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Copula-Based Multivariate Renewal Model for Life-Cycle Analysis of Civil Infrastructure considering Multiple Dependent Deterioration Processes

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Abstract

Civil infrastructure is subjected to multiple deterioration processes (e.g., gradual deterioration and shock deterioration) caused by environmental exposure and extreme events during its lifetime. To maintain performance and functionality, maintenance actions should be performed and the life-cycle cost may be affected. There is a need to explore the effect of maintenance actions and various uncertainties on the life-cycle performance of the system. This study proposes a probabilistic life-cycle analysis framework for civil infrastructure based on a set of performance indicators, e.g., reliability and maintenance cost. Stochastic uncertainties resulting from multiple dependent deterioration processes, system reliability, intervention actions, and maintenance cost are considered. In particular, the correlation between the maintenance interval and cost is highlighted. Previous studies generally assume they are independent. Such an assumption can be misleading and lead to inappropriate cost estimation. To address this concern, a copula-based multivariate renewal model is proposed to assess the life-cycle maintenance cost analytically and numerically. In addition to the expected cost, statistical moments (standard deviation, skewness, and kurtosis) are calculated to quantify uncertainties from higher-order moments. Two illustrative examples show that the dependence structure and uncertainties can have a large impact on the life-cycle cost, and decisions can be altered by considering statistical moments of the cost.

Keywords: Copula; life-cycle cost; higher-order moments; stochastic deterioration; structural reliability

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1. Introduction

During the lifetime, civil infrastructure systems are subjected to multiple deterioration processes, such as gradual deterioration resulting from environmental influence (e.g., corrosion and crack growth) and shock deterioration caused by extreme events (e.g., hurricanes and earthquakes) [1, 2]. The combination effects of deterioration processes may lead to damages and failure [3, 4], thus threatening public safety and resulting in considerable financial and social losses. In order to maintain the functionality of civil infrastructure, various intervention actions such as repair and replacement are required. The incurred maintenance cost increases the life-cycle cost and directly affects the subsequent decision-making process.

Due to various uncertainties associated with the life-cycle analysis, rational stochastic models and reliability analysis can be essential to assess the maintenance cost [5-7]. Although numerous studies have accounted for both gradual and shock deterioration processes [8-11], interaction and correlation between them are commonly neglected (i.e., assume they are independent). For instance, Sanchez-Silva *et al*. [12] studied the life-cycle performance of deteriorating structures by investigating the combined effects of progressive degradation and sudden events. Caballé and Castro [13] proposed a reliability-based analysis framework to assess the maintenance cost of the system with a finite lifetime subjected to internal continuous degradation and sudden events. The impact of dependent deterioration processes on system reliability has been explored in recent studies. For instance, Kumar *et al.* [14] proposed a stochastic framework for engineering systems to estimate the time to failure considering exposure to gradual degradation and sudden events. Wang *et al.* [15] developed a dependence framework to assess the time-dependent reliability of deteriorating structures considering the correlation between gradual and shock deterioration processes. Jia *et al.* [16] investigated the stochastic deterioration of reinforced concrete structures considering compound effects of corrosion, earthquakes, and ASR. However, the effects of dependent deterioration processes on maintenance planning and the associated cost have not been carefully explored in a life-cycle context.

In terms of the life-cycle cost analysis, most of the existing studies focus on one type of deterioration (either under gradual deterioration or shocks) and ignore their combined effects. For instance, Cheng *et al.* [17] presented an analytical framework to derive the probability distribution of maintenance cost of aging engineering systems subjected to gradual degradation by using the gamma process. Yang and Frangopol [1] assessed the life-cycle maintenance cost subjected to independent shock and deterioration processes using renewal models. A few recent studies take correlated deterioration effects into account. For instance, Jia and Gardoni [18] introduced state-dependent models to the life-cycle cost analysis subjected to earthquake and corrosion damage. Liu *et al.* [19] investigated dependent degradation processes using copulas and the resulting impact on the life-cycle cost. However, in these studies, the impact of dependent deterioration processes on the maintenance cost has not been explicitly discussed.

Despite considerable efforts on deterioration modeling and cost assessment, these studies commonly assume that the maintenance interval and cost are independent. The independence assumption has been widely used to simplify the analytical formulation associated with the renewal theory [17, 19, 20]. Neglecting the dependence and the associated uncertainties may result in an inappropriate estimation of the accumulative cost, thus misleading decision-makers during the life-cycle management. Pandey and Van Der Weide [20] also indicated that dependence between maintenance cost and renewal cycle cannot be ignored, especially when preventive maintenance is considered. To the best of the authors' knowledge, the dependent maintenance interval and cost have not been considered in the life-cycle analysis.

To incorporate the correlation between the maintenance interval and cost, statistical modeling of the joint probability distribution is essential. A conventional approach of multivariate modeling relies on an empirical multivariate joint distribution or a joint normal distribution [21, 22], but the approach is limited to a certain correlation relationship. Herein, a copula-based method is proposed. As an advanced mathematical tool, the copula model offers sufficient efficiency and flexibility in multivariate dependence modeling by considering the joint and marginal distributions separately [23, 24]. Due to this advantage, copulas have been increasingly applied in deterioration processes and reliability analysis. For instance, Goda [25] highlighted the importance of multi-variate seismic demand modeling by employing copulas. Fang *et al*. [24] provided an integrated approach to analyze the reliability of a degrading system by considering dependent component failures using the copula model.

In addition to uncertainties resulting from the dependence model, uncertainties associated with statistical moments (mean, standard deviation, skewness, and kurtosis) of the life-cycle cost have not been thoroughly explored. Although the minimum expected cost has been utilized as a standard decision criterion, the impact of the other statistical moments on the life-cycle cost and decision-making process has been rarely discussed. Pandey and Van Der Weide [20] indicated that the variance of the life-cycle damage cost could be significant to indicate the variability. Li *et al.* [26] stated the importance of higher-order moments (skewness and kurtosis) of the repair cost during system lifetime, as skewness and kurtosis imply potential tail risks. Hence, it is necessary to assess the statistical moments of the life-cycle maintenance cost.

This study presents a copula-based life-cycle cost analysis framework for deteriorating civil infrastructure systems. The developed copula-based approach allows various complex dependence structures between the maintenance interval and the cost in a renewal process. The impact of dependent deterioration processes on the life-cycle performance of a system is investigated. Furthermore, the effect of correlated maintenance cost (considering preventive and essential maintenance) and maintenance interval on the life-cycle maintenance cost is evaluated based on the renewal process. The proposed copula model allows including practical maintenance data into the life-cycle analysis, by identifying the correlation between maintenance cost and interval. The proposed framework can aid the decision-making associated with maintenance planning and optimization. The remainder of the paper is organized as follows. The following Section 2 introduces the life-cycle framework and the relevant dependence. Section 3 illustrates the stochastic modeling of deterioration processes and the time-dependent reliability assessment. In section 4, a copula-based renewal model is proposed to assess the life-cycle maintenance cost analytically and numerically. Subsequently, two illustrative examples are presented in Section 5. Conclusions are drawn in the last section.

2. Life-cycle analysis framework and dependence

Appropriate maintenance relies on the life-cycle analysis of the structural performance. Figure 1 shows the computational framework for the life-cycle maintenance cost assessment. There are various dependence relationships within the life-cycle analysis. To take the dependence into account, the analysis of deterioration and maintenance models also becomes more and more complicated. In previous studies, special attention has been paid to structural, economic, and stochastic dependencies of multi-component engineering systems [27, 28]. For instance, structural dependence has been widely explored to investigate the impact of the maintenance action of one component on other components [29, 9, 30]. Economic dependence refers to that the total maintenance cost of a system may be increased or decreased due to joint maintenance of components [31, 32]. Stochastic dependence has been considered for dependent deterioration or condition among components [9, 33]. In the life-cycle analysis of civil engineering, potential dependence associated with structural reliability can be related to structural status variables, external conditions, model parameters, and time [18, 19, 15]. In the maintenance aspect, potential dependence exists among preventive maintenance cost, corrective maintenance cost, total maintenance cost, and time.

In this study, the major contribution is associated with Stage 3 by considering the dependence between total maintenance cost within the maintenance interval and the interval. A copula-based renewal model is proposed. The deterioration modeling and the structural reliability analysis at Stages 1 and 2 aim to provide essential inputs of the expected maintenance cost and the maintenance interval for Stage 3. A gradual deterioration process and two shock processes are considered. Based on these inputs, the life-cycle maintenance cost can be assessed by using the copula-based renewal model.

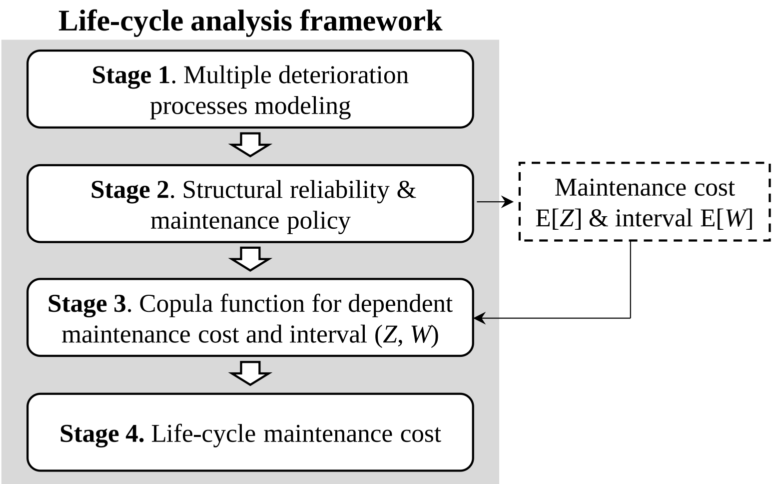


Figure 1. A flowchart of the proposed life-cycle analysis framework

3. Structural deterioration and reliability-based maintenance

3.1 Stochastic models of gradual deterioration and sudden events

3.1.1 Gradual deterioration

The stochastic gamma process has been widely used to model gradual deterioration [12, 34]. The gradual deterioration of an infrastructure system can be modeled by a stochastic gamma process. Over an interval $(0, s]$, the cumulative degradation follows the gamma distribution, and its probability density function (PDF) $\mathrm{ga}(q; \alpha s, \beta)$ and cumulative distribution function (CDF) $\mathrm{Ga}(q; \alpha s, \beta)$ are given by

$$\mathrm{ga}(q;\alpha s,\beta)=\frac{q^{\alpha s-1}\exp(-q/\beta)}{\beta^{\alpha s}\Gamma(\alpha s)} \tag{1}$$

$$\mathrm{Ga}(q;\alpha s,\beta)=\frac{\Upsilon(\alpha s,q/\beta)}{\Gamma(\alpha s)} \tag{2}$$

where αs and β are shape and scale parameters, respectively; $\Upsilon(\alpha s,\beta)=\int_0^{\beta} x^{\alpha s-1}e^{-x}dx$ is the lower incomplete gamma function; and $\Gamma(\alpha s)=\int_0^{\infty} x^{\alpha s-1}e^{-x}dx$ is the complete gamma function.

