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Single-pixel imaging through random media with automated adaptive corrections

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ABSTRACT

The existence of random media is challenging in optical imaging, as the existing approaches usually cannot work well when the optical channel exhibits a certain level of randomness. Here, we report an automated adaptive correction scheme for single-pixel imaging through random media. An alternating projection method is developed to reconstruct an object from light intensities recorded by a single-pixel detector. A series of scaling factors are incorporated into object reconstruction to correct wave distortions induced by random media. With the introduced scaling factors, an essential relationship between collected and theoretical light intensities is revealed. It is illustrated that the proposed corrections on the realizations do not require *prior* knowledge about random media, and can be adapted to various real-world scenarios. High-quality imaging through random media can always be realized in experiments, and the proposed approach opens up an avenue for high-quality imaging through random media in various applications.

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Optical imaging has become an important tool in many fields, such as life science, materials science, and astronomy. However, the ubiquity of random media in natural applications poses a challenge to achieving high-quality imaging.^{1–4} Typical examples of random media in optical imaging systems include biological tissues,¹ atmospheric disturbance,² and turbid water.³ Random media are typically characterized by spatial and temporal randomness inherent in their properties. Due to the randomly distributed refractive indices within random media, incident light is scattered into multiple secondary components with different phase constants.^{4–6} Therefore, optical waves are distorted, leading to a degradation in imaging performance and schemes have been developed to alleviate their effects in imaging systems. For instance, wavefront shaping allows for light focusing through scattering media by adjusting incident wavefront but could require *prior* knowledge about a reference point or object information.^{7–10} The advances in memory effect have enabled imaging through scattering media. The fundamental idea of memory effect is that the imaging system is shift-invariant within the memory effect range.^{11–13} However, imaging based on memory effect inherently suffers from restrictions in terms of the field of view (FOV) and the propagation distance within scattering media.¹⁴ Adaptive optics provides an alternative approach for focusing deep into scattering media. However, adaptive optics is primarily designed to correct low-order aberrations, and could require

the setup of a guide-star in an optical imaging system.^{15–17} Moreover, most existing studies have not addressed the issue of random media, and there are still some constraints on their applications.¹⁸

Optical waves could be scattered and absorbed as a light beam passes through random media.¹⁹ Direct recording of the output light would result in a combination of complex components having random amplitude and phase.⁶ The mixture of complex components in the output light makes it difficult to resolve an object. With this complexity, it is promising to address this challenge with an indirect imaging approach.^{20–22} The use of a single-pixel detector in an imaging system has emerged as an alternative solution, where the collected light intensities can be regarded as the sum of complex components.^{23,24} The computational means has been developed to reconstruct the object from single-pixel light intensities.^{25–30} As can be found in previous studies,^{30,31} single-pixel detection can facilitate optical imaging through scattering media. However, most random media in real-world scenarios simultaneously exhibit spatial and temporal randomness. The time-dependent nature of single-pixel detection makes the collected single-pixel light intensities susceptible to random variations in the wave propagation media over time. Therefore, the resulting single-pixel light intensities exhibit random fluctuations compared with those measured without scattering in the optical channel. The correlation between the illumination patterns and the realization is disturbed,

which compromises the reconstruction quality.^{30,31} Despite a tolerance to static distortion, the enhancement of single-pixel detection schemes is still desired to address the problems in random media.

In this Letter, we report an approach that imposes automated adaptive corrections on the distorted single-pixel light intensities for high-quality imaging through random media. An object reconstruction algorithm is developed using the concept of alternating projections.^{32–34} The proposed approach also enforces the corrections on the distorted single-pixel light intensities by introducing scaling factors,

which generalizes the effect of random media on the collected light intensities. Specifically, a sequence of scaling factors corresponding to the collected light intensities are estimated to approximate theoretical light intensities recorded without scattering in an optical channel. Then, the approximated intensities can be used for object reconstruction, and the effect of random media can be removed. The proposed approach allows for an automated correction on the realizations through incremental progress of alternating projections without *prior* knowledge about random media. Moreover, automated adaptive

