The following publication Y. Xiao, T. Ding, C. Mu, K. Pan, B. Zhang and M. Shahidehpour, "Convex Hull for Self-Scheduling Energy-Intensive Enterprises With Demand Response Regulations," in IEEE Transactions on Power Systems, vol. 40, no. 3, pp. 2003-2013, May 2025 is available at https://doi.org/10.1109/TPWRS.2024.3498102.

Convex Hull for Self-Scheduling Energy-Intensive Enterprises with Demand Response Regulations

Yang Xiao, *Student Member, IEEE*, Tao Ding, *Senior Member, IEEE,* Chenggang Mu, *Student Member, IEEE*, Kai Pan, *Member, IEEE*, Biyuan Zhang, *Student Member, IEEE*, Mohammad Shahidehpour, *Fellow, IEEE*

***Abstract*—The self-scheduling energy-intensive enterprise (EIE) has great potential to participate in demand response (DR) regulations. However, the multi-period self-scheduling model with the DR will bring computational burdens, since there are lots of binary decision variables. To address this challenge, this paper proposes a convex hull model for the self-scheduling model with the DR. Specifically, it presents the self-scheduling model of EIE as an integer programming (IP) model, then transforms this IP model into a dynamic programming (DP) model, and finally reformulates this DP model into a linear programming (LP) model. Furthermore, the proposed LP model is theoretically proved to be the convex hull of the self-scheduling EIE with the DR. Moreover, the benefits of the convex hull model are discussed, and extensive numerical experiments are carried out to demonstrate the excellent performance and efficiency of the convex hull model.**

***Index Terms*—Convex hull, energy-intensive enterprise, self-scheduling model, demand response**

NOMENCLATURE

Sets

Π	Set contains all discrete operation points of the energy-intensive enterprise
Σ	State space contains all states of the energy-intensive enterprise
$\Sigma_{\text{from}}(i)$	Set contains all possible states that can transfer from the state *i*.
$\Sigma_{\text{to}}(i)$	Set contains all possible states that can transfer to the state *i*.

Parameters

π_t	Electricity budget at the period *t*
λ_t	Real-time electricity price at the period *t*
β	Cost of each start-up and shut-down action
R	Ramping rate of the energy-intensive enterprise
TR	Maximum time as the energy-intensive enterprise remains the increasing state or the decreasing state
TM	Minimum time as the energy-intensive enterprise remains the maintenance state
T	Time scale
$\lvert\Pi\rvert$	Number of load operation points
$V(i,1)$	Payoff when the energy-intensive enterprise remains the state *i* at the first period
$V(i,j,t+1)$	Payoff when the energy-intensive enterprise remains the state *j* at the period *t*+1

Decision Variables

p_t	Load demand of the energy-intensive enterprise at period *t*
u_t/u_i	u_t/u_i=1 when the energy-intensive enterprise increases its load demand at period *t*/*i*; otherwise u_t/u_i=0
d_t/d_i	d_t/d_i=1 when the energy-intensive enterprise decreases its load demand at period *t*/*i*; otherwise d_t/d_i=0
m_t/m_i	m_t/m_i=1 when the energy-intensive enterprise maintains its load demand at period *t*/*i*; otherwise m_t/m_i=0
$v(i,1)$	Maximum payoff from the first period to the end when the energy-intensive enterprise remains the state *i* at the first period
$v(i,t)$	Maximum payoff from the period *t* to the end when the energy-intensive enterprise remains the state *i* at the period *t*
z	Payoff for the entire self-scheduling period

Functions

$\delta(\bullet)$	Function of the relationships between the load demand change and the number of the start-up/shut-down actions
$p(\bullet)$	Function of the relationships between the state and the load demand
$u(\bullet)$	Function of the relationships between the state and the load-increasing state
$d(\bullet)$	Function of the relationships between the state and the load-decreasing state
$x(\bullet)$	Function of the relationships between the state and the load-maintaining state

This work was supported in part by the National Natural Science Foundation of China [Grant No. 52277123], the Science and Technology Project of Guizhou Power Grid Company of China [Grant No. GZKJXM20222442], and the Research Grants Council of Hong Kong [Grant No. 15501221]. *(Corresponding author: Tao Ding, email: tding15@mail.xjtu.edu.cn)*

Yang Xiao, Tao Ding, Chenggang Mu, and Biyuan Zhang are with the Department of Electrical Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi, 710049, China.

Kai Pan is with the Department of Logistics and Maritime Studies, Hong Kong Polytechnic University, Kowloon, Hong Kong.

Mohammad Shahidehpour is with the ECE Department, Illinois Institute of Technology, Chicago, IL 60616 USA.

I. INTRODUCTION

ENVIRONMENTAL concerns with climate change and energy portfolios [1] have encouraged an energy transition that signifies an increasing proportion of clean energy [2]. Inevitably, the increasing penetration of renewable energy sources (RESs) requires more flexibilities of the power system.

Without enough flexible resources, the potential power imbalance [3] and transition instability [4] at some periods when renewable energy is fluctuating will arise. Thus, it would be imperative to provide adequate flexibility as power systems are proliferated with RES. Indeed, improving the installed generation capacity [5], ramp rate [6], peak load regulation [7], etc., are effective ways to provide generation flexibility as opposed to allocating redundant resources to cope with uncertainty [8]. Demand response (DR) offered by end-use customers, which has been considered to provide extra regulation resources, is considered an alternative approach [9], [10]. Ref. [11] analyzed the DR features and summarized the DR modeling for peaking and improving RES consumption. Ref. [12] proposed a day-ahead scheduling model considering the hourly DR for decreasing the operation cost. The DR uncertainty was investigated in [13], which enhances the unit commitment solution process. Ref. [14] suggested that the DR can be further utilized to reduce investment costs.

An energy-intensive enterprise (EIE), such as the electrolytic aluminum plant and the steel plant, is considered an end-use customer that consumes a lot of electricity [15]. Since the electricity cost is the main production cost, these EIEs are extremely sensitive to electricity prices. Therefore, they are significant potential participators in the DR [16]. Ref. [17] has investigated the EIE technical and economic potentials for the provision of the DR and pointed out that EIE can replace conventional plants to provide flexibility and economic benefits to electricity markets. However, most EIEs were not motivated to respond to the DR because of the lack of proper tools for measuring the DR merits [18]. Some scholars made progress in substantiating the EIE operations toward benefiting from the DR merits. Ref. [19] presented a model for electrolytic process industries to reduce electricity consumption to attain additional payoff during the period when electricity was expensive. Ref. [20] designed a coordinated scheduling approach between the EIE and the power system to achieve the final settlements. Ref. [21] constructed the merit-based self-scheduling model for the EIE as a mixed-integer programming (MIP) problem, which helped the EIE reduce its electricity consumption cost. Ref. [22] proposed a decision-making tool for EIEs to change their consumption patterns that would guide them to gain additional payoff. Recently, there were still a lot of papers on EIEs. Ref. [23] suggested that the EIE can enhance efficiency by production scheduling and presented a mixed-integer linear programming model for scheduling to improve the energy efficiency. Some other researchers also acknowledged that the EIEs will have a greater potential to participate in the friendly interactions between power supply and demand for lower electricity costs [24]-[26]. Besides, some researchers employed EIE self-scheduling to accommodate more RESs in [27]. However, these self-scheduling EIE problems are mainly established as MIP models that may bring low computational efficiency when addressing a large number of binary variables resulting from multiple periods.

In recent studies, several integer programming (IP) or MIP models were solved by their convex hull formulation (i.e., their convex hull model) to improve their computational efficiency. Such the idea was widely applied to the unit commitment problems, and brought its benefits. Ref. [28] presented the single-unit commitment (1UC) problem as a dynamic programming (DP) model and further reformulated this model into a linear programming (LP) model, enabling it to be solved in O(n^3). Ref. [29] refined this O(n^3) approach and developed a convex hull formulation of the 1UC problem, called extended formulation, to provide the unit commitment solution in O(n^2). Accordingly, a complete convex hull model was introduced for the 1UC problem with a pumped hydro storage unit [30]. Such significant performance of the convex hull model in computational efficiency motivates us to apply it in the EIE self-scheduling.