3.1.2 Shock deterioration: external shock and fatal shock

Different from gradual deterioration, shock deterioration indicates the abrupt decrease in the performance of a system caused by a shock event [13, 35]. There are two shock processes considered herein. One is the external shock process, which leads to the accumulation of shock deterioration and results in failure when the failure threshold is reached. The other one refers to a fatal shock process, which leads to immediate failure of the system. It is necessary to account for random fatal shocks, as the system can be subjected to extreme events with low-frequency and high-consequence during the lifetime. Two shock processes are modeled by the Poisson processes, in which the occurrence rate of a fatal shock process λ_{FaS} is much smaller than that of an external shock process λ_{ExS}. For a single shock process, the number of shocks follows a Poisson distribution, which gives

$$\mathrm{P}[N(t)=x]=\frac{(\lambda t)^{x}\exp(-\lambda t)}{x!} \tag{3}$$

where q is the number of shocks with x = 0, 1, 2, … and λ is the occurrence rate of a shock process. The intensity of shock deterioration is a random variable, e.g., ΔR^{ExS} denoting the intensity of a shock event is a random variable. In other words, the external shock and fatal shock processes are also compound Poisson processes. For the external shock, it is associated with the external demand. Herein, the external shock deterioration is assumed to follow lognormal distribution. The existence of a fatal shock results in an immediate failure, thus the distribution of the intensity is not specified herein.

3.2 System reliability analysis

3.2.1 Stochastic demand and capacity

The deterioration of a system has a direct influence on structural reliability. The time-dependent reliability analysis relies on the assessment of demand and capacity subjected to stochastic deterioration. During the period (0, s], the random occurrence of external loads imposes demand $\{D_i\}$ with $i = 1, 2, \ldots, n$ upon the system. The associated arriving times of demands are t_1, t_2, …, t_n. The external shock deterioration results from the demand due to external loading. In other words, the demand occurs at the same time as the external shock deterioration.

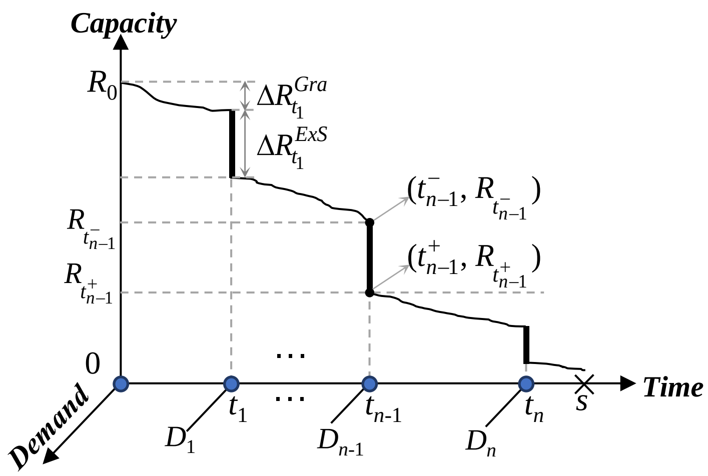


Figure 2. Capacity and demand of a system subjected to gradual deterioration and external shock process.

Figure 2 describes the demand and time-dependent resistance subjected to gradual deterioration and external shock. In this study, the gradual deterioration is modeled as a stochastic gamma process and the external shock is modeled by a Poisson process. The occurrence of demand is together with the external shock, thus also following a Poisson process. The period (0, s] is divided into n + 1 intervals by n number of load events, i.e., (0, t_1], (t_1, t_2], …, (t_{n-1}, t_n], (t_n, $t_{n+1} = s$]. The number of events n is a stochastic variable. The system resistance at time t_n can be denoted as R_{t_n}

$$R(t_n^-) = R_0 - R^{Gra}(0, t_n] - \sum_{i=1}^{N(t)-1} \Delta R_{t_i}^{ExS} \tag{4}$$

$$R(t_n^+) = R(t_n^-) - \Delta R_{t_n}^{ExS} \tag{5}$$

where R_0 represents the initial capacity of the system; t_n^- and t_n^+ are the time immediately before and after t_n; and $\Delta R_{t_i}^{ExS}$ is the external shock deterioration at time t_i. It should be noted that the demands and the shock deterioration are physically related. Herein, $\Delta R_{t_i}^{Gra} = R^{Gra}(t_{i-1}, t_i]$ denotes gradual deterioration within time interval $(t_{i-1}, t_i]$. Figure 3 describes the impact of gradual and shock deterioration on the system capacity at t_n (t_n^- and t_n^+).

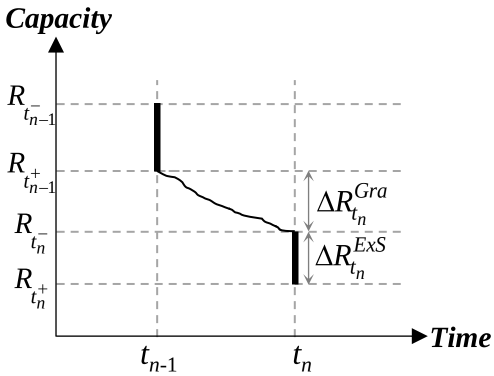


Figure 3. Schematic diagram of deterioration of system capacity.

For normalization, the capacity can be defined as the product of the deterioration function $G(t)$ and the initial capacity R_0, i.e., $R(t) = R_0 \cdot G(t)$. Accordingly, Eqs. (4) and (5) can be rearranged to the time-dependent deterioration functions, given by Eqs. (6) and (7)

$$G(t_n^-) = 1 - R^{Gra}(0, t_n] / R_0 - \sum_{i=1}^{n-1} \Delta R_{t_i}^{ExS} / R_0 \tag{6}$$

$$G(t_n^+) = 1 - R^{Gra}(0, t_n] / R_0 - \sum_{i=1}^{n} \Delta R_{t_i}^{ExS} / R_0 \tag{7}$$

Based on Figure 2, for a time instant δ, the deterioration function at time δ can also be described as

$$G(\delta^-) = 1 - R^{Gra}(0, \delta] / R_0 - \sum_{i=1}^{N(\delta)^-} \Delta R_{t_i}^{ExS} / R_0 \tag{8}$$

$$G(\delta^{+}) = 1 - R^{Gra}(0,\delta]/R_0 - \sum_{i=1}^{N(\delta)^{+}} \Delta R_{t_i}^{ExS} / R_0 \tag{9}$$

where N^{-} is the maximum integer j with $t_j < s$ and N^{+} is the maximum integer j with $t_j \leq s$. Herein, N^{+} equals N^{-}. $N^{+} = N^{-} + 1$ is only for $s = t_i$ ($i = 1, 2, \ldots, n$). In particular, let $\delta = s$, the deterioration function at time s can be given as

$$G(s) = G(\delta^{-}) = G(\delta^{+}) = 1 - R^{Gra}(0,s]/R_0 - \sum_{i=1}^{N(s)} \Delta R_{t_i}^{ExS} / R_0 \tag{10}$$

3.2.2 Failure mechanisms and limit state function

Two possible failure modes of the system are considered: one is that failure occurs when the demand exceeds its capacity, and the other one defines that the system fails when the cumulative deterioration or damage exceeds the threshold. For the first scenario, the system fails at the nth shock event with $R_{t_n^{-}} < D_{t_n}$ and the limit state function can be computed as

$$LS_n = R_0 - R^{Gra}(0,t_n]/R_0 - \sum_{i=1}^{n-1} \Delta R_{t_i}^{ExS} - D_{t_n} \tag{11}$$

Given Eq. (11), the failure occurs when LS_n is smaller than zero, i.e., $LS_n < 0$. The demand D_{t_n} is a random variable.

As the fatal shock process is taken into account, a fatal event results in immediate failure of the system. Additionally, failure may occur at an arbitrary time when the total deterioration exceeds the maximum deterioration level [14, 15]. For instance, the failure occurs when the total amount of deterioration caused by gradual deterioration and external shock exceeds the threshold (i.e., $1 - G(\delta^{+}) > G_{\max}$), as shown in Eq. (12)

$$R^{Gra}(0,t_n]/R_0 + \sum_{i=1}^{N(t)} \Delta R_{t_i}^{ExS} / R_0 > G_{\max} \tag{12}$$

3.2.3 Dependence between deterioration processes using copulas

Deterioration processes usually have interactive effects. For instance, cracks caused by external activities may accelerate the initiation and corrosion rate of reinforcement steel in terms of reinforced concrete structures [36]. The interactive effects in multiple stochastic processes have been widely investigated and different dependence models have been developed. For instance, Kumar *et al*. [14] introduced the correlation between the demand process and shock process by developing the joint probability density function. Wang *et al*. [15] modeled the dependence among gradual and shock degradation by using copula models. Liu *et al*. [19] modeled multiple dependent degradation processes using Gamma processes and copula functions.

Herein, the copula model is proposed to construct the multivariate dependency among parameters associated with deterioration processes. The advantage of using copula is that the simulation of multivariate probability distributions is separate from the univariate random variables, thus providing sufficient effectiveness during statistical modeling [37, 38]. A copula is a function that connects the multivariate distribution function of random variables to their marginal distributions [39]. For a sequence of continuous random variables $X_1, X_2, \ldots, X_n$ with marginal CDFs $F_{X_1}, F_{X_2}, \ldots, F_{X_n}$, their dependence structure can be defined by the joint CDF $H(x_1, x_2, \ldots, x_n)$. According to Sklar's theorem [40], there exists a unique n-dimensional copula C for all $(x_1, x_2, \ldots, x_n) \in [-\infty, \infty]^n$

$$H(x_1,\ldots,x_n) = C(F_{X_1}(x_1),\ldots,F_{X_n}(x_n)) \tag{13}$$

The associated joint probability density function can be written as

$$c(x_1,\ldots,x_n) = \frac{\partial^n C(x_1,\ldots,x_n)}{\partial x_1,\ldots,\partial x_n} \tag{14}$$

There are various copula functions. Commonly used copula families include elliptical copulas (e.g., Gaussian, Student's-*t*) and Archimedean copulas (e.g., Gumbel, Clayton, and Frank copulas) for multivariate and bivariate cases. When applying the copula model, the dependence between the investigated parameters should be measured by correlation coefficients. There are three common correlation coefficients to measure the association: Pearson's correlation coefficient, Kendall's tau, and Spearman's rho. Although Pearson's correlation coefficient may be the most popular one in previous studies, it is limited to a linear relationship [40]. Pearson's correlation coefficient for the correlated random vector (U, V) can be derived as

$$\gamma_d = \frac{\int (u-\bar{u})(v-\bar{v})\, f_{U,V}(u,v)dudv}{\sqrt{\int (u-\bar{u})^2 f_V(v)dv \int (v-\bar{v})^2 f_U(u)du}} \tag{15}$$

Due to the linear limitation, Kendall's tau τ and Spearman's rho ρ are more widely employed in recent studies, and given by [39]

$$\tau(\theta) = 4\int_{[0,1]^2} C_\theta(u,v)dC_\theta(u,v) - 1 \tag{16}$$

$$\rho(\theta) = 12\int_{[0,1]^2} uvdC_\theta(u,v) - 3 \tag{17}$$

Both these two coefficients are developed from the concept of concordance and give a similar interpretation of association in most cases [39, 42]. Given the correlation coefficient, the dependence parameter θ associated with a copula can be estimated. For instance, the maximum pseudo-likelihood method can be applied to compute the dependence parameter by maximizing the pseudo log-likelihood function [41-43]

$$L(\theta) = \sum_{i=1}^{n} \log\left[c_\theta\left(\frac{R_{U_i}}{n+1}, \frac{R_{V_i}}{n+1}\right)\right] \tag{18}$$

where R_{U_i} and R_{V_i} are ranks of the correlated random vector (U, V). R_{U_i} is the rank of U_i among U_1, U_2,… U_n and R_{V_i} is the rank of V_i among V_1, V_2,… V_n. The ranking process is performed by listing the monotone increasing U and V. For instance, when $n = 2$, there are two pairs of (U, V), i.e., (U_1, V_1) and (U_2, V_2). If the ranking of U_i and V_i gives $U_2 < U_1$, $V_1 < V_2$, then (R_{U_1}, R_{V_1}) = (2, 1) and (R_{U_2}, R_{V_2}) = (1, 2).

