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ORIGINAL RESEARCH

Optimal refund policy design for ship berthing appointment mechanism

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Abstract
Seaports, the nodal and bottleneck points in the global supply chain network, have been making effort to encourage shipping companies to book berths before ships arrive at the ports. The information and communication technology (ICT) system is essential for the success of this effort. We propose an appointment mechanism with a refund policy for the berth booking ICT system to mitigate port congestion. Two refund policies are investigated: cash refund policy and coupon refund policy. We develop a bi-level model that considers the interests of the port and shipping companies, with which the cash refund policy determines its optimal booking fee and the returned cash, and the coupon refund policy determines its optimal booking fee, the value of the coupon, and the shelf life of the coupon. Numerical experiments are conducted to analyze the two refund policies, the reactions of shipping companies with different characteristics, and how the port takes advantage of the characteristics of shipping companies to maximize profit. The proposed appointment mechanism achieves a win-win performance for the port and shipping companies as it increases the income of the port and reduces the delay cost of shipping companies. This study innovatively investigates the refund policy for the berth booking system and helps understand the mechanism of the system, thereby providing theoretical support for applying the berth booking system in the maritime industry. Moreover, this study contributes to alleviating port congestion and to environmental sustainability by reducing ship emissions caused by waiting at port and inappropriate sailing speed.

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1 Introduction

Maritime transport is the backbone of the global supply chain and carries over 80% of international trade by volume (UNCTAD, 2022). COVID-19 had once pushed the maritime industry to the emergency because of the continuous lockdowns in many ports. The impact of COVID-19 on the maritime industry even exceeds that of the economic crisis in 2008 (UNCTAD, 2020), which had been seen through the lens of unreasonably high freight rates and global supply chain disruptions. Fortunately, the maritime industry has been recovering as COVID-19 draws to a close. However, UNCTAD (2022) points out that maritime trade recovery faces unprecedented port congestion and unreliable schedules, which will undoubtedly increase shipping delays and thus affect the reliability of the global supply chain. The average container schedule delays doubled in 2021 and the port congestion persisted in 2022, e.g., port congestion caused 37% of world containership fleet capacity to be held up at ports by July 2022 (UNCTAD, 2022). It should be emphasized that any delay in ports may lead to disruptions in a wide range of manufacturing and service industries given the widely-adopted just-in-time policy, e.g., car manufacturing and semiconductors (Du et al., 2015; Zhen, 2016; Zhen et al., 2016). Therefore, apart from investments in port infrastructure, there is an urgent need to develop a managerial solution to port congestion, thereby reducing delays and improving shipping reliability. Moreover, as long waiting times for ships at the port lead to more emissions from ships, addressing the port congestion problem also helps to improve the air quality of port cities (Pratson, 2023).

Traditionally, shipping operators usually adopt the ‘hurry up and wait’ (HUW) (also a.k.a. “sprint-and-loiter”) strategy, which causes bunker fuel wastes at sea and long waiting times at ports (Du et al., 2015), and also exacerbates port congestion. To ease port congestion and lower emissions, the ‘virtual arrival’ (VA) (also a.k.a. “just-in-time arrival”) strategy, is advocated (Du et al., 2015; Jia et al., 2017). When there is a known delay in the focal port, the VA strategy allows a ship to reduce its sailing speed to meet a revised arrival time (Du et al., 2015; Jia et al., 2017). That is, the port will communicate the updated availability of berths to shipping operators and the shipping operators will accordingly adjust the sailing speeds of their ships with the aim of reducing waiting times at the port as well as decreasing the bunker fuel consumption at sea. With the development of information and communication technologies (ICTs), many online systems have been adopted to serve as decision-making tools for managers (Govindan et al., 2018; Govindan, 2023). Information system in port management is in its infancy (Lind et al., 2021). And shipping companies usually adopt port agencies to communicate with the port, which can not update information in real-time and relies on human effort. To assist the success of the VA strategy, an online berth booking system should be developed so that shipping companies are encouraged to book a berth (berthing window) online in advance and can obtain berth availability information in real-time. Accordingly, the port will better know in advance how many ships are about to arrive on different days and their specific berthing requirements, thereby making more informed resource allocation/scheduling plans. We argue that berths are valuable resources that serve shipping companies and berth bookings/appointments for specific time slots are chargeable, at least from an alternative reasonable view to existing practices. Borrowing the full or partial refund policy for product returns or service cancellations in the retail industry (Guo, 2009), we study an appointment mechanism that allows shipping companies to book berths in advance by paying the booking fee, which will be fully or partially refundable if ships do not arrive at the port on time. By investigating the appointment mechanism, our study helps understand the mechanism behind the berth booking system with a refund policy, thereby promoting the application of ICTs in port management.

We consider two refund policies in our study: (a) cash refund and (b) coupon refund. To explain it further, the port will implement a berth booking system and charge a booking fee. To support this booking system, one of the two refund policies will be adopted. The cash refund policy indicates that the booking fee will be returned in cash in full or in part if ships do not arrive at the port on time. The coupon refund policy means that the booking fee will be fully or partially returned to the shipping company in the form of coupons, each of which has an expiry date. As the uncertainty of the ship arrival times at a port is high (Wang & Meng, 2012; Ksciuk et al., 2023), it is necessary to develop an appropriate refund policy to encourage more shipping companies to book berthing windows in advance. The appointment mechanism can serve as an auxiliary tool for the VA strategy. That is, the port will release the available berths in advance and then shipping companies can decide whether to book. If the shipping company decides to make a reservation, the ship under investigation can adjust its sailing speed, arrive at the port according to the scheduled time, and start its loading and unloading operations on arrival. If the shipping company decides not to book, the ship may have to wait for a berth in anchorage after arrival at the port, and thus experience a delay and consume more bunker fuel because of speeding up for schedule on the voyage to the next port of call. Therefore, the shipping company operating a ship visiting the focal port needs to decide whether to book a berth according to its own operation characteristics, e.g., delay cost and ship's average on-time probability. The port aims to make a policy that maximizes its profit, i.e., the income earned through berth reservations.