As there is a lack of an efficient convex hull model to quickly provide a scheduling strategy for the self-scheduling EIE, this manuscript investigated a general convex hull framework for the EIE self-scheduling with the DR. The main contributions of this paper are summarized as follows.

1) A self-scheduling EIE is established as an IP model to represent the specific EIE operational requirements. However, there will be a large number of binary variables that will affect the computational performance of the proposed model. To address this problem, a convex hull of the self-scheduling EIE model with the DR is proposed, which gives an LP model and does not need any binary variable. The convex hull of the self-scheduling EIE with the DR is derived by transforming the proposed IP model into a DP model, and eventually reformulating the DP as an LP model.

2) The proposed LP model is strictly and theoretically proved to be the convex hull of the proposed self-scheduling EIE model. Moreover, the physical interpretation of the convex hull model for the self-scheduling EIE is presented to align the proposed convex hull model with practical applications.

This paper is organized as follows: Section II introduces the mathematical formulation of priced-based DR regulations and its corresponding convex hull. Section III provides several case studies. Finally, Section V concludes this paper.

II. Convex Hull Formulation of Self-Scheduling EIE

First, a self-scheduling EIE model is formulated as an IP to maximize the EIE's payoff by regulating its load demand in response to real-time prices considering several operational requirements. Second, a framework is established that can provide the convex hull of the self-scheduling EIE model.

A. IP Model for self-scheduling EIE

An EIE usually conducts self-scheduling as a price taker to improve its payoff. It aims to seek a solution by regulating the EIE load for the maximum payoff while satisfying the EIE operational requirements. Documented evidence suggested that frequent and uninterrupted load regulations tend to shorten the equipment service life and reduce product quality [31]. Thus, the frequent and uninterrupted load demand regulations should be avoided. This operational requirement can be satisfied with the following rules: 1) we can set a long enough maintenance time (i.e., *TM* in Fig. 1) between these two consecutive load regulations to avoid the frequent regulations; 2) we can set a suitable regulation time for load demand regulations to avoid uninterrupted regulation (i.e., *TR* in Fig. 1). Such representative requirements of the self-scheduling EIE with the DR are clarified in the following Fig. 1.

Furthermore, the load demand changes of the EIE are often accompanied by the start-up and shut-down actions of its devices. Therefore, the load demand cannot be continuously regulated. As a result, the self-scheduling EIE should operate at several discrete operation points [32]-[33], which are contained in the set Π. The start-up and shut-down costs due to the load demand changes can be calculated by the function $\delta(\bullet)$, which determines the number of start-up and shut-down actions due to the operation point changes. Moreover, it is also important to note that the shutdown of the entire production line must be avoided, as it would take several weeks or months for the entire production line to recover production [34]. By incorporating the electricity costs, the electricity budgets, and the start-up/shut-down costs into the objective function, the following IP model can be established for the self-scheduling EIE with the DR based on the operational requirements mentioned above:

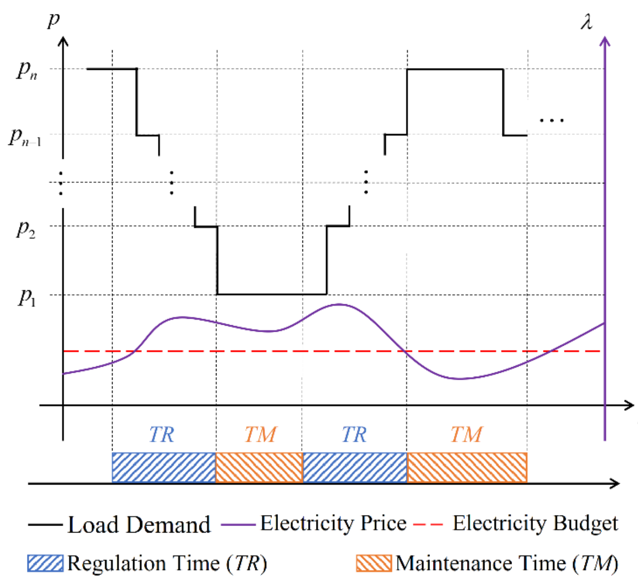


Fig. 1. Self-scheduling process of a single EIE

$$\max \ \sum_{t=1}^{T}\pi_t p_t - \sum_{t=1}^{T}\lambda_t p_t - \sum_{t=1}^{T-1}\beta\delta(p_{t+1}-p_t) \quad (1a)$$

$$\textbf{s.t.} \quad p_{t+1}-p_t \le (1-x_{t+1})R - d_{t+1}(R+\varepsilon), t\in\{1,...,T-1\} \quad (1b)$$

$$p_t - p_{t+1} \le (1-x_{t+1})R - u_{t+1}(R+\varepsilon), t\in\{1,...,T-1\} \quad (1c)$$

$$\sum_{i=t}^{t+TR} u_i \le TR, t\in\{1,...,T-TR\} \quad (1d)$$

$$\sum_{i=t}^{t+TR} d_i \le TR, t\in\{1,...,T-TR\} \quad (1e)$$

$$\sum_{i=t+1}^{t+TM} m_i \ge TM(m_{t+1}-m_t), t\in\{1,...,T-TM\} \quad (1f)$$

$$\sum_{i=t+1}^{T} m_i \ge (T-t)(m_{t+1}-m_t), t\in\{T-TM+1,T\} \quad (1g)$$

$$u_{t+1}+d_t \le 1, t\in\{1,...,T-1\} \quad (1h)$$

$$u_t+d_{t+1} \le 1, t\in\{1,...,T-1\} \quad (1i)$$

$$u_t+d_t+m_t = 1, t\in\{1,...,T\} \quad (1j)$$

$$u_t, d_t, m_t, u_i, d_i, m_i \in\{0,1\}, p_t \in \mathrm{P}. \quad (1k)$$

where constraints (1b)-(1c) are ramping rate limits, where ε in them denotes an arbitrarily small positive number. This small positive number restricts the load demand to keep regulating during the regulation time. Constraints (1d)-(1e) limit the maximum regulation time for the load demand regulations. Constraints (1f)-(1g) limit the minimum maintenance time between these two consecutive load demand regulations. Constraints (1h)-(1j) restrict the logical relationships of the EIE's operational states, which only allows the load demand to turn its state from maintenance to regulation or from regulation to maintenance. Constraints (1k) restrict the values of the load-increasing states, the load-decreasing states, the load-maintaining states, and the load demands.

B. *Mathematical Formulation of Convex Hull*

Since all EIE operational states are described as the load-increasing states, the load-decreasing, and the load-maintaining states, the self-scheduling solution of the EIE can be represented as a combination of these states. Thus, the self-scheduling EIE model with the DR can be formulated as a DP model. Then, we reformulate this DP model as its equivalent LP model, and the convex hull of the self-scheduling model of the EIE will be obtained. Specifically, the construction of the convex hull model is summarized in the following three parts:

(i) State Transition Process of EIEs

Here, all operational states of the EIE can be uniquely denoted as the tuple (p,u,d,x,l), whose components present the load demand, the load-increasing state, the load-decreasing state, the load-maintaining state, and the duration for the current state, respectively.

