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Model the ionospheric gradients between satellites in network RTK

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Abstract (exclude this section from numbering)

As the 25th solar cycle commences, users of network RTK (Real-Time Kinematic) encounter significant positioning errors originating from the ionosphere, particularly in low-latitude regions. A significant factor contributing to these errors is the inaccurate broadcast of ionospheric delays from the system to the user. Our analysis has shown that, despite the utilization of double-differencing, ionospheric delays of baselines less than 10km can still reach several decimeters, which posing a considerable challenge for ambiguity resolution and positioning accuracy at the user's end. To tackle this challenge, accurately estimating user ionospheric delays with appropriate methodology becomes crucial. After a thorough analysis, it has been identified that existing linear interpolation algorithms have limitations in capturing ionospheric errors induced by satellite movements. Consequently, a novel approach is proposed, which considers inter-satellite ionospheric variations. First, the underlying mechanism of this proposed approach is theoretically analyzed and explained. Then, the process and data used for interpolation within this new method are described. To validate its effectiveness, tests were conducted using data from two active days, March 14th and 24th, 2024. The results indicate that this new interpolation method could significantly reduce the double-differenced ionospheric delay estimation between the master station and the user. It is acknowledged that this method has its limitations. While it provides more accurate estimates of ionospheric delays, the increased observation noise and the difficulty in resolving ambiguities due to cycle slips during ionospheric scintillation periods remain challenging issues for future research. Addressing these challenges will require advancements in both system-end and user-end algorithms.

1. INTRODUCTION

In 2024, during the 25th solar cycle, solar activity is anticipated to peak, marking the "Solar Maximum" (Jha & Upton, 2024). This period brings about instability and unpredictability in the ionosphere, leading to positioning errors that can significantly compromise the accuracy, availability, and continuity of Real-Time Kinematic (RTK) systems (Dutta et al., 2022; Follestad et al., 2021). Traditional modeling techniques for ionospheric delays in RTK become inadequate during this volatile phase of the ionosphere, posing a significant challenge for Global Navigation Satellite System (GNSS) users, whether on land, sea, or air. The impact on RTK accuracy is profound, highlighting the urgency of investigating the mechanisms that lead to ionospheric degradation in RTK positioning and developing effective strategies to mitigate its adverse effects.

Unmodeled ionospheric delays in network RTK are one of the main concerns. Specifically, during solar active years, the ionosphere's maximum spatial gradient can reach up to 50 mm/km on approximately 70% of the days at low latitudes (Weng et al., 2015). That is, for a baseline with 20km, the positioning error would be larger than 1m. This frequently leads to inaccuracies in the broadcasted ionospheric corrections. Currently, the linear interpolation method (Wanninger, 1995a, 1995b) employed in network RTK could effectively mitigating spatially linear double-differenced ionospheric delays. However, these delays still suffer from spatially non-linear distance-dependent errors, particularly in regions around the magnetic equator.

When attempting to fit non-linear spatial ionospheric delays using low-order polynomials during interpolation, which builds upon traditional linear methods, suboptimal performance is often observed, similar to linear interpolation. While this issue is partly due to problems like the Runge Phenomenon (Fornberg & Zuev, 2007) that arise from polynomial fitting, the primary concern is that current interpolation methods only consider the distance between stations and ignore the distance between satellites. Essentially, the coordinates of the reference station are served as the sole independent variable, with the double-differenced ionospheric delay acting as the dependent variable. However, in reality, the double-differenced ionospheric delay is influenced by both the ionospheric pierce point (IPP) distance between single-differenced stations and satellites. The ionospheric variations caused by satellite movement are introduced into the ionospheric delays broadcast to users during the double-differencing process.

To address this issue, it is appropriate to investigate the modeling of spatial gradients caused by satellite movement. In the next section, we present the ionospheric gradient of the satellite, followed by an introduction to the modeling method. Finally, we compare the new modeling method with existing methods to determine its effectiveness.

2. PRESENTION OF THE IONOSPHERIC GRADIENTS BETWEEN SATELLITES

To model the ionospheric delay that cannot be addressed by linear interpolation methods, it is crucial to first understand the double differenced ionospheric delays. As depicted in **FIGURE 1**, from left to right, the diagram represents low and middle latitudes. When the signal from Satellite 1 traverses the ionosphere and is received by Receiver 1, the ionospheric delay of that signal is denoted as I_1^1, and the remaining notations follow suit accordingly.