In this research, we study the optimal design of the berth appointment mechanism by a bi-level mathematical programming model (which is finally solved by two mixed-integer linear programming models) and investigate which refund policy is better in practice. We also analyze the reactions of different shipping companies with different operational characteristics. Our research answers the following three key research questions (RQs) facing the port and the shipping companies in a berth booking system:

RQ1: Which refund policy is more profitable for the port, the cash refund policy or the coupon refund policy?

RQ2: What are the optimal booking fee and returned cash under the cash refund policy? What are the optimal booking fee, coupon value, and shelf life of the coupon under the coupon refund policy?

RQ3: What is the reaction of different shipping companies under the cash refund policy and the coupon refund policy, respectively?

The rest of the paper is organized as follows. Section 2 reviews the relevant literature and identifies the contributions of our research. Section 3 describes the problem and develops the mathematical models. Section 4 mathematically transforms the proposed model into solvable mixed-integer linear programming models. Section 5 conducts numerical experiments and sensitivity analyses and interprets the experimental results. Finally, conclusions are drawn in Sect. 6.

2 Literature review and research contributions

2.1 Literature review

In this section, we mainly review two streams of studies that are closely related to our study: (1) the berth allocation planning problem (BAP) and (2) port congestion-related topics.

BAP is a classic research topic in maritime transport (Jauhar et al., 2023; Tan & He, 2021). In the existing literature, BAP involves two categories of research: static BAP and dynamic BAP (Buhrkal et al., 2011). The static category supposes that ships are already at the port at the beginning of the berth planning horizon while the dynamic category treats ship arrivals as dynamic. Dynamic BAP is more in line with practice and attracts scholars' more attention. Nishimura et al. (2001) develop a genetic algorithm-based approach to solve the dynamic BAP in the public berth system. Imai et al. (2001) adopt a Lagrangian relaxation to solve the dynamic BAP and the experiments in their study show the adoptability of the proposed heuristic algorithm. Wang and Lim (2007) develop an innovative multiple-stage search method to address dynamic BAP. By and large, the existing studies usually develop heuristic algorithms to get sub-optimal solutions of dynamic BAP because of its computational complexity (Arango et al., 2011). Moreover, many scholars extend the BAP by considering many other factors and constraints. Imai et al. (2003) take service priorities into account in BAP and present a genetic algorithm to solve it. Du et al. (2015) reformulate the BAP by modeling the impacts of the tide on seaside operations and they find that the virtual arrival policy can bring both economic and environmental benefits. De et al. (2020) consider the ship fuel cost associated with waiting time in their proposed mixed-integer mathematical model in solving the dynamic BAP. Guo et al. (2023a) innovatively study BAP under a cooperative environment.

The core of BAP is to rationally allocate berths and reduce port congestion. If the arrival times of ships can be obtained in advance, it will greatly improve the rationality of the berth allocation plan. In this study, we propose an appointment mechanism that allows shipping companies to book berths in advance and further design the optimal refund policy for the port. Different from BAP, our study makes contributions to reducing port congestion by encouraging shipping companies to be involved in an innovative berth booking system and thus introduce additional decision freedom/space to port resource planning.

Instead of paying attention to optimizing port resource utilization after the arrivals of ships, many studies directly predict port congestion. AbuAlhaol et al. (2018) innovatively use Automatic Identification System (AIS) data to mine Port Congestion Indicators (PCIs), which provide early warning of congestion to port authorities. Peng et al. (2023) use AIS data and a Long Short-Term Memory (LSTM) neural network model to predict port congestion. Zhang et al. (2023) use AIS data in Yangshan port to predict port status and ship's turnaround time in port. They find that the port congestion level fluctuates little during the week. Some scholars focus on the economic issues brought about by port congestion. Jiang et al. (2017) explore the congestion 'knock on' from one port to the next port and they find that the marginal congestion costs of terminals influence a liner shipping company's operation. Bai et al. (2022) find that port congestion informs freight rates. Steinbach (2022) show that port congestion can reduce the competitiveness of one country's businesses in foreign markets.

By and large, port congestion affects the operational efficiency of a port, the reliability of the global supply chain, and affects the economic indicators of a nation from a long-term perspective. Reducing port congestion is a key concern in both the academic community and the maritime industry. Our study encourages the port to adopt an appointment mechanism to know in advance when ships will arrive and thus make more informed operational plans. Ships that book an available berth can adjust their sailing speeds based on the scheduled times and thus reduce fuel consumption by converting their waiting time at the port to more sailing time at sea at more economic speeds. The appointment mechanism helps to improve the port management level and operational efficiency. Moreover, as the existing studies usually focus on developing information systems for hinterland port logistics (Van Riessen et al., 2016) or for shipping routing (Fagerholt, 2004), our study innovatively proposes to use a decision support information system for berth booking.

