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From conventional to automated terminals: The optimal upgrade decision under demand and technological improvement uncertainties

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ABSTRACT

The age of automation and intelligence requires upgrading conventional terminals (CTs) with automated ones in the port industry. This paper examines how to optimally decide the size and timing of this upgrade under demand and technological uncertainties. We use the real options approach to derive the optimal upgrade decisions for two options: the upgrade option when the port decision maker upgrades CT by building a new Automated Terminal (AT) without interrupting the operation of CT and the upgrade option when the port decision maker abandons the capacity of existing CT and replaces the exited capacity by investing in AT. We then conduct numerical experiments to analyze how uncertain technological improvement and demand affect the optimal solutions. The results suggest that in the additional replacement option, high technological improvement and demand uncertainties will delay the AT investment but will not change the capacity investment choice for the AT. In the replacement upgrade option, high technological improvement will postpone the investment for AT and enlarge the exit capacity choice for the CT and the investment capacity choice for the AT. However, the exit capacity choice for the CT first increases and then decreases with increasing demand uncertainty. Finally, we extend our study to the public ownership of the port that adopts the upgrade option. Our study provides practical guidance for upgrading the CT and investing in the AT. It also contributes to the theoretical literature on automated technology adoption under uncertainty.

1. Introduction

Ports are critical interfaces that connect land and sea transport in the multimodal transport network, and they promote international trade and regional economies (Notteboom and Haralambides, 2020; Bai et al., 2022). Global trade has grown considerably in the past twenty years, and international merchandise trade values have increased from 645.2 billion US dollars in 2000 to 24.9 trillion US dollars in 2022 (UNCTAD, 2023). Moreover, since over 90 % of cargo volumes in international trade are transported by maritime shipping (Yang et al., 2019), the global container port throughput has also increased from 237 million TEUs in 2000 to 862 million TEUs in 2022 (Statista, 2024). To meet the rapidly growing demand for port service and expand the port hinterland, port with

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conventional manual terminals and equipment has traditionally expanded its handling capacity to attract shippers (Balliauw et al., 2019a). However, with the increased operating costs of manual terminals and equipment and the maturity of automation and intelligence technologies, the port with conventional terminals (CTs) have become less competitive in the multiport coastal regions (Guo et al., 2021; Guo and Jiang, 2022). As a result, the port becomes less attractive to shippers and shipping carriers and waste port resources. Furthermore, the shortage of labor and quarantine measures during the COVID-19 pandemic from 2020 to 2022 in the CTs also significantly reduced the port's operational efficiency and productivity (Merk et al., 2022). Therefore, it is vital for port decision-maker to know how to upgrade its CTs and propose practical measures for CT upgrading.

Automated terminals (ATs), which use automated equipment and processes to replace conventional manual operations, have gained considerable attention in recent years due to the emergence and rapid application of Industry 4.0, automation technology, and digital technologies (such as the Internet of Things, cloud computing, and artificial intelligence (AI)) (Kon et al., 2021; Li et al., 2023). Adopting ATs is a vital development for upgrading CTs in the port, and a recent survey gathered from Navis' (a top automated terminal operations system provider) 54 customers in 2020 indicates that 94 % of customers believe the AT is of great importance for keeping the port competitive in the next 3–5 years. AT offers many benefits to port agencies (Rodrigue and Notteboom, 2021), such as lowering overall terminal operating costs, enhancing operational efficiency and productivity in dealing with the increasing scale in ship size (which can increase by 25 to 50 percent compared with CTs), and attracting more shippers in the competitive overlapping hinterland by providing port services without delay and disruption.

Although AT plays a pivotal role in helping the port agency to pursue more efficient and resourceful operations, the share of ATs in all main terminals worldwide was still very low, and the total number of automated container terminals in the world was only 53 in 2021 (ITF, 2021). The main reasons why decision-maker hesitates to invest in ATs are the high irreversible investment cost of automated terminal construction and the technological improvement uncertainty of automation technology (Rodrigue and Notteboom, 2021). When a new port automation technology emerges, it usually entails a high cost of introducing the technology to construct the automated infrastructure. Therefore, port decision-maker face a technology adoption dilemma by considering the uncertainty of automation technological improvement: Should they adopt the high-cost new automation technology now or wait for future technological improvement with cost reduction? Since technological improvement is typically hard to predict and quite uncertain, it plays a vital role in shaping the AT investment decision for upgrading the CT.

Demand uncertainty is another critical factor influencing AT adoption for CT replacement, besides the uncertainty of automation technological improvement. Port demand depends on trade volume, which may be affected by many uncertain factors (Xia and Lindsey, 2021), such as economic and trade fluctuations (e.g., the economic crisis in 2008), public health emergencies (e.g., the COVID-19 pandemic in 2020), military conflict and war (e.g., the Russia-Ukraine conflict in 2022), and natural disasters (e.g., the Hurricane Katrina in 2005). Uncertain port demand will influence the profit of both types of terminals and then further affect AT adoption for CT upgrade. Thus, we must also include the uncertainty of demand in the CT terminal upgrade decision. Nowadays, the port decision-maker has adopted two popular AT investment strategies for upgrading the CTs. The one is that the decision maker invests in new ATs within the port areas without interrupting the operation of existing CTs. The other one is that the decision-maker abandons existing CTs' partial or full capacities and then upgrades the idle CT capacities by investing in ATs.[1] Under the above two upgrade strategies, the port decision-maker will address different problems for CT upgrading in the context of technological improvement and demand uncertainties. In the case of investing in a new AT, the decision maker has to determine when and what size should the port decision-maker invest in an AT? However, when replacing the existing CT, the decision maker should address two crucial problems: (1) When and what size should an existing CT exit the port market? (2) When and what size should the port decision-maker invest in an AT for replacing CT?

In the above context, using real options theory, this study aims to investigate the optimal upgrade decisions from CT to AT within a port under technological improvement and demand uncertainties. To achieve this aim, we first derive the optimal investment decision in an additional investment upgrade option that the port decision maker invests in new AT without interrupting the operation of CT. We then derive optimal exit and investment decisions in a replacement upgrade option in which the port decision maker abandons the capacity of the existing CT and replaces the exited capacity by investing in AT. Finally, we present numerical examples to analyze the analytic solutions obtained from the two upgrade options and provide a sensitivity analysis to further discuss the impacts of several essential parameters on optimal upgrade decisions.

Our study makes the following main contributions: (1) Our research is one of the first to enrich the literature on conventional port upgrade decisions under uncertainty based on the real options model, in which the port demand uncertainty and technological improvement uncertainty are incorporated. (2) Our study addressed the exit decision of the CT and the investment decision of the AT under two upgrade options (i.e., the additional upgrade option and the replacement upgrade option). The interactions among terminal congestion, terminal substitution effect, exit timing and size for the CT, and the investment timing and size for the AT are explicitly disclosed. (3) The numerical experiments illustrate how demand and automation technological improvement uncertainties influence the two CT upgrade options under the given parameters based on the European and North American private ports data.

The remainder of this paper is organized as follows. Section 2 reviews the literature on port/terminal investment and technology

[1] Although the existing installed port capacities are not easy to abandon as the port assets have the characteristics of fixed heavy assets and long-term depreciation period, the exit of existing CT capacities is still a useful strategy to prepare for the challenges of trade or economic fluctuation. For example, the Dalian Port, a top seaport in mainland China in terms of port throughput, has gained substantial economic benefits by abandoning the terminals that are near the CBD of Dalian city. By developing the real estate and modern service industries with the idle land of abandoned terminals, the Dalian Port has earned enormous economic benefits facing the world economic crisis in 2009.

adoption under uncertainty. Section 3 presents the basic models for measuring the uncertain port demand and investment cost. Section 4 presents the optimal upgrade option for additional investment in new AT and gives the corresponding analytic solutions. Section 5 presents the optimal upgrade option for replacing the CT with a new AT and gives the corresponding analytic solutions. Section 6 gives numerical experiments to illustrate further the optimal decisions and the corresponding analytic solutions obtained from Sections 4 and 5. Section 7 presents the extension of the upgrade options by considering the public ownership of the port. Section 8 concludes the study and gives the policy implications.

2. Literature review

Two streams of research are relevant to our study: the port/terminal investment and the technology adoption problem under uncertainty. In this section, we review the studies most relevant to our work regarding the two streams of research, based on which we also identify the contribution of this paper.

2.1. Port/terminal investment

This paper examines the AT investment problem for upgrading the CT, and the port/terminal investment problem is a longstanding topic in the port and maritime literature. This problem involves determining the optimal investment decision for ports in response to the increasing demand for international and regional trade and the rising threats of natural disasters and climate change (Musso et al., 2006). The existing literature on port investment can be classified into two categories, based on the conditions that ports face when making the investment decision: deterministic and uncertain. Regarding the studies on port investment under deterministic conditions, Allahviranloo and Afandizadeh (2008) determined the optimal investment plan for port development from the perspective of national investment, using a fuzzy integer-programming model that considered the type of design ships and design berths. Wan et al. (2016) focused on the investment decision of local governments on regional landside accessibility, taking into account two different port ownerships (i.e., public port aiming to maximize social welfare and private port aiming to maximize profit) and the impact of seaport competition. Wu et al. (2016) studied the final equilibrium for stopping the investment from the perspective of port companies and local governments, addressing the influence of local government on port investment. Guo et al. (2018) proposed a multiperiod port investment and exit strategy to cope with the oversupplied port capacity in a multi-port region in recent years, by maximizing the social welfare of the MPR. Yang et al. (2019) extended the work of Guo et al. (2018) to consider the industrial transformation and upgrading of oversupplied ports.

The studies on port investment under uncertainty can be further classified into two categories based on the type of uncertainty that ports face: demand and congestion uncertainties, and climate change and disaster uncertainties. Under demand and congestion uncertainties, previous studies have used game theory and real options theory to analyze port investment decisions. For example, Chen and Liu (2016) applied a two-stage game to address the problem of facility investment level and service prices for risk-averse ports under uncertain demand and congestion. Balliauw et al. (2019a) leveraged the Cournot competition and real options theory to investigate the two competing ports' capacity investment decisions (including capacity investment scale and timing) under demand uncertainty and congestion. Balliauw et al. (2020) used the real options theory to study the optimal investment size and timing of individual port investment decisions under congestion and demand uncertainty. Moreover, Guo et al. (2021) and Guo and Jiang (2022) also applied the real options theory to study how port investment under demand uncertainty affects the capacity integration for multiple ports in a given port cluster. The empirical results conducted in these papers indicate that the option value derived from investing under uncertainty accelerates the integration of port capacity. Under climate change and disaster uncertainties, previous studies have used integrated economic models and real options models to investigate port adaptation investment decisions. For instance, Xiao et al. (2015) used an integrated economic model to study the disaster-prevention investment, considering the uncertainty of disaster occurrence and the associated return of prevention investment. They found that the probability of disasters determines the timing of port investments. Wang and Zhang (2018) examined the effect of inter-and intra-port competition and cooperation on the port adaptation investment under the Knightian uncertainty regarding the emerging probability of a natural disaster. Randrianarisoa and Zhang (2019) developed a two-period real options game model to investigate the climate change adaptation investment size and timing with two "landlord" ports (consisting of a port authority and a downstream terminal operator company), facing the climate change impacts on the ports. Xia and Lindsey (2021) investigated the optimal timing and size of port capacity investment and the port charges under uncertain climate-related threats and demand. Zheng et al. (2021a, 2021b) compared the effects of subsidy policy and adaptation sharing under the minimum requirement on the seaport adaptation investment to climate change disasters. Moreover, Zheng et al. (2022) extended Zheng et al. (2021a, 2021b) by incorporating the asymmetric information regarding the actual disaster damage.

The above review reveals that mathematical programming models are commonly used to address port investment under deterministic conditions, while real options theory is widely adopted to address port investment under uncertainty. Real options theory is a powerful tool that can measure the option values of delaying or deferring investment under uncertainty (Sun et al., 2020; Guo et al., 2023). Therefore, we choose to adopt the real option theory to investigate the AT investment problem for upgrading the CT.

2.2. Technology adoption under uncertainty

Our paper also relates to the literature on technology adoption under uncertainty, which examines when and how to switch from old to new technology under uncertain technological improvement. This problem has attracted increasing attention in recent years.

Murto (2007) analyzed how technological uncertainty with a Poisson form process affects the investment timing of irreversible investment and how different types of uncertainty influence investment timing. Smith and Ulu (2012) modeled the uncertainty in quality and costs as a Markov process and studied how future innovation in quality and costs affects the consumers' technology adoption decision. Flor and Hansen (2013) addressed the optimal investment decision of a firm under the case that uncertain technological advance reduces the investment cost and earnings. Hagspiel et al. (2015) extended the traditional technology adoption under uncertainty problem by assuming that the arrival rate in the Poisson process is uncertain. They showed that the uncertain arrival rate in the Poisson process changes the optimal technology time significantly. Nunes et al. (2021) applied real options theory to address the investment problem under two sources of uncertainty (i.e., prices and technology), where the uncertain price is simulated by a geometric Brownian motion and technology innovation is simulated by a Poisson process. Huang et al. (2021) used real options framework to address the investment timing problem of carbon emission reduction technology, where the technology investment cost is measured by a jump-diffusion process. Unlike the above studies that assume the uncertain technological improvement follows a Poisson process, Armerin (2023) modeled the reduction of investment cost as a non-decreasing Lévy process and then presented the general result of optimal investment timing in the investment problem.

Another stream of literature has incorporated the exit decision of the old technology into the technology adoption problem. For instance, Kwon (2010) used stochastic analysis to decide whether to continue investing or stop and exit the current project when facing an uncertain declining profit stream. Similarly, Hagspiel et al. (2016) studied both capacity investment decision (containing capacity size and timing) and exit timing decision for replacing the old established product with an innovative product in an uncertain declining profit stream. They showed that higher uncertainty of the profit stream would defer the new innovative product investment timing and call for a larger capacity investment size. Furthermore, Hagspiel et al. (2020) extended their replacement strategy by considering adding an innovative product to the product portfolio. That is, both products compete as the established product is not entirely replaced by the new product.

2.3. The contribution of this paper

Despite the efforts of previous literature on addressing the port investment and technology adoption under uncertainty problems, some issues are still not well addressed. First, most previous port investment studies only consider the optimal investment decision for a new port or terminal and neglect the exit decision for the existing port or terminal. However, the existing port or terminal can still affect the optimal investment decision for the new port or terminal, as it can earn a profit even in a declining profit stream. Therefore, we need to include the exit decision of the existing conventional terminal in the investment decision of the automated terminal. Second, our paper is related to Hagspiel et al. (2016, 2020), but they have some gaps that we fill. Hagspiel et al. (2016) did not account for the impact of technological improvement on the optimal replacement strategy, nor did they study the competition problem when both products coexist in the market. Hagspiel et al. (2020) analyzed the impact of technological improvement on the optimal replacement options, but they did not address the capacity decisions in the optimal replacement strategy. Third, most previous studies addressing the exit decisions only focus on the product replacement problem in a declining market for the old technology. However, in an increasing market for the old product, the decision-makers may also prefer to replace the old product, as it is less attractive to consumers. In other words, replacing old products with new products may be more beneficial for decision-makers even if the old product faces an increasing market.

To fill the above research gaps, this paper addresses the conventional port upgrade decisions under the demand and technological improvement uncertainties based on the real option theory. We derive the optimal exit decision of the CT and the optimal investment decision of the AT under two upgrade options (i.e., the additional upgrade option and the replacement upgrade option). We explicitly disclose the interactions among terminal congestion, terminal substitution effect, exit timing and size for the CT, and the investment timing and size for the AT. Based on the European and North American private ports data, we conduct numerical experiments to illustrate how demand and automation technological improvement uncertainties influence the two CT upgrade options under the given parameters. Furthermore, we extend our study to the case of public ownership of the port and then derive the optimal upgrade decisions for the CT.

