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Article

Construction Waste Transportation Planning under Uncertainty: Mathematical Models and Numerical Experiments

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Abstract: Annually, over 10 billion tons of construction and demolition waste is transported globally from sites to reception facilities. Optimal and effective planning of waste transportation holds the potential to mitigate cost and carbon emissions, and alleviate road congestion. A major challenge for developing an effective transportation plan is the uncertainty of the precise volume of waste at each site during the planning stage. However, the existing studies have assumed known demand in planning models but the assumption does not reflect real-world volatility. Taking advantage of the problem structure, this study adopts the stochastic programming methodology to approach the construction waste planning problem. An integer programming model is developed that adeptly addresses the uncertainty of the amount of waste in an elegant manner. The proposed stochastic programming model can efficiently handle practical scale problems. Our numerical experiments amass a comprehensive dataset comprising nearly 4300 records of the actual amount of construction waste generated in Hong Kong. The results demonstrate that incorporating demand uncertainty can reduce the transportation cost by 1% correlating with an increase in profit of 14% compared to those that do not consider the demand uncertainty.

Keywords: construction waste transportation; demand uncertainty; stochastic programming

MSC: 90-10

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1. Introduction

Construction waste, also known as construction and demolition waste, consists of solid waste, such as rock, rubble, boulder, earth, and bamboo, arising from construction, renovation, and demolition projects [1]. Construction waste is the outcome of overordering leading to leftover materials, change in design, damages to the element during transportation, storage, laying, and cutting [2]. The escalating irreversible urbanization in many countries, resulting in the blooming of construction activities, has led to the generation of unprecedented amount of construction waste, over 10 billion tons of construction waste around the globe annually [3]. Furthermore, the proportion of construction waste in the total amount of solid waste landfilled is also significant [4]; for example, in the United States, the proportion is 25–40% [5], meanwhile, in the United Kingdom, Australia, and Japan, the proportions are 50%, 44%, and 36%, respectively [6], whereas in the European Union [7], the average proportion is more than a third of the total volume of waste generated. Generally, construction waste is generated at construction sites, then transported to off-site and centralized reception facilities for subsequent processes such as sorting, recycling, and landfilling.

Construction waste transportation is costly due to the sheer amount of waste involved [8]. Assuming 10 billion tons of construction waste generated worldwide annually is transported by vehicles with a capacity of 10 tons each consuming 26.4 L of diesel per

100 km, and assuming the average transportation distance from construction sites to waste reception facilities is 30 km, the annual fuel consumption is 7.92 billion liters (i.e., $26.4 \div 100 \times 30 = 7.92$).This quantity equates roughly to the same amount of fuel consumption in Denmark [9], resulting in a fuel cost that is approximately as high as USD7.92 billion, assuming the price of fuel is at 1 USD/L. Therefore, an optimal waste transportation system is vital to enhance the sustainability of the construction industry in terms of cost and carbon emissions [8]. In addition, issues of sustainability have emerged as a growing significant global concern [10,11].

In the current practice, the transportation company decides the dispatch of vehicles to collect the waste from a predetermined number of construction sites and transport it to a waste reception facility with the aim of minimizing the transportation cost. One of the primary challenges of developing the transportation plan is the uncertainty of the amount of waste generated at each construction site during the initial planning stage (e.g., the site manager can only provide an estimate of the weight or volume of the waste to the waste transportation company), resulting in the complexities in determining the capacities and trips of the planned vehicles in order to minimize the expected sum of cost for the planned and potential additional vehicles. However, most existing studies on waste transportation planning hold a strong assumption that the transportation company knows the actual amount of waste generated in each construction site. This paper addresses the above challenge by examining the transportation planning of construction waste with waste amount uncertainty. Specifically, a stochastic program is designed to capture the uncertainty in the amounts of construction waste at construction sites. The optimization objective from the perspective of a waste transportation company is to minimize the total expected cost for waste transportation, considering both the planned vehicles and the potential need for supplementary vehicles.

The novelties of this research are threefold: (1) From a modeling perspective, a novel stochastic programming model is devised for the transportation planning of construction waste. The proposed model considers the uncertainty in the amount of waste generated in each construction site, thereby eliminating the assumption identified in the existing literature (i.e., planners know the actual transportation amount when making their decisions). This novelty makes the proposed model more aligned with the stochastic decision-making situations faced by planners in real-life scenarios; (2) from an algorithmic perspective, a number of appealing model properties are analyzed for the development of an effective method capable of providing optimal solutions for waste transportation problems of practical scales in one second. While many established heuristic algorithms can yield high-quality feasible solutions, this paper contributes to the literature by illustrating how to derive optimal solutions through the exploration of problem structure; (3) from a practical perspective, real data from Hong Kong are collected for experiment validation. The results of the numerical experiments show that considering uncertainty can significantly reduce transportation costs. With that, our study contributes to the literature on optimization in construction management.

The subsequent sections of this paper are organized as follows. Section 2 provides an overview of the related literature. Section 3 describes the waste transportation planning problem using real examples. Section 4 proposes a stochastic programming model and outlines the proposed solution method. Section 5 reports the data for parameter setting and demonstrates the results of extensive numerical experiments. Finally, Section 6 summarizes the conclusions of this study.

2. Literature Review

Numerous papers have focused on the identification of potential locations for construction waste reception facilities, primarily aimed at minimizing the total construction cost of the facilities and the associated transportation cost. Xu et al. [12] proposed a mixed-integer linear programming model to aid in deciding the locations for waste collection and classification, remanufacturing, and landfilling facilities, in which the construction waste is first transported from construction sites to a waste collection and classification facility, then directed either to a remanufacturing facility or a landfilling facility. Lin et al. [13] developed a model to study the selection of locations for construction waste treatment facilities and landfilling facilities, where the construction waste is transported from construction sites to a treatment facility, or a landfilling facility, or disposed of illegally. Meanwhile, Rahimi and Ghezavati [14] formulated a bi-objective model aimed at minimizing both the total cost and environmental burden of a construction waste transportation network, taking into account the selection of facility locations as well as the risk preferences of decision makers. Pan et al. [15], on the other hand, formulated a multi-objective model to determine the number, locations, and sizes of waste treatment plants incorporating multiple stakeholders' objectives. Specifically, the model addresses the government's objective of maximizing the proportion of waste recycled, maximizing profit for the waste treatment facility operators and the construction contractors' objective of minimizing costs. Furthermore, in the model proposed by Pan et al. [15], the construction waste is transported from construction sites to a treatment facility, after which the recyclable part is directed to the sales market while the non-recyclables are sent to a landfilling facility (the locations of landfilling facilities are fixed and are not decision variables). Yang and Chen [16] investigated a similar predicament but comprising the uncertainty of the processing capacities of the facilities, while Ahmed and Zhang [17] proposed a mixed-integer linear programming model to determine the locations of waste collection, recycling, and landfilling facilities, in which the construction waste is first transported from construction sites to a waste collection facility, then to a recycling facility and eventually to a landfilling facility. However, the previous studies on location-related issues took the assumption of a constant cost of transportation between the two locations (e.g., facilities and construction sites). In contrast, the focus of this study is leaning towards the cost of transportation considering the dispatching of vehicles and uncertain amounts of waste at sites.

