Please cite this paper as:

Cai Q, Zhu S and Ke S. (2020). Can we unify vibration control and energy harvesting objectives in energy regenerative tuned mass dampers?. *Smart Materials and Structures*. 29(8): 087002.

https://doi.org/10.1088/1361-665X/ab92de

Can we unify vibration control and energy harvesting objectives in energy regenerative tuned mass dampers?

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Abstract

As an emerging concept, energy-regenerative tuned mass dampers (ERTMDs) have recently been proposed to perform vibration control and energy harvesting functions simultaneously. This study aims to answer a fundamental question whether these two intended functions are consistent in an ERTMD. The closed-form solutions for the optimal parameters of an ERTMD installed in a damped structure are derived first, wherein the optimization objectives, namely, minimize the kinetic energy of the controlled structure and maximize the harvested power from ERTMDs, are both considered. Results reveal that the optimal parameters for the two scenarios are identical and thus prove that the two performance objectives can be optimized simultaneously in an ERTMD. The effects of mass ratios of ERTMDs are evaluated based on the derived analytical solution, and the results demonstrate that a large mass ratio benefits both vibration control and energy harvesting functions of ERTMDs under random excitations. The effectiveness and accuracy of the analytical solution are validated through a numerical case study. Numerical results also indicate that the interested power terms of ERTMDs are likely insensitive to the parameter detuning or the optimization criteria adopted.

Keyword: tuned mass damper; energy regenerative; energy harvesting; vibration control; mass ratio; closed-form solution

1. Introduction

Tuned mass dampers (TMDs), since their first introduction by Frahm [1], have become one of the most effective and popular vibration control strategies to suppress unfavorable vibrations of civil and mechanical structures [2-4]. In the emerging field of vibration-based energy harvesting, a novel type of TMDs, termed energy-regenerative TMDs (ERTMDs), have been developed recently [5, 6]. ERTMDs convert structural vibration energy into electrical energy that can be stored or utilized instead of being dissipated directly. The introduction of the new energy harvesting function into classical TMDs provides a potential solution to address the power supply issues of wireless sensors in structural health monitoring. Electromagnetic dampers are employed as energy transducers in ERTMDs because of their high output power that ranges from the mW to kW level and their controllable damping that can be conveniently adjusted by varying the load resistance [7].

From the vibration control perspective, a well-known family of control criteria for TMDs includes H_∞ for minimizing the maximum displacement of structures [8], H_2 for minimizing the root mean square (RMS) displacement of structures over a frequency band [9], and stability maximization [10]. From the energy harvesting perspective, Cheng et al. [11] and Cammarano et al. [12] derived an overall impedance theory for single-degree-of-freedom (SDOF) electromagnetic energy harvesters in electrical and mechanical domains, respectively. Cai and Zhu [13] proposed a unified impedance optimization strategy that is applicable to both SDOF and multiple-degree-of-freedom (MDOF) electromagnetic energy harvesters under harmonic and random excitations.

An ERTMD is a device with dual functions, namely, vibration control and energy harvesting, and this feature raises a fundamental question on whether the optimizations for the two objectives are consistent or contradictory. For a structure with viscous dampers, Shen et al. [14] presented the consistency between the two functions under random excitations. However, the dual-function ERTMDs present a complicated optimization problem because numerous parameters, such as frequency and damping ratios, need to be determined. For harmonic excitation cases, Harne [15] and Gonzalez-Buelga et al. [16] reported a similar conclusion, that is, the two objectives of ERTMD are inconsistent in terms of the optimal mass ratio and load resistance. Meanwhile, for white noise excitation cases, Zuo and Cui [17] indicated numerically that the optimal parameters of ERTMDs for vibration control and energy harvesting are close. Zilletti et al. [18] pointed out that the minimization of structural kinetic energy and the maximization of the absorbed power of TMDs are consistent in slightly damped structures, but they did not provide a closed-form solution, while Tigli [19] derived analytically the optimal solution for minimization of velocity variance of the main structure but did not mention any information about the optimal absorbed power. A large mass ratio of TMD benefits the vibration control effect. Several researchers thus believe that a large mass ratio of ERTMD may reduce the harvested power because of the significantly suppressed vibrations of the primary structures.

