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ABSTRACT

Understanding the emergence and maintenance of cooperation in multiplayer games is a significant challenge across various theoretical disciplines. In this paper, we introduce an innovative model to study the impacts of environmental feedback in systems with critical thresholds. Different from prior studies on public goods games with environmental feedback, we propose that the system holds expectations for collective behavior, and the dynamic incentives are equal for all group members. Our findings reveal that dynamic incentives driven by environmental feedback significantly enhance cooperation, particularly in scenarios with low synergy factors. As incentives increase, the system shifts from the non-cooperative to cooperative state. Moreover, a faster rate of incentive growth leads to a higher level of cooperation, demonstrating a strong positive correlation between dynamic incentive levels and overall cooperation within the system. Counterintuitively, our study finds that introducing dynamic incentives from environmental feedback not only effectively promotes cooperation under high expectation levels but also surprisingly increases the success rate of cooperation as expectations rise.

Understanding the emergence and maintenance of cooperation in multiplayer games poses a significant challenge across various theoretical fields, particularly in the contexts of public goods and social dilemmas. This study introduces a novel model that integrates environmental feedback into systems with critical thresholds, challenging the assumptions of traditional static models. We conceptualize environmental feedback as a dynamic incentive that adjusts rewards or penalties based on collective behavior and examine how this feedback- –activated when certain expectations are met—can significantly enhance cooperation, particularly under low synergy conditions. Surprisingly, our findings indicate that higher expectations do not impede cooperation, instead, they unexpectedly foster it, highlighting the powerful role of dynamic incentives in driving cooperation. This research offers a fresh perspective on the mechanisms underlying cooperation, with broad implications for economics, biology, and social sciences.

I. INTRODUCTION

Understanding the emergence and maintenance of cooperation in social dilemmas is a significant challenge across various theoretical disciplines,[1–9] including biology, economics, and evolutionary psychology, all of which are essential for explaining critical social and natural systems. Multiplayer games on networks, such as the public goods game, play a pivotal role in these fields,[10–13] often exemplifying the well-known "tragedy of the commons."[14–16] In these games, players engage in localized interactions with their neighbors, sharing a common resource and independently deciding whether to contribute. While full cooperation maximizes collective welfare, the temptation of higher individual payoffs through defection introduces risks of resource depletion and the complex problem of second-order free-riding.[17–19] Furthermore, network structures introduce higher-order interactions, adding layers of complexity to the system.[20,21]

Given these challenges, it is imperative to develop effective mechanisms that promote cooperation while alleviating the negative effects of free-riding behavior.[22] Current research has primarily focused on reciprocity mechanisms, distilled into five well-known rules:[2] direct reciprocity,[23] indirect reciprocity,[24] kin selection,[25] spatial reciprocity,[26] and group selection.[27] Direct reciprocity relies on mutual assistance and is effective in repeated interactions, while indirect reciprocity uses social reputation and network effects to encourage individual cooperation. Spatial or network reciprocity leverages individuals' positions within network structures to enhance cooperative behavior, particularly in networked environments. Numerous studies have been built on these foundational mechanisms to develop various strategies centered on reputation,[28,29] inequality,[30] voluntary participation,[31] alliance formation,[32,33] as well as reward and punishment. These mechanisms hold significant theoretical importance and have demonstrated notable effectiveness in practice. For example, reward and punishment mechanisms[34–43] provide external incentives that can curb free-riding behavior and protect public goods. These studies demonstrate the crucial role that external stimuli play in fostering group cooperation. Although implementing such institutionalized mechanisms may require additional resources for monitoring and enforcement, as Ref. 44 suggests, these mechanisms function as "tools that provide incentives to help humanity overcome social dilemmas."

Although previous research has shed light on the role of various types of external stimuli in the evolution of cooperation, many of these studies rely on static environmental assumptions, where the intensity of stimuli remains constant during behavioral evolution, which fails to accurately reflect numerous real-world scenarios.[45] Recently, the impact of the environment itself on collective behavior has attracted considerable attention, with co-evolutionary game models[46] recognizing the close relationship between individual payoffs and environmental conditions. For instance, Weitz *et al.*(author?) [47] have suggested that environmental feedback alone may lead to the tragedy of the commons and oscillations in strategic states. Shao *et al.*(author?) [48] have found that an asymmetric feedback evolutionary game system promotes cooperation only when the relative rate of change in the cooperator multiplier exceeds a certain threshold. The time-delay effect[49] has also been investigated, showing periodic oscillations between strategies. Additionally, adaptive environmental feedback gradually balances reward intensity and cooperation levels, guiding the system toward a stable internal equilibrium point—even if the feedback speed changes, the final evolutionary outcome remains unaffected.[50]

However, these studies often assume that environmental feedback is asymmetric and frequently overlook the environment's expectations for collective behavior. In many real-world scenarios, public resources are typically shared, and their increase or decrease affects all group members equally. For instance, improvements in environmental protection or public infrastructure benefit the entire community, regardless of individual contributions, making this impact broad and fair. Additionally, there is still limited research on how environmental feedback mechanisms influence collective behavior under different conditions. Although Liu *et al.*(author?) [51] considered how environmental feedback in collective risk scenarios can induce sustained oscillations, where the collective goal represents the environment's expectations for group behavior, they(author?) [51] primarily focused on the co-evolution of risk and did not address the issue of adaptive environmental feedback.