Apart from that, research on routings of construction waste has been gaining momentum among academicians. For instance, Qiu et al. [18] noticed that construction waste transportation vehicles pose a severe threat to safe transportation and, hence, developed a method for finding the optimal route from origin to destination that considers the risk attitude of decision-makers and minimizes safety cost. Ahmed and Zhang [19] further elaborated on the transportation cost components associated with transporting construction waste from construction sites to collection facilities, recycling facilities, and landfilling facilities. Elshaboury and Marzouk [20] examined the transportation fleet requirement for construction waste using a genetic algorithm. Moreover, we concur that our problem is relevant to the well-known vehicle routing problem [21–26], bin packing problem [27–29], and their variants [30–33]. A key distinction between this study and the aforementioned ones is that we account for the uncertainty in the amounts of construction waste at sites, which is a practical factor that decision-makers must take into consideration when they design the optimal waste transportation plans.

Studies on transportation planning with uncertain demand have been conducted for decades. Bertsimas [34] examined several special cases of uncertain demand distributions, e.g., all customers have the same discrete demand distribution, and derived lower and upper bounds on a route. Additionally, Dror [35] derived theoretical properties of the problem, such as no arc will be traversed twice and a customer is visited at most n-1 times where n is the number of customers in total, without presenting a solution method. Mendoza et al. [36] applied a penalty to a vehicle route once the total demand of the customers on the route exceeds the capacity of the vehicle, without considering how to handle the excess demand. Moghaddam et al. [37] used an adjusted demand in their design of heuristics for vehicle routing, while Wang et al. [38] introduced different scenarios to address uncertain demands. Other approaches, such as robust optimization [39] and fuzzy set approaches [40], have also been utilized. The model proposed in this paper is capable of dealing with any bounded distribution of random demand and offers an exact solution that minimizes the total expected cost.

3. Problem Description

Consider a construction waste transportation company that operates a fleet of different types, denoted by set V, of vehicles. We assume that the number of vehicles of each type is infinite. The capacity of a vehicle of type $v \in V$ is W_v (tons) and the cost of using a vehicle of type v is c_v (\$/min). For simplicity, we let $W_1 \leq W_2 \leq \cdots \leq W_{|V|}$ and $c_1 \leq c_2 \leq \cdots \leq c_{|V|}$. We define $v(w)$ as the type of smallest vehicle whose capacity is no less than w, that is

$$v(w) = \min\{v' \in V \big| W_{v'} \geq w\}. \tag{1}$$

The vehicles are located at a depot that is next to a construction waste reception facility.

The transportation company transports construction waste from construction sites to a construction waste reception facility. Large construction sites produce significant amounts of waste and usually full truckloads of waste are transported. The challenge is planning the transport of construction wastes from small construction sites that produce less than a full load of the largest truck, which is our focus. When the construction work at a small construction site is finished, the site manager will call the transportation company to transport the waste to a waste reception facility. At the beginning of a day, the transportation company plans the transportation of construction waste from a set of small construction sites, denoted by N. Define the depot and the construction waste reception facility as location 0 since they are at the same location and define set $N_0 = N \cup \{0\}$ to include both the construction sites and the depot/waste reception facility. The waste at the construction sites must be transported to the waste reception facility. The travel time from location $n_1 \in N_0$ to location $n_2 \in N_0$ is $t_{n_1 n_2}$ (min). The actual amount of waste to transport from construction site $n \in N$ is unknown at the time of transportation planning and, hence, is modelled as a continuous random variable, denoted by $\widetilde{w}_n$. The exact amount is known after the waste is loaded onto a truck as the truck has pressure gauge that can measure the weight. $\widetilde{w}_n$ has a lower bound w_n^L, an upper bound w_n^U, and a cumulative distribution function $F_n(\cdot)$. That is, $\Pr\left(w_n^L \leq \widetilde{w}_n \leq w_n^U\right) = \int_{w_n^L}^{w_n^U} dF_n(w_n) = 1$. We assume that $w_n^U \leq W_{|V|}$ for all $n \in N$, that is, the largest vehicle is capable of transporting all the waste from any single site with probability 1. At the planning stage, the transportation company knows w_n^L, w_n^U, and $F_n(\cdot)$ based on the estimate of the construction site manager and historical records, as shown in the example below.

Example 1. *The construction site manager makes a phone call to the transportation company and informs them that "I estimate we have 5 tons of construction waste to be transported tomorrow." Suppose further that the transportation company has three historical records of similar sites and similar amounts of waste, in which the construction site managers estimated 4 tons, 6 tons, and 7 tons, whereas their actual weights are 4.3 tons, 5.3 tons, and 4.9 tons, respectively (107.5%, 88.3%, and 70.0% of the estimates, respectively). Then, the transportation company may estimate that, for example, the lower bound of the amount of the waste is 3.5 tons (70.0% of the estimate by the construction site manager), the upper bound is 5.375 tons (107.5% of the estimate by the construction site manager), and the amount follows a uniform distribution (we assume uniform distribution simply because there are few data here; of course, other distributions can be used if a large historical dataset is available, depending on the pattern of the data).*

At the planning stage, the transportation company needs to decide, for each vehicle, the construction sites to be visited and the sequence of the visits. Since the amount of waste at a construction site is random, it is possible that the planned vehicle does not have sufficient capacity to carry all the waste from the sites assigned to it. Extra trucks will be dispatched to the sites whose waste are not completely collected yet, and the types and routes of the extra trucks depend on the upper bounds of the amounts of waste at the sites. Moreover, if the planned vehicle does not have sufficient capacity to carry all the waste from a site, the amount of remaining waste at the site is still unknown. Example 2 below illustrates this point.

Example 2. *Suppose that at the planning time, the transportation company decides that a truck of type v visits construction sites 5, 3, and 8 sequentially. Four cases may occur, as shown in Figure 1, and we analyze them one by one.*

Case (i): the total actual amount of waste at the three sites is less than W_v (note, we assume random variable $\widetilde{w}_n$ is continuous and, hence, it makes no difference whether the total actual amount of waste at sites 5, 3, and 8 is less than W_v or the total actual amount is not greater than W_v). Then the truck travels from the depot to site 5, site 3, site 8, and the construction waste reception facility. The resulting cost is $c_v(t_{05} + t_{53} + t_{38} + t_{80})$.

Case (ii): the total actual amount of waste at site 5 and site 3 is less than W_v, but the total amount at the three sites is over W_v. Then the truck travels from the depot to site 5 (loads all the waste at site 5), site 3 (loads all the waste at site 3), site 8 (loads part of the waste at site 8), and the construction waste reception facility. The resulting cost for this planned truck is $c_v(t_{05} + t_{53} + t_{38} + t_{80})$. However, there is still some waste remaining at site 8 and the transportation company has to dispatch an extra truck to collect it. Denoted by $\hat{w}_5$ and $\hat{w}_3$ are the actual amounts of waste at site 5 and site 3, respectively. The amount of waste collected by the truck from site 8 is $W_v - \hat{w}_5 - \hat{w}_3$. Therefore, the maximum possible amount of waste remaining at site 8 is $w_8^U - (W_v - \hat{w}_5 - \hat{w}_3)$, the type of the extra truck that should be dispatched is $v\left(w_8^U - (W_v - \hat{w}_5 - \hat{w}_3)\right)$, and the resulting cost is $c_{v(w_8^U - (W_v - \hat{w}_5 - \hat{w}_3))}(t_{08} + t_{80})$.

