Hierarchical Seismic Metamaterial Design Towards Enhancing Multidirectional Seismic Attenuation

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The advent of seismic metamaterials (SMs) presents a new technology for protecting infrastructures from earthquakes. Their unique strength in guiding seismic wave propagation results in superior energy absorption compared to conventional methods. Despite the recent surge in seismic metamaterial research, achieving high-performance multidirectional wave attenuation upon actual SM deployment remains a challenge. To tackle this issue, we develop a high-fidelity modelling approach in this research that allows one to characterize the multidirectional seismic attenuation performance of SMs in real-world scenarios. This approach is further integrated with tailored features to achieve rapid performance assessment. Using this approach as a backbone, a hierarchical design, aiming at identifying the optimal unit cells and their arrangement pattern, is conducted to enhance the multidirectional seismic attenuation performance. This study offers a novel perspective on the design of SMs by integrating realistic considerations, which demonstrates practical applicability and significance. The effectiveness of the proposed framework is thoroughly validated by practicing an SM design implementation. Specifically, an embedded unit cell made of steel and rubber is developed, followed by the pattern design to synthesize the SM. SMs incorporating these unit cells in 2D graded patterns show a significant improvement in multidirectional seismic attenuation performance compared to those with non-graded and 1D graded patterns.

Keywords: Seismic metamaterials (SMs); bandgap; transmission; multidirectional seismic attenuation; embedded unit cell; graded pattern

1. Introduction

The accurate forecast of seismic events (i.e., earthquakes) is challenging due to the lack of understanding of the cause of the earthquake occurrence. As a result, passive methods to minimize the infrastructure damage caused by earthquakes have become prevalent [1-4]. Traditional passive methods involve the use of dampers and seismic isolation bearings for mitigating earthquake-induced vibration [5-9]. However, these devices may result in substantial residual displacement of infrastructures, adversely affecting the post-earthquake performance and posing challenges for necessary maintenance. Furthermore, engineering practices have demonstrated the limited capability of traditional methods in seismic energy dissipation, leading to insufficient protection of infrastructures during earthquakes.

In recent years, seismic metamaterials (SMs) have garnered significant attention in academic research due to their unique ability to interact with seismic waves of very long wavelengths and effectively control seismic wave propagation [10-13]. Consequently, they are being considered as the next generation of seismic-resistant technology. SMs are engineered structures composed of specifically arranged unit cells, with the underlying principle to manipulate the seismic wave propagation, and eventually to achieve the seismic energy absorption [14-16]. This mechanism enables SMs to exhibit superior wave attenuation capacities compared to the above conventional methods. Despite the potential of SMs in engineering applications, meticulous design of SMs is still essential to ensure their desired performance. Unit cells, involved in SMs, play a significant role in determining wave shielding performance. Based on the ideal assumption that an SM is comprised of an infinite number of identical unit cells, the specific unit cell configuration allows for the evaluation of an important characteristic of the SM, known as the bandgap [17-20]. The bandgap essentially represents a frequency range in which seismic waves cannot propagate. This information serves as a valuable reference for engineers to conduct the initial SM design. Fundamentally, the bandgap can be generated by two common mechanisms, namely Bragg scattering and local resonance [21-24]. Bragg scattering, a unique phenomenon of periodic structures, obstructs seismic waves through complex wave reflection, refraction, and diffraction in the medium. To activate the Bragg scattering mechanism, SMs however need to be designed with a large size comparable to the seismic wavelength, making them difficult to implement in actual practice [25]. On the other hand, local resonance can overcome the issue of the excessive SM size, facilitating its widespread use in the SM development. SMs built upon local resonance generally consist of host structures (i.e., soil substrates) and local resonators (i.e., key parts of unit cells) that generate bandgaps and dissipate seismic energy through resonance effects [26, 27]. Considering the low dominant frequency components carried by seismic waves, i.e., below 20 Hz [28], the primary goal of the unit cell design is to realize ultra-low-frequency bandgaps for the synthesized SM.

SMs based on local resonance have been extensively employed owing to the abovementioned advantages of the local resonance mechanism. The typical design strategy for the unit cell in this type of SMs involves arranging pillar arrays, referred to as local resonators, above the soil, which is cost-effective in terms of SM fabrication due to the accessibility of raw materials. The typical cross sections of unit cells include built-up steel sections [29], rectangular sections [30], cylindrical sections [31], and tuning-fork shaped sections [32], etc. Researchers have also explored the prospects of adopting material combination strategies, with emphasis on the local resonator design utilizing various raw materials, such as steel, concrete, rubber, and auxetic foam, etc. [33-36]. The basic idea of material combination is to amplify the local resonance effect by alternating the inherent coupling among components with different materials, thus attaining broad bandgaps. Furthermore, another common approach in the design of SMs is to focus on determining the topology or geometry of the local resonator. The introduction of complex topology designs, such as nested structures or fractal geometries, may lead to sophisticated local resonance, potentially resulting in wider bandgaps [37-39].

SMs based on nonlinear systems, so-called nonlinear SMs, also have demonstrated potential in seismic wave attenuation. Lou et al. [40] proposed a nonlinear meta-surface consisting of an array of Duffing oscillators attached to the free surface of an elastic half-space to mitigate seismic waves and discovered that a softening nonlinearity can yield a low-frequency bandgap. Based on vertical nonlinear and horizontal linear resonators, a nonlinear SM was designed by Lou et al. [41] The analysis results demonstrated that a wide bandgap for the hybrid Rayleigh wave can be achieved by the designed nonlinear meta-surface. By introducing the nonlinear resonant unit cell consisting of two Duffing oscillators connected in series, Lou et al.

[42] designed a nonlinear SM. Two low-frequency bandgaps were generated due to motion coupling of an elastic substrate with the dynamics of attached masses. The nonlinear SMs have also been extended to layered soil substrates to explore their feasibility in more sophisticated engineering scenarios [43].