2.2 Innovations and contributions

The existing studies usually focus on optimally allocating port resources to reduce port congestion, developing indicators to measure port congestion, and assessing the economic impact of port congestion. To the best of our knowledge, no study has paid attention to adopting an appointment mechanism with a refund policy to reduce port congestion by a berth booking system. The theoretical and practical contributions of our research are summarized as follows.

Theoretical contributions. First, our study proposes an innovative appointment mechanism with a refund policy to engage shipping companies in a berth booking system. This is the first study that considers the cash refund policy and the coupon refund policy in berth booking. Second, through building a bi-level model and further transforming it into two mixed-integer linear programming models, we investigate which refund policy can bring the maximum profit to the port by offering the optimal booking fee and returned cash or coupon. Third, we observe the reactions of different shipping companies and analyze how the optimal refund policy changes according to the characteristics of the port and shipping companies. Our study innovatively and comprehensively explores the berth appointment mechanism and demonstrates the benefits brought by a booking system. This represents a new perspective on addressing port congestion.

Practical contributions. Port congestion and ship emissions caused by ship fuel consumption are two main concerns of maritime practitioners. Our research addresses these two industry concerns by initiating a berth booking system and answering the key theoretical research questions in implementing this type of booking system. First, port staff can reasonably arrange port operations through reservations, thereby reducing port congestion caused by the lack of information from shipping companies. Meanwhile, by booking berths in advance, shipping operators can remedy the HUW strategy and adjust the sailing speeds of their ships to achieve just-in-time arrivals at the port. Thus, the number of ships accumulated in the port decreases and port congestion is avoided or alleviated. Second, as fewer ships will wait for a berth at the port, the fuel consumption and emissions of their auxiliary engines during waiting at anchorage will be reduced. Moreover, compared to the HUW strategy, the just-in-time sailing speeds with a berth booking system normally reduce ships' bunker fuel consumption. Therefore, the proposed berth booking mechanism can help the maritime industry achieve environmental sustainability maritime by reducing ship emissions associated with fuel consumption. Moreover, we believe our study can prompt the application of ICTs in maritime transport and thus help improve port operational efficiency.

3 Problem description and model development

A port is willing to devise an appointment mechanism to encourage shipping companies to book berths in advance. The port will charge the shipping companies booking fees when they book berthing space. By implementing the appointment mechanism, the focal port can facilitate berth management while gaining additional profits, and the shipping companies can reduce the uncertainty of the waiting times of their ships after arriving at the port, thereby reducing the delay cost. It is well-known that ship sailing schedule has great uncertainty, which is mainly caused by bad weather and the delay propagation from upstream ports of call (Wang & Meng, 2012; Ksciuk et al., 2023). Therefore, ships at risk of missing the booked times may lose their booking fees given the appointments made by managers. For addressing this problem, the port designs a partial refund policy—if a ship books berth in advance but does not arrive on time, the port will make a cash refund or provide a coupon for the next booking made by the same shipping company within a certain period. The port will decide which refund strategy to adopt to maximize its profit. And shipping companies will then decide whether to book berthing space in advance with the aim of minimizing cost. For the focal port, it aims to engage more shipping companies in the berth appointment mechanism because the port can have more income and make more informed berthing plans to reduce port congestion and improve operational efficiency. For shipping companies, if they have a low probability of arriving at the port on time, booking in advance may not be a good choice because they will lose part of the booking fee; in contrast, if they are confident in operating their ships and making the ships arrive at the port on time, booking a berth in advance can reduce the uncertainty of waiting times, thereby reducing costs generated by port delays.

The design of the refund policy for the ship berthing time window appointment is a two-stage problem. In the first stage, the port will decide on the refund policy by taking the shipping companies' reactions into account. In the second stage, shipping companies decide whether to book berths according to their own situations, e.g., the cost caused by delays and the probability of arriving on time.

As ship berthing is a complicated issue in practice, several reasonable assumptions are made to allow us to focus on the essential features of models and the berth booking mechanism. We will discuss how to relax the first three assumptions in Sect. 4.3.

- *Assumption 1*. Ships from the same shipping company are homogeneous in delay cost, arrival frequency, and the probability of arriving on time. And thus ships from the same shipping company will adopt the same strategies, i.e., booking or not booking berths.
- *Assumption 2*. Berths are ample at the time of booking and there is no overbooking in the port. However, when a ship arrives at the port without booking, it may have to wait as there are many other ships that are out of (do not use) the berth booking system.
- *Assumption 3*. The booking fee is a constant that does not depend on the reservation/booking lead time.
- *Assumption 4*. If a ship with a booked berth does not arrive at the port on time, it loses its berthing priority and may have to wait for a berth together with other ships at anchorage when it arrives. This assumption is reasonable from the perspective of service system practice. If a customer reserves a time slot of a resource through an online appointment system but does not show up at the scheduled time, she/he needs to join the queuing system without priority (Liu et al., 2019).