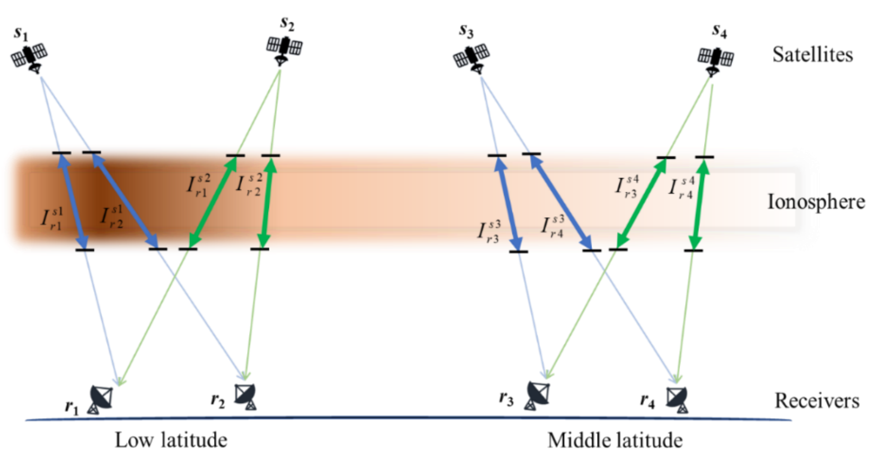

FIGURE 1 Ionospheric errors that remain uncorrected after the operation of double-difference: a schematic view of the comparison between low and mid-latitudes

Regarding ionospheric delay, assuming that between satellite single differences are first calculated, followed by between station single differences, the double-difference operation can effectively be expressed as:

$$f_{\Delta}^{\nabla}(I) = (I_1^1 - I_1^2) - (I_2^1 - I_2^2)$$

From the form of f_{Δ}^{∇}, we can know that the double-difference ionospheric delay comprises two types of distance dependent errors: one introduced by the between satellite single difference and the other by the between station single difference. Current linear interpolation algorithms utilize only station coordinates as independent variables and the double-difference ionospheric delay as the dependent variable (Wanninger, 1995a, 1995b). In mid-latitude regions, due to the relatively smooth nature of the ionosphere, the spatial variation of ionospheric delay with changes in distances between station is generally similar to the variation with distances between satellite. Therefore, linear interpolation is generally feasible in mid-latitudes. However, in low-latitude regions, the significant spatial gradient and its variation lead to spatial inconsistencies that render linear interpolation insufficient to fully capture the ionospheric delay between reference stations and user stations (Jongrujinan & Satirapod, 2020; Yuan et al., 2020).

To visualize the satellite ionospheric gradient better, we calculate the gradients of ionospheric delays using precise point positioning (PPP) (Glaner & Weber, 2023), as illustrated in **FIGURE 2**. Typically, after time differencing, the ionospheric gradients do not exhibit significant spatial variation trends, leaving only random components. However, the data reveals a noticeable nonlinear variation in the ionosphere at low latitudes. It is observable that the spatial gradient becomes more pronounced as the location moves away from directly overhead Hong Kong, specifically, as the elevation angle decreases, especially when nearing the equator around 20°N.

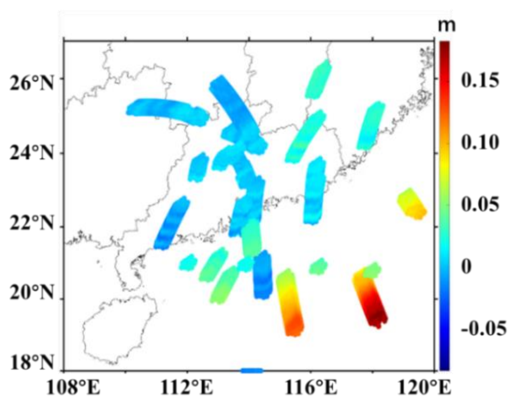

FIGURE 2 IPPs of all station-satellite pairs of GPS, GLONASS, Galileo, and Beidou over Hong Kong, along with their corresponding spatial ionospheric gradient variations, were monitored during the three-hour period at noon on January 1, 2023.