In addition to unit cell design, the arrangement pattern of unit cells also plays a crucial role in dictating the performance of SMs. The pattern design refers to the specific arrangement of unit cells with different geometric parameters (such as resonator height) following a certain pattern. Clearly, pattern design extends SM design to a higher dimension. However, considering the significant design effort associated with high-dimensional design space, researchers have explored graded patterns, which refer to the graded tendencies of unit cell geometric variables, to enhance wave attenuation performance. Previous studies have demonstrated that compared to periodic SMs composed of identical unit cells, the graded arrangement of unit cells along the direction of seismic wave propagation can enhance the wave shielding [44, 45]. Daradkeh et al. [46] fabricated square local resonators using various materials, including steel, rubber, concrete, tungsten, and carbon fiber reinforced polymer, and investigated the impact of the graded arrangement of SMs on wave attenuation. The results indicated that the application of the graded layout can expand the attenuation region, effectively filtering frequencies as low as 4.5 Hz and as high as 29 Hz. Moreover, they observed that when the SM was designed with triangular-like graded arrangement (double wedge), the seismic surface wave can be converted to bulk wave effectively, demonstrating excellent wave blocking performance [47]. It is worth noting that the pattern design usually disrupts the periodic nature of SMs, leading to the so-called non-periodic (aperiodic) SMs. As a result, the bandgap used as a metric to evaluate the performance of ideally periodic SMs is no longer effective for aperiodic SMs. To circumvent this challenge, frequency domain analysis (FDA) will be carried out to calculate the transmission spectrum, facilitating the performance evaluation of aperiodic SMs for unit cell pattern design.

As a numerically based technique, FDA is widely employed to evaluate the wave attenuation performance of periodic and graded seismic SMs. This approach is particularly advantageous due to its ability to quantify the degree of wave reduction, especially given the resource constraints associated with conducting full-scale experiments and the uncertainties inherent in sample fabrication and in-situ measurements. Currently, simplified FDA models are extensively utilized, consisting of a single array of unit cells through which seismic waves propagate in the direction of the unit cell arrangement to assess the unidirectional wave blocking performance of SMs. However, the simplified FDA has limitations. Firstly, periodic boundary conditions are applied in the direction perpendicular to the seismic wave propagation path, assuming an infinite number of rows of unit cells in that direction [29, 37, 38, 44]. This assumption is clearly impractical in real-world engineering applications and fails to accurately reflect the behavior of actual SMs. Additionally, when analyzing the performance of graded SMs, seismic waves are constrained to propagate only along the direction of the gradient arrangement of the unit cells. Given the unpredictable nature of seismic wave directions, this model is inadequate for evaluating the wave blocking capability of SMs when seismic waves transmit perpendicularly to the gradient direction, thereby hindering the design of more complex graded arrangements.

To tackle these challenges and develop more effective SMs capable of robustly protecting infrastructure in real-world scenarios, it is essential to investigate their multidirectional wave attenuation capabilities. This objective serves as the central motivation for our research and drives several key contributions. Firstly, we introduce a tailored modeling approach, termed full-scale FDA, which, unlike simplified FDA, considers a finite number of unit cells in practical configurations and enables comprehensive evaluations of multidirectional wave attenuation. Secondly, leveraging structural symmetry, we propose a computational acceleration strategy, the accuracy of which is confirmed through comparative studies. Building on this, we design multiple graded SMs by integrating unit cell design with arrangement patterns, progressing from one-dimensional graded layouts to more intricate two-dimensional graded configurations. The full-scale FDA model is then utilized to quantify their multidirectional wave-blocking performance, followed by comparative analyses to evaluate the effectiveness of our design concepts.

The remainder of this paper is structured as follows: Section 2 starts with the unit cell design, followed by the bandgap analysis to uncover the generation mechanism of bandgaps. In Section 3, we utilize the full-scale FDA method to quantify the wave attenuation effect. Specifically, different graded SMs are proposed building upon the designed unit cell, and their multidirectional wave attenuation capacities are analyzed. Comprehensive comparisons and discussions are subsequently provided. Section 4 is devoted to the conclusions.

2. Unit cell modelling, validation, and design

This section will undertake the initial phase of the hierarchical seismic metamaterial (SM) design process, specifically focusing on the design of the unit cell. We will begin with an introduction of the bandgap formulation theory. Following this theory, we then develop a unit cell model to obtain the bandgap information, in which the local resonance mechanism responsible for bandgap formulation will be investigated. Afterward, we will proceed with the unit cell design, aiming to maximize the total bandgap width. The improved performance of the designed unit cell will be verified by comparing it with those of existing unit cell configurations.

2.1. Bandgap formulation theory

For a homogeneous, isotropic, and fully linear elastic solid medium without damping considered, according to elastic wave theory, the governing equation for wave propagation is expressed as [29]:

$$\frac{E}{2(1+\nu)}\nabla^2\mathbf{u}(\mathbf{r})+\frac{E}{2(1+\nu)(1-2\nu)}\nabla\big(\nabla\cdot\mathbf{u}(\mathbf{r})\big)=-\rho\omega^2\mathbf{u}(\mathbf{r}) \tag{1}$$

where $\mathbf{u}$ denotes the displacement vector, $\mathbf{r}$ is the position vector, ω is the angular frequency with the unit of rad/s, E, ν, ρ denote the Young's modulus, Poisson's ratio, and mass density of the medium, respectively.