Case (iii): the actual amount of waste at site 5 is less than W_v, but the total amount at site 5 and site 3 is already over W_v. Then the truck travels from the depot to site 5 (loads all the waste at site 5), site 3 (loads part of the waste at site 3), and the construction waste reception facility. The resulting cost for this planned truck is $c_v(t_{05} + t_{53} + t_{30})$. The transportation company then has to dispatch extra trucks to collect the remaining waste at site 3 and the waste at site 8. There are two options. Option 1: two extra trucks are dispatched, one to site 3 and the other to site 8. The type of truck that should be dispatched to site 3 is $v\left(w_3^U - (W_v - \hat{w}_5)\right)$ and the resulting cost is $c_{v(w_3^U - (W_v - \hat{w}_5))}(t_{03} + t_{30})$. The type of truck that should be dispatched to site 8 is $v\left(w_8^U\right)$ and the resulting cost is $c_{v(w_8^U)}(t_{08} + t_{80})$. Option 2: one extra truck is dispatched to collect the waste from both site 3 and site 8. The type of truck that should be dispatched is $v\left(w_3^U - (W_v - \hat{w}_5) + w_8^U\right)$ and the resulting cost is $c_{v(w_3^U - (W_v - \hat{w}_5) + w_8^U)} \min\{t_{03} + t_{38} + t_{80}, t_{08} + t_{83} + t_{30}\}$, where the "min" operator exists because it is possible to visit site 3 first or to visit site 8 first. The lower-cost option will be selected for implementation.

Case (iv): the actual amount of waste at site 5 is already over W_v. Then the truck travels from the depot to site 5 (loads part of the waste at site 5), and the construction waste reception facility. The resulting cost for this planned truck is $c_v(t_{05} + t_{50})$. The transportation company then has to dispatch extra trucks to collect the remaining waste at site 5 and the waste at site 3 and site 8. There are five options. Option 1: The transportation company dispatches three extra trucks, one to site 5, one to site 3, and another to site 8. The resulting cost is $c_{v(w_5^U - W_v)}(t_{05} + t_{50}) + c_{v(w_3^U)}(t_{03} + t_{30}) + c_{v(w_8^U)}(t_{08} + t_{80})$. Option 2: The transportation company dispatches two extra trucks, one to site 5, and the other to site 3 and site 8. The resulting cost is $c_{v(w_5^U - W_v)}(t_{05} + t_{50}) + c_{v(w_3^U + w_8^U)} \min\{t_{03} + t_{38} + t_{80}, t_{08} + t_{83} + t_{30}\}$. Option 3: The transportation company dispatches two extra trucks, one to site 3, and the other to site 5 and site 8. The resulting cost is $c_{v(w_3^U)}(t_{03} + t_{30}) + c_{v(w_5^U - W_v + w_8^U)} \min\{t_{05} + t_{58} + t_{80}, t_{08} + t_{85} + t_{50}\}$. Option 4: The transportation company dispatches two extra trucks, one to site 8, and the other to site 5 and site 3. The resulting cost is $c_{v(w_8^U)}(t_{08} + t_{80}) + c_{v(w_5^U - W_v + w_3^U)} \min\{t_{05} + t_{53} + t_{30}, t_{03} + t_{35} + t_{50}\}$. Option 5: The transportation company dispatches one extra truck to site 5, site 3, and site 8. The resulting cost is $c_{v(w_5^U - W_v + w_3^U + w_8^U)} \min\{t_{05} + t_{53} + t_{38} + t_{80}, t_{05} + t_{58} + t_{83} + t_{30}, t_{03} + t_{35} + t_{58} + t_{80}, t_{03} + t_{38} + t_{85} + t_{50}, t_{08} + t_{85} + t_{53} + t_{30}, t_{08} + t_{83} + t_{35} + t_{50}\}$. The lowest-cost option will be selected for implementation.

The objective of the transportation company is to design a transportation plan that incurs the lowest expected total cost, including the cost of the planned vehicles and the cost of the extra vehicles. The definition of transportation plan is as follows.

Definition 1. *A transportation plan is decisions of (i) the set of trucks to use (excluding the extra trucks), (ii) the set of sites assigned to each truck, and (iii) the sequence of the sites serviced by each used truck (due to uncertainty, not all sites in the plan can be serviced, extra trucks have to be dispatched).*

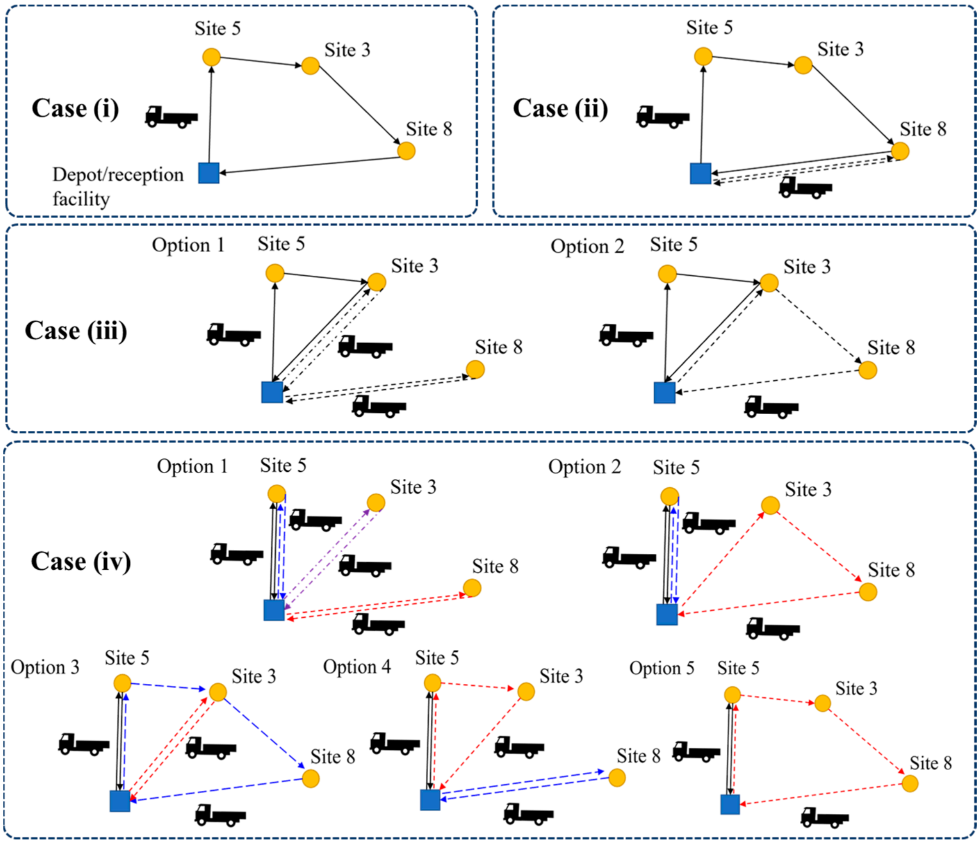

Figure 1. Cases of the route of the planned truck and the routes of extra trucks (note that in Option 2 of Case (iii), the extra truck may visit site 8 before site 3; this also applies to options 2, 3, 4, and 5 of Case (iv), which are plotted).

4. Stochastic Programming Model

To facilitate model formulation, we define a "trip" of construction sites below.

Definition 2. *A trip is a set of sites serviced by the same truck in the plan in a given sequence.*

For example, site 5 $\rightarrow$ site 3 $\rightarrow$ site 8 is a different trip from site 5 $\rightarrow$ site 3 and is a different trip from site 3 $\rightarrow$ site 8 $\rightarrow$ site 5. We denote by B the set of all possible trips, π_b the number of sites in trip $b \in B$, n_{bi} the ID of the site that is the ith site on the trip, $i = 1, \ldots, \pi_b$, and ρ_{bn} a binary indicator that is 1 if site $n \in N$ is in trip b and 0 otherwise. For instance, in the trip of site 5 $\rightarrow$ site 3 $\rightarrow$ site 8, $\pi_b = 3$, $n_{b1} = 5$, $n_{b2} = 3$, $n_{b3} = 8$, $\rho_{b1} = 0$, $\rho_{b2} = 0$, $\rho_{b3} = 1$, $\rho_{b4} = 0$.

