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Surrogate-assisted Seismic Performance Assessment Incorporating Vine Copula Captured Dependence

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Abstract:

Performance-based earthquake engineering (PBEE) is an advanced philosophy for the design, assessment and decision-making of structures under seismic hazards. Improving the accuracy and efficiency of PBEE assessment is of great importance. Traditionally, a linear relationship conditioned on scalar seismic intensity measure (IM) is used to predict the seismic demand. In addition, there exists dependence within PBEE, whereas multivariate normality of logarithmic values is widely assumed for modeling the dependence in previous studies. By interconnecting several advanced techniques, this paper proposes a hybrid and novel framework to improve the PBEE, and the proposed framework can reduce uncertainties while capturing more realistic dependence. The vector IM and surrogate models are then coupled to predict the seismic demand with satisfying accuracy. Vine copula could characterize complex nonlinear dependence structures, and it is adopted to model the dependence of demands and IMs. Seismic performance can be assessed confidently. The proposed framework is illustrated on a portfolio of bridges under seismic hazards. The results show that the proposed framework could improve accuracy significantly and better capture complex dependence. Additionally, the effect of dependence modeling on high-order moments of performance is investigated. The large difference of high order moments of performance is observed by using conventional assumption and vine copula, which further highlights the necessity of implementing the proposed framework.

Keywords: Performance-based earthquake engineering; Vector intensity measure; Surrogate model; Vine copula; Dependence.

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1 Introduction

After the 1994 Northbridge and 1995 Kobe earthquakes, it is found that the indirect loss (e.g., downtime) and direct loss (e.g., repair cost) are tremendous, even though the bridges were designed to satisfy safety requirements [1]. Performance-based earthquake engineering (PBEE) was then developed to aid the design and decision-making of structures considering performance objectives (e.g., economic loss, fatality, and downtime) concerned by stakeholders [2–4]. PBEE generally involves probabilistic hazard analysis, seismic demand prediction, and consequence evaluation [5]. Due to the existent of uncertainty and complex dependence, confident performance assessment is challenging. Improving the accuracy and confidence of performance assessment is an essential task. This paper aims to propose an updated seismic performance assessment framework by reducing uncertainty and capturing more realistic dependence.

Developing a probabilistic seismic demand model (PSDM) serves as the basic step in PBEE and directly affects the accuracy of performance assessment. PSDM can be used to compute the probabilistic distribution of seismic demand under various hazard intensity levels [6]. Within this process, a linear relationship between logarithmic scalar intensity measure (IM) and the logarithmic mean of demand is widely used in previous studies for demand prediction [7,8]. The scalar IM is used as the only predictor. However, the linear equation may not be adequate to represent the complex relationship between hazard intensity and demand. Additionally, the single IM may not be adequate to reflect the complex characteristics of the ground motion time history [9] and it could result in biased estimation [10]. To address these limitations, an advanced surrogate model representing the relationship between input and response could be adopted based on a learning process. The developed surrogate model can then facilitate efficient and accurate reliability analysis [11]. In this way, multiple predictors (e.g., IMs and structural parameters) could be incorporated in surrogate models to perform a more accurate performance assessment. Surrogate models have been applied in engineering problems with satisfying accuracy [12–16]. The polynomial chaos expansion (PCE) is one type of surrogate model that consists of spectral representations [17]. Some terms of PCE are insignificant for the prediction as the high-order interaction effect is usually negligible [18]. The sparse PCE (SPCE) which only contains the selected significant terms was then proposed. Compared with PCE, SPCE requires fewer training points under the same accuracy requirement [18]. Besides using the surrogate model, another way to improve the accuracy of performance assessment is incorporating more hazard information in demand prediction. Vector IM contains more information on the ground motion and could reflect multiple characteristics of the earthquake. Vector IM could reduce the standard deviations of logarithmic demand significantly [19], improve the predictive ability of structural demand [20], and facilitate more accurate probabilistic demand analysis of structures [21]. Considering more than one IM could improve the sufficiency and efficiency in seismic slope displacement prediction [22–24]. However, the vector IM has not been well incorporated in the surrogate model to improve the predictive ability within the PBEE framework. In this paper, vector IM and SPCE are coupled to jointly improve the accuracy of seismic performance assessment.

Within the PBEE, there exists dependence from multiple sources (e.g., the demand side and IM side). The assumption of multivariate normality of logarithmic IMs is widely used for probabilistic seismic hazard analysis [25–27]. The assumption of multivariate normality of logarithmic demands is also widely adopted in PSDM [28,29]. This assumption lacks comprehensive validation, and it may not be the optimal dependence structure for IMs and demands if another dependence modeling approach is applicable. Copula is a flexible approach for modeling the dependence of variables. In this approach, the joint distribution is decomposed as marginal distributions and dependence models [30]. Compared with the assumption of multivariate normality, the copula could incorporate more dependence characteristics (e.g., central-, lower-, and upper-tail dependence) and reflect more realistic dependence features [24,31,32]. However, with respect to multivariate variables, the conventional copula approach uses the same dependence structure for modeling all pairs of random variables. This constraint limits the modeling of multiple structures and characteristics of dependence among multivariate variables. Vine copula was then proposed to address this issue [33,34]. In the vine copula approach, the joint distribution is decomposed into marginal distributions, and the multiple dependence structures among multivariate variables are captured using a system of pair copulas. The widely used assumption of multivariate normality of logarithmic IMs and demands can be considered as a specific case in the vine copula approach, where the pair copulas are all Gaussian copulas [24]. To the authors' best knowledge, the vine copula approach has not been adopted for the dependence modeling of both IMs and demand surrogate models within an integrated PBEE framework. The vine copula approach is used in this study to model the complex dependence from multiple sources within PBEE.

The hazard analysis, structural analysis, damage analysis, and loss analysis are four components within PBEE, and any of them could directly affect the performance assessment, thus affect decision making. However, the advanced techniques, which can facilitate these four components, have not been well interconnected to formulate an integrated PBEE framework. To address these issues, a novel and updated PBEE (UPBEE) framework is proposed herein to improve the accuracy and confidence, by interconnecting vector IM, surrogate model, and vine copula. Specifically, the vector IM is incorporated in the surrogate model to improve the confidence of performance assessment. The vine copula is used to model the complex dependence of both multivariate IMs and multivariate seismic demands. An updated and integrated PBEE framework is developed with improved accuracy and confidence. The effect of dependence modeling on high-order moments of performance is investigated. The remainder of this paper is organized as follows. The conventional PBEE framework is discussed in Section 2. The methodology of UPBEE is introduced in section 3. The proposed UPBEE framework is introduced in Section 4. An illustrative example is presented in section 5. Section 6 contains conclusions.

2 Performance-based earthquake engineering (PBEE): A review

PBEE is a new generation philosophy for the assessment and decision-making of structures. In this engineering philosophy, the structures are expected to satisfy performance objectives (e.g., direct loss, indirect loss, and fatality, etc.). The conventional procedures of the PBEE framework can be summarized as follows. Probabilistic seismic hazard analysis is performed to identify the potential IM levels and corresponding probabilities. A linear relationship between logarithmic scalar IM and the logarithmic mean of demand is used to predict the seismic demand under different IM levels. For the dependence modeling among multiple demands, multivariate normality of logarithmic values is assumed. Vulnerability is computed based on PSDM. Then, the probabilistic performance can be computed. A general expression indicating the probability that a decision variable exceeding *DV* under a given IM can be written as [35]

$$G(DV \mid IM) = \iint G(DV \mid DM) dG(DM \mid EDP) dG(EDP \mid IM) \tag{1}$$

where *G* function is the complementary cumulative distribution function; *DM* represents damage measure; and *EDP* is engineering demand parameter. Seismic repair loss is one of the seismic performance indicators [2,36]. The ratio of repair loss to the construction cost of the structure is defined as the repair loss ratio. Herein, the repair loss ratio is considered to illustrate the proposed approach, the proposed approach could be updated by considering other performance aspects (e.g., sustainability and resilience). Each damage state is associated with a defined repair loss ratio [37,38]. The probability of structure being in each damage state can be calculated based on vulnerability. The repair loss ratio under given hazard intensity is calculated as the sum of weighted repair loss ratios associated with all damage states [39].

