), and the exact convex hull price (\$$0.101\times10^5$) while showing that these latter three prices are better than LMPs in incentive compatibility, especially DWPs and CHPs. In addition, uplift costs involve not only the opportunity cost but also the financial transmission right. This right, denoted by the second term of (34), describes the difference between the implied value of the available transmission capacity at specific prices and the value of the transmitted flows, as the instructions, which is non-uniform and will be solely settled by ISOs. Taking this right into account, the resulting cost, which is the uplift cost, is the payment that ISOs should pay. The uplift cost associated with the exact convex hull price (obtained from M-DWP and M-CHP) is \$$0.107\times10^5$, which is lower than those of LMPs (\$$6.652\times10^5$), LRPs (\$$2.831\times10^5$), and IRPs (\$$0.531\times10^5$). Therefore, LMPs and approximations (LRPs and IRPs) are less effective than the exact convex hull price (DWPs and CHPs) in achieving market transparency. In addition, as the above results indicate, the M-CHP yields the same results as the employed M-DWP, which means that the M-CHP can develop the exact convex hull, outperforming other approximations.

Moreover, the proposed M-CHP not only produces the exact convex hull price but also maintains excellent computational efficiency. Following the comparisons of total costs, Fig. 4 presents the time consumption results of these five methods. As shown in sub-plots (a) to (e), the M-LMP, M-IRP, and M-CHP only take 4.52s, 3.94s, and 10.49s. This is expected, as none of these methods require iterative procedures. In contrast, M-LRP and M-DWP, which rely on iterative processes to approach or achieve the exact convex hull price, require substantially more time, 1698.65s and 2085.59s, due to the large number of iterations involved. In addition to solving time, the total time consumption (i.e., solving time plus modeling time) is shown in sub-plots (f)–(j). Consistent with the earlier results, the total time required by M-LMP (1152.62s), M-IRP (1125.04s), and M-CHP (1164.41s) is significantly lower than that of M-LRP (3439.50s) and M-DWP (4150.30s). The longer total time of the latter two methods is attributed to their iterative nature and the repeated model reconstruction in each iteration.

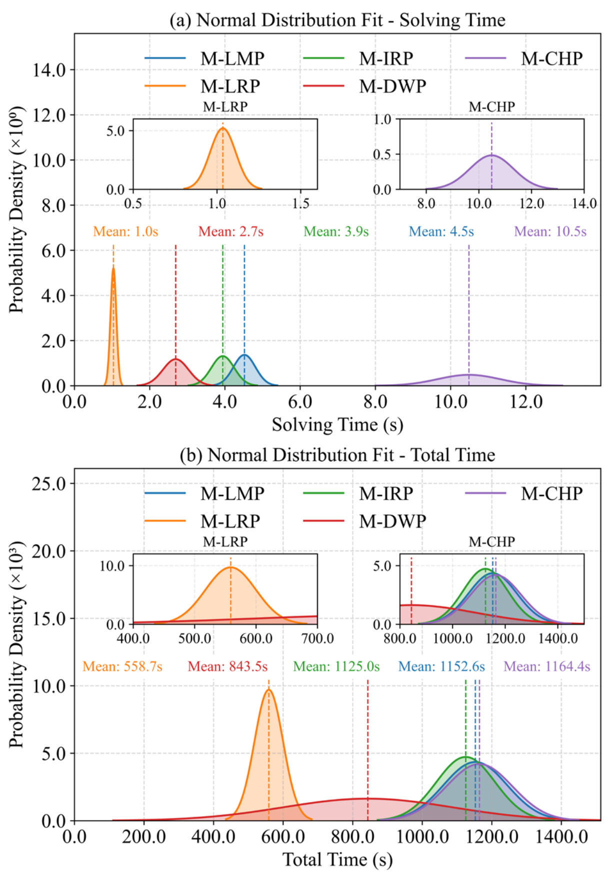

Fig. 5. Time Consumption of M-LMP, M-IRP, M-CHP, and the first iteration for M-LRP and M-DWP.