Yet, this naturally raises a series of thought-provoking questions: In systems with critical thresholds (expectations), can environmental feedback itself promote cooperative behavior toward the desired goal without introducing other mechanisms? Do rising expectations make cooperation more difficult?

To address these questions, we introduce an innovative model grounded in the spatial public goods game, where environmental feedback is conceptualized as a dynamic incentive that allows the shared resource to generate additional bonuses in response to the collective actions of group members. More specifically, we conceptualize every public good as a shared resource that holds expectations for contributions from group members. This resource incorporates a dynamic incentive mechanism: when contributions meet expectations, it provides additional rewards; conversely, when contributions fall short, the returns are reduced. This dynamic incentive mechanism can capture the satisfaction experienced by group members when contributing to public welfare, seen as intrinsic motivation from the environment. Different from prior studies on public goods games with environmental feedback, we consider the dynamic incentive equal to all group members; put differently, this mechanism avoids direct rewards or penalties for individual players, whether they are cooperators or defectors. Instead, it emphasizes how environmental feedback fosters cooperation without relying on direct rewards or punishments. We find that dynamic incentives from environmental feedback significantly enhance cooperation, particularly in scenarios where the synergy factor is low. As incentives increase, the system shifts from the non-cooperative to cooperative state. Moreover, a faster rate of incentive growth leads to a higher level of cooperation, demonstrating a strong positive correlation between dynamic incentive levels and overall cooperation within the system. Interestingly, intuitively, as the expected value of public goods contributions increases, the level of cooperation in the system tends to decrease,[52] primarily due to the increased challenge of obtaining positive incentives. However, counterintuitively, our research reveals that introducing dynamic incentives from environmental feedback not only effectively promotes cooperation under high expectation levels but also surprisingly increases the success rate of cooperation as expectations rise. In this paper, we commence with a brief background and summarization of our main contributions. Then, Sec. II delineates our public goods game with environmental feedback in detail. Advancing further, Sec. III presents an in-depth elaboration of our empirical insights into the co-evolutionary dynamics between group behavior and environmental feedback. Finally, Sec. IV synthesizes our research findings, offering a summary and discourse on the implications and future work.

II. MODEL

In a standard public goods game, a group of players undertakes a collective project, such as environmental protection and social welfare investment. Such a game g theoretically is a single-stage, k-player game. Every player i faces a binary action choice $a_i \in A = \{C, D\}$: either to cooperate ($a_i = C$) by contributing an investment c (for simplicity, let $c = 1$) to the shared pool or to defect ($a_i = D$) by contributing nothing. After all players take actions, the total investment in the shared pool will be enhanced by a fixed synergy factor of r and then distributed equally among players of the group, yielding a marginal utility of investment of $\frac{r}{k+1}$.[36] This enhancement is strict and brutal, with no additional rewards or penalties due to the collective behavior of the players.

In this study, we employ a modified public goods game model on an $L \times L$ square grid network structure with periodic boundary conditions. Each node $i \in N = \{1, 2, \ldots, n\}$ in the network represents an independent individual player who is able to interact with their $k = 4$ von Neumann neighbors.[53] In other words, each player not only contributes and gains in the shared pool centered on her but also engages in the common pool centered on her four neighbors. As a result, each player is associated with a unique shared pool, leading to a one-to-one correspondence between the number of players and the number of groups. For ease of notation, each group g_i is indexed according to its central player i identifier. For convenience, we define Ω_i to denote the set of players within the group centered on player i.

Here, considering the impact of collective behavior on the environment, we extend beyond traditional public goods game protocol by introducing dynamic public goods that are able to provide environmental feedback on collective behavior. Within this framework, the environmental feedback from the shared pool g_i brings an additional bonus w_{g_i} to every player in the group. As a result, the payoff for cooperators and defectors from this specific group is calculated as follows:

$$\begin{cases} \Pi^{g_i}(C) = \dfrac{r}{k+1} N_C^{g_i} c + w_{g_i} - c, \\ \Pi^{g_i}(D) = \dfrac{r}{k+1} N_C^{g_i} c + w_{g_i}, \end{cases} \tag{1}$$

where $N_C^{g_i}$ denotes the number of cooperators within the group g_i. In the context of the network, every player i accumulates the total payoff from all interactions in related groups, which is represented by

$$\Pi_i = \sum_{j \in \Omega_i} \Pi^{g_j}(a_i). \tag{2}$$

The dynamics of the game on the network follows a series of iterative Monte Carlo simulations, where each iteration of the simulation involves a set of elementary steps for $L \times L$ players. Initially, the network is evenly populated with cooperators and defectors, and the environmental feedback for all groups is set to zero. As the simulation progresses, a player i is randomly selected from the network

