

1 **Spin-Orbit-Locking Vectorial Metasurface Holography**

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37

38 Abstract: Vectorial metasurface holography, allowing for independent control over the  
39 amplitude, phase and polarization distribution of holographic images enabled by  
40 metasurfaces, plays a crucial role in the realm of optical display, optical and quantum  
41 communications. However, previous research on vectorial metasurface holography has  
42 typically been restricted to single degree of freedom input and single channel output,  
43 thereby demonstrating a very limited modulation capacity. In this work, we present a  
44 novel method to achieve multi-channel vectorial metasurface holography by harnessing  
45 spin-orbit-locking vortex beams. In each channel, the optical vectorial field is encoded  
46 with a pair of total angular momentums (TAMs) featuring two orthogonal spin angular  
47 momentums (SAMs) independently locked with arbitrary orbital angular momentums  
48 (OAMs). Our methodology relies on a modified Gerchberg-Saxton (GS) algorithm,  
49 enabling the encoding of various TAM channels within a single phase profile.  
50 Consequently, a pure geometry-phase metasurface with a non-interleaved approach can  
51 be used to support such multi-channel vectorial holography, achieving high selectivity  
52 of both SAM and OAM, and offering precise routing and manipulation of complex light  
53 channels. Our work presents a paradigm shift in the field of holography, offering  
54 promising avenues for high-density optical information processing and future photonic  
55 device design.

56

## 57 **1. Introduction**

58 Vectorial holography<sup>[1-4]</sup> offers not only control over the amplitude and phase of the  
59 holographic image but also the ability to arbitrarily tailor its polarization distribution,  
60 which is challenged to be achieved using conventional holographic setup due to the limited  
61 degrees of freedom (DoFs)<sup>[5]</sup>. Metasurfaces, which consist of arrays of 2D subwavelength  
62 structures, provide exceptional precision in manipulating the phase, polarization and  
63 amplitude of light<sup>[6-21]</sup>. These properties have made metasurfaces an essential platform  
64 for achieving compact, efficient, and adaptable vectorial holography, known as vectorial  
65 metasurface holography<sup>[22-26]</sup>. However, existing studies are typically constrained by  
66 offering a single DoF input and a static single-channel output, which significantly limits  
67 their modulation capacity.

68           Orbital angular momentum (OAM) constitutes an additional DoF in optical field<sup>[27-</sup>  
69   <sup>30]</sup>, characterized by the formula  $e^{j(l\theta)}$ , where  $j^2 = -1$ ,  $l$  and  $\theta$  denote its topological charge  
70 (TC) and azimuth angle, respectively. Unlike other optical DoFs, OAM theoretically provides  
71 unlimited orthogonal set of helical modes represented by an infinite range of TCs,  
72 showcasing significant potentials for enhancing operational channels and information  
73 capacity in optical and quantum communications. By exploiting the sampling technique  
74 in the holographic algorithms, it becomes feasible to generate OAM holography that can  
75 transfer the OAM properties from an incident OAM beam to a holographic image<sup>[31-35]</sup>.  
76 Despite the fact that OAM holography is compatible with other DoFs to enhance  
77 multiplexing capabilities, such as spin-selective OAM holography<sup>[36]</sup> and wavelength-  
78 selective OAM holography<sup>[37]</sup>, leveraging the OAM for generating multiplexed vectorial  
79 holography remains elusive. It is anticipated that the successful implementation of OAM  
80 vectorial holography could lead to substantial advancements in the performance of  
81 holographic systems and introducing new DoFs for manipulating vectorial holography.

82           In this study, we present a novel approach to achieve spin-orbit-locking vectorial  
83 metasurface holography (SOL-VMH) by harnessing total angular momentum (TAM)  
84 with a value of  $|\sigma, l\rangle$  defined as the locking of spin angular momentum (SAM)  $\sigma$  and  
85 OAM  $l$ . Each channel utilizes a pair of TAMs, corresponding to two orthogonal SAM  
86 states, with each state independently locked with an arbitrary OAM, to encode the  
87 optical vectorial field. We employ a modified Gerchberg-Saxton (GS) algorithm<sup>[3]</sup> to  
88 calculate the phase profiles of each TAM channel, enabling the generation of vectorial  
89 field with independent control over image intensity, polarization azimuth, and ellipticity  
90 angles. To accomplish multi-channel vectorial holography, a collective phase profile is  
91 derived by summing all the individual phase profiles from each TAM channel. The  
92 resulting phase profile for achieving multi-channel vectorial holography can be perfectly  
93 supported by a geometry-phase metasurface using a non-interleaved method<sup>[38]</sup>. Due to the  
94 conservation of OAM, and the nature of geometry-phase acting on spin conversion of  
95 SAM, only specific pairs of inverse TAMs ( $|\sigma, -l\rangle$ ) excitation can reconstruct the  
96 optical vectorial field in each channel, showcasing exceptional spin-orbit selectivity.  
97 The proposed SOL-VMH not only broadens the scope of light manipulation techniques  
98 but also demonstrates a high degree of selectivity. This selectivity resulting from the  
99 precise control over both SAM and OAM allows for the independent routing and  
100 manipulation of different light channels. Our approach, therefore, offers a promising

101 avenue for the future development of optical and quantum devices requiring  
 102 sophisticated control and multiplexing capabilities.

103

## 104 2. Results

### 105 2.1. Design principle of SOL-VMH.

106 The schematic of SOL-VMH is illustrated in **Figure 1**. Each input comprises a pair of TAMs  
 107 ( $|\sigma, l\rangle$ ) defined as the locking of SAM  $\sigma$  and OAM  $l$ , where SAM has two states ( $\sigma_+ = 1$   
 108 and  $\sigma_- = -1$ , representing left- (LCP) and right-circular polarization (RCP), respectively), and  
 109 OAM has unbounded and orthogonal set of states with integer TCs. Such a TAM pair can  
 110 be represented by a matrix  $[\sigma_+, l_+; \sigma_-, l_-]$  as shown in Figure 1a, consisting of two  
 111 orthogonal SAM states ( $\sigma_+ = -\sigma_-$ ), where each SAM state is independently locked with  
 112 an arbitrary OAM, also known as higher-order Poincare sphere beam<sup>[39]</sup> as shown in  
 113 Figure 1b. Notably, the TAMs with different input matrices are orthogonal to each other,  
 114 laying the foundation for multiplexing vectorial holography on a single metasurface as  
 115 shown in Figure 1c. Only when the TAM pair with corresponding input matrix is  
 116 projected onto the metasurface, a specific channel of vectorial holography can be  
 117 reconstructed, yielding a 2D point-like field distribution with desired amplitude ( $A$ ),  
 118 polarization azimuth ( $\psi$ ) and ellipticity ( $\chi$ ) distributions. For instance, when Input1,  
 119 represented by matrix  $[1, 1; -1, 2]$ , is illuminated onto the metasurface, only the vectorial  
 120 field encoded in Output1 can be distinctly recognized and retrieved, revealing clear  
 121 details of three images across three optical dimensions: amplitude, polarization azimuth,  
 122 and ellipticity angle.

123

### 124 2.2. Algorithm development and simulated results.

125 The methodology of SOL-VMH is illustrated in **Figure 2a**. Each modulated vectorial field  
 126 ( $U^i$ ) in channel  $i$  is obtained by multiplying the original vectorial field with a 2D Dirac  
 127 comb function ( $C^i$  with a constant period of  $d$ ). Notably, each channel is positioned at  
 128 distinct spatial locations across the optical field, with the full configuration not shown  
 129 in Figure 2a for better visualization **and detailed arrangement of the vector fields can be**  
 130 **found in Supplementary Note 1**. The value of  $d$  can be arbitrarily chosen as long as the  
 131 point-like field remains discernible. In each TAM channel, two desired phases ( $\phi_{\sigma_{\pm}}^i$ )  
 132 corresponding to opposite SAM states are calculated from the previous modulated vectorial  
 133 field in the spatial frequency domain using a modified GS algorithm<sup>[3]</sup> (see more details in  
 134 Supplementary Note 2). An arbitrary OAM helical phase ( $\phi_{l_{\pm}}^i$ ) is then superimposed with

135  $\varphi_{\sigma_{\pm}}^i$  to achieve the purpose of spin-orbit-locking. Subsequently, the superimposed phase  
 136 associated with the  $\sigma_+$  SAM state is inverted due to the opposite sign of geometry-phase  
 137 for the two SAM states. As a result, the final phase profile supporting all channels can be  
 138 expressed as,

$$139 \quad \varphi_{SOL-VMH} = \arg \left[ \sum_{i=1}^N \left( e^{j(\varphi_{\sigma_-}^i + \phi_{l_-}^i)} + e^{j(-\varphi_{\sigma_+}^i - \phi_{l_+}^i)} \right) \right] \quad (1)$$

140 where channel  $i = 1, 2, \dots, N$ , with  $N$  being the total number of channels,  $\phi_{l_-}^i$  and  $\phi_{l_+}^i$  are the  
 141 helical phases corresponding to TCs of OAM  $l_-^i$  and  $l_+^i$ , respectively, which are added into the  
 142 calculated phases  $\varphi_{\sigma_-}^i$  and  $\varphi_{\sigma_+}^i$  for their respective SAM states.