3. MODEL THE IONOSPHERIC GRADIENTS BETWEEN SATELLITES

To address this challenge, we propose an algorithm that models the satellite-induced ionospheric gradient. Firstly, the double-difference ionospheric delay between every two stations is calculated (Chen et al., 2001). Subsequently, we determine the satellite-specific (Chang et al., 2021; Li & Jiang, 2024) or between satellite (Supriadi et al., 2022) spatial gradient of the ionosphere. At each epoch, this gradient is detrended using the IPPs differences between the reference satellite and other satellites. The detrended results is then incorporated into the single-difference between stations, while maintaining the traditional linear interpolation approach based on station coordinates. The flowcharts illustrating this process are presented in **FIGURE 3**.

The primary reason for this approach is that the double-difference ionospheric delay is a distance-dependent term. One dependency is on the station coordinates, which correlate with local ionospheric variations. The other dependency is on the satellite position or IPP, reflecting ionospheric changes over a broader spatial scale. When these two dependencies align, detrending using only station coordinates suffices. However, when they diverge, the introduction of IPP becomes necessary to account for the trends introduced by inter-satellite differences.

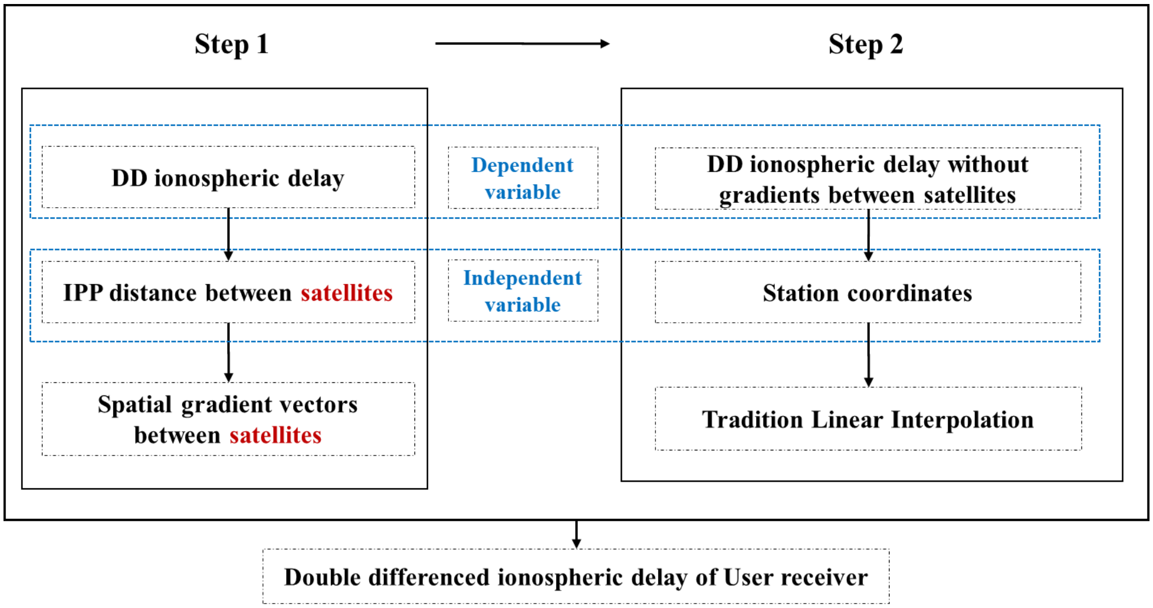

FIGURE 3 The work flow of the new interpolation method with the existing interpolation method

4. IMPLEMENTATION AND RESULTS

Multiple testing days were utilized to validate the efficacy of the proposed algorithm. For the purpose of experimentation, all ground stations located in Hong Kong were employed. Specifically, HKST served as the primary base station, while HKSC functioned as the user station. As illustrated in **FIGURE 4**,

As depicted in **FIGURE 4**, each set of three adjacent stations forms a triangular configuration. The baseline distance between each station and its neighboring stations is calculated, during which the double-difference ionospheric delay between those two stations is precisely estimated. Once the double-difference ionospheric delays between all neighboring stations have been determined, the double-difference ionospheric delay between the master station and the user station can be subsequently calculated, with the approximate location of the user, through the novel methodology.

