Zhang, P., & Dai, J.-G. (2024). New Mesoscale Phase Field Model for Analysis of FRP-to-Concrete Bonded Joints. Journal of Composites for Construction, 28(3), 04024007.

A new meso-scale phase field model for analysis of FRP-to-concrete bonded joints

Peng Zhang[a] and Jian-Guo Dai[b]

[a] *Post doctoral fellow, Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong, China*

[b] *Professor, Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong, China (Corresponding author, E-mail address:* cejgdai@polyu.edu.hk)

Abstract Externally bonded fiber-reinforced polymer (EB-FRP) laminates have become a popular technique for strengthening existing reinforced concrete (RC) structures. However, the high tensile strength of the FRP laminate is often not fully utilized due to premature debonding failure of the FRP-to-concrete interface, typically occurring in a thin layer beneath the bond interface. Numerical simulations have gained significant attention as a supplement to experimental tests, as they have the ability to provide valuable insights into the debonding process. However, most existing numerical models for EB-FRP joint debonding are unable to explicitly consider cracks within different concrete phases (i.e., mortar and interfacial transition (ITZ)), or precisely capture the corresponding failure mechanisms involving mortar cracking, ITZ debonding and kinking. This study proposes a novel meso-scale phase field model for

concrete, which is capable of accurately modeling complex failure behaviors, including mixed-mode fracture in both the mortar and ITZ, as well as friction on cracked surfaces. The ITZ is regularized using an auxiliary interface phase field and then the overall mixed-mode failure behaviors in both the mortar and ITZ are modeled using a unified damage phase field. To validate the proposed meso-scale model, three pull-off tests of FRP-to-concrete bonded joints, which were well reported in existing literature, are simulated. Moreover, the model is used to investigate the effects of adhesion and the FRP laminate on the debonding behavior of the FRP-to-concrete joints.

Keywords: FRP-to-concrete bonded joints; meso-scale modeling; phase field model; Pull-off test

Introduction

Externally bonded fiber-reinforced polymer (EB-FRP) laminates have become one of the most favored techniques for strengthening reinforced concrete (RC) structures. In comparison to traditional strengthening materials, such as steel plates, FRP laminates can offer some outstanding advantages such as high strength-to-weight ratio, superior corrosion resistance, and customizable material properties. However, numerous studies have shown that the high tensile strength of the FRP laminate is often not fully utilized due to premature debonding failure of the FRP-to-concrete interface (Chen and Teng, 2001, Teng et al., 2002, Teng et al., 2003, Dai et al., 2005, Lu et al., 2005a, Ali-Ahmad et al., 2006, De Lorenzis and Teng, 2007). This debonding failure is influenced by various factors, such as bond length, FRP to concrete width ratio, FRP axial stiffness, adhesive modulus and concrete strength (Chen and Teng, 2001, Yao et al., 2005, Dai et al., 2009, Wu and Jiang, 2013). Additionally, Pan and Leung (2007) found that the debonding process was also affected by concrete composition, as the interfacial friction resulting from aggregate interlocking within the fracture process zone could prevent damage evolution. However, it is very challenging to precisely evaluate the effect of aggregate distribution, shape and size on the bond capacity through experimental tests due to uncertainties that might be involved in preparation and test processes, such as surface treatment, adhesive thickness control, and load eccentricity. Therefore, advanced numerical simulation can serve as a supplement to experimental tests in investigating the underlying debonding mechanisms of FRP-to-concrete bonded joints (Yang et al., 2003, Lu et al., 2005b, Wu and Jiang, 2013, Tao and Chen, 2015, Kai et al., 2022). Despite significant efforts dedicated to this research area, the influence of adhesive properties, bending stiffness of the FRP laminate, and concrete composition on debonding mechanisms, particularly the complex mixed-mode fracture in the thin layer and the resultant interfacial fracture energy, which is defined as the area beneath the shear stress-slip relationship of the FRP-to-concrete interface assuming an overall Mode II loading condition, remains controversial (Dai et al., 2005, Lu et al., 2005a, Wu and Jiang, 2013).

Extensive research indicates that debonding of FRP laminates usually occurs within a thin layer in the concrete prism, approximately 1-5 mm away from the adhesive interface.(Lu et al., 2005b, Yao et al., 2005). Considering the commonly employed coarse aggregate size distribution in concrete, this debonding behavior clearly occurs within the mesoscopic length scale. Therefore, many mesoscale modellings of FRP-to-concrete debonding tests have been conducted over the last two decades. Those simulations can be broadly classified into two groups: homogeneous and heterogeneous meso-scale models. In homogeneous models, the concrete is assumed to be a uniform material with properties determined using homogenization techniques. Then to capture the meso-scale failure patterns beneath the adhesion layer, failure modellings are conducted using very small element sizes. This modeling strategy has been employed in several studies, including (Lu et al., 2005b, Lin and Wu, 2016, Wu and Jiang, 2013, Tao and Chen, 2015, Li and Guo, 2019), which have shown good agreement with experimental results. Although using elements that are one-order smaller than the thickness of the fracture layer seems to be a plausible approach for modeling debonding behaviors, there are still some concerns that need to be addressed. For instance, it remains unclear whether the homogenized material properties can accurately capture localized failures, such as small cracks that are parallel or inclined to the adhesion layer. Additionally, it is uncertain whether this modeling strategy can effectively describe the influence of aggregates near the bonded surface. Some of these factors were investigated by Coronado and Lopez (2010). They employed a crack band model to simulate the debonding process and assigned distinct properties to the crack band level and the concrete. An alternative modeling strategy is the heterogeneous model, which treats concrete as a multi-phase material comprising of mortar, aggregate and the interfacial transition zone (ITZ). By considering the debonding failure length scale, this strategy can provide a more precise and comprehensive understanding of the debonding process in FRP-to-concrete joints. However, there are currently only a limited number of numerical studies available in this field. Li et al. (2021) investigated the impact of coarse aggregate distribution on debonding behavior using a meso-scale model. However, they did not account for the ITZ, which significantly contributes to concrete damage. In fact, one major factor limiting the wide application of the heterogeneous model is the numerical difficulty. To be an effective heterogeneous model, it should possess the following characteristics based on the debonding behavior of the FRP-to-concrete joint: (1) the ability to handle complex failure patterns, including branching and connecting; (2) the ability to account for mixed-mode failure and subsequent friction; and (3) the ability to consider the interaction of failure within different phases, such as interfacial cracking and kinking into the mortar. One of the most promising methods that can fulfill these requirements is the phase field model of fracture.

The phase field model of fracture is a non-local smeared crack model based on the Francfort-Marigo variational principle (Francfort and Marigo, 1998). This model utilizes a continuous scalar damage phase field to regularize cracks and treats damage evolution as a competition between deformation and fracture energies. As a result, it can handle complex failure patterns without requiring additional failure criteria or crack tracking strategies. This makes it one of the most promising methods in computational fracture mechanics. Furthermore, one prominent advantage of the phase field model of fracture, compared to traditional smeared crack models, is its ability to theoretically reproduce the discrete crack surface area by integrating the crack surface density function, which enables the model to quantitatively investigate the failure process of materials. Phase field models of fracture have been successfully applied to various fracture problems, including brittle fracture (Bourdin et al., 2000, Miehe et al., 2010b, Ambati et al., 2015b), ductile fracture (Borden, 2012, Miehe et al., 2015, Ambati et al., 2015a) and composites fracture (Zhang et al., 2019a, Quintanas-Corominas et al., 2019, Bui and Hu, 2021) problems. The author recently proposed a meso-scale phase field model for multi-phase materials (Zhang et al., 2019b, Zhang et al., 2020, Zhang et al., 2023), which has been demonstrated to accurately hand complex failure patterns and the interactions of failure between different phases. However, this model is primarily designed for analyzing tensile-dominated failure.

This paper aims to propose a new meso-scale phase field model capable of considering mixed-model failure and the resulting friction between crack surfaces. The next section presents the proposed phase field model, which incorporates the effect of crack angle into the constitutive law and the damage phase field driving force, which distinguishes it from traditional phase field models of fracture. The proposed model constructs energy densities that contribute to the evolution of both tensile and shear failure by using a local crack coordinate. These densities are integrated into the variational principle (Francfort and Marigo, 1998) to derive a non-local damage phase field evolution equation. It is important to note that, to improve computational efficiency and mitigate convergence issues, the crack angle is only calculated at the onset of damage, such calculation is only valid under monotonic loads. Then, the construction of a phase field model-based meso-scale modeling framework is presented. A total of 17 pull-off tests from existing literature are simulated to validate the proposed model. The influences of adhesion and FRP properties on FRP-to-concrete joints are then investigated. Finally, the conclusions of the present study are provided.

Proposed phase field model for mixed mode failure

Constitutive law

Consider a continuum solid material occupying a domain $\Omega \in R^{\mathrm{dim}}$ as shown in Fig. 1(a), where *dim* is the dimension. Ω is subjected to a prescribed traction $\bar{\boldsymbol{t}}\left(\boldsymbol{x},t\right)$ and a prescribed displacement $\bar{\boldsymbol{u}}\left(\boldsymbol{x},t\right)$. The domain may be subjected to a body force per volume $\bar{\boldsymbol{b}}\left(\boldsymbol{x},t\right)$. Assuming that the material is isotropic and linear elastic, the constitutive relationship is

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor. $\boldsymbol{\varepsilon}$ is the strain tensor, and $\mathbf{C}$ is the fourth-order elasticity tensor.