143 This metasurface comprises an array of elliptical titanium dioxide ( $\text{TiO}_2$ ) meta-  
 144 pillars on a glass ( $\text{SiO}_2$ ) substrate, each with 600 nm in height and arranged in a square  
 145 lattice with a periodicity of 300 nm. These meta-pillars are designed as half-wave plates  
 146 with axis lengths of 100 nm and 260 nm, enabling them to convert the spin of circularly  
 147 polarized (CP) beams. The rotation of the meta-pillar, determined by angle  $\delta$ , facilitates  
 148 this CP conversion:  $|\sigma_+\rangle \rightarrow e^{j2\delta}|\sigma_-\rangle$  and  $|\sigma_-\rangle \rightarrow e^{-j2\delta}|\sigma_+\rangle$ , *i.e.*, the beams are converted to  
 149 the opposite spin with a geometry-phase (or Pancharatnam-Berry (PB) phase) of  $2\delta$  and  $-2\delta$ ,  
 150 respectively. Consequently, the two-dimensional rotation angle profile ( $\varphi_{\delta}$ ) of the  
 151 metasurface can be derived as,

$$152 \quad \varphi_{\delta} = \frac{1}{2}(\varphi_{SOL-VMH} - \beta) \quad (2)$$

153 where  $\beta$  is the propagation phase of the meta-pillar, which is constant for all meta-pillars  
 154 due to their uniform sizes. For simplicity, we ignore the propagation phase and set it as  
 155  $\beta = 0^\circ$ . Therefore, by rotating the meta-pillars, a pure geometry-phase can be obtained.  
 156 The phase profile derived from the aforementioned process can be encoded in a geometry-phase  
 157 metasurface using a non-interleaved method<sup>[38]</sup>. When illuminated by incident waves  
 158 possessing both  $\sigma_+$  and  $\sigma_-$  SAM, the metasurface generates a symmetric pair of  
 159 diffracted beams as shown in Figure 2b, allowing for precise control over amplitude and  
 160 polarization by superimposing both SAM states. In the  $i^{\text{th}}$  channel, the optical vectorial field  
 161 is encoded with a TAM pair  $[\sigma_-, l_-^i; \sigma_+, l_+^i]$ . To reconstruct the vectorial holographic images  
 162 from the metasurface, an inverse TAM pair  $[\sigma_+, -l_-^i; \sigma_-, -l_+^i]$  with opposite sign of SAM and  
 163 OAM is used for illumination due to the conservation of OAM and the cross-polarization  
 164 conversion of SAM. When illuminating with a vortex beam carrying TAM  $|\sigma_+, -l_-^i\rangle$  onto the  
 165 metasurface (Figure 2b(I)), the output optical field  $F_{\sigma_-}(\varphi_{\sigma_-}^i)$  in the Fourier plane on the left  
 166 side corresponds to the  $\sigma_-$  component of the desired vectorial field, where  $F$  represents the

167 Fourier transform. Similarly, illuminating a vortex beam with TAM  $|\sigma_-, -l_+^i\rangle$  (Figure 2b(II))  
 168 yields the output optical field  $F_+(\varphi_{\sigma_+}^i)$  on the left constituting the  $\sigma_+$  component. Due to the  
 169 opposite response of the geometry-phase on the two CP beams, conjugate vectorial holographic  
 170 images are produced on the right side, which are beyond the scope and thus ignored in the  
 171 following discussions. Finally, by illuminating an incident beam possessing the TAM pair of  
 172  $[\sigma_+, -l_-^i; \sigma_-, -l_+^i]$ , the desired vectorial field can be reconstructed on the left side (Figure  
 173 2b(III)).

174 Remarkably, our approach enables theoretically unlimited channels for SOL-VMH.  
 175 As a proof of concept, we demonstrate simulation results with four channels as an  
 176 example. As depicted in Figure 2a, we employ four optical vectorial fields, each encoded  
 177 with three binary patterns representing amplitude ( $A = 0.5, 0.866$ ), azimuth angle ( $\psi =$   
 178  $-45^\circ, 45^\circ$ ), and ellipticity angle ( $\chi = -22.5^\circ, 22.5^\circ$ ). The four encoded TAM pairs are [-  
 179 1, -12; 1, 11], [-1, 10; 1, -7], [-1, 8; 1, 9], and [-1, -16; 1, 5]. Therefore, four inverse  
 180 TAM pairs of [1, 12; -1, -11], [1, -10; -1, 7], [1, -8; -1, -9], and [1, 16; -1, -5] are utilized  
 181 as the incident light to reconstruct the desired vectorial fields as shown in **Figure 3(I)**.  
 182 The resulting amplitude profiles are shown in Figure 3(II), exhibiting point-like  
 183 distributions with designed patterns of “triangle”, “square”, “circle”, and “heart” shapes.  
 184 The corresponding polarizations of the output fields are further analyzed. Figures 3(III)  
 185 and 3(IV) illustrate the azimuth angle distributions of “fish”, “snail”, “octopus”, and  
 186 “rabbit”, and the ellipticity angle distributions of “starfish”, “butterfly”, “Mickey”, and  
 187 “turtle” in each channel, respectively, corresponding to the intentionally designed  
 188 patterns in Figure 2a. Detailed results can be referred to Supplementary Note 3). **It can**  
 189 **be observed that in the designated corresponding channel, the correct beam incident will**  
 190 **cause the corresponding spot array pattern to emerge, allowing for the further**  
 191 **identification of polarization without considering the crosstalk. A detailed analysis of**  
 192 **the crosstalk is provided in Supplementary Note 4.**

193

### 194 **2.3. Experimental results with identical OAMs in each TAM channel ( $l_+^i = l_-^i$ )**

195 In the following experiment, we concentrated on encoding the polarization state of the  
 196 optical vectorial field with a uniform intensity distribution to ensure optimal  
 197 experimental polarization outcomes. The footprint of metasurface supporting the SOL-  
 198 VMH is  $240 \mu\text{m} \times 240 \mu\text{m}$  consisting of  $800 \times 800$  pixels of nano-pillars. Scanning  
 199 electron microscope (SEM) images depicting the top and perspective view of the

200 fabricated metasurface are presented in **Figure 4a** (see more fabrication details in  
 201 Methods section). The experimental schematic diagram is shown in Figure 4b (see more  
 202 details in Methods section).

203 We first demonstrate identical OAM in each TAM pair, i.e.,  $l_+^i = l_-^i$ . As a proof of  
 204 concept, we utilized two TAM channels ( $N = 2$ ) for the multiplexing, encoded with TAM  
 205 pairs of  $[-1, 2; 1, 2]$  and  $[-1, 1; 1, 1]$ . In this scenario, each TAM pair degenerates into a linearly  
 206 polarized vector beam with a single OAM. Therefore, the experimental conditions can be  
 207 simplified, and vortex beams with TC of -2 or -1 in the horizontal polarization are illuminated  
 208 onto the metasurface to reconstruct the desired vectorial fields. In Figure 4b, the first quarter-  
 209 wave plate (QWP1) and one of the spatial light modulators (SLM2) in red dashed boxes are  
 210 blocked. As depicted in Figures 4c(I) and 4d(I), upon illuminating vector beams with  
 211 TCs of -2 and -1 in horizontal polarization, point-like intensity distributions are  
 212 observed in the corresponding regions. In contrast, donut-shaped intensity distributions  
 213 are observed when there is a TAM mismatch between input and output, as shown in  
 214 Figures 4c,d(IV). To assess the polarization properties, a polarizer is placed before the  
 215 camera for analysis. **Unlike previous studies of OAM multiplexed holography that utilize a**  
 216 **fundamental mode filtering aperture array for post processing<sup>[31]</sup>, in order to better reproduce**  
 217 **the experimental results, we present the original measured optical images in the following**  
 218 **discussions.** Figures 4c,d(II) and (III) present the measured intensity profiles with the  
 219 polarizer oriented at  $0^\circ$  and  $90^\circ$ , respectively, revealing complementary patterns due to  
 220 the orthogonality of horizontal and vertical polarizations. The polarization ellipticity  
 221 and azimuth angle distributions of the vectorial fields are determined via Stokes  
 222 parameter measurements (see more details in Supplementary Note 5). The reconstructed  
 223 patterns in channel 1, depicting a “deer” and a “bird”, and in channel 2, depicting a “fish”  
 224 and a “bat”, shown in the second row of Figures 4e and 4f, respectively, closely match  
 225 the target images displayed in the first row. **More experimental results of more channels**  
 226 **can be referred to Supplementary Note 6.**

227

#### 228 **2.4. Experimental results with distinct OAMs in each TAM channel ( $l_+^i \neq l_-^i$ )**

229 We proceed to demonstrate distinct OAMs in each TAM pair channel, i.e.,  $l_+^i \neq l_-^i$ .  
 230 Specifically, two channels are encoded with TAM pairs of  $[-1, 3; 1, 4]$  and  $[-1, 1; 1, 2]$ ,  
 231 respectively, each carrying different OAMs. The optical setup is shown in Figure 4b,  
 232 where a half-wave plate is employed to adjust the ratio of optical intensities reflected  
 233 by two SLMs, ensuring equivalent intensities within each TAM pair. Two sets of input



234 fields with each carrying a pair of TAMs, denoted as  $[1, -3; -1, -4]$  and  $[1, -1; -1, -2]$ , are  
235 used to reconstruct the optical vectorial fields, resulting in point-like intensity distributions  
236 in the corresponding areas without putting any optical analyzer as shown in **Figures**  
237 **5a,b(I)**. Figures 5a,b(II) and (III) display the optical output intensity distributions under  
238 two opposite SAM states, revealing complementary patterns due to the orthogonality of  
239 these two states. For a more direct comparison, the vectorial images designed in this  
240 experiment are identical to those in the preceding experiment shown in Figure 4. The  
241 reconstructed azimuth and ellipticity angle patterns for channel 1 (a “deer” and a “bird”)  
242 and channel 2 (a “fish” and a “bat”), as displayed in Figures 5a,b(IV) and (V), exhibit a  
243 strong agreement with their respective target images presented in the first row of Figures  
244 4e and 4f. To furtherly validate the robustness of the selectivity, vortex beams with  
245 various TCs ranging from 0 to -4 in horizontal polarization are projected onto the  
246 metasurface. The corresponding intensity distributions within the designed channels are  
247 illustrated in Figures 5c and 5d. Each of these images exhibits two types of donut  
248 patterns or their superimposed structures, which showcases the supposition of a pair of  
249 spin-orbit-locking holographic fields, thus underscoring high reliability and security of  
250 the proposed approach.

251

### 252 **3. Discussion**

253 We have successfully implemented spin-orbit-locking multi-channel vectorial  
254 metasurface holography using TAM as the input. Our approach allows for a pair of  
255 desired TAMs serving as the input to project onto a geometry-phase metasurface for the  
256 reconstruction of the corresponding optical vectorial field. This field exhibits a 2D  
257 point-like intensity distribution with designed profile of amplitude, polarization azimuth  
258 and ellipticity angles. The system demonstrates high selectivity, effectively preventing  
259 the acquisition of the optical vectorial field in case of incorrect inputs or mismatches  
260 between input and output. Only binary values for amplitude, azimuth, and ellipticity  
261 angles are presented to enhance visualization in this work. However, it’s important to  
262 note that these three parameters can take any arbitrary value, as they can be  
263 independently adjusted.

264 In this study, a geometry-phase metasurface consisting of  $\text{TiO}_2$  nanopillars exhibits  
265 an inherent broadband response of intensity and polarization in the visible range,  
266 ensuring robust performance across this range<sup>[6]</sup>. **The use of the geometric phase in a**  
267 **single unit cell with identical structures allows for high tolerance to fabrication errors**



268 in the metasurface. This results in holographic images deviating from the incident  
269 direction<sup>[40]</sup>. With improved fabrication techniques, more solutions can be introduced.  
270 The combination of geometric phase and propagation phase also enables the independent  
271 design of the required phase profiles for both spin states using the GS algorithm<sup>[41]</sup>.  
272 Furthermore, a supercell metasurface can bypass the GS algorithm and instead use the  
273 faster Fourier transform algorithm directly<sup>[42]</sup>.