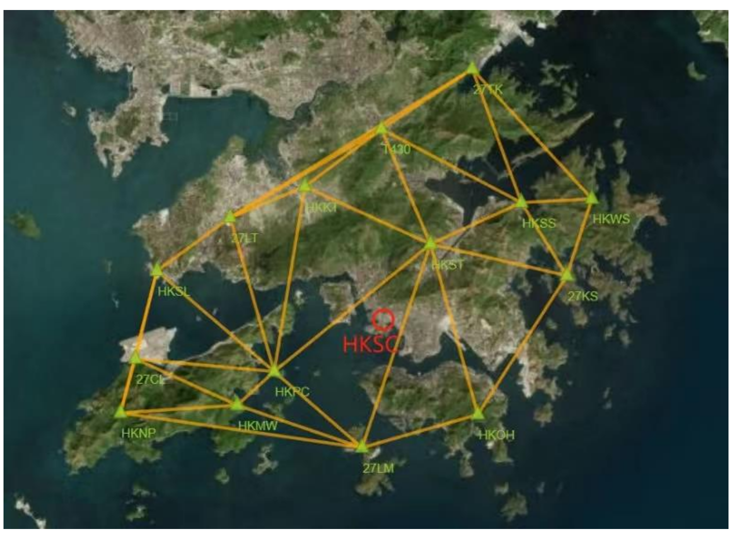

FIGURE 4 Reference stations and user locations

4.1 Double differenced ionospheric delay calculation

We examined the results of the double-difference ionospheric delays for various baselines after network formation. Taking HKST as the reference station, the double-difference ionospheric delays of its adjacent stations were calculated, as depicted in **FIGURE 5**. It is evident that for most baselines, starting from approximately 13:00, significant variations in the delays occurred, with the HKST-HKOH baseline exceeding 0.3 meters. The ionospheric delays remained stable until around 23:00 in the evening. Notably, some epochs exhibited apparent cycle slips or signal loss, which could impact the final interpolation results. Therefore, some epochs with low-quality observations were excluded. The reference satellite, which represents a delay of 0, is also indicated in the figure.

