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Full Length Article

Urban rail transit resilience under different operation schemes: A percolation-based approach

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ABSTRACT

To assess the resilience of urban rail transit (URT) systems under various operational conditions accurately and enhance their operation, this study develops a percolation model for nonfree flow transportation networks on the basis of percolation theory, which integrates multisource information and operational characteristics. Our model accounts for the state evolution of different hierarchical structures within the network and identifies nonlinear features. Specifically, we observed significant percolation transitions in the URT network, with distinct differences in critical percolation thresholds at different times, leading to multistate behavior. Network bottlenecks spatially shift with network phase transitions, exhibiting power-law frequency characteristics. On the basis of the full-day resilience assessment results, we analyzed the impact of different operational schemes on network resilience during the morning peak, the period with the lowest resilience. The results demonstrate that our resilience analysis framework effectively evaluates URT network resilience, providing theoretical support for enhancing operational management efficiency and accident prevention measures.

1. Introduction

Since the opening of the first underground line in London in 1863, urban rail transit (URT) has undergone more than 150 years of development. It has become a vital transportation mode for residents and visitors of large cities because of its convenience, punctuality, efficiency, and high capacity. Specifically, it plays a significant role in alleviating traffic congestion and satisfying daily commute demands (Chakrabarti, 2022). However, with the emergence of megacities such as Beijing, New York, and Tokyo, URT has evolved into a complex and sophisticated system that involves uncertain passenger demands (Shakibayifar et al., 2017), stochastic external disturbances (Borjigin et al., 2023), and operational accidents (Kyriakidis et al., 2012). Therefore, ensuring the resilient operation of URT systems and being able to respond and recover quickly in the event of disturbances are crucial in a metropolitan city. Failure to do so can result in significant traffic congestion, economic losses, and even casualties.

In the assessment of URT resilience, researchers often examine the static network topology properties of the system (Chopra et al., 2016; Ma et al., 2022; Yang et al., 2015) and variations in resilience under specific incident scenarios with passenger flow (Zhang et al., 2021). Both the network structure and passenger flow considerations can provide insight into the resilience of the URT network and guide the protection of critical stations and lines in the event of incidents. Enhancing resilience typically involves optimization problems such as train timetable optimization (Hassannayebi et al., 2017), optimal recovery strategies (Zhang et al., 2018), passenger flow assignment models, and optimal line design (Nian et al., 2019; Qu et al., 2022). Additionally, percolation theory has been extensively applied in the study of network properties, but research has focused mainly on artificial networks, such as random networks (Allard et al., 2015), scale-free networks (Serrano et al., 2011), and lattice networks (de Noronha et al., 2018). Investigations into transportation networks based on percolation theory are still in their infancy, with little research on the properties of URT networks based on percolation theory.

URT networks and road networks exhibit significant differences in both their operational mechanisms and topological structures. From the perspective of operational mechanisms, URT operates on fixed timetables, whereas road participants can freely enter or exit the network at will. In terms of topological structures, URT networks are characterized by spatially nonoverlapping lines, meaning that the closure of a transfer

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station does not disrupt train operations on individual lines. In contrast, road intersections, when obstructed, can induce congestion across connected road segments. From the perspective of complex network theory, URT networks feature a large number nodes of degree 2, whereas road networks are predominantly composed of degree-4 nodes. These fundamental differences render percolation models originally developed for road networks inapplicable to URT systems without modification.

Existing resilience metrics predominantly focus on either network topology or passenger travel demand, with most analyses centered on system performance loss and recovery processes (Ilalokhoin et al., 2023; Meng et al., 2020). However, few studies have simultaneously considered both network topology and passenger flow from the perspective of percolation theory to evaluate the operational resilience of URT systems. Moreover, many existing studies assess resilience from only a single dimension, failing to capture its multidimensional nature (Mattsson and Jenelius, 2015). To address these challenges, this study introduces a percolation-based framework that classifies URT resilience into three perspectives: connectivity, robustness, and fragmentation. This framework integrates network topology, passenger travel behavior, and percolation dynamics to provide a more comprehensive evaluation of system resilience. Specifically, we propose a percolation model for urban rail transit networks, which model the network as a dynamically evolving weighted network throughout a full day of operation. This approach quantitatively analyzes the phase transition processes during the percolation of the urban rail transit network. Through this model, we examine the dynamic characteristics of the network's full-day operation, including its division into two states within a day and the evolution of bottlenecks at different times during the percolation process. Additionally, changes in certain line operation schemes often do not impact network robustness. These findings validate the effectiveness of the framework proposed in this study and contribute to a deeper understanding of urban rail transit systems. The framework is illustrated in Fig. 1, with the main contributions as follows.

1) We propose a percolation model for nonfree flow transportation networks, such as URT networks, and investigate the evolution of network bottlenecks.
2) We develop a resilience assessment framework based on the characteristics of the network percolation process, which integrates multi-source data to analyze network resilience from various dimensions.
3) Using the Beijing urban rail transit system as a case study, we analyze the impact of different operational schemes on network resilience.

2. Literature review

This section primarily reviews the existing major works in two areas: the application of percolation theory in the transportation field and methods for resilience assessment in transportation. This study also highlights the limitations of current methods when they are applied to nonfree flow transportation networks.

2.1. Application of percolation theory in the transportation field

The percolation problem, initially proposed in 1957 by Broadbent and Hammersley (1957), was designed to distinguish it from diffusion theory and to offer a new way to describe the process of medium propagation in a network. In the late 20-th century, with the formulation of small-world networks and scale-free networks by Watts and Strogatz (1998) and Barabási and Albert (1999), network science experienced a