To solve Eq. (1), Floquent-Bloch theorem [48] is adopted to analyze wave propagation in an infinite space. According to the theorem, when unit cells are infinitely arranged in a space, the solution to Eq. (1) can be written as follows,

$$\mathbf{u}(\mathbf{r},t) = \mathbf{u}_{\mathbf{k}}(\mathbf{r})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \tag{2}$$

where $\mathbf{k}$ is the Bloch wavevector within the range of the first Brillouin zone, $\mathbf{u}_{\mathbf{k}}(\mathbf{r})$ is the displacement modulation function, and t represents time. When the unit cells are periodically arranged with the period of the lattice constant a, $\mathbf{u}_{\mathbf{k}}(\mathbf{r})$ is a periodic function. Hence, it can be expressed as follows,

$$\mathbf{u}_{\mathbf{k}}(\mathbf{r}) = \mathbf{u}_{\mathbf{k}}(\mathbf{r}+\mathbf{a}) \tag{3}$$

where $\mathbf{a} = (a_x, a_y)$ is the lattice constant vector, a_x and a_y are lattice constants along the *x* and *y* directions in a 2D plane, respectively. In this research, they are both equal to the constant denoted as a in this study.

Substituting Eq. (3) into Eq. (2) yields

$$\mathbf{u}(\mathbf{r}+\boldsymbol{a},t) = \mathbf{u}_{\mathbf{k}}(\mathbf{r}+\mathbf{a})e^{i(\mathbf{k}\cdot(\mathbf{r}+\mathbf{a})-\omega t)} = e^{i\mathbf{k}\cdot\mathbf{a}}\mathbf{u}(\mathbf{r},t) \tag{4}$$

Eq. (4) implies that the phase difference between the input and output waves is correlated with the vector dot product $\mathbf{k}\cdot\mathbf{a}$. Given the periodic boundary condition, the structure is assumed to consist of an infinite number of periodically arranged unit cells. The vibration modes of this ideal structure can therefore be estimated based on the vibration modes of a single unit cell. This significantly reduces the model scale and enhances computational efficiency.

Substituting Eq. (4) into Eq. (1), an eigenvalue problem can be obtained as [49]

$$\big(\boldsymbol{\Phi}(\mathbf{k}) - \omega^2\mathbf{m}(\mathbf{k})\big)\mathbf{u} = \mathbf{0} \tag{5}$$

where $\boldsymbol{\Phi}$ and $\mathbf{m}$ are the stiffness and mass matrices of a single unit cell, respectively, with respect to the wavevector $\mathbf{k}$, and $\mathbf{u}$ represents the displacement vector of the unit cell.

Solving the above eigenvalue problem yields the natural frequency ω (Note: ω^2 is eigenvalue) for a given wavevector $\mathbf{k}$. In order to investigate wave propagation modes in all directions, i.e., 0-360°, the wavevector $\mathbf{k}$ should be varied across the entire first irreducible Brillouin zone (FIBZ) (see the highlighted area in Figure 1a to obtain the relationship between ω and $\mathbf{k}$, that is, dispersion curves. Γ, X and M are the high-symmetry points in the FIBZ. When a wavevector has no corresponding frequency, it is known as the bandgap, where the wave cannot transmit in the medium theoretically. In contrast, a wavevector with a definite frequency value is a passband, allowing the wave to propagate in the medium.

2.2. Unit cell modeling and validation with examination of local resonance effect on bandgap formulation

The majority of local resonators in existing SMs have been deployed entirely above the soil, and such deployed SMs are referred to as non-embedded SMs. A popular non-embedded SM can be found in the study by Xu et al. [31], with the non-embedded SM unit cell composed of a soil substrate and an overground cylindrical steel column as a local resonator, as depicted in Figure 1b. Based on the detailed configuration of this unit cell, we construct its finite element (FE) model via COMSOL Multiphysics and conduct a bandgap analysis based on the theory and approach detailed in Section 2.1. The sound cone method [50, 51] is employed to extract the bandgap from the dispersion curves. This method divides dispersion curves into two regions by a sound line (red line in Figure 1c). Because the velocity of the surface wave is smaller than that of the shear wave, the surface wave modes and bandgaps (depicted in grey color in Figure 1c) can be retrieved in the region below the sound line. From Figure 1c, the bandgap derived from our analysis closely agrees with the one reported in [31], thereby validating the effectiveness of unit cell modeling procedures used in this research. This serves as a solid foundation for the subsequent SM design process.

Previous studies have demonstrated the performance of non-embedded column-type SMs in seismic wave attenuation. In addition to non-embedded SMs, embedded SMs [52, 53], characterized by local resonators buried into the soil, also warrant research attention due to their potential advantages, such as not obstructing traffic and not impacting urban aesthetics. For non-embedded SMs, if a local resonator is placed directly on the ground without additional connections, the interface between the resonator and the soil may separate during significant seismic events, thereby invalidating the assumption of complete bonding. However, this assumption, often idealized, is typically not thoroughly examined in existing research. Moreover, under substantial external loads, resonators of non-embedded SMs may experience overturning due to rotational moments. These limitations are effectively mitigated in embedded SMs, where the resonators are firmly supported by the surrounding soil. However, the local resonance effect of these resonators may be also limited due to the constraints imposed by the surrounding soil, leading to insufficient resonance. To clearly illustrate this issue, the cylindrical column-type local resonator embedded in the soil from [31] is shown in Figure 1d, with the corresponding bandgap analysis results presented in Figure 1e. It is evident that no bandgaps are formed within the sound line area, which aligns with the findings reported in the reference [54]. The lack of bandgaps could be attributed to the limited local resonance, as the resonator is fully constrained by the soil. To further investigate this, specific mode points (points P1 and P2) on the dispersion curves are extracted and their mode shapes are displayed in Figure 1f. These visualizations confirm that the column is hindered by the soil, disabling local resonance.