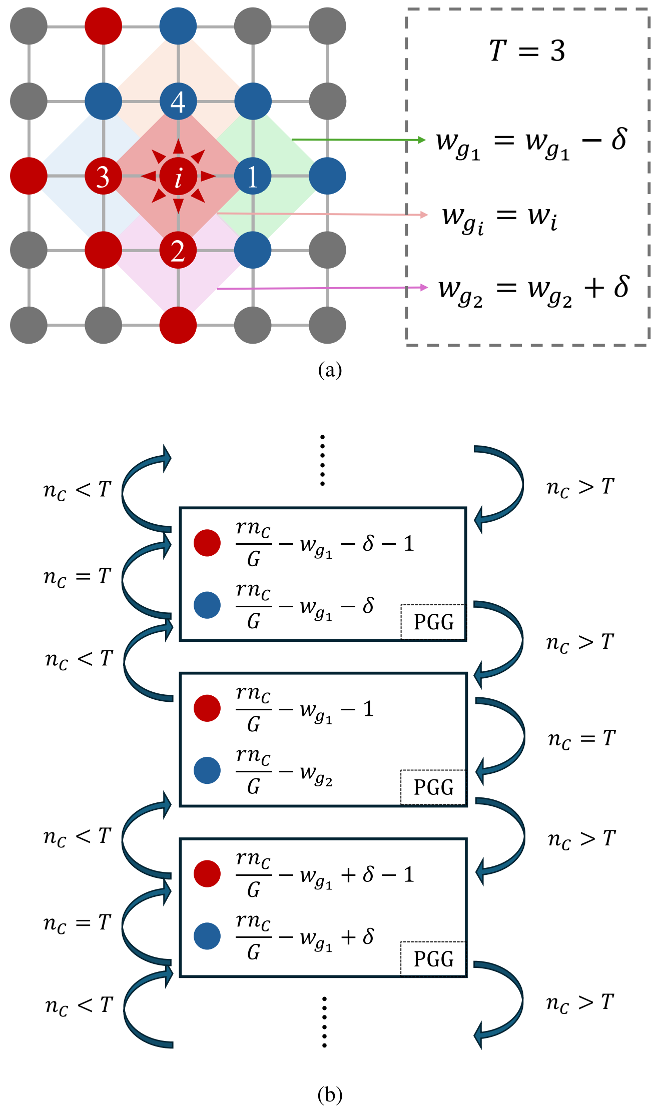


FIG. 1. The public goods game with environmental feedback. (a) Each node in the square grid network represents an independent player who participates in five public goods games. Each player faces a binary action choice: cooperation (*C*, red node) and defection (*D*, blue node). (b) At each time step, the player's strategy and those of the surrounding neighbors together determine the impact of the environment on the payoff in the next sequential time step.

at each Monte Carlo step. The additional bonus from the shared pool adjusts based on the performance of cooperators relative to the expected level of cooperation. Specifically, if the number of cooperators exceeds the expected threshold $T \in [0, k+1]$, the bonus from the shared resource will increase, e.g., group g_1 in Fig. 1(a); if the number of cooperators equals the threshold, the bonus remains unchanged, e.g., group g_i in Fig. 1(a); and if the number is below the threshold, the bonus will decrease, e.g., group g_2 in Fig. 1(a). The update of bonuses follows this rule,

$$w_{g_{j\in\Omega_i}} = \begin{cases} w_{g_j} + \delta, & N_C^{g_j} > T, \\ w_{g_j}, & N_C^{g_j} = T, \\ w_{g_j} - \delta, & N_C^{g_j} < T, \end{cases} \tag{3}$$

where $\delta \in [0, 1]$ represents the step size of the environmental feedback update, determining the speed at which the environmental feedback for each pool in the network is updated. The step-by-step updating illustration is shown in Fig. 1(b). Subsequently, this selected player's payoff Π_i is determined according to Eq. (2). Upon receiving the reward, the player randomly selects a neighbor z and decides whether to adopt the opponent's action. This decision is based on a comparison of their payoffs using the following probability function:

$$P_{a_i \leftarrow a_z} = \frac{1}{1 + \exp\left[(\Pi_i - \Pi_z)/K\right]}, \tag{4}$$

where K is the temperature coefficient of the Fermi function, indicating the level of noise of irrationality.[1,54] When K approaches zero, it signifies perfect rationality, where players consistently imitate only those neighbors with higher payoffs. As K increases, the probability of imitation becomes more random, and the individual becomes more irrational. Without losing generality, we set $K = 0.5$ following previous research.[55] The simulation of the game dynamics is described in Algorithm 1. In this study, the synergy factor r is constrained to $[1, k+1)$ to ensure that the marginal utility ratio $\frac{r}{k+1}$ does not exceed unit one. This configuration underscores that without external incentives, the increments in collective investment may not always yield returns proportional to the additional costs incurred, leading to a Nash equilibrium on mutual defection. In addition, the bonus w_i is always confined in $[0, 1]$ to ensure the social dilemma.

ALGORITHM 1. PGG-VE, Variable Environment-based PGG.

Require: a set of $L \times L$ players, the *MC_Steps*, two available actions $A = \{C, D\}$

initialize players' strategy $s_x \leftarrow$ according to equal probability
initialize the change of environment $w_i \leftarrow 0$
for $t \leq MC\_Steps$ **do**
  for $x \in L \times L$ **do**
    for $x \in \Omega$ **do**
      calculate the payoff p_x^i according to Eq. (1)
      update w_i by 3
    end for
    randomly choose a neighbor z of x
    calculate the payoff P_x and P_z according to Eq. (2)
    update x's action by Eq. (4)
  end for
end for