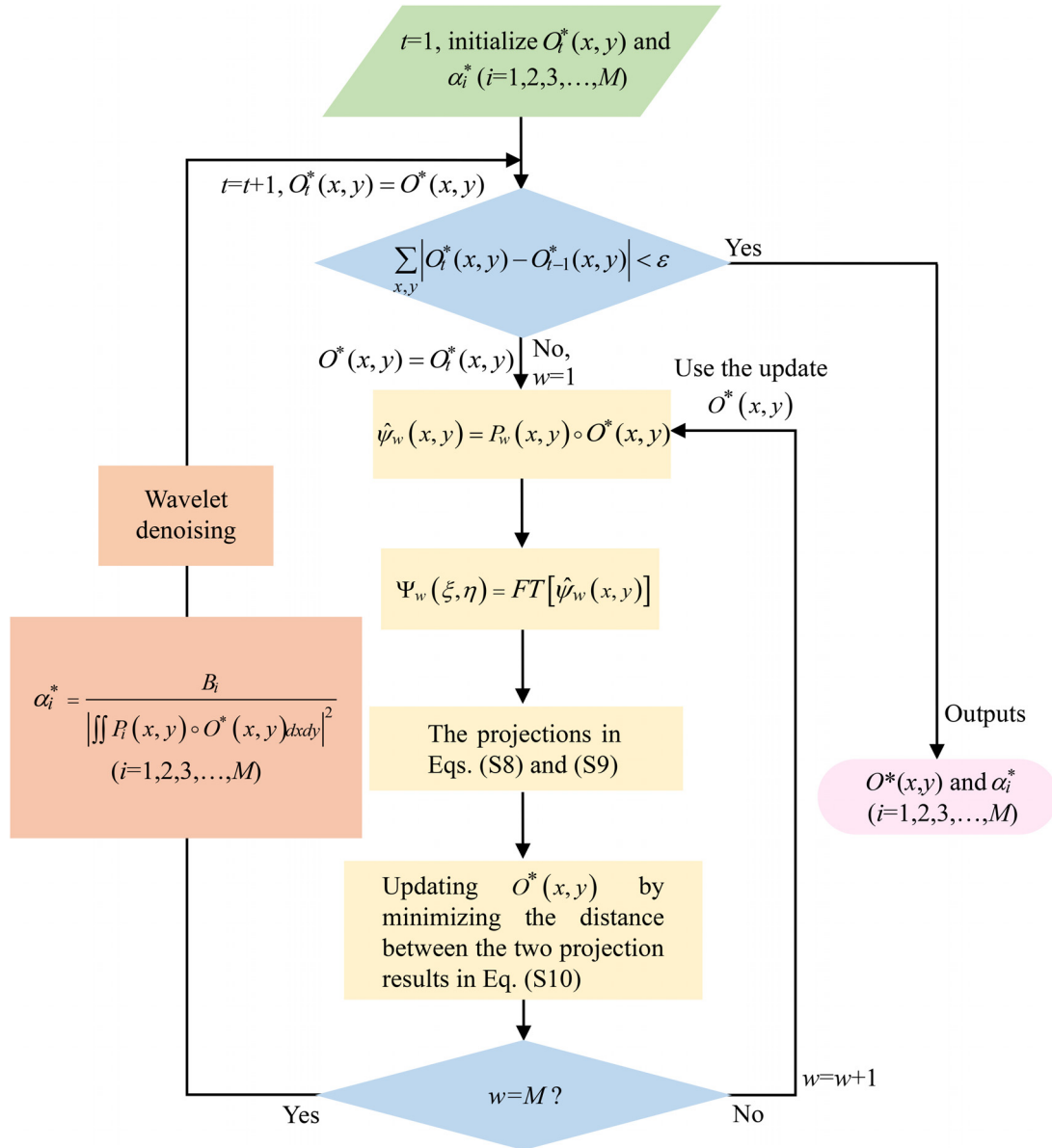


FIG. 1. A flow chart of the proposed algorithm. ϵ , a pre-defined threshold of 1.5; t , the iteration number; $w = 1, 2, 3, \dots, M$; M , the total number of the collected single-pixel light intensities; FT , Fourier transform. In the 1st iteration, $O_0^*(x, y)$ is set as a matrix with all ones. The series of scaling factors is initialized as a vector with all ones, and $O_t^*(x, y)$ is set as a random matrix. Each estimated scaling factor is regularized in the range of 0–1.0. Wavelet denoising³⁵ is applied to process the series of estimated scaling factors, and noise can be suppressed.

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corrections can be used to overcome the limitation of single-pixel detection, when rapidly temporal changes occur in random media. Optical experiments are conducted to demonstrate that the proposed scheme can be adapted to various scenarios, and can remain robust against different types of realization distortions. The proposed approach overcomes the challenge of realizing high-quality imaging through random media with a simple optical setup, and high adaptability is achieved, which broadens the applications of optical imaging in real-world scenarios.

Alternating projection methods^{32–34} have received much attention in different areas, and several modified algorithms have been developed, such as hybrid input–output.³² However, a direct application of the alternating projection methods cannot solve the problem of beam distortions induced by random media. We unravel the existence of a series of dynamic and random scaling factors that generalize the absorption and scattering effects in random media. A robust object reconstruction algorithm is reported, which adopts a constraint on the realizations using the estimated scaling factors. A flow chart of the proposed algorithm is given in Fig. 1, and more details on the proposed algorithm are given in Supplementary Note 1 in the supplementary material.