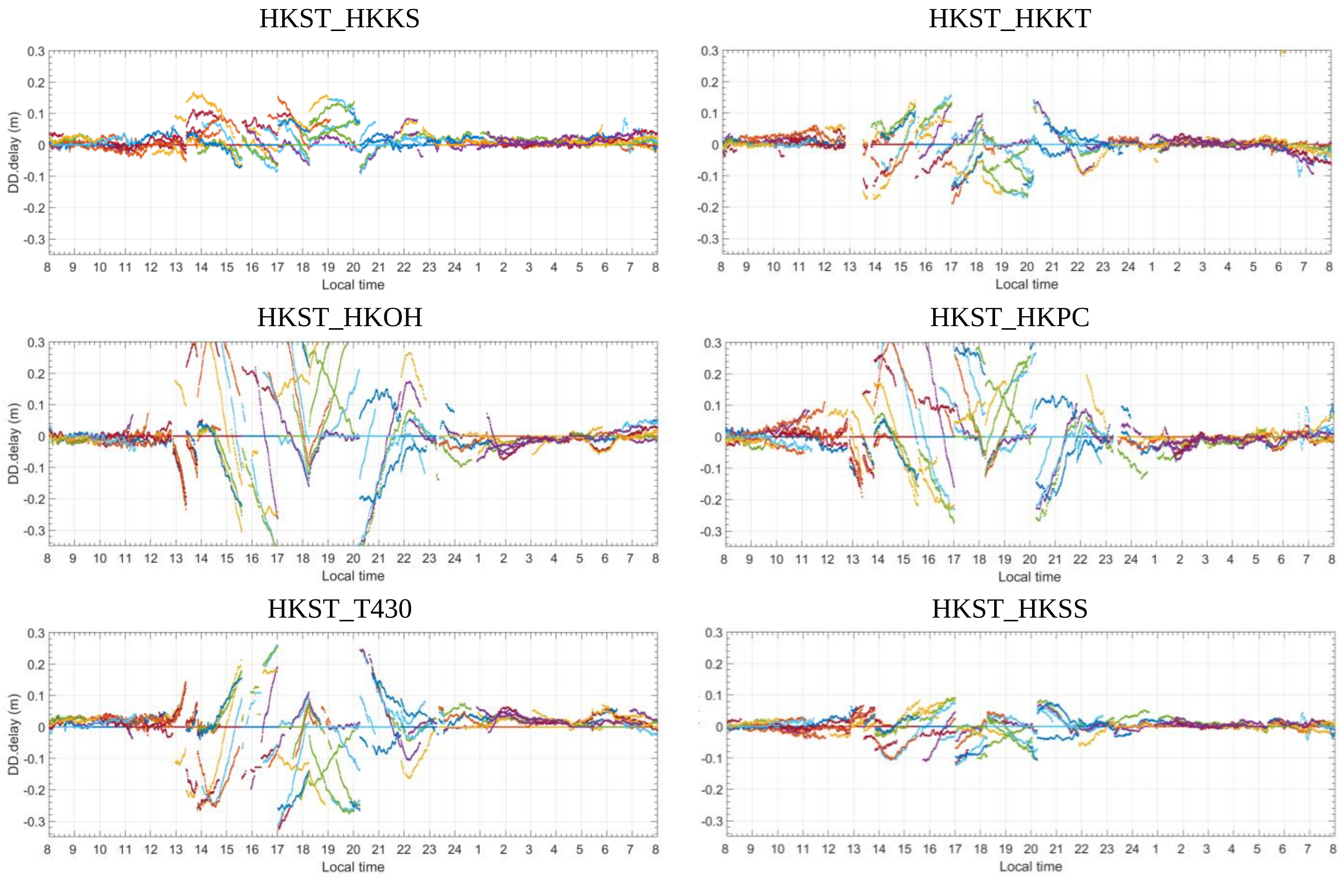

FIGURE 5 The six subplots represent the double-difference ionospheric delays calculated between HKKS, HKKT, HKOH, HKPC, T430, and HKSS, respectively, with HKST as the reference station. The horizontal axis indicates the local time in UTC+8, while the vertical axis represents the magnitude of the ionospheric delay. Different GPS satellites are represented by different colors. The date is March 24, 2024.

4.2 Satellite ionospheric delay component

As shown in **FIGURE 6**, the double-difference delay between the reference station HKST and the simulated user station HKSC increases from a few centimeters to over 0.2 meters between noon (12:00) and approximately midnight (24:00), and then gradually decreases. The linear interpolation depicted in the upper right figure adequately reflects the overall trend of change, yet its results are consistently a few centimeters smaller than the true values. This indicates that a portion of the ionospheric delay is not fully modeled by linear interpolation. As illustrated in the lower right figure, modeling the between satellite ionospheric gradient produces a magnitude of just a few centimeters, and its overall trend is similar to the true values.