274 The spin-orbit-locking phenomena can significantly enhance the hologram's capacity and  
275 security. This is due to the OAM, which theoretically provides an unlimited set of orthogonal  
276 helical modes represented by an extensive range of TCs. Additionally, the distinct OAMs in  
277 each TAM pair lead to asymmetric responses in the two orthogonal spin states. Combined with  
278 advanced techniques such as electrically tunable metamaterials and nonlinear optical responses,  
279 this feature can further enhance the application potential, particularly in the realms of optical  
280 encryption and video recording<sup>[31, 43, 44]</sup>. Moreover, integration with other DoFs, such as  
281 wavelength, is feasible by introducing varied meta-pillar structures<sup>[37]</sup>. The optical  
282 vectorial field in each channel can be further selectively positioned in different planes  
283 for generating three-dimensional SOL-VMH by incorporating an Fourier transform (FT)  
284 lens with chosen focal lengths<sup>[31]</sup> (see more details of numerical demonstration in  
285 Supplementary Note 7). Such a feature can enhance the integration density of the device,  
286 thereby providing more information with greater DoFs for broader applications such as  
287 OAM-selected virtual/augmented reality (VR/AR) and OAM-selected light detection  
288 and range (LiDAR)<sup>[45]</sup>. So far, the fabrication of metasurfaces is relatively expensive,  
289 making large-scale production challenging. In the future, improvements in  
290 manufacturing processes<sup>[46-48]</sup>, including DUV lithography and nanoimprint lithography,  
291 as well as the development of more efficient materials applicable to various wavelength  
292 bands, will lead to cost-effective mass production of metasurfaces. This will greatly  
293 unleash the application potential of metasurfaces.

294 Overall, the development of SOL-VMH enables the simultaneous and independent  
295 control of SAM and OAM, introduces multiple degrees of selectivity due to the spin-  
296 orbit-locking feature. This advancement holds promise to revolutionize optics by  
297 offering locally controlled complex optical fields through various TAM pairs. The  
298 increased capacity and security in optical communication systems achieved through such  
299 precise control could enhance the information encoding reliability. Furthermore, the  
300 intricate manipulation of polarization, phase, and intensity paves the way for enhanced  
301 interactions with other research fields, such as quantum computing and optical force

302 manipulation, where the control and manipulation of light at such granular levels are  
303 paramount.

304

## 305 **Methods**

### 306 **Sample fabrication**

307 The metasurface fabrication process involves electron beam lithography (EBL) and  
308 reactive ion etching (RIE) techniques. Initially, a 600-nm-thick titanium dioxide ( $\text{TiO}_2$ )  
309 film is deposited on a polished glass ( $\text{SiO}_2$ ) substrate via electron beam evaporation  
310 (EBE). A polymethyl methacrylate (PMMA) layer is then spin-coated onto the  $\text{TiO}_2$  film.  
311 After EBL, the PMMA resist is developed, and a chromium (Cr) film is deposited onto  
312 the patterned PMMA using EBE. The unexposed PMMA is removed, and the  $\text{TiO}_2$  is  
313 etched using RIE. The process concludes with the removal of the Cr mask through  
314 chemical etching.

### 315 **Optical setup**

316 The experimental schematic diagram for the vectorial holography is Fig. 3b. A collimated  
317 continuous-wave coherent laser beam with a wavelength of 488nm (OBIS, Coherent, USA) is  
318 first polarized by a half wave plate (HWP) and then directed onto a beam splitter (BS),  
319 which divides it into two distinct beams. Each beam is then projected perpendicularly  
320 onto a spatial light modulator (SLM, HOLOEYE PLUTO VIS056, German). A polarizer  
321 (P1) with horizontal polarization is placed in front of one SLM (SLM1), and the other polarizer  
322 (P2) with the vertical polarization is placed in front of the remaining SLM (SLM2), enabling  
323 the modulation of these two beams with orthogonally linear polarizations. These two  
324 beams are modulated and reflected by SLMs and reunified into a single beam at the  
325 original BS. A quarter-wave plate (QWP1) is positioned behind the beam splitter (BS)  
326 to transform horizontally polarized light into a SAM state of  $\sigma^-$  and vertically polarized  
327 light into a SAM state of  $\sigma^+$ . Then it is slightly focused on the metasurface using a lens (L1,  
328  $f = 50$  mm) to generate optical vector field. An aspheric lens (L2,  $f = 16$  mm) with a  
329 numerical aperture (NA) of 0.65 is placed behind the metasurface to execute a Fourier  
330 transform on the optical field in the metasurface plane. The optical field in the focal  
331 plane of L2 is transmitted to a scientific CMOS (sCMOS) camera (Dhyana 400BSI V2,  
332 Tucson, China) via a 4-f system, which includes a pair of lenses (L3 and L4, not shown  
333 in the figure for simplicity). To analyze the polarization state of the optical field, another  
334 QWP (QWP2) and a polarizer (P3) are strategically positioned in front of the sCMOS  
335 camera.

336 **Supporting Information**

337 Supporting Information is available from the Wiley Online Library or from the author.

338

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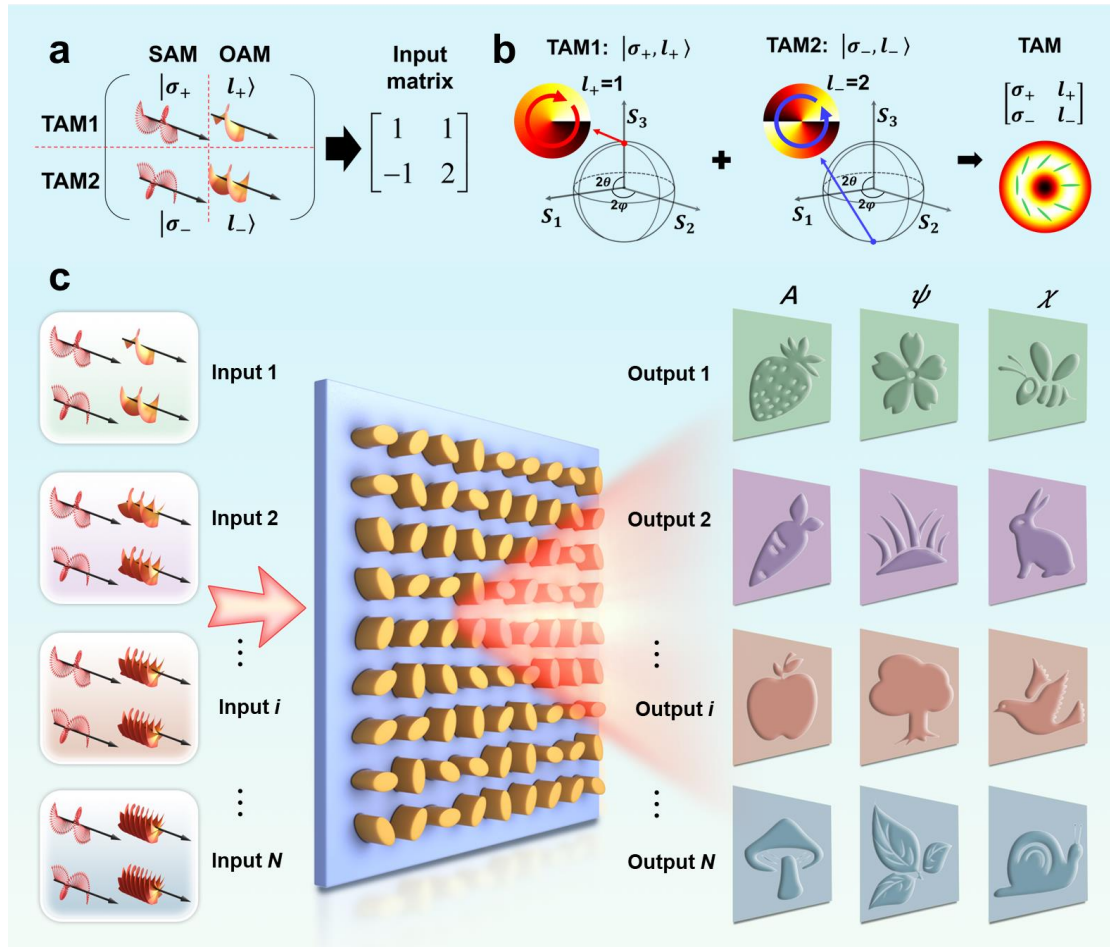
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353 **References**