This study derives the closed-form solutions for the optimal parameters of an ERTMD installed in an SDOF structure in consideration of structural damping, with the objectives to maximize the harvested power of the ERTMD and minimize structural kinetic energy, which are essentially consistent in this optimization problem. Subsequently, the general expression of the power efficiency in the ERTMD is presented, and the optimal power efficiency is derived. The influence of the mass ratio of ERTMD and the inherent damping ratio of the primary structure on the power efficiency is investigated mathematically. An SDOF numerical example is analyzed to validate the efficacy and accuracy of the analytical study.

2. System Modeling

Fig. 1 shows a typical configuration of a damped SDOF structure with a TMD. In the figure, m, k, and c denote the mass, stiffness, and damping coefficients, respectively. The subscripts 1 and 2 stand for the primary structure and TMD, respectively. For the realization of the energy harvesting function, an electromagnetic transducer is employed to provide damping in the TMD and forms an ERTMD that converts the damping power to electricity [20, 21]. Although the coils of electromagnetic transducer have inherent resistance and inductance, the coil inductance is typically small, and its effect is often negligible considering the typically low vibration frequency of civil structure [21]. When the electromagnetic transducer is connected to a pure resistor, the transducer can provide the damping required by the TMD, and the corresponding damping coefficient c_2 can be computed as

$$c_2 = \frac{K_{\mathrm{eq}}^2}{R_{\mathrm{coil}} + R_{\mathrm{load}}} + c_{\mathrm{p}}, \tag{1}$$

where R_{coil} and R_{load} are the coil and load resistance of the electromagnetic transducer, respectively; and K_{eq} and c_{p} are the machine constant and parasitic damping of the electromagnetic transducer, respectively. In this study, the TMD damping power is regarded as the potential harvested power, namely the gross ERTMD output power. The influence of the transducer parameters on the power efficiency will be discussed later.

3. Closed-form Solution under Force Vibration

When the primary structure is subjected to force excitation, the governing equation of the damped SDOF structure and TMD system is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}, \tag{2}$$

where x_1 and x_2 are the absolute displacement responses and the dot stands for the differential with respect to time. Since the electromagnetic transducer in this study only provides equivalent damping effect, Eq. (2) is always adoptable and independent of the strength of coupling effect in an ERTMD. Accordingly, the non-dimensional transfer functions of the relative velocity between the TMD and structure, and the velocity of the SDOF structure are respectively given as follows:

$$H_{1\text{-}2} = \left(\frac{1}{m_1\omega_1}\right)\frac{B_0 + B_1(sj) + B_2(sj)^2 + B_3(sj)^3}{A_0 + A_1(sj) + A_2(sj)^2 + A_3(sj)^3 + A_4(sj)^4}, \tag{3.1}$$

$$H_1 = \left(\frac{1}{m_1\omega_1}\right)\frac{C_0 + C_1(sj) + C_2(sj)^2 + C_3(sj)^3}{A_0 + A_1(sj) + A_2(sj)^2 + A_3(sj)^3 + A_4(sj)^4}, \tag{3.2}$$

where $s = \omega/\omega_1$, ω_1 is the natural frequency of the SDOF structure, ω is the excitation frequency, and

$$\begin{array}{lll} A_0 = \gamma^2 & B_0 = 0 & C_0 = 0 \\ A_1 = 2\xi_1\gamma^2 + 2\xi_2\gamma & B_1 = 0 & C_1 = \gamma^2 \\ A_2 = \gamma^2 + 1 + \mu\gamma^2 + 4\xi_1\xi_2\gamma & B_2 = 0 & C_2 = 2\xi_2\gamma \\ A_3 = 2\xi_2\mu\gamma + 2\xi_2\gamma + 2\xi_1 & B_3 = 1 & C_3 = 1 \\ A_4 = 1 & & \end{array}$$