1) When the load demand is increasing, any corresponding states are involved in:

$$(p,u,d,x,l)\in \mathrm{P}\times 1\times 0\times 0\times\{1,...,TR\} \quad (2)$$

2) When the load demand is decreasing, any corresponding states are involved in:

$$(p,u,d,x,l)\in \mathrm{P}\times 0\times 1\times 0\times\{1,...,TR\} \quad (3)$$

3) When the load demand is maintaining, any corresponding states are involved in:

$$(p,u,d,x,l)\in \mathrm{P}\times 0\times 0\times 1\times\{1,...,TM\} \quad (4)$$

The state space of the EIE consists of these above three sets, which can be denoted as Σ. With this state space Σ, all possible state transitions can be derived as the following rules based on the operational limits, which are described in model (1):

1) As the EIE is increasing its load demand and $TR = 1$, the following state transition is true: ① when the state is $(p,1,0,0,1)$, it only can transfer to the state $(p,0,0,1,1)$;

2) As the EIE is increasing its load demand and $TR > 1$, the following state transitions are true: ① when the state is $(p,1,0,0,1)$, it can transfer to the state $(p,0,0,1,1)$ or it also can transfer to the state $(p',1,0,0,2)$ with $0 \le p' - p \le R$; ② when the state is $(p,1,0,0,l)$ and $1 < l < TR$, it can transfer to the state $(p,0,0,1,1)$ or it also can transfer to the state $(p',1,0,0,l+1)$ with $0 \le p' - p \le R$; ③ when the state is $(p,1,0,0,TR)$, it only can transfer to the state $(p,0,0,1,1)$;

3) As the EIE is decreasing its load demand and $TR = 1$, the following state transition is true: ① when the state is $(p,1,0,0,1)$, it only can transfer to the state $(p,0,0,1,1)$;

4) As the EIE is decreasing its load demand and $TR > 1$, the following state transitions are true: ① when the state is $(p,0,1,0,1)$, it can transfer to the state $(p,0,0,1,1)$ or it also can transfer to the state $(p',0,1,0,2)$ with $0 \le p - p' \le R$; ② when the state is $(p,0,1,0,l)$ and $1 < l < TR$, it can transfer to the state $(p,0,0,1,1)$ or it also can transfer to the state $(p',0,1,0,l+1)$ with $0 \le p - p' \le R$; ③ when the state is $(p,0,1,0,TR)$, it only can transfer to the state $(p,0,0,1,1)$;

5) As the EIE is maintaining its load demand and $TM = 1$, the following state transition is true: ① when the state is $(p,0,0,1,1)$, it can transfer to the state $(p,0,0,1,1)$ or it also can transfer to the state $(p',1,0,0,1)$ with $0 \le p' - p \le R$ and state $(p',0,1,0,1)$ with $0 \le p - p' \le R$;

6) As the EIE is maintaining its load demand and $TM > 1$, the following state transitions are true: ① when the state is $(p,0,0,1,1)$, it only can transfer to state $(p,0,0,1,2)$; ② when the state is $(p,0,0,1,l)$ and $1 < l < TM$, it only can transfer to the state $(p,0,0,1,l+1)$; ③ when the state is $(p, 0,0,1,TM)$, it can transfer to state $(p,0,0,1,TM)$ or it also can transfer to the state $(p',1,0,0,1)$ with $0 \le p' - p \le R$ and state $(p',0,1,0,1)$ with $0 \le p - p' \le R$;

The following is a diagram to roughly describe these above state transitions, i.e.,:

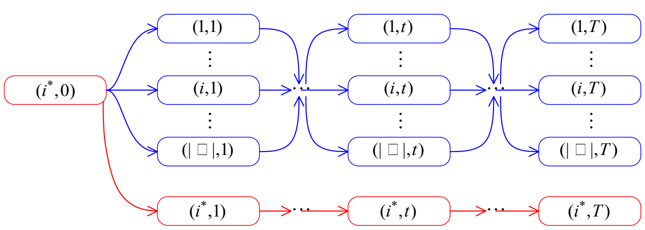


Fig. 2. State transitions diagram

where the tuples describing the states of the EIE are uniquely denoted as $1,\dots, i,\dots,|\Sigma|$ in the Fig. 2, and the (i,t) represents the EIE remains state i at period t. It should be mentioned that the arrow in this figure represents the transition between the states that are connected together, and this arrow is realized only if the conditions mentioned above are satisfied between these states.

(ii) DP Model for the Self-scheduling EIEs

The self-scheduling EIE model aims to maximize the payoff. To realize this goal, the EIE can construct the Bellman equations for its DP model using the following formulations of optimal state value functions and rewards. First, the optimal state value functions are represented by state variables (i.e., $v(i,t)/v(j,t)$), where all states in the state space Σ are uniquely represented as i/j. These state variables are further applied to indicate the maximum payoff from period t+1 to the end T when the EIE is in the state i/j at period t. Second, the rewards are represented by $V(i,j,t+1)$, which contain any possible payoffs when the EIE is in the state i/j at period t+1, respectively. Then, with the optimal state value functions and the rewards, we obtain the bellman equation for the DP model of the EIE as:

$$v(i,t) = \max_{\forall j \in \mathsf{S}_{\text{from}}(i)} \{V(i,j,t+1) + v(j,t+1)\} \quad ,i \in \mathsf{S}, t \in \{1,\dots,T-1\} \tag{5}$$

where $\Sigma_{\text{from}}(i)$ denotes a set, which contains all possible states that can transfer from the state i. These rewards, i.e., $V(i,j,t)$, are called payoff blocks henceforth, which can be expressed as:

$$V(i,j,t+1) = (\pi_{t+1} - \lambda_{t+1})\,p(j) - \beta\left|\delta(p(i) - p(j))\right| \tag{6}$$

where $p(i)$ and $p(j)$ in (6) indicate the load demands when the EIE remains the states i and j, respectively.

Assuming that the EIE has been maintaining enough time and satisfying the maintenance limitation before the first period, the maximum payoff of the whole self-scheduling model according to these bellman equations (2) can be represented as:

$$z = \max \quad \{V(i,1) + v(i,1) \mid i \in \mathsf{S}\} \tag{7}$$

where z in (7) refers specifically to the maximum payoff that the EIE can earn from the whole scheduling duration. $V(i,1)$ denotes the payoff of the EIE when it remains state i at the first period. Then, the DP model of the self-scheduling EIEs is described as follows:

$$z = \max \quad \{V(i,1) + v(i,1) \mid i \in \mathsf{S}\} \tag{8a}$$

$$v(i,t) = \max_{\forall j \in \mathsf{S}_{\text{from}}(i)} \{V(i,j,t+1) + v(j,t+1)\} \quad ,i \in \mathsf{S}, t \in \{1,\dots,T-1\} \tag{8b}$$

(iii) Convex Hull for the Self-scheduling EIE Model

Notice that all state transitions, which are denoted in the DP model (8), exist the formulation, "$v = \max\{v_1,\dots, v_N\}$". It can be equivalently replaced as "$\min\{v \mid v \ge v_1,\dots, v \ge v_N\}$". With this replacement, the DP model (8) can be reformulated into the following LP model:

$$\min \quad z \tag{9a}$$

$$\textit{\textbf{s.t.}} \quad z \ge V(i,1) + v(i,1) : w(i,1), i \in \mathsf{S} \tag{9b}$$

$$v(i,t) \ge V(i,j,t+1) + v(j,t+1) : w(i,j,t+1) \quad ,i \in \mathsf{S}, j \in \mathsf{S}_{\text{from}}(i), t \in \{1,\dots,T-1\} \tag{9c}$$

where $w(i,1)$ and $w(i,j,t+1)$ are the dual variables of the corresponding inequalities.

This LP model (9) can provide the maximum payoff, which is the optimal value of its objective function, but the optimal self-scheduling solution is still unknown. This is because the optimal solution of this LP model corresponds to the optimal state value functions (i.e., $v(i,t)/v(j,t)$) of the DP model instead of the optimal self-scheduling solution of the EIE. It is challenging to recover the optimal self-scheduling solution of the EIE from the optimal state value functions. Fortunately, some payoff blocks in the LP model (9) (i.e., $V(i,j,t+1)$) can help derive the optimal self-scheduling solution (see the following **Proposition 1**). Specifically, these payoff blocks (i.e., $V(i,j,t+1)$) not only indicate the payoff between consecutive states, but also denote the states of the EIE at the period t+1. Since the maximum payoff is composed of these payoff blocks, the optimal self-scheduling solution can be derived from the combination of the specific states denoted by these payoff blocks.

Proposition 1: The maximum payoff of the model (9) is composed of payoff blocks (i.e., $V(i,j,t+1)$).