The port decides the booking fee x and determines which of the two refund policies to adopt: 1) returning p dollars in cash, $0 \leq p \leq x$; 2) giving a coupon of y dollars that is useful within period Δ, $0 \leq y \leq x$. We use binary decision variable v to represent which policy is adopted and v equals 1 if the port implements the policy of returning cash and 0 otherwise. We use set $I = \{1, \dots, |I|\}$ to denote the shipping companies, $i \in I$. Ships belonging to the same shipping company are homogeneous and different shipping companies have different numbers of ships, denoted by N_i, which can be regarded as the parameters to measure the size of the shipping companies. For a shipping company i, its decision is whether to make an appointment. The binary decision variable z_i equals 1 if the shipping company i decides to book a berth in advance and 0 otherwise. The historical on-time arrival probability t_i and the delay cost are two issues that shipping companies mainly consider when making decisions. t_i is a parameter that can be measured by historical data of shipping company i. We use c_i to denote the delay cost per unit of time and the corresponding cost caused by the delay of shipping company i is $c_i \times \widetilde{q}$ (Desaulniers & Villeneuve, 2000), where $\widetilde{q}$ is a random variable that denotes the time the ship will wait at the port if it does not make an appointment and $\widetilde{q} \sim \mathrm{N}(\alpha, \beta^2)$ (Amin-Naseri & Baradaran, 2015). We suppose that the arrival frequency of ships in shipping company i is once in $\widetilde{d_i}$ days and $\widetilde{d_i} \sim \mathrm{U}(a_i, b_i)$ (Amin-Naseri & Baradaran, 2015).

The problem forms a typical bi-level model: the leader is the port as it makes refund policies at the first stage; the followers are shipping companies and they make decisions on whether to book. Next, we introduce the bi-level model in reverse order. As we assume that the ships belonging to the same shipping company are homogeneous, shipping company i can minimize its cost by minimizing the mean cost generated by a single ship.
[Follower: shipping company-i]

$$\min\ \mathbb{E}[(1 - z_i)c_i\widetilde{q} + z_i(t_i x + (1 - t_i)R_i)] \tag{1}$$

subject to

$$R_i = c_i\widetilde{q} + v(x - p) + (1 - v)(x - P_i y) \tag{2}$$

$$P_i = \begin{cases} 0, & 0 \le \Delta < a_i \\ \dfrac{\Delta - a_i}{b_i - a_i}, & a_i \le \Delta \le b_i \\ 1, & \Delta > b_i \end{cases} \tag{3}$$

$$z_i \in \{0, 1\}. \tag{4}$$

Constraint (2) defines auxiliary decision variable R_i that represents the cost if the ship does not arrive on time. Constraint (3) gives the probability that the coupon can be used. In detail, if the coupon has a shelf life of less than a_i days, the coupon is useless for the shipping company; if the coupon has a shelf life of more than b_i days, the coupon definitely can be used for the next booking; if $a_i \le \Delta \le b_i$, then the coupon is useful with probability $\frac{\Delta - a_i}{b_i - a_i}$. Constraint (4) restricts the domain of the decision variable z_i.
[Leader: port]

$$\max \sum_{i \in I} z_i \frac{2N_i}{a_i + b_i}(v(t_i x + (1 - t_i)(x - p)) + (1 - v)(t_i x + (1 - t_i)(x - P_i y))) \tag{5}$$

subject to

$$P_i = \begin{cases} 0, & 0 \le \Delta < a_i\ , i \in I \\ \dfrac{\Delta - a_i}{b_i - a_i}, & a_i \le \Delta \le b_i\ , i \in I \\ 1, & \Delta > b_i\ , i \in I \end{cases} \tag{6}$$

Table 1 Notations

Sets	
I	Set of shipping companies, indexed by i
Parameters	
$\widetilde{q}$	A random variable that indicates the waiting time for a ship at the port if it does not make an appointment and $q \sim \mathrm{N}(\alpha, \beta^2)$
t_i	The probability that shipping company i's ship will arrive at the port on time
$\widetilde{d_i}$	The ships in shipping company i arrive at the port once in $\widetilde{d_i}$ days and $\widetilde{d_i} \sim \mathrm{U}(a_i, b_i)$
c_i	The delay cost of shipping company i per unit of time
N_i	The size of the shipping company i, i.e., the number of ships in the shipping company i
T	The booking lead time
ϕ_T	The uncertainty of the estimation of t_i, $\phi_T \in [0, 1]$
ρ_T	The probability of booking an available berth, $\rho_T \in [0, 1]$
Decision Variables	
v	Binary decision variable that equals 1 if the port implements a policy of returning some of the cash and 0 otherwise
x	The booking fee
p	Cash returned
y	The value of the coupon
Δ	The shelf life of the coupon
z_i	Binary decision variable that equals 1 if shipping company i decides to book a berth in advance and 0 otherwise
R_i	Auxiliary decision variable that represents the cost if the ship does not arrive on time
P_i	Auxiliary decision variable that gives the probability of the coupon being used

$$v \in \{0, 1\} \tag{7}$$
$$p \leq x \tag{8}$$
$$y \leq x \tag{9}$$
$$x, y, p, \Delta \in N. \tag{10}$$

The port aims to maximize its profit and it will only receive money from the shipping company that decides to book, i.e., $z_i = 1,\ i \in I$. $\frac{a_i+b_i}{2}$ is the expected value of the random variable $\widetilde{d_i}$ and we need to take the arrival frequency $\frac{2}{a_i+b_i}$ into account in the objective function. Constraints (7)–(10) give the domain of decision variables. We restrict the value of the booking fee, the returned cash, the value of the coupon, and the shelf life of the coupon to non-negative integers.

The main notations used in our study are shown in Table 1.

4 Solution method

The bi-level model is computationally hard to solve as there are nonlinear terms (e.g., product terms in Objective function (1) and (5)) and a piecewise linear function in Constraint (3). In this section, we take advantage of the problem structure and mathematically transform the nonlinear bi-level model into two single-level mixed-integer linear programming models.