where $\gamma = \omega_2/\omega_1$ is the frequency tuning ratio between the TMD and structure, $\mu = m_2/m_1$ is the mass ratio, and $\xi_1 = c_1/2m_1\omega_1$ and $\xi_2 = c_2/2m_2\omega_2$ are the damping ratios of the structure and TMD, respectively. When the structure is subjected to white noise excitation, the gross output power P_{d} of the ERTMD and the kinetic energy P_{k} of the SDOF structure can be expressed as

$$P_d = 2S_0\omega_1^2 m_1\xi_2\gamma\mu\int_{-\infty}^{+\infty}|H_{1\text{-}2}|^2\,ds\,, \tag{4}$$

$$P_k = \frac{1}{2}S_0 m_1\omega_1\int_{-\infty}^{+\infty}|H_1|^2\,ds\,, \tag{5}$$

where S_0 is the constant power spectral density of the random force excitation (unit: $N^2\cdot s/rad$). The kinetic energy P_k is essentially proportional to the square of the H_2 norm of the structural velocity (i.e., the steady-state variance of structural velocity under white noise excitation). Eqs. (4) and (5) represent two key performance indices of the ERTMD for energy harvesting and vibration control, respectively. Adopting the derivatives of Eqs. (4) and (5) with respect to the damping ratio ξ_2 and frequency tuning ratio γ can provide the optimal conditions. It is interesting to find that the optimal conditions for these two performance indices are identical, which clearly demonstrates the consistency between energy harvesting and vibration control in the ERTMD. Although this conclusion was previously reported by Zilletti et al. [18], only approximate solutions were provided in their study. Following their observation, the present study provides the exact closed-form solutions for the two optimal conditions as follows:

$$\xi_{2,\mathrm{opt}} = \frac{\sqrt{2}\mu^2(1+\mu)}{4\xi_1\left\{\left[\frac{\mu}{\xi_1}\left(\lambda-\frac{1}{2}(1+\mu)\right)\sqrt{\mu\lambda}-\mu^2\lambda\right]\left(\frac{\mu}{\xi_1}\sqrt{\mu\lambda}-\mu^2\right)\right\}^{1/2}}\,, \tag{6.1}$$

$$\gamma_{\mathrm{opt}} = \left\{\frac{\mu^2-\frac{\mu}{\xi_1}\sqrt{\mu\lambda}}{2\left[\frac{\mu}{\xi_1}\left(\frac{1}{2}(1+\mu)-\lambda\right)\sqrt{\mu\lambda}+\mu^2\lambda\right]}\right\}^{1/2}\,, \tag{6.2}$$

where $\lambda = 1 + \mu - \xi_1^2$ is a dimensionless parameter. Eq. (6.2) provides a real frequency ratio when $\xi_1 < ((1 + \mu - (\mu^2+\mu)^{1/2})/2)^{1/2}$. Notably, this analytical solution has also been derived by Tigli [19].

When the entire system is subjected to white noise excitation, the total excitation power can be calculated as [22]

$$P_{ex} = P_d + P_s = \frac{\pi S_0}{m_1}\,, \tag{7}$$

where P_{ex} is the excitation power, and it is equal to the sum of the TMD damping power P_d (i.e., ERTMD output power) and structural damping power P_s. In a stationary response, the change rate of structural vibration energy is approximately zero. The inherent damper power of the primary structure is given by

$$P_s = 2S_0\xi_1 m_1\omega_1^2\int_{-\infty}^{+\infty}|H_1|^2\,ds = 4\xi_1\omega_1 P_k\,. \tag{8}$$

Such a relationship determines the gross power efficiency that is defined as the ratio of the output power to the total excitation power.

$$\eta = \frac{P_d}{P_{ex}} = \frac{\xi_2\gamma\mu\left(4\xi_1\xi_2^2\gamma+4\xi_1^2\xi_2\gamma^2+\xi_1\gamma^3+\xi_2+\xi_1\gamma^3\mu\right)}{(\xi_2+\xi_1\gamma)(\xi_2\gamma+\xi_1+\xi_2\gamma\mu)\left(4\xi_1\xi_2\gamma+1+\gamma^2+\gamma^2\mu\right)-\gamma(\xi_2+\xi_1\gamma)^2-\gamma(\xi_2\gamma+\xi_1+\xi_2\gamma\mu)^2} \tag{9}$$