2.3. Bandgap-oriented embedded unit cell design

In this research, to enable the local resonance effects of the SM to improve the wave energy dissipation, we aim to invent an embedded SM unit cell. The particular configuration of the unit cell invented in this research is given in Figure 2a. The unit cell consists of a soil substrate, an outer steel column, an inner steel column, and rubber plates used to connect them. The lattice constant a equals 2.5 m, together with other geometric parameters provided in Table 1. h_o, d and w_o are the height, width, and thickness of the outer column, respectively. h_i, l_1, w_1, l_2, and w_2 are the height, web length, web thickness, flange length, and flange thickness of the inner column, respectively. t is the thickness of the bottom steel plate. h_{r1} and h_{r2} are the thickness of the rubber plates at the bottom and the mid-height of the inner column, respectively. H is the thickness of the soil substrate. It is worth noting that Lamb waves may arise from the superposition of waves reflected from the upper and lower interfaces of a thin plate [38, 39]. Therefore, to capture surface wave modes, H should be large enough to decouple Lamb waves from surface waves, which in our study is set as $20a$. For simplicity, the unit cell materials are assumed to be homogeneous, isotropic, and elastic [29]. Table 2 presents the material properties used in the research.

Using the same modelling procedures as validated previously and with all parameters defined above, the FE model of the proposed embedded unit cell is developed (Figure 2b). The bandgap analysis results of this unit cell are shown in Figure 2c. Three bandgaps are generated, ranging from 2.05 to 2.95 Hz, 3.56 to 5.41 Hz and 5.93 to 16 Hz, respectively, with a total width of 12.8 Hz, accounting for approximately 80% of the area below the sound line. Three typical local resonance modes (A1, A2 and A3 in Figure 2d) are extracted, demonstrating their significant contribution to bandgap generation. The white arrows shown in Figure 2d indicate the vibration directions. Point A1 is at the lower boundary of the first bandgap, where a significant lateral bending resonance of the inner steel column is observed. The modal displacement distribution implies that the local resonance is primarily concentrated at the upper part of the inner column and gradually decreases along the depth direction.

In addition, the local resonance mode at point A2 is characterized by the torsion of the inner column along the vertical axis. In contrast, the local resonance mode at point A3 exhibits a bending resonance. Differing from that at point A1, the local resonance of the inner column is mainly confined at the bottom end, while the modal displacement at the upper end is minimal.

Evidently, the soft coupling facilitated by the rubber plates and the space between the outer and inner columns allows the inner column to resonate with minimal restriction. Overall, the inner column is pivotal in generating low-frequency bandgaps due to its strong local resonance effects. It is also worth noting that the local resonance of the outer column is considerably less than that of the inner column, as its other sides are constrained by the surrounding soil. Nevertheless, its contribution to the bandgap should not be overlooked, as it is instrumental in preserving the structural integrity of the system. In our design, the outer column acts as a protective casing for the inner column, shielding it from direct soil contact and enabling the rubber to enhance local resonance. As previously stated, the bandgap information of the unit cell can be utilized for initial SM design. However, considering that it is an ideal metric for assessing wave attenuation performance, we will subsequently implement the FDA method, and use the resulting transmission as introduced in Section 3 to continue the SM pattern design for performance enhancement.

3. Multidirectional wave attenuation-oriented seismic metamaterial (SM) pattern design

As outlined in Section 1, a hierarchical design strategy is adopted to develop the SM with improved wave attenuation performance in this research. In particular, following the unit cell design described in Section 2, we will proceed with the design of SM patterns. To ensure the effectiveness of the deployed SM without prior knowledge of the direction of the incident seismic wave, we propose using multidirectional wave attenuation performance as the key design metric. This metric can be evaluated using the full-scale FDA developed in this research. Building on this foundation, SMs with different unit cell layout patterns will be designed followed by a comprehensive performance assessment and comparison.

3.1. Wave attenuation evaluation using improved FDA

Although the bandgap derived from Section 2 provides valuable information for SM design, it is calculated given an assumption of an infinite number of identical unit cells in the SM. This assumption contradicts the actual SM composed of a finite number of unit cells, rendering the bandgap unmeasurable in real testing. Furthermore, the bandgap only provides frequency information without quantifying the specific level of vibration mitigation for seismic energy absorption.

Recall the discussion in Section 1. The current investigations primarily utilize simplified FDA. This approach assumes that there are an infinite number of unit cells in the direction perpendicular to the unit cells array, which is a clear mismatch with the real SM deployment in engineering practice. To mimic the actual SM, a full-scale modeling strategy is developed in this research, as depicted in Figure 3a. For FDA, it is assumed that the interactions between the soil substrate and steel columns are perfectly bonded, the materials are homogeneous and linearly elastic, and damping is neglected to focus exclusively on the wave attenuation effect provided by the SM itself [46,54]. In the full-scale FDA model, n rows and m columns of unit cells are arranged along the *x*- and *y*-axis, respectively, resulting in a total of $n \times m$ unit cells. To simulate the wave propagation, an excitation with frequency sweep is used as input in the model, and the displacement amplitudes on the right side of the model are recorded as output [37]. Through a literature investigation, the distance between the excitation and the SM is set as $10a$ [12, 37, 38, 55, 56]. This can ensure the rapid dissipation of seismic bulk waves along the depth of the soil only the surface wave interacts with the SM, allowing surface waves to interact with the SMs. In addition, to minimize unnecessary wave reflection, the Perfectly Matched Layer (PML) [57] with a thickness of $3a$ is applied on the four sides and bottom of the soil substrate to reduce unwanted wave reflection. The PML is a specific boundary condition designed to absorb the bulk wave and prevent it from reflecting into the soil, ensuring that only the surface wave is manipulated by the SM. The full-scale FDA model not only allows one to investigate the multidirectional wave attenuation effects, but also agrees with the actual layout of the SM in engineering applications. By performing harmonic sweeping, one can get output displacement amplitude values at varying frequencies. An indicator, known as the transmission, thus can be calculated [26, 30, 31, 58],

$$T(f) = 20\log_{10}\left(\frac{u_1(f)}{u_2(f)}\right) \tag{6}$$

where f is the frequency in the unit of Hz, which can be calculated using $f = \omega/2\pi$, $T(f)$ denotes the transmission value at the frequency of f, $u_1(f)$ and $u_2(f)$ denote the displacement amplitudes at the output area of the models with and without the SM, respectively. The transmission essentially represents a frequency spectrum. Clearly, a negative value of $T(f)$ indicates the seismic wave attenuated by the SM. The smaller the value of $T(f)$, the better the wave attenuation performance of the SM at the frequency f.