Then, consider a cohesive crack $\Gamma_c \in R^{\mathrm{dim}}$ as shown in Fig. 1(a). For a point $\boldsymbol{x}$ at the crack surface, Fig. 1(b) gives the corresponding 2D stress state. Within the phase field model framework, the influence of cracks on the constitutive law can be considered using the damage phase field d, which characterizes the material's damage state, as

$$\boldsymbol{\sigma} = \mathbf{C}_d\left(d,\theta\right):\boldsymbol{\varepsilon} \tag{2}$$

where $\mathbf{C}_d$ is termed as the damaged elasticity tensor.

To simplify the derivation of the damaged elasticity tensor $\mathbf{C}_d$, the principal stresses σ_1 and σ_2 are considered with $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ the corresponding principal stress axes and assume that the crack angle is α with respect to $\boldsymbol{n}_2$. As shown in Fig. 2(b), a local coordinate system $x'Oy'$ can be built, through which the vectors that are along and normal to the crack can be specified by

$$\boldsymbol{m} = \boldsymbol{R}\cdot\boldsymbol{n}_1,\ \ \boldsymbol{n} = \boldsymbol{R}\cdot\boldsymbol{n}_2 \tag{3}$$

where $\boldsymbol{R}$ is the rotation matrix. Accordingly, the stress and strain vectors at the local coordinate can be specified as

$$\begin{Bmatrix}\sigma_{x'}\\ \sigma_{y'}\\ \tau_{x'y'}\end{Bmatrix} = \boldsymbol{T}_\sigma\cdot\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix},\ \begin{Bmatrix}\varepsilon_{x'}\\ \varepsilon_{y'}\\ \gamma_{x'y'}\end{Bmatrix} = \boldsymbol{T}_\varepsilon\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \tag{4}$$

where the stress and strain rotation matrices are

$$\boldsymbol{T}_\sigma = \begin{bmatrix} l^2 & k^2 & 2kl\\ k^2 & l^2 & -2kl\\ -kl & kl & l^2-k^2\end{bmatrix},\ \boldsymbol{T}_\varepsilon = \begin{bmatrix} l^2 & k^2 & kl\\ k^2 & l^2 & -kl\\ -2kl & 2kl & l^2-k^2\end{bmatrix} \tag{5}$$

in which $l = \boldsymbol{n}_1(1)$ and $k = \boldsymbol{n}_1(2)$. At the local coordinate, the softening of the material stiffness resulting from tensile and shear damage can be characterized by the damage phase field d using two degradation functions: $g_I(d)$ for tensile failure and $g_{II}(d)$ for shear failure and friction. Explicit forms of the two degradation functions will be discussed in the following sections. Accordingly, the damaged elasticity matrix at the local coordinate can be specified by

$$\boldsymbol{C}_d^{loc}(d,\alpha) = \begin{bmatrix} C_{11} & g_I g_{II} C_{12} & 0 \\ g_I g_{II} C_{21} & g_I C_{22} & 0 \\ 0 & 0 & g_{II} C_{33} \end{bmatrix} \tag{6}$$

where C_{ij} (i,j=1,2,3) is the (i,j)th value of the intact elasticity matrix. Therefore, the damaged elasticity matrix at the global coordinate can be obtained as

$$\boldsymbol{C}_d(d,\alpha) = \boldsymbol{T}_\sigma^{-1} \cdot \boldsymbol{C}_d^{loc} \cdot \boldsymbol{T}_\varepsilon \tag{7}$$

Phase field model for mixed-mode failure

This section proposes a phase field model for mixed-mode failure. The main feature that distinguishes the present model from tradition phase field models of fracture is the incorporating of the mixed-model failure constitutive law constructed in the previous section. The determination of the crack angle and energy densities driving different failure patterns will be given in the following derivations.

Consider a system as shown in Fig. 3(a) and the corresponding smeared crack case characterized by the damage phase field d as shown in Fig. 3(b), in which d=0 and d=1 represent intact and totally damaged states, respectively. The elastic energy W_e and the external energy W_t can be specified as

$$W_e = \int_\Omega \psi_e(\boldsymbol{\varepsilon}, d, \alpha)\mathrm{d}V,\ \ W_t = \int_{\Gamma_t} \bar{\boldsymbol{t}} \cdot \boldsymbol{u}\mathrm{d}S + \int_\Omega \bar{\boldsymbol{b}} \cdot \boldsymbol{u}\mathrm{d}V \tag{8}$$

where ψ_e is the strain energy density. For a time-depended system, the kinetic energy W_k can be specified as

$$W_k = \frac{1}{2}\int_{\Omega} \rho \dot{\mathbf{u}} \cdot \dot{\mathbf{u}} \,\mathrm{d}V \tag{9}$$

where ρ is the mass density.

According to (Bourdin et al., 2000) the fracture energy with respect to the smeared crack description can be given by

$$W_f = \int_{\Omega} G_c \gamma_d(d, \nabla d) \mathrm{d}V \tag{10}$$

where $\gamma_d(d, \nabla d)$ is termed as the crack surface density function, whose integration over the entire domain gives the real crack surface area. G_c is the critical energy release rate. It should be noticed that Eq. (10) is the fracture energy for the traditional phase field model, which can only consider the tensile failure. In the following, a modified form will be proposed to consider the mixed-mode failure. Following (Bourdin et al., 2000), the general form of the crack surface density function can be given by

$$\gamma_d(d, \nabla d) = \frac{1}{c_0}\left[\frac{1}{l_0}\omega(d) + l_0 \nabla d \cdot \nabla d\right] \tag{11}$$

where l_0 is the damage phase field internal length scale characterizing the width of the smeared crack. $\omega(d)$ and c_0 are the crack geometry function and model parameter, respectively (Wu, 2017).

An viscosity part related to the rate of the damage phase field can be given by (Zhang et al., 2021)

$$W_v^d = \int_t \int_{\Omega} \frac{1}{2}\kappa \cdot \left\langle \dot{d} \right\rangle_+^2 \mathrm{d}V \mathrm{d}t \tag{12}$$

where the bracket operator is defined as $\left\langle x \right\rangle_{\pm} = \left(x \pm |x|\right)/2$. κ is the artificial phase field viscosity (Miehe et al., 2010b).

The Lagrange functional of the considered system can be specified as

$$L(\boldsymbol{q},\dot{\boldsymbol{q}})=W_k+W_v^d+W_t-W_e-W_f \tag{13}$$

where $\boldsymbol{q}=[\boldsymbol{u},d]$. Then according to the Lagrange dynamical equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right)-\frac{\partial L}{\partial \boldsymbol{q}}=\boldsymbol{0} \tag{14}$$

the governing equations and the boundary conditions can be obtained as

$$\nabla\cdot\frac{\partial \psi_e(\boldsymbol{\varepsilon},d,\alpha)}{\partial \boldsymbol{\varepsilon}}+\bar{\boldsymbol{b}}=\rho\ddot{\boldsymbol{u}} \quad \text{in }\Omega \tag{15}$$

$$-\frac{\partial \psi_e(\boldsymbol{\varepsilon},d,\alpha)}{\partial d\cdot G_c}-\frac{\partial \gamma_d(d,\nabla d)}{\partial d}=\kappa\left\langle \dot{d}\right\rangle_+, \quad \text{in }\Omega \tag{16}$$

$$\frac{\partial \psi_e(\boldsymbol{\varepsilon},d,\alpha)}{\partial \boldsymbol{\varepsilon}}\cdot\boldsymbol{p}=\bar{\boldsymbol{t}}, \quad \text{at }\Gamma_t \tag{17}$$

$$\nabla d\cdot\boldsymbol{p}=0, \quad \text{at }\Gamma \tag{18}$$

where $\boldsymbol{p}$ denotes the outward unit normal vector of the boundary Γ It is noteworthy that the crack angle α in Eqs. (15)-(17) is an internal variable that depends on the stress state. The expression for calculating this angle will be given in the following sections. Therefore, it is not considered in the variational operation.