Notably, Eq. (9) is a general form for the TMD power efficiency that is suitable for different design parameters of the TMD. By substituting the optimal conditions in Eq. (6) into Eq. (9), the optimal power efficiency is,

$$\eta_{\mathrm{opt}}=\frac{P_{\mathrm{d}}}{P_{\mathrm{ex}}}=\frac{4\xi_1\left(\xi_1^2-\lambda\right)\sqrt{\mu^3\lambda}+\mu^4+2\mu^3\left(1-2\xi_1^2\right)+\mu^2\left(1+4\xi_1^4\right)+4\mu\xi_1^2\left(1-\xi_1^2\right)}{\left[\mu^2+\mu\left(1-4\xi_1^2\right)+4\xi_1^4-4\xi_1^2\right]^2}. \tag{10}$$

Given a slightly damped primary structure (i.e., ξ_1 is small), the high-order terms of ξ_1 are negligible, and Eq. (10) can be approximately simplified as

$$\eta_{\mathrm{opt}}\approx 1-\frac{4\xi_1}{\sqrt{\mu(1+\mu)}}. \tag{11}$$

An empirical threshold of $\xi_1 < 0.1(\mu^2+\mu)^{1/2}$ is suggested because Eq. (11) is only suitable for low structural damping. It is evident that a larger mass ratio μ of the ERTMD enhances the energy harvesting performance, whereas a larger inherent damping ratio ξ_1 of the structure has a negative impact on the power efficiency. Meanwhile, it is well known that a larger mass ratio of TMD offers a better control effect, regardless of which control criterion is applied. A review of the optimized vibration response can be found in the study [9]. Therefore, it can be concluded that a larger mass ratio of ERTMD benefits the energy harvesting and vibration control performance simultaneously. To the best of the authors' knowledge, such a conclusion has never been reported in the literature.

From the vibration control perspective, this power efficiency reveals the control performance of TMD under a given criterion to some extent. In the optimal cases, the structural damping power and kinetic energy are minimized, and they can be approximately expressed under low structural damping as follows:

$$P_{\mathrm{s,\,opt}}\approx\frac{4S_0\xi_1\pi}{m_1\sqrt{\mu(1+\mu)}}, \tag{12.1}$$

$$P_{\mathrm{k,\,opt}}\approx\frac{S_0\pi}{m_1\omega_1\sqrt{\mu(1+\mu)}}. \tag{12.2}$$

A part of the excitation power is inevitably dissipated by the primary structure because of the existence of ξ_1. However, a relatively large mass ratio of TMD can effectively reduce the structural damping power or kinetic energy, revealing the benefit from the vibration control perspective. This result may shed light on the optimization of power distribution in TMD-controlled structures.

According to Eq. (1), the target optimal damping coefficient $c_{2,\mathrm{opt}}$ governs the selection of the electromagnetic transducer. The upper and lower limits of the achievable damping coefficients of an electromagnetic transducer correspond to $R_{\mathrm{load}} = 0$ and ∞, respectively. Therefore, a proper selection of the transducer parameters should be done to meet the requirement $c_{\mathrm{p}} < c_{2,\,\mathrm{opt}} < (c_{\mathrm{p}} + K_{\mathrm{eq}}^2/R_{\mathrm{coil}})$.

In addition, as aforementioned, the total TMD damping power is regarded as the gross output power of ERTMD. In reality, only a portion of the gross output power can be finally harvested in an energy storage element. The net output power of ERTMD depends on another power conversion efficiency η_{em} [13, 21], which is determined by the characteristics of the transducer and the load resistance of the energy harvesting circuit. If the power of the load resistance R_{load}

is regarded as the net output power, the power conversion efficiency inside the electromagnetic transducer can be expressed as [13, 21],

$$\eta_{\text{em}} = \frac{K_{\text{eq}}^2 R_{\text{load}}}{c_{\text{p}} \left(R_{\text{load}} + R_{\text{coil}}\right)^2 + K_{\text{eq}}^2 \left(R_{\text{load}} + R_{\text{coil}}\right)} . \tag{13}$$