3. Basic model

3.1. Uncertain port and terminal demand

As indicated by Balliauw et al. (2019a) and following Huisman and Kort (2015), we can use the multiplicative inverse demand function to simulate the relationship between the port price and the uncertain port demand. Since congestion can lead to potential loss of demand (Jansson and Shneerson, 1982), we thus have to include the congestion cost in the inverse demand function (Balliauw et al., 2019b). If the port decision maker has not adopted the port upgrade decision, then the sole type of CT is located in the port. In this case (denoted as case s), the port demand function is expressed as follows:

$$P_{st} = X_{st}\left(1 - \eta q_{st} - \gamma \frac{q_{st}}{K_{st}}\right), \tag{3.1}$$

$$dX_{st} = \mu_s X_{st} dt + \sigma_s X_{st} dz_t, \tag{3.2}$$

where P_{st} is the port price at time t in case s; X_{st} is the demand shift parameter, which is assumed to be exogenous by considering the derived demand characteristics of port demand and satisfies the geometric Brownian motion (GBM) according to the popular setting in the literature on transport investment under uncertainty (Galera and Soliño, 2010; Chow and Regan, 2011); μ_s is the drift parameter in GBM in case s; σ_s is the volatility parameter in GBM in case s; dz_t is the increment of a standard Wiener process; q_{st} is the port throughput at time t in case s; η is a slope parameter; K_{st} is the port capacity at time t in case s. Following the congestion cost equations shown in Álvarez-SanJaime et al. (2015), Guo et al. (2018), and Yang et al. (2019), the term $\gamma q/K$ is used to measure the generalized port congestion cost, where q/K is used to capture the port capacity utilization level and γ reflect the port operating efficiency. According to the term $\gamma q/K$, we can find that both the capacity utilization level and port operating efficiency will determine the port congestion cost. If the port has a high operating efficiency, the port users may not have to pay a high congestion cost even if the port utilization level nears saturation. Moreover, the term $\gamma q/K$ can also effectively reflect the difference in congestion cost between CT and AT, and the terminal congestion costs of the two types of terminals may not be equal even though the two terminals have the same capacity utilization level, as the AT usually has a higher operating efficiency than the CT.

Furthermore, when the CT is upgraded by AT, then two types of terminals will be located in the port. In this case (denoted as case m), the exogenous port demand (denoted as X_m) is equal to the total demand of the types of terminals (i.e., $X_m = X_{m1} + X_{m2}$), where subscripts 1 and 2 represent the CT and AT, respectively. Moreover, we have to simulate the competition relationship of the two types of terminals by applying a game-theoretic approach. Since the terminals are heterogeneous, the demand functions for the two types of terminal services are given by

$$P_{m1t} = X_{mt}\left(1 - \eta_1 q_{m1t} - \varepsilon q_{m2t} - \gamma_1 \frac{q_{m1t}}{K_{m1t}}\right), \tag{3.3}$$

$$P_{m2t} = X_{mt}\left(1 - \eta_2 q_{m2t} - \varepsilon q_{m1t} - \gamma_2 \frac{q_{m2t}}{K_{m2t}}\right), \tag{3.4}$$

$$dX_{mt} = \mu_m X_{mt} dt + \sigma_m X_{mt} dz_t, \tag{3.5}$$

where $\varepsilon(\in [0,1])$ is the differentiation parameter, taking the value of 1 representing the perfect substitution of two types of terminals and 0 representing the perfect complementarity. Moreover, the other notations in the two equations have the same definitions in the port demand function shown in Eqs. (3.1) and (3.2).

3.2. Uncertain investment cost

Furthermore, we must consider the impacts of uncertain technological improvement on the investment cost for AT construction. The investment cost for port automation technology adoption may be reduced due to technological improvement. Therefore, we use the compound Poisson process to simulate the evolution of the investment cost of ATs. Specifically, the investment cost that considers the uncertain impact of technological improvement is given by (Murto, 2007)

$$I_t(K_{m2t}, \theta_t) = I_0(K_{m2t})\phi^{\theta_t}, \tag{3.6}$$

where $I_t(K_{m2t}, \theta_t)$ is the investment cost of AT at time t; $I_0(K_{m2t})$ is the investment cost at time $t = 0$; $\theta_t > 0$ is a Poisson random variable with arrival rate $\lambda > 0$; and $\phi \in [0, 1)$ is a constant parameter reflecting the magnitude of technological improvement. With the above setting, the expected value of $I_t(K_{m2t}, \theta_t)$ can be expressed by an exponentially declining function of time, and the detailed expression is as follows:

$$\mathrm{E}[I_t(K_{m2t}, \theta_t)] = I_0(K_{m2t})e^{-\lambda t(1-\phi)} = I_0(K_{m2t})e^{-\xi t}, \tag{3.7}$$

where $\xi \equiv \lambda(1-\phi)$. Moreover, as shown in Dekker (2005), Guo et al. (2018), and Yang et al. (2019), the port or terminal investment is characterized by the economies of scale, i.e., the port or terminal capacity increases with investment at a decreasing rate. Then, based on the popular setting in port investment studies for measuring economies of scale in port investment, we have:

$$I_0(K_{m2t}) = w \cdot (K_{m2t})^u, \tag{3.8}$$

where ω is the scale parameter and u is the constant parameter reflecting the scale economics of terminal investment, $0 < u < 1$. In the numerical experiments in Sections 6 and 7, we will give the specific values of w and u in Eq. (3.8) with the investment costs and capacities data of the studied ports. Unless otherwise specified, we will drop the subscript t for the variables and parameters in the following sections.

4. Additional investment option for upgrading the CT

As mentioned by Xia and Lindsey (2021), port may have different ownership structure with different levels of private and public authorities' involvement. In the following Sections 4 and 5 for deriving the optimal upgrade options, we focus mainly on the private port owned by private authority. However, we will extend the optimal upgrade options to the case of public ownership of the port in Section 7.

In the additional investment upgrade option without disturbing the operation of CT, the optimal strategies are to determine the optimal investment size and timing for the AT. In the following subsections, we derive the optimal investment threshold of the additional investment upgrade option and then give the analytic solutions in a special case.

4.1. Optimal investment decision

As shown in Eqs. (3.1), (3.3), and (3.4), the generalized port or terminal congestion cost has measured the impact of port or terminal operating efficiency on the port or terminal price. For simplicity, we thus do not propose an individual operating cost parameter to investigate the influence of operating efficiency on the port or terminal profit. Assuming that the port/terminal profit is denoted as π, we can then derive the following expressions for calculating the profits of the two types of terminals in case m with Eqs. (3.3) and (3.4):

$$\pi_{m1}(X_m) = X_m\left(1 - \eta_1 q_{m1} - \varepsilon q_{m2} - \gamma_1\frac{q_{m1}}{K_{m1}}\right)q_{m1}, \tag{4.1}$$

$$\pi_{m2}(X_m) = X_m\left(1 - \eta_2 q_{m2} - \varepsilon q_{m1} - \gamma_2\frac{q_{m2}}{K_{m2}}\right)q_{m2}. \tag{4.2}$$

Furthermore, if the port decision-maker did not invest in the AT in case s, the port profit is equal to the profit of CT, i.e.:

$$\pi_s(X_s) = \pi_{s1}(X_s) = X_s\left(1 - \eta_1 q_{s1} - \gamma_1\frac{q_{s1}}{K_{s1}}\right)q_{s1}. \tag{4.3}$$

With the above setting, if the port decision-maker adopts the additional investment option, the port decision maker will solve the following problem to determine the optimal investment strategy according to the real options theory:

$$F(X_s, X_m, I) = \sup_\tau \boldsymbol{E}\left[\int_0^\tau e^{-rt}\pi_s(X_s)dt + \int_\tau^{+\infty} e^{-rt}\pi_m(X_m)dt - e^{-r\tau}I\right], \tag{4.4}$$

where $F(X_s, X_m, I)$ is the value function of the additional investment option; τ is the optimal investment timing of the AT; $\pi_m(X_m)(= \pi_{m1}(X_m) + \pi_{m2}(X_m))$ is the total profit of the two types of terminals in case m; and r is the discount rate. Suppose that r is a positive constant parameter and satisfies $r > \mu_s$ and $r > \mu_m$.

To get the maximum value of $F(X_s, X_m, I)$ in Eq. (4.4), $\pi_s(X_s)$ and $\pi_m(X_m)$ are then calculated as follows:

$$\pi_s(X_s) = \pi_{s1}(X_s) = X_s(1 - \gamma_1 - \eta_1 K_{s1})K_{s1},$$

$$\pi_m(X_m) = X_m[(1 - \gamma_1 - \eta_1 K_{m1})K_{m1} + (1 - \gamma_2 - \eta_2 K_{m2})K_{m2} - 2\varepsilon K_{m1}K_{m2}].$$

Therefore, if the sum future port profits in cases s and m are denoted as $D_s(X_s)$ and $D_m(X_m)$, respectively. Then, $D_s(X_s)$ and $D_m(X_m)$ can be calculated as follows:

$$D_s(X_s) = \int_0^\tau \pi_s(X_s)dt = \frac{X_s(1 - \gamma_1 - \eta_1 K_{s1})K_{s1}}{r - \mu_s},$$

$$D_m(X_m) = \int_\tau^{+\infty} \pi_m(X_m)dt = \frac{X_m[(1 - \gamma_1 - \eta_1 K_{m1})K_{m1} + (1 - \gamma_2 - \eta_2 K_{m2})K_{m2} - 2\varepsilon K_{m1}K_{m2}]}{r - \mu_m}.$$

Then Eq. (4.4) can be rewritten as:

$$F(X_s, X_m, I) = \sup_\tau \boldsymbol{E}\left[e^{-r\tau}\left(\frac{\pi_s(X_s)}{r - \mu_s} + \frac{\pi_m(X_m)}{r - \mu_m} - I\right)\right]. \tag{4.5}$$

Since the investment of AT will increase the port competitiveness and then further attract more port users, we thus have $X_m > X_s$. Moreover, according to the derived demand characteristics of port demand, the port's geographical location has been pointed out by many studies as a key determinant of port choice. The investment of AT in a sole port could not change the port's geographical location, and we thus can assume that the exogenous port demand X_m in the two terminals' case is linearly related to the port demand X_s in the sole CT case (i.e., $X_m = \upsilon X_s (\upsilon > 1)$). This assumption indicates that the port with two types of terminals has a larger attraction towards port users than the port with sole CT when considering the feedback effect of increased competitiveness. According to the linear relationship between X_m and X_s, the value of $F(X_s, X_m, I)$ is then only determined by X_m and I.

Therefore, to determine the optimal investment decision for AT, we can divide the decision state space $\vartheta \triangleq \mathbb{R}^+ \times \mathbb{R}^+$ into two parts: the stopping region (denoted as S_A) $S_A := \{(X_m, I) \in \mathbb{R}^+ \times \mathbb{R}^+ : F(X_m, I) = I\}$ and the continuation region (denoted as C_A) $C_A := \{(X_m, I) \in \mathbb{R}^+ \times \mathbb{R}^+ : F(X_m, I) > I\}$. Let τ^* be the time that the decision state variable first enters the stopping region, i.e., $\tau^* = \inf\{t \geqslant 0; (X_{mt}, I) \in S_A\}$, and thus τ^* is the optimal timing for the AT investment.

Below, we give the property of the optimal investment decision based on $F(X_m, I)$ in Eq. (4.5).

Proposition 4.1. *The stopping region satisfies the following*:

$$S_A = \left\{ (X_m, I) \in \vartheta \middle| \frac{X_m}{I} \geqslant x_m^* \right\},$$

where x_m^* is a constant to be determined.

Proof. The proof is shown in the Appendix.

According to Proposition 4.1 and the corresponding proof, we can conclude that the investment threshold x_m^* is unique, and x_m^* has divided the decision space into two parts, i.e., the stopping and continuation regions. As shown in Fig. 1, a ray starting from the origin divides the stopping and continuation regions. Moreover, according to Eq. (4.5), $F(kX_m, kI) = kF(X_m, I)$ $(\forall k > 0)$. Therefore, $F(X_m, I)/I = F(X_m/I, 1)$, and $F(X_m/I, 1)$ is just a function of X_m/I. This property and Proposition 4.1 can transform the above two-dimensional optimal stopping problem (determined by X_m and I) into a one-dimensional problem (determined by X_m/I), realizing the aim of dimensionality reduction to further ensure that the optimal analytic solutions can be derived.

Let $x_m = X_m/I$ and $g(x_m) = F(x_m, 1)$. If $(X_m, I) \in S$, $g(x_m) = \frac{\pi_s(x_m)}{\upsilon r - \mu_m} + \frac{\pi_m(x_m)}{r - \mu_m} - 1$. If $(X_m, I) \notin S$, i.e., whenever $x_m < x_m^*$, $g(x_m)$ should satisfy the following partial differential equation based on the Bellman equation and Ito's Lemma in real options theory:

$$\frac{\partial g(x_m)}{\partial x_m}\mu_m x_m + \frac{1}{2}\frac{\partial g^2(x_m)}{\partial x_m^2}\sigma_m^2 x_m^2 - (r+\lambda)g(x_m) + \lambda\phi g\left(\frac{x_m}{\phi}\right) = 0. \tag{4.6}$$

Moreover, when $x_m = x_m^*$, $g(x_m)$ must satisfy the following value matching and smooth-pasting conditions[2] based on Dixit and Pindyck (1994):

$$g(x_m^*) = \frac{\pi_s(x_m^*)}{\upsilon r - \mu_m} + \frac{\pi_m(x_m^*)}{r - \mu_m} - 1, \tag{4.7}$$

$$\frac{\partial g(x_m)}{\partial x_m}\Big|_{x_m = x_m^*} = \frac{\pi_s(x_m^*)}{x_m^*(\upsilon r - \mu_m)} + \frac{\pi_m(x_m^*)}{x_m^*(r - \mu_m)}. \tag{4.8}$$

Furthermore, if x_m approaches zero, the value of $g(x_m)$ must approach zero, and thus, we have:

$$\lim_{x_m \to 0} g(x_m) = 0. \tag{4.9}$$

With Eqs. (4.6) to (4.9), the problem of automated terminal investment is now well defined. We must find a specific function $g(x_m)$ and positive x_m^* such that $g(x_m)$ and x_m^* satisfy Eqs. (4.6) to (4.9). Substituting Eqs. (4.7)–(4.9) into Eq. (4.6) yields the threshold x_m^*:

$$x_m^* = \frac{r + \xi}{\frac{r-\mu_m}{\upsilon r - \mu_m}[(1-\gamma_1-\eta_1 K_{s1})K_{s1}] + [(1-\gamma_1-\eta_1 K_{m1})K_{m1} + (1-\gamma_2-\eta_2 K_{m2})K_{m2} - 2\varepsilon K_{m1}K_{m2}]}.$$

According to Eq. (3.5), the uncertain port demand is given by $X_{mt} = X_{m0} \cdot \exp((\mu_m - \frac{\sigma_m^2}{2})t + \sigma_m z_t)$, where X_{m0} is the port demand in the two terminals' case at the initial time and $\boldsymbol{E}[z_t] = 0$. Since $x_m = X_m/I$, then $\boldsymbol{E}[x_m] = \frac{\boldsymbol{E}[X_m]}{\boldsymbol{E}[I]}$.Assume that the expected value of x_m^* is denoted by $\widehat{x}_m^*$. Then, we have $\widehat{x}_m^* = \frac{\boldsymbol{E}[X_{m\tau^*}]}{\boldsymbol{E}[I_{\tau^*}]} = \frac{X_{m0}\cdot\exp((\mu_m - \frac{\sigma_m^2}{2})\tau^*)}{I_0 e^{-\xi\tau^*}}$, and thus $\tau^* = \frac{\ln(\widehat{x}_m^* I_0/X_{m0})}{\mu_m - \sigma_m^2/2 + \xi}$.

After getting the expected investment threshold $\widehat{x}_m^*$ and τ^*, we can derive the optimal capacity choice by solving the first-order condition $\partial(g(\widehat{x}_m^*)\cdot I_{\tau^*})/\partial K_{m2} = 0$, and the following equation determines the optimal capacity choice K_{m2}^*:

$$I_{\tau^*}\frac{\partial g(\widehat{x}_m^*)}{\partial K_{m2}^*} + +g(\widehat{x}_m^*)\cdot\frac{\partial I_{\tau^*}}{\partial K_{m2}^*} = 0. \tag{4.10}$$

Since $\widehat{x}_m^*$ is equal to $\widehat{x}_m^* = \frac{X_{m0}\cdot\exp((\mu_m - \frac{\sigma_m^2}{2})\tau^*)}{I_0 e^{-\xi\tau^*}}$, implying that $\widehat{x}_m^*$ is determined by the optimal investment timing τ^* under the given parameters X_{m0}, I_0, μ_m, and σ_m. Since $\tau^* = \inf\{t \geqslant 0; (X_{mt}, I) \in S_A\}$, and if S_A is empty, indicating that the port decision maker would never invest in AT, i.e., $\int_\tau^{+\infty}\pi_m(X_m)dt < I$ for all $(X_{mt}, I) \in \vartheta$. However, the increased competitiveness of upgrading the CT with AT ensures that there exist $(X_{mt}, I) \in \vartheta$ which invalidates the inequality and results in a contradiction. Therefore, it is obvious that S_A is not empty, and then there is obviously a unique solution for $\widehat{x}_m^*$. Unfortunately, there is no analytic solution for $g(\widehat{x}_m^*)$ that satisfies the above all conditions as I is not unique under technological improvement uncertainty. Section 4.2 will show the analytic solutions for $\widehat{x}_m^*$ and K_{m2}^* in the special case of deterministic technological improvement.