Proof: Since there are all recursions in the LP model, the state value functions (i.e., $v(i,t)$) can be eliminated as follows:

$$v(i,t) \Rightarrow V(i,j,t+1) + v(j,t+1) \quad ,i \in \mathsf{S}, j \in \mathsf{S}_{\text{from}}(i), t \in \{1,\dots,T-1\} \tag{10}$$

We can perform the above elimination process repeatedly until any "$v(i,t)$" on the right-hand side is replaced by "$V(i,j,T)+v(j,T)$". Then, except "$v(j,T)$", all state value functions (i.e., $v(i,t)$) are eliminated, and the LP model (9) will be reformulated as:

$$\min \quad \left\{ z \middle| z \geq P_1 + \cdots + P_n, [P_1 \cdots P_n]^{\mathrm{T}} \in \Theta_{\boldsymbol{P}} \right\} \tag{11}$$

where P_n in the model (11) expresses the payoff blocks (i.e., $V(i,j,t)$) and $\Theta_{\boldsymbol{P}}$ is a set involving all elimination results.

Note that the optimal value of (11) comprises these payoff blocks. Since the LP model (9) is equivalent to (11), the maximum payoff (i.e., the optimal value of the LP model (9)) also comprises payoff blocks (i.e., $V(i,j,t)$), proving the **Proposition 1** statement.

(Q.E.D)

Proposition 1 means that the maximum payoff of the model (9) is composed of an exact payoff block combination (i.e., an exact combination of $V(i,j,t)$). However, this exact combination is still not directly extracted from the LP model (9). Its dual model can provide the exact combination, as mentioned in **Proposition 2**. This is because the dual model can present the same optimal value of the objective function as the LP model (9) (i.e., maximum payoff) since strong duality always holds for a feasible LP model [35]. Meanwhile, for the dual model, the objective function coefficients are composed of these constant terms, i.e., payoff blocks, which are the constant terms on the right-hand sides of constraints in the original LP model. This means the dual objective function presents a combination of payoff blocks for maximizing the payoff. Therefore, solving the dual model of the LP model (9) (i.e., model (12)), its optimal solution can provide the exact combination of payoff blocks for maximizing the payoff, which can derive its optimal self-scheduling solution.

$$\max \quad \sum_{i\in \mathsf{S}}\left[w(i,1)V(i,1) + \sum_{t=2}^{T}\sum_{j\in \mathsf{S}_{\text{from}}(i)} w(i,j,t)V(i,j,t) \right] \tag{12a}$$

$$\textit{\textbf{s.t.}} \quad \sum_{i\in \mathsf{S}} w(i,1) = 1 \tag{12b}$$

$$-w(i,1) + \sum_{j\in \mathsf{S}_{\text{from}}(i)} w(i,j,2) = 0, i \in \mathsf{S} \tag{12c}$$

$$-\sum_{k\in \mathsf{S}_{\text{to}}(i)} w(k,i,t) + \sum_{j\in \mathsf{S}_{\text{from}}(i)} w(i,j,t+1) = 0 \\ ,i \in \mathsf{S}, t \in \{2,...,T-1\} \tag{12d}$$

$$w(i,1) \geq 0, i \in \mathsf{S} \tag{12e}$$

$$w(i,j,t+1) \geq 0, i \in \mathsf{S}, j \in \mathsf{S}_{\text{from}}(i), t \in \{1,...,T-1\} \tag{12f}$$

Proposition 2: The optimal solution of dual model (12) (e.g., $w^*(i,1)$ and $w^*(i,j,t+1)$) will provide the exact combination of payoff blocks for maximizing the payoff, deriving the optimal self-scheduling solution.

Proof: After representing the LP model (9) by (11), the resulting equivalent model and the dual form of this equivalent model can be expressed as (13) and (14), respectively:

Equivalent model:

$$\min \quad \left\{ z \middle| z \geq \mathbf{1}^{\mathrm{T}} \boldsymbol{P}_i : \alpha_i, \boldsymbol{P}_i \in \Theta_{\boldsymbol{P}} \right\} \tag{13}$$

Dual form of the equivalent model:

$$\max \quad \left\{ \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i \left(\mathbf{1}^{\mathrm{T}} \boldsymbol{P}_i \right) \middle| \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i = 1, \alpha_i \geq 0 \right\} \tag{14}$$

where α_i indicates the dual variable of its corresponding inequality, $\boldsymbol{P}_i$ indicates one of all possible payoff block combinations, and $\Theta_{\boldsymbol{P}}$ denotes all possible combinations of payoff blocks.

Since a feasible LP model always satisfies the strong duality, the problems stated in (13) and (14) have the same optimal values, such that

$$z^* = \mathbf{1}^{\mathrm{T}} \boldsymbol{P}^* = \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i^* \left(\mathbf{1}^{\mathrm{T}} \boldsymbol{P}_i \right) \tag{15}$$

where z^* is the optimal value of (13), $\boldsymbol{P}^*$ denotes the exact combination of payoff blocks for maximizing the payoff (i.e., $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*=z^*$), and α_i^* is the optimal solution of (14).

Moreover, since $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*$ indicates the optimal value of (13), then $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i \leq \mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*$, where $\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}$. Combining this with (15) and with the given $\sum_{\boldsymbol{P}_i\in\Theta_{\boldsymbol{P}}}\alpha_i^*=1$ and $\alpha_i^* \geq 0$, we have

$$\mathbf{1}^{\mathrm{T}} \boldsymbol{P}^* = \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i^* \left(\mathbf{1}^{\mathrm{T}} \boldsymbol{P}_i \right) \leq \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i^* (\mathbf{1}^{\mathrm{T}} \boldsymbol{P}^*) = \mathbf{1}^{\mathrm{T}} \boldsymbol{P}^* \tag{16}$$

This indicates that $\sum_{\boldsymbol{P}_i\in\Theta_{\boldsymbol{P}}}\alpha_i^* (\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*)=\sum_{\boldsymbol{P}_i\in\Theta_{\boldsymbol{P}}}\alpha_i^*(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i)$, which can be rewritten as:

$$\sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i^* (\mathbf{1}^{\mathrm{T}} \boldsymbol{P}_i^* - \mathbf{1}^{\mathrm{T}} \boldsymbol{P}_i) = 0 \tag{17}$$

Since $\alpha_i^* \geq 0$ and $(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i) \geq 0$, then $\alpha_i^*(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i) \geq 0$. Combining $\alpha_i^*(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i) \geq 0$ with equality (17), then we have $\alpha_i^*(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i)=0$. Then, we further present the following discussion to determine α_i^*. To avoid any confusion, subscripts j and k are applied to distinguish these two situations:

1) if $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_j \neq 0$, then $\alpha_j^*=0$;
2) if $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_k=0$, then $\alpha_k^*=1$.

where α_j^*/α_k^* and $\boldsymbol{P}_j/\boldsymbol{P}_k$ are involved in α_i^* and $\boldsymbol{P}_i$, respectively. For the first situation, $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_j \neq 0$ requires that $\alpha_j^*=0$ because $\alpha_i^*(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i)=0$. For the second situation, $\alpha_k^*=1$ can be proven by a contradiction method. Since $\boldsymbol{P}^*$ is the unique exact combination of payoff blocks for maximizing the payoff, it suggests that $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_k=0$ if and only if $\boldsymbol{P}_k=\boldsymbol{P}^*$. Suppose that $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_k=0$ while $\alpha_k^* \neq 1$, and this $\alpha_k^* \neq 1$ requires that there must exist one $\alpha_j^*>0$ when $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_j \neq 0$ under the given condition of $\sum_i\alpha_i^*=1$. However, this result is a contradiction since $\alpha_i^*(\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_i)=0$ for any α_i^*. Therefore, this case suggests that if $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_k=0$, then $\alpha_k^*=1$. So far, both situations (i.e., if $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_j \neq 0$, then $\alpha_j^*=0$ and if $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_k=0$, then $\alpha_k^*=1$) are always true.