The conventional PBEE framework can be further updated. The linear relationship used for seismic demand prediction may not satisfy the accuracy requirement due to its simplicity. The scalar IM used as the only predictor may not provide adequate information of the earthquakes, resulting in a relatively large amount of prediction uncertainty. The assumption of multivariate normality of logarithmic values is widely used in PBEE for dependence modeling, and it cannot well capture nonlinear dependence characteristics. These issues could jointly affect the confidence and accuracy of PBEE.

3 Efficient uncertainty quantification and modeling of nonlinear dependence within PBEE

To improve the confidence and accuracy of PBEE, this study proposes an updated PBEE framework by interconnecting SPCE, vine copula, and vector IM. The specific techniques are introduced in the following parts.

3.1 Sparse polynomial chaos expansion (SPCE) – surrogate model

For assessing the vulnerability of structures, it is necessary to compute the joint probabilistic distribution of multiple seismic demands under different IM levels. Conventionally, a linear relationship between logarithmic scalar IM and the logarithmic mean of demand is used [8]. In this study, the relationship between the input vector $\boldsymbol{X} \in \mathbb{R}^M$ and multiple outputs $[Y_1, Y_2, \ldots, Y_W]$ is established using surrogate model. Generally, multivariate surrogate model can be expressed as [28]

$$\hat{\boldsymbol{Y}} = \bar{\boldsymbol{Y}}(\boldsymbol{X}) + \boldsymbol{\varepsilon}_S(\boldsymbol{X}) \tag{2}$$

where $\hat{\boldsymbol{Y}} \in \mathbb{R}^W$ is the prediction from the model; $\bar{\boldsymbol{Y}} \in \mathbb{R}^W$ is the estimation from a trend model; and $\boldsymbol{\varepsilon}_S \in \mathbb{R}^W$ is the correlated model error.

SPCE is one type of surrogate model, and it performs well in uncertainty quantification and data-driven prediction [40,41]. SPCE is used in this study as a surrogate model. The random input vector of a computational model $\boldsymbol{M}$ is represented by a joint PDF $\boldsymbol{f_X}$. The output of the computational model $\boldsymbol{M}(\boldsymbol{X})$ is associated with finite variance, the PCE of $\boldsymbol{M}(\boldsymbol{X})$ is written as [42]

$$\boldsymbol{M}(\boldsymbol{X}) = \sum_{\alpha \in \mathbb{N}^M} c_\alpha \Psi_\alpha(\boldsymbol{X}) \tag{3}$$

where $\Psi_\alpha(\boldsymbol{X})$ are the multivariate polynomials orthonormal with respect to $\boldsymbol{f_X}$; $\alpha \in \mathbb{N}^M$ is a set of indices mapping to the components of the $\Psi_\alpha(\boldsymbol{X})$; and c_α are the coefficients.

The multivariate polynomial is computed as

$$\Psi_{\alpha}(\mathbf{x}) = \prod_{i=1}^{M} \phi_{\alpha_i}^{(i)}(x_i) \tag{4}$$

where $\phi_{\alpha_i}^{(i)}$ is the univariate orthogonal polynomial for the i^{th} variable in degree α_i.

The original PCE can be truncated as

$$\boldsymbol{M}^{\boldsymbol{PC}}(\boldsymbol{X}) = \sum_{\alpha \in \mathrm{k}} c_{\alpha} \Psi_{\alpha}(\boldsymbol{X}) \tag{5}$$

where k is the truncated set of multi-indices of multivariate polynomials. The PCE can be truncated by defining a maximum total degree p of all the polynomials associated with the input variables as [43]

$$\mathrm{k}^{M,p} = \left\{ \alpha \in \mathbb{N}^{M} : |\alpha| \leq p \right\}, card\mathrm{k}^{M,p} \equiv P = \frac{(p+M)!}{p!M!} \tag{6}$$

Then, the least-square solution is used to compute the coefficients of PCE as [18,44]

$$\hat{\boldsymbol{C}} = (\boldsymbol{\Phi}^{T}\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^{T}\boldsymbol{Y}_{i,v}, \boldsymbol{\Phi} = \begin{pmatrix} \Psi_0(\mathbf{x}^{(1)}) & \cdots & \Psi_{card\mathrm{k}^{M,p}-1}(\mathbf{x}^{(1)}) \\ \vdots & \ddots & \vdots \\ \Psi_0(\mathbf{x}^{(N)}) & \cdots & \Psi_{card\mathrm{k}^{M,p}-1}(\mathbf{x}^{(N)}) \end{pmatrix} \tag{7}$$

where $\hat{\boldsymbol{C}}$ is the computed vector of coefficients; and $\boldsymbol{Y}_{i,v}$ is the vector of the model evaluations associated with i^{th} demand at N input vectors $\mathbf{x}^{(1)}$, …, $\mathbf{x}^{(N)}$; and $\Psi_j(.), j = 0,...,card\mathrm{k}^{M,p}-1$ are the basis functions.

It is found that the SPCE, which contains the selected significant terms, performs better in some cases. Under the same accuracy requirement, SPCE requires a smaller size of training data compared with full PCE [18]. This study utilizes an algorithm orthogonal matching pursuit (OMP) [45] to develop SPCE. In OMP, the significant basis functions are iteratively selected from the candidate set and added to the model. For each iteration, the algorithm selects a basis function from the candidate set which is most correlated with the residual, this basis function is selected by solving

$$h(k) = \text{argmax}_{i\in\mathbb{C}_k} \frac{|\langle \boldsymbol{\psi}_i, \mathbf{r}_{k-1} \rangle|}{\|\boldsymbol{\psi}_i\|_2} \tag{8}$$

where $\Psi_{h(k)}$ is the selected basis function at iteration k; $\mathbb{C}_k$ is the updated dictionary at iteration k by excluding the basis function selected at iteration k – 1; $\boldsymbol{\psi_i}$ represent the evaluations using basis function i; and $\mathbf{r}_{k-1}$ represent the residual from the PCE associated with iteration k – 1.

After adding the selected basis functions at each iteration, the coefficients for active basis functions are computed using least square regression. The residual $\mathbf{r}_{k-1}$ is computed as

$$\mathbf{r}_{k-1} = \mathbf{\Phi}_{k-1}\boldsymbol{C}_{k-1} - \boldsymbol{Y}_{i,v} \tag{9}$$

where $\mathbf{\Phi}_{k-1}$ is the matrix containing the evaluations using the basis functions at iteration k – 1; and $\boldsymbol{C}_{k-1}$ are the coefficients obtained at iteration k – 1.

The algorithm for basis function selection is iteratively performed and stopped until $\|\mathbf{r}\|_2$ is below the predefined value. The algorithm stopping threshold of $\|\mathbf{r}\|_2$ is computed through the v-fold cross-validation technique. Given the data of probabilistic structural parameters, seismic IMs, and responses from finite element models, the SPCE could be developed using this algorithm.