4.2. Analytic solutions

As shown above, we cannot obtain the specific function of $g(\widehat{x}_m^*)$ and then derive the expected thresholds $\widehat{x}_m^*$ and K_{m2}^* in the above

[2] For the detailed proofs of the value matching and smooth-pasting conditions, please refer to Dixit and Pindyck (1994).

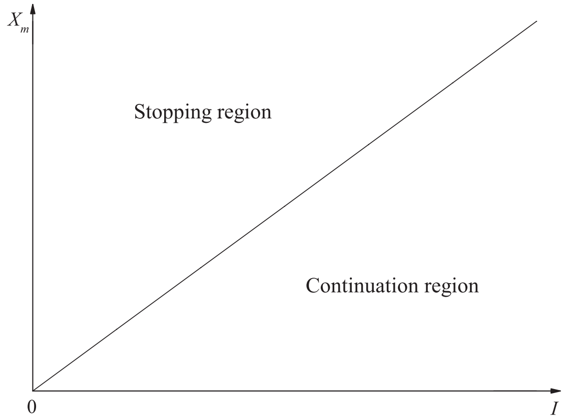

Fig. 1. The boundary of the stopping and continuation regions.

uncertain technological improvement setting. In this section, we solve the above free boundary problem to obtain the value of $F(X_m, I)$ and then derive the analytic solution in the deterministic technological improvement case by adjusting the parameters λ and ϕ.

We assume that the automated port technology has been dramatically improved and maintains a deterministic technological improvement progress, implying that the arrival rate of the Poisson random variable is increased ($\lambda \to +\infty$) and ϕ is increased ($\phi \to 1$). Therefore, the investment cost experiences a deterministic exponential declining trend:

$$I_t = I_0 e^{-\lambda t(1-\phi)} = I_0 e^{-\xi t},$$

where $\xi = \lambda(1-\phi)$ is always fixed as $\lambda \to +\infty$ and $\phi \to 1$.

Assume that the expected investment threshold, optimal investment timing, and capacity choice for AT in this special case are x_m^A, τ^A, and K_{m2}^A. Since ϕ is close to 1, we can expand the term $\lambda\phi g(\frac{x_m}{\phi})$ at point x_m with the Taylor expansion method, and then we have

$$\lambda\phi g\left(\frac{x_m}{\phi}\right) = \lambda\phi\left[g(x_m) + \left(\frac{x_m}{\phi} - x_m\right)g^{'}(x_m) + \frac{1}{2}\left(\frac{x_m}{\phi} - x_m\right)^2 g^{''}(x_m) + \ldots\right]. \tag{4.11}$$

As $\phi \to 1$, the second-order term $\frac{1}{2}(\frac{x_m}{\phi} - x_m)^2 g''(x_m)$ is close to 0. Therefore, $\lambda\phi g(\frac{x_m}{\phi}) \approx \lambda\phi g(x_m) + \lambda\phi(\frac{x_m}{\phi} - x_2)g'(x_m)$. Substituting this into Eq. (4.6) yields

$$\frac{1}{2}\sigma_m^2 g^{''}(x_m)x_m^2 + (\mu_m + \xi)x_m g^{'}(x_m) - (r+\xi)g(x_m) = 0. \tag{4.12}$$

Solving the above equation can derive the following general solution of $g(x_m)$:

$$g(x_m) = A_1 x_m^{\beta_1^A} + A_2 x_m^{\beta_2^A}, \tag{4.13}$$

where $\beta_1^A(>0)$ and $\beta_2^A(<0)$ are the two roots of $\frac{1}{2}\sigma_m^2\beta(\beta-1) + (\mu_m+\xi)\beta - (r+\xi) = 0$. Moreover, condition (4.9) indicates that $A_2 = 0$. Then, based on Eqs. (4.7) and (4.8), we have

$$A_1(x_m)^{\beta_1^A} = \frac{\pi_s(x_m)}{\upsilon r - \mu_m} + \frac{\pi_m(x_m)}{r - \mu_m} - 1, \tag{4.14}$$

$$\beta_1^A A_1(x_m)^{\beta_1^A - 1} = \frac{\pi_s(x_m)}{x_m(\upsilon r - \mu_m)} + \frac{\pi_m(x_m)}{x_m(r - \mu_m)}. \tag{4.15}$$

With these two equations, we can derive the following Corollary 4.1 regarding the unknown threshold x_m^A, τ^A, and K_{m2}^A.

Corollary 4.1. *x_m^A is given by*

$$x_m^A = \left(\frac{\beta_1^A}{\beta_1^A - 1}\right)\left[\frac{(1-\gamma_1-\eta_1K_{s1})K_{s1}}{\upsilon r - \mu_m} + \frac{(1-\gamma_1-\eta_1K_{m1})K_{m1} + (1-\gamma_2-\eta_2K_{m2})K_{m2} - 2\varepsilon K_{m1}K_{m2}}{r-\mu_m}\right]^{-1}, \tag{4.16}$$

and K_{m2}^A is determined by the following equation:

$$\frac{u}{K_{m2}^A} + x_m^A(1-\beta_1^A)\left[\frac{1-\gamma_2-2\eta_2K_{m2}^A - 2\varepsilon K_{m1}}{r-\mu_m}\right] = 0, \tag{4.17}$$

where $\tau^A = \frac{\ln(x_m^A I_0/X_{m0})}{\mu_m - \sigma_m^2/2 + \xi}$.

5. Replacement investment option for upgrading the CT

Apart from the additional investment option for upgrading the CT, the port decision maker may also choose to abandon the capacity of the existing CT and replace the existing capacity by investing in AT. In this situation, the port decision maker not only has to determine the exit and investment timings for the CT and AT, respectively, but also needs to determine the optimal exit capacity choice for the CT. In the following subsections, we first derive the optimal exit and investment decisions in the replacement investment option and then also give the analytic solution in the deterministic technological improvement case.

5.1. Optimal exit and investment decisions

Unlike the additional investment option without considering the exit capacity choice of the CT, the decision maker must solve the following optimal stopping problem in the replacement investment option:

$$H(X_s, X_m, K_{e1}, I) = \sup_{\tau_1} \boldsymbol{E}\left[\int_0^{\tau_1} e^{-rt}\pi_s^B(X_s)dt + \sup_{\tau_2:\tau_2\geqslant\tau_1, K_{e1}} \boldsymbol{E}\left[\int_{\tau_1}^{\tau_2} e^{-rt}\pi_s^A(X_s, K_{e1})dt + \int_{\tau_2}^{+\infty} e^{-rt}\pi_m(X_m)dt - e^{-r\tau_2}I\right]\right], \tag{5.1}$$

where τ_1 denotes the exit timing for the CT; τ_2 denotes the investment timing for the AT; and K_{e1} denotes the optimal exit capacity choice of CT ($K_{e1} \leqslant K_{s1}$).$\pi_s^B(X_s)$, $\pi_s^A(X_s, K_{e1})$, and $\pi_m(X_m)$ denote the profit of the CT before adopting the exit decision, the profit of the CT after adopting the exit decision but without investing in the AT, and the total profit of both types of terminals after adopting the exit and investment decisions, respectively.

To obtain the specific expression of $H(X_s, X_m, K_{e1}, I)$ and derive the optimal replacement option, we first determine the optimal investment decision for the AT. Since the port decision maker chose to replace the existing CT with AT, the investment capacity K_{m2} of AT is determined by the exit capacity of CT as the AT is installed in the port area that was occupied by the CT. Thus, we can assume that the investment capacity K_{m2} of the AT is given by $K_{m2} = \delta K_{e1}$, implying that K_{m2} is linearly related to the exited capacity K_{e1}. Moreover, we then have $K_{m1} = K_{s1} - K_{e1}$. $\pi_s^B(X_s)$ and $\pi_s^A(X_s, K_{e1})$ is thus calculated as follows:

$$\pi_s^B(X_s) = X_s(1 - \gamma_1 - \eta_1 K_{s1})K_{s1},$$

$$\pi_s^A(X_s, K_{e1}) = X_s(1 - \gamma_1 - \eta_1(K_{s1} - K_{e1}))(K_{s1} - K_{e1}).$$

Assume that the value function after the CT adopts the exit decision in $H(X_s, X_m, K_{e1}, I)$ is denoted by $Q(X_s, X_m, I)$, and we thus have

$$\begin{aligned} Q(X_s, X_m, I) = {} & \boldsymbol{E}\left[\int_{\tau_1}^{+\infty} e^{-rt}\pi_s^A(X_s, K_{e1})dt\right] \\ & + \sup_{\tau_2:\tau_2\geqslant\tau_1} \boldsymbol{E}\left[\int_{\tau_2}^{+\infty} e^{-rt}\left[\pi_s^A(X_s, K_{e1})\left(\frac{X_m}{X_s} - 1\right) + \pi_{m2}(X_m) - X_m\varepsilon(K_{s1} - K_{e1})\delta K_{e1}\right]dt - e^{-r\tau_2}I\right]. \end{aligned} \tag{5.2}$$

Assume that the total discounted sum profits of the two types of terminals after investing in the AT is denoted by $L(X_s, X_m, I)$, i.e.,

$$L(X_s, X_m, I) = \sup_{\tau_2, K_{e1}} \boldsymbol{E}\left[\int_{\tau_2}^{+\infty} e^{-rt}\left[\pi_s^A(X_s, K_{e1})\left(\frac{X_m}{X_s} - 1\right) + \pi_{m2}(X_m) - X_m\varepsilon(K_{s1} - K_{e1})\delta K_{e1}\right]dt - e^{-r\tau_2}I\right].$$

Since $X_m = \upsilon X_s$, and thus $L(X_s, X_m, I)$ is only determined by X_s and I. As in Eq. (4.5) in Section 4, $L(X_s, I)$ can be rewritten as follows:

$$L(X_s, I) = \sup_{\tau_2} \boldsymbol{E}\left[e^{-r\tau_2}\left(\frac{\pi_s^A(X_s, K_{e1})}{r - \mu_s} + \frac{\upsilon X_s\left(\frac{\pi_s^A(X_s, K_{e1})}{X_s} + \frac{\pi_{m2}(\upsilon X_s)}{\upsilon X_s} - \varepsilon(K_{s1} - K_{e1})\delta K_{e1}\right)}{r - \mu_m} - I\right)\right]. \tag{5.3}$$

As in the additional investment option in Section 4, we also can divide the decision state space $\psi \triangleq \mathbb{R}^+ \times \mathbb{R}^+$ for determining the optimal upgrade decision into two parts, namely, the stopping region (denoted as S_R)$S_R := \{(X_s, I) \in \mathbb{R}^+ \times \mathbb{R}^+ : L(X_s, I) = I\}$ and the continuation region (denoted as C_R)$C_R := \{(X_s, I) \in \mathbb{R}^+ \times \mathbb{R}^+ : L(X_s, I) > I\}$. Let τ_2^* be the time that the decision state variable first enters the stopping region, i.e., $\tau_2^* = \inf\{t \geqslant 0; (X_{st}, I_t) \in S_R\}$, and thus τ_2^* is the optimal timing for the AT investment. We can derive the following proposition regarding the optimal investment decision.

Proposition 5.1. *The stopping region satisfies the following*:

$$S_R = \left\{(X_s, I) \in \psi \middle| \frac{X_s}{I} \geqslant x_s^*\right\},$$

where x_s^* is a constant to be determined.

Proof. The proof is shown in the Appendix.

As in the Proposition 4.1, Proposition 5.1 and the corresponding proof also indicate that the threshold x_s^* is unique, and x_s^* also divided the decision space into two parts, i.e., the stopping and continuation regions. Moreover, according to Eq. (5.3), we have $L(kX_s, kI) = kL(X_s, I)(\forall k > 0)$. Therefore, $L(X_s, I)/I = L(X_s/I, 1)$, and thus $L(X_s/I, 1)$ is just a function of X_s/I. This property and Proposition 5.1

can also transform the above two-dimensional optimal stopping problem into a one-dimensional problem. Let $x_s = X_s/I$ and $l(x_s) = L(x_s, 1)$. If $(X_s, I) \in S_R$, then

$$l(x_s) = x_s\left[\frac{\pi_3^A(x_s, K_{e1})}{x_s(r-\mu_s)} + \frac{\upsilon\left(\frac{\pi_3^A(x_s,K_{e1})}{x_s} + \frac{\pi_{m2}(\upsilon x_s)}{\upsilon x_s} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1}\right)}{r-\upsilon\mu_s}\right] - 1.$$

If $(X_s, I) \notin S_R$, i.e., whenever $x_s \leqslant x_s^*$, $l(x_s)$ should satisfy the following partial differential equation based on the Bellman equation and Ito's Lemma in real options theory:

$$\frac{\partial l(x_s)}{\partial x_s}\mu_s x_s + \frac{1}{2}\frac{\partial l^2(x_s)}{\partial x_s^2}\sigma_s^2 x_s^2 - (r+\lambda)l(x_s) + \lambda\phi l\left(\frac{x_s}{\phi}\right) = 0. \tag{5.4}$$

Moreover, when $x_s = x_s^*$, $l(x_s)$ must satisfy:

$$l(x_s)\big|_{x_s=x_s^*} = x_s^*\left[\frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*(r-\mu_s)} + \frac{\upsilon\left(\frac{\pi_s^A(x_s^*,K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1}\right)}{r-\upsilon\mu_s}\right] - 1, \tag{5.5}$$

$$\frac{\partial l(x_s)}{\partial x_s}\big|_{x_s=x_s^*} = \frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*(r-\mu_s)} + \frac{\upsilon\left(\frac{\pi_s^A(x_s^*,K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1}\right)}{r-\upsilon\mu_s}. \tag{5.6}$$

Furthermore, the value of $l(x_s)$ will approach zero if x_s approaches zero, and thus we have:

$$\lim_{x_s\to 0} l(x_s) = 0. \tag{5.7}$$

With Eqs. (5.5)–(5.7), the AT investment problem under demand and technological improvement uncertainties is well defined. To obtain the optimal investment decision, we have to find a specific function $l(x_s)$ and positive x_s^* such that $l(x_s)$ and x_s^* satisfy Eqs. (5.5)–(5.7). Substituting Eqs. (5.5)–(5.7) into Eq. (5.4) yields the threshold x_s^*:

$$x_s^* = \frac{r+\xi}{(r-\mu_2)\left[\frac{M_1(K_{e1})}{(r-\mu_s)} + \frac{\upsilon(M_1(K_{e1})+M_2(K_{e1})-2\varepsilon(K_{s1}-K_{e1})\delta K_{e1})}{r-\upsilon\mu_s}\right]}, \tag{5.8}$$

where $M_1(K_{e1}) = (1-\gamma_1)(K_{s1}-K_{e1}) - \eta_1(K_{s1}-K_{e1})^2$ and $M_2(K_{e1}) = (1-\gamma_2)\delta K_{e1} - \eta_2(\delta K_{e1})^2$.

As in the expected investment timing for AT in Section 4, we assume that the expected investment threshold and the corresponding optimal investment timing for AT is denoted by $\widehat{x}_s^*$ and τ_2^*. Then we have $\widehat{x}_s^* = \frac{\boldsymbol{E}\left[X_{s\tau_2^*}\right]}{\boldsymbol{E}\left[I_{\tau_2^*}\right]} = \frac{X_{s0}\cdot\exp((\mu_s - \frac{\sigma_s^2}{2})\tau_2^*)}{I_0 e^{-\xi\tau_2^*}}$, and thus $\tau_2^* = \frac{\ln(\widehat{x}_s^* I_0/X_{s0})}{\mu_s - \sigma_s^2/2 + \xi}$. Section 5.2 will show the analytic solutions of $l(\widehat{x}_s)$ and $\widehat{x}_s^*$ in the deterministic technological improvement case.