Since $\alpha_k^*=1$ corresponds to $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_k=0$ (where if and only if $\boldsymbol{P}_k=\boldsymbol{P}^*$) and $\alpha_j^*=0$ corresponds to $\mathbf{1}^{\mathrm{T}}\boldsymbol{P}^*-\mathbf{1}^{\mathrm{T}}\boldsymbol{P}_j \neq 0$, then the exact combination of payoff blocks for the maximum payoff can be provided by

$$\boldsymbol{P}^* = \boldsymbol{P}_k = \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}} / \{\boldsymbol{P}_k\}} \alpha_j^* \boldsymbol{P}_j + \boldsymbol{P}_k = \sum_{\boldsymbol{P}_i \in \Theta_{\boldsymbol{P}}} \alpha_i^* \boldsymbol{P}_i \tag{18}$$

We accordingly have the following two conclusions. First, the optimal solution of the dual model (12) can provide the exact combination of payoff blocks for maximizing the payoff. Second, these payoff blocks (i.e., $V(i,j,t+1)$) also can denote the load-increasing, the load-decreasing, and the load-maintaining states of the EIE during the period $t+1$. Then, the optimal solution of the dual model (12) can derive the optimal self-scheduling solution.

(Q.E.D)

According to **Proposition 2**, we further summarize the physical interpretation of dual variables in Table I, which can guide the self-scheduling EIE to earn the maximum payoff.

TABLE I. PHYSICAL MEANING OF DUAL VARIABLES IN (12)

Var.	Physical Interpretation
$w(i,1)$	When $w(i,1)$=1, the EIE is required to remain the state i at period t; otherwise $w(i,1)$=0
$w(i,j,t+1)$	When $w(i,j,t+1)$=1, the EIE is required to remain the state j at period t+1; otherwise $w(i,j,t+1)$=0

Based on these physical meanings of dual variables, we can infer the load demand, the load-increasing states, the load-decreasing states, and the load-maintaining states as follows:

$$p_t = \begin{cases} \sum_{i\in \mathsf{S}} p(i)w(i,1) & t=1 \\ \sum_{j\in \mathsf{S}}\sum_{i\in \mathsf{S}_{\mathrm{to}}(i)} p(j)w(i,j,t) & t=\{2,...,T\} \end{cases} \tag{19}$$

$$u_t = \begin{cases} \sum_{i\in \mathsf{S}} u(i)w(i,1) & t=1 \\ \sum_{j\in \mathsf{S}}\sum_{i\in \mathsf{S}_{\mathrm{to}}(i)} u(j)w(i,j,t) & t=\{2,...,T\} \end{cases} \tag{20}$$

$$d_t = \begin{cases} \sum_{i\in \mathsf{S}} d(i)w(i,1) & t=1 \\ \sum_{j\in \mathsf{S}}\sum_{i\in \mathsf{S}_{\mathrm{to}}(i)} d(j)w(i,j,t) & t=\{2,...,T\} \end{cases} \tag{21}$$

$$m_t = \begin{cases} \sum_{i\in \mathsf{S}} m(i)w(i,1) & t=1 \\ \sum_{j\in \mathsf{S}}\sum_{i\in \mathsf{S}_{\mathrm{to}}(i)} m(j)w(i,j,t) & t=\{2,...,T\} \end{cases} \tag{22}$$

where $p(i)$ and $p(j)$ are load demands when the EIE remains the states i and j, respectively; $u(i)$ and $u(j)$ are load-increasing states when the EIE remains the states i and j, respectively; $d(i)$ and $d(j)$ are load-decreasing states when the EIE remains the states i and j, respectively; $m(i)$ and $m(j)$ are load-maintaining states when the EIE remains the states i and j, respectively. Based on the interpretations (19)-(22), the convex hull model can be denoted as the following formulation:

$$\max \quad \sum_{t=1}^{T}\pi_t p_t - \sum_{t=1}^{T}\lambda_t p_t - \sum_{t=1}^{T-1}\beta\delta(p_{t+1}-p_t) \tag{23a}$$

s.t. (12b)-(12f) (23b)

(19)-(22) (23c)

This is the final convex hull model of the self-scheduling EIE. With this proposed convex hull model, the maximum payoff and its optimal scheduling of the self-scheduling EIE can be quickly obtained with an average O(K[|Σ|×T]) computational time complexity by this convex hull constructed through this proposed LP model [36]. Here, K[•] in "O(K[|Σ|×T])" indicates the relationship between the number of variables and the time complexity, T is the time scale, |Σ| is the number of the states in the state space of the self-scheduling EIE.

Discussions: Replacing the IP model (1) with this equivalent convex hull (12) for the self-scheduling EIE model will present several advantages.

1) Time Complexity: The EIE manufacturing requirements usually introduce many binaries in its self-scheduling model, making its solution hard. The time complexity will be limited in the polynomial time complexity by constructing the convex hull for the self-scheduling EIE model. This supports the more complex and elaborate the EIE modeling.

2) Strong Duality: The marginal costs would guide the self-scheduling EIEs to make a payoff. Usually, marginal costs can be obtained by calculating the dual variables of the original problem. However, strong duality does not hold due to IP. For the convex hull model, the original IP model is equivalently reformulated into an LP model, which is a convex optimization. The strong duality always holds if the model is feasible and the dual information is easily obtained. This will help guide the self-scheduling EIE.

IV. CASE STUDY

In this section, the proposed convex hull provided by the LP model (12) is investigated to substantiate its performance in precision and computational efficiency. Moreover, this convex hull is eventually utilized in benefits analysis, illustrating the potential value of the self-scheduling EIE in response to the DR. All experiments are built with Python 3.11 and GUROBI 10.0.2 on a desktop equipped with an 13th Gen Intel(R) Core (TM) i7-13700K and a 32GB RAM.

A. Parameter Description

These critical parameters are provided in Table II for a self-scheduling EIE (i.e., an electrolytic aluminum plant [34]) composed of the load demand range, the ramp-up/ramp-down rate, the start-up/shut-down cost, the regulation time, and the maintenance time. Other major parameters including time granularity and electricity budget are presented in Table III. In addition, all these load demand setpoints are listed as 665MW+n×8.75MW (where n∈{0,1,2,3,4,5,6,7,8}), which are contained in the set Π involving all discrete operation points.

TABLE II. CRITICAL PARAMETERS OF SELF-SCHEDULING EIE

Parameter	Value
Minimum Load Demand (MW)	665
Maximum Load Demand (MW)	735
Ramp-up/ramp-down Rate (MW/h)	35
Start-up/shut-down Cost ($/8.75WM)	100
Regulation Time (h)	1
Maintenance Time (h)	2

TABLE III. MAJOR PARAMETERS IN THE MODELS

Parameter	Value
Time granularity (h)	0.25
Electricity Budget ($/MW)	50

B. Precision of Convex Hull Provided by LP Model

The precision of the convex hull provided by the LP model (12) can be estimated by comparing the relative solution errors between the IP model (1) and the convex hull. The relative errors are defined as:

$$error = \frac{z^{\mathrm{LP}} - z^{\mathrm{IP}}}{z^{\mathrm{IP}}} \tag{24}$$

where z^{LP} and z^{IP} denote the optimal costs, which are obtained by the LP model (12) and the IP model (1), respectively. This subsection calculates 1000 relative errors according to 1000 random electricity price series and presents the frequency and the probability density function of relative errors in Fig. 3. Note that the relative errors approximately obey the truncated distribution, with its probability density function shown in Fig. 3 that is fitted by a normal distribution, i.e.:

$$f(x|\mu,\sigma) = \frac{2}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{25}$$

where μ and σ, which indicate the location parameter and the scale parameter of this normal distribution, are -1.7113×10^{-13}%

and $6.1187\times10^{-12}\%$, respectively.

Based on the Pauta criterion, there are more than 99.73% of relative errors are located in the interval [$-1.8527\times10^{-11}\%$, $1.8185\times10^{-11}\%$]. Specifically, the maximum relative error is $5.9610\times10^{-11}\%$. This suggests that the relative solution error between LP and IP models is small enough to be ignored, indicating that the self-scheduling solution proposed by the convex hull is valid for the self-scheduling EIE operation.