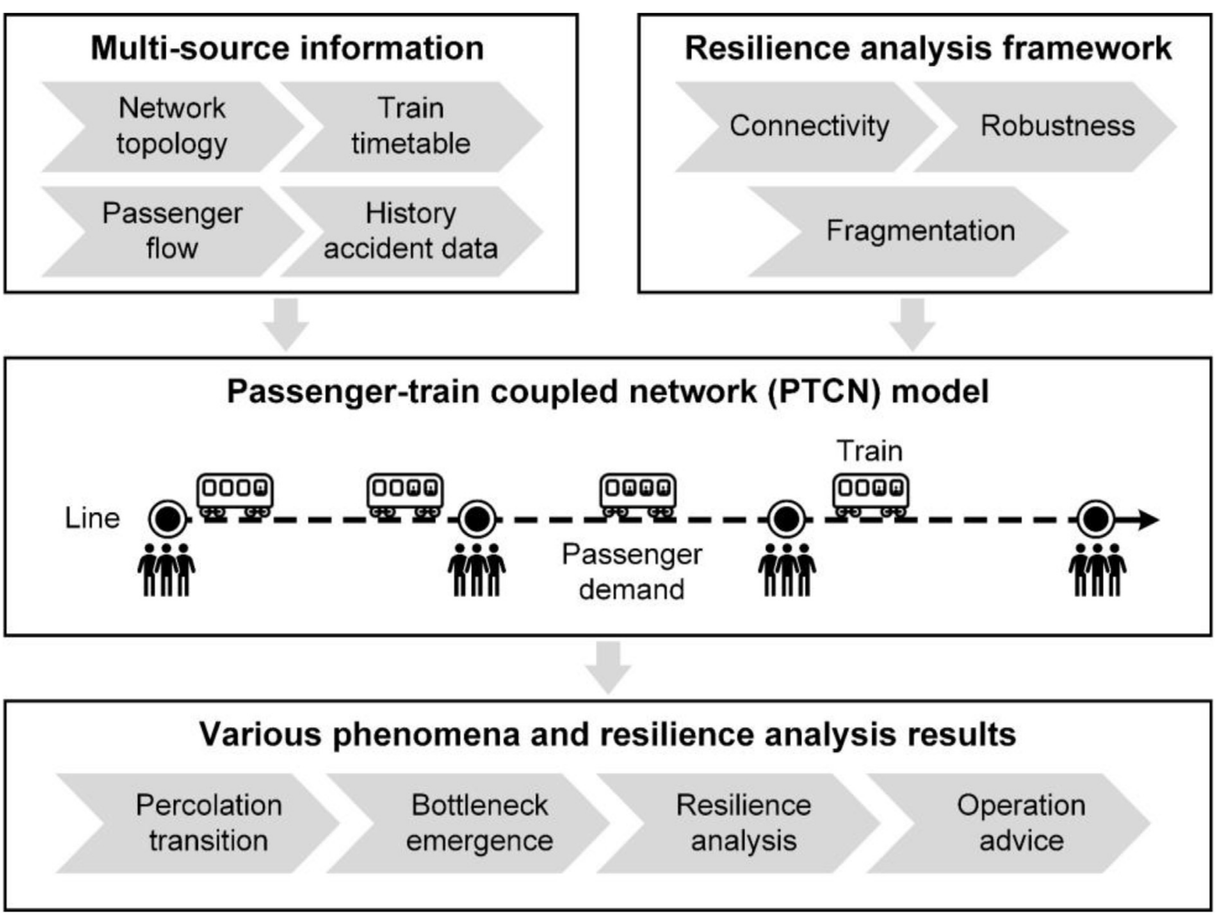


Fig. 1. Framework of URT resilience analysis.

surge in research interest, and percolation theory was increasingly applied to the investigation of various network properties, including transportation networks (Jiang et al., 2021). Consequently, percolation theory has become a prominent tool for analyzing the characteristics of transportation networks in recent years.

Callaway et al. (2000) researched the correlation between the average component size of a random network and its percolation threshold. Mohseni-Kabir et al. (2021) investigated and compared the percolation threshold variation for networks with different connectivity strengths, thereby providing an exemplary approach to designing networks with enhanced resilience. Schneider et al. (2011) validated the efficacy of their algorithm designed for improving network resilience on the European power grid. They also evaluated network resilience (R) via an essential metric proposed in their work, as expressed in Eq. (1):

$$R=\frac{1}{N}\sum_{Q=1}^{N}s(Q) \tag{1}$$

where N is the number of nodes in the network and where $s(Q)$ represents the fraction of nodes in the giant component after removing Q nodes. Additionally, a new resilience metric named percolation centrality was presented by Piraveenan et al. (2013), and the traffic flow quality in the percolation-backbone fractal network was analyzed by Nagatani (2020).

However, artificially constructed networks differ significantly from real-world transportation networks, which can result in a considerable difference in the observed percolation threshold (Dong et al., 2020). Consequently, many studies have been carried out to analyze the properties of transportation networks, with a primary focus on road networks. The relationship between traffic volume and the percolation threshold in a road network was investigated by Wang et al. (2015). Aleshkin (2021) developed an algorithm for calculating the percolation threshold and compared discrepancies in the percolation thresholds of several major cities worldwide under different network structures. Furthermore, Cogoni and Busonera (2021) investigated the transitions of traffic status in London via percolation theory and summarized patterns of congestion emergence at different time periods. Li et al. (2015) observed the percolation transition in urban road networks, and Zeng et al. (2020, 2019) investigated critical percolation modes and multiple metastable network statuses via percolation theory. Zhu et al. (2025) conducted a further analysis of bottleneck enhancement strategies and the propagation processes of bottlenecks within transportation networks. Similarly, Ruan et al. (2019) observed different critical percolation modes in the Hangzhou road network. Furthermore, researchers have investigated the percolation phenomenon on road networks under specific disaster scenarios (Dong et al., 2022; Guo et al., 2018).

Research on percolation theory applied to transportation networks has focused mainly on road networks, but recent studies have explored the applicability of this theory to other transportation modes. For example, Liu et al. (2020) established a temporal network model for airports and observed percolation transitions. Kim et al. (2023) designed a percolation rule that involves the removal of partial functionality of nodes. This rule randomly imposes restrictions on the capacity of some nodes in the URT network, and they observed changes in passenger travel time within the network.

However, current research has focused primarily on road networks, whereas other transportation modes exhibit significant differences in properties. The corresponding percolation models and methods are still in their nascent stages.

2.2. Resilience assessment approaches for URT

To date, the theory of network resilience assessment has been extensively studied (D'Lima and Medda, 2015; Nogal et al., 2019), resulting in numerous resilience metrics (Liu et al., 2023). Among the various metrics available, the most frequently utilized resilience metric (R_e) is that developed by Bocchini and Frangopol (2012), which can be mathematically expressed in Eq. (2):

$$R_e=\frac{\int_{t_0}^{t_0+t_\mathrm{d}}Q(t)\mathrm{d}t}{t_\mathrm{d}} \tag{2}$$

where t_0 is the time at which an incident occurs and where t_d is the duration of the incident. $Q(t)$ is a function that represents the system ability at times t and $t\in[t_0,t_0+t_\mathrm{d}]$. The metric is designed on the basis of the resilience triangle concept proposed by Bruneau et al. (2003). A higher value of this metric under disturbance indicates greater system resilience.

However, the ability of a system can be viewed from multiple perspectives, and as a result, the expression form of resilience may vary. For example, Wang et al. (2023) developed a resilience assessment framework from the perspectives of robustness, adaptability, and recoverability, using the Beijing urban rail transit system as a case study, whereas Mudigonda et al. (2019) assessed the resilience of the New Jersey subway network from a vulnerability perspective on the basis of extensive subway operational data. Therefore, evaluating the ability of a network via a uniform resilience assessment metric is challenging (Zhu et al., 2024a). Currently, there are two primary approaches for resilience assessment: the topology-based approach and the simulation approach (Chen et al., 2022).

The topology-based approach, developed from complex network theory, has been widely used to assess network resilience. This measurement approach relies on the static structure of the network and calculates topology metrics to evaluate the network's robustness, vulnerability, and efficiency (Lu and Lin, 2019). To assess more accurate results of resilience variations, the topology-based approach often incorporates dynamic factors, such as passenger flow (Ma et al., 2024; Wang et al., 2024).

The simulation approach is frequently used to simulate the behavior of a system under a particular incident event, such as a terrorist attack (Cong et al., 2022), earthquake (Oboudi et al., 2023), operational accident (Yan et al., 2020), or signal failure. Using the cascade failure theory, Shen et al. (2021) simulated the redistribution of passenger flow in the event of a station failure and conducted robustness optimization. The findings provide a strategy for dynamically evaluating safety and managing passenger flow emergencies. Zhang et al. (2021) established a resilience assessment framework based on integrated coupled map lattice theory and evaluated the vulnerability of a network under two attack modes.

The majority of existing resilience assessment indicators are based on the network structure and other related factors, and only a limited number consider the percolation state of the links (Zhu et al., 2024b).

Existing studies rarely consider the percolation status of nodes or links when assessing the resilience of URT networks. Resilience under different operating schemes is often characterized in an optimized form without integrating the overall system state, and it is difficult to rapidly provide results for network scale issues. Therefore, under different operating schemes, integrating multisource information from within and outside the system and considering the percolation status of the network can more accurately characterize the resilience of URT. The relevant research status is summarized in Table 1.