After obtaining the discounted sum profits of the CT and AT when choosing to exit, we have to derive the optimal exit decision for the CT. According to the derived expression of $l(x_s)$ and Eq. (5.2), Eq. (5.1) can be rewritten as follows:

$$H(X_s, K_{e1}) = \sup_{\tau_1, K_{e1}} \boldsymbol{E}\left[\int_0^{\tau_1} e^{-rt}\pi_s^B(X_s)dt + \int_{\tau_1}^{+\infty} e^{-rt}\pi_s^A(X_s, K_{e1})dt + Il(x_s^*)\right]. \tag{5.9}$$

According to the Bellman equation and Ito's Lemma in real options theory, then $H(X_s, K_{e1})$ should satisfy the following partial differential equation:

$$\frac{\partial H(X_s, K_{e1})}{\partial X_s}\mu_s X_s + \frac{1}{2}\frac{\partial H^2(X_s, K_{e1})}{\partial X_s^2}\sigma_s^2 X_s^2 - rH(X_s, K_{e1}) + \pi_s^B(X_s) + \pi_s^A(X_s, K_{e1}) + Il(x_s^*) = 0. \tag{5.10}$$

Solving the above equation can derive the following specific expression of $H(X_s, K_{e1})$:

$$H(X_s, K_{e1}) = \begin{cases} \frac{X_s[(K_{s1}-K_{e1})(1-\gamma_1-\eta_1(K_{s1}-K_{e1}))]}{r-\mu_s} + \frac{Il(x_s^*)}{r} + N(K_{e1})X_s^{\beta_3}, & 0\leqslant X_s < X_s^*, \\ \frac{X_s(1-\gamma_1-\eta_1 K_{s1})K_{s1}}{r-\mu_s}, & X_s \geqslant X_s^*. \end{cases} \tag{5.11}$$

where X_s^* is the demand threshold for the CT choosing to exit under the replacement investment option and $N(K_{e1})$ is a parameter to be determined. When $0\leqslant X_s < X_s^*$, it is optimal to choose the exit decision for the CT, and the first term of $H(X_s, K_{e1})$ represents the discounted expected profit of the conventional terminal after exiting from the port market; the second term of $H(X_s, K_{e1})$ represents the discounted value of the AT investment; and the third term $N(K_{e1})X_s^{\beta_3}$ represents the value of the option to exit. When $X_s \geqslant X_s^*$, it is optimal to maintain the capacity for the CT as the demand for the CT is not extremely low. Therefore, $\tau_1^* = \sup\{t : 0 \leqslant X_s < X_s^*\}$. $\beta_3(>0)$ is the positive root of equation $\frac{1}{2}\sigma_s^2\beta(\beta-1) + \mu_s\beta - r = 0$.

With the specific expression of $H(X_s, K_{e1})$, we can derive the optimal exit capacity choice by solving the first-order condition $\partial H(X_s, K_{e1})/\partial K_{e1} = 0$, and the following equation determines the optimal exit capacity choice K_{e1}^*:

$$\frac{2\eta_1(K_{s1} - K_{e1}^*) + \gamma_1 - 1}{r - \mu_s} + \frac{I}{r}\frac{\partial l(x_s^*)}{\partial K_{e1}} + X_s^{\beta_1}\frac{\partial N(K_{e1})}{\partial K_{e1}} = 0. \tag{5.12}$$

Knowing K_{e1}^*, we can derive X_s^* and $N(K_{e1})$ based on the following conditions:

$$H(X_s, K_{e1}^*)\Big|_{X_s = X_s^*} = \frac{X_s^*(1 - \gamma_1 - \eta_1 K_{s1})K_{s1}}{r - \mu_s}, \tag{5.13}$$

$$\frac{\partial H(X_s, K_{e1}^*)}{\partial X_s}\Big|_{X_s = X_s^*} = \frac{(1 - \gamma_1 - \eta_1 K_{s1})K_{s1}}{r - \mu_s}. \tag{5.14}$$

Solving the two equations (5.13) and (5.14) can derive the specific expressions of X_s^* and $N(K_{e1})$:

$$X_s^* = \left(\frac{\beta_3}{\beta_3 - 1}\right)\frac{Il(x_s^*)(r - \mu_s)}{rK_{s1}(1 - \gamma_1 - \eta_1 K_{s1}) - r(K_{s1} - K_{e1})(1 - \gamma_1 - \eta_1(K_{s1} - K_{e1}))}, \tag{5.15}$$

$$N(K_{e1}) = (X_s^*)^{-\beta_3}\left(\frac{1}{\beta_3 - 1}\right)\frac{Il(x_s^*)}{r}. \tag{5.16}$$

5.2. *Analytic solutions*

We also assume that the automated port technology maintains a deterministic technological improvement progress and then derive the analytic solution. Assume that the expected investment threshold and the corresponding optimal investment timing for AT in this special case are denoted by x_s^D and τ_2^D, respectively. Moreover, let the expected exit demand threshold and the corresponding optimal exit capacity choice be X_s^D and K_{e1}^D, respectively.

As in the analytic solution in the additional investment option shown in Section 4.2, the solution of $l(x_s)$ will follow the below equation as $\phi \to 1$:

$$\frac{1}{2}\sigma_s^2 l^{''}(x_s)x_s^2 + (\mu_s + \xi)x_s l^{'}(x_s) - (r + \xi)l(x_s) = 0. \tag{5.17}$$

Solving the above equation can derive the following general solution of $l(x_s)$:

$$l(x_s) = D_1 x_s^{\beta_3^D} + D_2 x_s^{\beta_4^D}, \tag{5.18}$$

where D_1 and D_2 are parameters to be determined; and $\beta_3^D(>0)$ and $\beta_4^D(<0)$ are the two roots of $\frac{1}{2}\sigma_s^2\beta(\beta-1) + (\mu_s + \xi)\beta - (r + \xi) = 0$. Condition (5.7) indicates that $D_2 = 0$, and thus $l(x_s) = D_1 x_s^{\beta_3^D}$. Then, based on Eqs. (5.5) and (5.6), we have

$$D_1(x_s^D)^{\beta_3^B} = x_s^D\left[\frac{\pi_s^A(x_s^D, K_{e1})}{x_s^D(r - \mu_s)} + \frac{\upsilon\left(\frac{\pi_s^A(x_s^D, K_{e1})}{x_s^D} + \frac{\pi_{m2}(\upsilon x_s^D)}{\upsilon x_s^D} - \varepsilon(K_{s1} - K_{e1})\delta K_{e1}\right)}{r - \upsilon\mu_s}\right] - 1, \tag{5.19}$$

$$\beta_3^D D_1(x_s^D)^{\beta_3^D - 1} = \frac{\pi_s^A(x_s^D, K_{e1})}{x_s^D(r - \mu_s)} + \frac{\upsilon\left(\frac{\pi_s^A(x_s^D, K_{e1})}{x_s^D} + \frac{\pi_{m2}(\upsilon x_s^D)}{\upsilon x_s^D} - \varepsilon(K_{s1} - K_{e1})\delta K_{e1}\right)}{r - \upsilon\mu_s}. \tag{5.20}$$

With these two equations, we can derive the unknown threshold x_s^D and parameter D_1:

$$x_s^D = \frac{\beta_3^D}{(\beta_3^D - 1)\left(\frac{M_1(K_{e1})}{(r - \mu_s)} + \frac{\upsilon(M_1(K_{e1}) + M_2(K_{e1}) - 2\varepsilon(K_{s1} - K_{e1})\delta K_{e1})}{r - \upsilon\mu_s}\right)}, \tag{5.21}$$

$$D_1 = \frac{(x_s^D)^{-\beta_3^D}}{(\beta_3^D - 1)}. \tag{5.22}$$

According to Eqs. (5.12) and (5.15), we can derive the following Corollary 5.1 regarding the analytic solution of X_s^D, τ_2^D, and K_{e1}^D.

Corollary 5.1. *The exit demand threshold X_s^D for the conventional terminal satisfies:*

$$X_s^D = \frac{\beta_3^D I_{\tau_2^D}(r-\mu_1)}{\left(\beta_3^D-1\right)^2\left[rK_{s1}(1-\gamma_1-\eta_1K_{s1}) - r(K_{s1}-K_{e1})(1-\gamma_1-\eta_1(K_{s1}-K_{e1}))\right]}, \tag{5.23}$$

$$\tau_2^D = \frac{\ln\left(x_s^D I_0/X_{s0}\right)}{\upsilon\mu_s - \sigma_s^2/2+\xi}.$$

Moreover, K_{e1}^D is the positive root of the following equation:

$$\eta\left(K_{s1}-K_{e1}^D\right)^2 + (\gamma_1-1)\left(K_{s1}-K_{e1}^D\right) - \frac{\beta_3^D I_{\tau_2^D}(r-\mu_s)}{r\left(\beta_3^D-1\right)^2} + K_{s1}(1-\gamma_1-\eta_1K_{s1}) = 0. \tag{5.24}$$

6. Numerical experiments

This section will present numerical examples based on the European private port to illustrate how demand and automation technological improvement uncertainties influence the optimal upgrade decisions for the CT under the given parameters. The numerical examples can better show the analytic solutions and offer complementary findings that are not easily observed through the above results.

We first estimate the parameters w and u that determine the investment cost for the AT at the initial time shown in Eq. (3.8). With the public data in the annual reports regarding the investment costs and capacities of existing European and North American automated terminals in the Rotterdam Port (Port of Rotterdam Authority, 2023), the Liverpool Port (Port of Liverpool, 2023), and the Long Beach Port (Port of Long Beach, 2023), we calibrate the log form of Eq. (3.8) and then obtain $\ln I_0 = 6.32 + 0.91\ln K$. Therefore, w and u are equal to 80 million dollars/million tons and 0.91, respectively. Suppose that the initial capacity K_{s1} of the CT is 0.3 billion tons. Moreover, the interest rate r is set to 10 % sourced from the European Central Bank (European Central Bank, 2024).

Table 1 shows the parameters for capturing the demand functions and the relationship between the demand and capacity with sole CT and the demand with two types of terminals. Rather than sourcing from any specific port case, the values of the parameters shown in Table 1 are used to produce reasonable demand elasticities with respect to the terminal operating efficiency, congestion effect, substitution effect, and spilling effect after investing in AT. Moreover, regarding the drift parameter μ and volatility parameter σ for measuring uncertain demand in two different upgrade options, we will specify the values in the following numerical experiments.

6.1. Additional investment option for upgrading CT

As mentioned in Section 4, the decision maker will choose an additional AT investment option for upgrading the CT without disturbing the operation of CT in the port. Therefore, we estimate the drift parameter $\mu_m(=0.05)$ and $\sigma_m(=0.1)$ with the demand data (measured by port throughput) of the Port of Rotterdam from 2018 to 2023 that adopts the additional investment option (Port of Rotterdam Authority, 2023). Suppose that the initial capacity K_{s1} of the CT is 0.3 billion tons and the initial demand X_{m0} for the port that contains two kinds of terminals at the start time is 0.3 billion tons. With the above data and the optimal investment decision described in Section 4, we can obtain the following numerical results.

As mentioned in Section 4.1, we cannot obtain the analytic solutions of the investment decision under the uncertain technological improvement situation, as the parameter ξ for measuring the technological improvement is uncertain. Therefore, in this numerical experiment case, we assume that the parameter ϕ in ξ for measuring the uncertain technological improvement is set to 0.5. Then, we can analyze the impacts of the Poisson variable λ in ξ on the optimal investment decision, and the results of $\widehat{x}_m^*$, τ^*, and K_{m2}^* with increasing λ are shown in Fig. 2. With Fig. 2, we observe that both the expected investment threshold $\widehat{x}_m^*$ and optimal investment capacity choice K_{m2}^* are increasing in λ, while the optimal investment timing τ^* is decreasing in λ. Moreover, as λ increases, both τ^* and K_{m2}^* become stable and converge to a constant value. This trend implies that when the uncertain technological improvement approaches deterministic technological improvement, i.e., $\lambda \to +\infty$, the optimal investment strategy for AT becomes stable.

As described in Section 4.2, when technological improvement for AT investment approaches a deterministic technological improvement process (i.e., $\lambda \to +\infty$ and $\phi \to 1$), the optimal AT investment decision is only determined by the uncertain port demand. In this case, we can derive the analytic solutions for optimal AT investment decision, and the analytic solutions of x_m^A, τ^A, and K_{m2}^A are equal to 2 billion tons/billion dollars, 12.7 years, and 0.49 billion tons.

Since uncertain port demand is the only factor that affects the optimal AT investment decision, we then further analyze the impacts of the demand uncertainty on the optimal investment decision for AT under deterministic technological improvement, as shown in

Table 1
Baseline values of the parameters used to capture the demand functions in two upgrade options.

Parameter	Value	Parameter	Value
η_1	0.1	γ_1	0.5
η_2	0.1	γ_2	0.4
υ	1.5	δ	1.5
ε	0.1	/	

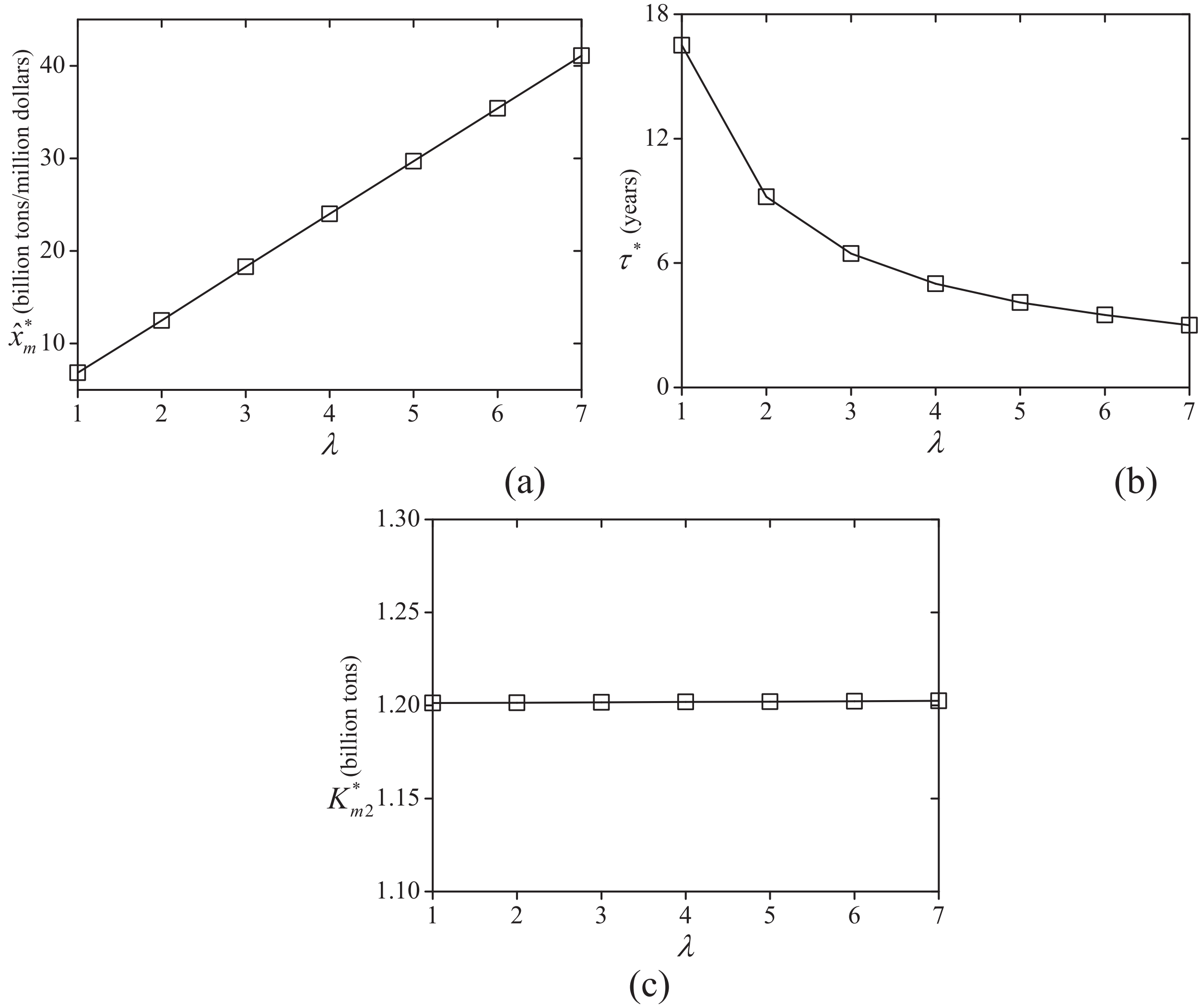

Fig. 2. The results of (a)$\hat{x}_m^*$, (b)τ^*, and (c)K_{m2}^* with increasing parameter λ.

Fig. 3. The results presented in Fig. 3 indicate that both the expected investment threshold x_m^A and the optimal investment timing τ^A for AT are increasing in σ_m, implying an increase in the uncertainty for port demand. However, the optimal investment capacity choice K_{m2}^A for AT has not changed with the increasing σ_m, implying that the demand uncertainty cannot affect the investment size for AT. Furthermore, we also observe that higher σ_m will lead to a sharp increase in the expected investment threshold x_m^A and investment timing τ^A. The above trend indicates that higher port demand uncertainty will postpone AT's investment decision even though the AT investment size does not change when facing higher demand uncertainty.