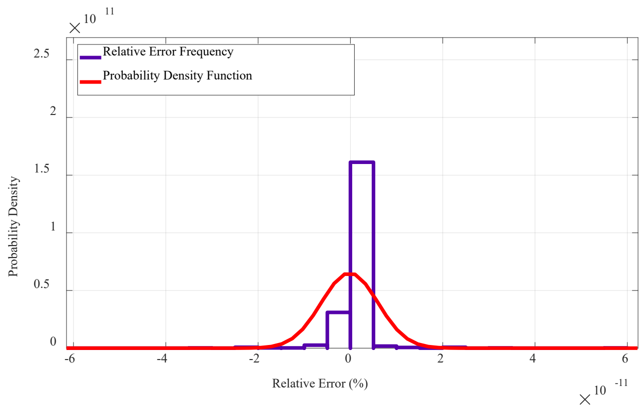


Fig. 3. Frequency histogram of the relative errors.

C. Computational Efficiency of the Convex Hull Model

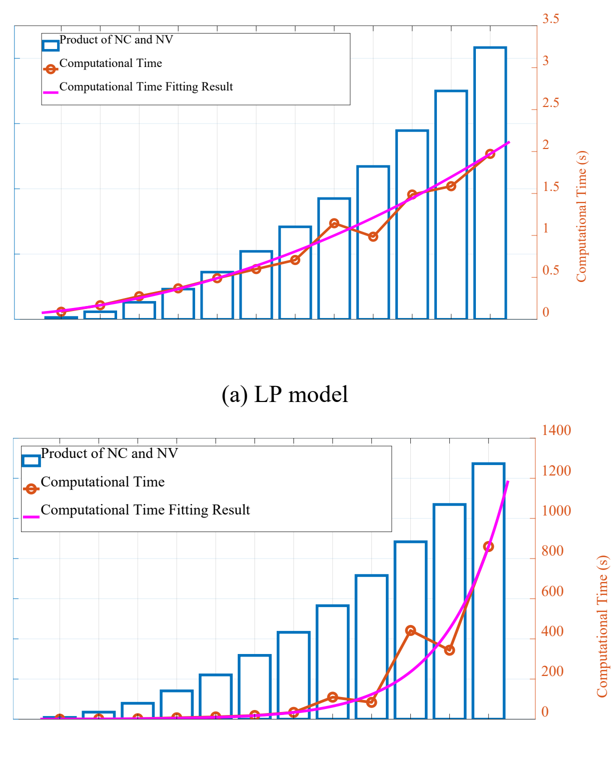
The model size and computational time are discussed in this subsection to illustrate that the convex hull is efficient. Here, Table IV lists the numbers of constraints and variables in the LP (12) and IP model (1) for different time scales, including 1 week, 2 weeks, …, and 12 weeks.

TABLE IV. NUMBER OF CONSTRAINTS AND VARIABLES

Time scales	No. of Constraints		No. of Variables	
	LP	IP	LP	IP
15min×4×24×7×1	14207	1137	20432	768
15min×4×24×7×2	28415	2289	40784	1536
15min×4×24×7×3	42623	3441	61136	2304
15min×4×24×7×4	56831	4593	81488	3072
15min×4×24×7×5	71039	5745	101840	3840
15min×4×24×7×6	85247	6897	122192	4608
15min×4×24×7×7	99455	8049	142544	5376
15min×4×24×7×8	113663	9201	162896	6144
15min×4×24×7×9	127871	10353	183248	6912
15min×4×24×7×10	142079	11505	203600	7680
15min×4×24×7×11	156287	12657	223952	8448
15min×4×24×7×12	170495	13809	244304	9216

As Table IV presents, the numbers of constraints in LP and IP models are approximately equal. However, the LP model will contain a greater number of continuous variables, indicating that more continuous variables are required when providing this convex hull. Although this LP model contains more continuous variables than the IP model, it is still well-performed and computationally efficient because the LP model does not need any integer variable. Fig. 4 shows the LP model superiority of the computation time and model sizes between LP and IP models. Note that the size of LP and IP models is stated as the product of NC and NV, denoting the numbers of constraints and variables, respectively. As Fig. 4 shows, with the time scale growing from 1 week to 12 weeks, the LP model increases its size from 2.903×10^{8} to 4.165×10^{10} and the IP model increases its size from 7.928×10^{6} to 1.273×10^{8}. Concerning the model sizes, the LP model is much greater than the IP model. However, the computational time of LP and IP models present linear and exponential growth with the time scale increasing, respectively. The results indicate that the computational time for solving the IP model will increase faster than that of the LP model with the growth of the time scale. For instance, it will take about 0.090s to solve the LP model on the 1 week time scale and about 1.972s on the 12 weeks time scale, while solving the IP model on the 1 week time scale will take about 0.144s and about 859.939s on the 12 weeks time scale.

Fig. 5 illustrates this result through the computational time proportions of the LP model to the IP model. As the proportion fitting result in Fig. 5 shows, the proportions are decreasing with the growth of time scale, indicating that the LP brings increasingly obvious improvement in computational efficiency as the model size expands. This suggests that the convex hull provided by the LP model performs well in computational efficiency, which will become more apparent as the model size increases when dealing with more complex EIE models.



(a) LP model

(b) IP model

Fig. 4. Computation time and model size of the models.

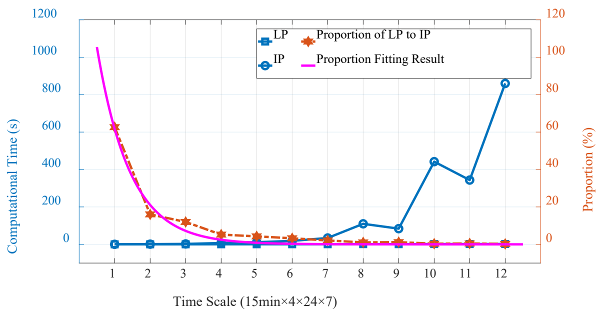


Fig. 5. Computation time relation for the models.

Furthermore, we employ a case, whose time scale is ranging from 1 week to 52 weeks (i.e., a whole year), to test the performance of our convex hull model in computational efficiency. The computational time is shown in Fig.6. The convex hull model is very efficient, even though the time scale is 52 weeks (i.e., a whole year), which completes its solving in only 6.983s. Moreover, as the time scale is ranging from 1 week to 52 weeks, the required computational time of the convex hull model is ranging from 0.090s to 6.983s. This suggests that increasing the model scale will not lead to an explosion in the computational time. This evidence further validates that the application prospect of the convex hull model in improving computational efficiency.

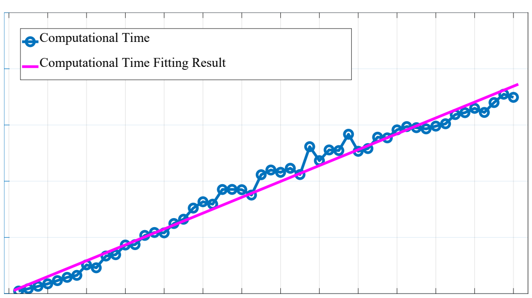


Fig. 6. Computation time for the convex hull model with varying time scales.