III. RESULTS

We start by studying the steady-state cooperation level of the system. Figure 2 shows the cooperation levels under different combinations of synergy factor r and update step size δ. It is evident that regardless of the expected contribution threshold T of public goods and the value of the update step size δ, the cooperation level in the system increases as the synergy factor r increases. Specifically, when $\delta = 0$, regardless of whether the group's cooperation level meets the expected threshold, the environmental feedback remains unchanged at zero. In this case, the system degrades into a standard public goods game model, and cooperation becomes feasible only when the synergy factor r reaches a sufficiently high level (specifically, $r = 3.75$). With the introduction of dynamic feedback from public goods ($\delta > 0$), the level of cooperation of the system increases, especially in regions with low synergy coefficients r. When δ increases from 0 to about 0.1, the system is sensitive to changes in δ, and the level of cooperation increases significantly. This observation suggests that an increase in the update step size favors a faster response of the environmental feedback mechanism to the state of the group's contribution, thus increasing the level of cooperation. However, beyond 0.1, the response of the system tends to flatten, and the system may have reached an equilibrium state. The level of cooperation is no longer changing, indicating that further increases

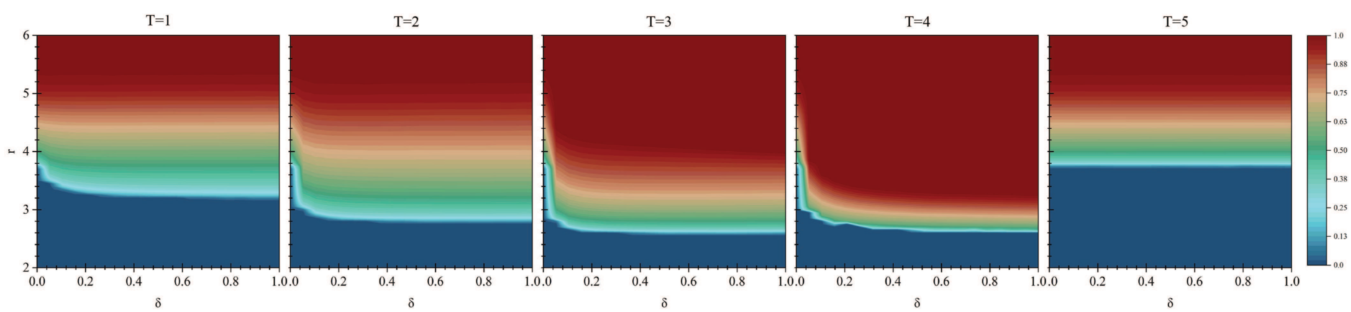


FIG. 2. The steady-state cooperation level ρ_C under different combinations of synergy factor r and update step size δ with respect to the expectation contribution T. The expected contribution T is set sequentially from left to right as $1, 2, 3, 4, 5$. The gradient from blue to red in the graph represents the stable outcome of cooperation level ranging from all defection to all cooperation. Compared to the traditional public goods game ($\delta = 0$ or $T = 5$), as the expectation T increases, the system becomes increasingly inclined to cooperation over defection.

in δ have negligible effects on cooperation. This trend is consistent across all combinations of synergy factor r and expected contribution T. Therefore, unless otherwise stated, we control $\delta = 0.1$ in the following research.

Additionally, we uncover a nontrivial and important phenomenon: as the expectation T increases, the system becomes increasingly inclined to cooperation over defection, as illustrated in Fig. 2 from left to right. Traditional intuition might suggest that a more considerable expectation T suggests that public goods require more members to forgo personal benefit maximization in favor of contributing to public goods, which seems challenging and typically requires a more significant synergy factor r. However, our findings contradict this intuition: as T increases, introducing dynamic incentives through environmental feedback paradoxically lowers the required synergy factor r for the emergence and sustainability of cooperation. It is also noteworthy that as the expectation value T rises, the incremental increase in the r required to propel the system toward full cooperation gradually diminishes. This observation indicates that at higher levels of T, even a slight enhancement in r can effectively advance the system toward full cooperation. However, when $T = 5$, the environmental feedback is constrained to an initial value of 0 and restricted to be non-negative, resulting in no updates to incentives. Consequently, the system reverts to the classical public goods game model, where cooperation is difficult to achieve and sustain. These findings demonstrate that the proposed dynamic incentive mechanism effectively promotes cooperation when combined with a system expectation.