In this research, the FDA model is set with 8×8 unit cells (i.e., $m = 8$ and $n = 8$ in Figure 3a). The transmission is calculated, as shown in Figure 4a. The bandgaps of the proposed unit cell obtained from Section 2.3 are also involved for comparison. Since the unit cells in the full-scale FDA model are periodically arranged, the bandgaps match well with the frequency spans that have significant attenuation in the transmission. Any minor discrepancies may be due to the different assumptions used in the analysis. The transmission corresponding to the first bandgap exhibits significantly less attenuation than other bandgaps. Previous studies have demonstrated that the displacement of the seismic wave can be reduced by over 90% when the SM dimension along the wave propagation direction is similar to the seismic wavelength [32, 35]. In this study, 8 rows of unit cells are positioned along the wave propagation direction, resulting in a total dimension of 20 m. This is considerably smaller than the seismic wavelength, with frequencies falling into the first bandgap. As a result, relatively weak wave attenuation is observed within this specific frequency range.

It should be highlighted that the computational time demanded by the full-scale FDA model is considerably extensive, i.e., approximately 37.8 hours recorded in this study, due to the large number of elements of the model. To substantially improve computational efficiency without compromising accuracy, we propose a symmetry-based computational acceleration strategy for the full-scale FDA, with the details included in Figure 3b. This scheme capitalizes on the symmetry of SM configurations. A symmetric plane is constructed to divide the entire model into two identical halves. Given that the displacements of the nodes in one half can mirror those in the other half, when symmetric boundary conditions are applied on the symmetric plane (Figure 3b), only one half of the full-scale model needs to be retained for analysis, thus considerably reducing the computational cost. This concept is similar to the degree-of-freedom (DOF) condensation for model scale reduction in structural dynamics [59-62]. To validate the feasibility of the computational acceleration scheme, a comparative analysis is conducted accordingly, with a comparison of transmissions shown in Figure 4b. The results indicate that the symmetry-based FDA model can achieve the same level of accuracy as the full-scale model. Figure 4c compares the computational time costs of the full-scale and symmetry-based FDA models. Clearly, the number of elements in the symmetry-based model is reduced by approximately half, leading to an approximately 50% decrease in computational time. Whenever feasible, we can leverage the symmetry of the SM arrangement to substantially speed up the transmission evaluation, which greatly facilitates the subsequent SM pattern design.

Moreover, we conduct time history analysis to examine the wave-blocking effect of the non-graded SM with an 8×8 unit cell configuration (Figure 3a) under actual seismic waves. The results, presented in Appendix A, demonstrate that, consistent with the findings from FDA, the placement of the SM in the wave propagation path significantly reduces the acceleration amplitudes of seismic wave components within the bandgap frequency ranges at the output area. Consequently, in the following section, due to the high computational cost associated with time history analysis, we will focus exclusively on frequency domain analysis to assess the wave reduction performance of graded SMs.

3.2. Seismic metamaterial (SM) pattern design and performance assessment

Using the full-scale FDA formulated in Section 3.1 as the backbone, we can further carry out SM pattern design in this subsection. Taking into account the balance between the design effort associated with the design space and the anticipated performance, we choose a typical aperiodic layout strategy, known as the graded pattern, as the starting point for this design exploration to achieve wave attenuation enhancement. Employing the accelerated full-scale FDA, we propose six graded SM layouts for performance evaluation, encompassing three 1D and three 2D graded patterns.

3.2.1. 1D pattern design, analysis, and performance evaluation

Building upon the unit cell geometry proposed in Section 2.3, three 1D graded SM layout patterns including a 1D linear and two different 1D polynomial patterns, are developed. The specific pattern profiles are determined by varying the heights of the local resonators along the x-axis, while keeping the same heights along the y-axis, as shown in Figures. 5b-d. For detailed information on the formulated 1D graded patterns, please refer to Appendix B, which allows the interested audience to duplicate the model construction and conduct relevant investigations. It is noteworthy that the non-graded (periodic) SM (Figure 5a) composed of identical unit cells analyzed in Section 3.1, is also involved for performance comparison.

Following the above 1D pattern profiles, we construct the respective FDA models and evaluate the transmissions under different incident wave directions. These results, along with those from the non-graded SM (Figure 5a), are combined for comparison. It is worth noting that the transmissions assessed for cases where the incident waves are transmitted along the positive *y*-axis and negative *y*-axis are identical due to the symmetry of the SM. For the presentation simplicity, we use *y*+ direction and *y*- direction to denote the incident wave direction along the positive *y*-axis and negative *y*-axis, respectively. This convention can also be applicable for defining wave propagation along the *x*-axis.

When incident waves propagate along the *x*+ direction, as can be seen in Figure 6b, three 1D graded SMs all exhibit remarkably better wave attenuation than the non-graded SM in the frequency range of 4.5-14 Hz, with the most notable improvement at the frequency of 11 Hz. To evidence this observation, we examine the displacement fields of the non-graded SM and the graded SM with 1D polynomial pattern-2 (Figure 5d) at the frequency of 11 Hz, as shown in Figure 7a. The wave conversion effect [25, 45, 63], where surface waves are transformed into bulk waves due to the SMs, is observed. To better illustrate this phenomenon, the direction of wave transformation is indicated by the red arrows in Figure 7a. As can be seen, when the wave propagates along the *x*+ direction, the SM with 1D polynomial pattern-2 exhibits a more pronounced wave conversion effect, resulting in a reduced displacement amplitude at the output area compared to the non-graded SM. This enhanced effect is likely due to the progressive increase in the height of the unit cells along the *x*+ direction.