3.2 Vine copula-based dependence modeling

3.2.1 Vine copula model

Copula is a powerful tool in characterizing the complex dependence associated with multiple variables. Let d random variables X_1, …, X_d have marginal distribution functions $F_i(x_i)$ and joint cumulative distribution function (CDF) $F(x_1, \ldots, x_d)$, i = 1, …, d, the joint CDF of these variables can be expressed as [30]

$$F(x_1,...,x_d) = P[X \le x_1,...,X_d \le x_d] = C(F_1(x_1),...,F_d(x_d)\,|\,\boldsymbol{\theta}) = C(u_1,...,u_d\,|\,\boldsymbol{\theta}) \tag{10}$$

where P[.] is the corresponding probability; $C(u_1, \ldots, u_d|\boldsymbol{\theta})$ is the copula function with copula parameters $\boldsymbol{\theta}$; and $u_i = F(x_i)$.

The joint probability density function (PDF) of $X_1, \ldots, X_d$ is expressed as

$$f(x_1,\ldots,x_d) = \frac{\partial^d C(F_1(x_1),\ldots,F_d(x_d)\,|\,\boldsymbol{\theta})}{\partial x_1 \ldots \partial x_d} = c(F_1(x_1),\ldots,F_d(x_d)\,|\,\boldsymbol{\theta}) \cdot \prod_{i=1}^{d} f_i(x_i) \tag{11}$$

$$c(F_1(x_1),\ldots,F_d(x_d)\,|\,\boldsymbol{\theta}) = \frac{\partial^d C(F_1(x_1),\ldots,F_d(x_d)\,|\,\boldsymbol{\theta})}{\partial u_1 \ldots \partial u_d} \tag{12}$$

where $c(u_1, \ldots, u_d|\boldsymbol{\theta})$ represents the copula density function; and $f_i(x_i)$ is the marginal PDF of x_i.

Many copula families could be used to characterize the dependence of random variables [30,46]. In the conventional copula approach, the same dependence structure is used for all pairs of variables, which is inflexible for describing the different dependence structures among multiple random variables. Vine copula [33] is used to address this issue. It is a more flexible approach to model the complex dependence structures of high-dimensional random variables. By using vine copula, the joint PDF is decomposed into the product of bivariant copula density functions, thus various copula families could be used for dependence modeling of high-dimensional variables.

The joint PDF of $X_1, \ldots, X_d$ can be expressed as

$$f(x_1,\ldots,x_d) = f_1(x_1) f_{2|1}(x_2\,|\,x_1) \ldots f_{d|1,\ldots,d-1}(x_d\,|\,x_1,\ldots,x_{d-1}) \tag{13}$$

where $f(x|\mathbf{v})$ is the conditional PDF and can be expressed as the product of pair copulas and conditional PDF as

$$f(x\,|\,\mathbf{v}) = c_{x,v_j|\mathbf{v}_{-j}}(F(x\,|\,\mathbf{v}_{-j}), F(v_j\,|\,\mathbf{v}_{-j}); \theta_{x,v_j|\mathbf{v}_{-j}})\, f(x\,|\,\mathbf{v}_{-j}) \tag{14}$$

where v_j is one variable of $\mathbf{v}$; $\mathbf{v}_{-j}$ is the vector excluding v_j; and $c_{x,vj|\mathbf{v}-j}(.)$ is the copula density function. The conditional CDF can be expressed as

$$F(x\,|\,\mathbf{v}) = \frac{\partial C_{x,v_j|\mathbf{v}_{-j}}(F(x\,|\,\mathbf{v}_{-j}), F(v_j\,|\,\mathbf{v}_{-j}); \theta_{x,v_j|\mathbf{v}_{-j}})}{\partial F(v_j\,|\,\mathbf{v}_{-j})} \tag{15}$$

where $C_{x,vj|v\text{-}j}(.)$ is the copula function. Eq. (13) could be decomposed as the product of copula density functions and marginal PDFs by using Eq. (14). The conditional CDF of x on univariant v can be expressed as

$$F(x\,|\,v) = \frac{\partial C_{x,v}(F_x(x), F_v(v); \theta_{x,v})}{\partial F_v(v)} \tag{16}$$

The $F(x|v)$ can be written as h-function

$$F(x\,|\,v) = \frac{\partial C_{x,v}(u_x, u_j; \theta_{x,v})}{\partial u_v} = h(u_x, u_v; \theta_{x,v}) \tag{17}$$

The copula functions and h-functions for various copula families are provided in the literature [30]. A drawable vine (D-vine) copula consists of a set of trees, each tree consists of several nodes and edges. Each edge is represented by a pair copula function. The PDF of a D-vine copula is expressed as

$$f(x_1, \ldots, x_d) = \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j|i+1,\ldots,i+j-1}(F(x_i\,|\,x_{i+1},\ldots,x_{i+j-1}), F(x_{i+j}\,|\,x_{i+1},\ldots,x_{i+j-1}); \theta_{i,i+j|i+1,\ldots,i+j-1}) \prod_{k=1}^{d} f(x_k)$$

(18)

3.2.2 Inference of vine copula from data

Given the vine copula structure and a set of samples $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(N)}$, $\mathbf{x}^{(i)} = (x_1^i, \ldots, x_d^i)$, the parameters of vine copula can be computed using joint maximum likelihood estimation [33]. The joint maximum likelihood estimation simultaneously computes all the parameters of a vine copula by maximizing the log-likelihood. The parameters of a given vine copula structure under a set of samples can be estimated as

$$\hat{\boldsymbol{\theta}} = \arg\max_{\Theta} LL(\mathbf{x}^k; \boldsymbol{\theta}) \tag{19}$$

where $\hat{\boldsymbol{\theta}}$ is the estimated vector of vine copula parameters; Θ is the range of copula parameters; and $LL(\mathbf{x}^k; \boldsymbol{\theta})$ is the log-likelihood for a given sample set.

The different conditioning order and copula families result in different structures of vine copulas. It is necessary to determine the optimal vine copula within the candidates. The Akaike Information Criterion (AIC) could be used to select the optimal copula [47,48]. For a given vine copula and sample set, the AIC is computed as

$$AIC = -2LL(\mathbf{x}^k; \hat{\boldsymbol{\theta}}) + 2np \tag{20}$$

where $LL(\mathbf{x}^k; \hat{\boldsymbol{\theta}})$ is the log-likelihood of the fitted vine copula; and np is the number of parameters in a vine copula.

Bayesian Information Criterion (BIC) is the other criterion to determine the optimal vine copula, it can be expressed as

$$BIC = -2LL(\mathbf{x}^k; \hat{\boldsymbol{\theta}}) + np \ln N_v \tag{21}$$

where N_v is the number of samples used for developing vine copula. The optimal vine copula is determined as the one associated with minimum AIC and BIC values. Once the optimal vine copula is inferred, the joint distribution of multivariant variables considering dependence could be determined [49].

3.3 Dependence modeling PBEE

There exists dependence associated with multiple sources within PBEE. For instance, a complex system usually consists of multivariant demands, the dependence among multiple demands could affect the system vulnerability. When vector IM is used, the dependence among multiple IMs could affect the joint exceeding frequency. In this study, the dependence from two sides (e.g., IMs and demands) is considered. Joint normality of logarithmic values is widely assumed in previous studies for dependence modeling within PBEE. However, the multivariate normal distribution could not reflect the complex nonlinear dependence characteristics. This simple assumption may result in inaccurate assessment and mislead decision-making of structures. Vine copula, which could capture complex nonlinear dependence characteristics, is adopted in this study to model more realistic dependence structures.