As discussed previously, the fracture energy W_t in Eq. (10) is defined for the traditional phase field model (Francfort and Marigo, 1998, Bourdin et al., 2000, Miehe et al., 2010a). In order to extend the traditional phase field model to account for mixed-mode failure, a modified phase field evolution equation is introduced to replace Eq. (16).

$$F-\frac{\partial \gamma_d(d,\nabla d)}{\partial d}=\kappa\left\langle \dot{d}\right\rangle_+ \tag{19}$$

where F is the driving force for damage phase field evolution. To account for different failure modes, a mixed driving force form is adopted as

$$F = -\frac{\partial g_I(d)}{\partial d} \cdot \frac{\psi_I^n(\boldsymbol{\varepsilon})}{G_I} - \frac{\partial g_{II}(d)}{\partial d} \cdot \frac{\psi_{II}^n(\boldsymbol{\varepsilon})}{G_{II}} \tag{20}$$

where G_I and G_{II} are the critical energy release rates for tensile and shear damage, respectively. $g_{II}(d)$ is the degradation function used to characterize the softening of the material stiffness due to shear failure. Therefore, $g_{II}(d)$ is a component of the shear-friction degradation function $\tilde{g}_{II}(d)$ introduced in Eq. (6), and the explicit expression will be given in the following sections. The elastic energy densities corresponding to tensile and shear failure are denoted as ψ_I^n and ψ_{II}^n, respectively. These energy densities can be determined using the constitutive law. Referring to the local coordinate illustrated in Fig. 2(b), the elastic stress components parallel and perpendicular to the crack surface are denoted as $\tau_m^n = \tau_{x'y'}$ and $\sigma_n^n = \sigma_{y'}$, respectively. Accordingly, the tensile energy density can be specified as

$$\psi_I^n = \frac{\left\langle \sigma_n^n \right\rangle_+^2}{2E} \tag{21}$$

The shear strength of materials can be characterized by Mohr-Coulomb criterion based on the relationship between shear stress and normal stress on a plane of failure as

$$\overline{Y}_{sh} = g_I(d)\left\langle \sigma_n^n \right\rangle_- \cdot \tan\phi + Y_o \tag{22}$$

where ϕ is the friction angle and Y_o is the cohesion strength. The first term of the right-hand side of the equation represents the contribution due to the normal compressive stress at the plane of failure, which is referred to as the reduced friction stress $\tau_f = g_I(d)\left\langle \sigma_n^n \right\rangle_- \cdot \tan\phi$ in this study. Accordingly, the shear energy density can be defined by

$$\psi_{II}^{n} = \frac{\left(\tau_{II}^{n}\right)^{2}}{2G} \tag{23}$$

where

$$\tau_{II}^{n} = \left\langle \left|\tau_{m}^{n}\right| - \left|\tau_{f}\right| \right\rangle_{+} \tag{24}$$

is the component of shear stress that excludes the contribution from friction stress. As the evolution of damage in a material is closely related to the contact condition between crack surfaces, the shear energy density should be calculated incrementally during this process, which can be given by

$$\left(\psi_{II}^{n}\right)^{k} = \left(\psi_{II}^{n}\right)^{k-1} + \int_{\gamma_{x'y'}^{k-1}}^{\gamma_{x'y'}^{k}} \tau_{II}^{n} \mathrm{d}\gamma \tag{25}$$

where the superscripts k and k-1 denote the current and previous loading increments, respectively.

To prevent physically unrealistic self-healing resulting from local unloading, two history-dependent variables are introduced as

$$H_{I} = \max_{j\in[0,t]}\left\{\psi_{I}^{n}(\boldsymbol{\varepsilon}, j)\right\},\quad H_{II} = \max_{j\in[0,t]}\left\{\psi_{II}^{n}(\boldsymbol{\varepsilon}, j)\right\} \tag{26}$$

Accordingly, the new phase field driving force in Eq. (20) can be rewritten as

$$F = -\frac{\partial g_{I}(d)}{\partial d}\cdot\frac{H_{I}}{G_{I}} - \frac{\partial g_{II}(d)}{\partial d}\cdot\frac{H_{II}}{G_{II}} \tag{27}$$

Crack angle, damage initiation and evolution

The unified phase field model of fracture proposed by Wu (2017) suggests that the influence of tensile-dominated cohesive damage on the softening of the material stiffness can be characterized by a parametric degradation function, which is defined as

$$g_I(d)=\frac{(1-d)^2}{(1-d)^2+a_1 d-0.5a_1 d^2} \tag{28}$$

where a_I is related to material properties. For tensile failure, it can be defined by

$$a_1=\frac{4EG_I}{c_0 l_0 Y_I^2} \tag{29}$$

where G_I and Y_I are the tensile critical energy release rate and strength, respectively.

According to the constitutive law in Eq. (2) and shear energy density in Eqs. (22) -(25), an interpolative degradation for shear-friction failure is defined by

$$\tilde{g}_{II}(d)=g_{II}(d)+\left[1-g_{II}(d)\right]\cdot\left|\frac{\tau_f}{\tau_{\max}}\right| \tag{30}$$

where $\tau_{\max}=\max\limits_{j\in[0,t]}\left\{\tau_m^n(\boldsymbol{\varepsilon},j)\right\}$ is the historical maximum shear stress and a_I can be defined as

$$a_1=\frac{4GG_{II}}{c_0 l_0 Y_o^2} \tag{31}$$

Eq. (30) shows that when the friction stress vanishes (i.e., $\tau_f=0$), $\tilde{g}_{II}=g_{II}$, indicating that $\tilde{g}_{II}$ is related only to shear failure. On the other hand, when $\tau_f\neq 0$ and $g_{II}=0$ (indicating total damage), $\tilde{g}_{II}$ is only dependent on frictional sliding. These relationships highlight the dependence of $\tilde{g}_{II}$ on both shear failure and frictional sliding and demonstrate how it varies under different conditions.

Eq. (27) shows that the damage driving force F is closely related to the crack angle α. To determine α, two assumptions are made in this paper. First, it is assumed that the crack will always evolve in the direction of the maximum damage driving force. Second, to improve computational efficiency and mitigate convergence issues, it is assumed that the crack angle is only calculated at the onset of damage. This means that the proposed model considers monotonic loads only. Accordingly, the crack angle can be determined by

$$\alpha = \arg\max_{\alpha}\{F\} = \arg\max_{\alpha}\left\{-\frac{\partial g_I(d)}{\partial d}\cdot\frac{H_I}{G_I} - \frac{\partial g_{II}(d)}{\partial d}\cdot\frac{H_{II}}{G_{II}}\right\}\Bigg|_{d=0} \tag{32}$$

Substituting the degradation functions and energy densities into Eq. (32), the above equation can be rewritten as

$$\alpha = \arg\max_{\alpha}\left\{\frac{2}{c_0 l_0}\cdot\left[\frac{\langle\sigma_n^n\rangle_+^2}{Y_I^2} + \frac{\langle|\tau_m^n| - |\tau_f|\rangle_+^2}{Y_0^2}\right]\right\} \tag{33}$$

It is noteworthy that for traditional phase field models of fracture, the damage initiation criterion is implicitly embedded in the damage evolution equation (Bourdin et al., 2000). However, we have proven that by properly defining the energy densities and the degradation functions, some widely adopted initiation criteria (such as Hashin damage criterion) can be implicitly embedded into the phase field model (Zhang et al., 2021). For the present model, the corresponding damage initiation criterion can be obtained through

$$F\big|_{d=0} - \frac{\partial\gamma_d(d,\nabla d)}{\partial d}\bigg|_{d=0} = 0 \tag{34}$$

Substituting the driving force (Eq. (27)) into Eq. (34), a damage initiation criterion that is implicitly embedded in the proposed phase field model can be obtained as

$$\frac{\langle\sigma_n^n\rangle_+^2}{Y_I^2} + \frac{\langle|\tau_m^n| - |\tau_f|\rangle_+^2}{Y_0^2} = 1 \tag{35}$$

The above equation indicates that a quadratic stress-based damage initiation criterion is automatically embedded in the model. If friction stress is not considered, Eq. (35) will degenerate into the widely used stress-based initiation criterion for cohesive zone models (Turon et al., 2006). Moreover, by comparing Eq. (35) and the equation used to determine the crack angle (i.e., Eq. (33)), it can be seen that the embedded damage initiation criterion is consistent with the crack angle determination procedure. This consistency indicates that the potential crack direction is also where the damage tends to initiate, which is physically reasonable.

Meso-scale phase field model

Meso-scale concrete model and ITZ regularization

Concrete is a heterogeneous material composed of different components: mortar, aggregates and the ITZ. These components have different properties and can be modeled as isotropic materials at the meso-scale (Ren et al., 2015, Kai et al., 2023b, Kai et al., 2023a). However, the material heterogeneity can significantly affect the internal failure pattern of concrete. To addressed this, we propose a meso-scale phase field model that can capture the failure of both the mortar and the ITZ within a unified phase field framework. For simplicity, only coarse aggregates larger than 2.4 mm are considered, while other fine aggregates, along with cement are regarded as the mortar phase (Xi et al., 2018, Hu et al., 2022). A typical three-segment gradation size distribution is employed, as described by (Neville, 1995, Xi et al., 2018): 2.40-4.76mm (8.08%), 4.76-9.52mm (15.96%), and 9.52-19.05mm (15.96%). In the simulations, aggregates are represented using polygons with 5 to 8 sides. And the corresponding random aggregate distribution is generated by a put and take algorithm, with further details available in (Xi et al., 2018).