Combining Eqs (1) and (13) yields,

$$\eta_{\text{em}} = \frac{\left(c_{2,\text{opt}} - c_{\text{p}}\right)\left(c_{\text{p}} + K_{\text{eq}}^2 / R_{\text{coil}} - c_{2,\text{opt}}\right)}{c_{2,\text{opt}} K_{\text{eq}}^2 / R_{\text{coil}}} . \tag{14}$$

Therefore, the selection of the transducer parameters represents another complex optimization problem to achieve the maximum power conversion efficiency η_{em}, which cannot be analytically discussed because of the lack of the empirical relations among K_{eq}, R_{coil} and c_{p} of electromagnetic transducers. A general conclusion is that a large ratio of $K_{\text{eq}}^2/R_{\text{coil}}$ and a small parasitic damping c_{p} of an electromagnetic transducer can enhance the power conversion efficiency η_{em}. The unity power conversion efficiency $\eta_{\text{em}} = 1$ only occurs when R_{coil} and c_{p} approach zero. The numerical optimization of η_{em} is out of the scope of this analytical study, and thus only the gross output power of ERTMD is discussed in this paper.

4. Numerical Validation

A numerical case study is conducted to validate the conclusions. The structural parameters are as follows: $m_1 = 3$ kg and $\omega_1 = 18.25$ rad/s. All subsequent power results are normalized by the input excitation power; thus, the ERTMD output power hereinafter is equivalent to the power efficiency. The conversion efficiency η_{em} inside the transducer is regarded as 1.

Fig. 2 illustrates the performance indices of the ERTMD under the fixed structural damping ratio $\xi_1 = 0.03$ and the fixed mass ratio of ERTMD $\mu = 0.03$. The overall trends of ERTMD output power and structural kinetic energy are nearly opposite with variation of damping and frequency tuning ratio. The optimal points for maximization of ERTMD output power and minimization of structural kinetic energy are identical, with parameters $\xi_2 = 0.0874$ and $\gamma = 0.9887$, which matches exactly with the theoretical prediction by Eq. (6). The corresponding ERTMD output power and structural kinetic energy after normalization are $P_{\text{d}} = 0.558$ and $P_{\text{k}} = 0.20$, respectively, which are consistent with Eq. (10).

Numerous optimization criteria have been proposed for TMDs. Therefore, TMDs or ERTMDs may be designed using different criteria other than the optimal conditions proposed in this study. The optimal condition presented in Eq. (6) is equivalent to the H_2 minimization of structural velocity. Three other control criteria that disregard structural inherent damping, as listed in Table 1, are also considered in this section for comparison.

Fig. 3 shows the variations of the ERTMD output power and structural kinetic energy with the increasing structural inherent damping, wherein the mass ratio of TMD is fixed at $\mu = 0.03$. It can be observed that (1) the optimal condition (i.e., Eq. (6)) offers a superior vibration control and energy harvesting performance over the other design criteria (i.e., Eq. (15)) that ignore structural inherent damping in the optimization. This finding demonstrates the accuracy and effectiveness of the closed-form derivation in this study; (2) the existence of structural inherent damping degrades the ERTMD output power but improves the vibration control effect; (3) at a low level of structural inherent damping, however, the different TMD design criteria lead to a similar vibration control and energy harvesting performance.

Fig. 4 shows a performance comparison under different TMD mass ratios. It can be observed that (1) a larger mass ratio benefits the energy harvesting efficiency and vibration control effects simultaneously, regardless of which TMD design criterion is applied; (2) with an increment in the mass ratio, the superiority of the proposed optimal conditions in terms of ERTMD output power and structural kinetic energy becomes increasingly evident in comparison with the other TMD design criteria.

Fig. 5 shows the effect of the detuned TMD parameters on the harvesting performance. The optimal parameters are determined according to Eq. (6). The ERTMD output power is insensitive to the slight detuning of the damping and frequency ratios. However, when the damping ratio deviates from the optimal value significantly (ξ_2 = 2 $\xi_{2,\mathrm{opt}}$), the performance degradation becomes increasingly apparent.