3.3.1 Probabilistic seismic hazard analysis for vector IM considering the dependence

Conventionally, scalar seismic IM is used in performance assessment [8]. Scalar IM can only reflect part of the information regarding amplitude, spectrum characteristics, and duration of ground motion. Due to the complexity of the ground motion, the demands predicted using a single seismic IM usually involve a relatively large amount of uncertainty. Compared with scalar IM, vector IM contains more information on ground motion, thus it could reduce the uncertainty of seismic demand prediction [10]. To further improve the accuracy of seismic demand prediction, the vector IM is used in this study.

To quantify the probabilistic performance, the joint probabilistic distribution of seismic intensities should be identified, and it is achieved by probabilistic seismic hazard analysis. For given magnitude and distance, the seismic intensity is uncertain. The ground motion prediction model (GMPM) is used to predict probabilistic seismic intensity [50]. The GMPM can be generally expressed as

$$\ln IM = \mu_{\ln IM}(R, M, \Omega) + \varepsilon_{IM}\sigma_{\ln IM} \tag{22}$$

where ln IM is the natural logarithm of an earthquake intensity; M is magnitude; R is the source to site distance; Ω are other parameters used to describe an earthquake scenario (e.g., region of the earthquake and shear wave velocity averaged over top 30 *m,* etc.); $\mu_{\ln IM}(R, M, \Omega)$ is the mean of ln IM for given R, M, and Ω; $\sigma_{\ln IM}$ is the standard deviation of ln IM; and ε_{IM} is normalized residual term.

There is dependence among IMs. The ε_{IM} represents the record-to-record aleatory variability [20] and is considered to follow a standard normal distribution (Baker *et al*., 2007). By using ε_{IM}, the correlation models were developed to account for the dependence among IMs [25]. The logarithmic IMs are assumed to follow a multivariate normal distribution in previous studies and the Pearson correlation coefficient is widely used [27]. As indicated previously, vine copula is a flexible approach and could capture complex dependence characteristics. This study utilizes the vine copula approach to model the dependence of IMs. By using the approach mentioned in section 3.2, the vine copula model can be inferred based on ε_{IM} and historical data. Once the vine copula model is established, the joint PDF of vector IM for given earthquake magnitude and distance $f(IM_1, IM_2, IM_3|m, r)$ can be computed based on GMPM and Eq. (18). The probability of IM_1, IM_2, IM_3 exceeding im_1, im_2, and im_3 for a given earthquake scenario can be expressed as

$$P_{(IM_1>im_1, IM_2>im_2, IM_3>im_3|m,r)} = \iiint_{im_1, im_2, im_3} f(IM_1, IM_2, IM_3 \mid m, r) dim_1 dim_2 dim_3 \quad (23)$$

Considering uncertain scenarios, the joint mean rate of the three IMs exceeding im_1, im_2, and im_3 is computed based on the total probability theorem as [24,27]

$$\lambda(im_1, im_2, im_3) = \lambda_{m_{\min}} \iint P_{(IM_1>im_1, IM_2>im_2, IM_3>im_3|m,r)} f_M(m) f_R(r) dmdr \quad (24)$$

where $\lambda_{m\min}$ is the annual rate of occurrence of earthquakes exceeding considered minimum magnitude; and $f_M(m)$ and $f_R(r)$ are the PDFs of the magnitude and distance, respectively.

3.3.2 Joint probabilistic seismic demands considering the dependence

For an engineering system, multiple seismic demands are usually of interest. It is necessary to compute the joint probabilistic demands considering dependence for system vulnerability analysis. The dependence among multiple structural demands can be described by the dependence of $\boldsymbol{\varepsilon_S}$ [28]. Modeling of $\boldsymbol{\varepsilon_S}$ plays an important role in uncertainty propagation and reliability analysis. It incorporates the consideration of the difference between finite element model evaluations and trend model predictions, as well as the uncertainty associated with ground motion. Normal distribution with a mean of zero is a widely acceptable consideration for modeling the marginal distribution of $\boldsymbol{\varepsilon_S}$ [28,40]. The multivariate normal distribution is widely assumed for the dependence modeling of $\boldsymbol{\varepsilon_S}$ [28]. In this study, the vine copula is used to model the dependence of $\boldsymbol{\varepsilon_S}$, as it could capture more complex dependence characteristics. After establishing the SPCE, the residual of the structural demands could be computed. Then, the residual from multiple demands is used to infer the vine copula based on the method mentioned in section 3.2. The joint distribution of multivariant demands incorporating dependence could be generated from the established vine copula model. The process of dependence modeling within PBEE is presented in Figure 1.

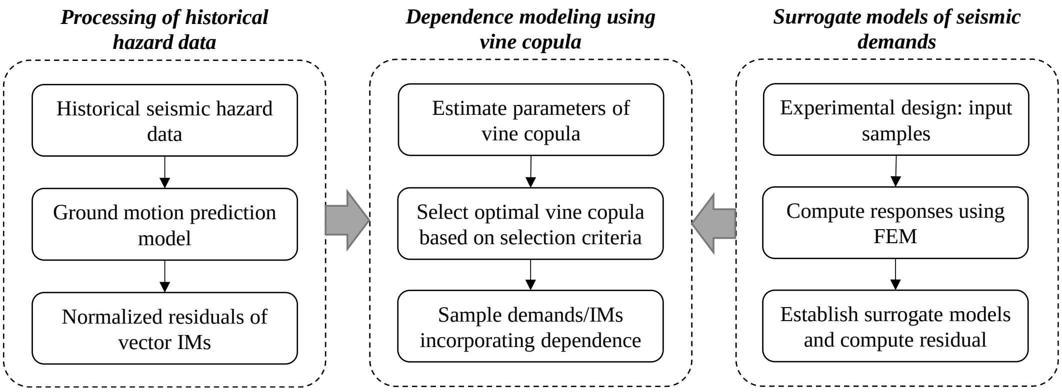

Figure 1. Process of dependence modeling within PBEE

4 Framework of updated performance-based earthquake engineering (UPBEE)

By integrating the above-mentioned techniques, this study proposes a framework of improved PBEE, which could aid more accurate and confident seismic performance assessment considering complex dependence. Vector IM is adopted within PBEE as it reflects more information on the hazard characteristics compared with conventional scalar IM. The probabilistic seismic hazard analysis is performed for vector IM. The Vine copula is used to capture the complex dependence among multiple IMs. The seismic demand surrogate models incorporating vector IM are established using a learning algorithm. The vine copula is used for the second time to model the dependence among multiple demands. The system vulnerability considering dependence can be computed using the surrogate model and vine copula. Then, the performance indicators can be computed.

Compared with conventional PBEE, the proposed UPBEE framework has several advantages. The assumption of multivariate normality of logarithmic values, which is widely used in PBEE for dependence modeling, can only capture one of many possible solutions and may produce severely biased results [52]. In the proposed framework, two vine copula models are established for demands and IMs, respectively. More realistic dependence structures associated with both IMs and demands are captured by vine copula models. In addition, the uncertainty associated with seismic demand prediction is reduced by using a learning algorithm and vector IM. The necessity and superiority of the proposed approach are illustrated in the case study. The computational process of the proposed approach is illustrated in Table 1. A comparison of the conventional PBEE framework and the UPBEE framework is presented in Figure 2.

The major contribution of this paper is to develop an updated and integrated PBEE framework by interconnecting several novel techniques. As indicated in Figure 2, the improvement is achieved within hazard analysis, structural analysis, damage analysis, and loss analysis. Confident seismic performance assessment can be accomplished by using the developed UPBEE framework.