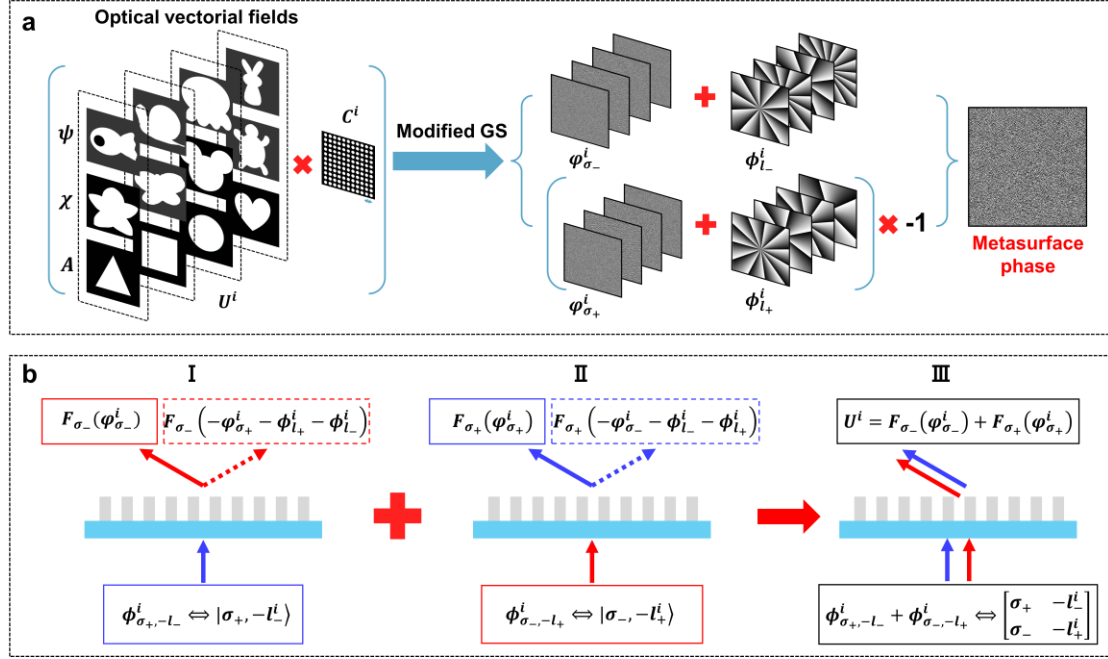
- 354 [1] Z.-L. Deng, Z.-Q. Wang, F.-J. Li, M.-X. Hu, X. Li, *Nanophotonics* **2022**, 11, 1725.  
 355 [2] Q. Song, X. Liu, C.-W. Qiu, P. Genevet, *Appl. Phys. Rev.* **2022**, 9, 011311.  
 356 [3] Q. Song, A. Baroni, P. C. Wu, S. Chenot, V. Brandli, S. Vezian, B. Damilano, P. de  
 357 Mierry, S. Khadir, P. Ferrand, P. Genevet, *Nat. Commun.* **2021**, 12, 3631.  
 358 [4] Z. Yang, P.-S. Huang, Y.-T. Lin, H. Qin, J. Zúñiga-Pérez, Y. Shi, Z. Wang, X. Cheng,  
 359 M.-C. Tang, S. Han, *Nat. Commun.* **2024**, 15, 232.  
 360 [5] T. Todorov, L. Nikolova, N. Tomova, *Appl. Optics* **1984**, 23, 4309.  
 361 [6] Q. Song, A. Baroni, R. Sawant, P. Ni, V. Brandli, S. Chenot, S. Vezian, B. Damilano,  
 362 P. de Mierry, S. Khadir, P. Ferrand, P. Genevet, *Nat. Commun.* **2020**, 11, 2651.  
 363 [7] J. P. Balthasar Mueller, N. A. Rubin, R. C. Devlin, B. Groever, F. Capasso, *Phys. Rev.*  
 364 *Lett.* **2017**, 118, 113901.  
 365 [8] M. Liu, W. Zhu, P. Huo, L. Feng, M. Song, C. Zhang, L. Chen, H. J. Lezec, Y. Lu, A.  
 366 Agrawal, *Light Sci. Appl.* **2021**, 10, 107.  
 367 [9] Q. Song, M. Odeh, J. Zúñiga-Pérez, B. Kanté, P. Genevet, *Science* **2021**, 373, 1133.  
 368 [10] A. C. Overvig, S. Shrestha, S. C. Malek, M. Lu, A. Stein, C. Zheng, N. Yu, *Light Sci.*  
 369 *Appl.* **2019**, 8, 92.  
 370 [11] B. Xiong, Y. Liu, Y. Xu, L. Deng, C.-W. Chen, J.-N. Wang, R. Peng, Y. Lai, Y. Liu,  
 371 M. Wang, *Science* **2023**, 379, 294.  
 372 [12] Z. L. Deng, Q. A. Tu, Y. Wang, Z. Q. Wang, T. Shi, Z. Feng, X. C. Qiao, G. P. Wang,  
 373 S. Xiao, X. Li, *Adv. Mater.* **2021**, 33, 2103472.  
 374 [13] S. So, J. Kim, T. Badloe, C. Lee, Y. Yang, H. Kang, J. Rho, *Adv. Mater.* **2023**, 35,  
 375 2208520.  
 376 [14] T. Naeem, J. Kim, H. S. Khaliq, J. Seong, M. T. S. Chani, T. Tauqeer, M. Q.  
 377 Mehmood, Y. Massoud, J. Rho, *Adv. Opt. Mater.* **2023**, 11, 2202278.

- 378 [15] P. Georgi, Q. Wei, B. Sain, C. Schlickriede, Y. Wang, L. Huang, T. Zentgraf, *Sci. Adv.*  
379 **2021**, 7, eabf9718.
- 380 [16] S. Latif, J. Kim, H. S. Khaliq, N. Mahmood, M. A. Ansari, X. Chen, J. Akbar, T.  
381 Badloe, M. Zubair, Y. Massoud, *Nano Lett.* **2024**, 24, 708.
- 382 [17] H. Zhou, B. Sain, Y. Wang, C. Schlickriede, R. Zhao, X. Zhang, Q. Wei, X. Li, L.  
383 Huang, T. Zentgraf, *ACS Nano* **2020**, 14, 5553.
- 384 [18] Z. Yu, H. Li, W. Zhao, P.-S. Huang, Y.-T. Lin, J. Yao, W. Li, Q. Zhao, P. C. Wu, B.  
385 Li, *Nat. Commun.* **2024**, 15, 2607.
- 386 [19] S. A. Schulz, R. Oulton, M. Kenney, A. Alù, I. Staude, A. Bashiri, Z. Fedorova, R.  
387 Kolkowski, A. F. Koenderink, X. Xiao, *Appl. Phys. Lett.* **2024**, 124.
- 388 [20] Q. Song, S. Khadir, S. Vézian, B. Damilano, P. de Mierry, S. Chenot, V. Brandli, R.  
389 Laberdesque, B. Wattellier, P. Genevet, *Nanophotonics* **2020**, 10, 697.
- 390 [21] Z. Yang, P.-S. Huang, Y.-T. Lin, H. Qin, J. Chen, S. Han, W. Huang, Z.-L. Deng, B.  
391 Li, J. Zúñiga-Pérez, *Nano letters* **2024**, 24, 844.
- 392 [22] S. Zhang, L. Huang, X. Li, R. Zhao, Q. Wei, H. Zhou, Q. Jiang, G. Geng, J. Li, X. Li,  
393 *Acs Photonics* **2021**, 8, 1746.
- 394 [23] D. Wang, F. Liu, T. Liu, S. Sun, Q. He, L. Zhou, *Light Sci. Appl.* **2021**, 10, 67.
- 395 [24] Z.-L. Deng, J. Deng, X. Zhuang, S. Wang, K. Li, Y. Wang, Y. Chi, X. Ye, J. Xu, G. P.  
396 Wang, *Nano Lett.* **2018**, 18, 2885.
- 397 [25] N. Mao, G. Zhang, Y. Tang, Y. Li, Z. Hu, X. Zhang, K. Li, K. Cheah, G. Li, *Proc.*  
398 *Natl. Acad. Sci. U. S. A.* **2022**, 119, e2204418119.
- 399 [26] Z. L. Deng, M. Jin, X. Ye, S. Wang, T. Shi, J. Deng, N. Mao, Y. Cao, B. O. Guan, A.  
400 Alù, *Adv. Funct. Mater.* **2020**, 30, 1910610.
- 401 [27] Y. Shen, X. Wang, Z. Xie, C. Min, X. Fu, Q. Liu, M. Gong, X. Yuan, *Light Sci. Appl.*  
402 **2019**, 8, 90.
- 403 [28] L. Allen, M. W. Beijersbergen, R. J. Spreeuw, J. P. Woerdman, *Phys. Rev. A.* **1992**,  
404 45, 8185.
- 405 [29] A. M. Yao, M. J. Padgett, *Adv. Opt. Photonics* **2011**, 3, 161.
- 406 [30] A. H. Dorrah, N. A. Rubin, M. Tamagnone, A. Zaidi, F. Capasso, *Nat. Commun.* **2021**,  
407 12, 6249.
- 408 [31] H. Ren, X. Fang, J. Jang, J. Bürger, J. Rho, S. A. Maier, *Nat. Nanotechnol.* **2020**, 15,  
409 948.
- 410 [32] H. Ren, G. Briere, X. Fang, P. Ni, R. Sawant, S. Héron, S. Chenot, S. Vézian, B.  
411 Damilano, V. Brändli, S. A. Maier, P. Genevet, *Nat. Commun.* **2019**, 10, 2986.
- 412 [33] X. Fang, H. Ren, M. Gu, *Nat. Photonics* **2020**, 14, 102.
- 413 [34] Z. Shi, Z. Wan, Z. Zhan, K. Liu, Q. Liu, X. Fu, *Nat. Commun.* **2023**, 14, 1869.
- 414 [35] L.-J. Kong, Y. Sun, F. Zhang, J. Zhang, X. Zhang, *Phys. Rev. Lett.* **2023**, 130, 053602.
- 415 [36] H. Yang, P. He, K. Ou, Y. Hu, Y. Jiang, X. Ou, H. Jia, Z. Xie, X. Yuan, H. Duan,  
416 *Light Sci. Appl.* **2023**, 12, 79.
- 417 [37] G. He, Y. Zheng, C. Zhou, S. Li, Z. Shi, Y. Deng, Z.-K. Zhou, *Light Sci. Appl.* **2024**,  
418 13, 98.
- 419 [38] Z. H. Jiang, L. Kang, T. Yue, H. X. Xu, Y. Yang, Z. Jin, C. Yu, W. Hong, D. H.  
420 Werner, C. W. Qiu, *Adv. Mater.* **2020**, 32, 1903983.
- 421 [39] G. Milione, H. Sztul, D. Nolan, R. Alfano, *Phys. Rev. Lett.* **2011**, 107, 053601.
- 422 [40] X. Zhang, S. Yang, W. Yue, Q. Xu, C. Tian, X. Zhang, E. Plum, S. Zhang, J. Han, W.  
423 Zhang, *Optica* **2019**, 6, 1190.
- 424 [41] Y. Xu, H. Zhang, Q. Li, X. Zhang, Q. Xu, W. Zhang, C. Hu, X. Zhang, J. Han, W.  
425 Zhang, *Nanophotonics* **2020**, 9, 3393.
- 426 [42] T. Wu, Q. Xu, X. Zhang, Y. Xu, X. Chen, X. Feng, L. Niu, F. Huang, J. Han, W.  
427 Zhang, *Adv. Sci.* **2022**, 9, 2204664.