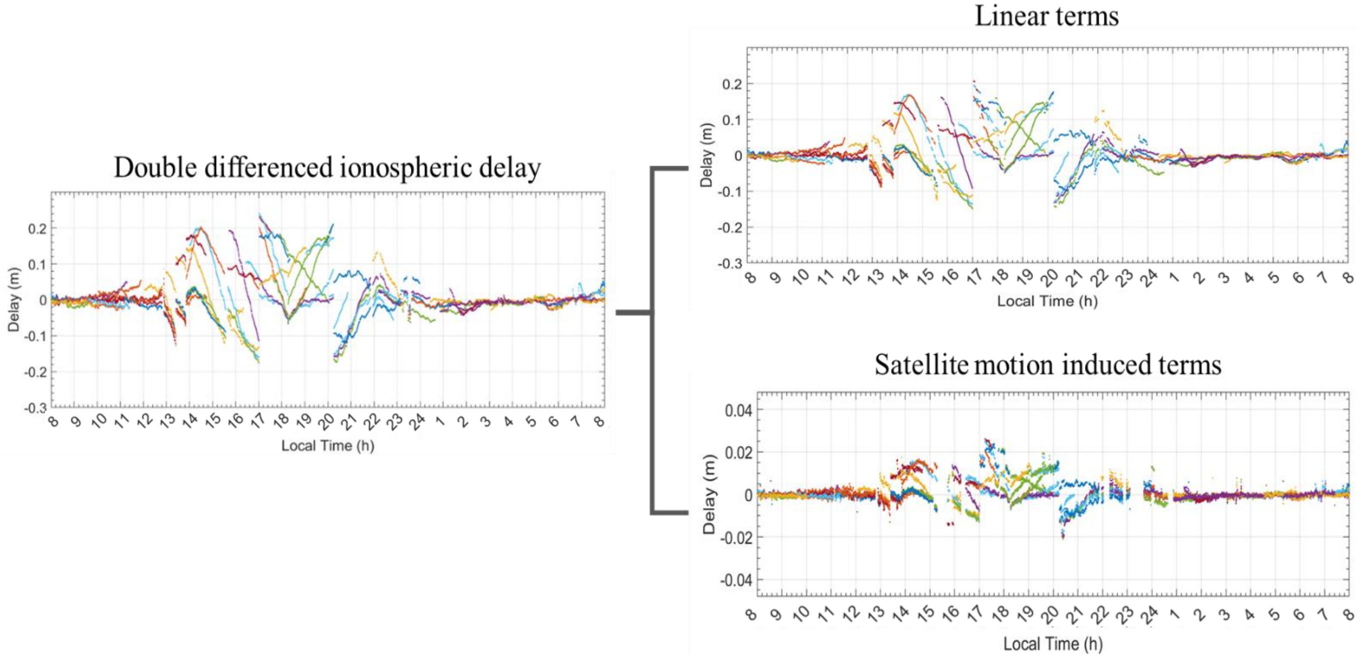

FIGURE 6 Results of the calculations conducted on March 24, 2024. The left figure depicts the double-difference ionospheric delay between the reference station HKST and the simulated user station HKSC. The upper right figure represents the results calculated using the traditional linear interpolation method, specifically the linear correlation term with station coordinates. The lower right figure illustrates the computed inter-satellite spatial gradient term. Different GPS satellites are represented by distinct colors. The horizontal axis denotes local time, while the vertical axis measures the ionospheric delay in meters.

4.2 Comparison with linear interpolation method

Testing was conducted using both the traditional linear interpolation method and the new proposed approach. The input data consisted of the double-differenced ionospheric delays for all baselines, and the output data was the estimated delay between the reference station and the user station.

As shown in **FIGURE 7**, which depicts the results for March 24, 2024, it is evident that by considering the between satellite ionospheric motion, the final ionospheric delay is significantly reduced. Specifically, during the period from approximately 13:00 to 15:00, errors that had persisted around 3 cm for several satellites were eliminated. At around 16:00-17:00, the significant trend observed for one satellite (depicted in purple) was nearly completely removed. Afterward, the delay values for many satellites became more stable.

To further validate the results, an additional test was performed using data from March 14, 2024, as shown in **FIGURE 8**. It can be observed that during the period from mid-afternoon to sunset, the pronounced trends observed for multiple satellites were mitigated.

During data processing, the low-latitude ionosphere during active years introduces numerous jumps in observational data from stations. This frequently necessitates changes in the reference satellite, yet these changes are not synchronized across all baselines. To rectify this, a unified reference satellite selection was implemented for all baselines on the system side. However, despite this adjustment, certain intractable jumps persist in the double-differenced ionospheric delays between base stations. It is postulated that these jumps are attributed to an increase in short arcs stemming from consecutive cycle slips and the challenges in resolving wide-lane ambiguities due to heightened pseudorange noise.

Overall, the new ionospheric interpolation algorithm that takes into account inter-satellite motion can more accurately estimate the user's double-differenced ionospheric delay, potentially enhancing the success rate of ambiguity resolution and improving positioning accuracy.