A schematic experimental setup for the developed optical single-pixel imaging system is shown in Fig. 2. A green laser (MGL-III-532 nm) with the maximum power of 200.0 mW and wavelength of 532.0 nm is used as light source to illuminate a spatial light modulator (SLM, Holoeye HED 6001) with a pixel pitch of 8.0 μm . Random amplitude patterns are sequentially displayed by SLM and projected onto an object by a $4f$ system consisting of a lens L_2 with a focal length of 50.0 mm and lens L_3 with a focal length of 150.0 mm. Each random amplitude pattern, having a size of 128×128 pixels, is pre-generated

numerically. After illuminating the object, optical wave propagates through random media and then is collected by a single-pixel photodetector (Thorlabs, PDA100A2). The achievable resolution of the optical imaging system is 24.0 μm , and the USAF 1951 resolution target is tested in the experiments. Three different random media are individually applied to verify the proposed approach as shown in Fig. 2. The first instance involves two cascaded moving ground glass diffusers (Thorlabs, DG10-1500). The second instance contains a non-line-of-sight (NLOS) imaging where a corner with a separation distance of 10.0 mm is placed behind a moving ground glass diffuser (Thorlabs, DG10-1500). An ordinary A4 paper is used for light reflection, and a black cardboard is applied to block partial light. The third instance employs a complex scenario with the combination of a dynamic and turbid water environment and a moving ground glass diffuser (Thorlabs, DG10-1500). The ground glass diffusers have a thickness of 2.0 mm, and are randomly moved in the transverse domain during the experiments. A water tank used in the third instance has a dimension of 10.0 cm (length) \times 30.0 cm (width) \times 30.0 cm (height), and initially contains 6000-ml clean water. 3.0-ml milk diluted with 500.0-ml clean water is being dropped into water tank during the experiments, and a stirrer operating at 600.0 rpm is placed inside water tank.

In the designed imaging system with single-pixel detection, the i th single-pixel light intensity B_i can be described by

$$B_i = \alpha_i \left| \iint P_i(x, y) \circ O(x, y) dx dy \right|^2, \quad (1)$$

where α_i denotes the i th scaling factor, i denotes an integer (1, 2, 3, ..., M), M denotes the total number, $O(x, y)$ denotes the intensity transmittance of an object, $P_i(x, y)$ denotes the i th random amplitude pattern, and \circ denotes the Hadamard product. In Eq. (1), a random

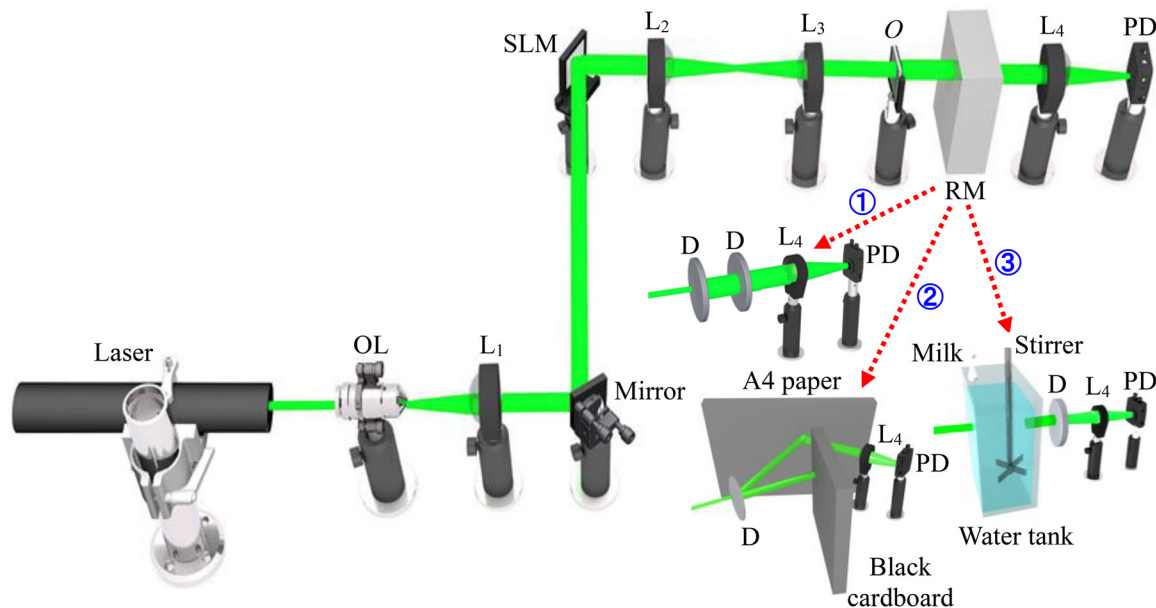


FIG. 2. A schematic experimental setup and illustration of random media. A setup for the proposed optical imaging with single-pixel detection: OL, objective lens; SLM, amplitude-only spatial light modulator; L_1 – L_4 , lenses; RM, random media; D, a diffuser randomly moving in the transverse domain without rotation; PD, a photodetector with single-pixel detection. Typical examples (i.e., the insets) of random media used in the experiments: (i) two cascaded randomly moving diffusers, (ii) a randomly moving diffuser around a corner, and (iii) a dynamic and turbid water tank with a randomly moving diffuser.