First, the decision of the port is which refund policy to adopt and we define a binary decision variable v in Sect. 3 to represent the choice. Since the profits of adopting different refund policies are independent, we can separately solve the maximum profit of the port under the two policies—cash refund policy and coupon refund policy, and then compare them to obtain a final decision. We use Π_1 to denote the maximum profit of adopting the cash refund policy and Π_2 to denote the maximum profit of adopting the coupon refund policy. Then the bi-level model in Sect. 3 can be converted into two bi-level models. One of them obtains Π_1:
[Cash refund policy–follower: shipping company-i]

$$\min \ \mathbb{E}[(1-z_i)c_i\widetilde{q} + z_i(t_i x + (1-t_i)(c_i\widetilde{q} + x - p))] \tag{11}$$

subject to (4).
[Cash refund policy–leader: port]

$$\max \sum_{i\in I} z_i \frac{2N_i}{a_i+b_i}(t_i x + (1-t_i)(x-p)) \tag{12}$$

subject to

$$x, p \in N \tag{13}$$

(8).

Π_2 can be obtained by the following bi-level model:
[Coupon refund policy–follower: shipping company-i]

$$\min \ \mathbb{E}[(1-z_i)c_i\widetilde{q} + z_i(t_i x + (1-t_i)(c_i\widetilde{q} + x - P_i y))] \tag{14}$$

subject to (3) and (4).
[Coupon refund policy–leader: port]

$$\max \sum_{i\in I} z_i \frac{2N_i}{a_i+b_i}(t_i x + (1-t_i)(x-P_i y)) \tag{15}$$

subject to

$$x, y, \Delta \in N \tag{16}$$

(6) and (9).

By comparing the value of Π_1 and Π_2, the port will draw a conclusion: if $\Pi_1 > \Pi_2$, the cash refund policy will be adopted; otherwise, the coupon refund policy will be adopted. Next, we linearize the two bi-level models to two single-level mixed-integer linear programming models, which can be solved by the off-the-shelf optimization solvers.

4.1 Linearizing the model to solve the profits of cash refund policy

The shipping company makes decisions with the goal of minimizing its cost. That is, it chooses the one with lower cost from reservation and non-reservation. Therefore, the Objective function (11) can be transformed into a constraint in Model [Cash refund policy–leader: port], and the bi-level model is converted to a single-level model:
[M1]

Objective function (12)

subject to

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[t_i x + (1-t_i)(c_i\widetilde{q} + x - p)] \le z_i M_1,\ i \in I \tag{17}$$

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[t_i x + (1-t_i)(c_i\widetilde{q} + x - p)] \ge (z_i - 1) M_1,\ i \in I \tag{18}$$

$$z_i \in \{0, 1\},\ i \in I \tag{19}$$

(8) and (13),

where M_1 is a big constant and the value of M_1 will be illustrated in the experiment part.

We now prove the Objective function (11) can be equivalently transformed to Constraints (17) and (18).

Proof Because of the linearity of expectation of random variables, the Objective function (11) is equivalent to:

$$\min\ \mathbb{E}[(1-z_i)c_i\widetilde{q}] + \mathbb{E}[z_i(t_i x + (1-t_i)(c_i\widetilde{q} + x - p))]. \tag{20}$$

The Objective function (20) is linear in the random variable $\widetilde{q}$. Therefore, it can be formulated as:

$$\min\ (1-z_i)\mathbb{E}[c_i\widetilde{q}] + z_i\mathbb{E}[t_i x + (1-t_i)(c_i\widetilde{q} + x - p)]. \tag{21}$$

Given that the objective function is to minimize the cost, $z_i = 0$ if $\mathbb{E}[c_i\widetilde{q}] \le \mathbb{E}[t_i x + (1 - t_i)(c_i\widetilde{q} + x - p)]$ and $z_i = 1$ otherwise. Therefore, we introduce a big constant M_1 to convert the Objective function (21) into Constraints (17) and (18).□

However, Model [M1] is still difficult to solve because there are nonlinear terms $z_i(x - (1-t_i)p),\ i \in I$ in the Objective function (12). We define an auxiliary decision variable $k_i = z_i(x - (1-t_i)p)$ to address the nonlinear term. By adding Constraints (23)–(26) and introducing a big constant M_k, Model [M1] can be linearized as follows.

[M2]

$$\max \sum_{i \in I} \frac{2N_i}{a_i + b_i} k_i \tag{22}$$

subject to

$$k_i \le x - (1-t_i)p,\ i \in I \tag{23}$$

$$k_i \le M_k z_i,\ i \in I \tag{24}$$

$$k_i \ge x - (1-t_i)p - M_k(1-z_i),\ i \in I \tag{25}$$

$$k_i \ge 0,\ i \in I \tag{26}$$

(8), (13), (17)–(19).

Therefore, the value of Π_1, i.e., the maximum profit of adopting the cash refund policy, can be obtained by solving Model [M2], which is a mixed-integer linear programming model. The decisions of booking fee x and the refund cash p can accordingly be obtained. □

4.2 Linearizing the model to solve the profits of coupon refund policy

The bi-level model of solving the profits of coupon refund policy can also be combined into one single-level model by introducing a big constant M_2:

[M3]

Objective function (15)

subject to

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[t_i x + (1-t_i)(c_i\widetilde{q} + x - P_i y)] \le z_i M_2,\ i \in I \tag{27}$$

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[t_i x + (1-t_i)(c_i\widetilde{q} + x - P_i y)] \ge (z_i - 1) M_2,\ i \in I \tag{28}$$

(6), (9), (16), (19).