Next, we explore the relationship between dynamic incentives and cooperation levels. In other words, does a higher dynamic incentive always make cooperation easier under different T values and environmental conditions? We investigate average incentives from environmental feedback $\bar{w}$ from public goods at the steady state, as shown in the Fig. 3. Combined with the results from Fig. 2, we identify a significant positive correlation between public goods' dynamic incentives and the system's overall cooperation level. Specifically, as the environmental incentive $\bar{w}$ increases from 0 to around 0.3, the cooperation level undergoes a noticeable transition from non-cooperation to cooperation state. Subsequently, as $\bar{w}$ continued to increase, the cooperation level also increased. These findings hold true under lower expectation values. Nonetheless, when $T = 4$, even in a fully cooperative state, $\bar{w}$ shows variations. This indicates different evolutionary patterns of strategies under different expectation values. In the subsequent discussion, we will not only analyze based on expectation values but also categorize $\bar{w}$ into low, medium, and high levels for further analysis. Above, we discussed the overall state of the system. Next, we delve into the distribution details of behaviors and incentives within the network. Figure 4(a) illustrates the steady-state distribution of cooperation strategies (orange) and defection strategies (blue) under different expected values T. In the absence of a dynamic incentive mechanism (e.g., at $T = 5$), defection strategies dominate in the low-incentive region, while cooperation strategies gradually diminish. However, with the introduction of a dynamic incentive mechanism and as T increases, the impact of incentive levels on strategy distribution becomes more pronounced. At $T = 1$, although cooperation strategies increase in the high-incentive region, defection strategies still prevail in the low and medium-incentive regions. When T increases to 3 and 4, high-incentive levels almost fully promote the adoption of cooperation strategies, while defection strategies maintain a certain presence in the low incentive region. This indicates that as the expected value T increases, especially at high incentive levels, the adoption of cooperation strategies significantly rises. Conversely, in the low-incentive region, defection strategies continue to dominate even as T increases. Further analysis reveals that under the condition of a high expected value $T = 4$, although the cooperation rate remains consistent, the average incentive level varies with different r parameters, suggesting an inconsistency in incentive distribution. As shown in Fig. 4(b), at low synergy factor $r = 2.5$, incentive levels are primarily concentrated in the low region, indicating that most participants tend to choose low incentives. As r increases to 2.9, medium-incentive levels begin to emerge, and high incentive levels also see a slight increase. When r reaches 3.2, the distribution of incentives evolves toward balanced, with low-incentive levels, while medium and high incentive levels rise, respectively. This shift reveals that as r increases, the system's incentive mechanism transitions from a predominantly low-incentive distribution to a more diversified and balanced one. At $r = 3.75$, the distribution of the three incentive levels becomes even more balanced, reflecting greater diversity and equilibrium in the incentive mechanism at higher r values.

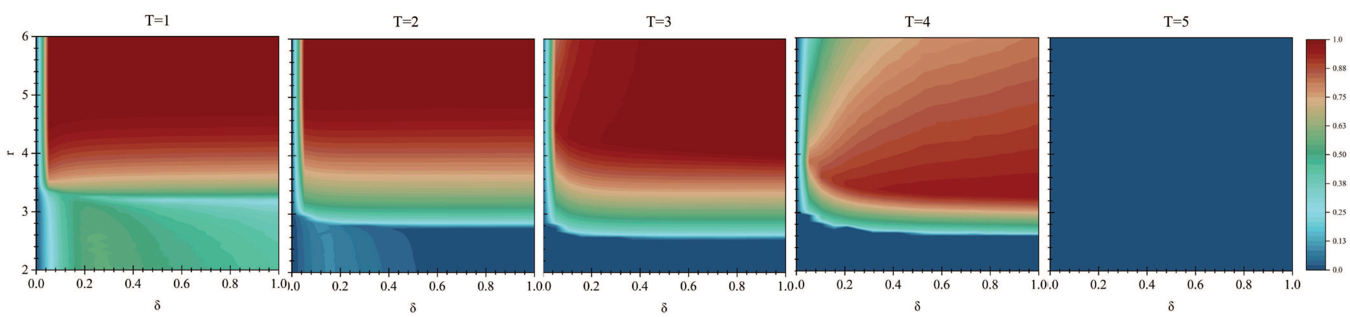
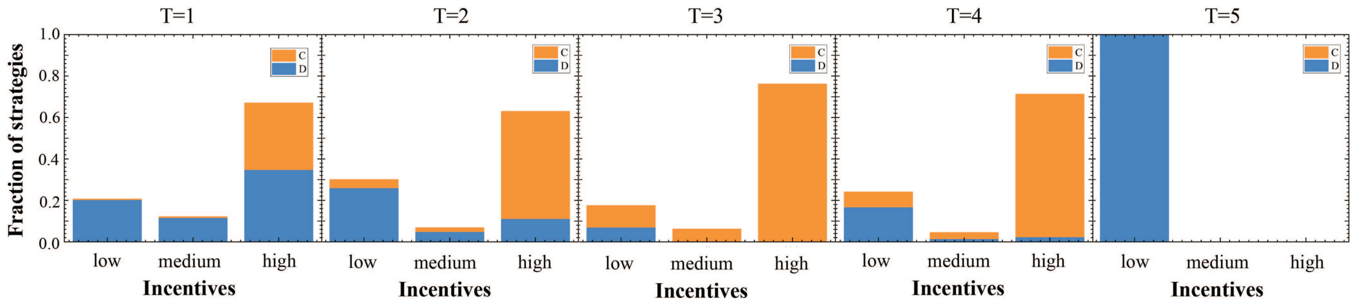
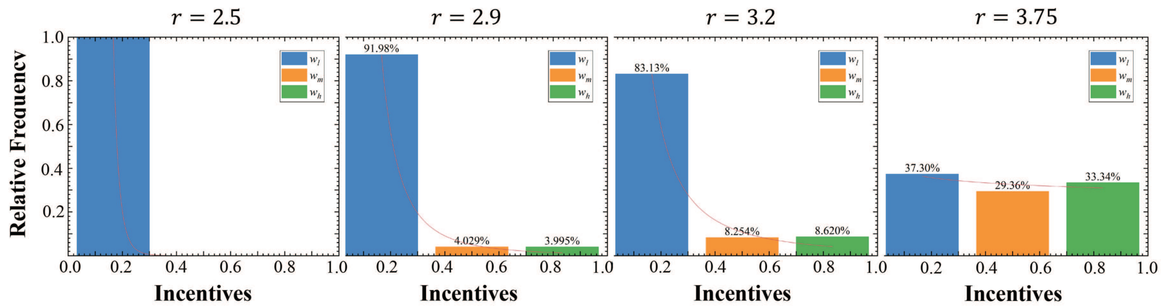


FIG. 3. The steady-state average incentives from environmental feedback $\bar{w}$ under different combinations of synergy factor r and update step size δ with respect to the expectation contribution T. Combined with the results from Fig. 2, there is a significant positive correlation between the dynamic incentives provided by public goods and the overall cooperation level within the system.