6.2. Replacement investment option for upgrading the CT

Unlike the additional investment option shown in Section 6.1, which does not consider the exit decision for the CT, we have to determine the optimal exit and investment decisions for the two types of terminals in the replacement investment option. We also estimate the drift parameter $\mu_s(=0.05)$ and volatility parameter $\sigma_s(=0.15)$ with the demand data of the Port of Long Beach from 2018 to 2023, which adopts the replacement investment option (Port of Long Beach, 2023). Suppose that the initial capacity K_{s1} of the CT in case s is also 0.3 billion tons and the initial demand X_{s0} in case s at the start time is 0.1 billion tons. With the above data and settings, we then derive the following numerical results regarding the optimal exit and investment decisions for the types of terminals.

As mentioned in Section 6.1, in the case of demand and technological improvement uncertainties, we also can analyze the impacts of the Poisson variable λ on the optimal exit capacity K_{e1}^*, expected investment threshold $\hat{x}_s^*$, optimal investment timing τ_2^* for the AT, and the exited demand threshold X_s^* for the CT. The detailed results are illustrated in Fig. 4.

From Fig. 4, we can observe that λ does not have a significant impact on the optimal exit demand threshold X_s^*, and X_s^* is always

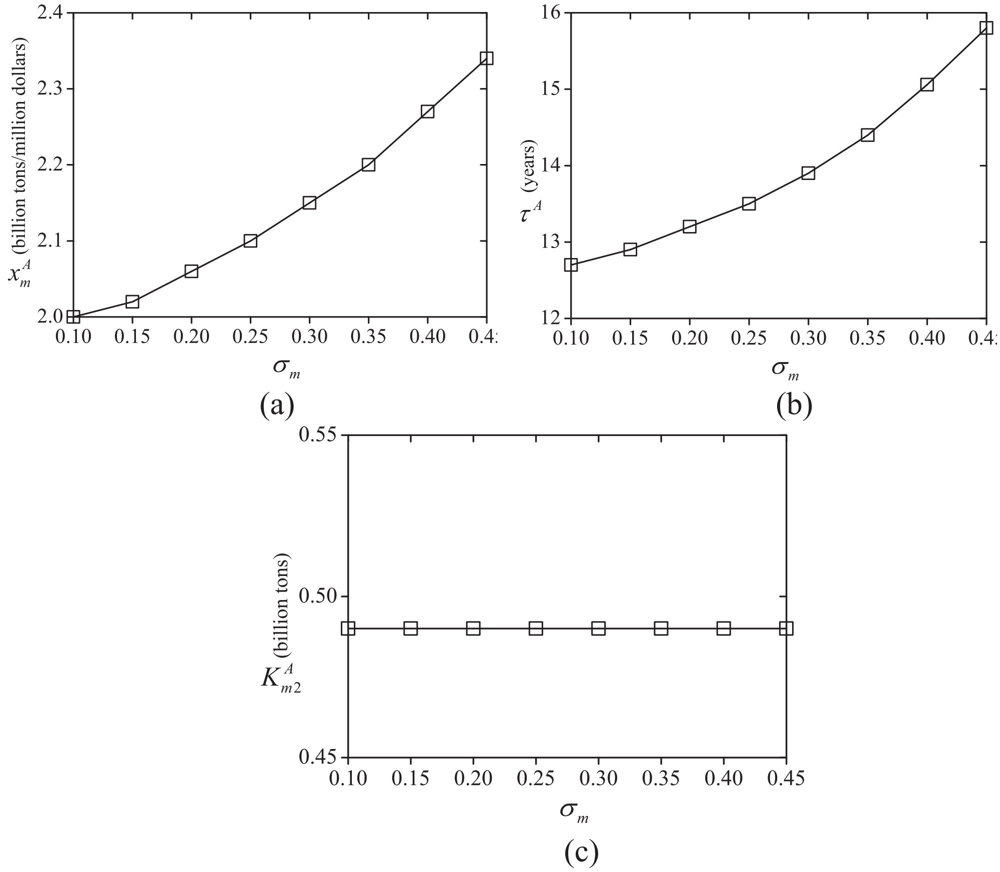

Fig. 3. The results of (a)x_m^A, (b)τ^A, and (c)K_{m2}^A with increasing parameter σ_m.

equal to 0.1 billion tons with the increasing λ. The results imply that uncertain automation technology for AT investment cannot influence the exit timing of the CT, and the decision maker can ignore the impact of technological improvement uncertainty on the exit timing decision for the CT. Nevertheless, regarding the impacts of λ on the optimal investment timing τ_2^* for AT, the expected investment threshold $\hat{x}_s^*$, and the exit capacity choice K_{e1}^*, they are quite different from that of the impact on X_s^*. The investment threshold $\hat{x}_s^*$ is increasing in λ, whereas the optimal investment timing τ_2^* and capacity choice K_{e1}^* are decreasing in λ. Moreover, as λ increases, the values of τ_2^* and K_{e1}^* become stable, so the optimal investment timing and exit capacity choice are convergent. We thus conclude that higher technological uncertainty of automated port technology will defer the investment timing of the AT and increase the exit capacity choice for the CT.

Furthermore, we also obtain the analytic solutions of the optimal exit and investment decisions under the deterministic technological improvement process. Unlike the special case of the deterministic technological improvement process in the additional investment option, both uncertain port demand and the exit capacity choice of the CT will affect the optimal investment threshold x_s^D, optimal investment timing τ_2^D, and optimal exit demand threshold X_s^D in the case of deterministic technological improvement process in this replacement investment option. According to the analytic solutions shown in Section 5.2, we derive the optimal results of the exit and investment decisions in the replacement investment option. The optimal expected investment threshold x_s^D, the optimal investment timing τ_2^D for the AT, the optimal exit capacity K_{e1}^D for the CT, and the optimal exit demand threshold X_s^D for the CT are equal to 0.3 billion tons/billion dollars, 28.7 years, 0.24 billion tons, and 0.2 billion tons.

Moreover, we also further analyzed the impacts of volatility parameter σ_s on the x_s^D, τ_2^D, K_{e1}^D, and X_s^D, and the results are illustrated in Fig. 5. As we can see from Fig. 5, the impacts of the volatility parameter σ_s on the above four variables are quite different. As σ increases, the expected investment threshold x_s^D and optimal investment timing τ_2^D gradually increase. In other words, higher demand uncertainty will notably defer the AT investment. Moreover, as in the special case in Section 6.2, the volatility parameter σ_s will also not affect the exit demand threshold X_s^D based on the results shown in Fig. 5. Regarding the impact of σ_s on the exit capacity choice K_{e1}^D, K_e^D first increases but then decreases with the increasing σ_s. When σ_s is equal to 0.2, K_{e1}^D reaches the highest value of 0.249 billion tons. This nonlinear trend implies that there exist an optimal σ_s such that K_{e1}^D reaches the maximum value.

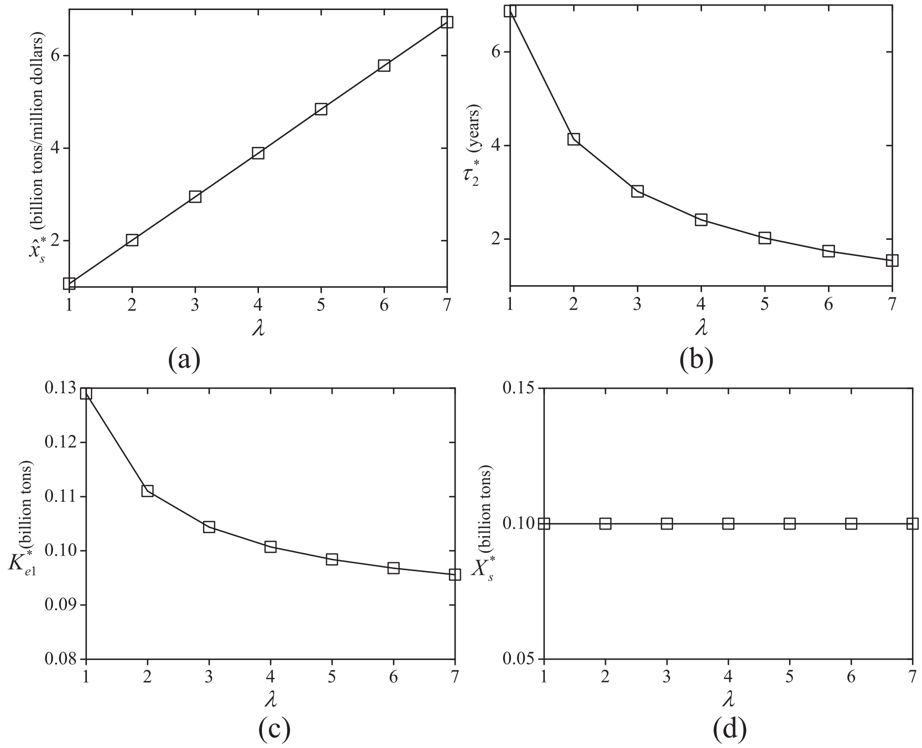

Fig. 4. The results of (a)$\widehat{x}_s^*$, (b)τ_2^*, (c)K_{e1}^*, and (d)X_s^* with increasing λ.

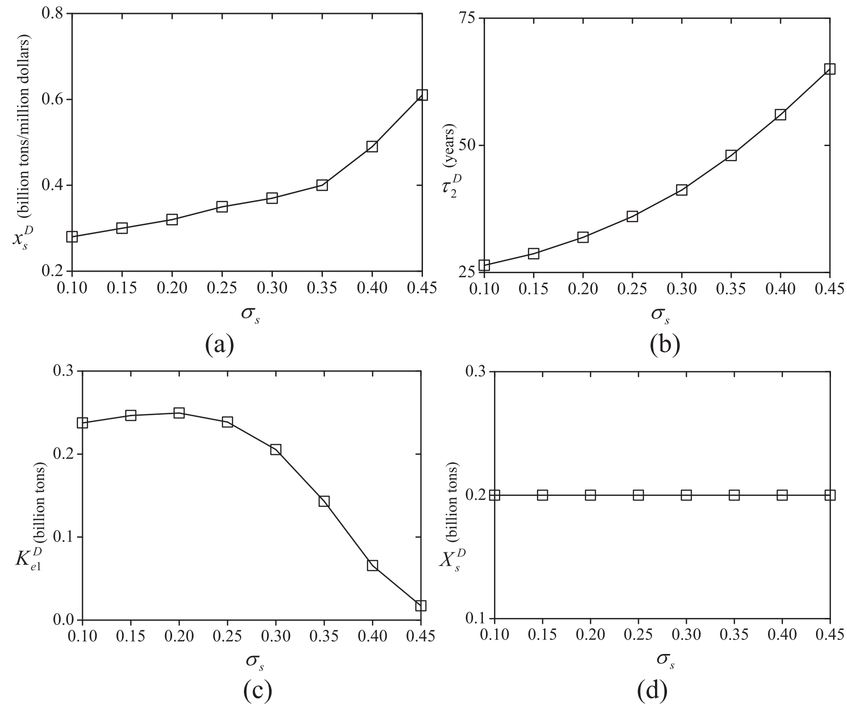

Fig. 5. The results of (a)x_s^D, (b)τ_2^D, (c)K_{e1}^D, and (d)X_s^D with increasing σ_s.

6.3. Sensitivity analysis

As shown in Section 3, γ and ε are two important factors in terminal demand function reflecting the terminal operating efficiency and the substitution effect of the two types of terminals, respectively. Moreover, the discount rate r will also influence the discounted profits of the terminals. Therefore, we further conduct sensitivity analyses to investigate the impacts of these three parameters on the optimal upgrade decisions.

(1) Impacts of parameter γ on the optimal upgrade decisions

In Table 1, parameters γ_1 and γ_2 are equal to 0.5 and 0.4, respectively, which means that the efficiency of the port with two types of terminals is 20 % percent higher than that of the port with sole CT. In this subsection, we assume that the parameter γ_1 is always equal to 0.5, and then we derive the optimal upgrade decisions when parameter γ_2 is equal to 0.3, 0.2, and 0.1 (i.e., the port efficiency with two types of terminals is 40 %, 60 %, and 80 % percents higher than that of the port with sole CT). According to the above sensitivity analysis, we can investigate how the AT efficiency affects optimal upgrade decisions.

Fig. 6 shows the optimal investment decision for the AT with increasing γ_2 in the additional investment option. From Fig. 6, we observe that when γ_2 decreases from 0.4 to 0.1, the expected investment thresholds ($\hat{x}_m^*$ and x_m^A) gradually decrease, but the investment capacity choices (K_{m2}^* and K_{m2}^A) for the AT increase progressively. Moreover, as γ_2 decreases, the optimal investment timing (τ^* and τ^A) for the AT has not significantly changed. These results mean that the increasing efficiency of the AT within the port with two types of terminals will increase the investment size of the AT. That is, the decision-maker is more willing to invest in AT if the AT can significantly improve the port efficiency.

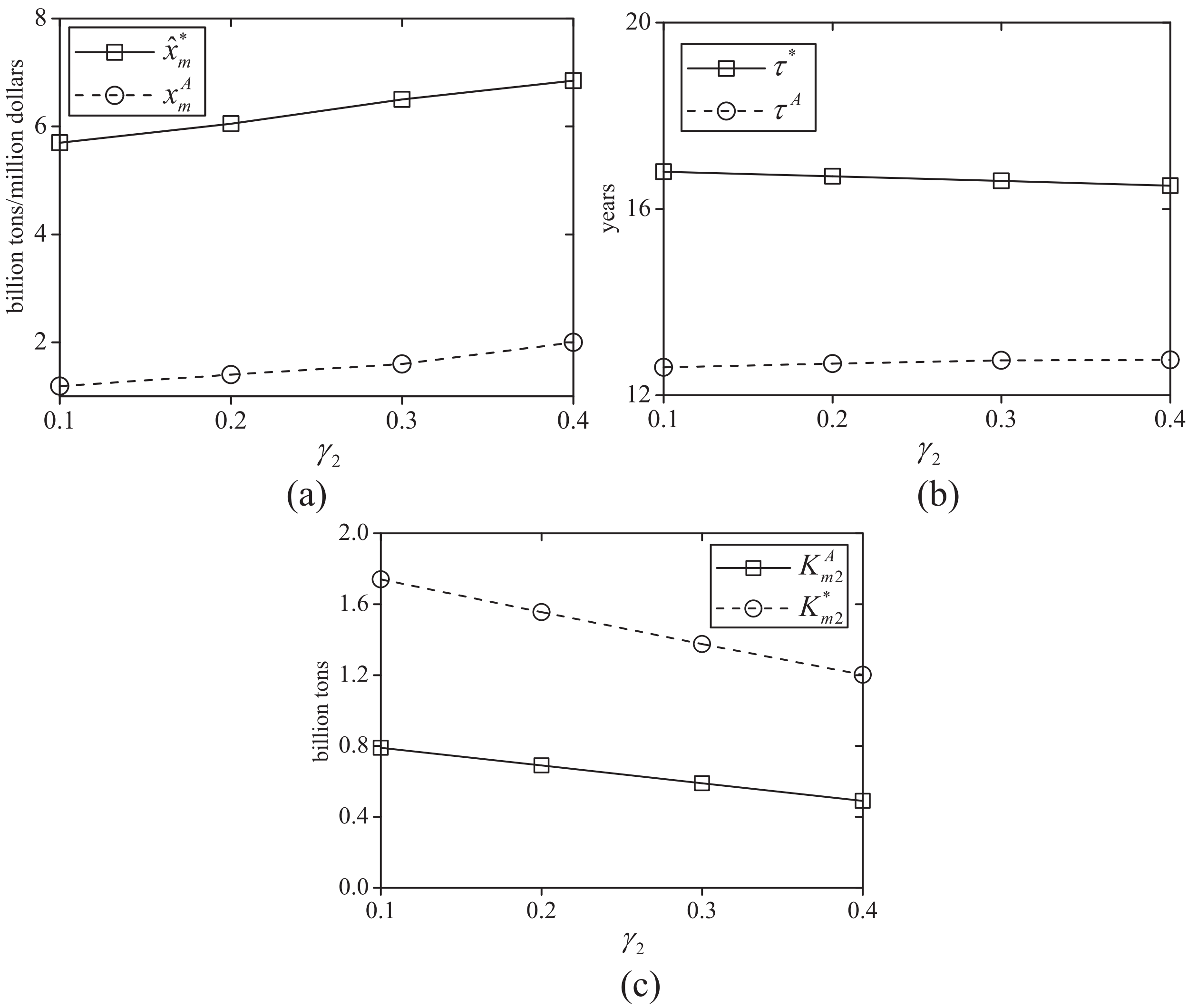

Fig. 6. Optimal investment decision for the AT with increasing γ_2 in the additional investment option

We also analyzed the optimal exit and investment decisions with increasing γ_2 in the replacement investment option, as illustrated in Fig. 7. The results shown in Fig. 7 indicate that when γ_2 decreases from 0.4 to 0.1, both the expected investment thresholds ($\hat{x}_s^*$ and x_s^D) and investment timings (τ_2^* and τ_2^D) for the AT gradually decrease, but the exit demand thresholds (X_s^* and X_s^D) for the CT have not changed. In addition, the optimal exit capacity choices (K_{e1}^* and K_{e1}^D) for the CT do not have the same changing trends. As γ_2 decrease, K_{e1}^* first increase but then decrease. When γ_2 is equal to 0.2, K_{e1}^* reaches the highest value of 0.29 billion tons. Moreover, as γ_2 decreases, K_{e1}^D gradually increases. The above results imply that higher AT efficiency will accelerate the exit of the CT for realizing the AT transition and encourage the decision-maker to invest early in the AT.