To further show computational differences, Fig. 5 compares the time consumption of the M-LMP, M-IRP, M-CHP, and the first iteration of the M-LRP and M-DWP. In these tests, the iterative procedures of the M-LRP and M-DWP are simplified by minimizing the complexity of model updating in subsequent iterations. Hence, the majority of the modeling time is concentrated in the first iteration, becoming the representative of the overall modeling effort in each method. Under the settings, the first iteration of the M-LRP and the M-DWP requires 1.0s and 2.7s for solving, and 558.7s and 843.5s for total time (including modeling). Compared with these results, non-iterative methods (M-LMP, M-IRP, and M-CHP) spend slightly higher one-time addressing durations but benefit from avoiding the cumulative overhead of multiple iterations. This indicates that even when model updates are streamlined, iterative methods still incur significant modeling costs from the outset. Moreover, as the number of iterations increases, often depending on system characteristics and convergence criteria, the total time grows substantially. In conclusion, the M-LMP, M-IRP, and M-CHP outperform the iterative methods. Also, while the M-LMP and M-IRP are faster, none of can guarantee the exact convex hull price. In contrast, the M-CHP achieves both efficiency and exact convex hull pricing, matching the result from the M-DWP while avoiding its iterative heavy burden, eventually making the M-CHP effective for convex hull pricing in practical applications.

V. Conclusion

This paper proposes an explicit Lagrangian dual formulation of the NCUC, where a convex hull model is developed for the 1UC problem, making the proposed formulation implementable and further allowing this optimal Lagrangian dual solution to be obtained using two simple LP models, eventually finishing convex hull pricing. Numerical experiments show that the proposed method achieves uplift cost reductions with less computational time. On one hand, the proposed pricing method (M-CHP) has achieved uplift cost reductions of 48.4% (vs. M-LRP) and 12.5% (vs. M-IRP) on the IEEE 30 system, with more significant reductions, being less than one-sixtieth of the M-LRP and one-fifth of the M-IRP observed on the large-scale Polish 2383 system. On the other hand, the total time consumption of the proposed M-CHP is reduced to around 25% of the M-DWP, which can yield the exact convex hull price on the Polish-2383 system. In conclusion, from the viewpoint of reducing the uplift costs, the M-CHP can yield the exact convex hull price, which is the same as that obtained from the M-DWP, being better than the M-LRP and M-IRP in achieving market transparency. In addition, by further comparing the total time consumption, the M-CHP only needs to spend around 25% of the time required by the M-DWP, showing its excellent computational efficiency.

In practice, the proposed method can provide the exact convex hull price. It not only has strong economic interpretability and incentive compatibility, but also does well in computational efficiency by avoiding traditional iterations. As a result, it has offered ISOs a theoretically reasonable, computationally efficient, and scalable convex hull pricing approach.

Appendix A

Specifically, $\boldsymbol{S}_g$ is employed to denote the state space, whose elements are vectors. In order, the components of these vectors are power output, online state, start-up instruction, shut-down instruction, and duration of the current online state.

To represent this state space conveniently, $\boldsymbol{S}_g$ is characterized by two sub-sets (denoted by $\boldsymbol{S}_g^+$ and $\boldsymbol{S}_g^-$), whose details are:

• Since T_g^{on}=1, $\boldsymbol{S}_g^+$ would be denoted as:

$$\mathcal{P}_g \times 1 \times \{0,1\} \times 0 \times 1 \tag{39}$$

• Since T_g^{on}>1, $\boldsymbol{S}_g^+$ would be denoted as:

$$\mathcal{P}_g \times 1 \times 1 \times 0 \times 1 + \mathcal{P}_g \times 1 \times 0 \times 0 \times \{2,\dots,T_g^{\text{on}}\} \tag{40}$$

• Since T_g^{off}=1, $\boldsymbol{S}_g^-$ would be denoted as:

$$0 \times 0 \times 0 \times \{0,1\} \times 1 \tag{41}$$

• Since T_g^{off}>1, $\boldsymbol{S}_g^-$ would be denoted as:

$$0 \times 0 \times 0 \times 1 \times 1 + 0 \times 0 \times 0 \times 0 \times \{2,\dots,T_g^{\text{off}}\} \tag{42}$$