Consider a domain Ω containing an aggregate as shown in Fig. 4. In typical normal strength concrete, the aggregates are generally stronger than the mortar and the ITZ. As a result, this paper only considers mortar cracking and ITZ debonding (Fig. 4(a)). The mixed mode phase field model, which is proposed in the previous section, can be directly used to simulate mortar cracking. However, accurately modeling ITZ debonding remains challenging due to the ITZ's thickness, which is normally tens of microns (Barnes et al., 1979). Such a thin ITZ thickness necessitates the use of a very dense element mesh, which may distort elements in adjacent areas. Currently, there are two modeling categories for ITZ debonding: (a) using zero-thickness cohesive elements (Ren et al., 2015, Yılmaz and Molinari, 2017, Xi et al., 2018), and (b) using an approximate ITZ thickness of about 0.1 mm – 1 mm (Šavija et al., 2013, Du et al., 2014, Huang et al., 2015, Zhou et al., 2017). In this paper, we adopt a combined ITZ modeling strategy. During the elastic stage, ITZ is regarded to have zero thickness, which implies that the mortar and the aggregate are perfectly bonded. During the debonding evolution stage, the ITZ's fracture properties, such as its strengths, critical energy release rates, and friction angle, are regularized in the adjacent mortar phase. This interface regularization method is inspired by the traditional phase field model of fracture and can be naturally integrated into the current modeling framework. Previous numerical results have demonstrated that this strategy is capable accurately capturing ITZ debonding (Nguyen et al., 2016, Zhang et al., 2019b, Zhang et al., 2020, Li et al., 2020, Hu et al., 2022).

Fig. 4(b) shows that an interface phase field, denoted as η, is introduced to regularize the ITZ. The corresponding governing equation and boundary conditions for η are (Zhang et al., 2019b)

$$\eta - l_i^2 \Delta \eta = 0, \quad \text{in } \Omega \tag{36}$$

$$\eta(\boldsymbol{x}) = 1, \quad \text{at } \Gamma_i \tag{37}$$

$$\nabla \eta \cdot \boldsymbol{p} = 0, \quad \text{at } \Gamma \tag{38}$$

where l_i is internal length scale of the the interface phase field, which generally can be set to be the same as that of the damage phase field, i.e., l_i=l_0 (Zhang et al., 2019b, Hu et al., 2022). Γ_i is the aggregate boundary. It is important to note that the interface phase field can be solved using either an implicit or an explicit solution scheme. However, once η is determined, no further updates are necessary, regardless of the solution scheme used. As a result, this additional degree of freedom (DOF) will have minimal impact on the overall computational efficiency. For more details of the interface phase field, refer to (Zhang et al., 2019b, Zhang et al., 2020).

Following the work conducted by Hu et al. (2022) a sharp transition between different phases is employed as

$$\begin{cases} \eta(x) \geq \overline{\eta} & \text{ITZ} \\ \text{Otherwise} & \text{Mortar} \end{cases} \tag{39}$$

We would like to reiterate that in Eq. (39), the ITZ and the mortar only represent the corresponding fracture properties, as the ITZ is not considered during the elastic stage. $\bar{\eta}$ is a specified value that controls the boundary between different phases. In this paper, we set the thickness of the ITZ to be the same as the smeared crack width. It is well known that in phase field models, the sharp crack surface area can be reproduced by integrating the crack density function γ_d over the smeared crack region. Therefore, the current approach to treating ITZ thickness ensures that the ITZ's fracture energy can be accurately captured during the modeling. According to (Wu, 2017), the width of the fully developed smeared crack can be given by

$$D = c_0 \cdot l_0 \tag{40}$$

Using the one-dimensional analytical solution of the interface phase field,

$$\eta = \exp\left(-\frac{|x|}{l_i} \right) \tag{41}$$

the specified value $\bar{\eta}$ can be obtained as $\bar{\eta} = \exp(-c_0)$.

Meso-scale modeling

Material properties determination and verification

According to Euro Code (2005), the Young's modulus of the aggregate can be determined by

$$E_a = 22\left(\frac{f_a}{10} \right)^{0.3} \tag{42}$$

where f_a is the cubic compressive strength of the aggregate.

The Young's modulus of the homogenized concrete can determined using the

Chinese code for design of concrete structures as (Lu et al., 2005a)

$$E_c = \frac{100}{2.2 + 34.74 / f_c} \tag{43}$$

where f_c is the cubic compressive strength of the concrete.

According to the Mori-Tanaka homogenization theory (Mori and Tanaka, 1973, Li et al., 2021), the Young's modulus of the mortar E_m can determined using the following relationship

$$E_c = E_m + \frac{V_a(E_a - E_m)}{1 + (1 - V_a) f_m} \tag{44}$$

where V_a is the aggregate volume fraction and the parameter f_m is

$$f_m = \frac{E_a - E_m}{E_m + 4\mu_m / 3}, \quad \mu_m = \frac{E_m}{2(1 + \upsilon_m)} \tag{45}$$

in which υ_m is the Poisson's ratio of the mortar.

As shown in earlier sections, the meso-scale modeling requires knowledge of the fracture properties of the mortar and the ITZ. According to (Sideris et al., 2004), the cylinder compressive strength of the mortar can be specified by

$$f_m^{'} = \frac{E_m - 12.4147}{0.2964} \tag{46}$$

Then according to the work conducted by Nagai et al. (Nagai et al., 2005), the tensile strength and the critical energy release rates for tensile failure of the mortar can be specified by

$$Y_I^m = 1.4 \cdot \ln\left(f_m^{'}\right) - 1.5 \tag{47}$$

$$G_I^m = \left(0.0469 d_a^2 - 0.5 d_a + 26\right) \cdot \left(\frac{f_m^{'}}{10}\right)^{0.7} \tag{48}$$

where d_a=2.36 mm is adopted (Li et al., 2021). According to (Pina‑Henriques and Lourenço, 2006, Prakash et al., 2020), the critical energy release rate for shear failure of the mortar can be taken as four times that of the tensile failure, i.e., $G_{II}^m = 4G_I^m$. According to (Nagai et al., 2005), the tensile strength Y_I^i and the cohesion strength Y_o^i of the ITZ can be obtained by

$$Y_I^i = -1.44C_w + 2.3 \tag{49}$$

$$Y_o^i = -2.6C_w + 3.9 \tag{50}$$

where

$$C_w = \frac{1}{0.047 f_m^{'} + 0.5} \tag{51}$$

It is important to note that, to the best of the authors' knowledge, there is currently no available method to directly evaluate the cohesion strength Y_o^m of the mortar. Therefore, in this study, this parameter is determined by $Y_o^m = Y_I^m / Y_I^i \cdot Y_o^i$. Furthermore, following the suggestions in (López et al., 2008, Huang et al., 2016), the critical energy release rates of the ITZ are set to be half that of the mortar, i.e., $G_I^i = 0.5G_I^m$ and $G_{II}^i = 0.5G_{II}^m$. In this study, the aggregate cubic compressive strength f_a=122.63 MPa used in (Contrafatto et al., 2016, Li et al., 2021) is adopted. A typical friction angle ϕ=35° is adopted for both the mortar and the ITZ (Nagai et al., 2005).

This section outlines the relationships among different material properties of concrete as adopted in existing literature. To further validate these relationships, three-point bending tests conducted by Hoover et al. (2013) are simulated here by the proposed meso-scale phase field model. According to (Hoover et al., 2013), the concrete used in these tests had a cylinder compressive strength of 55.6 MPa, a Young's modulus of 41.24 GPa and a Poisson's ratio of 0.172. By utilizing these known properties, the other material properties of concrete required for the simulations can be obtained based on the above-mentioned relationships.

The geometry and boundary conditions for the tests are depicted in Fig. 5. Four different cases are considered for the verification, including two specimen sizes: D=40 mm and D=93 mm, and two crack lengths: λ=0.15 and λ=0.3 for each specimen size. The predicted force versus crack mouth opening displacement (CMOD) curves, along with the corresponding crack patterns, are illustrated in Fig. 6. Experimental results reported by Hoover et al. (2013) are also included as grey regions for comparison. It can be seen that the predicted force-CMOD curves exhibit an initial elastic stage, followed by a softening stage after reaching the maximum value. These stages align well with the experimental data, indicating not only the validation of the proposed meso-scale phase field model but also the effectiveness of the material properties obtained through the relationships outlined in this section.

Pull-off test FE model

In the references (Lu et al., 2005b, Tao and Chen, 2015, Lin and Wu, 2016, Li and Guo, 2019), numerical simulations were conducted for the pull-off test, in which an FRP laminate is bonded to a concrete prism and subjected to tension. It should be noted that, for the sake of simplicity, most of the simulations were conducted using two-dimensional models with the plane stress hypothesis. However, it is important to acknowledge that the debonding of the FRP-to-concrete joint is not a two-dimensional case in theory, as the bond width of the FRP laminate is typically smaller than the width of the bonded surface of the concrete prism. Experimental results have demonstrated that the ratio between the widths of the FRP laminate and the prism concrete significantly influences the bond strength. To account for this three-dimensional width effect, it is common practice to incorporate a width factor β_w into a two-dimensional model. According to Lin et al. (2017) this width factor can be given by

$$\beta_w = 1 + f_{co}^{0.385}\left[8(E_f t_f)^{-0.438} + 0.001\right]\left(1 - b_f / b_c\right)^{0.5} / \left(1 + 0.01 b_f^{1.7}\right) \quad (52)$$

where f_{co} is the compressive strength of concrete; E_f and t_f are the Young's modulus and thickness of FRP laminate, respectively; b_f and b_c are the widths of the FRP laminate and concrete prism, respectively.