Furthermore, compared to the non-graded SM, the 1D graded SMs exhibit a broader wave attenuation region. Based on the insights from the reference [58], the enhanced wave attenuation observed in graded SMs may stem from the broadening of the so-called overlapping bandgap. The overlapping bandgap arises from the superposition of bandgap areas from various unit cells, each featuring a unique resonator height. Note that the resonator height in this research is represented by the height of the outer column (i.e., h_o in Figure 2a). The overlapping bandgap is obtained through iterative calculation of the bandgaps of unit cells involved in 1D graded SMs, with resonator heights increasing from $3a$ to $5a$. Following this analysis procedure, the bandgaps of all unit cells in 1D graded SMs are calculated, as depicted in Figure 6a. The overlapping bandgap, ranging from 1.62 to 16 Hz with a total range of 14.38 Hz, is identified from the bandgap data and represented as the grey areas in Figures. 6b-d. The good correspondence between the overlapping bandgap and the significant attenuation regions in the transmissions of 1D graded

SMs fundamentally elucidates the rationale behind using graded patterns to achieve the desired wave attenuation across an extended frequency range.

Figure 6c presents the transmissions of seismic wave propagation along the *x*- direction, combined with the overlapping bandgap for comparison. Clearly, when the wave propagates along this direction, the SMs with 1D graded patterns still exhibit wider frequency attenuation bands compared to the non-graded SM. It is noteworthy that wave propagation along the *x*- direction exhibits a different behavior compared to that along the *x*+ direction. Within the 9.5-16 Hz frequency range, the non-graded SM demonstrates more pronounced attenuation than SMs with 1D graded patterns. This effect is especially noticeable at a frequency of 16 Hz, as illustrated by the displacement fields of SMs with the non-graded pattern and the 1D linear pattern in Figure 7b. For the non-graded SM, the surface wave is almost entirely transformed into a bulk wave by the time it reaches the sixth row of unit cells. In contrast, for the SM with a 1D linear pattern, some of the wave energy persists even after passing through the entire SM. This indicates that when the wave propagates in the opposite direction of the gradient, it may be less effective for the transformation of surface waves into bulk waves. In addition to evaluating performance along the pattern gradient directions, we also investigate wave propagation and attenuation along other directions, i.e., along the *y*- or *y*+ direction. As seen in Figure 6d, the SMs with 1D graded patterns exhibit better wave attenuation than the non-graded SM in the frequency range of 4.5-14 Hz. At the frequency of 6 Hz, the SM with 1D polynomial pattern-1 shows a notable improvement over the non-graded SM, which is further clarified by the corresponding displacement fields shown in Figure 7c.

These findings are further analyzed from an alternative perspective, as depicted in Figures 8a-c, where the transmissions of the same SM under different incident wave directions are compared. It is apparent that a significant improvement in wave attenuation over nearly the entire frequency range is achieved by employing SMs with 1D graded patterns, especially when the wave travels along the *x*+ direction. On the other hand, the attenuation performance decreases when the wave propagates along the opposite direction, even though it still moves along the gradient of the pattern. These results imply a strong interdependence between the orientation of the pattern gradient and the direction of wave propagation in determining the effectiveness of wave attenuation.

Since the wave attenuation performance is assessed based on the transmission calculated, it is dependent on the specific frequency component of interest. To quantify the wave attenuation performance, we further establish the following metrics,

$$T_a(i) = \frac{1}{f_s}\int_0^{f_s} T(f)df \tag{7}$$

$$T_a = \frac{1}{N}\sum_{i=1}^{N} T_a(i) \tag{8}$$

where i denotes the propagation direction of the incident wave, which can be along the *x*+ direction, *x*- direction, *y*+ direction, or *y*- direction, $T_a(i)$ represents the average transmission along the i direction, $T(f)$ represents the transmission along the respective direction depending on the frequency f defined in Eq. (6), f_s is the upper bound of the frequency range below the sound line, which in this research is 16 Hz, T_a denotes the overall average transmission considering N wave propagation directions. N in this research is 4. As the negative value of the transmission signifies the effective wave attenuation, a smaller value of $T_a(i)$ indicates the better wave attenuation along the i direction, and a smaller value of T_a indicates an enhanced overall wave attenuation capacity, i.e., multidirectional wave attenuation performance.

Based on the transmission results in Figures 6b-d, we use Eqs. (7) and (8) to calculate their metric values to further assess the unidirectional and multidirectional wave attenuation performances of the SMs with non-graded and 1D graded patterns. The results, found in Table 3, show that the SMs with 1D graded patterns outperform the non-graded SM in terms of unidirectional wave attenuation along the *x*+ and *y*+ directions. Notably, the SM with 1D polynomial pattern-2 achieves a $T_a(x+)$ of -14.1 dB, a 35.3% (i.e., $|-14.1-(-10.42)|/|-10.42|$) improvement compared to that of the non-graded SM. However, the performance of SMs with 1D graded patterns is similar to that of the non-graded SM for waves propagating along the *x*-direction, highlighting the impact of pattern orientation. Overall, all SMs with 1D graded patterns show enhanced multidirectional wave attenuation capacity due to their larger overall average transmissions. Among 1D graded SMs, the SM with 1D polynomial pattern-2 exhibits the best overall performance, with a T_a of -12.12 dB, which is 16.3% better than that of the non-graded SM.