Table 1. Computational procedures of the UPBEE framework

Procedures of UPBEE framework
Probabilistic hazard analysis for vector IM considering vine copula captured dependence
1. Process historical earthquake data to obtain normalized residuals of considered IMs
2. Determine copula families
3. Pair copula selection
4. Compute copula parameters by performing joint maximum likelihood estimation, subjected to residual data
5. Compute AIC and BIC
6. Obtain best-fit vine copula
7. Identify seismic hazard source
8. Sample dependent residuals using established vine copula
9. Compute $\mu_{\ln IM}$ (R, M, Ω) and $\sigma_{\ln IM}$ for the corresponding scenarios using the ground motion prediction model
10. Obtain joint distribution of vector IM considering the dependence
Surrogate-assisted vulnerability assessment considering vine copula captured dependence
11. Determine probabilistic structural parameters from inventory
12. Obtain a set of structure samples
13. Perform nonlinear time history analysis for the sampled structures
14. Record the demands of interest
15. Perform leaning algorithm to establish surrogate models of all demands using structure samples, vector IMs, and recorded demands
16. Compute residuals from surrogate models

17. Establish vine copula for demands using the residuals (invoke line 2-6)
18. Compute vulnerability using surrogate model and vine copula

Performance assessment

19. Compute the probabilities of structures being in each damage state
20. Determine consequences associated with each damage state
21. Compute probabilistic performance

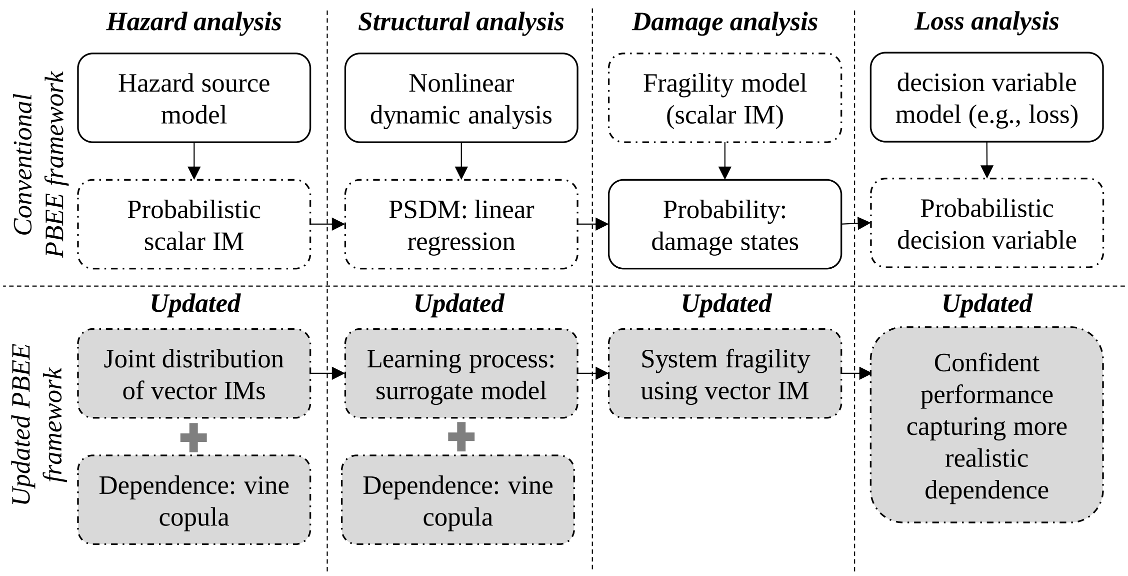

Figure 2. Conventional PBEE framework and updated PBEE framework

5 Illustrative example

For regional seismic performance assessment, the performance of portfolios of bridges is of concern. The proposed framework is applied to a portfolio of bridges subjected to seismic hazards. Probabilistic seismic hazard analysis for vector IM is performed and vine copula is used to capture dependence. Bridge samples are generated from corresponding distributions (e.g., bridge inventory). Nonlinear time history analysis of the bridge samples is performed in OpenSees to obtain the training data. Seismic demand surrogate models are established, and vine copula is used to model the dependence among multiple demands. Finally, system vulnerability and probabilistic performance are computed. The illustration of the computational process is shown in Figure 3.

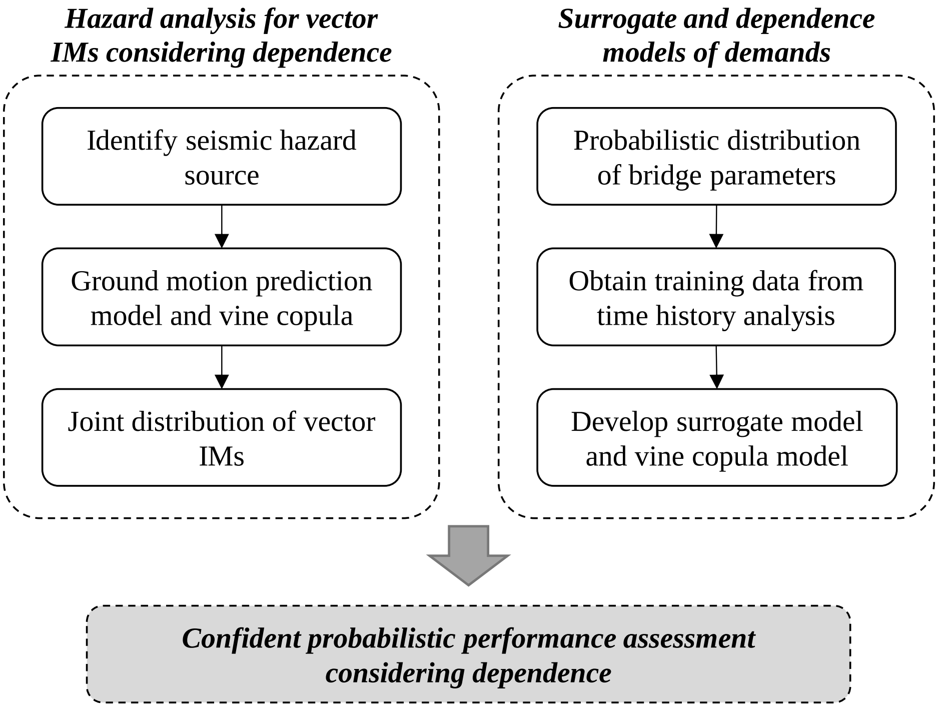

Figure 3. Illustration of the computational process

5.1 Probabilistic seismic hazard for vector IM incorporating vine copula captured dependence

The peak ground acceleration (PGA), spectral acceleration at the period of 0.2s (Sa0.2), and spectral acceleration at the period of 1.5s (Sa1.5) are used as vector IM in this example. There is dependence among multiple seismic IMs. Dependence modeling is necessary for probabilistic seismic hazard analysis for multiple IMs. Conventionally, the dependence is modeled based on the assumption of multivariate normality of logarithmic values. In this study, the dependence modeling of IMs is accomplished by using vine copula as it could capture nonlinear and complex dependence characteristics. The historical ground motion data [50] is used to establish the vine copula model. The normalized residuals are computed using the ground motion prediction model [50]. The marginal distribution of IM residual is considered to follow the standard normal distribution [51]. Then, a vine copula model capturing the dependence of IMs is established using IM residuals, as indicated in section 3.3.

Given different seismic scenarios, the mean and standard deviation of ln IM can be computed using the ground motion prediction model. The residual samples of IMs are generated from the established vine copula model, then the joint probabilistic distribution of the three IMs associated with a given scenario could be computed. In structural decision-making, the IMs at certain return periods are of interest. The structures are expected to satisfy different performance levels under different return period earthquakes. This aspect is considered herein. The magnitudes considered are 5.5-8, the distance is considered as 6 km. Two million vector IM samples considering dependence are generated using the ground motion prediction model and vine copula. For each vector IM sample, the return period can be calculated. For a considered return period, the target vector IM samples can be determined [26].