We define trip because it has several nice properties.

Property 1. *In reality, a truck can only serve a few (one, two or three) sites on a trip. Therefore, we can enumerate all possible trips. For instance, if there are 30 sites and a trip has at most three sites, then the number of all possible trips is* $P^1_{30} + P^2_{30} + P^3_{30} = 25,260$, *where P means permutation.*

Property 2. *The expected total cost of serving all the sites on a trip (i.e., the sum of the expected cost of using the truck in the plan and the extra trucks) is independent of the other trips. In other words, the expected total cost of serving all the sites on a trip is only dependent on the type of truck planned for the trip and the sequence of the sites in the trip.*

Based on the above two properties, we elaborate on the computation of the expected cost of a trip with a planned vehicle of type v, that is, the expected total cost of serving all the sites on the trip when a truck of type v is planned to serve the trip. For better readability, we describe the computation of the expected costs of trips with one, two, and three construction sites separately.

Case (i): trip b has only one construction site n_{b1} and a truck of type v is planned to serve the trip. The expected cost of the trip, denoted by C_{bv}, is

$$C_{bv} = \Pr\left(\widetilde{w}_{n_{b1}} < W_v\right) c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + \Pr\left(\widetilde{w}_{n_{b1}} > W_v\right)\left[c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right)\right],\ b \in B,\ \pi_b = 1,\ v \in V. \tag{2}$$

In Equation (2), if the actual weight of the waste at construction site n_{b1} is less than the capacity of the truck in the plan, then the cost will be $c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right)$. Otherwise, the cost $c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right)$ will still be incurred for the planned truck, and the cost $c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right)$ will be incurred for the extra truck.

Case (ii): trip b has two construction sites and a truck of type v is planned to serve the trip. Define $E[\cdot]$ as the expectation operator. The expected cost of the trip is

$$\begin{aligned}
&\Pr\left(\widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} \le W_v\right) c_v\left(t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}\right) + \Pr\left(\widetilde{w}_{n_{b1}} < W_v,\ \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} > W_v\right)\Big[c_v\left(t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}\right) + \\
&E\left[c_{v(w^U_{n_{b2}} - (W_v - \widetilde{w}_{n_{b1}}))}\middle|\widetilde{w}_{n_{b1}} < W_v,\ \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} > W_v\right]\left(t_{0n_{b2}} + t_{n_{b2}0}\right)\Big] + \Pr\left(\widetilde{w}_{n_{b1}} > W_v\right)\Big[c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + \\
&\min\Big\{c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + c_{v(w^U_{n_{b2}})}\left(t_{0n_{b2}} + t_{n_{b2}0}\right), c_{v(w^U_{n_{b1}} - W_v + w^U_{n_{b2}})}\min\left\{t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}, t_{0n_{b2}} + \right. \\
&\left. t_{n_{b2}n_{b1}} + t_{n_{b1}0}\right\}\Big\}\Big],\ b \in B,\ \pi_b = 2,\ v \in V.
\end{aligned} \tag{3}$$

Equation (3) has three terms. The first one calculates the cost when the total weight of waste at the two sites is less than the capacity of the planned truck; the second one calculates the cost when the weight of the waste at the first site is less than the capacity of the planned truck, but the total weight of waste at the two sites is more than the capacity of the truck in the plan (an extra vehicle is dispatched, its type depends on the random amount of waste at the first site, and the resulting expected cost is $E\left[c_{v(w^U_{n_{b2}} - (W_v - \widetilde{w}_{n_{b1}}))}\middle|\widetilde{w}_{n_{b1}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} > W_v\right]\left(t_{0n_{b2}} + t_{n_{b2}0}\right)$; the third one calculates the cost when the weight of the waste at the first site is more than the capacity of the planned truck (then we may dispatch two extra vehicles, resulting in a cost of $c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + c_{v(w^U_{n_{b2}})}\left(t_{0n_{b2}} + t_{n_{b2}0}\right)$, or one vehicle, resulting in a cost of $c_{v(w^U_{n_{b1}} - W_v + w^U_{n_{b2}})}\min\left\{t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}, t_{0n_{b2}} + t_{n_{b2}n_{b1}} + t_{n_{b1}0}\right\}$).

Case (iii): trip b has three construction sites and a truck of type v is planned to serve the trip. The expected cost of the trip is

$$\begin{aligned}
C_{bv} = {}&\Pr\left(\widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} + \widetilde{w}_{n_{b3}} < W_v\right) C_0 + \Pr\left(\widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} + \widetilde{w}_{n_{b3}} > W_v\right) C_1 + \Pr\left(\widetilde{w}_{n_{b1}} < W_v, \widetilde{w}_{n_{b1}} + \right. \\
&\left. \widetilde{w}_{n_{b2}} > W_v\right) C_2 + \Pr\left(\widetilde{w}_{n_{b1}} > W_v\right) C_3, b \in B, \pi_b = 3, v \in V.
\end{aligned} \tag{4}$$

Equation (4) has four terms, where C_0 is the cost when the total weight of waste at the three sites is less than the capacity of the planned truck, C_1 is the cost when the total weight of the waste at the first two sites is less than the capacity of the planned truck, but the total weight of waste at the three sites is more than the capacity of the planned truck, C_2 is the cost when the weight of the waste at the first site is less than the capacity of the planned truck, but the total weight of waste at the first two sites is more than the capacity of the planned truck, and C_3 is the cost when the weight of the waste at the first site is more than the capacity of the planned truck. The detailed formulae for Equation (4) as well as C_0, C_1, C_2, and C_3 is in Appendix A.

Evidently, the computation of the probabilities in Equation (2) involves univariate calculus and the computation of the probabilities and conditional expectations in Equations (3) and (4) involves multivariate calculus. As a result, it is generally challenging to derive a closed-form expression of C_{bv}. Nevertheless, the value of C_{bv} can be easily estimated with high accuracy using Monte Carlo simulation. A theoretical analysis is in Appendix B. Moreover, the estimation of C_{bv} can be carried out using parallel computing.

Before presenting the model, let us summarize all the assumptions in the study:

1. The number of vehicles of each type operated by the construction waste transportation company is infinite.
2. At the planning stage, the transportation company knows the lower bound, upper bound, and distribution of the amount of waste at each construction site.
3. The capacity of the largest vehicle is greater than the upper bound on the amount of waste at any single site.
4. If the planned vehicle does not have sufficient capacity to carry all the waste from a site, extra vehicles will be dispatched based on the upper bound of the amounts of wastes at the sites in the trip that are not served by the planned vehicle.
5. A trip has at most three sites.