3. Methodology

To address the challenges in existing research, this study proposes a passenger-train coupled network (PTCN) percolation model applicable directly to urban rail transit networks and constructs a resilience assessment framework that considers the network's percolation state. The traffic flow equilibrium model is used to allocate passenger demands within the network, enabling a comprehensive evaluation of the resilience of the urban rail transit network.

Table 1
Summary of relevant research status.

Ref.	Network type		Method		
	Free flow	Nonfree flow	Percolation	Simulation	Optimization
Wang et al. (2015)	Road	—	✓	✓	—
Aleshkin (2021)	Road	—	✓	—	—
Cogoni and Busonera (2021)	Road	—	✓	✓	—
Li et al. (2015)	Road	—	✓	✓	—
Zeng et al. (2019)	Road	—	✓	—	—
Zeng et al. (2020)	Road	—	✓	—	—
Ruan et al. (2019)	Road	—	✓	✓	—
Dong et al. (2022)	Road	—	✓	✓	—
Guo et al. (2018)	Road	—	✓	✓	—
Bocchini and Frangopol (2012)	Road	—	—	—	✓
Yan et al. (2020)	—	URT	—	✓	—
Ma et al. (2024)	—	URT	—	✓	—
Liu et al. (2020)	—	Aviation	✓	—	—
Kim et al. (2023)	—	URT	✓	—	—
Zhu et al. (2024b)	—	URT	✓	—	—
This study	—	URT	✓	✓	—

3.1. Construction of the dynamic PTCN model

As discussed in Section 1, URT exhibits distinct differences from other transportation modes, thus requiring the development of novel approaches to effectively capture its unique characteristics. In this subsection, we develop the dynamic PCTN model and its corresponding construction methodology for assessing the resilience of URT networks under different operation schemes.

As noted in Section 2, numerous studies have employed percolation theory to investigate road networks. Specifically, three distinct methods have been utilized to render nodes or links dysfunctional: (1) completely random, (2) based on node degree, and (3) based on node or link betweenness. However, these methods have difficulty accurately reflecting the real-world conditions of transportation networks. Random failures often lead to uncertain results, whereas deliberate failures do not necessarily match the various disturbances that occur in daily operations. For example, the most congested stations are often those that fail, which may not be the nodes with the highest degree or betweenness centrality. To address this issue, Li et al. (2015) and Zeng et al. (2020, 2019) developed an alternative method that characterizes road failure in terms of road capacity. As shown in Eq. (3), the road capacity v_{ij} is expressed as the ratio of the current average velocity $v_{\mathrm{mean}}^{\mathrm{now}}$ to the maximal velocity observed in the network's history $v_{\max}^{\mathrm{history}}$. On the basis of a predetermined threshold q, roads can be classified as functional when $v_{ij} \geq q$ and dysfunctional when $v_{ij} < q$:

$$v_{ij} = \frac{v_{\mathrm{mean}}^{\mathrm{now}}}{v_{\max}^{\mathrm{history}}} \tag{3}$$

In the URT system, train operating velocities are governed by timetables, rendering speed an unsuitable metric for measuring capacity. To address this limitation, we suggest defining the capacity (r_{ij}) of URT links as the ability to transport passengers. Specifically, this capacity is calculated via Eq. (4):

$$r_{ij} = 1 - \frac{p_{ij}}{mct_{ij}} \tag{4}$$

where p_{ij} is the number of passengers on the link that connects nodes i and j, t_{ij} is the number of trains on the link that connects nodes i and j, c is the train fixed capacity, and m is the maximum capacity utilization rate.

As illustrated in Fig. 2, within a given time frame, Train 101 travels from Station A to Station D, whereas Train 203 operates from Station B to Station E (Fig. 2a). Furthermore, there exists an origin-destination (OD) demand from A to D and from B to F (Fig. 2b). On the basis of this setup, the number of trains passing through each segment, as well as the corresponding flow, can be determined (Fig. 2c). Assuming that $c = 100$ and $m = 100\%$, the r_{ij} values for each segment can be calculated via Eq. (4). Additionally, if the computed $r_{ij} < 0$, its value is manually set to zero to indicate that the segment lacks transit capacity. By utilizing the train timetable and OD passenger flow data, we can compute the capacity ratio r_{ij} for each link and subsequently classify the links into functional ($r_{ij} \geq q$) or dysfunctional ($r_{ij} < q$) based on the predetermined threshold q.

3.2. Passenger assignment model

To characterize the behavior of passengers accurately during their travel process, we employ a stochastic user equilibrium (SUE) model to allocate the known OD flow in URT systems. The model considers passengers' actual waiting time, passenger walking time, number of transfers, and in-vehicle time as generalized costs (C_{od}^{k}), as shown in Eq. (5), and does not account for passenger abandonment of travel or the choice of alternative transportation modes.

$$C_{\mathrm{od}}^{k} = \left(2 - r_{ij}\right)\varphi_1 + \beta\varphi_2 + \varphi_3 + \varphi_4 \tag{5}$$

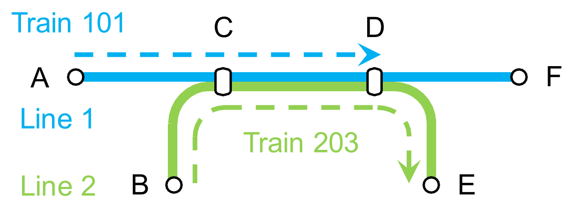
where the coefficient $2 - r_{ij}$ denotes the in-vehicle time φ_1 congestion factor, whereas β represents the penalty time associated with the number of transfers φ_2. Additionally, φ_3 and φ_4 correspond to the intrastation walking time and waiting time, respectively (in the present study, the departure interval is set at half its value to represent waiting time, with all transfer behaviors on the route being incorporated into the analysis), respectively. The term C_{od}^{k} represents the generalized cost associated with the k-th path connecting an origin to a destination, which is determined by employing the K-shortest path algorithm. By utilizing this cost function, it is feasible to calculate the travel cost for passengers on each route, and we assume that all the random errors of each travel path utility function are independent of each other and obey the Gumbel distribution. The passenger flow is subsequently allocated according to the logit function delineated in Eqs. (6) and (7) (Ma et al., 2024):

$$p_{\mathrm{od}}^{k} = \frac{\exp\left(-\theta C_{\mathrm{od}}^{k}\right)}{\sum\limits_{k \in K} \exp\left(-\theta C_{\mathrm{od}}^{k}\right)} \tag{6}$$

$$f_{\mathrm{od}}^{k} = f_{\mathrm{od}} p_{\mathrm{od}}^{k} \tag{7}$$

where θ represents a parameter in the logit function; a higher θ means that passengers have full awareness of the network, and the error in perceived path cost is small, typically set as 2 (Zhang et al., 2023); p_{od}^{k} denotes the proportion of passenger flow allocated to the k-th path between O and D; K represents the set of paths between origins and destinations; f_{od} represents the total traffic demand between O and D; and f_{od}^{k} represents the traffic allocated to the k-th path between O and D. With this logit-based passenger flow allocation model, the passenger flow along each path can be obtained. Finally, to achieve the SUE state, this study employs the method of successive weighted averages (MSWA) to solve for the SUE model.

(a)



(b)

O	D	Demand
Station A	Station D	75
Station B	Station F	100

(c)

Section	A–C	B–C	C–D	D–E	D–F
Capacity	100	100	200	100	0
Flow	75	100	175	0	100
r_{ij}	0.25	0	0.125	1	0

Fig. 2. Construction method of the dynamic PTCN model: (a) schematic representation of the rail lines and trains; (b) distribution of the OD demand; and (c) an example of the calculation process.