In this study, the dependence among multiple deterioration processes is simulated based on the model described in Wang *et al*. [15]. The deterioration modeling aims to provide inputs for the life-cycle maintenance cost assessment. However, the impact of shock events on the parameters of gamma process is not considered. Future studies are encouraged to incorporate the interactions. Herein, a series of demands $\{D_i\}$ are associated with external shock deterioration. Meanwhile, the shock-induced deterioration interacts with the gradual deterioration. Herein, the interaction among different deterioration processes is modeled by a multivariate probability distribution function. The interaction in terms of shock deterioration focuses on the external shock deterioration, as the fatal shock deterioration always results in immediate failure of the system. Let $A_{t_i} = R^{Gra}(t_{i-1}, t_i] / R_0$, $B_{t_i} = \Delta R_{t_i}^{ExS} / R_0$, and $\Psi_{t_i} = D_i / R_0$ represent the normalized gradual deterioration, external shock deterioration, and demand at time t_i, respectively. The joint CDF of the three correlated random variables $(A_{t_i}, B_{t_i}, \Psi_{t_i})$ can be denoted as $F_{A,B,\Psi}(a,b,d)$. The CDF of the random vector $(A_{t_i}, B_{t_i}, \Psi_{t_i})$ can either be derived by empirical models or the advanced copula approach.

By using the copula model, the joint CDF of the random vector can be expressed as

$$F_{A,B,\Psi}(a,b,d) = C(F_A(a), F_B(b), F_\Psi(d)) \quad (19)$$

where C is the copula function; $F_A(a)$, $F_B(b)$, and $F_\Psi(d)$ are the CDFs of the normalized gradual deterioration, external shock deterioration, and demand. The detailed explanation of copula theory and copula functions is provided in the following section. The detailed modeling of $F_{A,B,\Psi}(a,b,d)$ is provided in the illustrative example.

Dependence and interaction between deterioration processes can be complicated. In addition to the introduced approach, other methods can also be used to capture the interaction between deterioration processes. Future studies are needed to explore a more detailed dependence model of deterioration processes by considering the stochastic frequency and magnitude.

3.2.4 Assumptions

Deterioration processes and the dependence effect on the system can be complex. There can be different methods to model the stochastic deterioration and interaction between deterioration processes. In this section, the proposed reliability analysis is based on several assumptions:

1. The total deterioration of the system caused by gradual and shock deterioration is the sum of individual deterioration processes.
2. The occurrence of multiple deterioration processes relies on the occurrence of shock events.
3. The external shock process is the result of external load effects, while the load imposes the demand on the system. Thus, the occurrence of external shock and demand is simultaneous.
4. The gradual deterioration, external shock deterioration, and demand are dependent, and their dependence can be modeled by the dependence model introduced in Eq. (19).

3.2.5 Time-dependent reliability calculation

In this study, the time-dependent reliability is calculated by a double-loop Monte Carlo simulation method. To begin with, the total time T_{sum}, time interval d_{T}, and distribution parameters of all related random values and processes are determined. Then, at each time step $T_i = k \cdot d_{\mathrm{T}}$ ($k = 1, 2, \ldots$), the number of shocks n_T is sampled according to its Poisson distribution parameters (Eq.(3)). If the value of n_T is equal to zero, only the progressive deterioration is considered, and its deterioration value A_{t_i} is sampled. If the value of n_T exceeds zero, both the progressive deterioration and sudden damage are concerned, and their values of $A_{t_i}, i=1,...,n_T+1$ and $B_{t_i}, i=1,...,n_T$ are sampled. Next, the conditional failure probability of T_i, $P_f(T_i|A_{ti},B_{ti})$, could be expressed as

$$P_f\left(T_i \mid A_{t_i}, B_{t_i}\right) = \Phi\left[\frac{1}{a_{33}}\left(\Phi^{-1}\left(F_{\Psi_{t_i}}\left(1-\sum_{i=1}^{n_T} A_{t_i} - \sum_{i=1}^{n_T-1} B_{t_i}\right)\right) - a_{31}v_1 - a_{32}v_2\right)\right] \quad (20)$$

where Φ denotes the cumulative distribution function of standard normal distribution; $F_{\Psi_{t_i}}(\cdot)$ is the marginal CDF of the demand Ψ_{t_i} at time t_i; $\mathbf{A}=\{a_{ij}, i,j=1,2,3\}$ is the lower triangular matrix satisfying the coefficient matrix of $\mathbf{U}=(U_1, U_2, U_3)=\mathbf{A}\,\mathbf{A}^T$, which could be computed by the correlation matrix of $F_{A,B,\Psi}(a,b,d)$ and Nataf transformation [15].

Meanwhile, n_T, A_{ti}, and B_{ti} would be sampled N_{mcs} times to capture N_{mcs} of $P_f(T_i|A_{ti}, B_{ti})$ by using Eq. (20). Then, the failure probability of T_i, i.e., $P_f(T_i)$, could be approximately evaluated through the mean value of all $P_f(T_i|A_{ti}, B_{ti})$. Such a computational process could be repeated until T_i reaches T_{sum}. In order to obtain high accuracy results of failure probability $P_f(T_i)$, the sampling number N_{mcs} needs to be quite large. Such an algorithm is flexible to consider different deterioration scenarios and maintenance policies.

3.3 Maintenance policy and cost

The performance (e.g., capacity, reliability) of the system is reduced due to multiple deterioration processes, thus requiring maintenance actions to minimize potential failure risks and damage. The quantification of maintenance actions and costs relies on the performance of system. In this study, a reliability-based maintenance policy is proposed, consisting of preventive and essential maintenance interventions. The system reliability provided by the previous section is the main input for this section to determine the maintenance interval and cost. For instance, the probability of failure is taken as a performance indicator to implement the maintenance policy. Herein, preventive maintenance (PM) gives minimal repairs, while essential maintenance (EM) provides major repairs or replacement to enhance the system reliability to the initial level. Preventive maintenance is conducted when the probability of the system failure exceeds P_{PM}. The resulting cost of preventive action is C_{PM}. After a preventive maintenance action, the rate of gradual deterioration is reduced. Essential maintenance is performed when the probability of failure exceeds a threshold P_{EM} or the failure occurs. The cost of essential action is denoted as C_{EM}. Following the essential maintenance, the structural resistance is restored to the initial level R_0. In other words, the system is resumed and a renewal process is formed [17, 19].

To determine the maintenance interval and the maintenance cost associated with the renewal process, the system reliability should be determined. Given the maintenance policy, the time-dependent limit state function becomes

$$LS_n = R_0 - r_{pre} \cdot R^{Gra}(0, t_n] - \sum_{i=1}^{n-1} \Delta R_{t_i}^{ExS} - D_{t_n} \tag{21}$$

in which r_{pre} ($r_{pre} < 1$) is the changing rate in terms of the gradual deterioration after a preventive maintenance action. Regarding the time-dependent reliability analysis described by Eq. (21), the maintenance effects on deterioration values could be easily considered by modifying the A_{t_i} and B_{t_i} in Eq. (20). Figure 4 provides an illustrative diagram to describe the effect of preventive and essential maintenance actions on the probability of failure associated with the system.

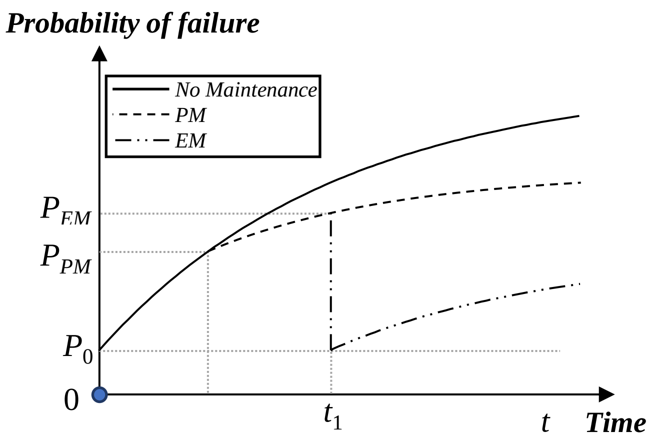
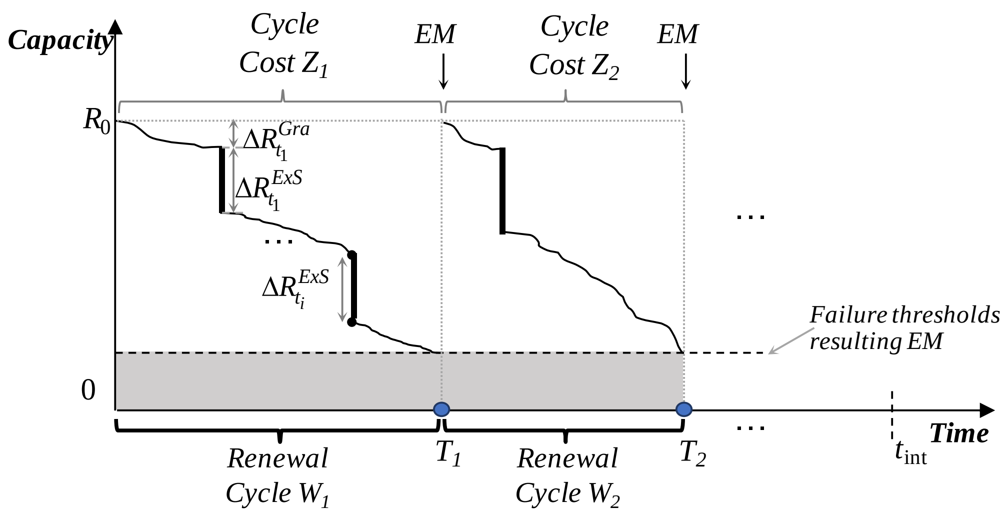
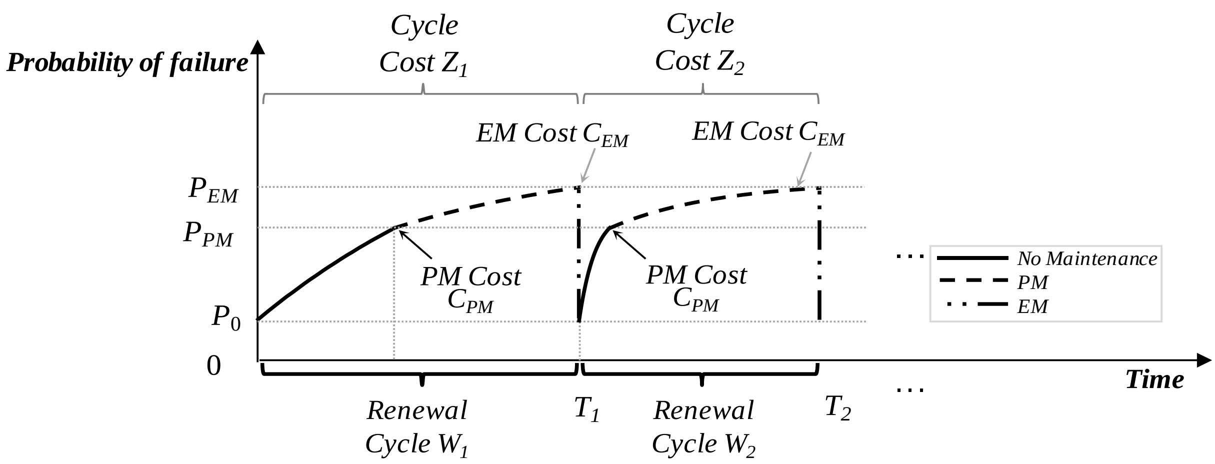


Figure 4. An illustrative diagram of reliability-based preventive maintenance (PM) and essential maintenance (EM) actions.