(2) Impacts of parameter ε on the optimal upgrade decisions

As shown in the demand functions in Section 3, the parameter ε is an important factor that reflects the substitution effect of the two kinds of terminals. In this sensitivity analysis subsection, we further investigate the impacts of the parameter ε on the optimal upgrade decisions, and the detailed results are presented in Figs. 8 and 9.

Fig. 8 shows the optimal investment decision for the AT with increasing ε in the additional investment option. As Fig. 8 shows, when ε increases from 0.1 to 0.4, the expected investment thresholds ($\hat{x}_m^*$ and x_m^A) gradually increase, while the investment capacities (K_{m2}^* and K_{m2}^A) for the AT decrease progressively. Moreover, as ε increases, the investment timings (τ^* and τ^A) of the AT have not significantly changed. These impacts imply that the high substitution between these two types of terminals will not affect the timing decisions for AT, but it will reduce the capacity investment size of the AT, as the AT did not have remarkable advantages in the high substitution situation.

Moreover, Fig. 9 presents the optimal exit and investment decisions with increasing ε in the replacement investment option. From Fig. 9, we observe that when ε increases from 0.1 to 0.4, the expected investment thresholds ($\hat{x}_s^*$ and x_s^D) gradually increase, while the exit capacities (K_{e1}^* and K_{e1}^D) for the CT decrease progressively. Moreover, as ε increases, both the investment timings (τ_2^* and τ_2^D) for the AT and the exit demand thresholds (X_s^* and X_s^D) for the CT have not changed. These trends indicate that although high substitution between these two types of terminals will not affect the exit and investment timings for the CT and AT, respectively, it will enlarge the maintained capacity of the CT and also decrease the investment in the AT.

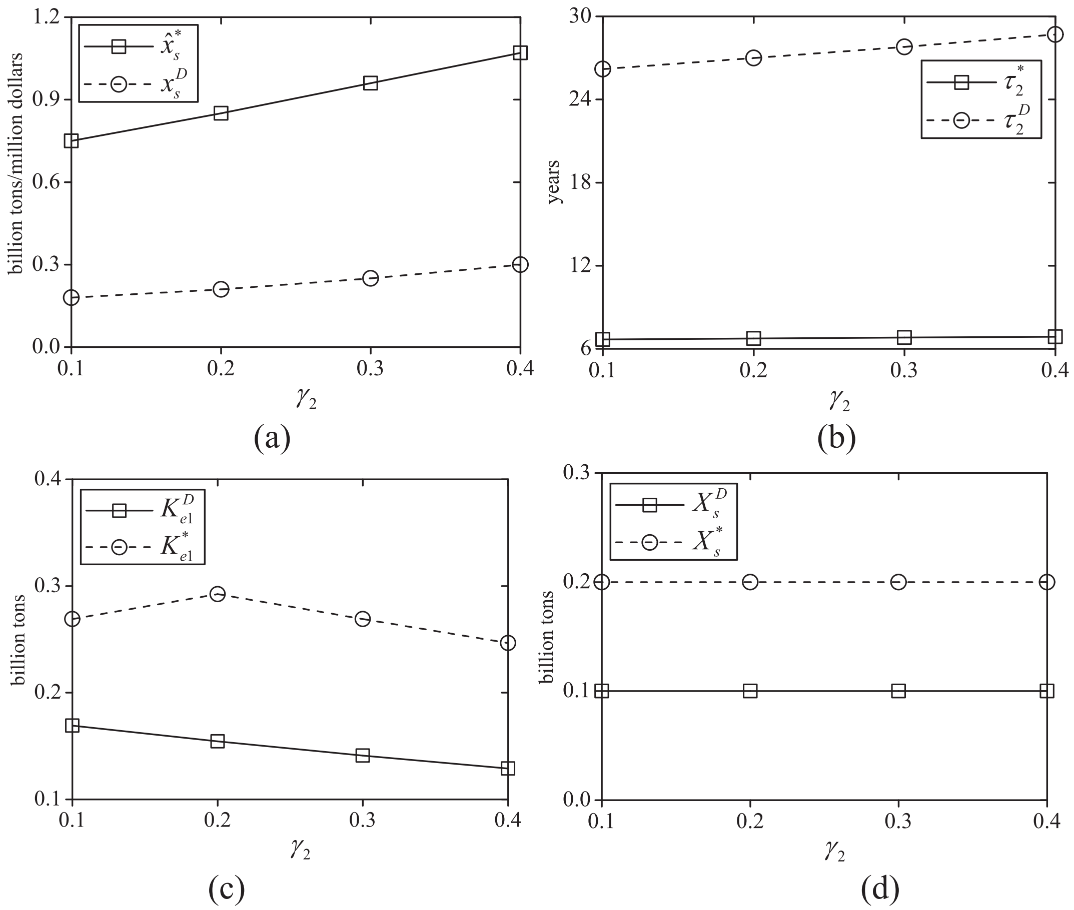

Fig. 7. Optimal exit and investment decisions with increasing γ_2 in the replacement investment option

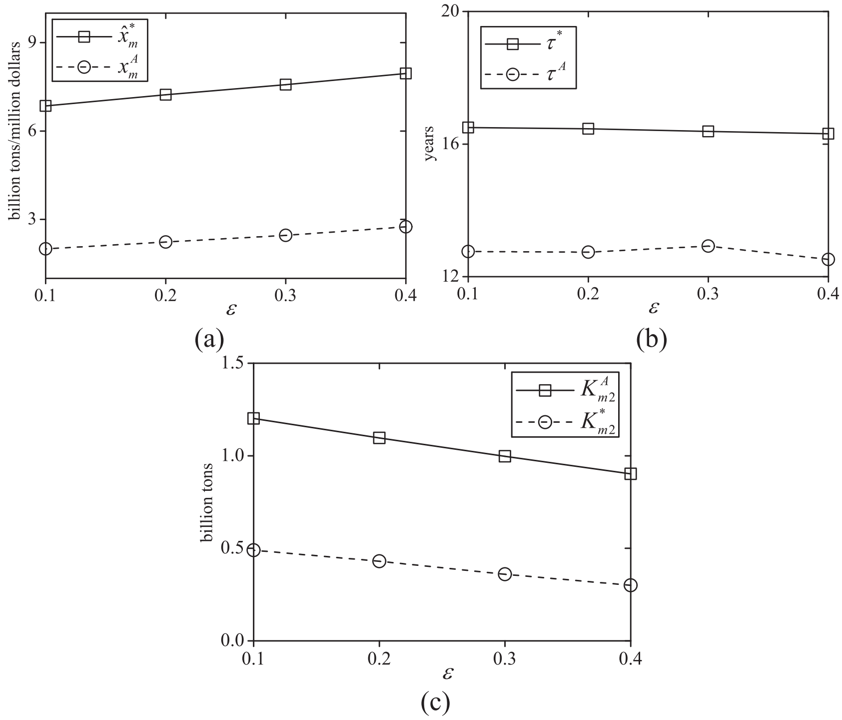

Fig. 8. Optimal investment decision for the AT with increasing ε in the additional investment option

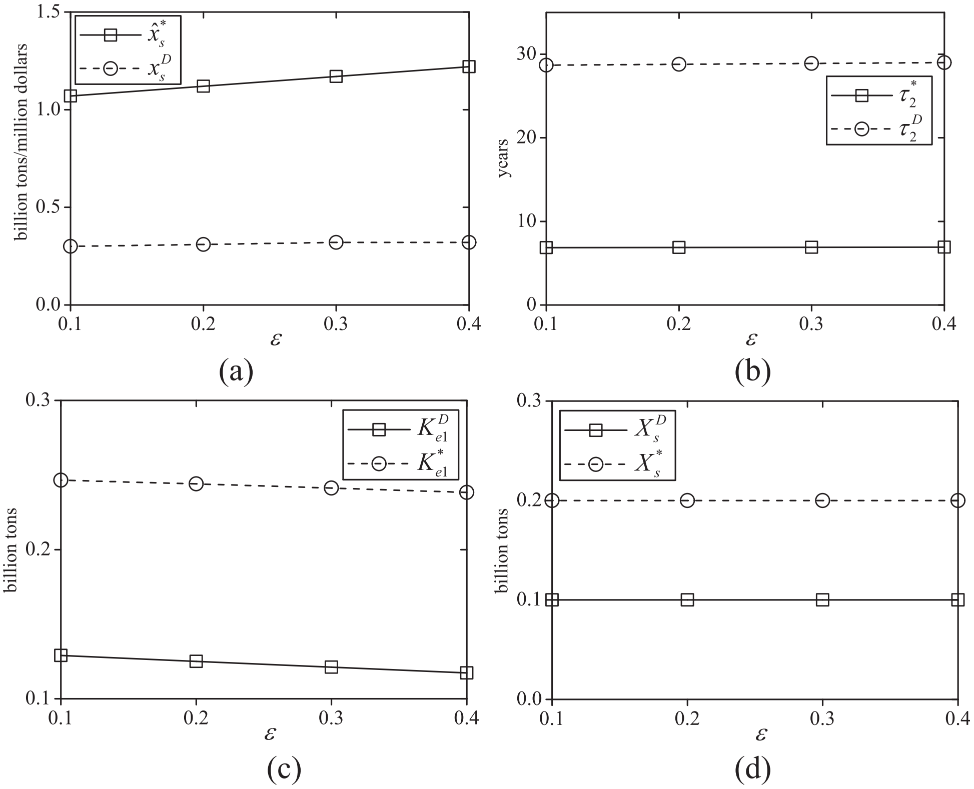

Fig. 9. Optimal exit and investment decisions with increasing ε in the replacement investment option

(3) Impacts of parameter r on the optimal upgrade decisions

Fig. 10 shows the optimal investment decision for the AT with increasing r in the additional investment option. As Fig. 10 shows, when r increases from 0.1 to 0.2, the expected investment threshold $\hat{x}_m^*$ and investment timing τ^* gradually decrease, while the investment capacity K_{m2}^* has not changed significantly. Moreover, in the case of deterministic technology improvement, as r increases, x_m^A, τ^A, and K_{m2}^* have not changed significantly. These impacts imply that although the high return of investment in AT resulting from the high discount rate cannot change the investment size for AT, it will speed up its investment. However, in the case of deterministic technology improvement, a high return of investment in AT cannot have a significant impact on the upgrade decision, as the existing CT can also obtain a high profit in the case of a high discount rate.

Fig. 11 presents the optimal exit and investment decisions with increasing r in the replacement investment option. From Fig. 11, we observe that when r increases from 0.1 to 0.2, the expected investment thresholds ($\hat{x}_s^*$ and x_s^D) gradually increase, while the exit capacities (K_{e1}^* and K_{e1}^D) for the CT, exit demand threshold X_s^*, and the investment timing τ_2^D in the case of deterministic technology improvement decrease progressively. Moreover, as r increases, both the investment timing τ_2^* for the AT and the exit demand thresholds X_s^D for the CT in the case of deterministic technology improvement have not changed. These trends show that a high discount rate will decrease the capacity exit size of the CT, as the existing CT can also obtain a high profit in the case of a high discount rate.

7. Extension: Considering the public ownership of the port

In addition to the private port we studied in the above sections, many ports worldwide (e.g., mainland China) are also public ports, pursuing the maximum social welfare for serving the hinterland. Therefore, we decided to extend our work to the case of public ownership of the port. In this section, we first derive the propositions for determining the optimal upgrade decisions and then validate the solutions with a numerical experiment conducted based on the data of Chinese public ports.

7.1. Optimal upgrade decisions

The social welfare generated by the individual terminal is the sum of the terminal profit and shippers' consumer surplus. Assume that the social welfare is denoted by SW, then the maximum social welfare generated by the two types of terminals in the additional investment option can be calculated as follows:

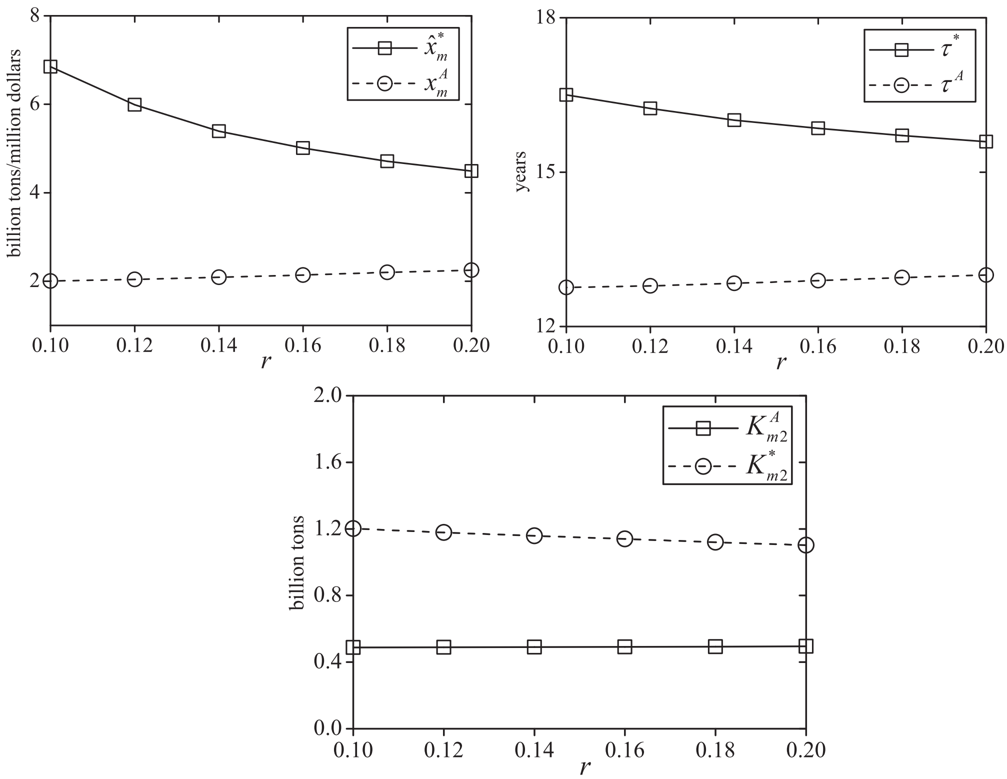

Fig. 10. Optimal investment decision for the AT with increasing r in the additional investment option