3.2.2. 2D pattern design, analysis, and performance evaluation

As demonstrated in Section 3.2.1, the comparison of the performance between the non-graded and 1D graded SMs indicates that waves are effectively attenuated when they propagate along the positive direction of the pattern gradient. Inspired by this observation, improved multidirectional wave attenuation performance may be attained by incorporating additional gradient tendencies in SM pattern design. Intuitively, we can investigate gradient patterns along other directions based on the current 1D graded SMs to create the so-called 2D graded SMs. Following this concept, we develop three 2D graded SMs, with the pattern profiles displayed in Figures 5e-g. In comparison to their 1D counterparts, the 2D graded SMs extend the gradients along both the x- and y-axis, with detailed geometric information provided in Appendix B. Due to the symmetry of the SMs, the transmissions for waves propagating along the x- and y- directions, and for waves propagating along the x+ and y+ directions, are identical.

The overlapping bandgap obtained from all unit cells in 2D graded SMs with resonator heights from $3a$ to $7a$, is analyzed, as depicted in Figure 9a. The overlapping region of bandgaps is from 1.38 to 16 Hz, covering a total width of 14.62 Hz. Figure 9b displays the transmissions of three 2D graded SMs under the x+ (or y+) wave. The overlapping bandgap is represented in a gray area. Similar to 1D graded pattern cases, the transmissions of SMs with 2D graded patterns exhibit significant seismic wave attenuation within the overlapping bandgap region. The SM with 2D polynomial pattern-1 exhibits the best wave attenuation performance in the frequency range of 5-8 Hz, particularly with a pronounced desired performance observed at 6 Hz, while the transmission values over other frequency ranges of all 2D graded SMs are similar. The displacement fields of the 2D graded SMs under the x+ (or y+) wave at the frequency of 6 Hz are presented in Figure 10a. At this frequency, the SM with 2D polynomial pattern-1 demonstrates an optimal wave conversion effect, resulting in a significant reduction of the displacement amplitude at the output area. The transmissions for wave propagation along the x- (or y-) direction follow a similar trend among different 2D graded SMs, as shown in Figure 9c, which is further illustrated by the displacement fields in Figure 10b.

Moreover, the transmissions of the same 2D graded SM under different wave propagation directions are illustrated in Figures 11a-c. The results reveal that all 2D graded SMs can provide enhanced wave attenuation in the frequency range of 9.2-13 Hz when the incident wave propagates along the x+ (or y+) direction compared to that when the wave propagates along the x- (or y-) direction. This finding further confirms that the unidirectional wave attenuation performance relies on the interrelation between the pattern orientation and wave propagation direction.

Following similar analysis procedures, we revisit Eqs. (7) and (8) to evaluate the unidirectional and multidirectional wave attenuation performance of 2D graded SMs. The results are presented in Table 4. All 2D graded SMs show significant wave attenuation improvement for wave propagation along the x+ (or y+) direction, with a T_a(x+ or y+) of around -14.5 dB. This is because the wave propagation along the x+ (or y+) direction aligns with the positive direction of the 2D gradient, leading to a more pronounced wave conversion effect. Despite the reduced wave attenuation performance for wave propagation along the x- (or y-) direction, the overall average transmission still demonstrates the desired multidirectional wave attenuation performance of all 2D graded SMs. The SM with 2D polynomial pattern-1 especially stands out for its exceptional multidirectional wave attenuation (i.e., with a T_a of -14.31 dB).

The unidirectional wave attenuation performances of the SMs with 2D and 1D graded patterns are compared in Figures. 12a-c. The results illustrate that the SMs with 2D linear pattern and 2D polynomial pattern-1 significantly outperform their 1D counterparts in terms of unidirectional wave attenuation capability. While the average transmission of the SM with 2D polynomial pattern-2 is slightly inferior to its 1D counterpart for wave propagation along the *y*- direction, its attenuation performance along other directions is noticeably improved.

Finally, we combine the overall average transmissions of all non-graded and graded SMs to indicate the multidirectional wave attenuation performance, as depicted in Figure 12d. Apparently, the graded SMs lead to a significant enhancement in multidirectional wave attenuation performance compared to non-graded SMs. Moreover, 2D graded SMs further elevate the performance, with the SM featuring a 2D polynomial pattern-1 demonstrating the most optimal multidirectional wave attenuation capability. Specifically, the overall average transmission improves by 3.5 dB (a 20% improvement) compared to the non-graded SM, and by 2.29 dB (a 19.05% improvement) compared to its 1D counterpart. The hierarchical design proposed in this research is expected to significantly improve the multidirectional wave attenuation performance of the developed SMs, offering a cost-effective solution for real-world seismic hazard mitigation.

4. Conclusions

In this paper, we propose a hierarchical framework for conducting the design of seismic metamaterials (SMs). This design process is of sequential nature, starting with the unit cell configuration design and proceeding to the unit cell arrangement pattern design. To ensure cost-effectiveness in practical SM deployment and operation, we propose using multidirectional wave attenuation performance as the design metric. A tailored modeling approach is developed accordingly to assess the multidirectional wave attenuation performance, which is further integrated with the tailored feature to expedite the computation and thus to facilitate the SM design.

Following such a design concept, we first propose an embedded unit cell. The primary concept behind the unit cell design in this research is to enhance local resonance in order to broaden the bandgap when the unit cell is buried into the soil. However, if not appropriately designed, the embedded unit cell may lack the bandgap, potentially limiting the surface wave attenuation ability of the synthesized SM. The designed embedded unit cell exhibits low-frequency bandgaps, with a total width of 12.8 Hz, covering approximately 80% of the region below the sound line. The soft coupling between the inner and outer columns is crucial in amplifying local resonance, and the gap between the two columns also provides sufficient space for the vibration of the inner column, thereby generating bandgaps.