Based on the time-dependent reliability analysis, the maintenance interval and the cost can be identified accordingly. As mentioned above, the system is renewed after an essential maintenance and the process of restoration can be modeled as a renewal process. Based on the stochastic renewal process, the occurrence interval of essential maintenance actions can be defined as a renewal cycle W. Within the renewal cycle W, the total maintenance cost can be denoted as the cycle cost Z. Then, the relationship between the system capacity and the renewal cycle W under multiple deterioration processes can be described as Figure 5(a). Both W and Z are random variables, and $(0, t_{\mathrm{int}}]$ is the service period of the system. To determine W and Z, Figure 5(a) should be further shifted to Figure 5(b) based on reliability analysis using Eq. (21). Figure 5(b) shows an illustrative sketch to demonstrate the impact of maintenance actions on the system failure probability, renewal cycle W, and the associated maintenance cost Z. It can be noted that the total maintenance cost Z (i.e., cycle cost) within a cycle W consists of preventive maintenance cost C_{PM} and essential maintenance cost C_{EM}, i.e., $Z = C_{PM} + C_{EM}$. Herein, it is assumed that C_{PM} and C_{EM} are deterministic. Subsequently, the mean values of renewal cycle W and the maintenance cost Z can be obtained based on the reliability analysis and Monte Carlo simulation. The obtained mean values of renewal cycle W and maintenance cost Z are the key inputs for the life-cycle maintenance cost analysis.



(a)



(b)

Figure 5. Illustrative diagrams of stochastic renewal cycle W and maintenance cost Z associated with (a) the system capacity under deterioration processes and (b) the system reliability and maintenance policy.

4. Life-cycle maintenance cost

4.1 A multivariate copula-based renewal model

After determining the renewal cycle and maintenance cost, the life-cycle maintenance cost within the period $(0, t_{\text{int}}]$ can be determined based on the stochastic renewal process. Meanwhile, the dependence between renewal cycle (i.e., maintenance interval) and maintenance cost (i.e., cycle cost) cannot be neglected during the life-cycle analysis. The maintenance cost Z naturally depends on the cycle length W as different maintenance actions are involved. Herein, the key challenge is to incorporate the dependence between random variables Z and W into the renewal process during the life-cycle maintenance cost assessment. A novel copula-based renewal model is proposed to address this problem. To the best of the authors' knowledge, the dependent maintenance interval and cost have not been considered in the life-cycle analysis. For the proposed copula model, two types of methods can be considered to model dependent maintenance cost Z and renewal cycle W in the life-cycle analysis, as described in Figure 6. One can be based on historical data to achieve the copula function, and the other one is directly based on correlation and copula function.

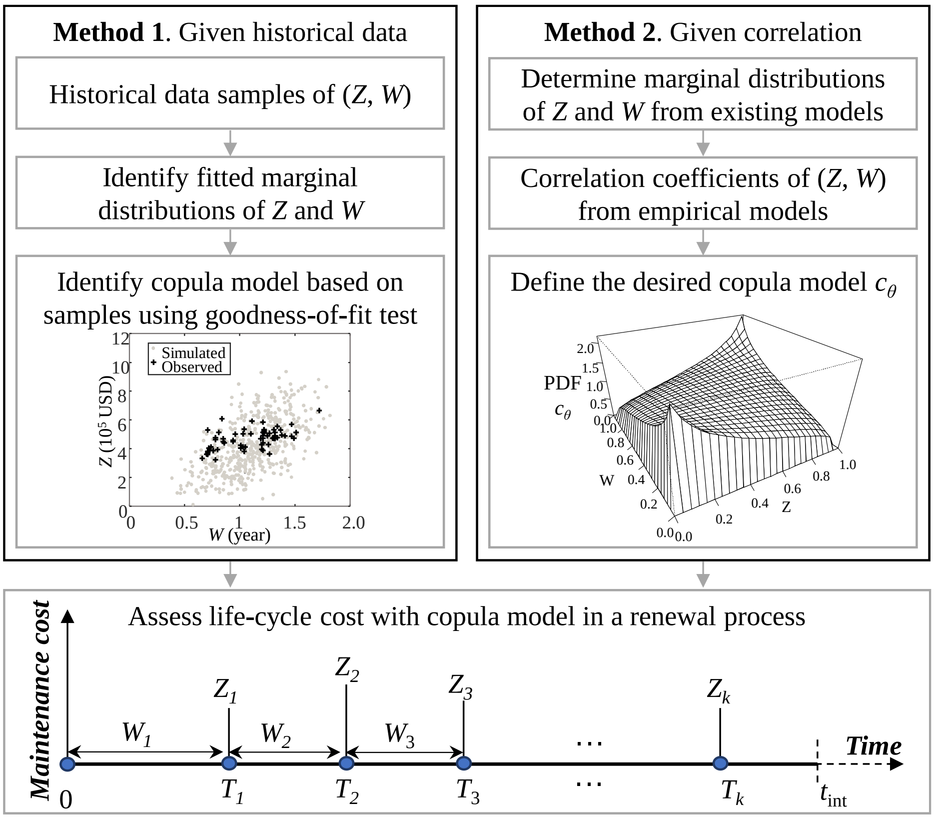


Figure 6. Assessment of life-cycle maintenance cost using the proposed copula-based multivariate renewal model

Based on the renewal process, during the investigated service period (0, t_{int}], there can be a series of renewal cycles {W_1, W_2, …, W_k} due to the essential maintenance. The maintenance costs associated with the renewal cycles can be {Z_1, Z_2, …, Z_k}. The chronological time in terms of the kth failure can be written as T_k, with $T_k = W_1 + W_2 + \ldots + W_k$. W_k and Z_k (k = 1, 2, …) are non-negative random variables. The life-cycle maintenance cost can be defined as $LCC(t_{\text{int}})$. The renewal cycle W_k and maintenance cost Z_k are dependent, while the joint probability distributions of (Z_i, W_i) are independent of (Z_k, W_k,) for any $i \neq k$. Given these parameters, the life-cycle maintenance cost $LCC(t_{\text{int}})$ is the accumulative cost of all the renewal cycles and gives

$$LCC(t_{\text{int}}) = \sum_{k=1}^{N(t_{\text{int}})} Z_k e^{-rT_k} \tag{22}$$

in which $N(t_{\text{int}})$ is the total number of essential maintenance actions and a discount rate r is used to discount the future expense to the present. Also, as mentioned in the previous section, the mean values of Z and W can be determined based on system reliability and maintenance strategy by Monte Carlo simulation. The maintenance cost Z consists of the cost of preventive maintenance C_{PM} and the cost of essential maintenance C_{EM}. The deterministic value of cost Z can be taken as the mean of Z, i.e., $E[Z] = C_{PM} + C_{EM}$, as indicated in Figure 5.

To model the dependence between the renewal cycle and maintenance cost, the bivariate copula is employed. The dependence structure between Z and W can be described by a joint CDF $F_{Z,W}(z, t)$. Based on the copula theory, the joint CDF of the bivariate random vector (Z_k, W_k) can be written as

$$F_{Z,W}(z,t) = C(F_Z(z), F_W(t)) \tag{23}$$

in which $F_Z(z)$ and $F_W(t)$ are CDFs of maintenance cost and renewal cycle, respectively. C is the CDF of a copula function. The PDF of the random vector $f_{Z,W}(z, t)$ is given as

$$f_{Z,W}(z,t) = c(F_Z(z), F_W(t))\, f_Z(z)\, f_W(t) \tag{24}$$

where c describes the PDF of a copula; f_Z and f_W are the univariate PDFs of maintenance cost and renewal cycle, respectively.

To determine the copula function, there are generally two methods in terms of the cases with and without data, as shown in Figure 6. When there are detailed historical records, the selection of the copula model can be data-based [44, 45]. The data-based method requires two main parts: quantification of marginal distributions (i.e., $F_Z(z)$ and $F_W(t)$) and selection of the most fitted copula by using the goodness-of-fit test. While there are limited data available, the dependence structure between variables is commonly determined according to correlation coefficients [24, 46]. Detailed descriptions of the two methods are shown in the section of illustrative examples. As practical data can be incorporated, the proposed copula approach can be significant for data-based decision-making during the life-cycle management of civil infrastructure.

After selecting the copula model and estimation of the dependence parameter, the life-cycle maintenance cost incorporating dependent maintenance cost and renewal cycle can be assessed. Due to complicated expressions of copulas, statistical modeling generally relies on numerical simulations. Simulations are flexible with various copulas but can be time-consuming and expensive. The algorithm to assess the life-cycle maintenance cost using a Monte Carlo simulation is summarized as follows:

Simulation algorithm

(1) Inputs: t_{int}, r, marginal PDFs or CDFs of Z and W (e.g., $F_Z(z)$, $f_Z(z)$, $F_W(t)$, $f_W(t)$);

(2) Establish dependence structure of the copula function and generate dependent random vectors (Z_k, W_k);

(3) Simulate a stochastic renewal process $\{N(t_{\text{int}})\}$ by using $\{W_1, W_2, \ldots, W_k\}$ generated from Step (2);

(4) Compute $\{T_1, T_2, \ldots, T_k\}$ of the process based on Step (3);

(5) Compute $LCC(t_{\text{int}})$ based on Eq. (22) by using $\{T_1, T_2, \ldots, T_k\}$ of Step (4), the associated $\{Z_1, Z_2, \ldots, Z_k\}$ generated from Step (2), and the number of events $N(t_{\text{int}})$ from Step (3);

(6) Repeat Step (2) to (5) for N_{MC} times based on Monte Carlo simulation; and

(7) Outputs: the mean, standard deviation, skewness, and kurtosis of $LCC(t_{\text{int}})$ based on N_{MC} samples.

4.2 Analytical case: life-cycle analysis with FGM copula

In addition to numerical modeling, an analytical case is developed in this section. The closed-form expressions of statistical moments of the life-cycle maintenance cost considering an FGM copula are derived. Derivations are based on the renewal theory and Laplace Transform. Due to its analytical characteristics, the FGM copula was employed by Eryilmaz [47] to model dependent degradation rates for the reliability analysis of systems. The FGM copula is the first-order Taylor approximation of the Frank copula and belongs to neither the elliptical family nor the Archimedean family [47].

The FGM copula demonstrates a weak correlation, including both positive and negative. The PDF of the FGM copula c_θ^{FGM} is given as

$$c_\theta^{FGM}(u,v) = 1 + \theta(1-2u)(1-2v) \tag{25}$$

where the dependence parameter θ is between [-1, 1] and $(u, v) \in [0, 1] \times [0, 1]$.