Constraint (6) is a piecewise function and we next linearize it for solving Model [M3]. We define four auxiliary decision variables w_i^1, w_i^2, w_i^3, and w_i^4 to represent Δ and P_i. And we introduce three binary auxiliary decision variables γ_i^1, γ_i^2, and γ_i^3 to restrict the interval of the piecewise function in Constraint (6). We can use the following constraints to make an equivalent replacement of Constraint (6):

$$\Delta = w_i^1 \times 0 + w_i^2 \times a_i + w_i^3 \times b_i + w_i^4 \times M_3,\ i \in I \tag{29}$$

$$P_i = w_i^1 \times 0 + w_i^2 \times 0 + w_i^3 \times 1 + w_i^4 \times 1,\ i \in I \tag{30}$$

$$w_i^1 + w_i^2 + w_i^3 + w_i^4 = 1,\ i \in I \tag{31}$$

$$0 \le w_i^1, w_i^2, w_i^3, w_i^4 \le 1,\ i \in I \tag{32}$$

$$\gamma_i^1 + \gamma_i^2 + \gamma_i^3 = 1,\ i \in I \tag{33}$$

$$w_i^1 \le \gamma_i^1,\ i \in I \tag{34}$$

$$w_i^2 \le \gamma_i^1 + \gamma_i^2,\ i \in I \tag{35}$$

$$w_i^3 \le \gamma_i^2 + \gamma_i^3,\ i \in I \tag{36}$$

$$w_i^4 \le \gamma_i^3,\ i \in I \tag{37}$$

$$\gamma_i^1, \gamma_i^2, \gamma_i^3 \in \{0, 1\},\ i \in I. \tag{38}$$

Constraints (29) use the separation point of the intervals in Constraint (6)—0, a_i, b_i, and M_3—to represent the decision variable Δ. Note that the decision variable Δ does not have an upper bound in Constraint (6) and thus we adopt M_3 to denote the upper bound of Δ. The value of M_3 will be illustrated in the experiment part because its value is case-specific. Constraints (30) give the value of P_i by calculating the value of the four separation points in Constraint (6) and multiplying them by the corresponding weights. Constraints (31)–(38) restrict the interval of Δ and its value. According to Constraints (30), the Objective function (15) can be rewritten as:

$$\max \sum_{i \in I} z_i \frac{2N_i}{a_i + b_i}(x - (1-t_i)(w_i^3 y + w_i^4 y)), \tag{39}$$

which is nonlinear. We suppose that the upper bound of the decision variable y is M_y. We define:

$$K = \lfloor \log_2 M_y \rfloor, \tag{40}$$

and we then define new binary auxiliary decision variables $\theta_k, k = 0, ..., K$. The value of y can be replaced with $\sum_{k=0}^{K} 2^k \theta_k$, and $0 \le \sum_{k=0}^{K} 2^k \theta_k \le M_y$. Thus, we have $w_i^3 y = w_i^3 \sum_{k=0}^{K} 2^k \theta_k$ and $w_i^4 y = w_i^4 \sum_{k=0}^{K} 2^k \theta_k$. That is, the Objective function (39) can be rewritten as:

$$\max \sum_{i \in I} z_i \frac{2N_i}{a_i + b_i}(x - (1-t_i)(\sum_{k=0}^{K}(w_i^3 + w_i^4) 2^k \theta_k)), \tag{41}$$

We define $\mu_i^k = (w_i^3 + w_i^4)2^k\theta_k$ and M_u to perform an equivalent transformation:
[M4]

$$\max \sum_{i \in I} z_i \frac{2N_i}{a_i + b_i}(x - (1 - t_i)\sum_{k=0}^{K} \mu_i^k) \tag{42}$$

subject to

$$\mu_i^k \le 2^k(w_i^3 + w_i^4),\ \ i \in I, k = 0, ..., K \tag{43}$$

$$\mu_i^k \le M_u\theta_k,\ \ i \in I, k = 0, ..., K \tag{44}$$

$$\mu_i^k \ge 2^k(w_i^3 + w_i^4) - M_u(1 - \theta_k),\ \ i \in I, k = 0, ..., K \tag{45}$$

$$\mu_i^k \ge 0,\ \ i \in I, k = 0, ..., K \tag{46}$$

$$0 \le \sum_{k=0}^{K} 2^k\theta_k \le M_y,\ \ k = 0, ..., K \tag{47}$$

$$\theta_k \in \{0, 1\},\ \ k = 0, ..., K \tag{48}$$

(9), (16), (19), and (27)–(38).

We further introduce an auxiliary decision variable $\eta_i = z_i(x - (1 - t_i)\sum_{k=0}^{K} \mu_i^k)$ to linearize the Objective function (42):
[M5]

$$\max \sum_{i \in I} \frac{2N_i}{a_i + b_i}\eta_i \tag{49}$$

subject to

$$\eta_i \le x - (1 - t_i)\sum_{k=0}^{K} \mu_i^k,\ \ i \in I \tag{50}$$

$$\eta_i \le M_\eta z_i,\ \ i \in I \tag{51}$$

$$\eta_i \ge x - (1 - t_i)\sum_{k=0}^{K} \mu_i^k - M_\eta(1 - z_i),\ \ i \in I \tag{52}$$

$$\eta_i \ge 0,\ \ i \in I \tag{53}$$

(9), (16), (19), (27)–(38), (43)–(48).

Therefore, the maximum profit of the coupon refund policy (Π_2) can be obtained by solving the mixed-integer linear model [M5].