The FE model and boundary conditions are depicted in Fig. 7. In this study, a displacement-controlled loading mode is employed. The proposed meso-scale phase field model is implemented in the commercial software ABAQUS (Version, 2011) through the users' subroutine VUEL. For more details regarding the implementation and source codes, please refer to (Hu et al., 2023). As depicted in the figure, the concrete prism has a thickness of 45 mm. Along the thickness direction, it is divided into two distinct parts: the meso-scale part, which comprises aggregates, mortar and ITZ, and the homogenized concrete part. Extensive research indicates that debonding usually occurs within a thin layer in the concrete prism, approximately 1 - 5 mm away from the adhesive interface. Hence, to ensure the computational efficiency while allowing sufficient depth for debonding evolution, the thickness of the meso-scale part is set to be t_m=15 mm. Another critical consideration in meso-scale modeling is the three-dimensional distribution of aggregates within the concrete prism. This heterogeneity cannot be adequately represented by a two-dimensional meso-scale model. Consequently, in this section, four different aggregate distributions are employed. These distributions can be viewed as two-dimensional slices extracted from the three-dimensional pull-off test, specifically along the width direction. Each slice case uses the plane stress hypothesis and the width factor given in Eq. (52). As a result, the load-slip curve of the pull-off test can be obtained by averaging the curves from the four aggregate distributions. The lengths of the FRP laminate and the concrete prism are L_f=150 mm and L_c=190 mm, respectively. There is an unbonded zone between the the loaded end and the right edge of the concrete prism, which has a length of L_r=25 mm. The restrained height, as shown in Fig. 7, is t_u=15 mm. When bonding FRP to concrete, there are two common methods: using a prefabricated laminate bonded with adhesive or utilizing dry fiber sheets using a wet lay-up process (Teng et al., 2002, Lu et al., 2005b). In the former method, the FRP laminate and the adhesive can be clearly distinguished. However, in the more widely adopted wet lay-up method, the boundary between the FRP laminate and the adhesive cannot be clearly distinguished. Hence, following the approach in (Lu et al., 2005b, Tao and Chen, 2015, Lin and Wu, 2016), we assume that the FRP laminate is perfectly bonded to the concrete prism. Of course, explicitly considering the interface effect would provide a more precise understanding of the related failure mechanisms. For instance, Jawdhari and colleagues (Jawdhari et al., 2018, Jawdhari et al., 2019, Kadhim et al., 2021, Kadhim et al., 2022) conducted both experimental and numerical investigations on the bond characteristics of carbon FRP rod panels adhered to concrete. They adopted interfacial elements to simulate the interfacial behavior to reproduce test results. In this study, cohesive elements (CEs) are also used to investigate the effect of adhesive on the bond behavior as shown in the parametric study section. The FRP laminate is treated as an isotropic elastic material, with a Poisson's ratio $\upsilon_f = 0.3$, as suggested by (Li et al., 2021). The element size in the meso-scale part is set to h_e=0.125 mm, hence the corresponding internal length scales are set to be 0.25 mm.

Verification and discussion

To validate the efficacy of the proposed meso-scale phase field model in predicting the ultimate loads of FRP-to-concrete joints, a total of 17 tests (Takeo et al., 1997, Ueda et al., 1999, Tan, 2002, Yao et al., 2005, Ali-Ahmad et al., 2006, Wu and Jiang, 2013) are simulated. Fig. 8 presents a comparison between the predicted results and experimental data, showing a good agreement between the numerical predictions and the test results. Furthermore, in order to showcase the capacity of the model in capturing more detailed characteristics of FRP-to-concrete joints, i.e., the load-slip curve, load FRP axial strain, and crack pattern within the concrete prism, four tests (Ali-Ahmad et al., 2006, Yao et al., 2005, Wu and Jiang, 2013) are chosen from the aforementioned database as illustrative examples. The material properties and geometric information for these tests are given in Table 1.

Fig. 9 showcases the predicted load-slip curves for the considered tests. As discussion in the previous section, four different aggregate distributions are employed to approach the mechanical behavior of the three-dimensionally distributed aggregates. It can be observed that the mean curves (represented by solid black lines) obtained from various distributions exhibit similar characteristics as slip increases: an initial linear stage followed by nonlinear growth with a gradually reduced rate, indicating the initiation of damage at this stage. Subsequently, the external load reaches a plateau, indicating a stable debonding propagation behavior. Finally, a sharp decrease in the loading capacity occurs, indicating complete debonding of the FRP laminate from the concrete prism. These stages are consistent with the observations from pull-off tests on FRP-to-concrete joints (Lu et al., 2005a, Yao et al., 2005). Moreover, the mean curves show a quantitative agreement with experimental results (referred to red dotted lines and dots).

Fig. 10 illustrates the predicted distributions of axial strains in FRP laminates for different tests. For the sake of clarity, only the mean strain obtained by averaging the strains from the four different distributions is provided, similar to Fig. 9. At low external load/slip, the strain distribution gradually decreases as the location moves away from the loaded end, indicating a linear or initiation of debonding stage. As the external load/slip increases, the strain distribution approaches a plateau near the loaded end, followed by a decrease as the location moves away, indicating a debonding propagation process. Moreover, Fig. 10 includes comparisons between the distributions of strains obtained from simulations and experimental measurements, demonstrating a good agreement between the predictions and experimental results. It is important to note that in Fig. 10(c), the strain distributions under different external loads are compared. However, considering certain characteristics of the debonding of the FRP-to-concrete joints, such as the inevitable differences in ultimate loads between simulations and experiments, as well as the significant variation in slip with a small change in external load during the debonding process, the comparison should be conducted carefully. During the elastic stage, a numerical strain distribution corresponding to a load level that is equal to the experimental load is selected. During the debonding propagation stage, the strain distribution used for comparison is chosen to ensure a similar effective stress transfer length to that of the experimental strain distribution. This treatment is similar to that employed in (Lu et al., 2005b, Lin and Wu, 2016).

Fig. 11(a) and (b) illustrate the predicted crack patterns for the tests conducted by Ali-Ahmad et al. (2006) and Wu and Jiang (2013), respectively. It is important to note that due to the lack of accurate information regarding the adhesive between the FRP laminate and mortar/aggregate, as well as the fact that debonding typically occurs within the concrete prism, a perfect bonding condition is assumed, as adopted in (Lu et al., 2005b, Tao and Chen, 2015, Lin and Wu, 2016). Furthermore, to avoid unrealistically deep debonding cracks caused by the assumption of perfect bonding between the big aggregates and FRP laminate, the aggregate distribution algorithm is modified to ensure that the depth of the aggregate bonded to the FRP laminate is less than 8 mm, a value close to the maximum crack depth reported by Lin and Wu (2016). In Fig. 11(a) and (b), only regions with the phase field value $d \geq 0.9$ are displayed to provide a clear representation of the crack patterns. It can be seen that in all cases, the debonded portion consisting of aggregates and mortar has a height less than 8 mm which is consistent with the findings reported in (Yao et al., 2005, Lin and Wu, 2016). Furthermore, several distinct failure behaviors associated with aggregates can be observed. Firstly, cracks tend to deviate from their original propagation paths and are significantly influenced by adjacent ITZ regions on the left sides of the aggregates, due to the weaker fracture properties of these regions. Secondly, the mortar on the right side of the aggregates is more prone to peel off, forming inclined cracks to the FRP laminate, as indicated by the yellow rectangle in the figure. These distinctive crack patterns are a result of considering the influence of compression and friction in these areas. In fact, similar crack patterns can also be observed in simulations conducted using homogeneous models (Lu et al., 2005b, Lin and Wu, 2016, Li and Guo, 2019), as shown in Fig. 11(c), although these models cannot explicitly illustrate the influence of aggregate on the debonding behavior.

Parametric studies

In the previous section, the proposed meso-scale model is verified through three pull-off tests. The corresponding results indicate that the debonding behavior of FRP-to-concrete joints is influenced by various factors. To gain a general understanding of how these different components affect the debonding behavior, parametric studies on adhesive properties, as well as the thickness and modulus of the FRP laminate are conducted in this section.

The effect of adhesive on the debonding behavior

The objective of this section is to investigate the influence of adhesive properties on the debonding behavior. We would like to reiterate that in the simulations in previous sections, a prefect bonding assumption, as suggested by Lu et al. (2005b), is adopted. In this section, to consider the effect of the adhesive layer, a layer of cohesive elements is embedded between the FRP laminate and the concrete prism, as depicted Fig. 7. Specifically, we consider three different adhesive moduli: (a) E_{ad}/E_f=3.3×10^{-3}; (b) E_{ad}/E_f=9.7×10^{-3}; (c) E_{ad}/E_f=9.7×10^{-2}, where E_{ad} and E_f are the Young's modulus of the adhesive and the FRP laminate, respectively. An adhesive strength of 52 MPa from (Shi et al., 2019) is adopted for all cases.