The joint probability of (Z, W) can be expressed as follows using the copula

$$\begin{aligned} f_{Z,W}(z,t) &= c_{\theta}^{FGM}(F_Z(z), F_W(t))\, f_Z(z)\, f_W(t) \\ &= [1+\theta(1-2F_Z(z))(1-2F_W(t))]\, f_Z(z)\, f_W(t) \end{aligned} \tag{26}$$

4.2.1 Expectation and variance of life-cycle maintenance cost

The expected life-cycle maintenance cost under a renewal process can be formulated by conditioning on the first arrival time y [48]

$$\begin{aligned} \mu_{LCC}(t_{\text{int}}) &= E[LCC(t_{\text{int}})] = E\left[E[e^{-ry}Z_1 + e^{-ry}LCC(t_{\text{int}}-y) \middle| W_1 = y] \right] \\ &= \int_0^{t_{\text{int}}} e^{-ry} E\left[Z \middle| W = y \right] f_W(y)dy + \int_0^{t_{\text{int}}} e^{-ry} E\left[LCC(t_{\text{int}}-y) \right] f_W(y)dy \end{aligned} \tag{27}$$

in which the first arrival time is equal to the first inter-arrival time $T_1 = y = W_1$. The conditional expectation of maintenance cost $E[Z|W = y]$ can be expressed by the conditional probability

$$E\left[Z \middle| W = y \right] = \int_0^{\infty} z\, f_{Z|W=y}(z)dz \tag{28}$$

where the conditional density function of maintenance cost $f_{Z|W=y}$ is associated with the bivariate joint probability $f_{Z,W}(z, t)$. Substituting the FGM copula according to Eq. (25), the conditional density function gives

$$f_{Z|W=y}(z) = \frac{f_{Z,W}(z,t)}{f_W(t)} = [1+\theta(1-2F_Z(z))(1-2F_W(t))]\, f_Z(z) \tag{29}$$

Substituting Eq. (29) into Eq. (28), the conditional expectation of maintenance cost gives

$$\begin{aligned} E\left[Z \middle| W = y \right] &= \int_0^{\infty} z[1+\theta(1-2F_Z(z))(1-2F_W(y))]\, f_Z(z)dx \\ &= E[Z](1-\theta(1-2F_W(t))) + \theta(1-2F_W(y))E[\Lambda] \end{aligned} \tag{30}$$

in which $E[\Lambda]$ is defined to combine the identical items

$$E[\Lambda] = \int_0^{\infty} z(2-2F_Z(z))\, f_Z(z)dz = \int_0^{\infty} (1-F_Z(z))^2\, dz \tag{31}$$

A Poisson process is the most common renewal process. It has exponentially distributed inter-arrival times. It gives that the inter-arrival time follows $W \sim \text{EXP}(\lambda)$ with an occurrence rate λ. Hence, the PDF of the inter-arrival time $f_W(t)$ gives

$$f_W(t) = \lambda \exp(-\lambda t) \tag{32}$$

Herein, let $\omega(t;\lambda)$ represent the PDF $f_W(t)$ of W [49, 50]. This parameter can help simplify the derivation process in the Laplace transform, especially in higher-order moments. Consequently, the expected life-cycle maintenance cost can be rearranged as

$$\begin{aligned}\mu_{LCC}(t_{\text{int}}) = E[Z]\int_0^{t_{\text{int}}} \frac{\lambda}{\lambda+r}\omega(y;\lambda+r)dy + \theta\left(E[\Lambda]-E[Z]\right)\int_0^{t_{\text{int}}} \frac{2\lambda}{2\lambda+r}\omega(y;2\lambda+r)dy \\ -\theta\left(E[\Lambda]-E[Z]\right)\int_0^{t_{\text{int}}} \frac{\lambda}{\lambda+r}\omega(y;\lambda+r)dy + \int_0^{t_{\text{int}}} \frac{\lambda}{\lambda+r}\omega(y;\lambda+r)\mu_{LCC}(t_{\text{int}}-y)dy\end{aligned} \tag{33}$$

Taking the Laplace transform of Eq. (33) on both sides, the Laplace transform of the expected life-cycle maintenance cost $\tilde{\mu}_{LCC}(\tau)$ can be written as

$$\begin{aligned}\tilde{\mu}_{LCC}(\tau) = E[Z]\frac{\lambda}{\lambda+r}\frac{\tilde{\omega}(\tau;\lambda+r)}{\tau} + \theta\left(E[\Lambda]-E[Z]\right)\frac{2\lambda}{2\lambda+r}\frac{\tilde{\omega}(\tau;2\lambda+r)}{\tau} \\ -\theta\left(E[\Lambda]-E[Z]\right)\frac{\lambda}{\lambda+r}\frac{\tilde{\omega}(\tau;\lambda+r)}{\tau} + \frac{\lambda}{\lambda+r}\tilde{\omega}(\tau;\lambda+r)\tilde{\mu}_{LCC}(\tau)\end{aligned} \tag{34}$$

where the Laplace transform of the PDF of inter-arrival time $\tilde{\omega}_{LCC}(\tau;\lambda)$ can be computed as

$$\tilde{\omega}(\tau\,;\lambda) = \frac{\lambda}{\lambda+\tau} \tag{35}$$

Substituting Eq. (35) into Eq. (34), the Laplace transform of expected life-cycle maintenance cost can be rearranged as

$$\tilde{\mu}_{LCC}(\tau) = \frac{E[Z]\lambda}{\tau(\tau+r)} + \frac{\theta\lambda\left(E[\Lambda]-E[Z]\right)}{\tau(2\lambda+r+\tau)} \tag{36}$$

By taking inverse Laplace transform of Eq. (36) on both sides, the expected life-cycle maintenance cost under dependency is obtained

$$\mu_{LCC}(t_{\text{int}}) = \frac{E[Z]\lambda}{r}(1-e^{-rt_{\text{int}}}) + \frac{\theta\lambda\left(E[\Lambda]-E[Z]\right)}{2\lambda+r}(1-e^{-(2\lambda+r)t_{\text{int}}}) \tag{37}$$

Following the similar procedure of the first moment, the second moment of life-cycle maintenance cost can be assessed by conditioning on the first arrival time y

$$E[LCC^2(t_{\text{int}})] = E\left[E[(e^{-ry}Z_1 + e^{-ry}LCC(t_{\text{int}} - y))^2 \middle| W_1 = y] \right]$$

$$= \int_0^{t_{\text{int}}} e^{-2ry} E\left[Z^2 \middle| W = y \right] f_W(y)dy + \int_0^{t_{\text{int}}} e^{-2ry} E[LCC^2(t_{\text{int}} - y)] f_W(y)dy \quad (38)$$

$$+2\int_0^{t_{\text{int}}} e^{-2ry} E\left[Z \middle| W = y \right] \mu_{LCC}(t_{\text{int}} - y) f_W(y)dy$$

Following similar procedures in terms of the Laplace transform approach, the second moment of the life-cycle maintenance cost can be derived accordingly. The key derivation process and results are shown in Appendix A. Consequently, the variance can be evaluated from the first two moments as shown in Eqs. (A4) and (A5).

When the dependence parameter is zero, the maintenance cost and renewal cycle become independent. The associated expectation and variance of life-cycle cost give identical outcomes as described in previous studies [26, 20], as shown in Eqs. (39) and (40)

$$\mu_{LCC}(t_{\text{int}}) = \frac{E[Z]\lambda}{r}(1 - e^{-rt_{\text{int}}}) \quad (39)$$

$$\sigma^2_{LCC}(t_{\text{int}}) = \frac{\lambda E[Z^2]}{2r}(1 - e^{-2rt_{\text{int}}}) \quad (40)$$

4.2.2 Higher-order moments of life-cycle maintenance cost

The *m*th order moment can also be evaluated using the Laplace transform approach. The *m*th order moment of life-cycle maintenance cost can be derived using the univariate distribution of inter-arrival time

$$E[LCC^m(t_{\text{int}})] = \int_0^{t_{\text{int}}} e^{-mry} E\left[Z^m \middle| W = y \right] f_W(y)dy$$

$$+\int_0^{t_{\text{int}}} e^{-mry} E\left[LCC^m(t_{\text{int}} - y) \right] f_W(y)dy \quad (41)$$

$$+\sum_{i=1}^{m-1} \binom{m}{i} \int_0^{t_{\text{int}}} e^{-mry} E\left[Z^i \middle| W = y \right] E\left[LCC^{m-i}(t_{\text{int}} - y) \right] f_W(y)dy$$

where $m \geq 1$ and $1 \leq i < m$.

Similar to the first two moments, the *m*th order conditional expectation of maintenance cost can be expressed as

$$E\left[Z^m \middle| W=y\right]=\int_0^\infty z^m f_{Z|W=y}(z)dz=\int_0^\infty z^m c_\theta^{FGM}(F_Z(z),F_W(t))\,f_Z(z)dz \tag{42}$$

Substituting Eq. (42) into Eq. (41), the *m*th order moment of life-cycle maintenance cost gives

$$\begin{aligned}E[LCC^m(t_{\text{int}})]&=\int_0^{t_{\text{int}}}\int_0^\infty e^{-mry}z^m f_{Z,W}(z,y)dzdy+\int_0^{t_{\text{int}}} e^{-mry}E[LCC^m(t_{\text{int}}-y)]\,f_W(y)dy\\&+\sum_{i=1}^{m-1}\binom{m}{i}\int_0^{t_{\text{int}}}\int_0^\infty e^{-mry}z^i f_{Z,W}(z,y)E[LCC^{m-i}(t_{\text{int}}-y)]dzdy\end{aligned} \tag{43}$$

Considering the exponential distribution associated with the inter-arrival time $f_W(t)$, the *m*th order moment becomes

$$\begin{aligned}&E[LCC^m(t_{\text{int}})]\\&=\lambda\int_0^{t_{\text{int}}}\int_0^\infty e^{-(\lambda+mr)y}z^m f_Z(z)c_\theta(F_Z(z),F_W(y))dzdy+\lambda\int_0^{t_{\text{int}}} e^{-(\lambda+mr)y}E[LCC^m(t_{\text{int}}-y)]dy\\&+\lambda\sum_{i=1}^{m-1}\binom{m}{i}\int_0^{t_{\text{int}}} e^{-(\lambda+mr)y}z^{-(m-i)}c_\theta(F_Z(z),F_W(y))\,f_Z(z)E[LCC^{m-i}(t_{\text{int}}-y)]dzdy\end{aligned} \tag{44}$$

Consequently, statistical moments can be derived analytically. The analytical case can be more effective than complicated numerical simulation. Based on the recursive moments (i.e., Eq. (44)) using an FGM copula, decision-makers can estimate the life-cycle cost under dependency effectively. Given more data, a more detailed dependence model can be further studied by using the proposed copula model. Future studies can investigate the multivariate distribution for the preventive maintenance cost, essential maintenance cost, and the renewal cycle by using the copula model.

5. Illustrative example

There are two illustrative examples provided to demonstrate the proposed copula-based life-cycle analysis framework. The first example focuses on the impact of different copula models and the effect of multiple deterioration processes on the life-cycle maintenance cost. The second example aims to show a decision-making process based on practical data using the proposed copula model. The significance of considering higher-order moments of the life-cycle maintenance cost is highlighted.