scaling factor is induced to account for the effects of absorption and scattering occurred in random media on each collected single-pixel light intensity (see supplementary Note 1 for the detailed method in the [supplementary material](#)). When the imaging system is free of scattering and absorption, the scaling factors could be assumed to remain a constant. Then, Eq. (1) can be interpreted as a sum of the exit wave behind the target, and the normalized light intensities would converge to a normal distribution.²⁴

In optical imaging through random media, the parameter α_i is regarded as a random variable ranging from 0 to 1, and each scaling factor depends on the current properties of the random media at the measurement instant. To render the distortion effect of random media, the collected single-pixel light intensities are shown in Figs. 3(a), 3(c), and 3(e). The histograms of these single-pixel light intensities are respectively shown in Figs. 3(b), 3(d), and 3(f), providing the effects induced by random media. The distributions do not exhibit symmetry or skewness around the center, indicating significant deviations from the expected normal distributions. As can be seen in Figs. 3(a)–3(f), single-pixel light intensities collected through random media are subject to random distortions, breaking the inherent correlation. Therefore, it is recognized that reconstructing the object from the distorted single-pixel light intensities is challenging, and effective

corrections on the realizations are essential for imaging through random media.

It is desirable to eliminate the effect of random scaling factors and recover theoretical correlations for object reconstruction. The proposed approach can effectively correct the realizations to become approximately a normal distribution that is consistent with a theoretical distribution of realizations obtained without scattering media in the optical channel. The probability distribution fitting is applied to validate the reliability of the realization corrections. The single-pixel light intensities after correction are shown in Figs. 4(a), 4(c), and 4(e). The corrected light intensities result from an algorithmic correction on the realizations in Figs. 3(a), 3(c), and 3(e), respectively. Compared to the collected light intensities, the corrected ones exhibit far fewer deviations. The histograms of the corrected realizations are shown in Figs. 4(b), 4(d), and 4(f), respectively. To verify the effectiveness of the realization corrections, each set of the corrected realizations is fitted to match a normal distribution model. It is demonstrated in Figs. 4(b), 4(d), and 4(f) that normal density functions with the expectation and standard deviation estimated from each set of corrected data can be generated (see Supplementary Note 1 for the detailed method in the [supplementary material](#)). As can be seen in Figs. 4(b), 4(d), and 4(f), the histograms of corrected realizations exhibit a strong resemblance

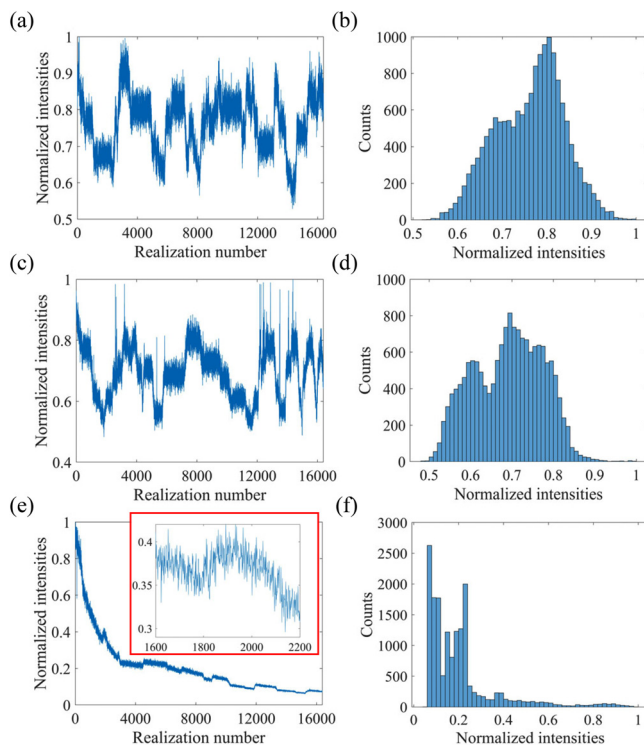


FIG. 3. The collected single-pixel light intensities in random media. (a), (c), and (e) The normalized single-pixel light intensities collected respectively through the two cascaded randomly moving diffusers, a randomly moving diffuser around a corner, and dynamic and turbid water with a randomly moving diffuser. Each set of realizations contains 16 384 elements. (b), (d), and (f) The histograms of single-pixel light intensities in (a), (c), and (e), respectively. The inset in (e) shows a part of the realizations.