Using the concept of trip, we define z_b as a binary decision variable that equals 1 if trip $b \in B$ is chosen and 0 otherwise. We define a binary decision variable y_{bv} to be 1 if trip b is served by a truck of type v, and 0 otherwise. The model is:

$$[\text{M1}] \text{ minimize } \textstyle\sum_{b \in B} \sum_{v \in V} C_{bv} y_{bv} \tag{5}$$

subject to

$$\sum_{b \in B} \rho_{bn} z_b = 1,\ n \in N \tag{6}$$

$$\sum_{v \in V} y_{bv} = z_b, b \in B \tag{7}$$

$$y_{bv} \in \{0, 1\}, b \in B, v \in V \tag{8}$$

$$z_b \in \{0, 1\}, b \in B. \tag{9}$$

Equation (5) minimizes the total cost. Equation (6) ensures that each construction site is included in exactly one chosen trip. Equation (7) enforce that each chosen trip uses exactly one truck. Equations (8) and (9) define the decision variables to be binary.

Model [M1] has the following properties, whose proofs are straightforward and hence omitted.

Property 3. *Let $\left(y_{bv}^*,\ z_b^*,\ b \in B,\ v \in\ V\right)$ be an optimal solution to [M1]. If a trip $b\prime$ is used, that is, $z_{b\prime}^* = 1$, then there exists a type of vehicle $v' \in argmin_{v \in V}\{C_{b\prime v}\}$ such that $y_{b\prime v\prime}^* = 1$. In other words, the choice of type of vehicle in the plan for a trip is independent of the decisions for the other trips.*

Property 4. *The integrality constraints on y_{bv} can be removed. That is, constraints (8) in [M1] can be replaced with $0 \le y_{bv} \le 1,\ b \in B, v \in\ V$. Such a replacement can reduce the number of integer decision variables in the model, simplifying the branch-and-bound tree in the solution algorithm.*

Based on Property 3, we can further simplify [M1]. Define $C_b = \min_{v \in V}\{C_{bv}\}$ as the minimum expected cost of trip $b \in B$, i.e., the expected cost of trip b when the optimal type of vehicle is planned. z_b is still a binary decision variable that equals 1 if trip $b \in B$ is chosen and the optimal type of vehicle is planned and 0 otherwise. The model is:

$$[\text{M2}] \text{ minimize } \textstyle\sum_{b \in B} C_b z_b \tag{10}$$

subject to

$$\sum_{b \in B} \rho_{bn} z_b = 1,\ n \in N \tag{11}$$

$$z_b \in \{0,1\}, b \in B. \tag{12}$$

Equation (10) minimizes the total cost. Equation (11) ensures that each construction site is included in exactly one chosen trip (the truck to use for a chosen trip is determined a priori). Equation (12) define the decision variables to be binary.

Property 5. *Consider two trips $b_1 \in B$ and $b_2 \in B$ in [M2] with the same set of sites ($\rho_{b_1 n} = \rho_{b_2 n}$, $n \in N$) and $C_{b_1} < C_{b_2}$. Then, $z^*_{b_2} = 0$ in all optimal solutions.*

Property 5 can be used to eliminate a number of trips from model [M2]. For example, there are six trips that contain sites 3, 5, and 8: site 3 → site 5 → site 8; site 3 → site 8 → site 5; site 5 → site 3 → site 8; site 5 → site 8 → site 3; site 8 → site 3 → site 5; and site 8 → site 5 → site 3; only one of them is needed in model [M2].

Property 6. *Consider a trip $b_1 \in B$ and a set of trips $B_1 \in B$ and $|B_1| \geq 2$. If the sites contained in the trips in B_1 are the same as the ones in b_1, that is, $\sum_{b \in B} \rho_{bn} = \rho_{b_1 n}$, $n \in N$, and the total cost of the trips in B_1 is less than that of b_1 ($\sum_{b \in B_1} C_b < C_{b_1}$). Then, $z^*_{b_1} = 0$ in all optimal solutions.*

We illustrate the implications of Property 6 using an example. Suppose that the cost of the trip site 5 → site 3 → site 8 is greater than the sum of the costs of the three trips: site 3 only, site 5 only, and site 8 only, then we will not use the trip site 5 → site 3 → site 8 in any optimal solution. Suppose that the cost of the trip site 5 → site 3 → site 8 is greater than the sum of the costs of the two trips: site 3 only and site 8 → site 5, then we will not use the trip site 5 → site 3 → site 8 in any optimal solution.

Suppose that the remaining set of trips after using Property 5 and Property 6 is $B\prime$. The resulting model is:

$$[\text{M3}]\ \text{minimize} \sum\nolimits_{b \in B'} C_b z_b \tag{13}$$

subject to

$$\sum_{b \in B'} \rho_{bn} z_b = 1,\ n \in N \tag{14}$$

$$z_b \in \{0,1\}, b \in B'. \tag{15}$$

Equation (13) minimizes the total cost. Equation (14) ensures that each construction site is included in exactly one chosen trip. Equation (15) defines the decision variables as binary.

It can be seen that [M3] is a weighted set cover problem and, thus, NP-hard. However, because in reality the cardinality of N is small and the number of construction sites in a trip is very limited, as demonstrated in Property 1, [M3] can be efficiently solved by off-the-shelf integer programming solvers.

5. Computational Experiments

In this section, extensive computational experiments based on the construction waste data of Hong Kong were conducted to test the efficiency of the proposed stochastic programming models and to verify their effectiveness. This approach has been widely used in the literature for methodology validation [41–45]. Note that although only relevant Hong Kong data were collected for case study purposes, our proposed model and method can be applied to various countries and regions for their cost-optimized transport planning of construction waste.

We selected Hong Kong as an instructive case study because of its representativeness. Hong Kong is characterized by high population density, limited space, and unmet housing demands, all of which are common features in countries and regions where a considerable amount of construction waste is likely to be generated. Figure 2 shows the construction waste reception facility at Chai Wan, Hong Kong. The construction waste transportation company operates a fleet of $|V|= 8$ types of vehicles, whose carrying capacity W_v and cost c_v are listed in Table 1. The transportation company will serve $|N|\in \{5, 10, 20, 30, 40\}$ construction sites in Hong Kong Island the next day, whose locations are randomly generated. The amount of waste at each site is randomly generated based on the actual amount of waste collected at the Chai Wan reception facility. The real data of construction waste generation in Hong Kong can be found at the website of the Environmental Protection Department (https://www.epd.gov.hk/epd/misc/cdm/scheme.htm, accessed on 10 July 2024).

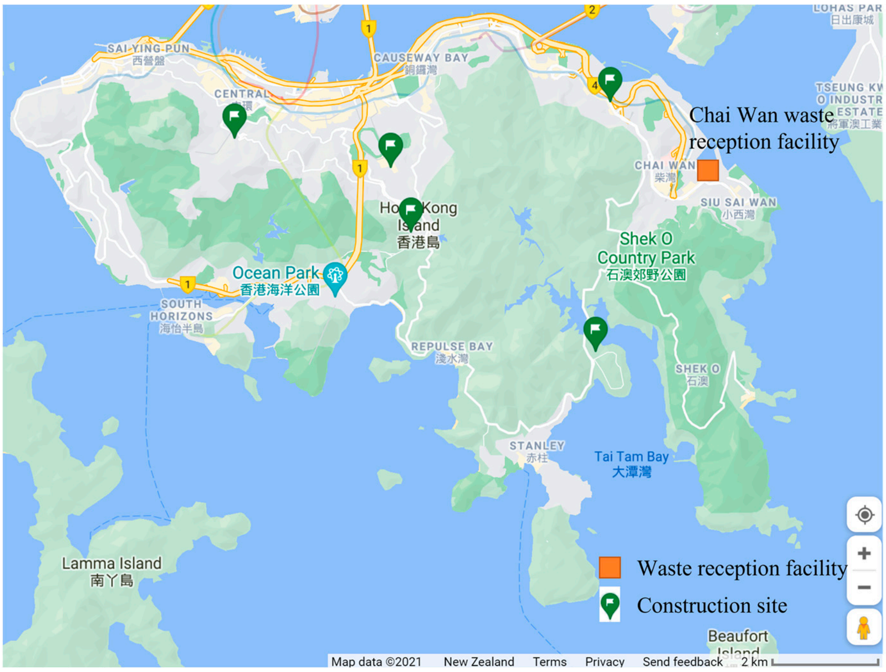

Figure 2. Locations of the construction waste reception facility and examples of construction sites.