5.1 Example 1: Life-cycle cost analysis of aging civil infrastructure

This example aims to show the assessment process of the life-cycle maintenance cost of a bridge considering reliability-based maintenance policy. The impact of different dependence structures (i.e., different copulas) on the life-cycle maintenance cost is investigated. The effects of fatal shocks and dependent deterioration processes on the maintenance interval, maintenance cost, and the associated life-cycle maintenance cost are explored.

The investigated bridge is subjected to dependent deterioration processes, such as gradual deterioration, external shock and fatal shock. For the gradual deterioration, a gamma process is employed. The associated shape parameter αs and scale parameter β of the gamma process are 0.04 and 0.16, respectively. The detailed computation of the deterioration parameters of aging bridges can be based on observation data [51]. An alternative way to define the inputs for gamma process can rely on the deterioration amount. For instance, the initial resistance of the investigated system is R_0. At the end of a time period of 40 years, the expected cumulative gradual deterioration is $0.2R_0$ with a coefficient of variation of 0.4, and the expectation of the cumulative gradual deterioration changes linearly with time [51]. For the external shock process, random shocks are caused by hazards and modeled by a Poisson process, with an annual occurrence rate of $\lambda_{ExS} = 0.3$. The resulting deterioration in terms of the external shock process is lognormally distributed. It has a mean of $0.03L$ and a coefficient of variation of 0.4. Meanwhile, hazards impose demands acting on the bridge, thus following the same Poisson process. It is assumed that demands follow a Gumbel distribution with a mean of $0.3L$ and a coefficient of variation of 0.3. Herein, the L can be associated with the external load on bridges such as the extreme wind load, wave and surge load caused by tropical cyclones, load caused by vehicles hitting the structures, etc. The demands and the shock deterioration are physically related. In civil engineering practice, they are usually associated with the loading effect. Herein, it assumes $L = R_0/3$. For the fatal shock process, the occurrence is also modeled by a Poisson process with an annual occurrence rate $\lambda_{FaS} = 1 \times 10^{-5}$. A low-frequency fatal event leads to the immediate failure of a system and results in essential maintenance. The maximum deterioration level $G_{\max}$ is 0.5.

Subsequently, the system reliability analysis can be performed. At this stage, the dependence structure among multiple deterioration processes is incorporated using the copula function, as described in Eq. (19). The multivariate dependence of the normalized gradual deterioration, external shock deterioration, and demand $(A_{t_i}, B_{t_i}, \Psi_{t_i})$, as shown in Eq. (45), is modeled by a Gaussian copula for illustrative purposes. The Gaussian copula has been widely applied in previous reliability studies due to its advantages in reducing the computational cost based on Nataf transformation [15, 52, 53]. Other copula models can be applied when there is more information provided. Based on the Gaussian copula, the joint CDF of the correlated random vector $(A_{t_i}, B_{t_i}, \Psi_{t_i})$ can be written as

$$\begin{aligned} F_{A,B,\Psi}(a,b,d) &= C_{Gau}(F_A(a), F_B(b), F_\Psi(d)) \\ &= \Phi_{\varsigma}(\Phi^{-1}(F_A(a)), \Phi^{-1}(F_B(b)), \Phi^{-1}(F_\Psi(d))) \end{aligned} \tag{45}$$

in which Φ(.) is the CDF of a multivariate normal distribution; ζ is the correlation matrix; and $\Phi^{-1}(.)$ is the inverse CDF of the standard normal distribution. The correlation between random vectors is positive [54], as a stronger external load results in a larger decrease in resistance due to damage (e.g., crack). Meanwhile, changes in resistance further accelerate the gradual deterioration process (e.g., corrosion in terms of reinforcement). Herein, the associations between every two random variables are described by Pearson's correlation coefficient with γ_d = 0.3. The assigned values are presented here for illustrative purposes and can be upgraded with specific problems.

In addition to the deterioration processes and system reliability analysis, the assessment of maintenance interval W and maintenance cost Z for the life-cycle analysis requires parameters associated with maintenance policy. Herein, maintenance actions are performed when the probability of the system failure hits the associated thresholds, i.e., $P_{PM} = 1\times10^{-5}$ for preventive maintenance and $P_{EM} = 1\times10^{-3}$ for essential maintenance, respectively. The changing rate r_{pre} on gradual deterioration after the preventive maintenance is 0.5, as described in Eq. (21). Additionally, as mentioned previously, it is assumed that the bridge would be restored to the initial status after essential maintenance. In this example, the costs of preventive maintenance C_{PM} and essential maintenance C_{EM} are given as 50,000 USD and 487,100 USD, respectively [55, 56]. The bridge has a service life of 100 years, i.e., $t_{\text{int}} = 100$. Given these inputs, the expected renewal cycle can be determined based on system reliability analysis using Monte Carlo simulation with 10^6 replications, as shown in Figure 7. The figure describes the computed probability of failure of the bridge subjected to multiple dependent deterioration. It can be identified that the bridge experiences nearly four cycles of essential maintenance and resulting in a renewal cycle (i.e., maintenance interval) of $E[W]$ = 25.6 years. The associated maintenance cost within a renewal cycle can also be obtained accordingly, i.e., $E[Z]$ = 537,100

USD. The reliability analysis confirms that $E[Z]$ consists of one preventive intervention and one essential maintenance, i.e., $E[Z] = C_{PM} + C_{EM}$.

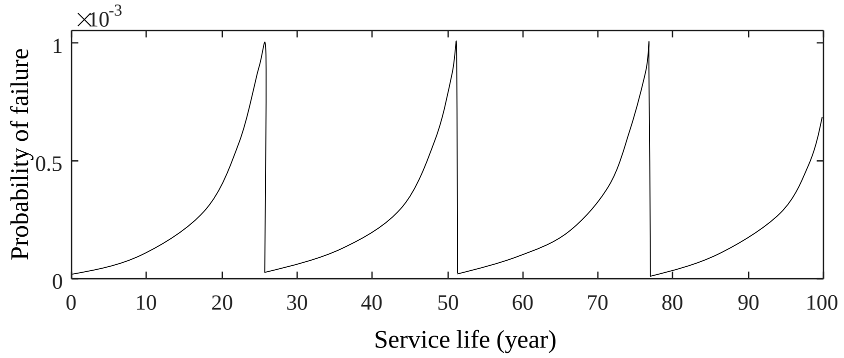
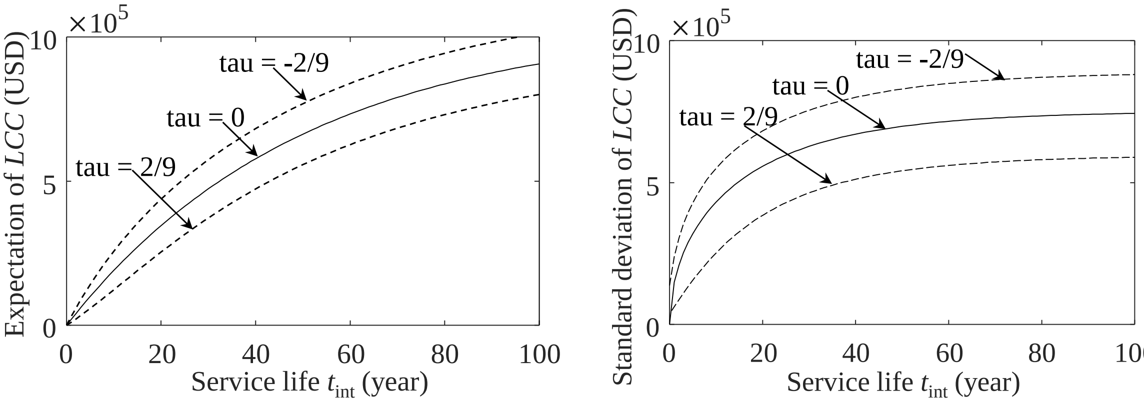


Figure 7. The probability of bridge failing subjected to multiple dependent deterioration processes considering preventive and essential maintenance actions.

Given the expected maintenance interval $E[W]$ and the maintenance cost $E[Z]$, the life-cycle maintenance cost can be evaluated. W and Z are random variables and they are assumed to follow exponential distributions herein. The monetary discount rate is 2%. In this example, the mean $E[LCC]$ and standard deviation $Std[LCC]$ of the life-cycle maintenance cost are of interest. The impact of dependent maintenance interval and cost on the $E[LCC]$ and $Std[LCC]$ are explored using the proposed FGM copula. As the FGM copula indicates the weak correlation, the maximum positive correlation refers to Kendall's tau at 2/9. The associated expectation and standard deviation of life-cycle maintenance cost are computed as 800,152 USD and 588,943 USD, respectively. If considering an independent case (i.e., tau of zero), the expectation and standard deviation of the life-cycle cost can be computed as 907,054 USD and 743,714 USD, respectively. The analytical results have been validated by using numerical modeling based on Monte Carlo simulation. Figure 8 demonstrates the difference in the life-cycle maintenance cost by considering dependent maintenance interval and cost associated with an FGM copula. A negative correlation may exist when there is a different maintenance policy.



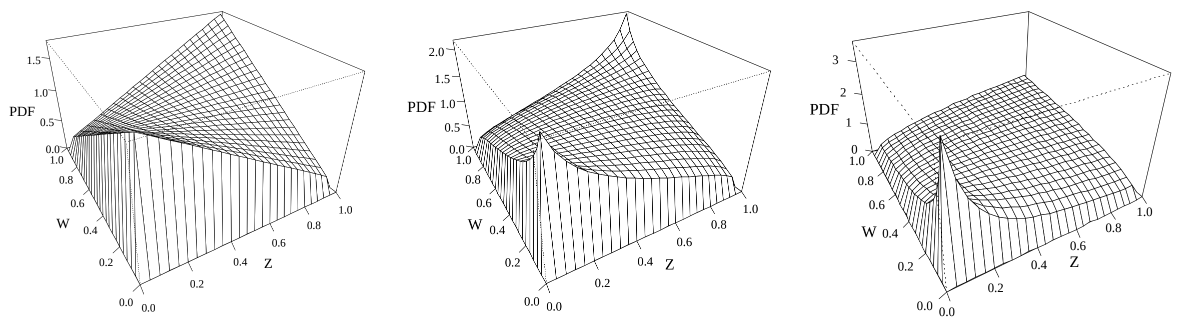
(a) (b)

Figure 8. (a) Expectation and (b) standard deviation of life-cycle maintenance cost with a FGM copula subjected to Kendall's tau at -2/9, 0, and 2/9

5.1.1 Effect of dependent correlated renewal sequences

Apart from the weak correlation associated with the FGM copula, different correlation relationships and copulas may influence the life-cycle maintenance cost. Herein, the dependence structures described by Gaussian and Clayton copulas are also investigated by using numerical modeling. Figure 9 shows the three-dimensional schematic PDFs of FGM, Gaussian, and Clayton copulas with Kendall's tau of 0.2. The PDF of the Gaussian copula can be written as

$$c_{\theta}^{Gau}(u,v)=\frac{1}{\sqrt{(1-\theta^2)}}\exp\left(\frac{2\theta\Phi^{-1}(u)\Phi^{-1}(v)-\theta^2(\Phi^{-1}(u)^2+\Phi^{-1}(v)^2)}{2(1-\theta^2)}\right) \tag{46}$$



(a) FGM copula (b) Gaussian copula (c) Clayton copula

Figure 9. Three-dimensional PDFs of different copulas with Kendall's tau = 0.2.