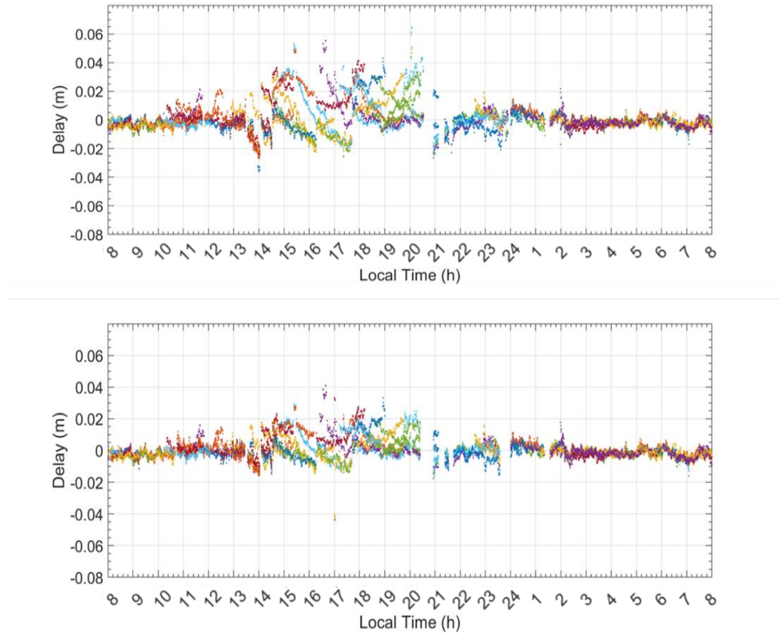

FIGURE 7 The upper plot represents the results of the traditional linear interpolation, while the lower plot shows the results after considering the variations in ionospheric delay caused by satellite motion. Different GPS satellites are represented by distinct colors. The horizontal axis denotes local time, while the vertical axis measures the ionospheric delay in meters. The data is March 24, 2024.

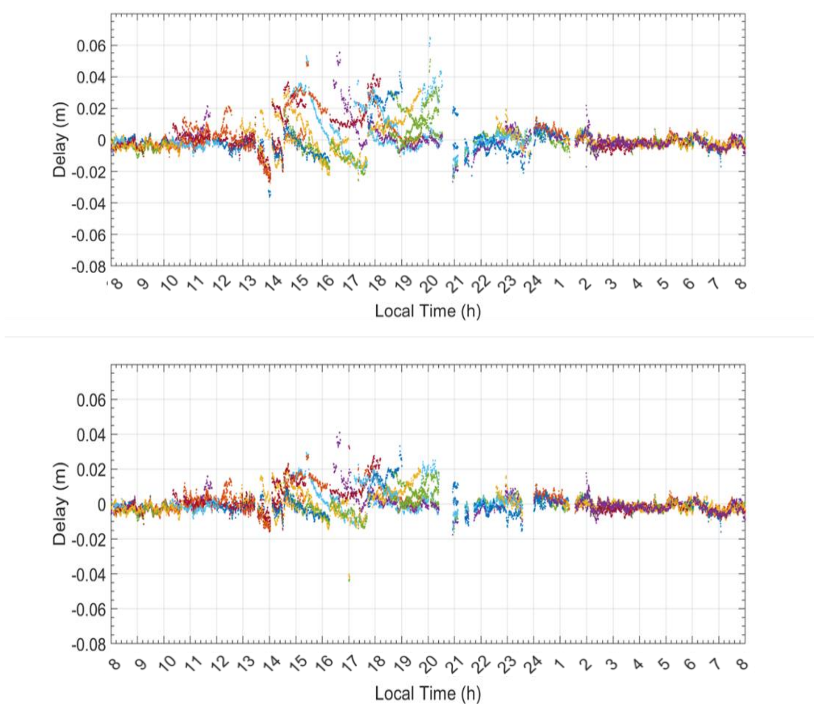

FIGURE 8 The upper plot represents the results of the traditional linear interpolation, while the lower plot shows the results after considering the variations in ionospheric delay caused by satellite motion. Different GPS satellites are represented by distinct colors. The horizontal axis denotes local time, while the vertical axis measures the ionospheric delay in meters. The data is March 14, 2024.

5. DISSCUSION

As observed from the final results, while the new method demonstrates some improvement, the double-differenced ionospheric estimates at the user end are still not pure white noise. This indicates that there is still room for further enhancement. Two primary factors should be considered: firstly, the north-south difference, as Hong Kong is located north of the magnetic equator, and the ionospheric gradient in the afternoon is primarily caused by the north-south direction (Kumar, 2020; Yang et al., 2024). The differences in the north-south and east-west directions should be modeled separately. Secondly, the handling of cycle slips or gross errors in the data processing should be more refined. During periods of active ionospheric activity, traditional cycle slip detection methods may not be applicable, which could lead to jumps in the double-differenced ionospheric delays at the system end, thereby affecting the final interpolation results (Breitsch & Morton, 2023; Zhao et al., 2024). In such cases, the new method may not be able to fully leverage its advantages.