Table 1. Carrying capacity and cost of the vehicle fleet.

Vehicle Type v	Carrying Capacity W_v (ton)	Cost c_v ($/min)
1	3	1.75
2	5	2.26
3	8	2.86
4	10	3.19
5	15	3.91
6	20	4.52
7	30	5.53
8	35	5.97

In particular, Figure 3 shows the distribution of the actual amounts of 4291 records, with the minimum weight of 0.58 ton and the maximum weight of 22.37 ton. It can be seen that the weights are concentrated at 5, 7.5, 11, 15, and 22 tons. We, therefore, assume the construction manager of a site will estimate 5, 7.5, 11, 15, and 22 tons if the actual amount is between 0.58 and 6.25 tons, between 6.25 and 9.25 tons, between 9.25 and 13 tons, between 13 and 18.5 tons, and between 18.5 and 22.37 tons, respectively. For each of the $|N|$ sites in the study, we assume its actual amount of waste (which is unknown at the planning stage) is randomly drawn from the distribution in Figure 3. For example, if the actual (but unknown yet) amount is 4.23 tons, the construction site manager will estimate 5 tons, and then the transportation company knows its lower bound (i.e., 0.58 tons), its upper bound (i.e., 6.25 tons), and its distribution based on the historical records.

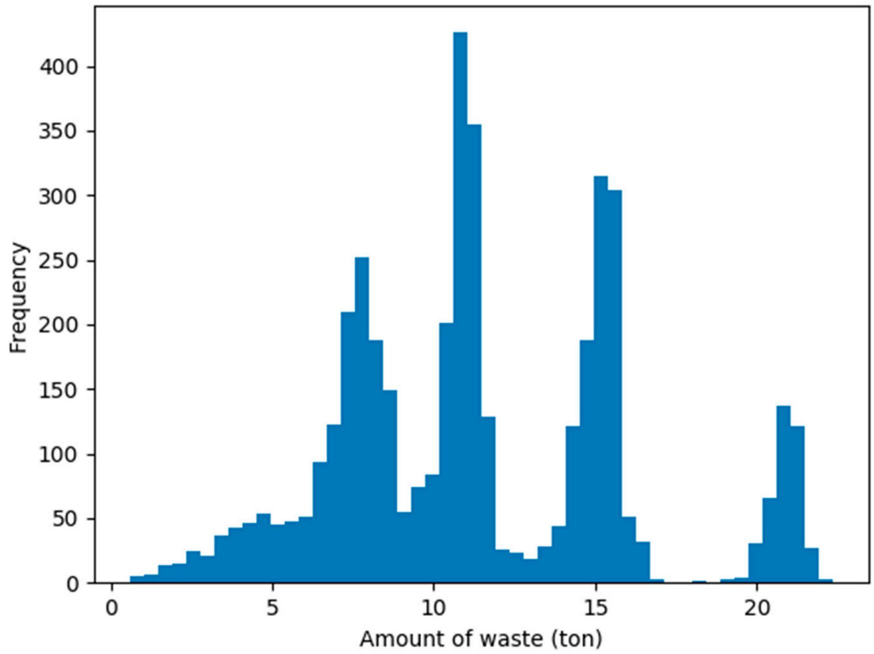

Figure 3. Distribution of the actual weight of construction waste collected at Chai Wan reception facility (data from Hong Kong Environmental Protection Department [46]).

For each value of $|N|$, five instances are randomly generated. For each instance, we first use a sample size of 10,000 to estimate the value of C_b using Monte Carlo simulation. The sample size 10,000 is chosen because in our trial experiments, we find this size is sufficient for practical purposes. Then, model [M3] is solved by integer programming solver CPLEX 12.10 on a Lenovo laptop with CPU of 1.10 GHz processing speed and 16 GB of memory.

5.1. Computational Analysis

We first examine the computational efficiency of model [M3]. Table 2 shows the results over five instances regarding the number of sites (column 1), the theoretical number of trips (column 2), the average actual number of trips after applying Properties 3, 5, and 6 (column 3), the ratio of the average actual number of trips after applying properties 3, 5, and 6 and the theoretical number of trips (column 4), and the average computation time for solving one instance of [M3] (column 5). Given $|N|$ sites and $|V|$ types of vehicles and a trip can have at most three sites, the theoretical number of trips is $|V|(\text{Combination}(|N|,1)+\text{Combination}(|N|,2)+\text{Combination}(|N|,3))$. Table 2 shows that the theoretical number of trips increases with $|N|$ roughly in a cubic manner, that is, when $|N|$ is doubled, the theoretical number of trips will be eight times as large. Applying Properties 3, 5, and 6, we can significantly reduce the number of trips that need to be considered. Moreover, for larger values of $|N|$, a higher proportion of trips can be excluded by Properties 3, 5, and 6. Note that the number of trips to be considered corresponds to the number of decision variables and, hence, a smaller number is highly desirable from computational perspective. Indeed, this can be reflected in the last column of Table 2: [M3] can be solved in 1 s, showing the computational efficiency.

Table 3 reports the detailed trips chosen and their vehicles used for an instance of 10 construction sites. As shown in the first row of Table 3, the trip that includes only site 3 (its estimated amount of waste is 7.5 tons) uses a vehicle of type 4 (its carrying capacity is 10 tons) instead of a vehicle of type 3 (its carrying capacity is 8 tons). This is because whereas the estimated amount of waste at site 3 is 7.5 tons, the actual amount of waste can be any value between 6.25 and 9.25 tons. If a vehicle of type 3 were used, there would be a high chance that an extra vehicle has to be dispatched. Similarly, as shown in the second row of Table 3, the trip that includes only site 6 (its estimated amount of waste is 15 tons) uses a vehicle of type 6 (its carrying capacity is 20 tons) instead of a vehicle of type 5 (its carrying capacity is 15 tons) because the actual amount of waste can be any value between 13 and 18.5 tons.

Table 2. Computational efficiency of [M3].

Number of Sites $\|N\|$	Theoretical Number of Trips	Average Actual Number of Trips	$\frac{\text{Actual}}{\text{Theoretical}}$	Average CPU Time (s)
5	200	15.4	7.7%	<1
10	1400	72.4	5.2%	<1
20	10,800	284.6	2.6%	<1
30	36,200	725.2	2.0%	<1
40	85,600	1348.8	1.6%	<1

Table 3. Trips chosen and their vehicles used for an instance of 10 construction sites (with consolidation and uncertain demand).

Trip	Vehicle Type
3	4
6	6
$2 \to 5$	6
$7 \to 8$	6
$9 \to 1$	6
$4 \to 10$	8

The trip that includes site 2 (its estimated amount of waste is 11 tons) and 5 (its estimated amount of waste is 7.5 tons) uses a vehicle of type 6 (its carrying capacity is 20 tons). This vehicle will always be able to transport all the waste from site 2 as its maximum amount is 13 tons, but may not be able to transport all the waste from site 5 because the sum of the maximum amounts at the two sites is 13 + 9.25 = 22.25 tons. The trips $7 \to 8$ and $9 \to 1$ are similar to the trip $2 \to 5$. In trip $4 \to 10$, both sites have an estimated amount of waste of 15 tons, and the used vehicle has a capacity of 35 tons, larger than the sum of estimated amounts at the two sites.