D. Merits of Self-Scheduling EIE with DR

This subsection utilizes a comparison example to show that self-scheduling EIE can respond to the DR to gain more payoff and provide additional flexibility to the power system. There are two examples in the comparison example. One is that the self-scheduling EIE takes the DR into consideration. This self-scheduling EIE can regulate its load demand according to the real-time electricity price to maximize its payoff. Another is the self-scheduling EIE without considering the DR, and its load demand is always maintained at the middle level, i.e., 700MW. The time resolution of these examples is set as 15 minutes, and the whole time scales are both 24 hours. That is, these examples are both single-day scheduling problems with 96 operational horizons. Among them, the applied real-time electricity prices are taken from MISO's public data [37]. Considering the DR, the self-scheduling EIE is willing to decrease its load demand when the electricity is expensive and increase its load demand when the electricity is cheap. TABLE V denotes the start-up and shut-down costs corresponding to the load demand regulation of these two examples. Furthermore, as Fig. 7 shows, the self-scheduling EIE decreases its load demand from 735 MW to 700 MW during the periods from 5:30 a.m. to 6:30 a.m., and from 700 MW to 665 MW during the periods from 9:30 a.m. to 10:30 a.m.. On the contrary, the self-scheduling EIE increases its load demand from from 665 MW to 700 MW during the periods from 2:30 p.m. to 5:30 p.m., and from 700 MW to 735 MW during the periods from 7:30 p.m. to 8:30 p.m.. Moreover, the load demand regulation motivated by the varying electricity prices can seek ±35 MW extra flexibilities to the power system, which indicates that the EIE can provide the flexibilities to the system through self-scheduling to respond to the DR.

TABLE V. START-UP/SHUT-DOWN COST AND LOAD DEMAND REGULATION

Period (×15min)	**Cost** ($)	**DR** (MW)	**Period** (×15min)	**Cost** ($)	**DR** (MW)
23	100	-8.75	67	100	8.75
24	100	-8.75	68	100	8.75
25	100	-8.75	69	100	8.75
26	100	-8.75	70	100	8.75
39	100	-8.75	79	100	8.75
40	100	-8.75	80	100	8.75
41	100	-8.75	81	100	8.75
42	100	-8.75	82	100	8.75

What's more, the payoff of the self-scheduling EIE has been investigated. Fig. 8 shows that the extra payoffs can be earned from the DR, where "Payoff with DR" and "Payoff without DR" denote the payoffs that the EIE presents at each period when the EIE considers and does not consider the DR, respectively; "Cumulative Extra Payoff" denotes the cumulative extra payoffs between the EIEs with and without the DR from the start period to the current period. After taking the DR into account, the total payoff of the self-scheduling EIE will increase by 4708.6 $ compared to that without the DR.

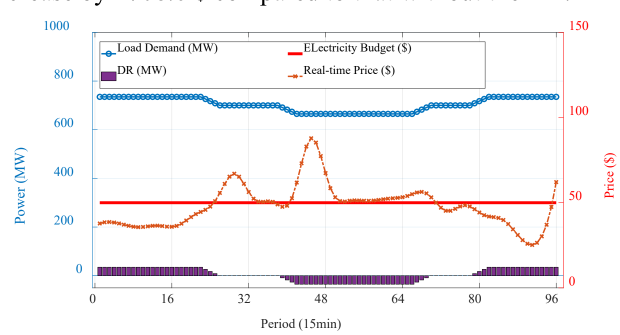


Fig. 7. EIE self-scheduling results with the DR.

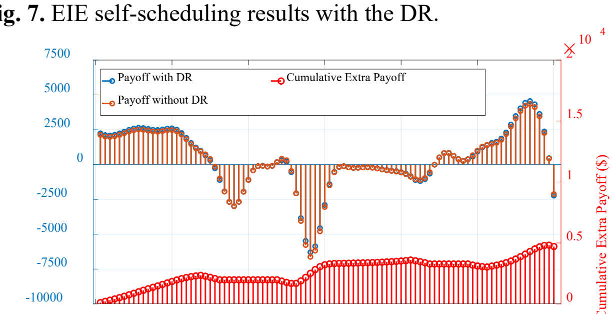


Fig. 8. Extra payoff earned from the DR.

V. Conclusion

The convex hull theory is increasingly attractive because it can be solved efficiently to provide precise dual information, which cannot otherwise be provided by IP. This paper proposes a convex hull model for the self-scheduling EIE with the DR. This framework transforms the presented basic model of self-scheduling EIE with the DR into a DP model and then reformulates the DP model into an LP model. Eventually, this LP model describes the convex hull of the self-scheduling EIE with the DR. Numerical cases are carried out to demonstrate the precision and computational efficiency of the proposed convex hull. For precision, the proposed convex hull is consistent with the IP model since more than 99.73% of relative errors are located in $[-1.8527, 1.8185]\times10^{-11}$%. For computational efficiency, the computation time of the LP model presents a linear growth as the model size increases, while the original IP model presents an exponential growth. This means that the proposed convex hull will perform better in computational efficiency as the model size is expanded beyond that of the IP model. In addition, the proposed convex hull is utilized to illustrate the benefits of the self-scheduling EIE in response to the DR. Numerical results show that, motivated by varying electricity prices, the self-scheduling EIE with the DR will regulate its load, better than that without the DR, to make an extra payoff and provide extra flexibilities to the power system.

References

[1] T. Ding, Y. Lin, G. Li, and Z. Bie, "A New Model for Resilient Distribution Systems by Microgrids Formation," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 4145–4147, Sep. 2017.

[2] S. Sen and S. Ganguly, "Opportunities, barriers and issues with renewable energy development – A discussion," *Renewable and Sustainable Energy Reviews*, vol. 69, pp. 1170–1181, Mar. 2017.

[3] S. Guo, Q. Liu, J. Sun, and H. Jin, "A review on the utilization of hybrid renewable energy," *Renewable and Sustainable Energy Reviews*, vol. 91, pp. 1121–1147, Aug. 2018.

[4] T. Ding, C. Li, Y. Yang, J. Jiang, Z. Bie, and F. Blaabjerg, "A Two-Stage Robust Optimization for Centralized-Optimal Dispatch of Photovoltaic Inverters in Active Distribution Networks," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 2, pp. 744–754, Apr. 2017.

[5] J. Ma, V. Silva, R. Belhomme, D. S. Kirschen, and L. F. Ochoa, "Evaluating and Planning Flexibility in Sustainable Power Systems," *IEEE Transactions on Sustainable Energy*, vol. 4, no. 1, pp. 200–209, Jan. 2013.

[6] S. Varghese, S. Dalvi, A. Narula, and M. Webster, "The Impacts of Distinct Flexibility Enhancements on the Value and Dynamics of Natural Gas Power Plant Operations," *IEEE Transactions on Power Systems*, vol. 36, no. 6, pp. 5803–5813, Nov. 2021.

[7] D. A. Tejada-Arango, G. Morales-España, S. Wogrin, and E. Centeno, "Power-Based Generation Expansion Planning for Flexibility Requirements," *IEEE Transactions on Power Systems*, vol. 35, no. 3, pp. 2012–2023, May 2020.

[8] Y. Zheng *et al.*, "Mixed Game Theory Based Coordinated Development and Planning of Wind Power, Thermal Power and Grid," *IOP Conf. Ser.: Earth Environ. Sci.*, vol. 647, no. 1, p. 012123, Jan. 2021.

[9] A. R. Jordehi, "Optimisation of demand response in electric power systems, a review," *Renewable and Sustainable Energy Reviews*, vol. 103, pp. 308–319, Apr. 2019.

[10] N. O′Connell, P. Pinson, H. Madsen, and M. O′Malley, "Benefits and challenges of electrical demand response: A critical review," *Renewable and Sustainable Energy Reviews*, vol. 39, pp. 686–699, Nov. 2014.

[11] W. Guo *et al.*, "Demand-side resource response characteristics and peak shaving scheduling response model," *Energy Reports*, vol. 8, pp. 586–593, Sep. 2022.

[12] H. Wu, M. Shahidehpour, and M. E. Khodayar, "Hourly Demand Response in Day-Ahead Scheduling Considering Generating Unit Ramping Cost," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2446–2454, Aug. 2013.

[13] Q. Wang, J. Wang, and Y. Guan, "Stochastic Unit Commitment With Uncertain Demand Response," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 562–563, Feb. 2013.

[14] M. Asensio, G. Muñoz-Delgado, and J. Contreras, "Bi-Level Approach to Distribution Network and Renewable Energy Expansion Planning Considering Demand Response," *IEEE Transactions on Power Systems*, vol. 32, no. 6, pp. 4298–4309, Nov. 2017.