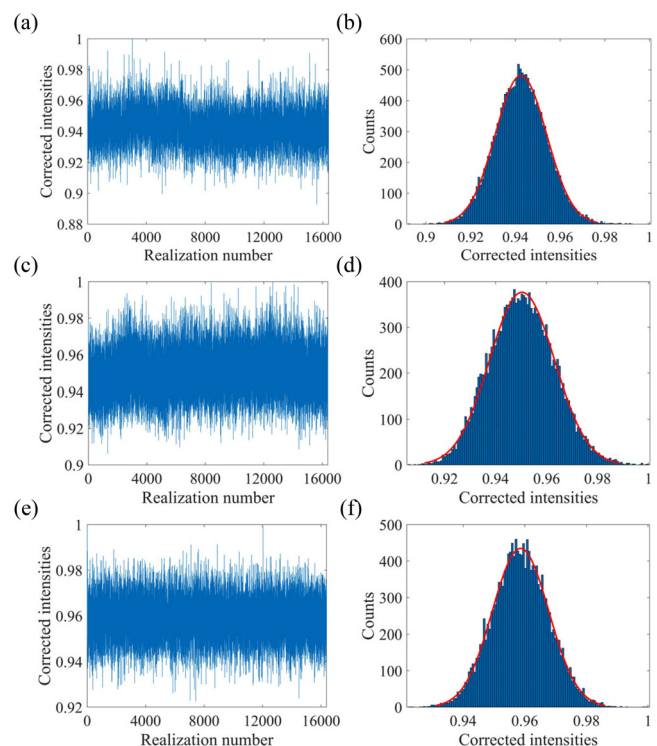


FIG. 4. The corrections on the collected single-pixel light intensities using the proposed approach. (a), (c), and (e) The corrected single-pixel light intensities (normalized). (b), (d), and (f) The histograms of the corrected single-pixel light intensities in (a), (c), and (e), respectively. Normal distributions with the estimated expectation and standard deviation from these sets of corrected data are indicated by red curves in (b), (d), and (f), respectively.

to normal density functions in terms of shape and skewness. The expectations of normal distributions used for the distribution fitting are 0.9427, 0.9504, and 0.9586, respectively. Standard deviations of normal distributions used for the distribution fitting are 0.0115, 0.0127, and 0.0091, respectively. These sets of corrected data are well fitted to normal distributions characterized by similar expectations and standard deviations, showing that the correction results are consistent and conform to the theories. It is worth noting that the realizations are obtained in random media with different configurations, and the properties of random media undergo random changes in spatial and temporal domains. The proposed approach can allow to correct wave distortions without *prior* knowledge about random media.

The effectiveness of realization corrections is further verified by comparing the quantiles of the corrected realizations and normal

distribution models. The results are given in Supplementary Note 2 in the [supplementary material](#). Moreover, a comparison between the corrected realizations in random media and the realizations obtained without scattering in the optical channel is conducted. As can be seen in Fig. S2 in the [supplementary material](#), only subtle errors occur. Therefore, the automated adaptive correction scheme can correct wave distortion such that the corrected realizations are in accordance with the single-pixel light intensities obtained without scattering in the optical channel.

With the presence of random media, conventional approaches fail to recover effective or clear information about the target (see Supplementary Note 3 in the [supplementary material](#) for details). This is because the correlation between illumination patterns and the realizations is disrupted by random scaling factors. By incorporating