3.3. Metrics of URT network resilience

In the context of transportation network analysis, resilience evaluation is essential for understanding how the network responds to disruptions. Therefore, in this subsection, we aim to investigate the resilience of URT networks via three metrics: connectivity, robustness, and fragmentation. These metrics capture different aspects of network resilience and provide a better understanding of the network's ability to maintain functionality under disruptions. All three metrics are considered in the context of the link percolation state, which captures the gradual failure of links in the network due to the accumulation of damage or congestion.

3.3.1. Connectivity metric

Connectivity is a crucial characteristic of URT networks, as it determines the network's ability to maintain performance even after some stations have failed. This attribute is similar to the concept of accessibility, as a network with greater connectivity is more likely to enable passengers to travel successfully, thereby enhancing overall accessibility. Consequently, connectivity plays a significant role in assessing the current degree of accessibility in a URT network. Therefore, it is essential to measure the level of connectivity in the network to ensure that it remains functional and resilient in the face of potential failures.

Schneider et al. (2011) suggested a network resilience metric, represented by Eq. (1), which is essentially a measure of network connectivity. However, the node failure method used in their metric is based on either the highest node degree or the maximum betweenness, which may not reflect the actual conditions of URT networks. Therefore, we have improved this metric to analyze network connectivity (R^t_{con}) based on real-world scenarios rather than deliberate attacks, as shown in Eq. (8). This metric considers the average proportion of the giant component throughout the entire network percolation process, with each edge in the network containing its capacity and flow information. Consequently, compared with simulation methods for deliberate attacks, this metric better characterizes network connectivity from a practical perspective.

$$R^t_{\text{con}} = S_q \sum_{q=0}^{1} \frac{G(q,t)}{N} \;\forall t \in T \tag{8}$$

where q is a threshold and is typically initialized to 0, N is the number of nodes in the URT network, $G(q,t)$ is the number of nodes in the giant component at time t, S_q is the step length of q, and T is the set of time instances. This means that we find the mean value of the percentage of nodes in its giant component for each increasing step of q at time t. During the percolation process, each increase in the threshold q results in the removal of edges with weights r_{ij} less than q. However, if certain edge weight differences are smaller than a specified step size S_q, multiple edges may be removed simultaneously as q increases, leading to inaccuracies in the percolation process. This inaccuracy becomes most pronounced when $S_q = 1$. To balance computational efficiency and accuracy, this study sets $S_q = 0.01$.

3.3.2. Robustness metric

Robustness is a fundamental aspect of URT resilience evaluation, as it measures a network's ability to withstand external disruptions. A highly robust URT system can maintain stable service levels in the event of minor disturbances and still meet basic passenger travel demands in the case of major incidents (e.g., disasters). As such, the network's tolerance to perturbations is a key determinant of its resiliency, with higher levels of robustness (R^t_{rob}) indicating greater resilience. This metric can be calculated via the formula provided in Eq. (9):

$$R^t_{\text{rob}} = S_q \sum_{q=0}^{1} LE(q,t) \;\forall t \in T \tag{9}$$

where $LE(q,t)$ is the network local efficiency at time t, which represents the mean network efficiency of all subgraphs within the network (Latora and Marchiori, 2001). Hence, the larger the value of this indicator is, the stronger the robustness of the network.

3.3.3. Fragmentation metric

In graph theory, as the nodes or links of a network are damaged and removed due to internal or external factors, the network may experience irreversible global collapse, and fragmentation (R^t_{fra}) is a metric that describes this phenomenon (Chen et al., 2007). In this study, the calculation of the fragmentation metric is illustrated in Eq. (10):

$$R^t_{\text{fra}} = S_q \sum_{q=0}^{1} \frac{C_N(q,t)}{N} \;\forall t \in T \tag{10}$$

where $C_N(q,t)$ is the number of clusters of threshold q at time t, and $R^t_{\text{fra}} \in [1/N, 1]$. R^t_{fra} takes a value of 1 when all links in the network fail, and $1/N$ when all nodes in the network are accessible. Notably, in this study, a cluster refers to the number of connected subgraphs, and isolated nodes are also considered a cluster. This metric calculates the average level of network discretization during the percolation process, with higher values indicating a greater level of network discretization and lower resilience of the URT network.

The utilization of three distinct resilience assessment metrics enables a more complete evaluation of URT resilience from various perspectives. This approach facilitates the provision of an accurate representation of the current state of the URT network, thereby offering valuable guidance for enhancing its resilience and preventing unforeseen incidents.

4. Case study

4.1. Dataset

The data used in this study comprise several components, including train timetables, network topology information, interval running time, and OD passenger volume data (see the Electronic Supplementary Material). The timetables and network topology information were sourced from the official website of the Beijing Subway (https://www.bjsubway.com), while the OD passenger volume data were obtained from the automatic fare collection (AFC) system. The data processing involved a sequence of two steps. First, incomplete data from certain lines, such as the two airport lines, two tram lines, the Yanfang line, and Line 19, were eliminated from the study. Next, we deduce the train routes from the train schedule data of each station. The study utilized a total of 348 stations, 19 lines of topological data, 1621 sets of train timetable data, and 112,290 sets of OD data.

The topological structure of the URT network in Beijing that we investigated is displayed in Fig. 3 presents the distribution of inbound passenger volume across the day, which is based on a statistical interval of 15 min and a time interval ranging from 4:00 to 23:45. We treat the inbound volume at each time point as a time series and employ the Fisher optimal partition algorithm to divide the study period into five segments: pretraffic peak (4:00–6:00), morning peak (6:00–9:00), off-peak (9:00–17:00), evening peak (17:00–19:00), and posttraffic peak (19:00–24:00). The plot shows two prominent peaks during the day, which correspond to the morning and evening rush hours, with an average hourly inbound passenger volume of 95,284 (the dashed line in Fig. 3). The volume during nonrush hours is significantly lower than the hourly average.

4.2. Percolation transition in the URT network

The intervals of a URT network are divided into upstream and downstream components, allowing for separate calculations of the r_{ij} values for each component. Therefore, we aggregate the r_{ij} values of the upstream and downstream components and take their average. The variable q is initialized to a value of 0, and its step size is set to $S_q = 0.01$. During each iteration, we increment q by one step. At each step, any link with a corresponding r_{ij} value less than q is considered dysfunctional, which means that passengers cannot pass through this interval in a reasonable amount of time; otherwise, the link is considered functional. Our analysis focuses on three distinct time points, specifically 8:00, 12:00, and 18:00, as displayed in Fig. 4. Each color represents a cluster, and we sort the clusters by their size. Only the top four clusters are colored, with the remaining clusters represented in gray.