- 428 [43] I. Kim, J. Jang, G. Kim, J. Lee, T. Badloe, J. Mun, J. Rho, *Nat. commun.* **2021**, 12,  
429 3614.
- 430 [44] C. Jung, G. Kim, M. Jeong, J. Jang, Z. Dong, T. Badloe, J. K. Yang, J. Rho, *Chem.*  
431 *Rev.* **2021**, 121, 13013.
- 432 [45] Y. Yang, J. Seong, M. Choi, J. Park, G. Kim, H. Kim, J. Jeong, C. Jung, J. Kim, G.  
433 Jeon, K.-i. Lee, D. H. Yoon, J. Rho, *Light Sci. Appl.* **2023**, 12, 152.
- 434 [46] J. Seong, Y. Jeon, Y. Yang, T. Badloe, J. Rho, *Int. J. Pr. Eng. Man-GT* **2024**, 11, 685.
- 435 [47] Y. Yang, H. Kang, C. Jung, J. Seong, N. Jeon, J. Kim, D. K. Oh, J. Park, H. Kim, J.  
436 Rho, *ACS Photonics* **2023**, 10, 307.
- 437 [48] H. Kang, D. Lee, Y. Yang, D. Kyo Oh, J. Seong, J. Kim, N. Jeon, D. Kang, J. Rho,  
438 *Photon. Insights* **2023**, 2, R04.
- 439
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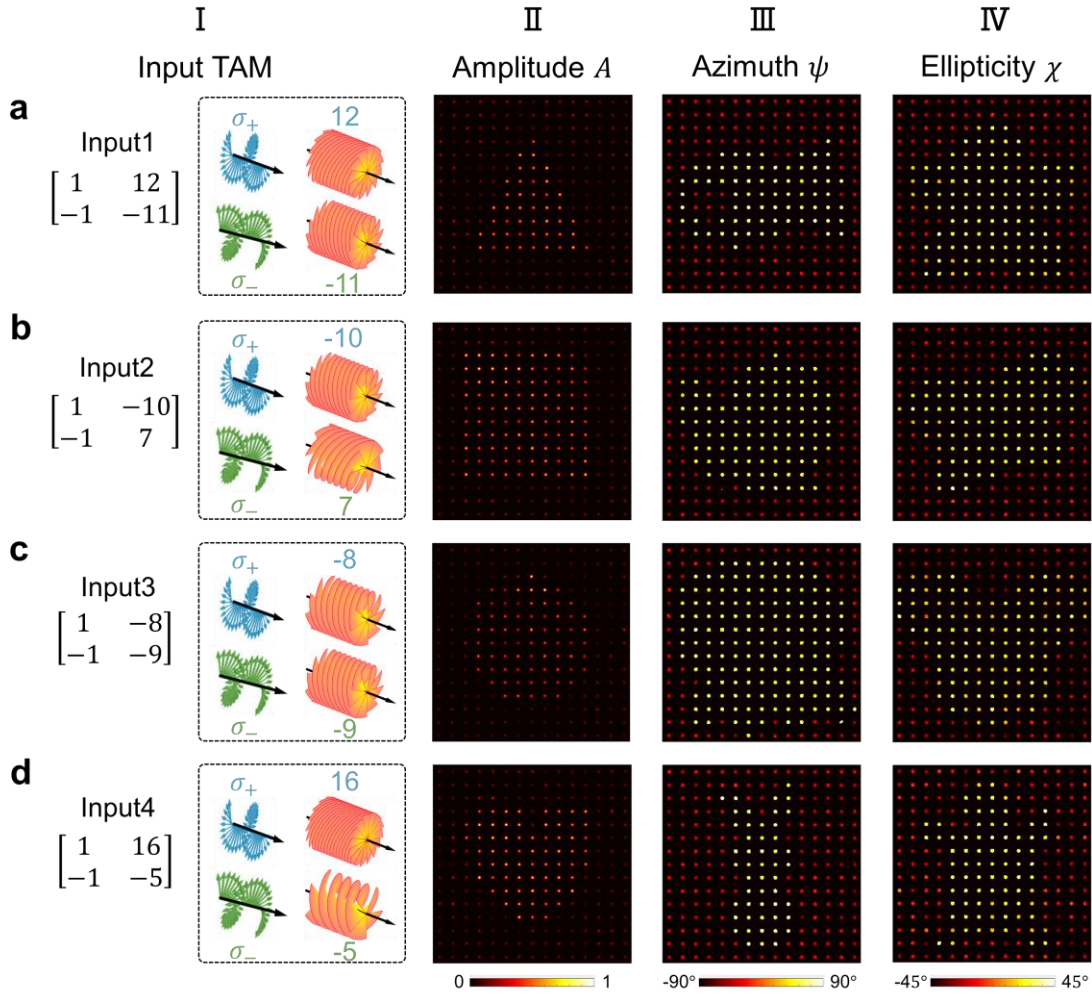


**Figure 1. Schematic illustration of the SOL-VMH.** (a) In each channel, the SOL-VMH is achieved by illuminating a desired input composed with a pair of TAMs (TAM1 and TAM2) that consist of orthogonal SAM states with each independently locked by an arbitrary OAM. The input can be simplified as  $[\sigma_+, l_+; \sigma_-, l_-]$ . (b) TAM1 and TAM2 can be represented as two higher-order Poincaré sphere beams. The superposition of them gives rise to a vectorial vortex beam with modulated polarization distribution. (c) The metasurface can be multiplexed with various channels of SOL-VMH. In each channel, the reconstructed vectorial field is characterized by desired amplitude ( $A$ ), polarization azimuth ( $\psi$ ) and ellipticity ( $\chi$ ) distributions.

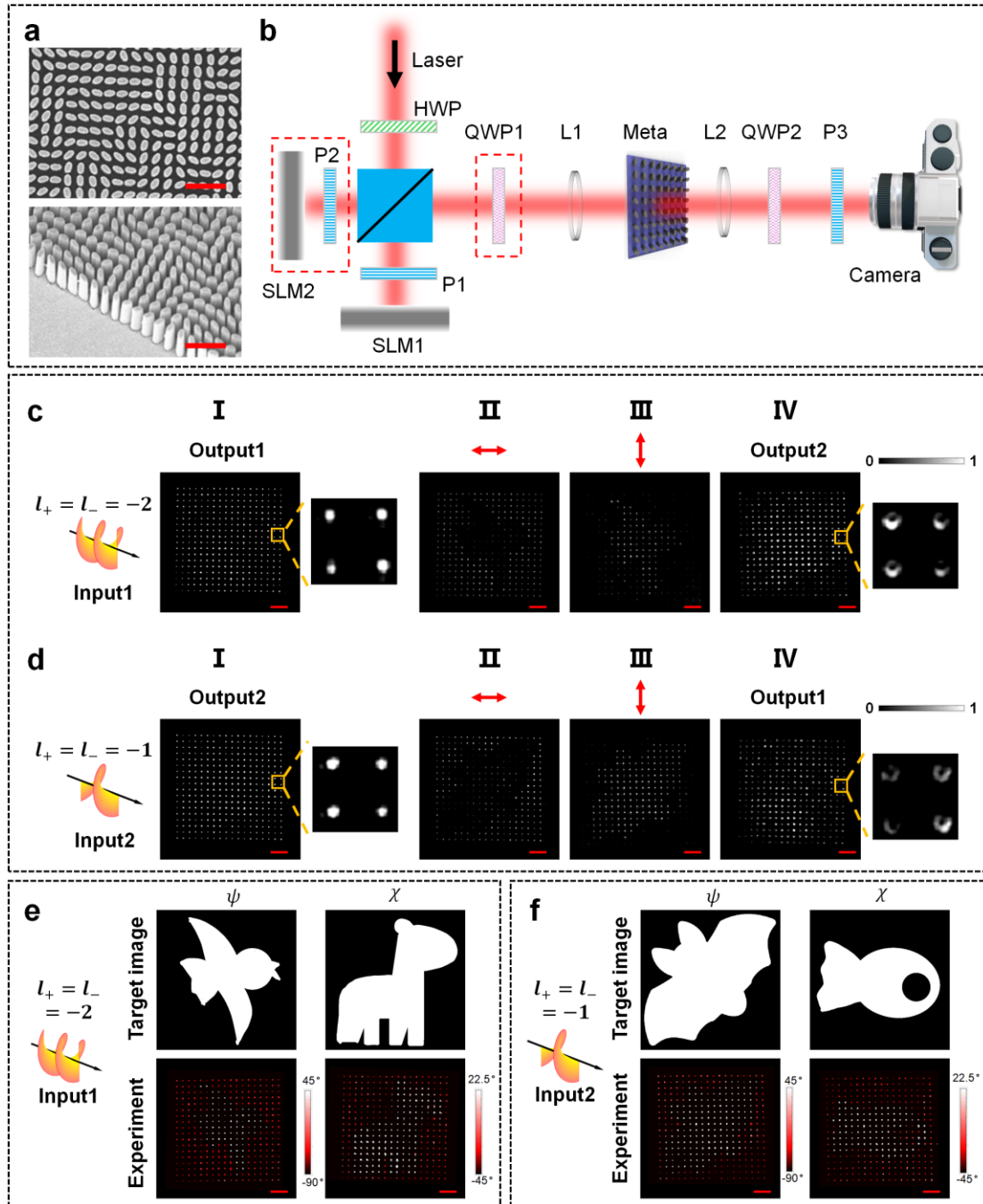


**Figure 2. Flowchart and design principle of the SOL-VMH.** (a) The flowchart for the design of four-channel SOL-VMH. In each channel, the optical vectorial field is encoded with three binary patterns of amplitude ( $A$ ), azimuth angle ( $\psi$ ) and ellipticity angle ( $\chi$ ) distributions. This field is then multiplied by a 2D Dirac comb function positioned at distinct spatial locations across the optical field. Subsequently, a modified GS algorithm is employed to extract a pair of phases in two opposite SAM states. The obtained phase is added with the corresponding OAM helical phase. The phase added in  $\sigma_+$  SAM state is further inverted. This yields a pair of processed phases in two opposite SAM states, forming the hologram for each channel. The final phase profile is superimposed by all holographic channels, leading to the design of multi-channel SOL-VMH. (b) The demonstration of a geometry-phase metasurface supporting the SOL-VMH with a non-interleaved method in each channel.  $F$  represents the Fourier transform.