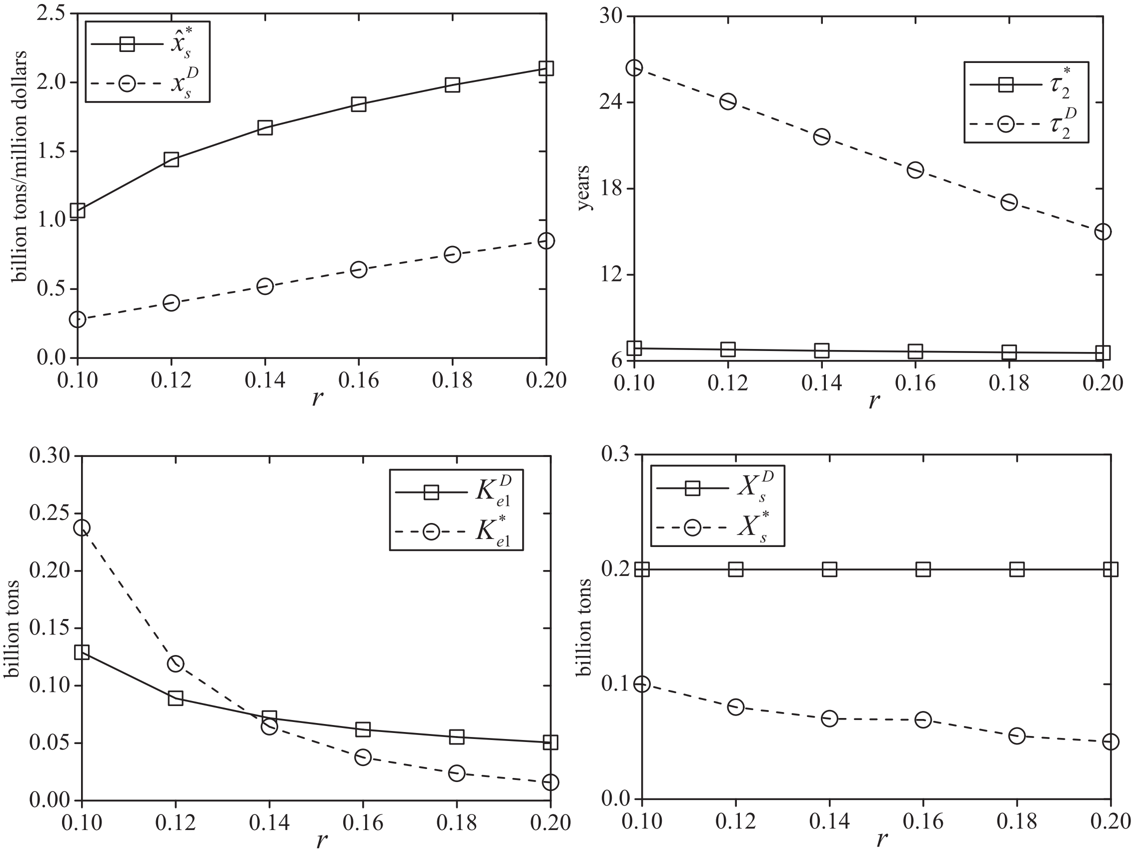

Fig. 11. Optimal exit and investment decisions with increasing r in the replacement investment option

$$SW_s(X_s) = X_s(1-\gamma_1-\eta_1K_{s1})K_{s1} + \left[\int_0^{K_{s1}} X_s(1-\gamma_1-\eta_1K_{s1})dy - X_s(1-\gamma_1-\eta_1K_{s1})K_{s1}\right] = \pi_s(X_s) + X_s\eta_1K_{s1}^2\Big/2,$$

$$SW_{m1}(X_m) = \pi_{m1}(X_m) + X_m\eta_1K_{m1}^2/2,$$

$$SW_{m2}(X_m) = \pi_{m2}(X_m) + X_m\eta_2K_{m2}^2/2.$$

Moreover, the social welfare of the two types of terminals in the replacement investment option can be expressed as follows:

$$SW_s^B(X_s) = \pi_s^B(X_s) + X_s\eta_1K_{s1}^2/2,$$

$$SW_s^A(X_s) = \pi_s^A(X_s) + X_s\eta_1(K_{s1}-K_{e1})^2/2.$$

Furthermore, the social welfare of each type of terminal after investing in the AT in the replacement investment option is the same as shown in the additional investment option. We then update the profit π with social welfare SW in the value functions in Sections 4 and 5 to derive the optimal upgrade decision for the public port. Suppose that AT's investment threshold and capacity investment choice in the additional investment option are denoted by $x_m^{'}$ and $K_{m2}^{'}$, respectively. Moreover, Let the optimal investment threshold for AT and the exit demand threshold and capacity choice for CT in the replacement investment option be $x_s^{'}$, $X_s^{'}$, and $K_{e1}^{'}$. Then, we can derive the following two propositions.

Proposition 7.1. *In the case of public port, the value function in the additional investment option is* $F(X_m, I) + \frac{X_m\eta_1K_{s1}^2}{2(vr-\mu_m)} + \frac{X_m\eta_2K_{m2}^2}{2(r-\mu_m)}$; *the investment threshold for AT is*

$$x_m^{'} = \frac{r+\xi}{\frac{r-\mu_m}{vr-\mu_m}\left[(1-\gamma_1-\eta_1K_{s1})K_{s1} + \frac{\eta_1K_{s1}^2}{2}\right] + \left[(1-\gamma_1-\eta_1K_{m1})K_{m1} + (1-\gamma_2-\eta_2K_{m2})K_{m2} - 2\varepsilon K_{m1}K_{m2} + \frac{\eta_2K_{m2}^2}{2}\right]}.$$

Moreover, the optimal capacity investment choice $K_{m2}^{'}$ *is determined by the following equation:*

$$I\frac{\partial g(x_m^{'})}{\partial K_{m2}} + +g(x_m^{'})\cdot\frac{\partial I}{\partial K_{m2}} = 0.$$

By comparing the values of $x_m^{'}$ and x_m in the case of private port, we can obtain the following corollary.

Corollary 7.1. *Public port prefers to adopt the earlier investment in AT compared to the private port in the additional investment option, as* $x_m^{'}$ *is always lower than* x_m.

Proposition 7.2. *In the case of public port, the value function of AT investment in the replacement investment decision is* $L(X_s, I) + \frac{\upsilon X_s\eta_1(K_{s1}-K_{e1})^2+\upsilon X_s\eta_2(\delta K_{e1})^2}{2(r-\upsilon\mu_s)}$*; the investment threshold* $x_s^{'}$ *is*

$$x_s^{'} = \frac{r+\xi}{(r-\mu_s)\left[\frac{M_1(K_{e1})}{(r-\mu_s)} + \frac{\upsilon\left(M_1(K_{e1})+M_2(K_{e1})-2\varepsilon(K_{s1}-K_{e1})\delta K_{e1}+\frac{1}{2}\eta_1(K_{s1}-K_{e1})^2+\frac{1}{2}\eta_2(\delta K_{e1})^2\right)}{r-\upsilon\mu_s}\right]}.$$

Moreover, the exit demand threshold $X_s^{'}$ *is*

$$X_s^{'} = \left(\frac{\beta_3}{\beta_3-1}\right)\frac{I\left[l(x_s^{'}) + \left(x_s^{'}\eta_1(K_{s1}-K_{e1})^2 + x_s^{'}\eta_2(\delta K_{e1})^2\right)/(2(r-\upsilon\mu_s))\right](r-\mu_s)}{rK_{s1}(1-\gamma_1+\eta_1K_{s1}/2) - r(K_{s1}-K_{e1})(1-\gamma_1+\eta_1(K_{s1}-K_{e1})/2)},$$

and the optimal exit capacity choice $K_{e1}^{'}$ *is determined by the following equation*:

$$\frac{2\eta_1(K_{s1}-K_{e1}^{'})+\gamma_1-1}{r-\mu_s} + \frac{I}{r}\frac{\partial\left[l(x_s^{'}) + \left(x_s^{'}\eta_2(\delta K_{e1}^{'})^2\right)/(2(r-\upsilon\mu_s))\right]}{\partial K_{e1}^{'}} + X_s^{\beta_3}\frac{\partial N(K_{e1}^{'})}{\partial K_{e1}^{'}} = 0.$$

As in the additional investment option, we also can derive the following corollary by comparing the values of $x_s^{'}$ and x_s in the case of private port.

Corollary 7.2. *Public port prefers to adopt the earlier investment in AT compared to the private port in the replacement investment option, as* $x_s^{'}$ *is always lower than* x_s.

Proof. The proofs are presented in the Appendix.

7.2. Numerical experiment with the data of Chinese public ports

As in the numerical experiments shown in Section 6, we further validate the solutions in the optimal upgrade decisions in the case of

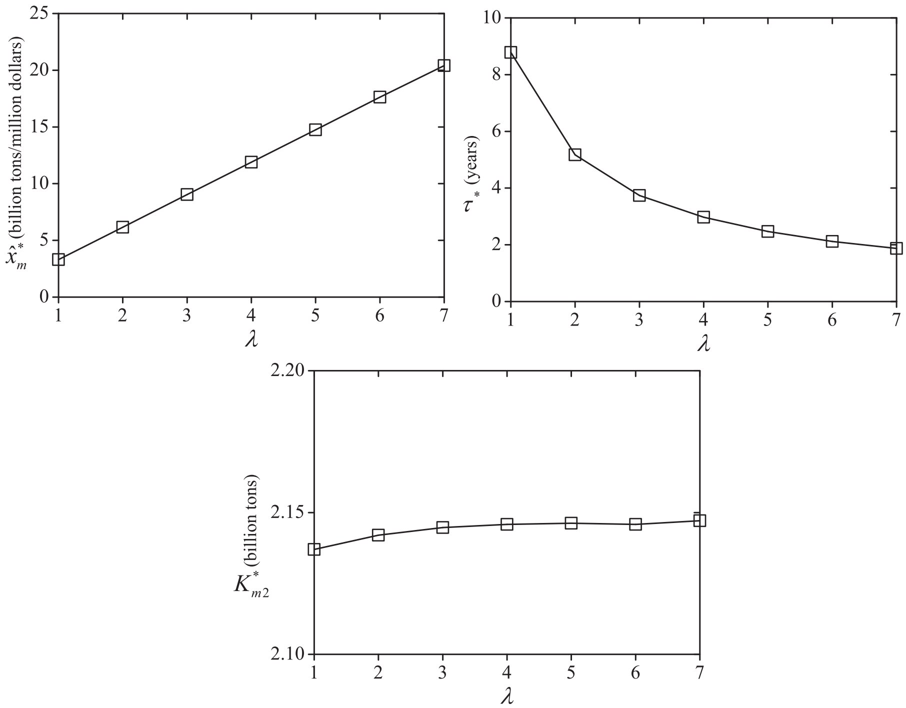

Fig. 12. The results of (a)$\hat{x}_m^*$, (b)τ^*, and (c)K_{m2}^* with increasing parameter λ in the case of public ownership of the port.

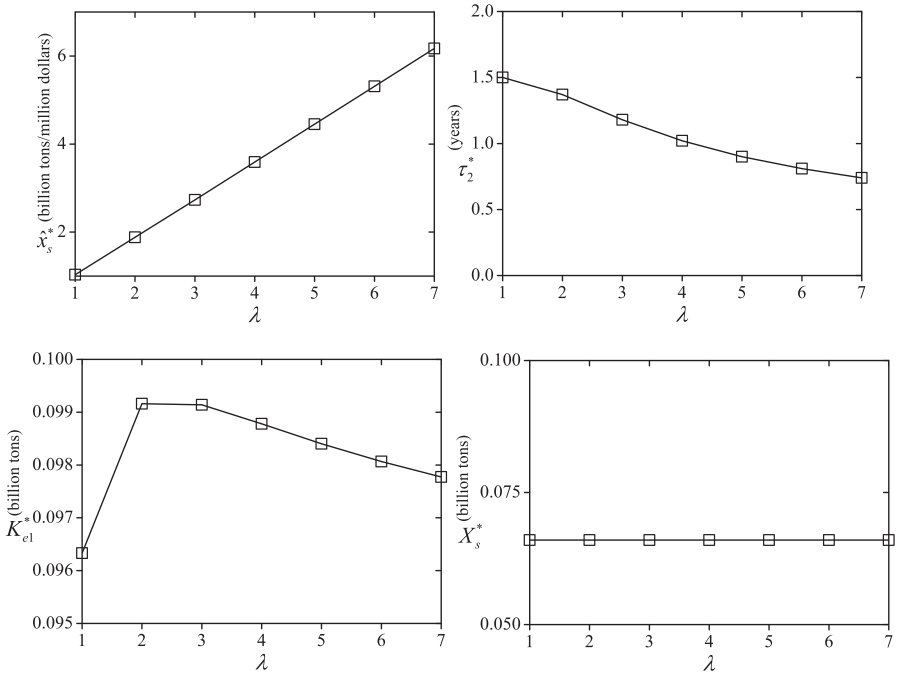

Fig. 13. The results of (a)$\widehat{x}_s^*$, (b)τ_2^*, (c)K_{e1}^*, and (d)X_s^* with increasing λ in the case of public ownership of the port.

the public ownership of the port. Since most Chinese ports are public ports, we chose to conduct a numerical experiment using data from Chinese public ports. We also estimate the parameters w and u in Eq. (3.8) with the public data in the annual reports regarding the investment costs and capacities of existing Chinese automated terminals in the Shanghai Port (Shanghai International Port Group, 2023), the Qingdao Port (Port of Qingdao, 2023), and the Shenzhen Port (Port of Shenzhen, 2023). We then obtain $\ln I_0 = 5.84 + 0.45\ln K$, and thus w and u are equal to 345 million dollars/million tons and 0.45, respectively. Moreover, the other parameters are set as the same as shown in Section 6. We can obtain the following numerical results with the above data and the Propositions illustrated in Section 7.1.

Figs. 12 and 13 illustrate the optimal upgrade decisions of the two replacement options in the case of the public port. From Fig. 12, we find that the optimal upgrade decision in the case of the public port has the same changing trend compared with the case of the private port in the additional replacement option shown in Fig. 2. Both the expected investment threshold $\widehat{x}_m^*$ and optimal investment capacity choice K_{m2}^* are increasing in λ, while the optimal investment timing τ^* is decreasing in λ. Moreover, as λ increases, both τ^* and K_{m2}^* become stable and converge to a constant value. Furthermore, by comparing the results of τ^* and K_{m2}^* in Figs. 2 and 12, we observe that the optimal investment timing for AT in the public port is lower than that of the private port. Still, the capacity investment size for AT in the public port is larger than that of the private port, implying that the decision maker in the public port prefers to adopt the earlier and larger investment in AT compared to the private port.

Fig. 13 presents the optimal upgrade decision in the replacement upgrade decision in the case of the public port, and the results in the Figure indicate that the results of the $\widehat{x}_s^*$, τ_2^*, and X_s^* in the case of public port have the same changing trend compared with the case of the private port. However, the impact of λ on the K_{e1}^* in the public port is quite different from that of the private port. In the case of the public port, as λ increases, K_{e1}^* first increase but then decrease, whereas K_{e1}^* is decreasing in λ in the case of private port. Moreover, by comparing the results of τ_2^* in Figs. 4 and 13, we also observe that the optimal investment timing for AT in the public port is lower than that of the private port, implying that the decision maker in the public port prefers to adopt the earlier investment in AT compared to the private port.

8. Conclusions and policy implications

This paper examines the problem of upgrading CT with AT under technological improvement and demand uncertainties. We use the real options theory to derive the optimal AT investment decision (i.e., including the investment timing and capacity choice) in the additional upgrade option, where the decision-maker does not have to abandon the existing capacity of the CT. We also derive the optimal CT exit decision (i.e., including the exit timing and capacity choice) and the optimal AT investment decision in the replacement upgrade option, where the decision-maker chooses to abandon the partial or full capacity of existing CT and replace the idle capacity of the CT with the AT. We then derive the analytic solutions for the optimal upgrade decisions for each option under the deterministic technological improvement process. We also conduct numerical experiments to verify the optimal upgrade decisions and the corresponding analytical solutions based on the European and North American private ports data. The numerical results indicate that in the additional replacement option, the higher technological improvement and higher demand uncertainties will delay the AT investment but will not change the capacity investment choice for the AT. In the replacement upgrade option, higher technological improvement will postpone the investment for AT and enlarge the exit capacity choice for the CT and the investment capacity choice for the AT. However, the impacts of the demand uncertainty on the upgrade decision are pretty different from those impacts arising from the uncertainty of technological improvement. Although higher demand uncertainty will delay the AT investment, the exit capacity choice for the CT first increases and then decreases with increasing demand uncertainty. We also perform sensitivity analyses regarding the impacts of the terminal operating efficiency, terminal substitution effect, and discount rate on the optimal upgrade options. The results suggest that higher AT efficiency will accelerate the exit of the CT for realizing the AT transition and encourage the decision-maker to make early investments in the AT. Moreover, high substitution between these two types of terminals will not change the exit and investment timings for the CT and AT, respectively; it will enlarge the CT's maintained capacity and reduce the investment in the AT. Furthermore, a high discount rate will decrease the capacity exit size of the CT, as the existing CT can also obtain a high profit in the case of a high discount rate. Finally, we extend our model to account for the public ownership of the studied port and then derive the corresponding optimal upgrade decisions.

Possible policy implications for policymakers and industry are proposed to guide the implementation of CT upgrade decisions. First, although replacing CTs with ATs is an inevitable trend for developing the port industry, our results indicate that the uncertain demand and technological improvement greatly affect the investment timings and sizes for the upgrade decisions. Therefore, the port decision-maker should set up a specialized department to capture the evolution trends of port demand and technological improvement so that the decision-maker can formulate a scientific upgrade plan. Second, the results shown in the numerical experiments indicate that in the replacement upgrade option, the exit capacity choice for the CT first increases and then decreases with increasing demand uncertainty. This trend suggests that facing an extremely uncertain trade market, the decision-maker should maintain a relatively high port capacity to stay competitive when attracting shippers. Therefore, the decision-maker is suggested to propose some supportive polices to promote the development of CT in the extreme uncertain environment. Third, the transition from CTs to ATs requires senior skilled workers to operate the automated equipment and machines in the ATs. Thus, training existing employees in CTs and recruiting new workers are also crucial to achieving a healthy replacement.