When $q = 0$, the URT network during morning and evening rush hours is not intact, and excessive commuting passenger flow causes this phenomenon. As the threshold q increases, the URT network experiences

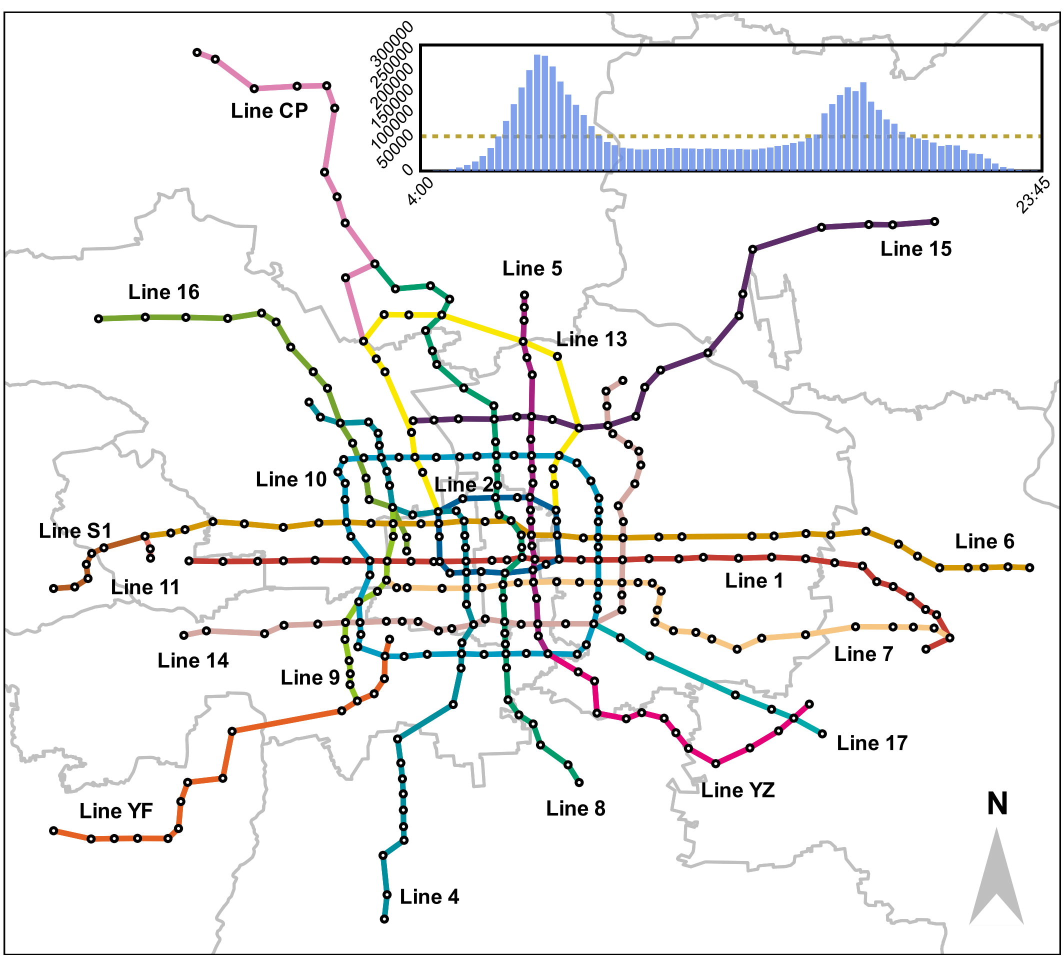


Fig. 3. Basic information of the Beijing URT system: Beijing URT network; the inset shows distribution of inbound passenger volume from 4:00 to 23:45.

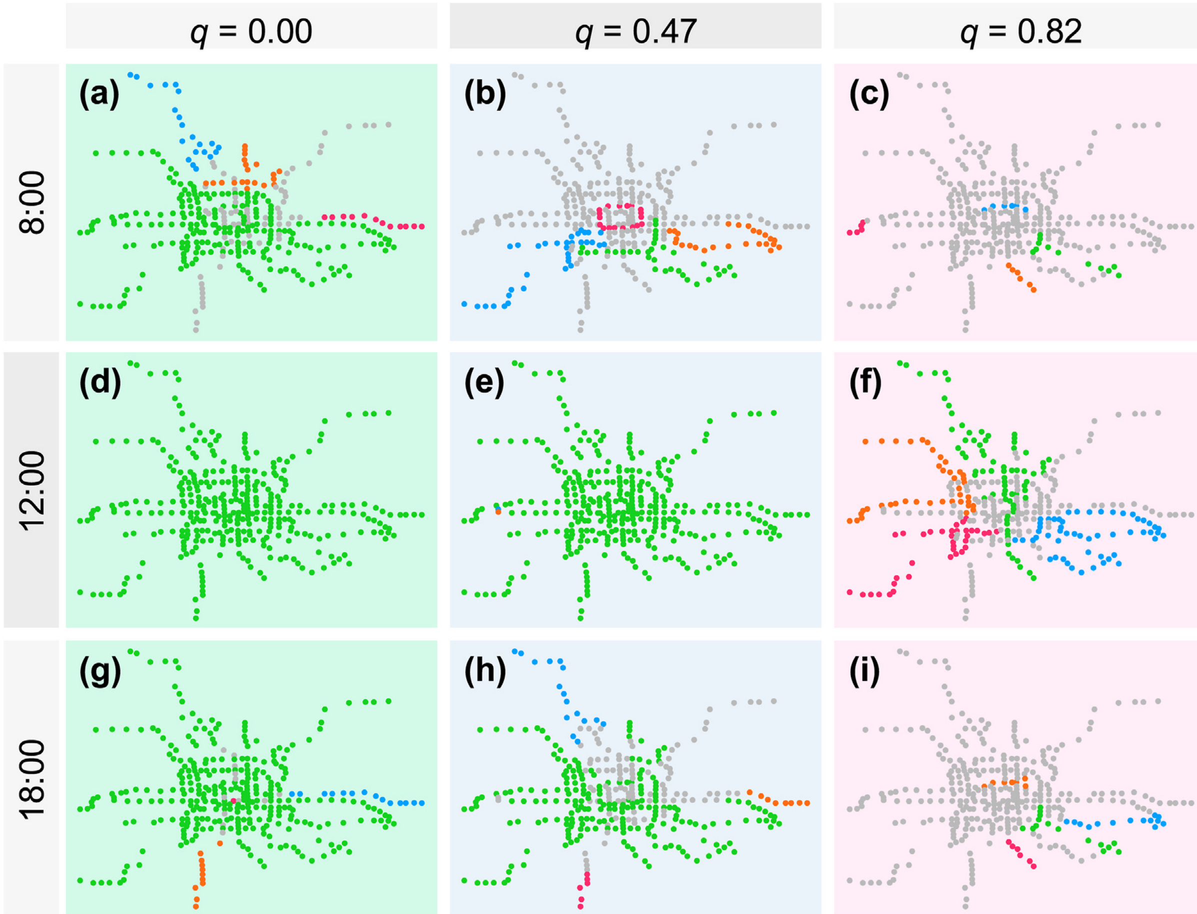


Fig. 4. URT network exhibits distinct percolation states during the morning and evening rush hours as well as during the off-peak periods of the day.