In this section, we introduce M_1, M_2, M_3, M_k, M_y, M_u, and M_η to address nonlinear terms and their value will be illustrated in the section of numerical experiments as these values depend on specific cases.

4.3 Relaxation of assumptions

We here discuss how to relax the assumptions made in Sect. 3 and we will further conduct numerical experiments in Sect. 5.4 to validate these relaxations.

- Relaxation of Assumption 1. In Sect. 3, we assume that ships from the same shipping companies are homogeneous and will adopt the same strategies in terms of whether they book berths in advance. Our proposed model and solution methods can handle non-heterogeneous situations by introducing "*virtual shipping companies*". In detail, ships can be categorized into different groups based on all the possible concerning characteristics (e.g., delay cost, arrival frequency, and the probability of arriving on time, ship type, engine model, and types of cargo carried by ship) if they are non-heterogeneous. In an extreme scenario, if a shipping company wants to customize a booking plan for each ship, we can view every ship as a virtual shipping company. And then our model can produce optimal solutions for each virtual shipping company.
- Relaxation of Assumption 2. Assumption 2 restricts that berths are ample at the booked times whenever the shipping companies call in for berth booking. However, when the berth reservation system is promoted in practice, there may be a large number of existing reservations resulting in reduced berth availability at the requested times given the reservation lead time. In this situation, berths are no longer plentiful and the shipping company will face the risk of not being able to book an available berth if it books too late. However, the shipping company will have a more accurate estimation of the on-time arrival probability of a ship if it books a berth when the ship is close to the port. It is understandable that the booking fee may increase as the booking lead time shrinks. Thus, the shipping companies have to make a trade-off between booking earlier with higher uncertainty in estimating on-time arrival probability but a lower booking fee and booking later with lower uncertainty in estimating on-time arrival probability but a higher booking fee. To relax Assumption 2, we first use ρ_T, where T is the booking lead time, to denote the probability of securing an available berth in booking. Obviously, ρ_T decreases when the booking lead time is close to the berthing time. We then revise the on-time arrival probability t_i to $\phi_T t_i$, where $\phi_T \in [0, 1]$ considers the uncertainty of the estimation of t_i. And ϕ_T increases when the booking time becomes closer to the berthing time. Thus, the original models can be viewed as a special case where ϕ_T is set to 1. The Objective function (1) (the objective of shipping company i) should be revised as follows:

$$\min \ \mathbb{E}[(1-z_i)c_i\widetilde{q} + z_i(\rho_T(\phi_T t_i x + (1-\phi_T t_i)R_i) + (1-\rho_T)c_i\widetilde{q})], \tag{54}$$

and the Objective function (5) (the objective of the port) is modified to:

$$\max \sum_{i\in I} z_i \frac{2N_i}{a_i+b_i}(v(\phi_T t_i x+(1-\phi_T t_i)(x-p))+(1-v)(\phi_T t_i x+(1-\phi_T t_i)(x-P_i y))). \tag{55}$$

For the solution process of the cash refund policy, Constraints (17) and (18) should be revised to:

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[\rho_T(\phi_T t_i x + (1-\phi_T t_i)(c_i\widetilde{q} + x - p)) + (1-\rho_T)c_i\widetilde{q}] \le z_i M_1, \ i \in I \tag{56}$$

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[\rho_T(\phi_T t_i x + (1-\phi_T t_i)(c_i\widetilde{q} + x - p) + (1-\rho_T)c_i\widetilde{q}] \ge (z_i-1)M_1, \ i \in I, \tag{57}$$

and Objective function (22) will be:

$$\max \sum_{i\in I} \frac{2N_i}{a_i+b_i} k_i, \tag{58}$$

where $k_i = z_i(x - (1 - \phi_T t_i)p)$. For the solution process of the coupon refund policy, Constraints (27) and (28) are modified to:

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[\rho_T(\phi_T t_i x + (1 - \phi_T t_i)(c_i\widetilde{q} + x - P_i y)) + (1 - \rho_T)c_i\widetilde{q}] \le z_i M_2, \ i \in I \tag{59}$$

$$\mathbb{E}[c_i\widetilde{q}] - \mathbb{E}[\rho_T(\phi_T t_i x + (1 - \phi_T t_i)(c_i\widetilde{q} + x - P_i y)) + (1 - \rho_T)c_i\widetilde{q}] \ge (z_i - 1)M_2, \ i \in I, \tag{60}$$

and Objective function (22) becomes:

$$\max \sum_{i \in I} \frac{2N_i}{a_i + b_i}\eta_i, \tag{61}$$

where $\eta_i = z_i(x - (1 - \phi_T t_i)\sum_{k=0}^{K} \mu_i^k)$.

- Relaxation of Assumption 3. Assumption 3 regulates that the booking fee is a constant that is independent of the reservation lead time. As we relax Assumption 2, it is logical for the booking system to charge the booking fee in a piecewise format according to the lead time: bookings made when it is closer to the arrival time are charged more. The models revised after relaxing Assumption 2 can handle this situation (further relax Assumption 3) by introducing the concept of "*virtual ports*". Specifically, we can set virtual ports according to the lead time. For example, if the booking system has set up a three-stage charging policy and booking charges are different for three different booking lead times (one week, one week to three days, and less than three days), we can create three virtual ports for the focal seaport with each virtual port corresponding to a booking lead time and then for each virtual port solve the models for the optimal solutions of the two refund policies. The port's decision is still to set the piecewise booking fee with the objective of maximizing its profit.