Fig. 12(a) illustrates the predicted load-slip curves for different cases. It can be seen that the ultimate slip, corresponding to the complete debonding of the FRP laminate from the concrete prism, exhibits a decreasing trend as the adhesive Young's modulus increases, indicating that weaker adhesion can lead to a higher ultimate slip. One advantage of phase field models of fracture, is their ability to theoretically reproduce the discrete crack surface area. This is achieved by integrating the crack surface density function γ_d across the entire computational domain. This characteristic allows for the introduction of a normalized crack length in the current simulation, representing the ratio between the crack surface area and the length of the FRP laminate. Fig. 12(b) depicts the normalized crack lengths for various cases. It can be observed that the smallest Young's modulus case has the largest normalized crack length, indicating that the concrete beneath the bonded FRP laminate experiences more damage. As the Young's modulus increases, the normalized crack length decreases and eventually approaches the perfectly bonding case. This trend is supported by the local crack patterns depicted in Fig. 13, where the case with the lowest Young's modulus tends to exhibit additional and deeper cracks compared to the other cases, as indicated by the yellow rectangle.

The effect of FRP thickness and axial stiffness on the debonding behavior

In pull-off tests, the thickness of the FRP laminate is typically much smaller than that of the concrete prism. To address meshing issues in the finite element (FE) model, previous studies (Lu et al., 2005b, Lin and Wu, 2016, Shi et al., 2019) have proposed using a nominal FRP laminate thickness, such as 1.0 mm. However, to maintain a constant axial stiffness ($E_f t_f$) of the FRP laminate, the Young's modulus needs to be adjusted accordingly. As shown in (Lu et al., 2005b, Lin and Wu, 2016, Shi et al., 2019), this approach of using a nominal thickness can increase the element size of the FRP laminate, reducing computational costs. This section aims to evaluate the validity of the nominal FRP thickness assumption in the meso-scale model and explore the impact of varying FRP thickness while keeping a constant Young's modulus on the debonding behavior.

To assess the validity of the nominal FRP thickness assumption, three thicknesses, i.e., t_f=0.25 mm, t_f=0.5 mm and t_f=1.0 mm, are considered, with the Young's moduli adjusted to maintain a constant axial stiffness. Fig. 14(a) illustrates the predicted load-slip curves. It shows that the curves for different thicknesses exbibit similar behavior until the slip reaches 0.75 mm. Beyond that point, clear deviations between the curves occur, with thicker laminate cases tending to have larger ultimate slips. One possible reason for this phenomenon could be the variation in bending stiffness, which can affect the stress conditions during debonding propagation. Fig. 14(b) depicts the corresponding normalized crack lengths. The increase in the normalized crack length quantitatively indicates that the thickness of the FRP laminate still influences the crack pattern, even when maintaining a constant axial stiffness. This trend is supported by the local crack patterns depicted in Fig. 15, where the thickest case exhibits more and deeper cracks compared to the other cases, as indicated by the yellow rectangle.

To investigate the impact of the thickness of the FRP laminate while keeping the Young’s modulus constant, three different thickness cases, i.e., t_f=0.25 mm, t_f=0.5 mm and t_f=1.0 mm, are considered. Fig. 16(a) illustrates the predicted load-slip curves corresponding to various thicknesses. It can be observed that increasing the thickness of the FRP laminate leads to a notable increase in the maximum load but a decrease in ultimate slip, aligning with the experimental findings reported by (Zhang and Smith, 2013). In Fig. 16(b), the normalized crack length is depicted, revealing a consistent decrease with increasing FRP thickness. Fig. 17 illustrates the local crack patterns. It can be seen that the trend in normalized crack length is influenced by the smoothness of the crack pattern. The thicker laminate case (Fig. 17 (c)) exhibits fewer dentiform cracks compared to thinner cases (i.e., Fig. 17 (a) and (b)), as indicated by yellow rectangles.

Conclusions

This paper proposes a novel meso-scale phase field model for accurately simulating the debonding behavior of FRP-to-concrete joints under monotonic loads. The proposed model has been successfully validated using pull-off tests reported in existing literature. The predicted results, including load-slip curves, axial strain distributions in FRP laminates, and debonding crack patterns, exhibit good agreement with experimental findings. One notable advantage of the proposed meso-scale phase field model, when compared to existing homogeneous models, is its explicit consideration of aggregate distribution. This feature enables the capture of complex failure mechanisms, such as mortar failure, ITZ failure, and frictional effects, leading to a more comprehensive understanding of the debonding process in FRP-to-concrete joints. Moreover, the incorporation of a crack density function within the phase field model allows for accurate reproduction of the surface area of cracks, facilitating quantitative investigations of crack density-related behaviors. Through numerical investigations, it has been found that the damage per unit area in the concrete beneath the bonded surface is influenced by the adhesive modulus. A smaller adhesive modulus will lead to more damage, and as the modulus increases, it tends to approach the perfectly bonding case. When the tensile/axial stiffness (i.e., Young's modulus × thickness) of the FRP laminate is kept constant, different FRP laminate thicknesses will lead to different debonding behaviors. Such differences become more significant when changing the laminate thickness while keeping the Young's modulus of the FRP laminate constant, indicating a prominent influence of the axial stiffness of the FRP laminate on the debonding behavior of FRP-to-concrete joints. It should be noted that while the pull-off test in this paper is simulated under a plane stress hypothesis, the actual debonding behavior is three-dimensional. Therefore, a nature extension of the proposed meso-scale model to three-dimensional cases can be pursued. The relevant work will be carried out in the future.

Data Availability Statement

Data will be made available on request from the corresponding author.

Acknowledgements

This research was supported by Guangdong Province R&D Plan for Key Areas (Project code: 2019B111107002), the Hong Kong Research Grants Council – Theme-based Research Scheme (Project code: T22-502/18-R), and The Hong Kong Polytechnic University through the Post-doctoral Fellowship (Project code: 1-W21R) and the

Research Institute for Sustainable Urban Development (No.1-BBWE).

Nomenclature

$\bar{\boldsymbol{b}}$, $\bar{\boldsymbol{t}}$	Body and boundary forces
b_f, b_c	Widths of the FRP laminate and concrete prism
$\boldsymbol{C}_d$, $\boldsymbol{C}_d^{loc}$	Global and local damaged elasticity matrices
d, $\dot{d}$	Damage phase field and its time derivative
E_a, E_m, E_c	Young's moduli for aggregates, mortar and concrete
E_f, E_{ad}	Young's moduli for FRP laminate and adhesive
f_a, f_c	Cubic compressive strengths for aggregates and concrete
$f_m^{'}$, $f_c^{'}$	Cylinder compressive strengths for mortar and concrete
g_I, g_{II}	Tensile and shear failure degradation functions
$\tilde{g}_{II}$	Shear-friction failure degradation function
G_I, G_{II}	Tensile and shear critical energy release rates
l_0, l_i	Length scales of damage and interface phase fields
$\boldsymbol{n}_1$, $\boldsymbol{n}_2$	Principal stress directions
t_f	Thickness of FRP laminate
$\boldsymbol{T}_\sigma$, $\boldsymbol{T}_\varepsilon$	Stress and strain rotation matrices
$\boldsymbol{u}$	Displacement field
Y_I, $\bar{Y}_{sh}$	Tensile and shear strengths
Y_o	Cohesion strength

α	Crack angle
β_w	Width factor
γ_d	Crack surface density function
$\boldsymbol{\varepsilon}$	Strain tensor
η	Interface phase field
κ	Artificial damage phase field viscosity
ρ	density
$\boldsymbol{\sigma}$	Stress tensor
σ_n^n, τ_m^n	Normal and shear stresses at crack surfaces
$\tau_{\max}$	Historical maximum shear stress
τ_f	Reduced shear stress at crack surfaces
τ_{II}^n	The component of shear stress that excludes friction stress
ϕ	Friction angle
ψ_I^n, ψ_{II}^n	Elastic energy densities governing tensile and shear damage

References

Ali-Ahmad, M., Subramaniam, K. and Ghosn, M. (2006) 'Experimental investigation and fracture analysis of debonding between concrete and FRP sheets', *Journal of engineering mechanics,* 132(9), pp. 914-923.

Ambati, M., Gerasimov, T. and De Lorenzis, L. (2015a) 'Phase-field modeling of ductile fracture', *Computational Mechanics,* 55(5), pp. 1017-1040.

Ambati, M., Gerasimov, T. and De Lorenzis, L. (2015b) 'A review on phase-field models of brittle fracture and a new fast hybrid formulation', *Computational Mechanics,* 55(2), pp. 383-405.

Barnes, B., DIAMOND, S. and Dolch, W. (1979) 'Micromorphology of the interfacial zone around aggregates in Portland cement mortar', *Journal of the American Ceramic Society,* 62(1-2), pp. 21-24.

Borden, M. J. (2012) *Isogeometric analysis of phase-field models for dynamic brittle and ductile fracture.* PhD dissertation, The University of Texas at Austin.

Bourdin, B., Francfort, G. A. and Marigo, J.-J. (2000) 'Numerical experiments in revisited brittle fracture', *Journal of the Mechanics and Physics of Solids,* 48(4), pp. 797-826.