The unit cell pattern design, specifically adopting the graded patterns, is subsequently implemented, where multiple SMs with 1D and 2D graded patterns are constructed and analyzed in terms of both unidirectional and multidirectional wave attenuation performances. It has been demonstrated that graded SMs indeed outperform non-graded SMs. However, the unidirectional wave attenuation performance of graded SMs inherently relies on the direction of incident wave propagation and pattern orientation. Specifically, for the same graded pattern, the wave attenuation performance of the SM is superior when the wave propagates in the positive direction of the gradient, attributed to the wave conversion effect. The strategy of incorporating gradients along two directions in SMs to establish the so-called 2D graded SMs, is implemented and thoroughly validated. There is a significant increase in the multidirectional wave attenuation performance of the resulting 2D graded SMs. One specific graded SM among them achieves up to a 20% performance improvement compared to the non-graded SM. Overall, the results from comparing the performance of all involved SMs clearly demonstrate the effectiveness of our proposed design framework and modeling approach.

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Appendix A. Time history analysis of the non-graded SM

The time history analysis model closely resembles the frequency response analysis model, with the key distinction that low-reflection boundaries are employed instead of perfectly matched layers (PMLs) in the time history analysis. In this study, we utilize the Landers earthquake wave (Figure A1(a)) [64] and the Oroville earthquake wave (Figure A1(b)) [65]. For comparative purposes, we also calculate the time domain response of a model without the SM subjected to the same earthquake waves. Figures A1(c) and (d) illustrate the vertical acceleration response at the output area under the influence of the Landers and Oroville earthquake waves, respectively. It is evident that for the model incorporating the SM, the acceleration amplitude at the output area is significantly reduced compared to the model without the SM. Furthermore, we perform a fast Fourier transform analysis on the acceleration signals recorded at the output area to quantify the attenuation effect of the SM on various frequency components of the seismic waves. As depicted in Figures A1(e) and (f), the deployment of the SM leads to a substantial decrease in the amplitudes of the seismic wave components within the bandgap ranges, which aligns with the findings from the frequency domain response analysis.

Appendix B. The geometric information of graded SMs

This section aims to provide detailed information on unit cell pattern development for readers interested in implementing the practice. Figure B1 shows the top view of unit cells. The unit cells are labeled from A to H and from 1 to 8 along the *x*- and *y*-axis, respectively. The coordinate origin is set at unit cell A1, and the distance between adjacent unit cells is equal to the lattice constant a (2.5 m). The 1D and 2D graded SMs are constructed by varying the heights of the local resonators. Note that the relationship between the height of the outer column (h_o in Figure 2a and Table 1) and the height of the inner column (h_i in Figure 2a and Table 1) in the SM follows the equation,

$$h_i = h_o - h_{r1} - t \tag{B.1}$$

where the values of h_{r1} and t are provided in Table 1 and remain constant throughout the study. Since the height of the outer column is larger than that of the inner column, the height of the local resonator is represented by the height of its outer column.

For the 1D linear pattern, 1D polynomial pattern-1, and 1D polynomial pattern-2, the *x* and *y* coordinate values of the unit cells can be substituted into Eqs. (B.2) -(B.4) to obtain the corresponding resonator heights, respectively. Similarly, the resonator heights for the 2D linear pattern, 2D polynomial pattern-1, and 2D polynomial pattern-2 can be obtained by substituting the coordinate values into Eqs. (B.5) -(B.7), respectively.

$$h_o = \frac{2}{7}x + 0y + 3a \tag{B.2}$$

$$h_o = -\frac{2}{49a}x^2 + \frac{4}{7}x + 0y + 3a \tag{B.3}$$

$$h_o = -\frac{2}{49a}x^2 + \frac{4}{7}x + 0y + 3a \tag{B.4}$$

$$h_o = \frac{2}{7}x + \frac{2}{7}y + 3a \tag{B.5}$$

$$h_o = -\frac{2}{49a}x^2 + \frac{4}{7}x - \frac{2}{49a}y^2 + \frac{4}{7}y + 3a \tag{B.6}$$

$$h_o = \frac{2}{49a}x^2 + \frac{2}{49a}y^2 + 3a \tag{B.7}$$

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Table 1. Geometric dimensions of the proposed unit cell.

a	h_o	d	w_o	h_i	l_1	w_1	l_2	w_2	t	h_{r1}	h_{r2}	H
2.5 m	$3a$	$0.8a$	$0.03a$	$2.8a$	$0.5a$	$0.03a$	$0.4a$	$0.04a$	$0.04a$	$0.16a$	$0.2a$	$20a$

Table 2. Material properties.

	Young's modulus (Pa)	Density (kg/m^3)	Poisson's ratio
Steel	2.1×10^{11}	7850	0.3
Soil	3×10^{7}	1800	0.3
Rubber	1×10^{6}	1300	0.47

Table 3. Comparison of $T_a(i)$ and T_a of SMs with non-graded and 1D graded patterns.

Pattern type	$T_a(x+)$ (dB)	$T_a(x-)$ (dB)	$T_a(y+$ and $y-)$ (dB)	T_a (dB)
Non-graded	-10.42	-10.42	-10.42	-10.42
1D linear	-13.66	-10.30	-11.78	-11.91
1D polynomial-1	-13.25	-10.61	-12.11	-12.0
1D polynomial-2	-14.10	-10.02	-12.23	-12.12

Table 4. Comparison of $T_a(i)$ and T_a of SMs with 2D graded patterns.

Pattern type	$T_a(x+$ or $y+)$ (dB)	$T_a(x-$ or $y-)$ (dB)	T_a (dB)
2D linear	-14.52	-12.52	-13.52
2D polynomial-1	-14.72	-13.89	-14.31
2D polynomial-2	-14.29	-11.42	-12.86