5.2 Surrogate models of seismic demands incorporating dependence

To establish the surrogate model of seismic demand, a set of training data should be obtained. Based on the probabilistic distributions listed in Table 2, 320 bridge samples are generated using the Latin hypercube sampling technique [53]. Each bridge realization is paired with a selected ground motion [54], and nonlinear time history analysis is performed in software OpenSees [55–57]. The finite element model of the bridge is presented in Figure 4. The demands associated with the column, bearing, and abutment are recorded. Thus, the training data set including the probabilistic input parameters and demands is obtained.

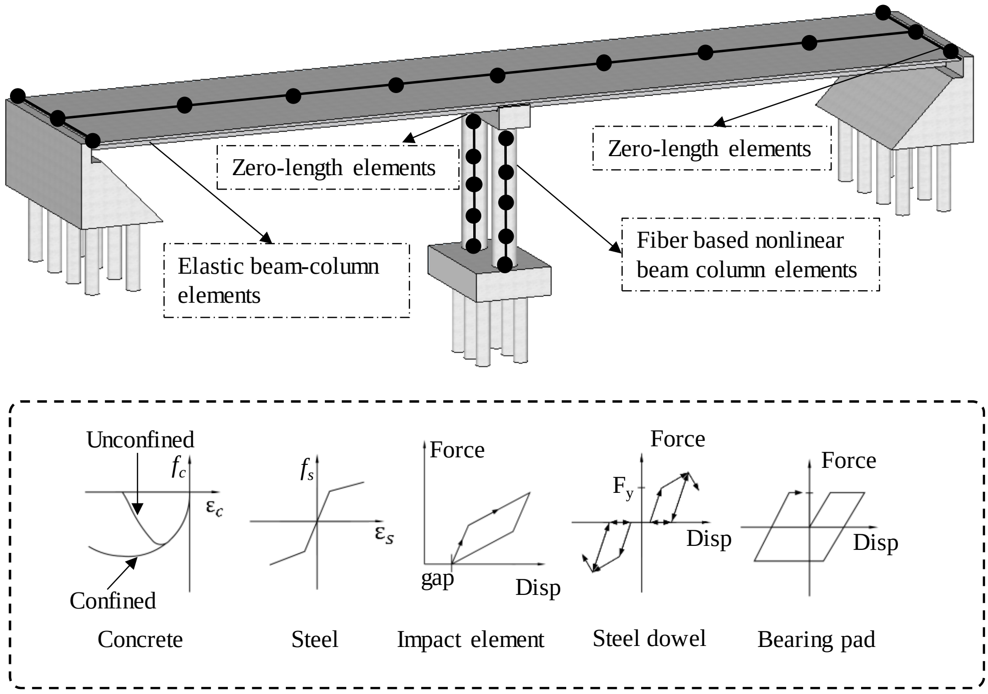

Figure 4. Finite element model of the bridge

Table 2. Probabilistic parameters used for the bridges

Parameters	Units	Distribution type	μ	σ	Ref.
Concrete compressive strength	MPa	Normal	29.03	3.59	[58]
Reinforcing steel yield strength	MPa	Lognormal	465.0	37.30	[58]
Span length	mm	Lognormal	31775	8738	[58]
Deck width	mm	Lognormal	11970	2418	[58]
Column height	mm	Lognormal	6625	865	[58]
Abutment backwall height	mm	Lognormal	2186	441	[58]
Bearing coefficient of friction	-	Normal	0.3	0.1	[58]
Strength of a composite of two dowels	kN	Lognormal	116	9.28	[29]
Abutment-deck gap	mm	Lognormal	23.5	12.5	[58]
Backfill initial stiffness at the benchmark backwall height	N/m/cm	Lognormal	384	138	[59]
Backfill ultimate capacity at the benchmark backwall height	kN/m	Lognormal	475	111	[59]
Damping		Normal	0.045	0.0125	[58]
Foundation translational spring stiffnesses	N/mm	Normal	140101	105076	[58]
Shear modulus of elastomeric pad	MPa	Uniform	1.365	0.407	[29]
Mass factor	-	Uniform	1	0.058	[29]
Longitudinal reinforcement ratio	(%)	Uniform	2.25	0.52	[58]

Note: μ = mean value and σ = standard deviation.

Given the training data set, the SPCE models of seismic demands are established using the approach indicated in section 3.1. Once the SPCE is established, it could be used for efficient seismic demand prediction. As discussed previously, the conventional approach may result in a relatively large amount of uncertainty in engineering applications. By using SPCE, the relationship between demand and IM is established by a learning process. Multiple bridge parameters could be incorporated as predictors. Additionally, multiple hazard characteristics (e.g., vector IMs), which provide a more comprehensive description of the hazard, could be incorporated into the demand prediction process. To illustrate the prediction performance of the SPCE and vector IM coupled approach, the mean squared errors (MSEs) on a test sample set are computed and presented in Table 3. When scalar IM is used, mean squared errors of the six demands are reduced by implementing SPCE, compared with the conventional method. The mean squared errors of the six demands are further reduced by using vector IM in SPCE. The relative improvement of accuracy of the proposed approach compared with the conventional approach (e.g., (MSE of the conventional method - MSE of proposed method)/MSE of conventional method) is shown in Figure 5. By using the developed approach, a significant improvement of accuracy is observed for the six demands. The reduction of error associated with the proposed approach could be interpreted from three aspects: (1) the implementation of SPCE for uncertainty propagation; (2) the incorporation of a more comprehensive description of hazard intensities by vector IM; and (3) the incorporation of multiple bridge parameters within demand prediction.

Table 3. Mean squared error on a test set

Methods	C1	C2	C3	C4	C5	C6
Linear regression	0.357	0.880	0.270	6.791	6.779	0.229
SPCE	0.263	0.700	0.199	6.735	6.705	0.176
SPCE and vector IM	0.245	0.467	0.176	4.971	4.932	0.162

Note: C1 is the column curvature ductility; C2 is the bearing longitudinal displacement; C3 is the bearing transverse displacement; C4 is the abutment active displacement; C5 is the abutment passive displacement; and C6 is the abutment transverse displacement.

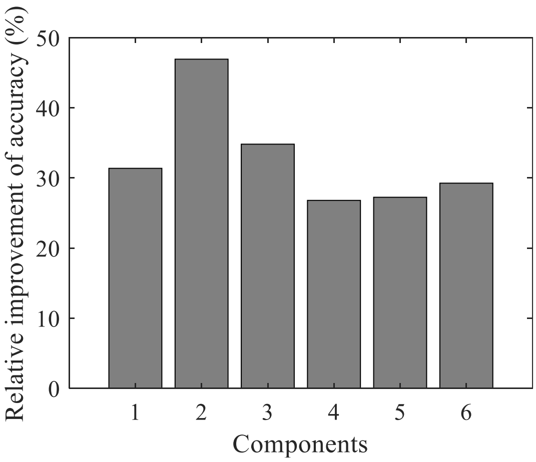

Figure 5. The relative improvement of accuracy comparing the proposed approach and conventional approach

The model error is used to characterize the dependence among multiple demands. The marginal distribution of these residuals is considered as a normal distribution with a mean of zero [28,40]. For dependence modeling, the multivariate normal distribution is widely assumed [28]. This study uses the vine copula approach to capture the complex nonlinear dependence characteristics of seismic demands. The residuals associated with six demands are computed from the surrogate models. Then, the vine copula model can be established using the residual data. To illustrate the performance of the best fit vine copula, the criterion values of AIC, BIC, and log-likelihood are shown in Figure 6. The best fit vine copula is associated with minimum AIC, BIC, and maximum log-likelihood, which indicates that the best fit vine copula performs best for the dependence modeling. The assumption of multivariate normality of logarithmic values is widely used in previous studies for both IMs and demands, it can be considered as a specific case in the copula approach, where the dependence is modeled using Gaussian copula. The results show that multivariate normality is not the optimal dependence structure, while vine copula performs better due to its flexibility. To further testify the necessity of using vine copula approach, it is necessary to know how much difference in performance calculated by conventional multivariate normality assumption and vine copula approach would be. This aspect is investigated in next section.