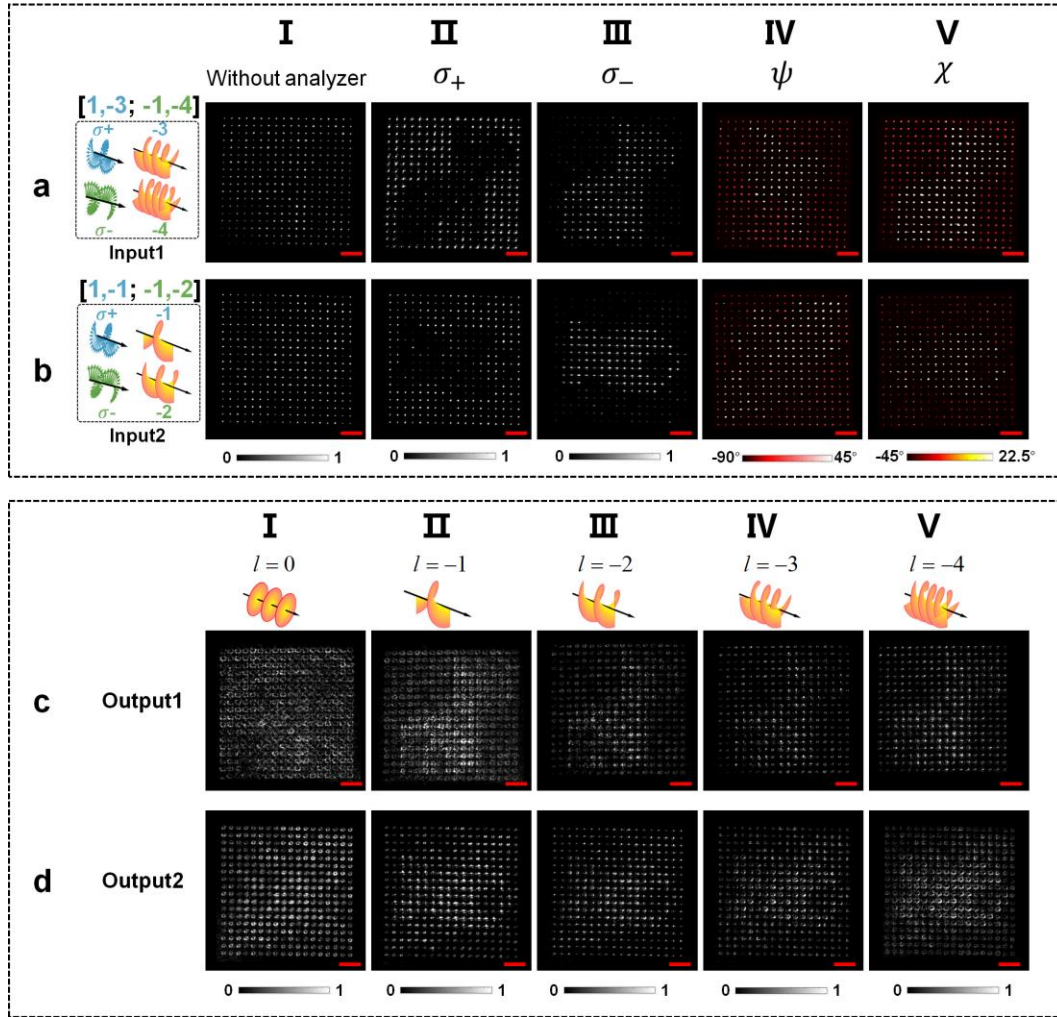


**Figure 3. Numerical demonstration of four-channel SOL-VMH with four sets of TAM pairs.** The four encoded TAM pairs are  $[-1, -12; 1, 11]$ ,  $[-1, 10; 1, -7]$ ,  $[-1, 8; 1, 9]$ , and  $[-1, -16; 1, 5]$ . Therefore, four inverse TAM pairs of (a)  $[1, 12; -1, -11]$ , (b)  $[1, -10; -1, 7]$ , (c)  $[1, -8; -1, -9]$ , and (d)  $[1, 16; -1, -5]$  are utilized as the incident light to reconstruct the desired vectorial fields. Column I show the input matrices. The images employed for the amplitude design are “triangle”, “square”, “circle”, and “heart” shapes as shown in column II. The images employed for the polarization azimuth design are “fish”, “snail”, “octopus”, and “rabbit” as shown in column III. The images employed for the polarization ellipticity design are “starfish”, “butterfly”, “Mickey”, and “turtle”.



**Figure 4. Experimental characterization with identical OAMs in each TAM pair channel ( $l_+^i = l_-^i$ ).** (a) SEM images of the fabricated metasurface under top and tilted view. Scale bar:  $1\mu\text{m}$ . (b) Experimental setup of the holographic measurement. HWP: half wave plate; L1-L2: lens; Meta: metasurface; P1-P3: polarizers; QWP1-QWP2: quarter-wave plate; SLM1-SLM2: spatial light modulators. Components in red dashed boxes should be removed in related experiments. (c) Results of optical intensity distributions by illuminating a horizontally polarized vortex beam with a TC of -2. (d) Results of optical intensity distributions by illuminating a horizontally polarized vortex beam with a TC of -1. (e) Target images (top) and measured azimuth and ellipticity angle distributions (bottom) by illuminating a horizontally polarized vortex beam with a TC of -2. (f)

Target images (top) and measured azimuth and ellipticity angle distributions (bottom) by illuminating a horizontally polarized vortex beam with a TC of -1. Scale bar: 500  $\mu\text{m}$ .



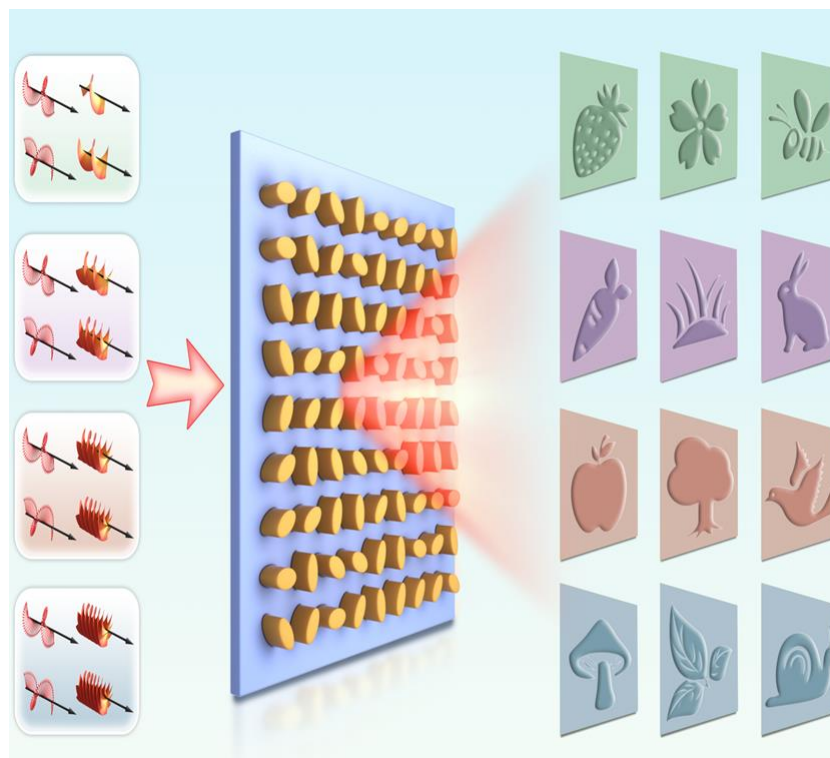
**Figure 5. Experimental characterization with distinct OAMs in each TAM pair channel ( $l_+^i \neq l_-^i$ ).** (a) Measured results by illuminating a TAM pair of [1, -3; -1, -4]; (b) Measured results by illuminating a TAM pair of [1, -1; -1, -2]; (c) Results of optical intensity distributions in the region of Output1 by illuminating a horizontally polarized vortex beam with TCs ranging from 0 to -4; (d) Results of optical intensity distributions in the region of Output2 by illuminating a horizontally polarized vortex beam with TCs ranging from 0 to -4. Scale bar: 500  $\mu\text{m}$ .

## Spin-Orbit-Locking Vectorial Metasurface Holography

A new multi-channel vectorial holography method is introduced by using spin-orbit-locking vortex beams, which encodes TAMs with SAMs and OAMs. This approach allows for high selectivity in SAM and OAM, enabling advanced optical information processing and photonic device design.

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### ToC figure



## Supporting Information

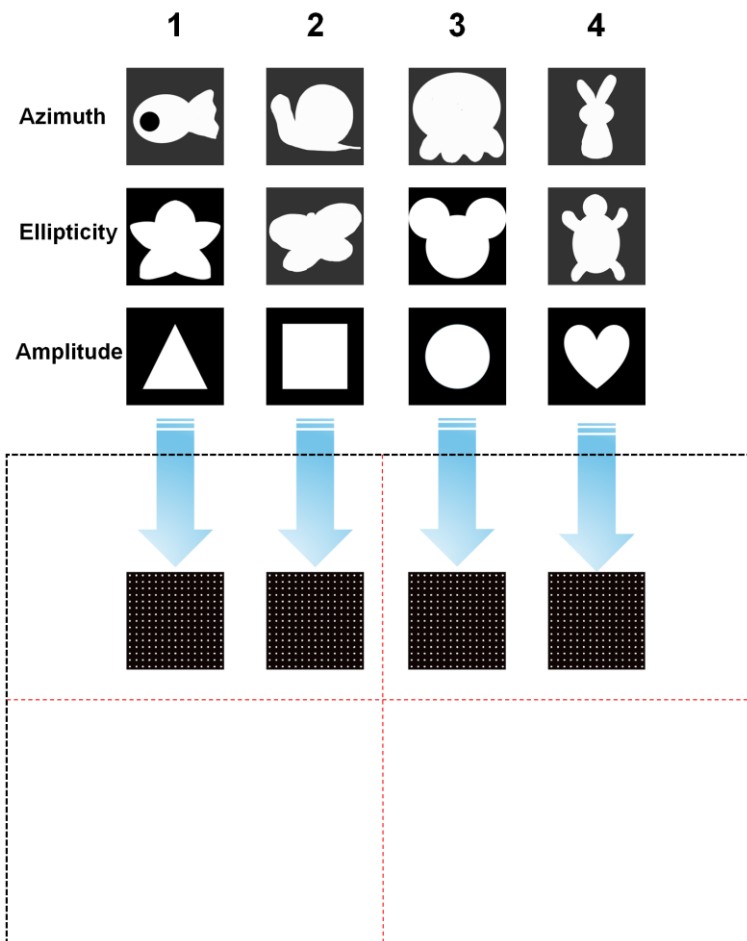
**Spin-Orbit-Locking Vectorial Metasurface Holography**

Zhipeng Yu, Xinyue Gao, Jing Yao, Haoran Li, Yuzhi Shi, Bo Li, Zhenwei Xie,

Xiaocong Yuan<sup>\*</sup>, Puxiang Lai,<sup>\*</sup> and Qinghua Song<sup>\*</sup>

**Supplementary Note 1: Detailed arrangement of the vectorial fields.**

Each vectorial field with a 2D point-like field distribution with desired amplitude ( $A$ ), polarization azimuth ( $\psi$ ) and ellipticity ( $\chi$ ) angle distribution is arranged at a distinct position with an equal interval.



**Figure S1. The arrangement of vector fields in the Fourier plane.**



Supplementary Note 2: Modified Gerchberg-Saxton (GS) algorithm.

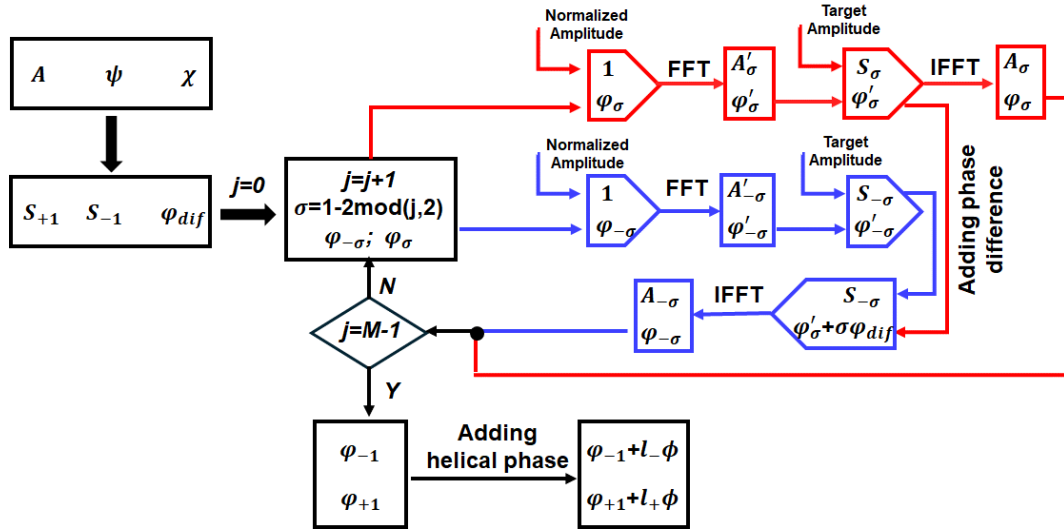


Figure S2. The flowchart of modified GS algorithm.