In addition, $\mathcal{P}_g$ can be obtained as follows:

$$\mathcal{P}_g = \mathcal{P}_{g,1} \cup \mathcal{P}_{g,2} \cup \mathcal{P}_{g,3} \cup \mathcal{P}_{g,4} \tag{43}$$

where $\mathcal{P}_{g,1}$, $\mathcal{P}_{g,2}$, $\mathcal{P}_{g,3}$, and $\mathcal{P}_{g,4}$ respectively are:

$$\{p_g^{\min} + nR_g^{\text{regu}} \mid nR_g^{\text{regu}} \in [0, P_g^{\max} - P_g^{\min}], n \in \mathbf{Z}\} \tag{44}$$

$$\{p_g^{\max} + nR_g^{\text{regu}} \mid nR_g^{\text{regu}} \in [P_g^{\min} - P_g^{\max}, 0], n \in \mathbf{Z}\} \tag{45}$$

$$\{R_g^{\text{init}} + nR_g^{\text{regu}} \mid nR_g^{\text{regu}} \in [P_g^{\min} - R_g^{\text{init}}, P_g^{\max} - R_g^{\text{init}}], n \in \mathbf{Z}\} \tag{46}$$

$$\{P_{g,i}^{\text{seg}} + nR_g^{\text{regu}} \mid nR_g^{\text{regu}} \in [P_g^{\min} - P_{g,i}^{\text{seg}}, P_g^{\max} - P_{g,i}^{\text{seg}}], n \in \mathbf{Z}, \forall i\} \tag{47}$$

Now, the state space $\boldsymbol{S}_g$ can be developed as $\boldsymbol{S}_g^+ \cup \boldsymbol{S}_g^-$.

Appendix B

Then, based on operational requirements involving the logic of online status (2c)-(2d), ramping limits (2f)-(2g), and minimum on/off time limits (2h)-(2k), all possible state transitions can be generated. Here, $[s_t]_i$ is also employed to denote the i-th component of the state s_t. $\boldsymbol{A}_g(s_t)$ contains all immediate successors of the state s_t, where $s_t \in \boldsymbol{S}_g$, which could be:

• If T_g^{on}=1, then $\boldsymbol{A}_g(s_t)$ could be developed as follows:

Given the state s_t where $[s_t]_2$=1, $[s_t]_3$=1, $[s_t]_4$=0, and $[s_t]_5$=1, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = \begin{cases} [s_{t+1}]_1 \times 1 \times 0 \times 0 \times 1, |[s_{t+1}]_1 - [s_t]_1| \le R_g^{\text{regu}} \\ 0 \times 0 \times 0 \times 1 \times 1, [s_t]_1 \le R_g^{\text{init}} \end{cases} \tag{48}$$

Moreover, $\boldsymbol{A}_g(s_t)$ also is (48) for the state s_t where $[s_t]_2$=1, $[s_t]_3$=1, $[s_t]_4$=0, and $[s_t]_5$=1.

• If T_g^{on}>1, then $\boldsymbol{A}_g(s_t)$ could be developed as follows:

Given the state s_t where $[s_t]_2$=1, $[s_t]_3$=1, $[s_t]_4$=0, and $[s_t]_5$=1, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = [s_{t+1}]_1 \times 1 \times 0 \times 0 \times 2, |[s_{t+1}]_1 - [s_t]_1| \le R_g^{\text{regu}} \tag{49}$$

Given the state s_t where $[s_t]_2$=1, $[s_t]_3$=0, $[s_t]_4$=0, and 1<$[s_t]_5$<T_g^{on}, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = [s_{t+1}]_1 \times 1 \times 0 \times 0 \times [s_t]_5 + 1, |[s_{t+1}]_1 - [s_t]_1| \le R_g^{\text{regu}} \tag{50}$$

Given the state s_t where $[s_t]_2$=1, $[s_t]_3$=0, $[s_t]_4$=0, and $[s_t]_5$=T_g^{on}, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = \begin{cases} [s_{t+1}]_1 \times 1 \times 0 \times 0 \times T_g^{\text{on}}, |[s_{t+1}]_1 - [s_t]_1| \le R_g^{\text{regu}} \\ 0 \times 0 \times 0 \times 1 \times 1, [s_t]_1 \le R_g^{\text{init}} \end{cases} \tag{51}$$

• If T_g^{off}=1, then $\boldsymbol{A}_g(s_t)$ could be developed as follows:

Given the state s_t where $[s_t]_1$=0, $[s_t]_2$=0, $[s_t]_3$=0, $[s_t]_4$=1, and $[s_t]_5$=1, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = \begin{cases} [s_{t+1}]_1 \times 1 \times 0 \times 0 \times 1, P_g^{\min} \le [s_{t+1}]_1 \le R_g^{\text{init}} \\ 0 \times 0 \times 0 \times 0 \times 1 \end{cases} \tag{52}$$

Moreover, $\boldsymbol{A}_g(s_t)$ also is (52) for the state s_t where $[s_t]_1$=0, $[s_t]_2$=0, $[s_t]_3$=0, $[s_t]_4$=0, and $[s_t]_5$=1.