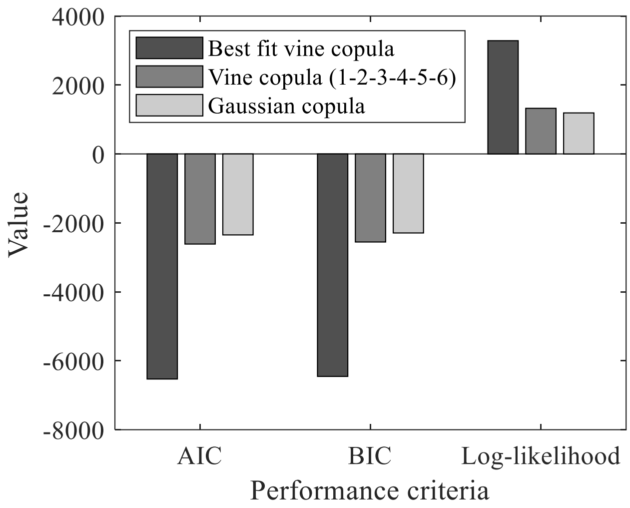

Figure 6. Performance of the best fit vine, arbitrary order vine, and Gaussian copulas

5.3 Seismic vulnerability and probabilistic performance

The surrogate model and vine copula are used to compute the seismic vulnerability of bridges. The probabilistic joint seismic demands are computed. The capacity samples are generated from corresponding distributions as listed in Table 4. Then, the bridge system vulnerability can be computed by comparing the demand and capacity samples. By repeating this process for a set of IM vectors, the vulnerability surfaces can be generated as shown in Figure 7.

Table 4. Damage states associated with different bridge components [29]

Component	Slight		Moderate		Extensive		Complete	
	med.	disp.	med.	disp.	med.	disp.	med.	disp.
Concrete Column (curvature ductility)	1.29	0.59	2.10	0.51	3.52	0.64	5.24	0.65
Elastomeric Bearing Fixed-Long (mm)	28.9	0.60	104.2	0.55	136.1	0.59	186.6	0.65
Elastomeric Bearing Fixed-Tran (mm)	28.8	0.79	90.9	0.68	142.2	0.73	195.0	0.66
Abutment-Passive (mm)	37.0	0.46	146.0	0.46	N/A	N/A	N/A	N/A
Abutment-Active	9.8	0.70	37.9	0.90	77.2	0.85	N/A	N/A

(mm)								
Abutment-Tran (mm)	9.8	0.70	37.9	0.90	77.2	0.85	N/A	N/A

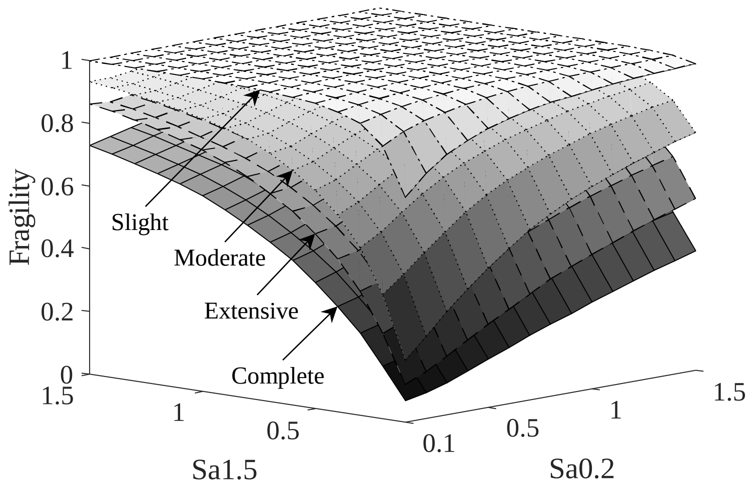

Figure 7. Fragility surfaces computed by SPCE and vine copula under PGA = 0.1g

Given the probabilistic distribution of the IM vector for different scenarios as computed in section 5.1 and vulnerability, the probabilistic loss ratio can be computed. Herein, the repair loss ratio for none, slight, moderate, extensive, and complete damage states are considered as 0, 0.03, 0.25, 0.75, and 1, respectively [37,57]. Statistical moments of loss ratio using a joint normal distribution (widely used in previous studies) and vine copula under different scenarios are listed in Table 5. For the investigated scenarios, the vine copula approach and joint normal distribution approach produce similar results in terms of the mean, standard deviation (STD), and kurtosis of the loss ratio, while the significant difference is observed in skewness. By using the joint normal distribution approach, the skewness is underestimated from 20% to 51% compared with the vine copula-based approach. This difference may be caused by the ignorance of nonlinear dependence characteristics in the joint normal distribution.

Table 5. Statistical moments of loss ratio using joint normal distribution and vine copula under different scenarios

Scenario	Method	Mean	STD	Skewness	Kurtosis	The relative difference of

						skewness (%)
M=7.8, R=5	Vine copula	0.655	0.190	-0.393	2.452	20
	Joint normality	0.668	0.185	-0.491	2.572	
M=7.8, R=10	Vine copula	0.544	0.201	-0.050	2.216	51
	Joint normality	0.556	0.200	-0.103	2.234	
M=7.8, R=15	Vine copula	0.465	0.200	0.190	2.272	42
	Joint normality	0.473	0.197	0.134	2.282	

Similarly, given the distribution of the IM vector for different return periods as presented in section 5.1, the probabilistic loss ratio can be computed. The density of loss ratios computed using vine copula subjected to different return periods is presented in Figure 8. With increasing return periods, the peaks of density shift from small loss ratios to large loss ratios. Statistical moments of loss ratio using joint normal distribution and vine copula subjected to return periods of 75, 120, 475, 975, and 2475 years are presented in Table 6. The relative difference of statistical moments of loss ratio by using joint normal distribution and vine copula is visualized in Figure 9 (a). A significant difference is observed for STD, skewness, and kurtosis values. As compared previously, the vine copula captures dependence better based on the criteria of AIC, BIC, and log-likelihood. By using the conventional joint normal distribution, biased high order moments could be obtained due to the ignorance of nonlinear dependence. The incorporation of uncertainty is necessary for the seismic performance assessment of structures [55,60]. Using only expected cost may not be appropriate when risk aversion is considered within the decision-making of structures [61]. The high order moments of performance indicator (e.g., variance, skewness, and kurtosis of loss) reflecting the information of probabilistic distribution are essential for risk-neutral decision-makers to incorporate different decision attitudes [62]. The decision-making of structures may not be optimal if only expected performance is considered, higher-order moments of performance should be incorporated to aid the rational decisions [63]. Thus, the structural assessment and decision-making may be misled by using the conventional joint normal distribution for dependence modeling.

The ratio of statistical moments of loss to the values associated with a return period of 75 years is presented in Figure 9 (b). An increasing trend with return periods is observed for most of the statistical moment values. Within the four statistical moments, mean values are associated with the most significant increasing trend. The density plots of loss ratios computed using joint normal distribution and vine copula subjected to return periods of 75 and 475 years are presented in Figure 10. The figures show that heavy and long tail behaviors are well captured by vine copula, while higher peaks of density are associated with the joint normal distribution.