Along with offering conclusions and implications, this paper leaves research directions for future study. One possible direction is to use a general process to simulate the technological improvement under uncertainty and compare its results with those of the existing Poisson process, which is the model that this paper employs and derives the relevant results from. Another possible direction is to estimate the future values of important parameters and analyze how they influence the optimal replacement options. Meanwhile, the extension of our work by considering the competing ports in the multiport region should derive different optimal replacement decisions. Furthermore, the port labor unions will influence the port or terminal operating efficiency and further impact the upgrading decisions. Thus, future studies can quantify the relationship between port labor unions and port operating efficiency and incorporate the influence of labor unions on upgrading decisions into the model formulation.

CRediT authorship contribution statement

Liquan Guo: Writing – original draft, Visualization, Methodology, Investigation, Formal analysis, Conceptualization. **Shiyuan Zheng:** Writing – review & editing, Validation, Project administration, Methodology, Investigation. **Changmin Jiang:** Writing – review & editing, Validation, Project administration, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

The mathematical proofs are presented as follows.

Proof of Proposition 4.1: Let $\overline{X}_m = kX_m$ and $\overline{I} = kI(k > 0)$, and let $\overline{\tau}$ be the optimal stopping time by solving the following maximization problem:

$$\sup_{\overline{\tau}}\boldsymbol{E}\left[e^{-r\overline{\tau}}\left(\frac{\pi_s(\overline{X}_m/\upsilon)}{r-\mu_s}+\frac{\pi_m(\overline{X}_m)}{r-\mu_m}-\overline{I}\right)\right]=k\sup_{\tau}\boldsymbol{E}\left[e^{-r\tau}\left(\frac{\pi_s(X_m/\upsilon)}{r-\mu_s}+\frac{\pi_m(X_m)}{r-\mu_m}-I\right)\right]. \tag{A.1}$$

Let the corresponding stopping region be $\overline{S}_A$, then we have $\overline{\tau} = \inf\{t\geqslant 0; (\overline{X}_{mt}, \overline{I}_t) \in \overline{S}_A\}$. It is easy to confirm that the stopping time problem (A.1) has the same solution as that in problem (4.5); thus, the optimal stopping region in problem (A.1) is also S_A and the optimal stopping time is equal, i.e., $\tau^* = \overline{\tau}$. This conclusion implies that the pairs (X_m, I) and (kX_m, kI) hit simultaneously, i.e., when $(X_m,$

$I) \in S_A$, then $(kX_m, kI) \in S_A$. Since k is a positive constant parameter, the boundary of S_A is a ray originating from origin, which divides the stopping and continuation regions. Let the ratio of X_m/I at the boundary of S_A be x_m^*. We next prove that there exists one and only one boundary ray.

Suppose that there exists another ray originating from the origin of the boundary of S_A. Then we have $\overline{x}_m = \frac{\overline{X}_m}{\overline{I}} > x_m^* = \frac{X_m^*}{I^*}$, implying that investing in the AT is not optimal. Thus, in the boundary rays of x_m^* and $\overline{x}_m$, we have $F(X_m^*, I^*) = \frac{\pi_s(X_m^*/\upsilon)}{r-\mu_s} + \frac{\pi_m(X_m^*)}{r-\mu_m} - I^*$ and $F(\overline{X}_m, \overline{I}) > \frac{\pi_s(\overline{X}_m/\upsilon)}{r-\mu_s} + \frac{\pi_m(\overline{X}_m)}{r-\mu_m} - \overline{I}$. That is, $F(x_m^*, 1) = \frac{\pi_s(x_m^*/\upsilon)}{r-\mu_s} + \frac{\pi_m(x_m^*)}{r-\mu_m} - 1$ and $F(\overline{x}_m, 1) > \frac{\pi_s(\overline{x}_m/\upsilon)}{r-\mu_s} + \frac{\pi_m(\overline{x}_m)}{r-\mu_m} - 1$. Multiplying $F(\overline{x}_m, 1) > \frac{\pi_s(\overline{x}_m/\upsilon)}{r-\mu_s} + \frac{\pi_m(\overline{x}_m)}{r-\mu_m} - 1$ by $\frac{x_m^*}{\overline{x}_m}$ yields $F(x_m^*, \frac{x_m^*}{\overline{x}_m}) > \frac{\pi_s(x_m^*/\upsilon)}{r-\mu_s} + \frac{\pi_m(x_m^*)}{r-\mu_m} - \frac{x_m^*}{\overline{x}_m}$, indicating that it is not optimal invest in $\frac{x_m^*}{\overline{x}_m}$ to get the profit $\frac{\pi_s(x_m^*/\upsilon)}{r-\mu_s} + \frac{\pi_m(x_m^*)}{r-\mu_m}$. Since $F(x_m^*, 1) = \frac{\pi_s(x_m^*/\upsilon)}{r-\mu_s} + \frac{\pi_m(x_m^*)}{r-\mu_m} - 1$, then it is optimal to invest in 1 to get profit $\frac{\pi_s(x_m^*/\upsilon)}{r-\mu_s} + \frac{\pi_m(x_m^*)}{r-\mu_m}$. However, this is contradictory, as $x_m^*/\overline{x}_m < 1$. So x_m^* is unique and the stopping region satisfies $S_A = \left\{ (X_m, I) \in \vartheta \middle| \frac{X_m}{I} \geqslant x_m^* \right\}$.

Proof of Proposition 5.1: Let $\overline{X}_s = kX_s$ and $\overline{I} = kI(k > 0)$, and let $\overline{\tau}_2$ be the optimal stopping time by solving the following maximization problem:

$$\begin{aligned}
&\sup_{\overline{\tau}_2} \boldsymbol{E}\left[e^{-r\overline{\tau}_2}\left(\frac{\pi_s^A(\overline{X}_s, K_{e1})}{r-\mu_s} + \frac{\upsilon\overline{X}_s\left(\frac{\pi_s^A(\overline{X}_s, K_{e1})}{\overline{X}_s} + \frac{\pi_{m2}(\upsilon\overline{X}_s)}{\upsilon\overline{X}_s} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - \overline{I} \right) \right] \\
&= k\sup_{\tau_2} \boldsymbol{E}\left[e^{-r\tau_2}\left(\frac{\pi_s^A(X_s, K_{e1})}{r-\mu_s} + \frac{\upsilon X_s\left(\frac{\pi_s^A(X_s, K_{e1})}{X_s} + \frac{\pi_{m2}(\upsilon X_s)}{\upsilon X_s} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - I \right) \right].
\end{aligned} \tag{A.2}$$

Let the corresponding stopping region be $\overline{S}_R$, then we have $\overline{\tau}_2 = \inf\{t \geqslant 0; (\overline{X}_{st}, \overline{I}_t) \in \overline{S}_R\}$. It is easy to confirm that the stopping time problem (A.2) has the same solution as that in problem (5.3), thus, the optimal stopping region in problem (A.2) is also S_R and the optimal stopping time is equal, i.e., $\tau_2^* = \overline{\tau}_2$. This conclusion implies that the pairs (X_s, I) and (kX_s, kI) hit simultaneously, i.e., when $(X_s, I) \in S_R$, then $(kX_s, kI) \in S_R$. Since k is a positive constant parameter, then the boundary of S_R is also a ray originating from origin. Let the ratio of X_s/I at the boundary of S_R be x_s^*. We next prove that the boundary ray x_s^* is unique.

Suppose that there exists another ray originating from the origin of the boundary of S_R. Then we have $\overline{x}_s = \frac{\overline{X}_s}{\overline{I}} > x_s^* = \frac{X_s^*}{I^*}$, implying that investing in the AT is not optimal. Thus, in the boundary rays of x_s^* and $\overline{x}_s$, we have

$$L(X_s^*, I^*) = \frac{\pi_s^A(X_s^*, K_{e1})}{r-\mu_s} + \frac{\upsilon X_s^*\left(\frac{\pi_s^A(X_s^*, K_{e1})}{X_s^*} + \frac{\pi_{m2}(\upsilon X_s^*)}{\upsilon X_s^*} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - I^*,$$

and

$$L(\overline{X}_s, \overline{I}) > \frac{\pi_s^A(\overline{X}_s, K_{e1})}{r-\mu_s} + \frac{\upsilon\overline{X}_s\left(\frac{\pi_s^A(\overline{X}_s, K_{e1})}{\overline{X}_s} + \frac{\pi_{m2}(\upsilon\overline{X}_s)}{\upsilon\overline{X}_s} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - \overline{I}.$$

That is,

$$L(x_s^*, 1) = \frac{\pi_s^A(x_s^*, K_{e1})}{r-\mu_s} + \frac{\upsilon x_s^*\left(\frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - 1,$$

and

$$L(\overline{x}_s, 1) > \frac{\pi_s^A(\overline{x}_s, K_{e1})}{r-\mu_s} + \frac{\upsilon\overline{x}_s\left(\frac{\pi_s^A(\overline{x}_s, K_{e1})}{\overline{x}_s} + \frac{\pi_{m2}(\upsilon\overline{x}_s)}{\upsilon\overline{x}_s} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - 1.$$

Multiplying $L(\overline{x}_s, 1) > \frac{\pi_s^A(\overline{x}_s, K_{e1})}{r-\mu_s} + \frac{\upsilon\overline{x}_s\left(\frac{\pi_s^A(\overline{x}_s, K_{e1})}{\overline{x}_s} + \frac{\pi_{m2}(\upsilon\overline{x}_s)}{\upsilon\overline{x}_s} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - 1$ by $\frac{x_s^*}{\overline{x}_s}$ yields

$$L\left(x_s^*, \frac{x_s^*}{\overline{x}_s}\right) > \frac{\pi_s^A(x_s^*, K_{e1})}{r-\mu_s} + \frac{\upsilon x_s^*\left(\frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m} - \frac{x_s^*}{\overline{x}_s},$$

indicating that it is not optimal invest in $\frac{x_s^*}{\overline{x}_s}$ to get a profit $\frac{\pi_s^A(x_s^*, K_{e1})}{r-\mu_s} + \frac{\upsilon x_s^*\left(\frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1}-K_{e1})\delta K_{e1} \right)}{r-\mu_m}$.

Since $L(x_s^*,\ 1) = \frac{\pi_s^A(x_s^*, K_{e1})}{r-\mu_s} + \frac{\upsilon x_s^* \left(\frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1} - K_{e1})\delta K_{e1} \right)}{r-\mu_m} - 1$, then it is optimal to invest in 1 to get profit $\frac{\pi_s^A(x_s^*, K_{e1})}{r-\mu_s} + \frac{\upsilon x_s^* \left(\frac{\pi_s^A(x_s^*, K_{e1})}{x_s^*} + \frac{\pi_{m2}(\upsilon x_s^*)}{\upsilon x_s^*} - \varepsilon(K_{s1} - K_{e1})\delta K_{e1} \right)}{r-\mu_m}$. However, this is contradictory, as $\frac{x_s^*}{\bar{\bar{X}}_s} < 1$. So x_s^* is unique and the stopping region satisfies $S_R = \left\{ (X_s, I) \in \psi \middle| \frac{X_s}{I} \geqslant x_s^* \right\}$.

Proof of Proposition 7.1: In the case of public port, the value function in the additional investment option is

$$\sup_{\tau} \boldsymbol{E} \left[e^{-r\tau} \left(\frac{\pi_s(X_s) + \frac{1}{2} X_m \eta_1 K_{s1}^2}{r - \mu_s} + \frac{\pi_m(X_m) + \frac{1}{2} X_m \eta_2 K_{m2}^2}{r - \mu_m} - I \right) \right] = F(X_m, I) + \frac{X_m \eta_1 K_{s1}^2}{2(\upsilon r - \mu_m)} + \frac{X_m \eta_2 K_{m2}^2}{2(r - \mu_m)}.$$

Applying the Ito's Lemma and the value matching and smooth pasting conditions, we can derive the following specific expression of x_m':

$$x_m' = \frac{r + \xi}{\frac{r - \mu_m}{\upsilon r - \mu_m} \left[(1 - \gamma_1 - \eta_1 K_{s1}) K_{s1} + \frac{X_m \eta_1 K_{s1}^2}{2} \right] + \left[(1 - \gamma_1 - \eta_1 K_{m1}) K_{m1} + (1 - \gamma_2 - \eta_2 K_{m2}) K_{m2} - 2\varepsilon K_{m1} K_{m2} + \frac{X_m \eta_2 K_{m2}^2}{2} \right]}.$$

After getting the investment threshold x_m', we can derive the optimal capacity choice by solving the first-order condition $\partial\left(x_m' \cdot I\right)/\partial K_{m2} = 0$, and the following equation determines the optimal capacity choice K_{m2}':

$$I \frac{\partial g\left(x_m'\right)}{\partial K_{m2}} + +g\left(x_m'\right) \cdot \frac{\partial I}{\partial K_{m2}} = 0.$$

Proof of Proposition 7.2: In the case of the public port by maximizing the social welfare in the replacement investment option, the value function of AT investment is expressed as follows:

$$\sup_{\tau_2} \boldsymbol{E} \left[e^{-r\tau_2} \left(\frac{\pi_s^A(X_s, K_{e1})}{r - \mu_s} + \frac{\upsilon X_s \left(\frac{\pi_s^A(X_s, K_{e1})}{X_s} + \frac{\pi_{m2}(\upsilon X_s)}{\upsilon X_s} - \varepsilon(K_{s1} - K_{e1})\delta K_{e1} \right)}{r - \mu_m} + \frac{\upsilon X_s \eta_1 (K_{s1} - K_{e1})^2 + \upsilon X_s \eta_2 (\delta K_{e1})^2}{2(r - \upsilon \mu_s)} - I \right) \right]$$

$$= L(X_s, I) + \frac{\upsilon X_s \eta_1 (K_{s1} - K_{e1})^2 + \upsilon X_s \eta_2 (\delta K_{e1})^2}{2(r - \upsilon \mu_s)}$$

With the above value function of AT investment, the value function for CT exit decision can be expressed as follows:

$$\sup_{\tau_1, K_{e1}} \boldsymbol{E} \left[\int_0^{\tau_1} e^{-rt} \pi_s^B(X_s) dt + \int_{\tau_1}^{+\infty} e^{-rt} \pi_s^A(X_s, K_{e1}) dt + Il\left(x_s^*\right) + \frac{\upsilon \mathrm{X}_s \eta_1 (\mathrm{K}_{s1} - \mathrm{K}_{e1})^2 + \upsilon \mathrm{X}_s \eta_2 (\delta \mathrm{K}_{e1})^2}{2(\mathrm{r} - \upsilon \mu_s)} \right]. \tag{A.3}$$

Applying the Ito's Lemma and the value matching and smooth pasting conditions in problem (A.3), we can derive the following specific expressions of x_s' and X_s':

$$x_s' = \frac{r + \xi}{(r - \mu_s) \left[\frac{M_1(K_{e1})}{(r - \mu_s)} + \frac{\upsilon \left(M_1(K_{e1}) + M_2(K_{e1}) - 2\varepsilon(K_{s1} - K_{e1})\delta K_{e1} + \frac{1}{2}\eta_1 (K_{s1} - K_{e1})^2 + \frac{1}{2}\eta_2 (\delta K_{e1})^2 \right)}{r - \upsilon \mu_s} \right]},$$

$$X_s' = \left(\frac{\beta_3}{\beta_3 - 1} \right) \frac{I\left[l\left(x_s'\right) + \left(x_s' \eta_1 (K_{s1} - K_{e1})^2 + x_s' \eta_2 (\delta K_{e1})^2 \right) / (2(r - \upsilon \mu_s)) \right] (r - \mu_s)}{r K_{s1} (1 - \gamma_1 + \eta_1 K_{s1}/2) - r(K_{s1} - K_{e1})(1 - \gamma_1 + \eta_1 (K_{s1} - K_{e1})/2)}.$$

Moreover, according to the first-order condition of problem (A.3) where the capacity choice K_{e1} is the decision variable, we can derive the following equation to determine the optimal capacity choice:

$$\frac{2\eta_1 \left(K_{s1} - K_{e1}'\right) + \gamma_1 - 1}{r - \mu_s} + \frac{I}{r} \frac{\partial\left[l\left(x_s'\right) + \left(x_s' \eta_2 \left(\delta K_{e1}'\right)^2 \right) / (2(r - \upsilon \mu_s)) \right]}{\partial K_{e1}'} + X_s^{\beta_3} \frac{\partial N\left(K_{e1}'\right)}{\partial K_{e1}'} = 0.$$

Data availability

Data will be made available on request.

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