5.2. Comparison with No Consolidation

Next, we assess the value of construction waste consolidation. The value of consolidation in logistics has been widely acknowledged [47]. However, the Hong Kong Environmental Protection Department [46] does not allow consolidation of construction waste from different sites on one vehicle and it requires a vehicle to report the ID of the construction site whose waste is being carried by the vehicle. One reason for this regulation is for the government to track the amount of waste transported to the reception facility so that if there is illegal dumping of waste from a construction site, the government can spot abnormality from its record. For instance, if a large construction site has only a small amount of waste transported to the reception facility, it is likely that the construction site has illegally dumped some waste. Therefore, we examine numerically the potential loss from forbidding construction waste consolidation so that the government can weigh the pros and cons of forbidding construction waste consolidation.

To this end, we solve the same instances with the requirement that each vehicle can only carry construction waste from one site (no consolidation). Note that when consolidation is allowed, construction waste from at most three sites can be carried on the same vehicle, as shown in Figure 1. Table 4 reports the average cost for instances of different numbers of construction sites. It can be seen that with consolidation, the cost can be significantly reduced. Moreover, the cost reduction increases with the number of construction sites. This is because with more construction sites, the average distance between two sites is smaller and, hence, it is easier to collect wastes from several neighboring sites.

Table 4. Value of construction waste consolidation.

Number of Sites $\|N\|$	Average Cost without Consolidation (Each Trip Contains One Site Only)	Average Cost with Consolidation (Each Trip Contains at Most Three Sites)	Cost Reduction
5	1421.55	1356.72	4.6%
10	2809.90	2662.69	5.2%
20	5960.36	5511.30	7.5%
30	8947.00	8220.79	8.1%
40	12,073.24	11,009.00	8.8%

Table 5 reports the detailed trips chosen and the vehicles used for the same instance of 10 construction sites as Table 3. For trips that include only one site in Table 3, their used vehicles are unchanged without consolidation. For trips that include more than one site in Table 3, they are broken down into multiple trips, using smaller vehicles, leading to a higher transportation cost.

Table 5. Trips chosen and their vehicles used for an instance of 10 construction sites without consolidation.

Trip	Vehicle Type
1	4
2	5
3	4
4	6
5	4
6	6
7	5
8	4
9	5
10	6

5.3. Comparison with Models That Do Not Consider Random Demand

Finally, we assess the value of considering the randomness of the amount of construction waste, as a comparison with studies in the literature that do not consider demand uncertainty [12,13]. To this end, we solve the same instances by assuming the amount of waste at each construction site is equal to the estimated value and then the chosen trips are evaluated by considering the uncertainty. Table 6 reports the average cost for instances of different numbers of construction sites. It can be seen that, taking into account the randomness, the cost can be reduced by about 1%. Note that the operating margin of the transport and logistics industry is about 7% [48]. Therefore, reducing the cost by 1% means increasing the profit by 14%. We notice that there are fluctuations in the percentage of cost reduction, which is solely due to randomness in the parameter settings in the numerical experiments. Moreover, even if the cost is reduced by only 0.4%, the profit will be increased by 5%.

Table 6. Value of considering the randomness of the amount of construction waste.

Number of Sites $\|N\|$	Average Cost without Considering Randomness	Average Cost Considering Randomness	Cost Reduction
5	1361.08	1356.72	0.3%
10	2726.32	2662.69	2.3%
20	5600.74	5511.30	1.6%
30	8378.53	8220.79	1.9%
40	11,051.32	11,009.00	0.4%

Table 7 reports the detailed trips chosen and their vehicles used for the same instance of 10 construction sites as Table 3. Without considering uncertain demand, the wastes from more construction sites are consolidated. Moreover, the capacities of the vehicles used will be determined solely based on the estimated amounts of waste at the sites in the trips. For instance, the estimated amount at site 1 is 7.5 tons and the estimated amount at site 6 is 15 tons, hence a vehicle of type 7 (carrying capacity 30 tons) is used for trip $1 \to 6$, and the estimated amount at site 3 is 7.5 tons and the estimated amount at site 9 is 11 tons, hence a vehicle of type 6 (carrying capacity 20 tons) is used for trip $3 \to 9$. As a result, considerably high costs of extra vehicles are incurred, compared with the plan in Table 3.

Table 7. Trips chosen and their vehicles used for an instance of 10 construction sites without considering uncertain demand.

Trip	Vehicle Type
$1 \to 6$	7
$3 \to 9$	6
$4 \to 7$	7
$5 \to 10$	7
$8 \to 2$	6

6. Conclusions

The amount of construction and demolition waste to transport from construction sites to reception facility is considerable, owning to the rapid urbanization all over the world. Optimal planning of the transportation for waste transportation companies can reduce cost and thereby decrease carbon emissions and alleviate road congestion. However, a major challenge for developing a transportation plan is the exact amount of waste at each site is unknown at the planning stage. Taking advantage of the problem structure, we define the concept of trip that handles the uncertainty in an elegant manner. Specifically, the calculation of the expected cost of a trip involves the amounts of waste at only a small number of sites and, hence, can easily be achieved using Monte Carlo simulation. Based on the concept of trip, we develop an integer programming model, which can be solved efficiently for problems of practical size. Using the construction waste data in Hong Kong, we carry out extensive numerical experiments. The results demonstrate that considering uncertainty can reduce about 1% of the transportation cost, which can be translated to about 14% of profit. To use the proposed model, a waste transportation company only needs to keep a record of the estimated amount by site manager and the actual amount of the waste at sites that have been served by the company. Therefore, the proposed model offers significant practical relevance to the construction waste transportation industry.

Some limitations of this study should be recognized. For example, only the uncertainty of the amount of waste is considered in the transportation planning of construction waste. However, in real practices, waste transportation companies are faced with diverse uncertain factors that can have a compound impact on the optimality of the generated transportation plan. Considering the uncertainty of both the amount of waste and the travel time is a worthwhile future search direction.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

We present the detailed formulae of Equation (4) below:

$$
\begin{aligned}
C_{bv} = {} & \Pr\left(\widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} + \widetilde{w}_{n_{b3}} < W_v\right) c_v\left(t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}n_{b3}} + t_{n_{b3}0}\right) + \Pr\left(\widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} + \right. \\
& \left. \widetilde{w}_{n_{b3}} > W_v\right) \left[c_v\left(t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}n_{b3}} + t_{n_{b3}0}\right) + E\left[c_{v(w^U_{n_{b3}} - (W_v - \widetilde{w}_{n_{b1}} - \widetilde{w}_{n_{b2}}))} \middle| \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} + \right.\right. \\
& \left.\left. \widetilde{w}_{n_{b3}} > W_v \right] \left(t_{0n_{b3}} + t_{n_{b3}0}\right) \right] + \Pr\left(\widetilde{w}_{n_{b1}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} > W_v\right) \left[c_v\left(t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}\right) + \right. \\
& \min\left\{ E\left[c_{v(w^U_{n_{b2}} - (W_v - \widetilde{w}_{n_{b1}}))} \middle| \Pr\left(\widetilde{w}_{n_{b1}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} > W_v\right)\right] \left(t_{0n_{b2}} + t_{n_{b2}0}\right) + c_{v(w^U_{n_{b3}})}\left(t_{0n_{b3}} + \right.\right. \\
& \left. t_{n_{b3}0}\right)\right), E\left[c_{v(w^U_{n_{b2}} - (W_v - \widetilde{w}_{n_{b1}}) + w^U_{n_{b3}})} \middle| \Pr\left(\widetilde{w}_{n_{b1}} < W_v, \widetilde{w}_{n_{b1}} + \widetilde{w}_{n_{b2}} > W_v\right)\right] \min\left\{ t_{0n_{b2}} + t_{n_{b2}n_{b3}} + t_{n_{b3}0}, t_{0n_{b3}} + t_{n_{b3}n_{b2}} + \right. \\
& \left.\left.\left. t_{n_{b2}0} \right\} \right\} \right] + \Pr\left(\widetilde{w}_{n_{b1}} > W_v\right) \left[c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + \min\left\{ c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + c_{v(w^U_{n_{b2}})}\left(t_{0n_{b2}} + t_{n_{b2}0}\right) + \right.\right. \\
& c_{v(w^U_{n_{b3}})}\left(t_{0n_{b3}} + t_{n_{b3}0}\right), c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + c_{v(w^U_{n_{b2}} + w^U_{n_{b3}})} \min\left\{ t_{0n_{b2}} + t_{n_{b2}n_{b3}} + t_{n_{b3}0}, t_{0n_{b3}} + t_{n_{b3}n_{b2}} + \right. \\
& \left. t_{n_{b2}0} \right\}, c_{v(w^U_{n_{b2}})}\left(t_{0n_{b2}} + t_{n_{b2}0}\right) + c_{v(w^U_{n_{b1}} - W_v + w^U_{n_{b3}})} \min\left\{ t_{0n_{b1}} + t_{n_{b1}n_{b3}} + t_{n_{b3}0}, t_{0n_{b3}} + t_{n_{b3}n_{b1}} + t_{n_{b1}0} \right\}, c_{v(w^U_{n_{b3}})}\left(t_{0n_{b3}} + \right. \\
& \left. t_{n_{b3}0}\right) + c_{v(w^U_{n_{b1}} - W_v + w^U_{n_{b2}})} \min\left\{ t_{0n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}, t_{0n_{b2}} + t_{n_{b2}n_{b1}} + t_{n_{b1}0} \right\}, c_{v(w^U_{n_{b1}} - W_v + w^U_{n_{b2}} + w^U_{n_{b3}})} \min\left\{ t_{0n_{b1}} + \right. \\
& t_{n_{b1}n_{b2}} + t_{n_{b2}n_{b3}} + t_{n_{b3}0}, t_{0n_{b1}} + t_{n_{b1}n_{b3}} + t_{n_{b3}n_{b2}} + t_{n_{b2}0}, t_{0n_{b2}} + t_{n_{b2}n_{b1}} + t_{n_{b1}n_{b3}} + t_{n_{b3}0}, t_{0n_{b2}} + t_{n_{b2}n_{b3}} + t_{n_{b3}n_{b1}} + \\
& \left.\left.\left. t_{n_{b1}0}, t_{0n_{b3}} + t_{n_{b3}n_{b1}} + t_{n_{b1}n_{b2}} + t_{n_{b2}0}, t_{0n_{b3}} + t_{n_{b3}n_{b2}} + t_{n_{b2}n_{b1}} + t_{n_{b1}0} \right\} \right\} \right], b \in B, \pi_b = 3, v \in V.
\end{aligned}
\tag{A1}
$$

Appendix B

We carry out a theoretical analysis of the accuracy of using Monte Carlo simulation to estimate the value of C_{bv} in Equation (2). The cases of Equations (3) and (4) can be analyzed in a similar way.

For brevity, define $p = \Pr\left(\widetilde{w}_{n_{b1}} < W_v\right)$, $C\prime = c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right)$, and $C'' = c_v\left(t_{0n_{b1}} + t_{n_{b1}0}\right) + c_{v(w^U_{n_{b1}} - W_v)}\left(t_{0n_{b1}} + t_{n_{b1}0}\right)$; $C\prime < C''$. Then, $C_{bv} = pC\prime + (1-p)C''$, $b \in B, \pi_b = 1, v \in V$.

To estimate C_{bv} using Monte Carlo simulation, we randomly generate a sample of M realizations of $\widetilde{w}_{n_{b1}}$ according to its distribution function $F_{n_{b1}}\left(w_{n_{b1}}\right)$, and the resulting estimate of C_{bv}, denoted by $\hat{C}_{bv}$, is $\hat{C}_{bv} = \hat{p}C\prime + (1-\hat{p})C''$, where $\hat{p}$ is the proportion of the M realizations that are less than W_v. Note that $\hat{C}_{bv}$ is random as it depends on the sample that is randomly drawn from the distribution of $\widetilde{w}_{n_{b1}}$.

Supposing that we hope $\Pr\left(\left|\frac{\hat{C}_{bv} - C_{bv}}{C_{bv}}\right| < 1\%\right) > 99\%$, that is, there is at least 99% chance that the relative estimation error is at most 1%, let us examine the choice of sample size M. To this end, we only need to guarantee $\Pr\left(\frac{\hat{C}_{bv} - C_{bv}}{C_{bv}} > 1\%\right) < 0.5\%$ and $\Pr\left(\frac{C_{bv} - \hat{C}_{bv}}{C_{bv}} > 1\%\right) < 0.5\%$.

We now examine the requirement $\Pr\left(\frac{\hat{C}_{bv} - C_{bv}}{C_{bv}} > 1\%\right) < 0.5\%$, i.e., $\Pr\left(\frac{[\hat{p}C\prime + (1-\hat{p})C''] - [pC\prime + (1-p)C'']}{pC\prime + (1-p)C''} > 1\%\right) < 0.5\%$. According to Hoeffding's inequality, which provides an upper bound on the probability that the sum of random variables deviates from its mean value by a given amount, $\Pr\left(\frac{[\hat{p}C\prime + (1-\hat{p})C''] - [pC\prime + (1-p)C'']}{pC\prime + (1-p)C''} > 1\%\right) \leq e^{-2M\left\{\frac{1\%[pC\prime + (1-p)C'']}{C'' - C\prime}\right\}^2} < e^{-2M\left\{\frac{1\%[pC\prime + (1-p)C'']}{C''}\right\}^2}$. Since $C\prime < C''$, $\Pr\left(\frac{[\hat{p}C\prime + (1-\hat{p})C''] - [pC\prime + (1-p)C'']}{pC\prime + (1-p)C''} > 1\%\right) < e^{-2M\left(\frac{1\%C\prime}{C''}\right)^2}$. To ensure $\Pr\left(\frac{\hat{C}_{bv} - C_{bv}}{C_{bv}} > 1\%\right) < 0.5\%$, we only need $e^{-2M\left(\frac{1\%C\prime}{C''}\right)^2} < 0.5\%$. Suppose $C'' = 5C\prime$, we only need to take a sample of size M not less than 662,290. The examination of the requirement $\Pr\left(\frac{C_{bv} - \hat{C}_{bv}}{C_{bv}} > 1\%\right) < 0.5\%$ yields the same sample size.

Note that the above derivation refers to the worst case and many steps in the derivation can be further strengthened. Therefore, in reality the sample size required is much smaller than the above theoretical value. In fact, in our trial experiments we find setting M at 10,000 is sufficient and, therefore, in the numerical study of the paper, we set M at 10,000.

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