Fig. 6 presents a comparison of the power efficiency computed using the approximate and exact expressions. The power efficiency is also evaluated numerically through a dynamic simulation of the entire system under broadband random excitation (0–500 Hz). The corresponding results computed directly from the input and output power are also presented in Fig. 6. An excellent agreement is observed between the numerical results and exact mathematical expression (i.e., Eq. (10)). Meanwhile, the approximation (i.e., Eq. (11)) offers a satisfactory evaluation of power efficiency only at a low level of structural inherent damping. The relative error is about 14% at the proposed empirical threshold (i.e., $0.1(\mu^2+\mu)^{1/2} \approx 1.8\%$). Such an approximation is inapplicable to a relatively large structural inherent damping.

5. Discussions

(1) When an ERTMD is optimized to minimize the H_2 norm of structural velocity in consideration of structural inherent damping, the performance objectives of vibration control and energy harvesting are identical and can be optimized simultaneously.

(2) When an ERTMD is designed using other TMD optimization criteria for vibration control, the energy harvesting performance of the ERTMD becomes sub-optimal. However, if the structural inherent damping and TMD mass ratios are small, then the deviation from the optimal performance will be limited, and the two performance objectives of the ERTMD will still be approximately consistent.

(3) Ignoring the structural inherent damping in the TMD optimization results in slightly sub-optimal parameters of the ERTMD. It may also cause a misleading conclusion that all the excitation power is absorbed by the ERTMD and the primary structure does not dissipate any power. The closed-form solution that considers structural inherent damping not only provides accurate optimal conditions, but also enables the analyses of the power distribution between the structure and ERTMD, and the corresponding power efficiency.

(4) Though the presented study is based on force excitations, it may also provide some insight into a structure subjected to random ground motions, wherein the total excitation power is proportional to the sum of the structural and ERTMD masses. A larger ERTMD mass enhances the total excitation power into the system and benefits the power harvesting. Since the total

excitation power is the sum of the ERTMD output power and structural inherent damping power, the maximization of the former leads to the minimization of the latter. This observation implies that the maximum output power of ERTMD is consistent with the minimum H_2 norm of the relative velocity between the controlled structure and the ground. However, the optimal conditions for a seismically excited structure will be different from those derived in this study and need to be numerically searched as the closed-form solution may not exist.

(5) The expressions of the power conversion efficiency and power distribution in Eqs. (10) and (11) are not only of interest to the novel ERTMDs, but also shed light on the optimization of classical TMDs with respect to power distributions.

(6) The power flow insides the electromagnetic transducer, which is influenced by the characteristics of the transducer, represents another optimization problem. The optimal selection of the transducer parameters can be done after the optimal damping $c_{2,\mathrm{opt}}$ is determined.

6. Concluding Remarks

This study presents a closed-form solution to the optimal parameters of an ERTMD installed in a damped structure subjected to random excitations, with the optimization objectives to minimize the kinetic energy of the controlled structure and to maximize the output power of the ERTMD. Several major results and conclusions are summarized as follows:

(1) Vibration control and energy harvesting are generally consistent in an ERTMD installed in a damped structure subjected to broadband random excitations, wherein the excitations may either be random forces or ground motions.

(2) A larger mass ratio of the ERTMD benefits both vibration control and energy harvesting performances. This observation is different from what has been reported in previous studies.

(3) The energy harvesting performance of the ERTMD is insensitive to slight parameter detuning.

(4) The general expressions of the power efficiency and power distributions will shed light on the power-based optimization for classical TMDs installed in damped structures.

Acknowledgement

The authors are grateful for the financial support provided by the Research Grants Council of Hong Kong through the NSFC/RGC Joint Research Scheme (N_PolyU533/17, 51761165022), the Research Impact Fund (PolyU R5020-18), and the Theme-based Research Scheme (T22-502/18-R). The findings and opinions expressed in this paper are from the authors alone and are not necessarily the views of the sponsors.