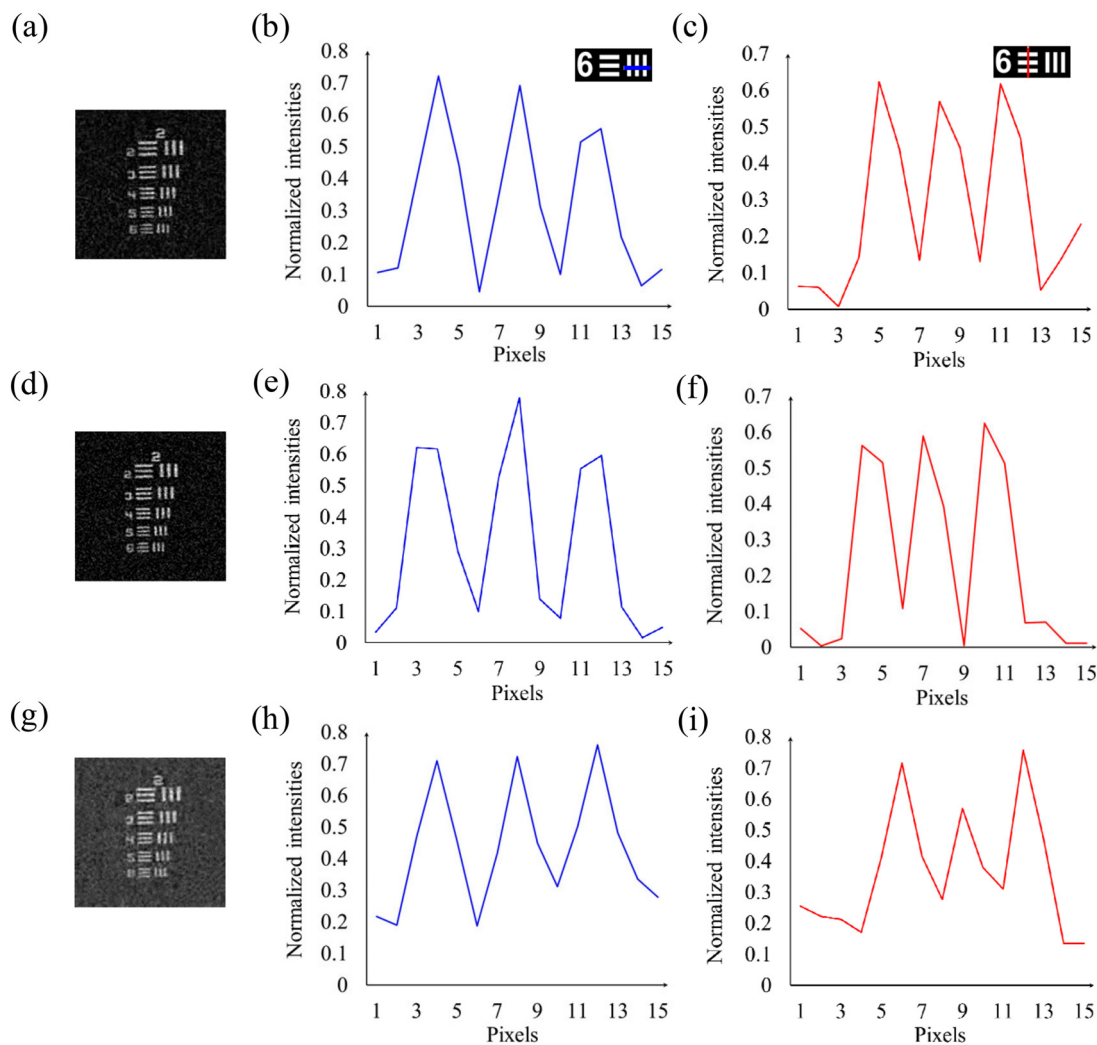


FIG. 5. Object reconstruction. (a) A reconstructed object through the two cascaded randomly moving diffusers. (d) A reconstructed object through a randomly moving diffuser around a corner. (g) A reconstructed object through dynamic and turbid water with a randomly moving diffuser. CNRs of the reconstructed object images in (a), (d), and (g) are 6.33, 5.65 and 5.31, respectively. (b), (e), and (h) The horizontal profiles (indicated by a blue line in the inset) in the reconstructed object images in (a), (d), and (g), respectively. (c), (f), and (i) The vertical profiles (indicated by a red line in the inset) in the reconstructed object images in (a), (d), and (g), respectively.

random scaling factors into the developed algorithm, high-quality objects can be recovered (see Supplementary Note 1 for the detailed method in the [supplementary material](#)). Figures 5(a), 5(d), and 5(g) show the reconstructed object images. The horizontal intensity profiles in the reconstructed objects are shown in Figs. 5(b), 5(e), and 5(h), and the vertical intensity profiles in the reconstructed objects are shown in Figs. 5(c), 5(f), and 5(i). It can be seen that the bars in the element 6 of group 2 are clearly distinguished vertically and horizontally. The standard resolution target is reconstructed with a resolved resolution of $140.3 \mu\text{m}$. Contrast-to-noise ratio (CNR)³⁶ is also calculated to quantitatively evaluate the quality of the reconstructed objects (see Supplementary Note 4 in the [supplementary material](#) for details). The variation of CNRs is presented to illustrate that the automated adaptive correction algorithm always converges to a satisfactory solution. The relationships between the number of iterations and CNR values are given in Figs. 6(a)–6(c). It can be seen that the proposed approach can always converge to a high CNR. More illustrations on algorithm convergence are given in Supplementary Notes 5 and 6 in the [supplementary material](#).

Figure 7 shows the experimental results of imaging another target through random media. The histograms of the collected single-pixel light intensities are shown in Figs. 7(a), 7(d), and 7(g). The variations

indicate that the changes due to wave propagation media are random and unpredictable. To illustrate superior capabilities of the automated adaptive correction scheme, a conventional algorithm, i.e., differential ghost imaging (DGI),²⁶ is applied to recover the object using the collected single-pixel light intensities, and the reconstruction results are shown in Figs. 7(b), 7(e), and 7(h). The failure is due to the reason that the correlation between illumination patterns and the realizations is disrupted. In contrast, the images reconstructed by using the proposed approach can clearly render spatial information of the target, as shown in Figs. 7(c), 7(f), and 7(i). Therefore, the proposed approach can overcome the challenges for high-quality imaging through random media by imposing an automated adaptive correction on the collected single-pixel light intensities. Moreover, the proposed approach remains effective when correcting different types of wave distortions, as it can deal with any randomly distorted realizations with variable probability distributions.