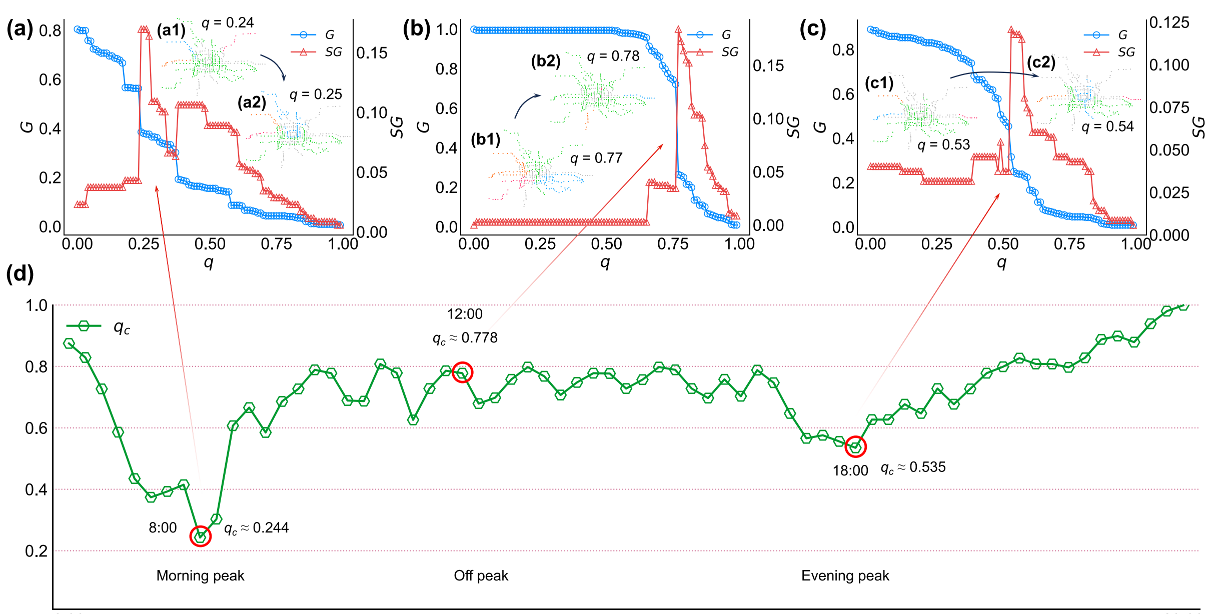


Fig. 5. Analysis of the critical percolation threshold q_c. (a, b, c) Percolation transition processes during the morning peak, off-peak, and evening peak periods for the URT system. (a1, a2) Network phase transitions occurring during the morning peak period. (b1, b2) Network phase transitions during the off-peak period. (c1, c2) Network phase transitions during the evening peak period. (d) Trend of q_c throughout the entire day.

gradual collapse at a given point in time. However, the same network exhibits substantial differences in terms of resilience for the threshold q.

In Fig. 4a, during the morning rush hour, owing to longer commuting times from suburban areas, traffic flows mainly from the fringes of the network toward its center at this time, with failure links predominantly clustered around the network periphery. Conversely, at noon, traffic is distributed homogeneously across the URT network, resulting in improved network connectivity (Fig. 4d). Fig. 4g shows that during the evening rush hour, the performance of the URT network is better than that during the morning rush hour. When $q = 0.47$ at 8:00:00 (Fig. 4b), the URT network collapses into several small clusters but remains functional at 12:00 and 18:00 (Figs. 4e and 4h). Notably, the URT network disintegrates into multiple clusters (Fig. 4f) and becomes entirely nonfunctional Figs. 4c and 4i).

As depicted in Figs. 5a, 5b, and 5c, the critical threshold q_c was computed for each point-in-time mentioned above. G represents the ratio of the number of nodes in the giant component to the total number of nodes, whereas SG represents the ratio of the number of nodes in the second giant component to the total number of nodes. In these three cases, G decreases gradually as q increases, and when $q = q_c$, SG skyrockets and G plunges suddenly. This means that some vital bottlenecks fail as q approaches q_c, which will cause the URT network to collapse into several clusters. These phenomena are manifested in Figs. 5a1, 5a2, 5b1, 5b2, 5c1, and 5c2, where at the moment the percolation threshold q surpasses the critical percolation threshold q_c, the network undergoes a phase transition. Different time periods clearly correspond to different values of q_c. During peak hours in the morning and evening, the network exhibits lower resilience, resulting in lower q_c values. Conversely, during off-peak periods, the network demonstrates stronger resilience, leading to higher q_c values.

The trend of q_c throughout the day was analyzed, as demonstrated in Fig. 5d. The value of q_c clearly exhibited a pronounced trough during the morning and evening rush hours but fluctuated less during the nonrush hours. Notably, the trend of q_c is negatively correlated with the trend of the distribution of the all-day inbound volume. Specifically, a greater number of passengers in the network results in a lower q_c value, indicating greater network vulnerability to disruption.

4.3. Identification of bottlenecks during phase transition

Identifying bottlenecks during the dynamic network evolution of URT is pivotal. Enhancing the capacity of bottlenecks and ensuring that they do not malfunction after accidental damage can prevent the network from breaking into several small clusters and improve connectivity at a reasonable cost. Therefore, we counted the number of times a bottleneck appeared in the network, as shown in Fig. 6. The abscissa represents the frequency of a certain interval becoming a bottleneck, whereas the ordinate represents the magnitude of the corresponding occurrence probability. The occurrence frequency and probability exhibit a typical power law distribution, indicating that the links that become bottlenecks more often are very rare, and most of them become bottlenecks only a few times within a day.

Here, we select the top ten sections on the basis of their appearance frequency, as shown in Table 2 and Fig. 6. The fourth column of the table represents the average passenger flow volume in each section throughout the entire day. Notably, the average passenger flow volume across all the sections in the URT network is 100,748. These sections are characterized by relatively high passenger flow volumes. Notably, in bottleneck identification of URT, there may be cases where consecutive intervals on a section are all bottlenecks. This is because a small number of passengers

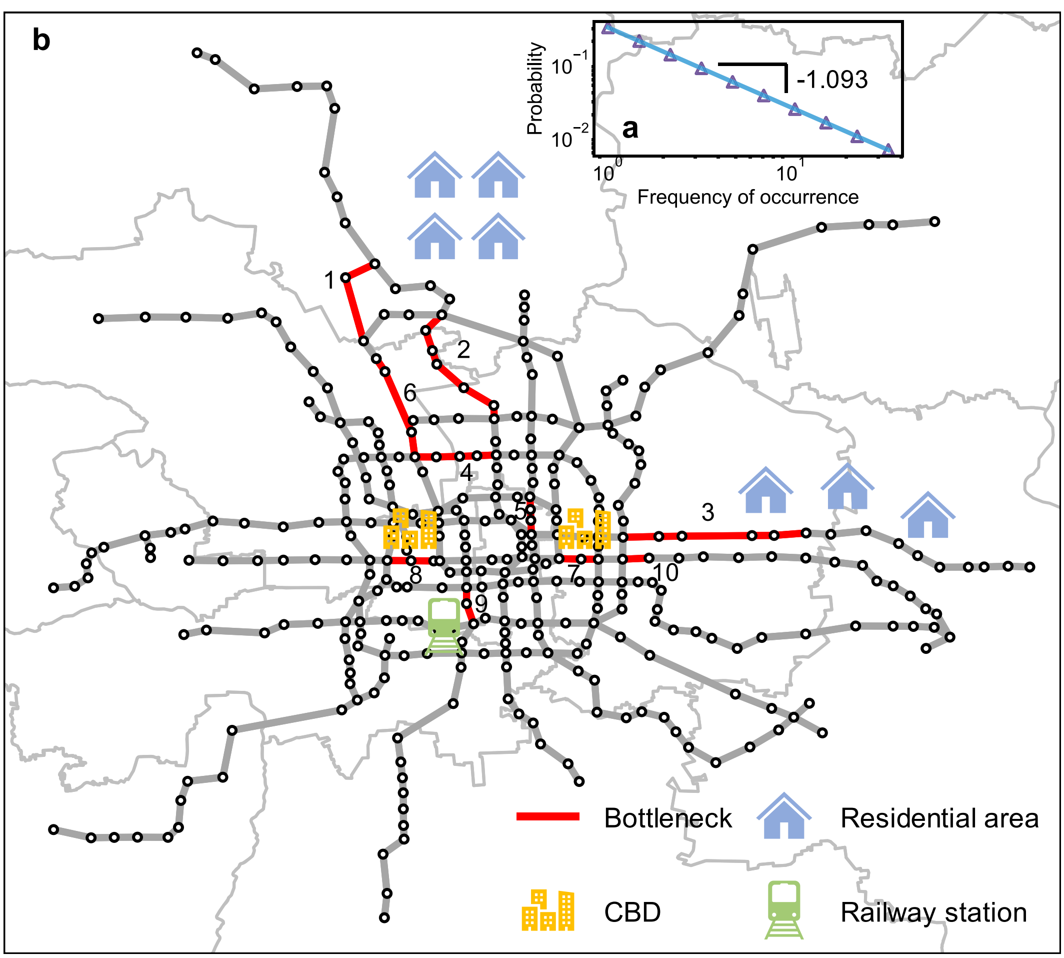


Fig. 6. Network bottleneck analysis: Geographical distribution of the top 10 bottleneck sections.; the inset shows the probability distribution of the bottleneck occurrence frequency.