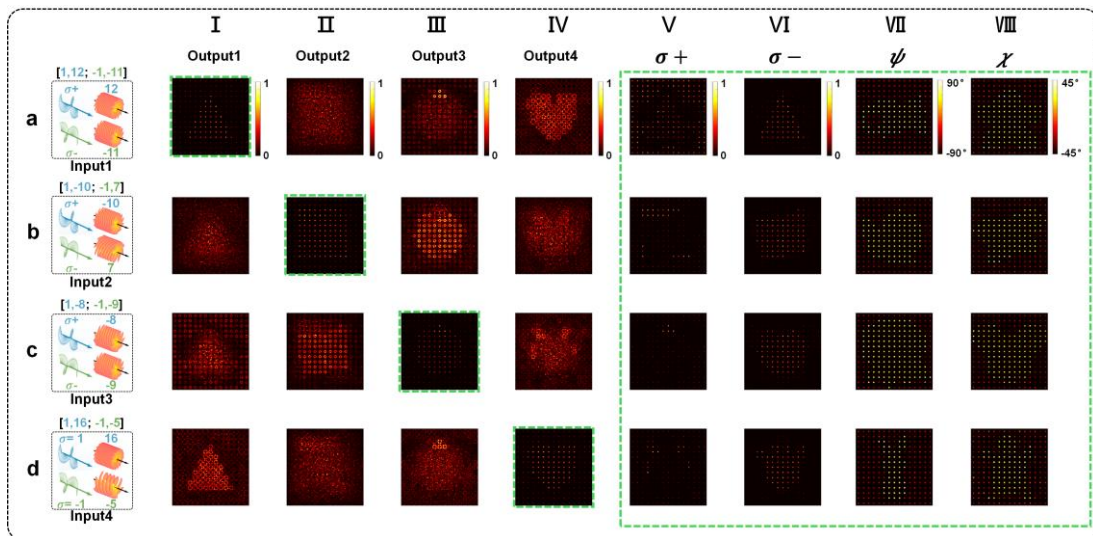
The modified GS algorithm flow chart for multiple OAM and polarization encryption is shown in Fig. S2. Two amplitude values ( $S_{+1}$  and  $S_{-1}$ ) with opposite spin states and phase difference ( $\varphi_{dif}$ ) between these two spin states in the Fourier plane can be calculated from a vectorial field with desired amplitude ( $A$ ), azimuth angle ( $\psi$ ) and ellipticity angle ( $\chi$ ). The initial phases with opposite spin states are randomly selected and then substituted into the algorithm for iteration. To demonstrate the process in detail, we use  $\sigma = 1$  (LCP) as a reference. In a single iteration cycle, the flowchart marked in red corresponds to deriving the phase profile of the spin state for  $\sigma = 1$ , while the one marked in blue corresponds to  $\sigma = -1$  (RCP). After performing a fast Fourier transform (FFT), the amplitudes and phases of the two spin states in the Fourier plane can be obtained. Then, the amplitudes of both spin states will be replaced with the desired amplitudes. The phase for  $\sigma = 1$  is maintained, while the phase for  $\sigma = -1$  will be replaced with the sum of the phase for  $\sigma = 1$  and the required phase difference ( $\varphi_{dif}$ ) between these two spin states. An inverse FFT is ultimately applied to these two optical fields to obtain the desired optical fields on the metasurface plane, with only the phases carried over to the next cycles, specifically for  $\sigma = -1$ . In each iteration, the processes for the two



spin states are alternated and linked by  $\varphi_{dif}$ , with their order determined by whether the iteration count is odd or even. Finally, two different OAM phases are added into the calculated phases for the respective spin states after the iteration process.

**Supplementary Note 3: Detailed simulation results of multi-channel TAM vectorial holography.**

As shown in Figs. S3(I-IV), the diagonal images, marked with green dotted boxes, exclusively exhibit point-like intensity distributions with the correct patterns for valid input-output pairs, which include, respectively, triangle, square, circle, and heart shapes. In contrast, the other images show either multiple donut-like ring structures or random superimpositions of these patterns. The polarizations of the output fields corresponding to the diagonal images will be be further analyzed. Figs. S3(V) and S3(VI) display the optical intensity distributions in two opposite SAM states, which exhibit complementary patterns due to the orthogonality of these states. Furthermore, Figs. S3(VII) and S3(VIII) show the ellipticity angle distributions of a five-point star, butterfly, Mickey, and turtle, and the azimuth angle distributions of fish, snail, octopus, and rabbit in each channel, respectively.



**Figure S3. Numerical demonstration of the TAM vectorial metasurface holography.**

### Supplementary Note 4: Analysis of crosstalk in the TAM vectorial metasurface holography.

Figure S4 shows the simulated results of the full optical intensity distribution when the metasurface is illuminated by different TAM pairs. It can be observed that in the designated channels, when the correct TAM is incident, the corresponding pattern exhibits a spot array. However, crosstalk from other channels will display donut patterns instead of spot array due to the mismatch in TAM. Therefore, by placing different vectorial field channels at different spatial locations, the desired image with spot array can be clearly identified. For example, in Fig. S4a, a designed triangular pattern is displayed for the desired channel (highlighted in the dashed box), where each spot in the pattern exhibits a Gaussian beam profile. In contrast, the other channels exhibit two types of donut patterns or their superimposed patterns, which can be easily distinguished. This characteristic allows us to filter out the desired image and effectively avoid the crosstalk.

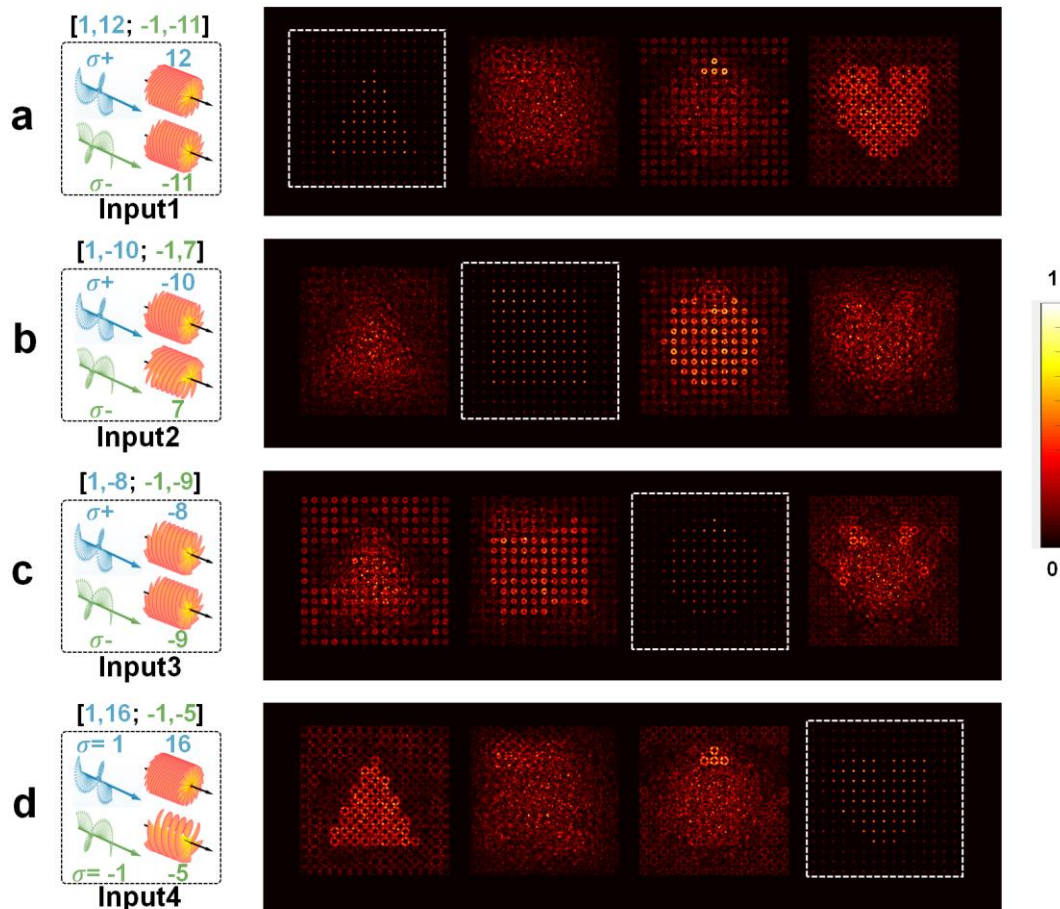


Figure S4. Analysis of crosstalk in the TAM vectorial metasurface holography.