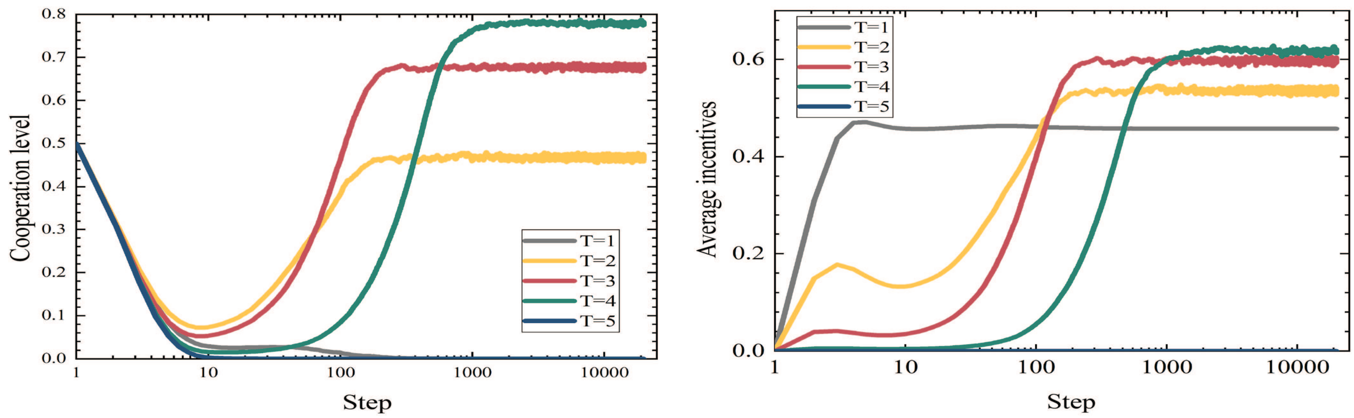
(a) Strategies distribution under different T



(b) Incentives distribution under different r

FIG. 4. The steady-state distribution of strategies and incentives. (a) Cooperation and defection distribution under different expectation values T (from left to right corresponding to T values of 1 to 5) at $r = 3.5$, $\delta = 0.1$. The x-axis represents the incentive levels from public goods, categorized into low $\left(0 \leq w < \frac{1}{3}\right)$, medium $\left(\frac{1}{3} \leq w < \frac{2}{3}\right)$, and high $\left(\frac{2}{3} \leq w \leq 1\right)$ classes. The y-axis shows the fraction of players in the system adopting cooperation (C, orange) and defection (D, blue) strategies. (b) Incentives distribution under different r at $T = 4$, $\delta = 0.1$, (a) strategies distribution under different T and (b) incentives distribution under different r.