Table 2
Most bottleneck-prone sections.

Number	Origin station	Destination station	No. of passengers
1	Zhuxinzhuang	Xi'erqi	184,907
2	Huoying	Olympic Park	149,920
3	Changying	Jintai Lu	209,304
4	Zhichun Lu	Beitucheng	250,113
5	Yonghegong Lama Temple	Dongsi	200,908
6	Qinghe	Zhichun Lu	153,007
7	Jianguomen	Guomao	208,589
8	Military Museum	Fuxingmen	199,811
9	Caishikou	Beijingnan Zhan	247,222
10	Sihui	Dawang Lu	169,049

Table 3
Number of bottleneck occurrences at different time.

Interval	Period			Sum
	Morning	Afternoon	Evening	
Jintai Lu–Shilipu	1	8	7	16
Qingnian Lu–Dalianpo	1	5	7	13
Shilipu–Qingnian Lu	0	5	8	13
Dalianpo–Huangqu	1	6	6	13
Huangqu–Changying	1	5	5	11
Hujialou–Jintai Lu	0	5	4	9
Zaoying–Chaoyang Park	11	1	0	12
Jintai Lu–Chaoyang Park	9	0	1	10
Dongfeng Beiqiao–Zaoying	7	0	1	8
Dawang Lu–Jintai Lu	7	1	0	8
Jiangtai–Dongfeng Beiqiao	6	1	1	8
Dawang Lu–Jiulongshan	6	2	1	9

deboard or transfer at that section, and most passengers need to transfer at farther stations or reach their destinations. Therefore, if the train routes are the same on a section and there are no transfer stations within the section, we consider all intervals within the section as one bottleneck. These bottleneck sections often appear in areas with high commuting demand (e.g., Zhuxinzhuang–Xi'erqi and Changying–Jintai Lu), city centers (e.g., Yonghegong Lama Temple–Dongsi and Jianguomen–Guomao), and next to railway stations (e.g., Caishikou–Beijingnan Zhan).

We selected a representative region and analyzed the bottlenecks that occurred during the morning, afternoon, and evening periods. The results are presented in Table 3. The links that function as bottlenecks during the morning rush hour are less likely to become bottlenecks in the afternoon or evening periods, and vice versa. This behavior can be attributed to the convergence of commuter traffic during the morning peak period, which tends to concentrate on specific links. However, in the afternoon or evening periods, passengers have more flexibility in choosing their routes and departure times, which results in a more dispersed and less concentrated traffic flow.

4.4. Resilience assessment under various operation schemes

Initially, we analyze the diurnal resilience trends of the URT network on the basis of the resilience assessment framework proposed in the preceding text, with the results depicted in Fig. 7. The results of the connectivity assessment are shown in Fig. 7a, which is a macrolevel metric that reflects the accessibility of the network, with a larger R^t_{con} indicating greater accessibility at that moment. The results revealed that network connectivity exhibited a trough during the morning and evening rush hours, followed by relatively stable fluctuations during the daytime flat hours. Furthermore, the connectivity gradually increases after the evening peak, and the R^t_{con} value approaches 1. These observations suggest that network connectivity experiences distinct temporal patterns throughout the day.

Fig. 7b portrays the diurnal variation trend of the URT network's robustness, demonstrating that lower values of R^t_{rob} in the face of specific disturbances indicate a more pronounced degradation in network resilience. The overall trend in network robustness is analogous to its connectivity performance. This correlation is attributed to the combined effect of passenger travel patterns and service scheduling. As the network faces increased passenger demand pressure, congestion within the network becomes more conspicuous. Consequently, even minute perturbations can precipitate extensive congestion, incur significant economic losses, or potentially result in accidents.

The network's fragmentation characterizes the current degree of network discretization, with higher values indicating greater fragmentation. The evaluation results are shown in Fig. 7c. In the best scenario, during the percolation process, each incremental elevation of the threshold value may result in the emergence of at most one new cluster (where isolated points are also considered individual clusters within the context of this study), signifying a slow-paced augmentation in the number of clusters within the network. This phenomenon is indicative of enhanced resilience. However, according to our observations, when the passenger traffic volume within the URT system is relatively high, the percolation process tends to yield a proliferation of clusters, thus rendering the network exceedingly discrete and significantly diminishing its resilience.

On the basis of the comprehensive resilience assessment results, it is evident that network resilience is weakest during the morning and evening peaks. To investigate the impact of various train operation schemes on network resilience, we conduct an analysis taking the morning peak period as a reference case.

Some lines within URT systems often reach their capacity limits in terms of departure intervals during peak periods, rendering it impossible to further reduce these intervals with current operational techniques and equipment. Given the impact of such factors, we categorize adjustments to the operational schemes into two scenarios: (1) adjustments are made exclusively to the operational schemes of congested lines, and (2) adjustments are applied to the operational schemes network-wide. Notably,

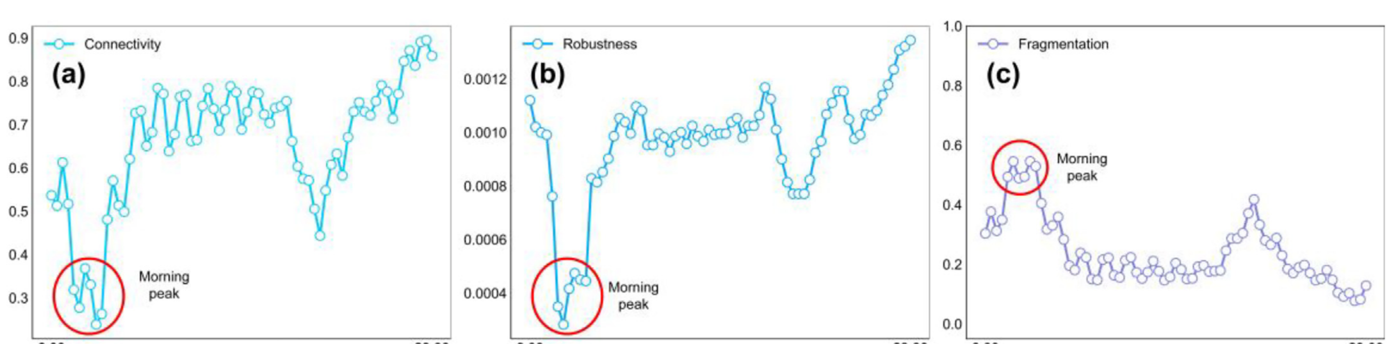


Fig. 7. Depiction of diurnal resilience trends in the URT network from three perspectives: (a) connectivity; (b) robustness; and (c) fragmentation.