**Supplementary Note 5: Stokes parameter measurement.**

In order to measure the polarization of the vectorial field accurately, an 8-step measurement is taken without removing or inserting any components during the process<sup>[1]</sup>. The optical setup of the polarization measurement is shown in Fig. S5. A quarter waveplate (QWP) with its fast axis along the x-axis and a linear polarizer (LP) with the transmission axis along the x-axis are placed before the holographic image. We need to select at least eight angles with equal intervals between 0° and 180° to measure the optical power. The measured powers at 8 different angles (shown in Table S1) can be used to calculate four Fourier coefficients (A, B, C, and D):

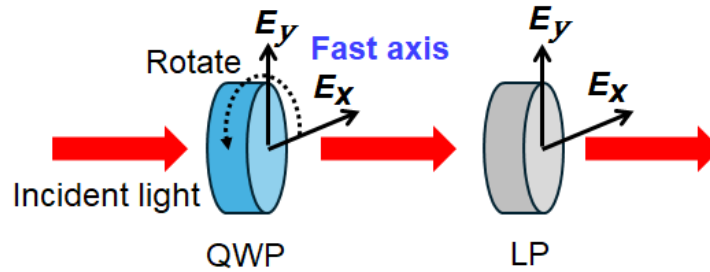
$$A = \frac{2}{N} \sum_{n=1}^N P_n; B = \frac{4}{N} \sum_{n=1}^N P_n \sin(2\theta_n); C = \frac{4}{N} \sum_{n=1}^N P_n \cos(4\theta_n); D = \frac{4}{N} \sum_{n=1}^N P_n \sin(4\theta_n) \quad (S1)$$

Thereafter, using these coefficients, we can calculate the Stokes parameters:

$$S_0 = A - C; S_1 = 2C; S_2 = 2D; S_3 = B \quad (S2)$$

The azimuth angle and ellipticity angle can be extracted from the Stokes parameter as:

$$\psi = \frac{1}{2} \tan^{-1}(S_2 / S_1); \chi = \frac{1}{2} \sin^{-1}(S_3 / S_0) \quad (S3)$$



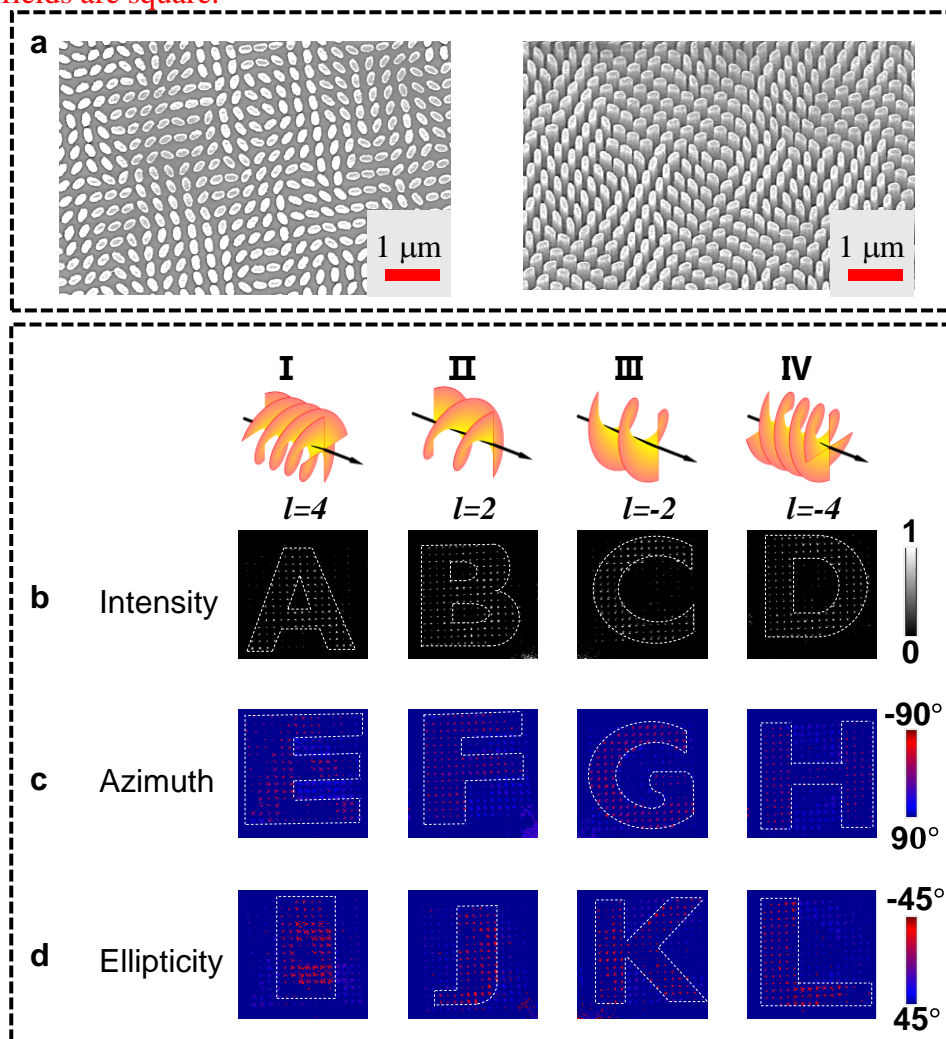
**Figure S5. Optical setup of the polarization measurement.** LP: linear polarizer; QWP: quarter waveplate.

n	1	2	3	4	5	6	7	8
Waveplate angle(°)	0	22.5	45	67.5	90	112.5	135	157.5
Measured power	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$

**Table S1 Eight powers at eight different rotation angles with an interval of 22.5°.**

**Supplementary Note 6: Experimental results of multiplexing four optical vectorial fields with each uniquely encoded with independent amplitude, polarization azimuth, and ellipticity angle.**

We have also made significant advancements by experimentally designing four optical vectorial fields. Each channel is uniquely encoded with independent amplitude, polarization azimuth, and ellipticity angles, enabling the transmission of twelve capital letters from “A” to “L”, as shown in Fig. S6. Scanning electron microscope (SEM) images depicting the top and perspective view of the new fabricated metasurface are presented in Fig. S6a. To present our results more clearly, we have outlined the contours of each pattern. Four optical spot arrays, each with distinct patterns labeled from “A” to “D”, can be observed in the corresponding channels when the metasurface is illuminated with vortex beams (VBs) featuring topological charges (TCs) ranging from -4 to 4, as shown in Fig. S6b. Additionally, Figs. S6c and S6d display the polarization azimuth and ellipticity angle distributions of the corresponding vector fields. All patterns can be clearly observed except for the letter “I”, which is primarily attributed to fabrication errors of the metasurface. In principle, the maximum number of vectorial channels is given by the formula  $\frac{N^2/n^2}{2}$ , where  $N$  and  $n$  are the pixel number along the width of the metasurface and each vectorial field, respectively, assuming the metasurface and all vectorial fields are square.



**Figure S6. Experimental characterization with identical OAMs in four TAM pair channels ( $l_+^i = l_-^i$ ).** (a) SEM images of the fabricated metasurface under top and tilted view. Optical intensity distributions (b), azimuth angle distributions (c), and ellipticity angle

distributions (**d**) are presented for each vector field when illuminated by a horizontally polarized vortex beam. The topological charges (TCs) of the vortex beam are -4, -2, 2, and 4, respectively. The dashed line in b-d is manually added to enhance the visualization.



**Supplementary Note 7: Simulation results of 3D multi-channel SOL-VMH.**

To illustrate the potential of our approach for 3D vectorial holography, four channels are designed at two image planes by further adding different FT lens functions to the phase information of the holograms. To be specific, these two planes have focal lengths of 1 mm and 2 mm, respectively, with each plane featuring two channels. In Plane 1, the vector fields at Outputs 1 and 2 can only be reconstructed when the correct illumination is applied (as shown in Figs. S7a and b); similarly, in Plane 2, the vector fields at Outputs 3 and 4 can only be reconstructed under the correct illumination conditions (as depicted in Figs. S7g and h).

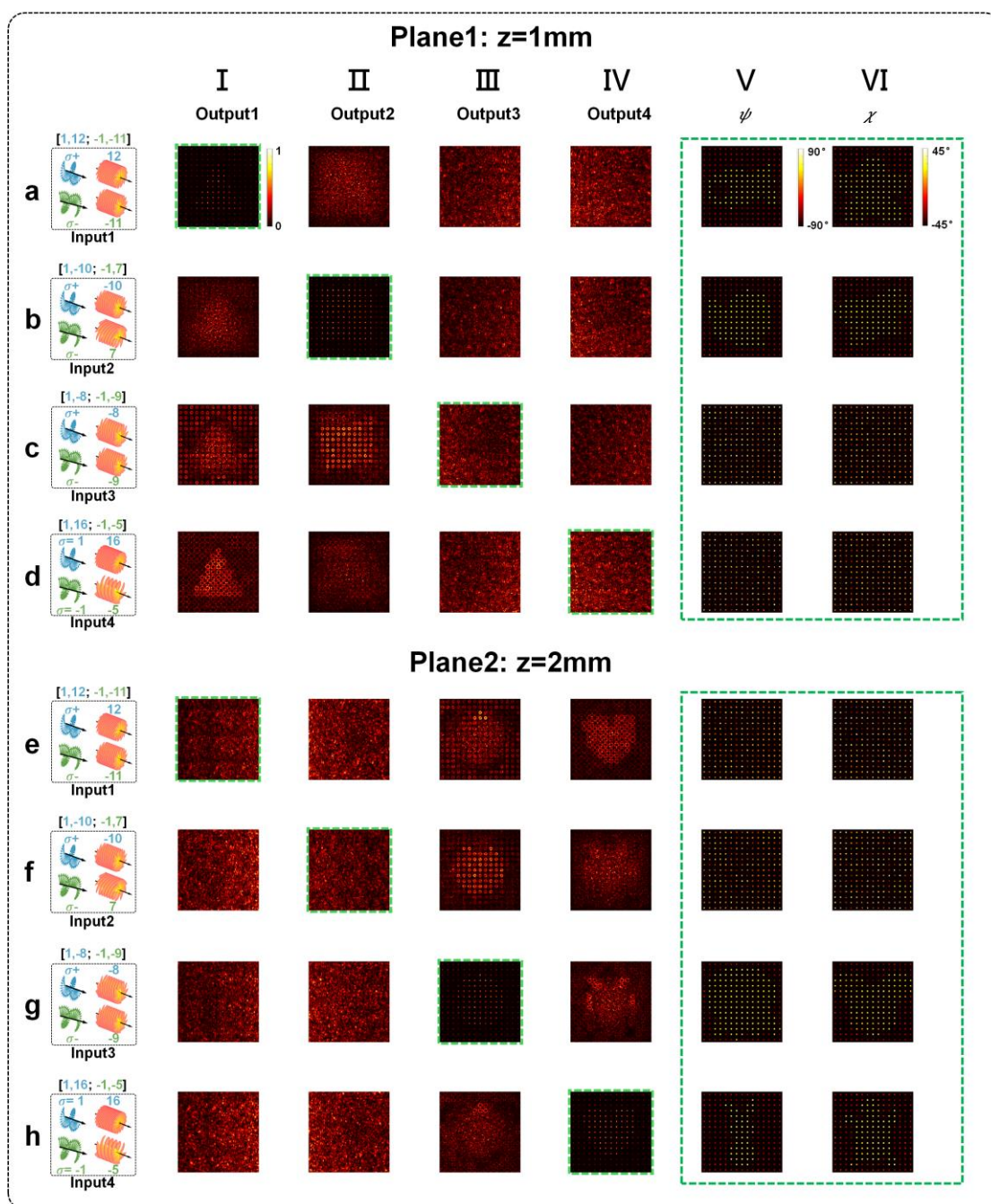


Figure S7. Numerical demonstration of the SOL-VMH in multi planes.



**REFERENCES**

- [1] Schaefer B, Collett E, Smyth R, Barrett D, Fraher B., *American Journal of Physics* 75, 163-168 (2007).