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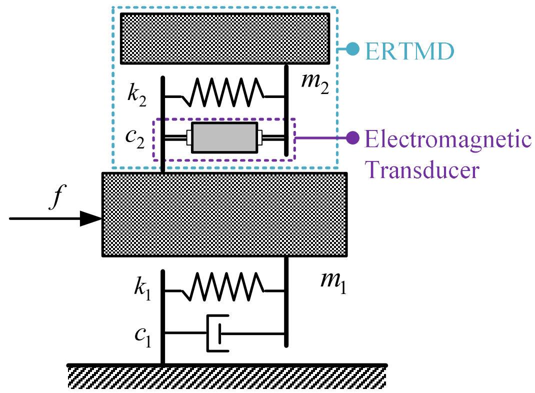

Fig. 1 Typical configuration of a damped SDOF structure with ERTMD

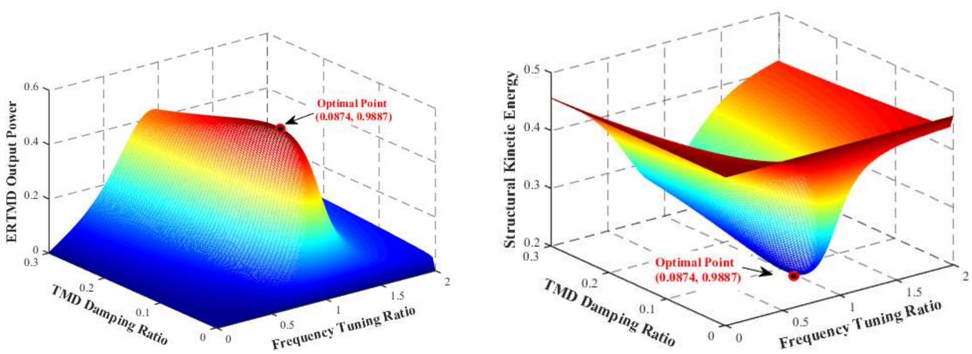

Fig. 2 Variation in the performance indices with different damping ratios (ξ_2) and frequency tuning ratios (γ) of ERTMD parameters ($\mu = \xi_1 = 0.03$) and the optimal conditions predicted by Eq. (6)

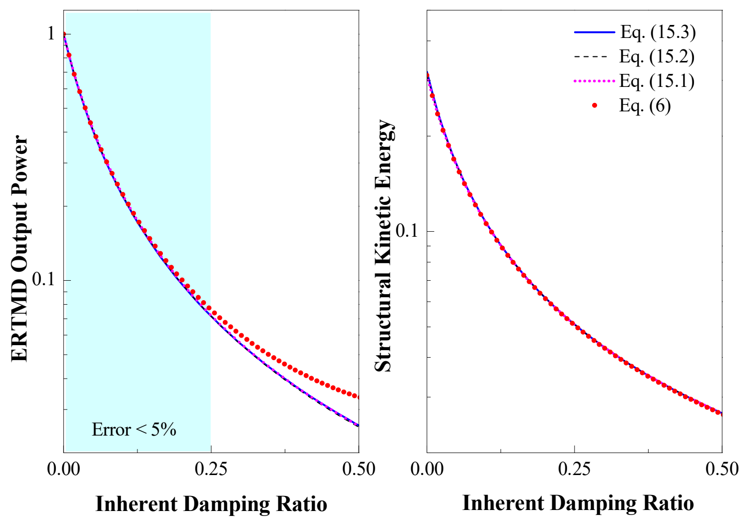

Fig. 3 Comparison of the performance of different TMD design criteria in consideration of structural inherent damping ($\mu = 0.03$)