We report an automated adaptive correction scheme for high-quality single-pixel imaging through random media. Random amplitude patterns are utilized for structured illumination in the designed optical imaging system, and light intensities are sequentially recorded by a single-pixel detector. It is demonstrated that the complex absorption and scattering effects caused by random media can be generalized

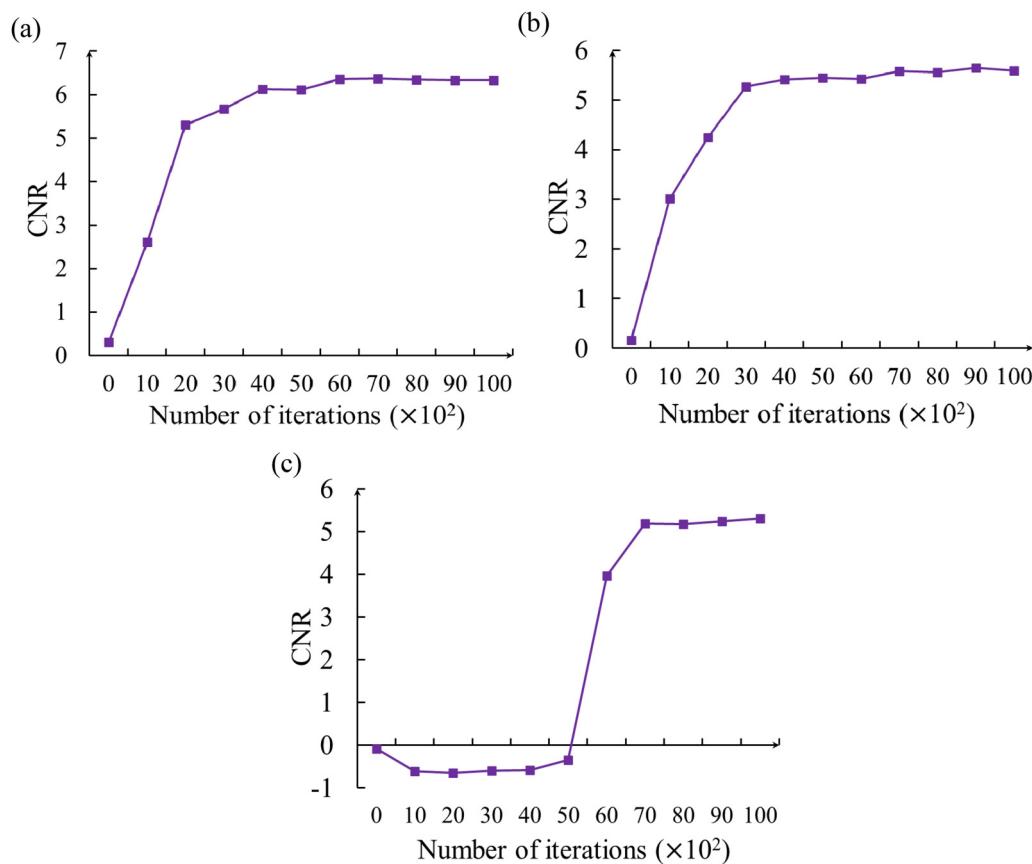


FIG. 6. Algorithm convergence. (a) A plot of the CNRs with the number of iterations as imaging through two cascaded randomly moving diffusers. (b) A plot of the CNRs with the number of iterations as imaging through a randomly moving diffuser around a corner. (c) A plot of the CNRs with the number of iterations as imaging through dynamic and turbid water with a randomly moving diffuser. For comparison, CNRs of the object images reconstructed by using DGI are 1.11, 0.53, and 0.22, respectively.