• If T_g^{off}>1, then $\boldsymbol{A}_g(s_t)$ could be developed as follows:

Given the state s_t where $[s_t]_1$=0, $[s_t]_2$=0, $[s_t]_3$=0, $[s_t]_4$=1, and $[s_t]_5$=1, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = 0 \times 0 \times 0 \times 0 \times 2 \tag{53}$$

Given the state s_t where $[s_t]_1$=0, $[s_t]_2$=0, $[s_t]_3$=0, $[s_t]_4$=0, and 1<$[s_t]_5$<T_g^{off}, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = 0 \times 0 \times 0 \times 0 \times [s_t]_5 + 1 \tag{54}$$

Given the state s_t where $[s_t]_1$=0, $[s_t]_2$=0, $[s_t]_3$=0, $[s_t]_4$=0, and $[s_t]_5$=T_g^{off}, then $\boldsymbol{A}_g(s_t)$ could be generated as follows:

$$\boldsymbol{A}_g(s_t) = \begin{cases} [s_{t+1}]_1 \times 1 \times 1 \times 0 \times 1, P_g^{\min} \le [s_{t+1}]_1 \le R_g^{\text{init}} \\ 0 \times 0 \times 0 \times 0 \times T_g^{\text{off}} \end{cases} \tag{55}$$

Appendix C

Here, the LP model (8a)-(8d) along with (9a)-(9e) can provide the optimal scheduling decision. Specifically, these equations (9a)-(9e), indeed, have no material effect on the optimal objective of the LP model (8a)-(8d). Therefore, this optimal objective still equals the original 1UC problem (3a)-(3b), having:

$$\sum\nolimits_{t=1}^{T} c_{g,t}^* = \sum\nolimits_{t=1}^{T} \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) V_g(s_t) \tag{56}$$

where $c_{g,t}^*$ is the optimal value of $c_{g,t}$ obtained from (3a)-(3b) and $\omega_g^*(s_t, s_{t+1})$ is the optimal value of $\omega_g(s_t, s_{t+1})$. Then, matching these values for the same period t, which yields:

$$c_{g,t}^* = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) V_g(s_t) \tag{57}$$

Now, taking $p_{g,t}^*$, $x_{g,t}^*$, $u_{g,t}^*$, and $d_{g,t}^*$ to be the optimal values of $p_{g,t}$, $x_{g,t}$, $u_{g,t}$, and $d_{g,t}$ derived from (3a)-(3b), $c_{g,t}^*$ could be:

$$c_{g,t}^* = C_g^{\text{u}} u_{g,t}^* + C_g^{\text{d}} d_{g,t}^* + f_g(p_{g,t}^*, x_{g,t}^*) \tag{58}$$

and replacing $c_{g,t}^*$ and $V_g(s_t)$ in (57) based on (6) and (58) gives:

$$c_{g,t}^* = \sum\nolimits_{t=1}^{T} \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) V_g(s_t) \tag{59a}$$

$$p_{g,t}^* = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) [s_t]_1 \tag{59b}$$

$$x_{g,t}^* = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) [s_t]_2 \tag{59c}$$

$$u_{g,t}^* = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) [s_t]_3 \tag{59d}$$

$$d_{g,t}^* = \sum\nolimits_{s_t \in \boldsymbol{S}_g} \sum\nolimits_{s_{t+1} \in \boldsymbol{A}_g(s_t)} \omega_g^*(s_t, s_{t+1}) [s_t]_4 \tag{59e}$$

Moreover, as mentioned in [48], if a polyhedron is the convex hull of the original MILP feasible region, it requires to verify that optimizing a linear function in any direction, i.e., utilizing any coefficients of the cost function, within this polyhedron and the MILP feasible region provides the same solution.

Therefore, this LP model (8a)-(8d), along with (9a)-(9e), can provide the same solution regardless of the coefficients, illustrating that the resulting feasible region can be then recognized as the convex hull model of the original 1UC problem (3a)-(3b).