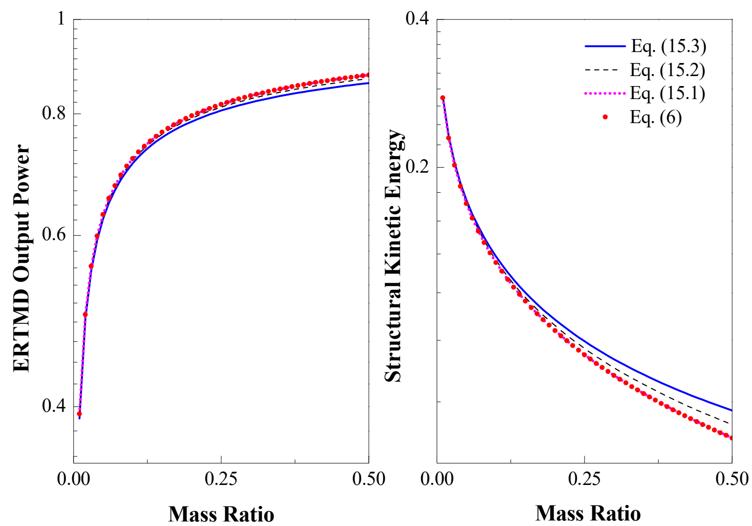

Fig. 4 Comparison of the performance of different TMD design criteria in consideration of various mass ratios of TMD ($\xi_1 = 0.03$)

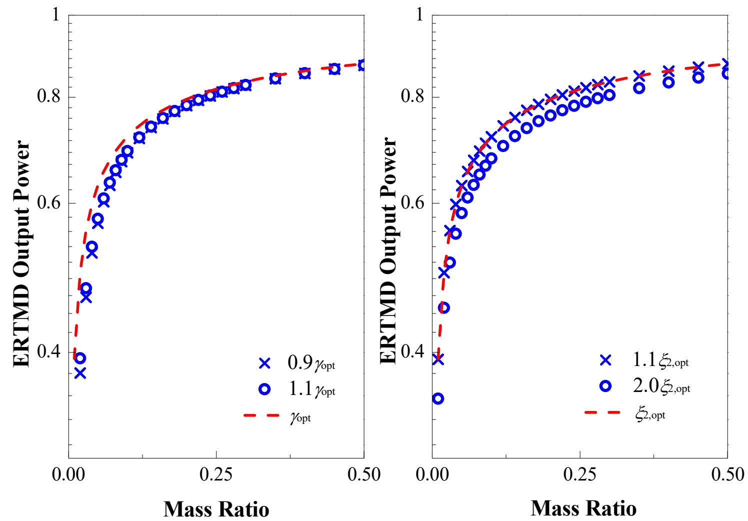

Fig. 5 Sensitivity of energy harvesting performance to detuned parameters

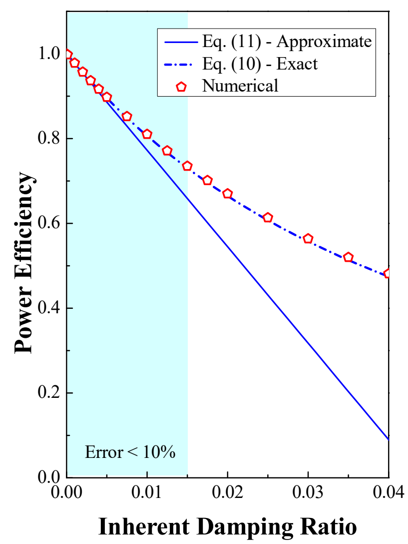

Fig. 6 Comparison of the power efficiency using approximate and exact expressions (μ = 0.03)

Table 1 Different optimization criteria of TMDs considering or ignoring structural inherent damping

Optimization Criterion	**Optimal Parameters**	**Eq.**
H_2 for velocity - Minimize structural kinetic energy - Consider structural inherent damping	See Eq. (6) proposed in this study	
H_2 for velocity [9] - Minimize structural kinetic energy; - Ignore structural inherent damping	$\xi_2 = \left\{\frac{\mu}{4}\right\}^{1/2}$, $\gamma = \left\{\frac{1}{1+\mu}\right\}^{1/2}$	(15.1)
H_2 for displacement [9] - Minimize RMS displacement - Ignore structural inherent damping	$\xi_2 = \left\{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}\right\}^{1/2}$, $\gamma = \frac{1}{1+\mu}\left\{\frac{2+\mu}{\mu}\right\}^{1/2}$	(15.2)
H_∞ for displacement [8] - Minimize the maximum displacement - Ignore structural inherent damping	$\xi_2 = \left\{\frac{3\mu}{8(1+\mu)}\right\}^{1/2}$, $\gamma = \frac{1}{1+\mu}$	(15.3)