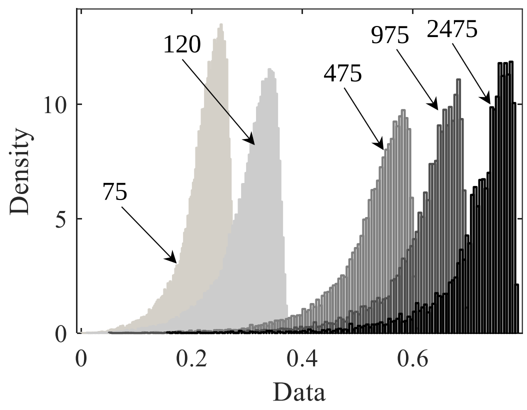

Figure 8. The density of loss ratios computed using vine copula subjected to return periods of 75, 120, 475, 975, and 2475 years seismic scenario

Table 6. Statistical moments of loss ratio using joint normal distribution and vine copula subjected to different return periods

Return period (years)	Dependence model	Mean	STD	Skewness	Kurtosis
75	Joint normality	0.2254	0.0311	-1.0966	4.2789
	Vine copula	0.2217	0.0398	-1.3236	5.0262
120	Joint normality	0.3043	0.0402	-1.1686	4.3502
	Vine copula	0.3030	0.0506	-1.4662	5.5370
475	Joint normality	0.5258	0.0565	-1.4637	5.2962
	Vine copula	0.5233	0.0698	-1.7884	7.2429

975	Joint normality	0.6212	0.0593	-1.6265	5.9130
	Vine copula	0.6202	0.0713	-2.0841	9.0426
2475	Joint normality	0.7243	0.0549	-1.8953	7.6103
	Vine copula	0.7178	0.0705	-2.4533	11.7571

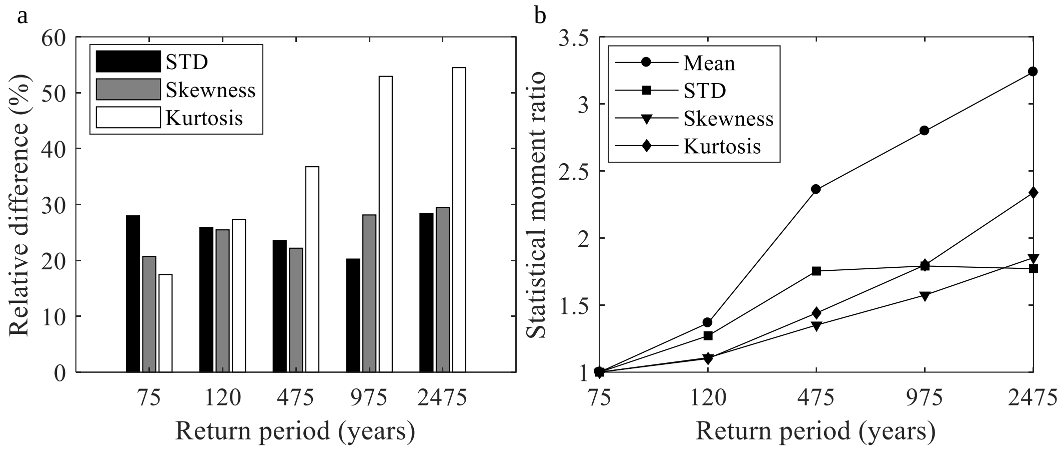

Figure 9. (a) The relative difference of statistical moments of loss ratio by using joint normal distribution and vine copula and (b) ratio of statistical moments of loss to the values associated with a return period of 75 years

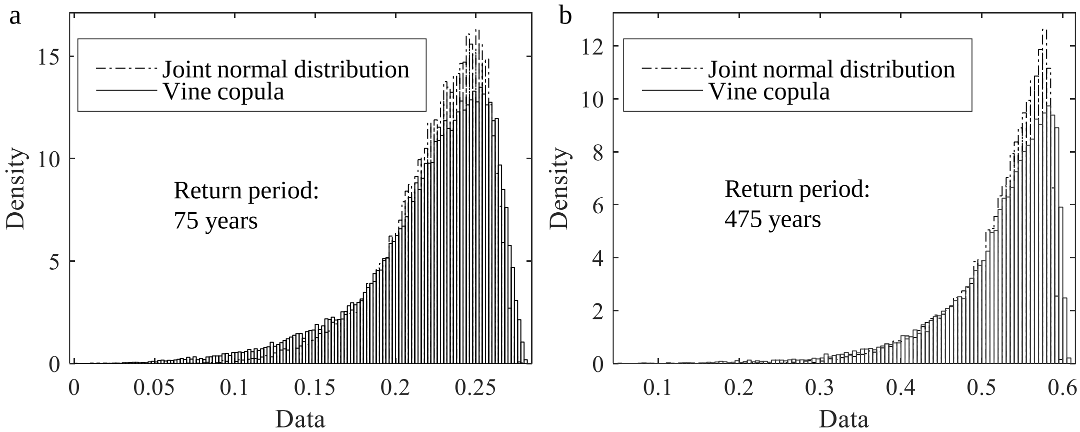

Figure 10. The density of loss ratios computed using the joint normal distribution and vine copula subjected to return periods of (a) 75 years and (b) 475 years

Overall, the proposed PBEE framework updates the conventional PBEE framework from several stages including hazard analysis, structural analysis, damage analysis, and performance analysis. In hazard analysis, the scalar IM used in most previous studies could only reflect limited information. The proposed approach adopts vector IM to describe the probabilistic hazards more comprehensively. The complex dependence among multiple IMs is captured by vine copula without using the assumption of joint normality of logarithmic values. In structural and damage analysis, linear regression is used in most of the previous studies to describe the relationship between scalar IM and demand. The dependence among multiple demands is not well considered. The proposed approach uses the surrogate model to describe this complex relationship, and vector IM reflecting more hazard information is incorporated in the surrogate model. Thus, the uncertainty of demand prediction can be reduced. The vine copula is used for the second time to model the complex dependence among multiple demands. The system vulnerability and performance can be computed more accurately and confidently.

6 Conclusions

This study proposes a hybrid framework for PBEE by interconnecting several advanced techniques. The SPCE is used as a surrogate model for seismic demand prediction. Vector IM, which contains more information on the hazard compared with scalar IM, is incorporated in the surrogate model to further improve the accuracy of prediction. The dependence from both the IM side and demand side is modelled using vine copula. The seismic performance can be computed using the proposed hybrid framework confidently. The framework is applied to an illustrative example. Several conclusions are drawn.

- Compared with the conventional method, SPCE and vector IM coupled approach could improve the accuracy of seismic demand prediction significantly. By using SPCE, a complex relationship of the input and demand can be captured, and multiple uncertain parameters can be incorporated in uncertainty propagation. The use of vector IM incorporates more hazard information in the analysis compared with scalar IM, thus, the uncertainty can be further reduced.
- The multivariate normality of logarithmic values is a widely used assumption for dependence modelling within PBEE. This study found that the multivariate normality of logarithmic values is not the optimal dependence structure for either seismic IMs or demands, the performance criteria show that vine copula performs better to capture complex dependence associated with both IMs and demands.

- The difference of the high order moments of loss derived from widely used multivariate normality assumption and the proposed vine copula-based approach is large. The structural assessment and decision-making may be misled by using the conventionally adopted assumption.
- The proposed approach updates the existing performance assessment framework from two aspects: improving accuracy and capturing a more realistic dependence structure. It could advance the rational assessment and decision-making of engineering systems under seismic hazards.

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