Bui, T. Q. and Hu, X. (2021) 'A review of phase-field models, fundamentals and their applications to composite laminates', *Engineering Fracture Mechanics*, pp. 107705.

Chen, J. F. and Teng, J. (2001) 'Anchorage strength models for FRP and steel plates bonded to concrete', *Journal of structural engineering,* 127(7), pp. 784-791.

Code, P. (2005) 'Eurocode 2: design of concrete structures', *British Standard Institution, London*.

Contrafatto, L., Cuomo, M. and Gazzo, S. (2016) 'A concrete homogenisation technique at meso-scale level accounting for damaging behaviour of cement paste and aggregates', *Computers & Structures,* 173, pp. 1-18.

Coronado, C. A. and Lopez, M. M. (2010) 'Numerical modeling of concrete-FRP debonding using a crack band approach', *Journal of composites for construction,* 14(1), pp. 11-21.

Dai, J.-G., Yokota, H. and Ueda, T. (2009) 'A hybrid bonding system for improving the structural performance of FRP flexurally strengthened concrete beams', *Advances in Structural Engineering,* 12(6), pp. 821-832.

Dai, J., Ueda, T. and Sato, Y. (2005) 'Development of the nonlinear bond stress-slip model of fiber reinforced plastics sheet-concrete interfaces with a simple method', *Journal of composites for construction,* 9(1), pp. 52-62.

De Lorenzis, L. and Teng, J.-G. (2007) 'Near-surface mounted FRP reinforcement: An emerging technique for strengthening structures', *Composites Part B: Engineering,* 38(2), pp. 119-143.

Du, X., Jin, L. and Zhang, R. (2014) 'Modeling the cracking of cover concrete due to non-uniform corrosion of reinforcement', *Corrosion Science,* 89, pp. 189-202.

Francfort, G. A. and Marigo, J.-J. (1998) 'Revisiting brittle fracture as an energy minimization problem', *Journal of the Mechanics and Physics of Solids,* 46(8), pp. 1319-1342.

Hoover, C. G., Bažant, Z. P., Vorel, J., Wendner, R. and Hubler, M. H. (2013) 'Comprehensive concrete fracture tests: Description and results', *Engineering fracture mechanics,* 114, pp. 92-103.

Hu, X., Tan, S., Xia, D., Min, L., Xu, H., Yao, W., Sun, Z., Zhang, P., Bui, T. Q. and

Zhuang, X. (2023) 'An overview of implicit and explicit phase field models for quasi-static failure processes, implementation and computational efficiency', *Theoretical and Applied Fracture Mechanics*, pp. 103779.

Hu, X., Xu, H., Xi, X., Zhang, P. and Yang, S. (2022) 'Meso-scale phase field modelling of reinforced concrete structures subjected to corrosion of multiple reinforcements', *Construction and Building Materials,* 321, pp. 126376.

Huang, Y., Yang, Z., Chen, X. and Liu, G. (2016) 'Monte Carlo simulations of meso-scale dynamic compressive behavior of concrete based on X-ray computed tomography images', *International Journal of Impact Engineering,* 97, pp. 102-115.

Huang, Y., Yang, Z., Ren, W., Liu, G. and Zhang, C. (2015) '3D meso-scale fracture modelling and validation of concrete based on in-situ X-ray Computed Tomography images using damage plasticity model', *International Journal of Solids and Structures,* 67, pp. 340-352.

Jawdhari, A., Fam, A. and Harik, I. (2018) 'Numerical study on the bond between CFRP rod panels (CRPs) and concrete', *Construction and Building Materials,* 177, pp. 522-534.

Jawdhari, A., Semendary, A., Fam, A., Khoury, I. and Steinberg, E. (2019) 'Bond characteristics of CFRP rod panels adhered to concrete under bending effects', *Journal of Composites for Construction,* 23(1), pp. 04018077.

Kadhim, M. M., Jawdhari, A., Adheem, A. H. and Fam, A. (2022) 'Analysis and design of two-way slabs strengthened in flexure with FRCM', *Engineering Structures,* 256, pp. 113983.

Kadhim, M. M., Jawdhari, A. and Peiris, A. 'Evaluation of lap-splices in NSM FRP rods for retrofitting RC members'. *Structures*: Elsevier, 877-894.

Kai, M.-F., Ji, W.-M. and Dai, J.-G. (2022) 'Atomistic insights into the debonding of Epoxy–Concrete interface with water presence', *Engineering Fracture Mechanics,* 271, pp. 108668.

Kai, M.-F., Li, G., Yin, B.-B. and Akbar, A. (2023a) 'Aluminum-induced structure evolution and mechanical strengthening of calcium silicate hydrates: an atomistic insight', *Construction and Building Materials,* 393, pp. 132120.

Kai, M.-F., Sanchez, F., Hou, D.-S. and Dai, J.-G. (2023b) 'Nanoscale insights into the interfacial characteristics between calcium silicate hydrate and silica', *Applied Surface Science,* 616, pp. 156478.

Li, G., Yin, B., Zhang, L. and Liew, K. (2020) 'Modeling microfracture evolution in heterogeneous composites: A coupled cohesive phase-field model', *Journal of the Mechanics and Physics of Solids,* 142, pp. 103968.

Li, W. and Guo, L. (2019) 'Dual-horizon peridynamics analysis of debonding failure in FRP-to-concrete bonded joints', *International Journal of Concrete Structures and Materials,* 13(1), pp. 1-15.

Li, Y.-Q., Chen, J.-F., Yang, Z.-J., Esmaeeli, E., Sha, W. and Huang, Y.-J. (2021) 'Effects of concrete heterogeneity on FRP-concrete bond behaviour: Experimental and mesoscale numerical studies', *Composite Structures,* 275, pp. 114436.

Lin, J.-P. and Wu, Y.-F. (2016) 'Numerical analysis of interfacial bond behavior of externally bonded FRP-to-concrete joints', *Journal of Composites for Construction,* 20(5), pp. 04016028.

Lin, J.-P., Wu, Y.-F. and Smith, S. T. (2017) 'Width factor for externally bonded FRP-to-concrete joints', *Construction and Building Materials,* 155, pp. 818-829.

López, C. M., Carol, I. and Aguado, A. (2008) 'Meso-structural study of concrete fracture using interface elements. II: compression, biaxial and Brazilian test', *Materials and structures,* 41(3), pp. 601-620.

Lu, X., Teng, J., Ye, L. and Jiang, J. (2005a) 'Bond–slip models for FRP sheets/plates bonded to concrete', *Engineering structures,* 27(6), pp. 920-937.

Lu, X., Ye, L., Teng, J. and Jiang, J. (2005b) 'Meso-scale finite element model for FRP sheets/plates bonded to concrete', *Engineering structures,* 27(4), pp. 564-575.

Miehe, C., Hofacker, M., Schänzel, L.-M. and Aldakheel, F. (2015) 'Phase field modeling of fracture in multi-physics problems. Part II. Coupled brittle-to-ductile failure criteria and crack propagation in thermo-elastic–plastic solids', *Computer Methods in Applied Mechanics and Engineering,* 294, pp. 486-522.

Miehe, C., Hofacker, M. and Welschinger, F. (2010a) 'A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits', *Computer Methods in Applied Mechanics and Engineering,* 199(45-48), pp. 2765-2778.

Miehe, C., Welschinger, F. and Hofacker, M. (2010b) 'Thermodynamically consistent phase-field models of fracture: Variational principles and multi-field FE implementations', *International journal for numerical methods in engineering,* 83(10), pp. 1273-1311.

Mori, T. and Tanaka, K. (1973) 'Average stress in matrix and average elastic energy of materials with misfitting inclusions', *Acta metallurgica,* 21(5), pp. 571-574.

Nagai, K., Sato, Y. and Ueda, T. (2005) 'Mesoscopic simulation of failure of mortar and concrete by 3D RBSM', *Journal of Advanced Concrete Technology,* 3(3), pp. 385-402.

Neville, A. M. (1995) *Properties of concrete.* Longman London.

Nguyen, T. T., Yvonnet, J., Zhu, Q.-Z., Bornert, M. and Chateau, C. (2016) 'A phase-field method for computational modeling of interfacial damage interacting with crack propagation in realistic microstructures obtained by microtomography', *Computer Methods in Applied Mechanics and Engineering,* 312, pp. 567-595.

Pan, J. and Leung, C. K. (2007) 'Effect of concrete composition on FRP/concrete bond capacity', *Journal of Composites for Construction,* 11(6), pp. 611-618.

Pina-Henriques, J. and Lourenço, P. B. (2006) 'Masonry compression: a numerical investigation at the meso-level', *Engineering computations*.

Prakash, P. R., Pulatsu, B., Lourenço, P. B., Azenha, M. and Pereira, J. M. (2020) 'A meso-scale discrete element method framework to simulate thermo-mechanical failure of concrete subjected to elevated temperatures', *Engineering Fracture*

Mechanics, 239, pp. 107269.