APPENDIX D

According to (29) and (33), $f_{l,t}^{\mathbf{P1}(*)}$ and $\Pi_{b(g),t}$ in (34) can be expanded, further yielding the following formulation:

$$\begin{Bmatrix} \max & \sum_{g=1}^{G}\sum_{t=1}^{T}(\lambda_t^{\mathbf{P2}(*)}p_{g,t}-\lambda_t^{\mathbf{P2}(*)}p_{g,t}^{\mathbf{P1}(*)}-c_{g,t}+c_{g,t}^{\mathbf{P1}(*)}) \\ & +\sum_{g=1}^{G}\sum_{t=1}^{T}p_{g,t}\sum_{l=1}^{L}H_{l,b(g)}(\beta_{l,t}^{\mathbf{P2}(*)}-\gamma_{l,t}^{\mathbf{P2}(*)}) \\ & -\sum_{g=1}^{G}\sum_{t=1}^{T}p_{g,t}^{\mathbf{P1}(*)}\sum_{l=1}^{L}H_{l,b(g)}(\beta_{l,t}^{\mathbf{P2}(*)}-\gamma_{l,t}^{\mathbf{P2}(*)}) \\ \textbf{s.t.} & (1d) \end{Bmatrix}$$
$$+\sum_{l=1}^{L}\sum_{t=1}^{T}[\beta_{l,t}^{\mathbf{P2}(*)}F_l+\beta_{l,t}^{\mathbf{P2}(*)}(\sum_{g=1}^{G}H_{l,b(g)}p_{g,t}^{\mathbf{P1}(*)}-\sum_{b=1}^{B}H_{l,b}P_{b,t}^{\text{load}})]$$
$$+\sum_{l=1}^{L}\sum_{t=1}^{T}[\gamma_{l,t}^{\mathbf{P2}(*)}F_l-\gamma_{l,t}^{\mathbf{P2}(*)}(\sum_{g=1}^{G}H_{l,b(g)}p_{g,t}^{\mathbf{P1}(*)}-\sum_{b=1}^{B}H_{l,b}P_{b,t}^{\text{load}})] \tag{60}$$

and performing elimination on the constant $p_{g,t}^{\mathbf{P1}(*)}$ in (60) yields:

$$\begin{Bmatrix} \max & \sum_{g=1}^{G}\sum_{t=1}^{T}(\lambda_t^{\mathbf{P2}(*)}p_{g,t}-\lambda_t^{\mathbf{P2}(*)}p_{g,t}^{\mathbf{P1}(*)}-c_{g,t}+c_{g,t}^{\mathbf{P1}(*)}) \\ & +\sum_{g=1}^{G}\sum_{t=1}^{T}p_{g,t}\sum_{l=1}^{L}H_{l,b(g)}(\beta_{l,t}^{\mathbf{P2}(*)}-\gamma_{l,t}^{\mathbf{P2}(*)}) \\ \textbf{s.t.} & (1d) \end{Bmatrix}$$
$$+\sum_{l=1}^{L}\sum_{t=1}^{T}(\beta_{l,t}^{\mathbf{P2}(*)}F_l-\beta_{l,t}^{\mathbf{P2}(*)}\sum_{b=1}^{B}H_{l,b}P_{b,t}^{\text{load}}) \tag{61}$$
$$+\sum_{l=1}^{L}\sum_{t=1}^{T}(\gamma_{l,t}^{\mathbf{P2}(*)}F_l+\gamma_{l,t}^{\mathbf{P2}(*)}\sum_{b=1}^{B}H_{l,b}P_{b,t}^{\text{load}})$$

Since the second part in (61) is a constant and it does not affect the maximization problem, it can be incorporated, yielding:

$$\begin{Bmatrix} \max & \sum_{g=1}^{G}\sum_{t=1}^{T}(\lambda_t^{\mathbf{P2}(*)}p_{g,t}-\lambda_t^{\mathbf{P2}(*)}p_{g,t}^{\mathbf{P1}(*)}-c_{g,t}+c_{g,t}^{\mathbf{P1}(*)}) \\ & +\sum_{l=1}^{L}\sum_{t=1}^{T}[\beta_{l,t}^{\mathbf{P2}(*)}(F_l+f_{l,t})+\gamma_{l,t}^{\mathbf{P2}(*)}(F_l-f_{l,t})] \\ \textbf{s.t.} & (1d) \end{Bmatrix} \tag{62}$$