It is worth mentioning that for days with severe ionospheric scintillation, the baseline solution at the system end may become unstable due to poor data quality. Under such conditions, any interpolation algorithm may fail. On the other hand, during periods of non-active ionospheric activity, the double-differenced ionospheric delays are typically very small, and the difference between using linear interpolation and the new interpolation method may not be significant. The new method is primarily applicable during active years. These discussions are illustrated in **FIGURE 9**.

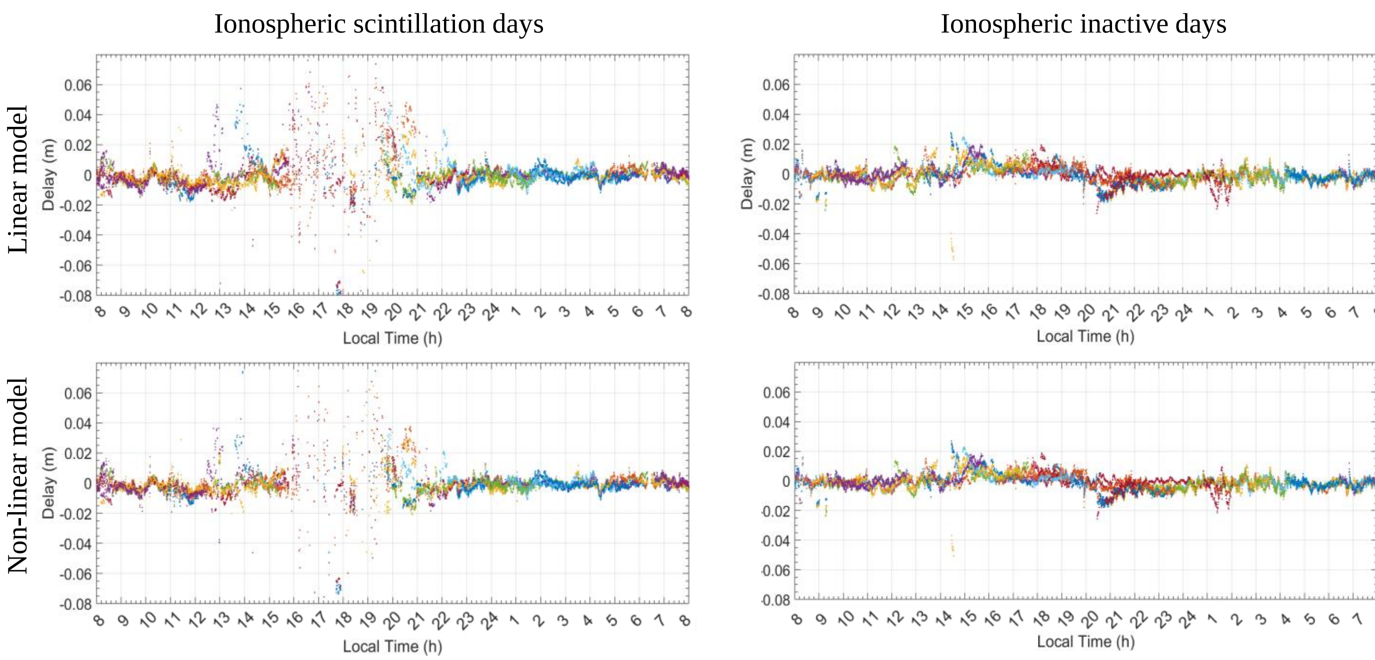

FIGURE 9 The two subplots on the left represent days with ionospheric scintillation, specifically January 21, 2024, while the right side depicts a day with inactive ionosphere, January 6, 2021. The top two graphs show the results of traditional linear interpolation, and the bottom two graphs illustrate the outcomes of the new method. The horizontal axis represents local time, and the vertical axis indicates the delay in meters. Different colors represent various GPS satellites.

6. CONCLUSION

During years of active ionosphere, users of network RTK positioning face the issue of significant positioning errors caused by the ionosphere, particularly severe in low-latitude regions of the Earth. One contributing factor is the inaccurate broadcast of ionospheric delays from the system end to the user end. In response, we have examined the existing linear interpolation algorithms and analyzed their inability to account for ionospheric errors induced by satellite movements. Subsequently, we propose a novel approach that considers inter-satellite ionospheric variations. The results indicate that our new method can effectively reduce the double-differenced ionospheric delay estimation between the master station and the user.

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