Quintanas-Corominas, A., Reinoso, J., Casoni, E., Turon, A. and Mayugo, J. (2019) 'A phase field approach to simulate intralaminar and translaminar fracture in long fiber composite materials', *Composite Structures,* 220, pp. 899-911.

Ren, W., Yang, Z., Sharma, R., Zhang, C. and Withers, P. J. (2015) 'Two-dimensional X-ray CT image based meso-scale fracture modelling of concrete', *Engineering Fracture Mechanics,* 133, pp. 24-39.

Šavija, B., Luković, M., Pacheco, J. and Schlangen, E. (2013) 'Cracking of the concrete cover due to reinforcement corrosion: A two-dimensional lattice model study', *Construction and Building Materials,* 44, pp. 626-638.

Shi, J.-W., Cao, W.-H. and Wu, Z.-S. (2019) 'Effect of adhesive properties on the bond behaviour of externally bonded FRP-to-concrete joints', *Composites Part B: Engineering,* 177, pp. 107365.

Sideris, K., Manita, P. and Sideris, K. (2004) 'Estimation of ultimate modulus of elasticity and Poisson ratio of normal concrete', *Cement and concrete composites,* 26(6), pp. 623-631.

Takeo, K., Matsushita, H., Makizumi, T. and Nagashima, G. (1997) 'Bond characteristics of CFRP sheets in the CFRP bonding technique', *Proceedings of Japan concrete institute,* 19(2), pp. 1599-1604.

Tan, Z. (2002) 'Experimental research for RC beam strengthened with GFRP', *Graduation thesis, Tsinghua Univ., Beijing, China (in Chinese)*.

Tao, Y. and Chen, J.-F. (2015) 'Concrete damage plasticity model for modeling FRP-to-concrete bond behavior', *Journal of composites for construction,* 19(1), pp. 04014026.

Teng, J., Chen, J.-F. and Yu, T. (2002) 'FRP-strengthened RC structures'.

Teng, J. G., Chen, J., Smith, S. T. and Lam, L. (2003) 'Behaviour and strength of FRP-strengthened RC structures: a state-of-the-art review', *Proceedings of the institution of civil engineers-structures and buildings,* 156(1), pp. 51-62.

Turon, A., Camanho, P. P., Costa, J. and Dávila, C. (2006) 'A damage model for the simulation of delamination in advanced composites under variable-mode loading', *Mechanics of materials,* 38(11), pp. 1072-1089.

Ueda, T., Sato, Y. and Asano, Y. 'Experimental study on bond strength of continuous carbon fiber sheet'. *4th International Symposium on Fiber Reinforced Polymer Reinforcement for Reinforced Concrete Structures*: American Concrete Institute, 407-416.

Version, A. (2011) '6.11 Documentation', *Dassault Systemes Simulia Corp., Providence, RI, USA*.

Wu, J.-Y. (2017) 'A unified phase-field theory for the mechanics of damage and quasi-brittle failure', *Journal of the Mechanics and Physics of Solids,* 103, pp. 72-99.

Wu, Y.-F. and Jiang, C. (2013) 'Quantification of bond-slip relationship for externally bonded FRP-to-concrete joints', *Journal of Composites for Construction,* 17(5), pp. 673-686.

Xi, X., Yang, S., Li, C.-Q., Cai, M., Hu, X. and Shipton, Z. K. (2018) 'Meso-scale mixed-mode fracture modelling of reinforced concrete structures subjected to non-uniform corrosion', *Engineering Fracture Mechanics,* 199, pp. 114-130.

Yang, Z., Chen, J. F. and Proverbs, D. (2003) 'Finite element modelling of concrete cover separation failure in FRP plated RC beams', *Construction and Building Materials,* 17(1), pp. 3-13.

Yao, J., Teng, J. and Chen, J. F. (2005) 'Experimental study on FRP-to-concrete bonded joints', *Composites Part B: Engineering,* 36(2), pp. 99-113.

Yılmaz, O. and Molinari, J.-F. (2017) 'A mesoscale fracture model for concrete', *Cement and Concrete Research,* 97, pp. 84-94.

Zhang, H. and Smith, S. T. (2013) 'Fibre-reinforced polymer (FRP)-to-concrete joints anchored with FRP anchors: tests and experimental trends', *Canadian Journal of Civil Engineering,* 40(11), pp. 1103-1116.

Zhang, P., Dai, J.-G., Das, C. S. and Zheng, J.-J. (2023) 'A fully coupled meso-scale electro-chemo-mechanical phase field method for corrosion-induced fracture in concrete', *International Journal of Solids and Structures,* 267, pp. 112165.

Zhang, P., Hu, X., Bui, T. Q. and Yao, W. (2019a) 'Phase field modeling of fracture in fiber reinforced composite laminate', *International Journal of Mechanical Sciences,* 161, pp. 105008.

Zhang, P., Hu, X., Yang, S. and Yao, W. (2019b) 'Modelling progressive failure in multi-phase materials using a phase field method', *Engineering Fracture Mechanics,* 209, pp. 105-124.

Zhang, P., Yao, W., Hu, X. and Bui, T. Q. (2020) '3D micromechanical progressive failure simulation for fiber-reinforced composites', *Composite Structures,* 249, pp. 112534.

Zhang, P., Yao, W., Hu, X. and Bui, T. Q. (2021) 'An explicit phase field model for progressive tensile failure of composites', *Engineering Fracture Mechanics,* 241, pp. 107371.

Zhou, R., Song, Z. and Lu, Y. (2017) '3D mesoscale finite element modelling of concrete', *Computers & Structures,* 192, pp. 96-113.

Tables

Table 1 Material properties and geometric details of the pull-off tests.

Experiment	$f_c^{'}$ (MPa)	b_c (mm)	b_f (mm)	t_f (mm)	E_f (GPa)
Ali-Ahmad et al. NO. 1	38.0	125	46	0.167	230.0
Yao et al. II-5	23.0	150	25	0.165	256.0
Wu and Jiang C50-250-1	46.1	150	50	0.167	248.3
Wu and Jiang C60-250-1	56.4	150	50	0.167	248.3

Figure Captions

Fig. 1 (a) Considered continuum domain with boundary conditions and a crack and (b) stress state at the point depicted in right figure.

Fig. 2 (a) Principal stress and (b) stress components at the local coordinate system with *x'-axis* along crack direction.

Fig. 3 Sharp and diffusive crack topology. (a) Shape crack embedded in the continuum domain Ω and (b) the regularized crack Γ_d represented by crack phase field *d*.

Fig. 4 Sketches of mortar cracking and ITZ debonding: (a) the discrete representation and (b) the regularized representation.

Fig. 5 Geometry and boundary condition of the TPB test.

Fig. 6 The experimental (Hoover et al., 2013) and predicted force-CMOD relationships, along with simulated crack patterns in meso-scale regions: (a) specimen of D=40 mm and λ=0.15; (b) specimen of D=40 mm and λ=0.3; (c) specimen of D=93 mm and λ=0.15; and (d) specimen of D=93 mm and λ=0.3.

Fig. 7 Two-dimensional pull-off test FE model.

Fig. 8 Comparison of the predictions with experimental results.

Fig. 9 Predicted and experimental load-slip curves: (a) specimen No. 1 in (Ali-Ahmad et al., 2006); (b) specimen II-5 in (Yao et al., 2005); (c) specimen C50-250-1 in (Wu and Jiang, 2013); (d) specimen C60-250-1 in (Wu and Jiang, 2013).

Fig. 10 Predicted and experimental axial strain distributions in FRP: (a) specimen No. 1 in (Ali-Ahmad et al., 2006); (b) specimen II-5 in (Yao et al., 2005); (c) specimen C50-250-1 in (Wu and Jiang, 2013)

Fig. 11 Predicted crack patterns: (a) proposed model for specimen No. 1 in (Ali-Ahmad et al., 2006); (b) proposed model for specimen C60-250-1 in (Wu and Jiang, 2013); (c) numerical simulations from (Lin and Wu, 2016, Lu et al., 2005b, Li and Guo, 2019) by using homogeneous models.

Fig. 12 Numerical predictions of the proposed model for various adhesions: (a) load-slip curves; (b) normalized crack lengths.

Fig. 13 Predicted crack patterns near the loaded end for various adhesions: (a) E_{ad}/E_f=3.3×10^{-3}; (b) E_{ad}/E_f=9.7×10^{-3}; (c) E_{ad}/E_f=9.7×10^{-2}.

Fig. 14 Numerical predictions of the proposed model for various FRP thicknesses (constant FRP axial stiffness): (a) load-slip curves; (b) normalized crack lengths.

Fig. 15 Predicted crack patterns near the loaded end for various FRP thicknesses (constant FRP axial stiffness): (a) t_f=0.25 mm; (b) t_f=0.5 mm; (c) t_f=1.0 mm.

Fig. 16 Numerical predictions of the proposed model for various FRP thicknesses (constant FRP Young's modulus): (a) load-slip curves; (b) normalized crack lengths.

Fig. 17 Predicted crack patterns near the loaded end for various FRP thicknesses (constant FRP Young's modulus): (a) t_f=0.25 mm; (b) t_f=0.5 mm; (c) t_f=1.0 mm.