Also, $c_{g,t}^{\mathbf{P1}(*)}$ is a constant that can be moved outside the maximum problem. Consequently, (62) can be reformulated as:

$$\sum_{g=1}^{G}\sum_{t=1}^{T}c_{g,t}^{\mathbf{P1}(*)}$$
$$+\begin{Bmatrix} \max & \sum_{g=1}^{G}\sum_{t=1}^{T}(\lambda_t^{\mathbf{P2}(*)}p_{g,t}-\lambda_t^{\mathbf{P2}(*)}p_{g,t}^{\mathbf{P1}(*)}-c_{g,t}) \\ & +\sum_{l=1}^{L}\sum_{t=1}^{T}[\beta_{l,t}^{\mathbf{P2}(*)}(F_l+f_{l,t})+\gamma_{l,t}^{\mathbf{P2}(*)}(F_l-f_{l,t})] \\ \textbf{s.t.} & (1d) \end{Bmatrix} \tag{63}$$

Note that the power balance is satisfied in **P1**, having:

$$\sum_{g=1}^{G}p_{g,t}^{\mathbf{P1}(*)}=\sum_{b=1}^{B}P_{b,t}^{\text{load}},\forall t \tag{64}$$

then the following relationship is true:

$$\sum_{t=1}^{T}\lambda_t^{\mathbf{P2}(*)}(\sum_{g=1}^{G}p_{g,t}^{\mathbf{P1}(*)})=\sum_{t=1}^{T}\lambda_t^{\mathbf{P2}(*)}(\sum_{b=1}^{B}P_{b,t}^{\text{load}}) \tag{65}$$

Now, taking (65) into (63) will generate:

$$\sum_{g=1}^{G}\sum_{t=1}^{T}c_{g,t}^{\mathbf{P1}(*)}$$
$$+\begin{Bmatrix} & -\sum_{g=1}^{G}\sum_{t=1}^{T}c_{g,t} \\ \max & +\sum_{t=1}^{T}\lambda_t^{\mathbf{P2}(*)}(\sum_{g=1}^{G}p_{g,t}-\sum_{b=1}^{B}P_{b,t}^{\text{load}}) \\ & +\sum_{l=1}^{L}\sum_{t=1}^{T}\beta_{l,t}^{\mathbf{P2}(*)}(F_l+f_{l,t}) \\ & +\sum_{l=1}^{L}\sum_{t=1}^{T}\gamma_{l,t}^{\mathbf{P2}(*)}(F_l-f_{l,t}) \\ \textbf{s.t.} & (1d) \end{Bmatrix} \tag{66}$$

Generally, maximizing an objective function is equivalent to minimizing its negative. Therefore, (66) can be denoted as:

$$\sum_{g=1}^{G}\sum_{t=1}^{T}c_{g,t}^{\mathbf{P1}(*)}$$
$$-\begin{Bmatrix} & \sum_{g=1}^{G}\sum_{t=1}^{T}c_{g,t} \\ \min & -\sum_{t=1}^{T}\lambda_t^{\mathbf{P2}(*)}(\sum_{g=1}^{G}p_{g,t}-\sum_{b=1}^{B}P_{b,t}^{\text{load}}) \\ & -\sum_{l=1}^{L}\sum_{t=1}^{T}\beta_{l,t}^{\mathbf{P2}(*)}(F_l+f_{l,t}) \\ & -\sum_{l=1}^{L}\sum_{t=1}^{T}\gamma_{l,t}^{\mathbf{P2}(*)}(F_l-f_{l,t}) \\ \textbf{s.t.} & (1d) \end{Bmatrix} \tag{67}$$

Note that the minimization problem is the Lagrangian dual function of **P1** with the dual variables fixed at $\boldsymbol{\lambda}^{\mathbf{P2}(*)}$, $\boldsymbol{\beta}^{\mathbf{P2}(*)}$, and $\boldsymbol{\gamma}^{\mathbf{P2}(*)}$, i.e., $\mathcal{D}^{\mathbf{P1}}(\boldsymbol{\lambda}^{\mathbf{P2}(*)},\boldsymbol{\beta}^{\mathbf{P2}(*)},\boldsymbol{\gamma}^{\mathbf{P2}(*)})$, thereby giving:

$$\sum_{g=1}^{G}\sum_{t=1}^{T}c_{g,t}^{\mathbf{P1}(*)}-\mathcal{D}^{\mathbf{P1}}(\boldsymbol{\lambda}^{\mathbf{P2}(*)},\boldsymbol{\beta}^{\mathbf{P2}(*)},\boldsymbol{\gamma}^{\mathbf{P2}(*)}) \tag{68}$$

Ultimately, (34) has been equivalently reformulated as (36).

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