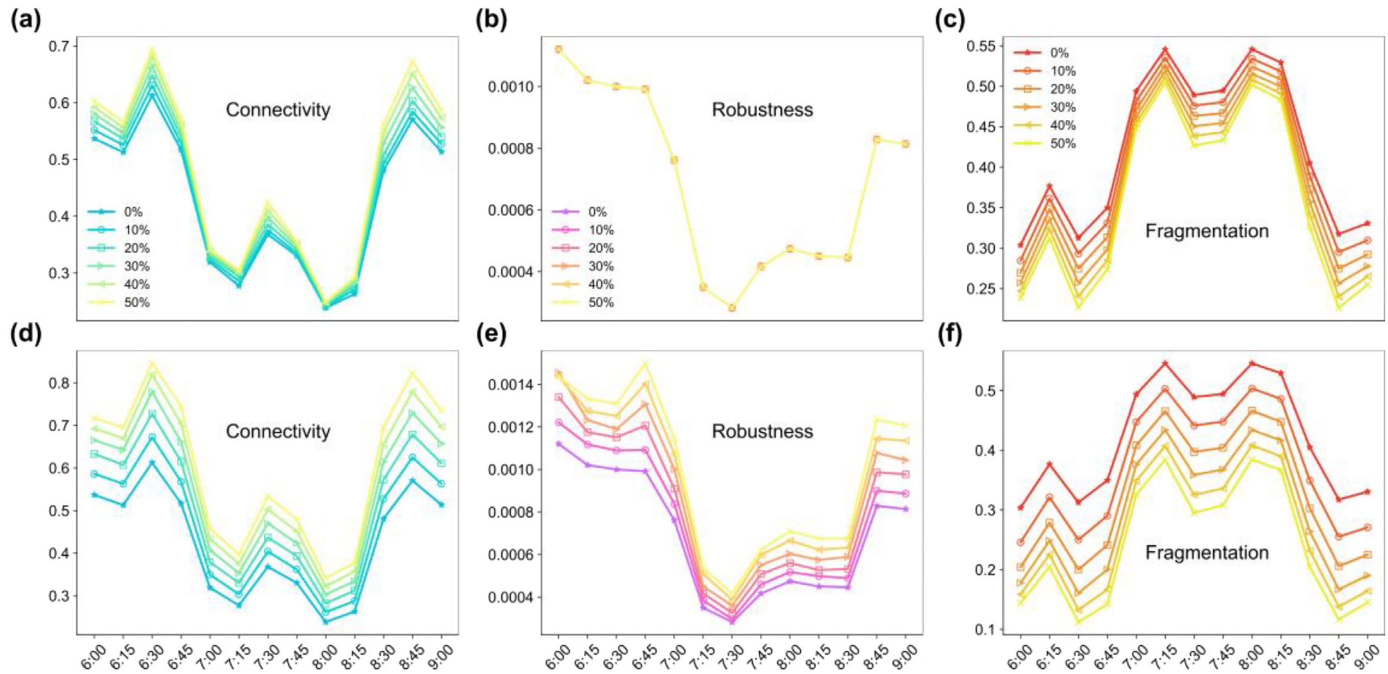


Fig. 8. Impact of different operational schemes on network resilience during morning rush hour: (a–c) influence of the first operational scheme on resilience; (d–f) effects of the second operational scheme on resilience.

within the scope of this study, the strategy for adjusting operational schemes solely considers the reduction in departure intervals as a means to augment capacity. Other scenarios will be further contemplated in future research. Moreover, the adjusted departure intervals are not less than the minimum departure intervals of the lines.

In the first operational scheme, the selected routes included Lines 1, 4, 5, 6, and 8, which experienced significant congestion during the morning peak hours, as depicted in Figs. 8a–8c. A baseline of 0 % represents the original network without any operational adjustment strategies applied. Conversely, Figs. 8d–8f illustrate the resilience alterations under the second operational scheme. From a connectivity perspective, it is apparent that merely reinforcing bottleneck routes becomes less effective in enhancing connectivity as the network's connectivity worsens, especially at 8:00 a.m., which showed almost no improvement; the second operational scheme, however, can more uniformly enhance network connectivity. Fragmentation exhibited similar characteristics. Surprisingly, under the first operational scheme, there was no change in robustness, indicating that isolated line adjustments fail to contribute to the network's robustness.

The aforementioned outcomes corroborate the efficacy of the methodology proposed in this study, laying a theoretical framework for the assessment of URT network resilience under various operational schemes. This resilience evaluation is not only applicable during the morning peak periods but also relevant for the evening rush hours.

5. Conclusions

This study commences with a review of the current research status of percolation theory in the field of transportation and, on the basis of existing limitations, develops a novel methodology to construct a dynamic PTCN model tailored for non-free-flow transportation networks. Taking the URT system of Beijing as a case study, this study identifies a pronounced percolation transition phenomenon within a non-free-flow transportation network and thoroughly investigates the spatiotemporal variations in bottlenecks within the network. Finally, this study analyzes the full-day network resilience trends from three perspectives and, by focusing on the network during the morning peak—the period of lowest resilience—observes the resilience changes of the URT network under different operational schemes.

This study yielded several key findings. First, the critical percolation threshold exhibited during the percolation transition process can serve as an important parameter for characterizing the resilience of the network. The frequency of bottleneck occurrences within the network demonstrates a power-law distribution, and significant differences are evident in the locations of bottlenecks during the peak morning and evening periods, providing a crucial basis for the formulation of train operation schemes. Finally, different operational schemes significantly affect network resilience, thus offering a theoretical foundation for enhancing the overall resilience of the network.

Our study facilitates the application of percolation theory to non-free-flow transportation systems, thereby revealing the evolutionary mechanisms of transport processes in different transport systems, with a particular focus on the emergency management of potential system collapses. Specifically, during actual operations, when the URT system is subject to internal passenger flow surges or external disturbances, the proposed framework allows for the rapid identification of network bottlenecks on the basis of real-time data. Leveraging this information, operators can implement targeted adjustments to operational plans, such as dispatching additional trains to alleviate congestion in critical areas while simultaneously modifying the timetables of bottlenecked lines to maintain their operational capacity. These measures collectively enhance the resilience of the URT network. Furthermore, for random or unforeseen incidents within the system, simulation-based approaches can be employed to quantitatively assess their impacts on system resilience and develop effective mitigation strategies. Thus, our study offers theoretical support for research on accident prevention in network resilience (Wei et al., 2024). We recommend further investigations into the spatial and temporal distribution patterns of interdependent transportation network bottlenecks, the influence of passenger behavior on network resilience (Yang et al., 2017), the dynamic coupling of timetables and passenger demand in URT systems, and the impact of different traffic parameters on network properties.

CRediT authorship contribution statement

Tianlei Zhu: Writing – original draft, Software, Methodology, Data curation. **Xin Yang:** Writing – original draft, Validation, Methodology, Conceptualization. **Yun Wei:** Visualization, Validation, Conceptualization. **Anthony Chen:** Writing – review & editing, Investigation, Formal analysis. **Jianjun Wu:** Supervision, Project administration, Investigation.

Replication and data sharing

All data used in this study can be obtained from the official website of Beijing Subway (https://www.bjsubway.com), with the exception of OD data. This is because OD data involves personal privacy, so it will be shared if there is a reasonable request. The code used for analysis in the study can be found in https://github.com/TLZhu1/BJURTPercolation.git.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Electronic Supplementary Material

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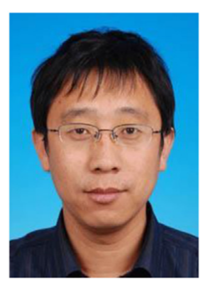

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