[15] Z. Wang, F. Gao, Q. Zhai, X. Guan, J. Wu, and K. Liu, "Electrical Load Tracking Analysis for Demand Response in Energy Intensive Enterprise," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 1917–1927, Dec. 2013.

[16] E. Gomez-Villalva and A. Ramos, "Optimal energy management of an industrial consumer in liberalized markets," *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 716–723, May 2003.

[17] M. Paulus and F. Borggrefe, "The potential of demand-side management in energy-intensive industries for electricity markets in Germany," *Applied Energy*, vol. 88, no. 2, pp. 432–441, Feb. 2011.

[18] M. H. Shoreh, P. Siano, M. Shafie-khah, V. Loia, and J. P. S. Catalão, "A survey of industrial applications of Demand Response," *Electric Power Systems Research*, vol. 141, pp. 31–49, Dec. 2016.

[19] C. A. Babu and S. Ashok, "Peak Load Management in Electrolytic Process Industries," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 399–405, May 2008.

[20] R. Chen, H. Sun, Q. Guo, H. Jin, W. Wu, and B. Zhang, "Profit-seeking energy-intensive enterprises participating in power system scheduling: Model and mechanism," *Applied Energy*, vol. 158, pp. 263–274, Nov. 2015.

[21] J. Shu, R. Guan, L. Wu, and B. Han, "A Bi-Level Approach for Determining Optimal Dynamic Retail Electricity Pricing of Large Industrial Customers," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2267–2277, Mar. 2019.

[22] F. Angizeh, M. Parvania, M. Fotuhi-Firuzabad, and A. Rajabi-Ghahnavieh, "Flexibility Scheduling for Large Customers," *IEEE Transactions on Smart Grid*, vol. 10, no. 1, pp. 371–379, Jan. 2019.

[23] M. Abedi, R. Chiong, N. Noman, X. Liao, and D. Li, "A Metaheuristic Framework for Energy-Intensive Industries With Batch Processing Machines," *IEEE Transactions on Engineering Management*, pp. 1–15, 2022.

[24] Y. Zhou, Q. Zhai, M. Zhou, and X. Li, "Generation Scheduling of Self-Generation Power Plant in Enterprise Microgrid With Wind Power and Gateway Power Bound Limits," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 2, pp. 758–770, Apr. 2020.

[25] J. Wang, Q. Wang, and W. Sun, "Quantifying flexibility provisions of the ladle furnace refining process as cuttable loads in the iron and steel industry," *Applied Energy*, vol. 342, p. 121178, Jul. 2023.

[26] S.-Y. Lee, G.-S. Lee, S.-I. Moon, and Y.-T. Yoon, "Optimization of iron and steel manufacturing plant considering electricity price tariff and electric arc furnace control," *IET Generation, Transmission & Distribution*, vol. 17, no. 22, pp. 5027–5040, 2023.

[27] H. Li, J. Wang, Z. Shen, C. Gu, Q. Li, and Q. Ren, "Optimal energy base planning method with integrated industry high-consumption demand response and dynamic line rating," *Electric Power Systems Research*, vol. 225, p. 109795, Dec. 2023.

[28] A. Frangioni and C. Gentile, "Solving Nonlinear Single-Unit Commitment Problems with Ramping Constraints," *Operations Research*, vol. 54, no. 4, pp. 767–775, Aug. 2006.

[29] Y. Guan, K. Pan, and K. Zhou, "Polynomial time algorithms and extended formulations for unit commitment problems," *IISE Transactions*, vol. 50, no. 8, pp. 735–751, Aug. 2018.

[30] M. Qu, T. Ding, Y. Sun, C. Mu, K. Pan, and M. Shahidehpour, "Convex Hull Model for a Single-Unit Commitment Problem with Pumped Hydro Storage Unit," *IEEE Transactions on Power Systems*, pp. 1–12, 2022.

[31] H. Cui and K. Zhou, "Industrial power load scheduling considering demand response," *Journal of Cleaner Production*, vol. 204, pp. 447–460, Dec. 2018.

[32] S. Mitra, I. E. Grossmann, J. M. Pinto, and N. Arora, "Optimal production planning under time-sensitive electricity prices for continuous power-intensive processes," *Computers & Chemical Engineering*, vol. 38, pp. 171–184, Mar. 2012.

[33] L. Gan, T. Yang, X. Chen, G. Li, and K. Yu, "Purchased Power Dispatching Potential Evaluation of Steel Plant With Joint Multienergy System and Production Process Optimization," *IEEE Transactions on Industry Applications*, vol. 58, no. 2, pp. 1581–1591, Mar. 2022.

[34]H. Jin, Z. Li, H. Sun, Q. Guo, R. Chen, and B. Wang, “A robust aggregate model and the two-stage solution method to incorporate energy intensive enterprises in power system unit commitment,” *Applied Energy*, vol. 206, pp. 1364–1378, Nov. 2017.

[35]S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge: Cambridge University Press, 2004.

[36]M. B. Cohen, Y. T. Lee, and Z. Song, “Solving Linear Programs in the Current Matrix Multiplication Time,” *J. ACM*, vol. 68, no. 1, p. 3:1-3:39, Jan. 2021.

[37]“MISO Market Data.” https://www.misoenergy.org/markets-and-operations/real-time--market-data (accessed Dec. 29, 2022).

Yang Xiao (Student Member, IEEE) received the B.S. degree from the School of Electrical Engineering, Xi’an Jiaotong University, Xi’an, China, in 2021. He is currently working toward the Ph.D. degree at Xi’an Jiaotong University. His major research interests include power system optimization and power system economics.

Tao Ding (Senior Member, IEEE) received the B.S.E.E. and M.S.E.E. degrees from Southeast University, Nanjing, China, in 2009 and 2012, respectively, and the Ph.D. degree from Tsinghua University, Beijing, China, in 2015. During 2013 and 2014, he was a Visiting Scholar in the Department of Electrical Engineering and Computer Science, University of Tennessee, Knoxville, TN, USA. During 2019 and 2020, he was a Visiting Scholar of the Robert W. Galvin Center for Electricity Innovation at Illinois Institute of Technology. He is currently a Professor in the State Key Laboratory of Electrical Insulation and Power Equipment, the School of Electrical Engineering, Xi’an Jiaotong University. His current research interests include electricity markets, power system economics and optimization methods, and power system planning and reliability evaluation. Dr. Ding is an editor of IEEE Transactions on Power Systems, IEEE Power Engineering Letters, IET Generation Transmission & Distribution and CSEE JPES.

Chenggang Mu (Student Member, IEEE) received the B.S. degree in 2021 from the School of Electrical Engineering, Xi’an Jiaotong University, Xi’an, China,where he is currently working toward the Ph.D. degree. His main research interests include energy market and blockchain.

Kai Pan (Member, IEEE) received the B.S. degree in industrial engineering from Zhejiang University, Hangzhou, China, in 2010, and the Ph.D. degree from the Department of Industrial and Systems Engineering, University of Florida Gainesville, FL, USA, in 2016. He is currently an Associate Professor with the Department of Logistics and Maritime Studies, The Hong Kong Polytech nic University, Hong Kong. His research interests include stochastic integer programming and data-driven optimization with applications in power system operations, transportation system scheduling, and supply chain management.

Biyuan Zhang (Student Member, IEEE) received the B.S. degree from the School of Electrical Engineering, Xi’an Jiaotong University, Xi’an, China, in 2024. He is currently working toward the Ph.D. degree at Xi’an Jiaotong University. He focuses on the acceleration algorithms of unit commitment.

Mohammad Shahidehpour (Fellow, IEEE) received the Honorary Doctorate degree from the Polytechnic University of Bucharest, Bucharest, Romania, in 2009. He is currently a University Distinguished Professor, a Bodine Chair Professor, and the Director of the Robert W.Galvin Center for Electricity Innovation, Illinois Institute of Technology, Chicago, IL, USA. He is a Member of the U.S. National Academy of Engineering, a Fellow of the American Association for the Advancement of Science, and a Fellow of the National Academy of Inventors.