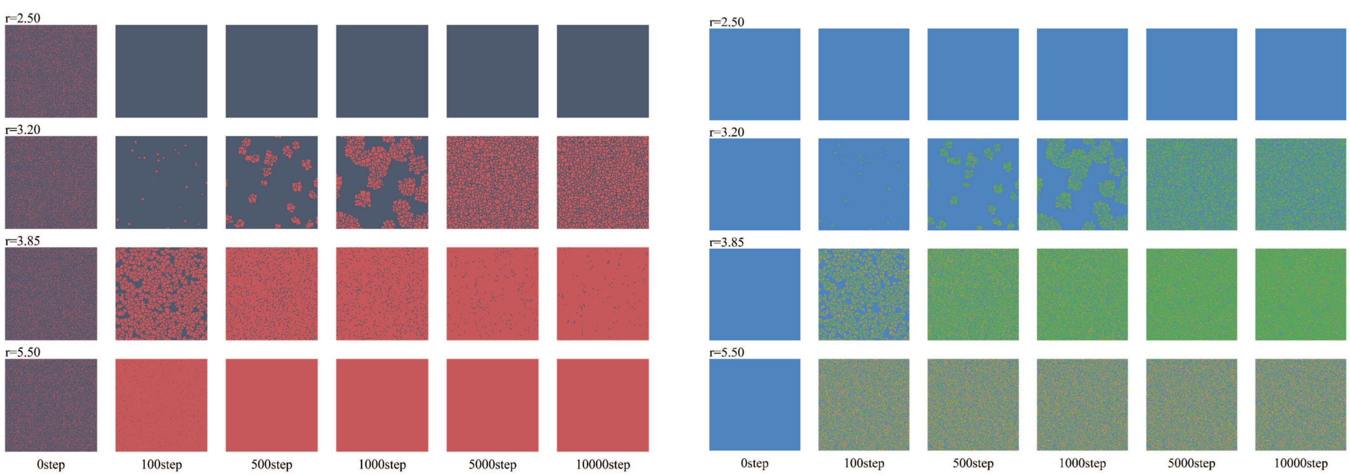
To further understand how public goods with expectations promote cooperation, we then study the temporal evolution of the system under different expectation values T. Throughout the process, without loss of generality, we maintain $r = 3.2$, as there are significant differences in cooperation levels observed under different expectation values, as shown in Fig. 2. Figure 5 illustrates the evolution of cooperation and incentives over time under different expectation values T. Overall, the dynamic process of cooperation levels and incentives can be divided into three stages. In the initial stage, it is clearly observed that the cooperation level rapidly dropped from 0.5 to nearly 0 across all expectation values T, completing this transition within approximately 10 steps. Notably, compared to the conventional scenario ($T = 5$), the incentive $\bar{w}$ gradually increases from zero, with smaller T values leading to faster growth. This is because smaller T values make it easier for collective behavior to meet expectations, thereby prompting the public goods to respond positively to collective behavior more quickly. In the second stage, dynamic incentives begin to play a significant role, promoting the emergence of cooperation. Although $\bar{w}$ grows most rapidly and reaches a steady state earliest at $T = 1$, the fluctuations in cooperation levels are not significant, leading to the eventual disappearance of cooperation after a brief appearance. As $T = 1$ increases, the continuous growth of $\bar{w}$ drives an increase in cooperation levels. Finally, in the third stage, when $\bar{w}$ stabilizes, the cooperation levels also stabilize, with all systems reaching a steady state around 1000 steps. This result reveals the role of expectation value T in encouraging cooperation by assessing collective behavior and spontaneously providing dynamic incentives. Next, we explore the evolutionary patterns of individual strategy and pool incentive distribution under a square network. Figure 6 presents characteristic snapshots of a 400×400 player square network at different time steps, with T fixed at 4 across different synergy factors. The results suggest that, regardless of the level of synergy factors, there is a clear co-evolution between cooperative behavior and incentives from common pools. Given the co-evolution of public goods feedback and player strategies observed in Fig. 6, we classify players based on the incentive levels of the common pool they belong to. Although players may participate in multiple common pools simultaneously, they usually mimic the strategies of their direct neighbors, who are also part of the same player-centered public pool and thus receive the same incentives. Therefore, we primarily classify players according to the incentive levels of this central common pool: players in low-incentive common pools $(0 \leq w < \frac{1}{3})$, players in medium-incentive common pools $(\frac{1}{3} \leq w < \frac{2}{3})$, and players in high-incentive common pools $(\frac{2}{3} \leq w \leq 1)$. Based on these classifications and the individual strategies of players (cooperation or defection), all players can



(a) Evolution of average cooperation (b) Evolution of average incentives

FIG. 5. The temporal evolution of cooperation frequency and overall reword w respectively when given $r = 3.2$, $\delta = 0.1$. (a) shows the change in cooperation level over time for different expected values T, while (b) depicts the corresponding variation in average incentives over time. As the number of steps increases, the impact of different T values on cooperation frequency and incentive levels becomes evident, with higher T values leading to significant increases in both cooperation levels and average incentives, (a) evolution of average cooperation and (b) evolution of average incentives.

be further categorized into six types: low-incentivized cooperators (w_lC), low-incentivized defectors (w_lD), medium-incentivized cooperators (w_mC), medium-incentivized defectors (w_mD), high-incentivized cooperators (w_hC), and high-incentivized defectors (w_hD). Despite Fig. 6 showing the co-evolution between individual cooperative behaviors and environmental incentives, the specific process by which they jointly promote group cooperation requires further exploration. As Fig. 7 highlights, environmental incentives play a crucial role in fostering cooperation by supporting the formation and expansion of cooperative clusters, a process illustrated through network snapshots. These snapshots from a 400×400 player square network at different time steps illustrate how player strategies and common pool incentive levels evolve. All common pool feedback incentives are initially zero, and cooperators and



(a) Strategies snapshots of players (b) Incentives snapshots of common pools

FIG. 6. Characteristic snapshots of a 400×400 player square network at different time steps, with T fixed at four across different synergy factors. (a) illustrates the evolution of player strategies, where red indicates cooperation and gray indicates defection. (b) depicts changes in pool incentive levels, with blue representing low incentives, yellow representing medium incentives, and green representing high incentives. Regardless of the level of synergy factors, the expansion of cooperative behavior is consistently associated with higher incentive levels, indicating the co-evolution of cooperation and incentives from the environment, (a) strategies snapshots of players and (b) incentives snapshots of common pools.

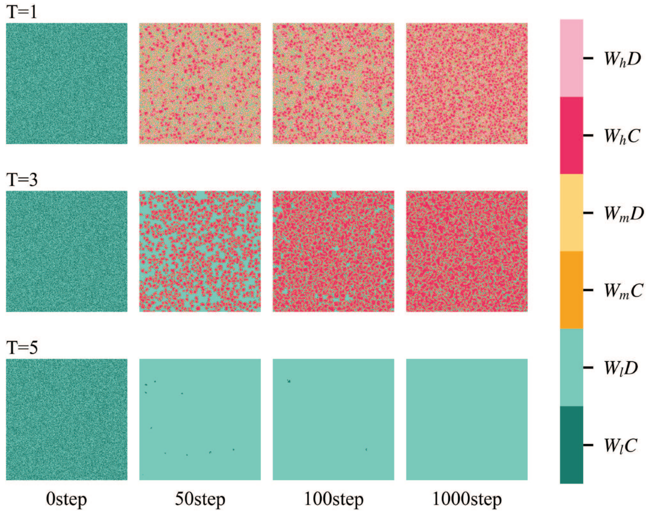


FIG. 7. Characteristic snapshots of a 400×400 player square network at different time steps, with r fixed at 3.5 across different expectations, illustrating the dynamic of player strategies and public pool incentive levels. The colors in the figure represent the incentive levels of the public pool: green indicates low incentives $\left(0 \leq w < \frac{1}{3}\right)$, yellow indicates medium incentives $\left(\frac{1}{3} \leq w < \frac{2}{3}\right)$, and red indicates high incentives $\left(\frac{2}{3} \leq w \leq 1\right)$. The color shades reflect player strategy types: darker colors denote cooperation (C), while lighter colors indicate defection (D).