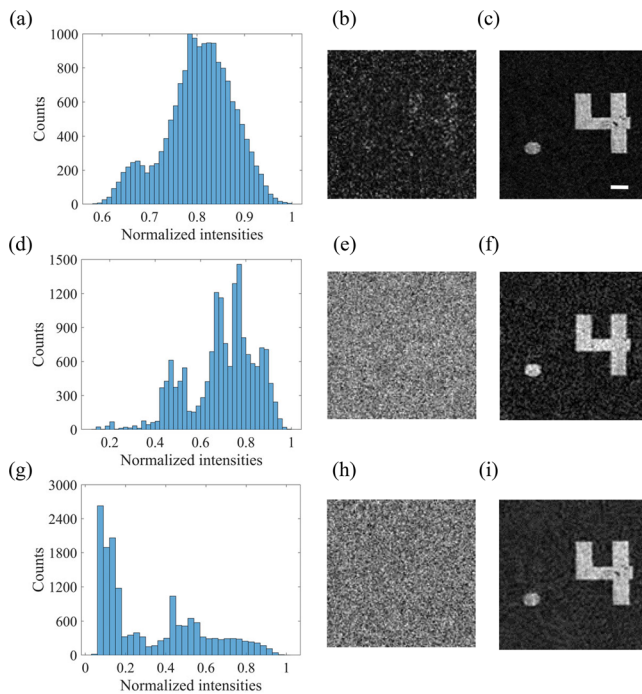


FIG. 7. Imaging another target through random media. (a) The histogram of single-pixel light intensities collected through the two cascaded randomly moving diffusers. (d) The histogram of single-pixel light intensities collected through a randomly moving diffuser around a corner. (g) The histogram of single-pixel light intensities collected through dynamic and turbid water with a randomly moving diffuser. (b), (e), and (h) The reconstructed objects obtained by using DGI. (c), (f), and (i) The reconstructed objects obtained by using the proposed approach. The CNRs of the reconstructed object images in (b), (e), and (h) are 0.95, 0.14, and 0.06, respectively. The CNRs of the reconstructed object images in (c), (f), and (i) are 7.89, 6.86, and 5.32, respectively. Scale bar in (c): 1.0 mm.

by a series of random scaling factors imposed on the collected single-pixel light intensities. By integrating the series of random scaling factors as a constraint in the reconstruction algorithm, the corrected single-pixel light intensities and a high-quality image of the target can be simultaneously acquired. Different types of random media have been studied in the experiments: cascaded randomly moving diffusers, NLOS imaging with a moving diffuser, and dynamic and turbid water. Experimental results demonstrate that high-quality object images can always be reconstructed from the randomly distorted realizations using the proposed approach. The proposed method is adaptive, and can also be applied in other scattering environments, e.g., light disturbance and detection fluctuation.

The proposed automated adaptive correction on wave distortions is effective, and spatial and temporal randomness is explored. The proposed approach corrects the distorted light intensities by recursively estimating dynamic scaling factors in random media. The proposed approach also possesses other advantages. The proposed approach enforces the completely algorithmic corrections on the distorted single-pixel light intensities, thus eliminating a need of *prior* knowledge about random media. Moreover, no physical corrections are implemented in the designed optical imaging system, offering adaptive and flexible features in

real-world scenarios. The FOV achieved is determined only by speckle patterns used for structured illumination.³⁰ Therefore, the proposed approach is not subject to a trade-off between the FOV and the imaging depth suffered by the methods based on memory effect when thick media are considered. Most importantly, an advanced solution is reported for imaging through random media that can change rapidly over time, whereas most existing methods deal with static or slowly varying scattering. Since the developed imaging system is built on a single-pixel detection framework, it possesses more advantageous features than the methods that use 2D array cameras. For instance, the proposed approach has a flexibility to be adapted to the different imaging modalities, e.g., infrared,^{37,38} x-ray,³⁹ and terahertz.⁴⁰ It is also possible to transform the developed imaging system for hyperspectral imaging,⁴¹ 3D imaging,⁴² or remote sensing⁴³ with reduced cost and complexity.

Since data acquisition is conducted in a sequential way, the sampling ratio could affect imaging performance. Although the capability of the proposed approach for high-quality imaging through random media is studied, it is noted that there is potential for further exploring the proposed approach. Compressive sensing (CS) is an efficient paradigm that exploits the sparsity of a signal and allows for recovering the signal with far fewer samples. Therefore, it is possible to use orthogonal basis patterns for structured illumination, and then a projection to the object in the proposed approach can be realized by using CS algorithms to achieve an enhanced reconstruction quality with a reduced sampling ratio. Deep learning can also be incorporated to reduce the sampling ratio by conducting the projections with neural networks. In a broad sense, the proposed approach can serve as a general framework to simultaneously implement the realization corrections and object reconstruction. Therefore, the projections in the proposed approach can be conducted flexibly with a computational means to meet different demands. Despite the aforementioned advantages, the proposed approach could have certain limitations. For instance, the sequential detection scheme could restrict the proposed approach from some potential applications in real-time imaging, which is inherent in single-pixel detection schemes. High-speed display devices, such as LED arrays, could be explored.

In conclusion, we have reported an automated adaptive correction scheme for high-quality single-pixel imaging through random media. An alternating projection method is developed to reconstruct an object from light intensities recorded by a single-pixel detector. A series of scaling factors are incorporated into object reconstruction to correct wave distortions induced by random media. With the introduced scaling factors, an essential relationship between collected and theoretical light intensities is revealed. It is experimentally illustrated that the proposed corrections on the realizations do not require *prior* knowledge about random media, and can be adapted to various real-world scenarios.^{44–47} Experimental results demonstrate that the approach always can realize high-quality imaging through random media, and could open up an avenue for high-quality imaging through random media in various applications.

See the [supplementary material](#) for details on the methods used and more experimental results.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yining Hao: Data curation (lead); Formal analysis (lead); Investigation (lead); Methodology (lead); Writing – original draft (lead). **Yin Xiao:** Formal analysis (equal); Investigation (equal). **Wen Chen:** Conceptualization (lead); Formal analysis (lead); Funding acquisition (lead); Methodology (lead); Project administration (lead); Resources (lead); Supervision (lead); Writing – review & editing (lead).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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