The expectation and standard deviation of life-cycle maintenance cost with respect to the three copulas are shown in Figure 10. Both weak (i.e., Kendall's tau of 0.2) and strong (i.e., Kendall's tau of 0.9) positive correlations are considered. The FGM copula only illustrates the weak correlation. Compared with the independent case, the positive correlation decreases the expected life-cycle maintenance cost and standard deviation. A stronger correlation can lead to a more significant reduction. The interpretation of such a trend is that increasing the maintenance cost (e.g., with more frequent preventive cost) leads to a longer maintenance interval, as more preventive actions delay the occurrence of essential maintenance. Consequently, the life-cycle maintenance cost is reduced. Such findings can assist researchers and decision-makers in exploring the optimization of maintenance policy by comparing the life-cycle cost. In Figure 10, with the same correlation coefficients (i.e., Kendall's tau), the expectation and standard deviations of the life-cycle maintenance cost are not significantly affected by different copula models. Under the weak correlation, the results associated with the FGM copula show similar estimates compared with the Gaussian and Clayton copulas. Therefore, the proposed analytical approach using an FGM copula provides an effective tool for decision-makers to estimate the life-cycle cost considering weak correlation. The analytical estimation significantly accelerates the computation process, as numerical modeling of copula functions can be complicated and time-consuming.

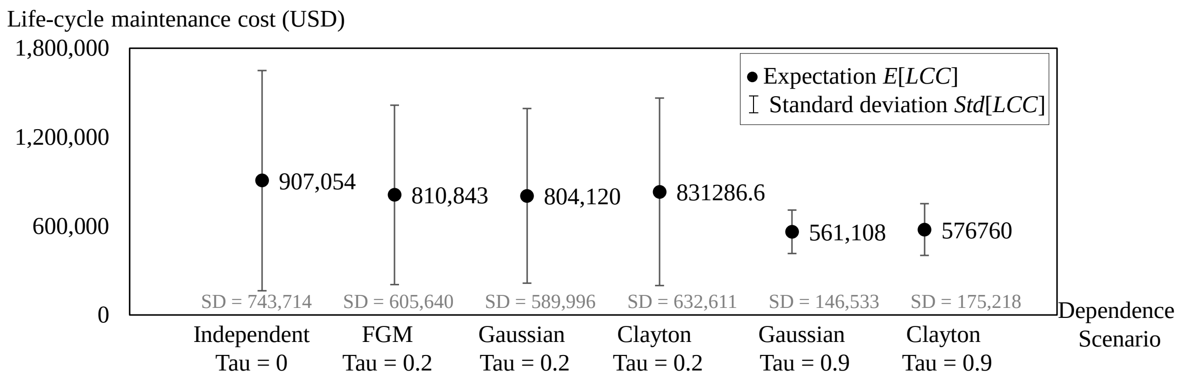


Figure 10. Expected life-cycle cost and standard deviation of different dependence scenarios.

5.1.2 Effect of fatal shock and dependent deterioration processes

In addition to the dependence structure, the interaction between deterioration processes affects the maintenance interval, maintenance cost, and life-cycle cost. For instance, the renewal cycle (i.e., maintenance interval) is particularly affected by deterioration processes. Figure 11 presents the probability of bridge failing subjected to deterioration under three scenarios: dependent deterioration processes (correlation coefficient $\gamma_d = 0.3$) with fatal shocks, dependent deterioration processes ($\gamma_d = 0.3$) without fatal shocks, and independent deterioration process ($\gamma_d = 0$) without fatal shocks. The expected maintenance interval $E[W]$ with respect to the three scenarios are 25.6, 26.8, and 29.2 years, respectively. The associated maintenance cost remains unchanged at 537,100 USD. Considering a FGM copula (Kendall's tau = 2/9), the expected life-cycle maintenance costs associated with the three scenarios are 800,153 USD, 760,552 USD, and 691,311 USD, respectively. It shows that dependent deterioration processes and fatal shocks slightly shorten the maintenance interval and increase the life-cycle maintenance cost.

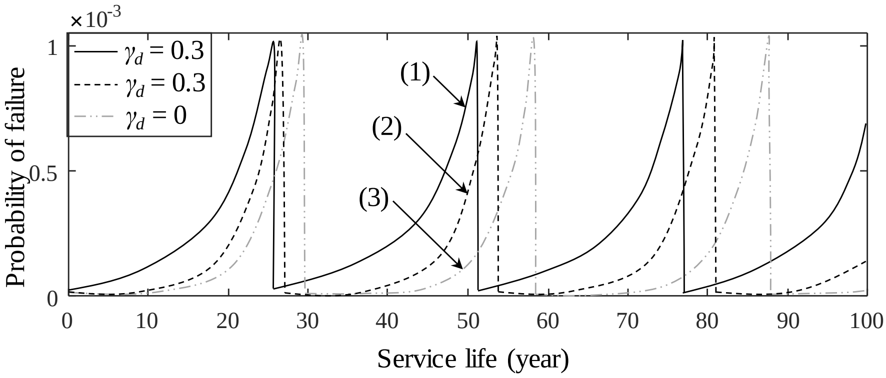


Figure 11. The probability of failure subjected to dependent gradual deterioration, external shock and fatal shock deterioration processes under three scenarios: (1) With dependence γ_d = 0.3 and with fatal shock; (2) With dependence γ_d = 0.3 and without fatal shock; and (3) Without dependence γ_d = 0 and without fatal shock.

The maintenance cost is more likely affected by the maintenance policy, e.g., maintenance threshold. For instance, if maintenance thresholds for preventive and essential action change to 1×10^{-5} and 0.1, respectively, the maintenance interval and cost can be significantly altered. The interval is extended to 56 years, while the maintenance cost remains unchanged. The maintenance cost changes with different preventive and essential maintenance actions. Considering the FGM copula (Kendall's tau = 2/9), the associated expected life-cycle cost becomes 328,906 USD with a standard deviation of 369,844 USD. Therefore, the maintenance interval can be sensitive to the maintenance thresholds. The associated parameters should be carefully examined during the life-cycle analysis.

5.2 Example 2: Maintenance decision-making using higher-order moments of the life-cycle cost

In previous studies, the minimum expected life-cycle cost has been broadly utilized as a standard criterion in the decision-making process. However, decisions exclusively based on the expected cost may not be optimal, as uncertainties associated with the other three statistical moments have been ignored [25]. Herein, an illustrative example is provided to apply statistical moments of the life-cycle maintenance cost in the decision-making process. Based on the proposed copula approach and historical records, a data-based decision-making process is provided to determine an appropriate maintenance policy for a reinforced concrete bridge.

There are two maintenance policies considered for the bridge, as shown in Figure 12. Maintenance Policy 1 is provided based on the historical records of 50 similar reinforced concrete bridges from the U.S. National Bridge Inventory (NBI) database [57]. The maintenance interval of Policy 1 has a mean of 16.14 years and a mean maintenance cost per unit deck area of 4298.02 USD/m^2. As the sizes of bridges vary significantly, the maintenance cost is conditioned on the unit deck area. In contrast, Maintenance Policy 2 is proposed based on [57] with engineering justification, in which the maintenance interval is extended by increasing the maintenance cost. Policy 2 has a mean maintenance interval of 24.10 years and a mean maintenance cost per unit deck area of 6390.55 USD/m^2. Data associated with Maintenance Policy 2 are provided for illustrative purposes. Between the two alternatives, decisions should be made to select an appropriate policy for the bridge by considering statistical moments of the life-cycle maintenance cost.

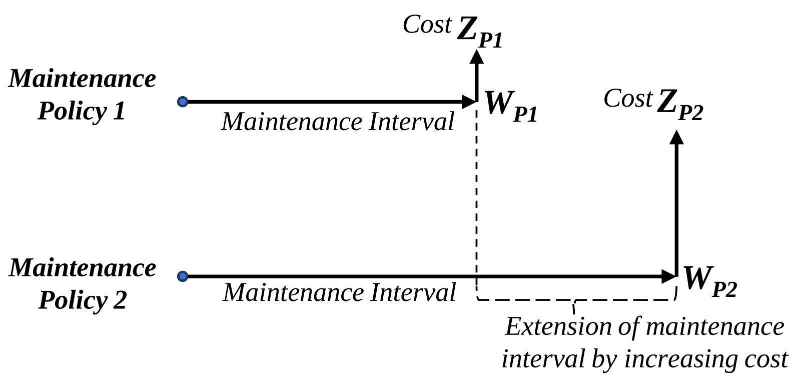


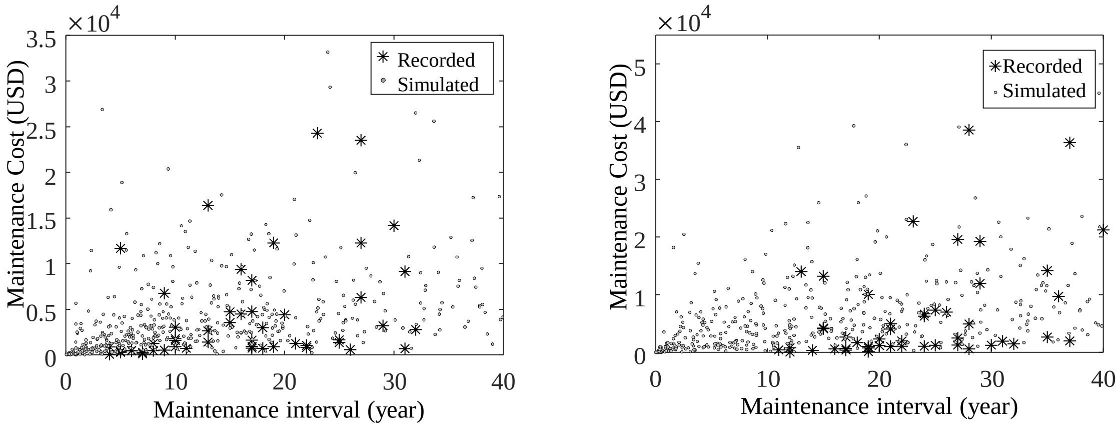
Figure 12. Two maintenance policies with different maintenance interval *W* and maintenance cost *Z*.

For maintenance policy 1, the dependence structure between the maintenance interval *W* and maintenance cost *Z* can be examined using the presented Method 1 as described in Figure 6. Firstly, marginal distributions of *W* and *Z* should be fitted. It is identified that there are many distribution alternatives due to limited data records. Herein, their marginal distributions are fitted into exponential distributions. Subsequently, the copula function for the correlated *W* and *Z* is assessed using the goodness-of-fit test [58, 59]. Based on the Akaike information criterion (AIC) and Bayesian information criterion (BIC), the Clayton copula is selected among candidates (i.e., Gaussian, Student's *t*, Clayton, Gumbel, and Frank copulas) for the two policies. Detailed fitting procedures and the goodness-of-fit test follow the process of copula selection described in Li *et al.* [37]. The PDF of the Clayton copula can be described as

$$c_{\theta}^{Clay}(u,v) = (\theta+1)(uv)^{-(\theta+1)}(u^{-\theta}+v^{-\theta}-1)^{-\frac{2\theta+1}{\theta}} \tag{47}$$

where θ is the dependence parameter.