defectors are uniformly distributed across the network. Consequently, at the start of the simulation, low-incentive cooperators (w_lC, dark green) and low-incentive defectors (w_lD, light green) populate the entire system, as shown in the first column panels of Fig. 7. In the traditional case of $T = 5$ where, without dynamic incentives from the environment, cooperation is fragile when cooperators are sparsely distributed, and defectors gain the upper hand through free-riding. However, when public goods have low expectations for collective behavior ($T = 1$, see the top row of panels in Fig. 7), cooperators within a group trigger positive feedback in the common pool, encouraging low-incentive cooperators to transition into medium-incentive cooperators (w_mC, dark yellow). This transition leads defectors with low incentives to be frequently surrounded by cooperators or defectors with moderate incentives, creating a supportive structure for cooperative clusters to expand. As these clusters grow, the number of qualifying pools increases and accumulated incentives further promote cooperative behavior. Medium-incentive cooperators eventually become high-incentive cooperators who encircle defectors and reinforce cooperative strategies. When expectations are higher ($T = 3$, see the middle panels in Fig. 7), clusters of defectors are penalized for failing to meet expectations (with pool incentives decreasing or remaining stagnant). In contrast, large clusters of cooperators, through sustained positive reinforcement, influence neighboring players, leading low-incentive cooperators to transition into high-incentive cooperators.

These findings demonstrate that, although dynamic incentives in the environment apply equally to both cooperators and defectors, the synergy between individual behaviors and environmental feedback effectively promotes cooperation. Interestingly, we find that the lowest expectations do not yield the highest levels of cooperation, challenging previous intuitions. Crucially, our mechanism aligns the distribution of environmental incentives with cooperative behavior, reinforcing cooperative clusters and facilitating the spread of cooperation throughout the system.

IV. CONCLUSION AND DISCUSSION

In this study, we propose a novel framework that integrates spatial public goods games and generates dynamic incentives in response to the collaborative efforts of group members. Within this framework, each public good has inherent expectations for member contributions. When group behavior meets these expectations, the public pool rewards all group members; conversely, penalties are imposed if the expectations are not met. By simulating interactions among players on a square lattice network, we explore how dynamic incentive mechanisms related to collective behavior influence the emergence and stability of cooperation. First, we find that dynamic incentives from public goods significantly enhance cooperation, particularly in scenarios where the synergy factor r is low. The introduction of feedback mechanisms that reward groups based on their cooperation levels encourages players to contribute to the common good, even when the immediate payoff for defecting might seem more attractive. This contrasts with traditional public goods games, where cooperation is hard to sustain without sufficiently high synergy factors. Our findings suggest that environmental feedback can shift the balance in favor of cooperation, making it a viable strategy in a wider range of conditions.

Second, we observe a strong positive correlation between the level of dynamic incentives and the overall cooperation in the system. As incentives increase, cooperation transitions from a non-cooperative state to a cooperative one, especially in cases with lower expectation thresholds. The distribution of strategies and incentives within the network reveals that cooperation tends to cluster in regions where incentives are higher, suggesting that feedback mechanisms help stabilize cooperative behavior over time. Third, the expectation threshold T, representing the collective behavior needed to trigger positive feedback, plays a critical role in determining cooperation levels. Interestingly, higher expectations paradoxically reduce the synergy factor required for cooperation to emerge and persist. This counterintuitive result suggests that setting higher collective expectations can, under the right conditions, lead to more cooperative behavior as the system penalizes groups that do not meet expectations to force players to cooperate to meet those expectations. However, when expectations are too high (e.g., $T = 5$), the system reverts to the classical public goods model, where cooperation is difficult to achieve. Finally, our study reveals that while dynamic incentives are applied equally to both cooperators and defectors, they effectively promote cooperation by shifting the distribution of incentives in a way that favors cooperative behavior. The co-evolution of player strategies and incentives demonstrates that environments with dynamic feedback can support and sustain cooperation, even in challenging conditions. However, the lowest expectations do not always result in the highest levels of cooperation. In conclusion, this research demonstrates the power of dynamic incentives in fostering cooperation in public goods games. By integrating environmental feedback mechanisms, we can design systems that naturally encourage cooperative behavior, making it more robust and sustainable in various contexts. Future work could explore different network topologies and more complex forms of feedback to further understand the conditions under which cooperation thrives.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Die Hu: Software (equal); Writing – original draft (equal). **Jinzhuo Liu:** Supervision (equal); Writing – original draft (supporting). **Chen Liu:** Resources (equal); Supervision (equal); Writing – review & editing (equal). **Chen Chu:** Resources (equal); Supervision (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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