The recorded and simulated maintenance interval and maintenance cost based on the fitted Clayton copula associated with two policies are shown in Figure 13. For Policy 1 (e.g., Figure 13(a)), the dependence parameter for the Clayton copula is 1.24, and the correlation between W and Z is measured by Kendall's tau as 0.38. For Policy 2 in Figure 13(b), the associated dependence parameter is 0.89, and Kendall's tau is computed to be 0.31. Given the fitted copula models, the life-cycle maintenance costs with respect to two policies can be assessed. The service life of the bridge is defined as 100 years. The associated expectation, standard deviation, skewness, and kurtosis are computed using the Monte Carlo simulation, as shown in Table 1.



(a) (b)

Figure 13. Scatter plots of the recorded and simulated data of the maintenance interval W and maintenance cost Z of (a) Maintenance Policy 1 and (b) Maintenance Policy 2.

Table 1. Mean, standard deviation (S.D.), skewness, and kurtosis of the life-cycle maintenance cost associated with two maintenance policies.

	Mean (USD/m^2)	S.D. (USD/m^2)	Skewness	Kurtosis
Maintenance Policy 1	10231.86	5555.48	1.04	1.89
Maintenance Policy 2	10068.05	7010.80	1.32	2.83

To determine an appropriate maintenance policy, four statistical moments are defined as four different decision criteria. For the investment in maintaining civil infrastructure, decision-makers may tend to be risk-averse [60], as they tend to avoid large variability and extreme cost. For instance, risk averters tend to seek a smaller standard deviation and a positive skewness of the investment return [26, 61].

In this example, the decision process is based on the multi-attribute utility theory. The multi-attribute utility theory generally consists of four steps: quantification of attributes, identification of utility functions, assessment of relative weights, and decision on the maximum utility [62]. Four statistical moments are considered as four attributes. As smaller expected life-cycle maintenance cost is preferred, the normalized attribute function of the mean can be defined as [63, 64]

$$\varepsilon = \frac{E[LTL]_{\min}}{E[LTL]} \tag{48}$$

in which $E[LTL]_{\min}$ is the minimum mean value between the considered maintenance policies. Based on the risk-averse attitude, a smaller standard deviation should be chosen. Meanwhile, risk averters avoid extreme events associated with low-probability and high-consequence. The extreme situation can be implied by the potential tail risk in terms of skewness and kurtosis [25, 26]. Therefore, attributes for skewness and kurtosis should be defined based on the aversion of a heavy tail associated with the huge cost. For the investigated case, as the life-cycle maintenance cost indicates negative investment return, smaller skewness and kurtosis are favored [61, 65]. Accordingly, similar to the mean attribute described in Eq. (46), the minimum values of the other three attributes (i.e., standard deviation, skewness, and kurtosis) are also preferred. Hence, all four attributes can be defined as the ratio of minimum value over the attribute value.

After defining attributes, the utility function of each attribute can be formulated. In this example, the same utility functions are utilized for the four attributes, as they are all statistical characteristics of the life-cycle maintenance cost. The utility function is commonly fitted by a few points in the utility curve, which is typically concave for risk averters [63, 66]. Herein, a risk-averse utility function is directly given for illustrative purpose [67], as shown in Eq. (49)

$$u(\varepsilon) = 5.5\exp(-2/\varepsilon) \tag{49}$$

Subsequently, the additive multi-attribute utility function can be formulated. The utility of each attribute is multiplied by the associated weighting factor and then summed over. The multi-attribute utility function can be described as Eq. (50)

$$u_{LTL}(mean, sd, skew, kurt) = w_{mean}u_{mean} + w_{sd}u_{sd} + w_{skew}u_{skew} + w_{kurt}u_{kurt} \tag{50}$$

where u_{mean}, u_{sd}, u_{skew}, and u_{kurt} are the utility values of the four attributes (i.e., mean, standard deviation, skewness, and kurtosis); w_{mean}, w_{sd}, w_{skew}, and w_{kurt} are weighting factors with respect to the attributes. Typically, weighting factors are allocated considering information provided by decision-makers [68]. Herein, the four weighting factors, w_{mean}, w_{sd}, w_{skew}, and w_{kurt}, are allocated as 0.40, 0.25, 0.20, and 0.15, respectively. These values can be adjusted based on the preferences of decision-makers.

Given these inputs of attributes, the utility of Maintenance Policy 1 and Policy 2 can be computed as 0.735 and 0.535, respectively. As Policy 1 gives the maximum utility value between alternatives, Policy 1 should be chosen as the appropriate maintenance policy for the bridge. However, if the decision is purely based on the mean value (i.e., the expected life-cycle maintenance cost) as shown in Table 1, Policy 2 should be selected due to a relatively lower expected cost. A different decision outcome is attained due to the consideration of statistical moments. Therefore, statistical moments should be considered during the life-cycle analysis and decision-making process. The proposed copula tool also provides an effective data-based model for decision-making.

6. Conclusions

This study proposed a copula-based life-cycle analysis framework for deteriorating civil infrastructure systems considering uncertainties and correlation effects (e.g., dependent maintenance interval and maintenance cost). Statistical moments associated with the life-cycle maintenance cost can be effectively estimated analytically and numerically using the copula approach. Multiple dependent deterioration processes are considered in the proposed framework, including gradual deterioration, external shock, and fatal shock. Reliability-based preventive and essential maintenance actions are performed based on system reliability. Several significant conclusions are drawn as follows:

1. The joint probability distribution of the maintenance interval and the maintenance cost can be effectively modeled by the proposed copula approach. An analytical case, i.e., the FGM copula, is employed to derive statistical moments of the life-cycle cost under the weak correlation, due to its unique mathematically trackable form. Results show that even only with a weak correlation, the dependence can significantly affect the life-cycle maintenance cost.
2. The proposed copula-based approach is flexible to incorporate practical data to determine the correlation between the maintenance interval and the cost, thus delivering data-based models for the life-cycle analysis. In addition to the expectation, the other statistical moments (i.e., standard deviation, skewness, and kurtosis) of the life-cycle maintenance cost should be considered during the life-cycle cost assessment, as different decision results can be attained due to the exclusion of the other three statistical moments.
3. In addition to the FGM copula, the Gaussian and Clayton copulas are also applied to explore the effect of different dependence structures on the life-cycle cost. Results show that the expectation and standard deviation of the life-cycle cost will decrease when the correlation increases. Under the same degree of dependence (i.e., with identical Kendall's tau), the life-cycle maintenance cost is not significantly affected by different copula models.
4. Dependent deterioration processes and maintenance policy affect the maintenance interval and maintenance cost, thus influencing the life-cycle maintenance cost. For instance, in the illustrative example, considering dependent deterioration processes and fatal shocks results in a significant decrease in the maintenance interval and an increase of the life-cycle maintenance cost. Changing maintenance thresholds also leads to considerable differences in the maintenance interval and the life-cycle maintenance cost.
5. Future studies are needed to explore the dependence model of deterioration processes by incorporating data and considering the stochastic frequency and magnitude. Future studies may investigate the impact of different intervention actions on the maintenance cost and the life-cycle cost. The implementation of higher-order moments during the life-cycle analysis and decision-making process needs to be explored. The employed model of dependent deterioration processes relies on several assumptions. Future

studies are encouraged to relax these restrictive assumptions and analytical solutions should be investigated.

Appendix A. Second moment of life-cycle cost with an FGM copula

Analytical formulation of the second moment of the life-cycle maintenance cost with an FGM copula is presented. Following Eq. (38), the conditional second moment of maintenance cost can be computed and rearranged as

$$E\left[Z^2 \middle| W=y\right]=\int_0^\infty z^2 f_{Z|W=y}(z)dz=E[Z^2]+\theta\left(E[\Lambda^2]-E[Z^2]\right)(1-2F_W(y)) \tag{A1}$$

where

$$E[\Lambda^2]=\int_0^\infty z^2(2-2F_Z(z))f_Z(z)dz=\int_0^\infty 2z(1-F_Z(z))^2 dz \tag{A2}$$

The PDF of the renewal cycle can be denoted as $\omega(t, \lambda)$. Consequently, the second moment of life-cycle maintenance cost can be computed as

$$\begin{aligned}
E[LCC^2(t_{\text{int}})] &= E[Z^2]\int_0^{t_{\text{int}}} \frac{\lambda}{\lambda+2r}\omega(y;\lambda+2r)dy \\
&+\theta\left(E[\Lambda^2]-E[Z^2]\right)\int_0^{t_{\text{int}}}\left[\frac{2\lambda}{2\lambda+2r}\omega(y;2\lambda+2r)-\frac{\lambda}{\lambda+2r}\omega(y;\lambda+2r)\right]dy \\
&+2E[Z]\int_0^{t_{\text{int}}}\frac{\lambda}{\lambda+2r}\omega(y;\lambda+2r)\mu_{LCC}(t_{\text{int}}-y)dy \\
&+2\theta\left(E[\Lambda]-E[Z]\right)\int_0^{t_{\text{int}}}\left[\frac{2\lambda}{2\lambda+2r}\omega(y;2\lambda+r)-\frac{\lambda}{\lambda+2r}\omega(y;\lambda+2r)\right]\mu_{LCC}(t_{\text{int}}-y)dy \\
&+\int_0^{t_{\text{int}}}\frac{\lambda}{\lambda+2r}\omega(y;\lambda+2r)E[LCC^2(t_{\text{int}}-y)]dy
\end{aligned} \tag{A3}$$

By taking the Laplace transform of Eq. (A3) on both sides and performing the associated inversion, the second moment of life-cycle cost under dependency can be derived as

$$
\begin{aligned}
E[LCC^2(t_{\text{int}})] = {} & \frac{\lambda E[Z^2]}{2r}(1-e^{-2rt_{\text{int}}}) + 2\lambda^2 E[Z]^2 \left(\frac{1-2e^{-rt_{\text{int}}}+e^{-2rt_{\text{int}}}}{2r^2}\right) \\
& +\theta\lambda\left(E[\Lambda^2]-E[L^2]\right)\left(\frac{1-e^{-(2\lambda+2r)t_{\text{int}}}}{2\lambda+2r}\right) \\
& +2\theta\lambda^2 E[Z]\left(E[\Lambda]-E[Z]\right)\left(\frac{e^{-(2\lambda+r)t_{\text{int}}}}{(2\lambda-r)(2\lambda+r)} - \frac{e^{-2rt_{\text{int}}}}{2r(2\lambda-r)} + \frac{1}{2r(2\lambda+r)}\right) \\
& +2\theta\lambda^2 E[Z]\left(E[\Lambda]-E[Z]\right)\left(\frac{e^{-(2\lambda+2r)t_{\text{int}}}}{2(2\lambda+r)(\lambda+r)} + \frac{1}{2r(\lambda+r)} - \frac{e^{-rt_{\text{int}}}}{r(2\lambda+r)}\right) \\
& +2\theta^2\lambda^2\left(E[\Lambda]-E[Z]\right)^2\left(\frac{1}{2(2\lambda+r)(\lambda+r)} - \frac{e^{-(2\lambda+r)t_{\text{int}}}}{r(2\lambda+r)} + \frac{e^{-(2\lambda+2r)t_{\text{int}}}}{2r(2\lambda+r)}\right)
\end{aligned} \quad \text{(A4)}
$$

Consequently, the variance can be evaluated from the first two moments

$$
\sigma^2_{LCC}(t_{\text{int}}) = Var[LCC(t_{\text{int}})] = E[LCC^2(t_{\text{int}})] - (\mu_{LCC}(t_{\text{int}}))^2 \quad \text{(A5)}
$$

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