5 Numerical experiment

5.1 Parameter setting

It is well-known that the shipping market is highly oligopoly. MSC, A.P. Moller-Maersk, and CMA CGM Group are the top three shipping companies in the world and dominate the global shipping market (Zheng & Luo, 2021). Without loss of generality, we set $I = \{1, 2, 3\}$ to conduct numerical experiments and analyze the results. Firstly, the mean waiting time of ships at the port is determined by many factors, e.g., the port congestion levels (Roberts et al., 2014; Guo et al., 2023b) and the ship size (Park & Suh, 2019). For example, the average wait time in Detroit in 2012 is less than one hour (Roberts et al., 2014). However, according Park and Suh (2019), the average wait time for a 20,000 TEU (Twenty-foot Equivalent Unit) container ship could be more than 100 h. In this study, we do not consider extreme scenarios; instead, we set the mean waiting time of ships at port based on the average level reported by UNCTAD (2023). According to UNCTAD (2023), the mean waiting time of container ships in developed countries from 2016 to 2020 ranges from 2 h to 4 h; and the mean waiting time of container ships in developing countries from 2016 to 2020 ranges from 6 h to 8 h. During the COVID-19 period, the mean waiting times of developed countries and developing countries both increased, with an approximate rise of around 2 h. Therefore, to capture the ordinary scenario, we first set the mean waiting time of ships at port to 5 h, and then conduct sensitivity analysis to analyze the impact of the mean waiting time of ships at port. Drawing upon assumptions from academic research on port waiting times, we assume that $\widetilde{q}$ follows a normal distribution (Wang et al., 2023), with a variance set at 0.5, i.e., $\widetilde{q} \sim \mathrm{N}(5, 0.5^2)$.

Secondly, regarding the probabilities of arriving on time, and the distribution of delay costs for each company, these two parameters are internal metrics of the shipping companies and are not disclosed to the public. There are many studies that focus on estimating vessel arrival times based on real-world data and obtain good performance (Chu et al., 2023). We believe that each shipping company is equipped with professional expertise to predict vessel arrival times, thereby achieving a high probability of on-time arrivals. Thus, we first set the probability of arriving on time t_1, t_2, and t_3 to 0.8, 0.9, and 0.7, respectively. We also conduct sensitivity analysis to examine the influence of the probability of arriving on time. Thirdly, the delay cost of the shipping company is proportional to the probability of arriving on time, which is in line with practice because the shipping company with the higher delay cost is more inclined to arrive on time. Therefore, we set $c_1 = \$800/\mathrm{h}$, $c_2 = \$900/\mathrm{h}$, and $c_3 = \$700/\mathrm{h}$. Fourthly, we initially set the number of ships for each company to be the same ($N_1 = N_2 = N_3 = 1000$) and we will show the impacts of company size in the sensitivity analysis.

Next, we introduce the values of M_1, M_2, M_3, M_k, M_y, M_u, and M_η. The values of these big-M parameters should be large enough to ensure that the constraints are satisfied by the optimal solution to the problem. With the condition of ensuring optimality, a smaller value is preferable because the computation time could be saved. According to Constraint (17) and (18), the lower bound of M_1 is:

$$M_1 \geq \max_{i \in I} \{\mathbb{E}[c_i \widetilde{q}] - \mathbb{E}[t_i x + (1 - t_i)(c_i \widetilde{q} + x - p)]\}. \tag{62}$$

Because of Constraint (8), the right side of the inequality satisfies:

$$\max_{i \in I} \{\mathbb{E}[c_i \widetilde{q}] - \mathbb{E}[t_i x + (1 - t_i)(c_i \widetilde{q} + x - p)]\} \leq \max_{i \in I} \{\mathbb{E}[c_i \widetilde{q}] - \mathbb{E}[(1 - t_i) c_i \widetilde{q}]\} = 2430. \tag{63}$$

Thus, we can set $M_1 = 2430$.

The value of M_2 is also set to 2430 because according to Constraint (9), (27), and (28), we have:

$$M_2 \geq \max_{i \in I} \{\mathbb{E}[c_i \widetilde{q}] - \mathbb{E}[t_i x + (1 - t_i)(c_i \widetilde{q} + x - P_i y)]\}, \tag{64}$$

and

$$\max_{i \in I} \{\mathbb{E}[c_i \widetilde{q}] - \mathbb{E}[t_i x + (1 - t_i)(c_i \widetilde{q} + x - P_i y)]\} \leq \max_{i \in I} \{\mathbb{E}[c_i \widetilde{q}] - \mathbb{E}[(1 - t_i) c_i \widetilde{q}]\} = 2430. \tag{65}$$

M_3 denotes the upper bound of Δ. We set $M_3 = 30$, i.e., the maximum shelf life of the coupon is one month. The total costs of a ship's port call are very high considering the costs of berthing and operating expenses (Zheng et al., 2022) and it costs millions for a large container ship to dock. We suppose that the shipping company is willing to spend at most 1% of the total costs for berth reservation. Therefore, the maximum values of M_k, M_y, M_u, and M_η are set to 10,000. We also observe whether the optimal solutions change as we increase these big-M values in our experiment and the results turn out that 10,000 guarantees the optimality of our model. The experiments were run on a laptop computer equipped with 2.60 GHz of

Intel Core i7 CPU and 16 GB of RAM, and models were solved by IBM ILOG CPLEX Optimizer 20.1.0 via Python API.

5.2 Basic experimental results

The basic experimental results are shown in Table 2. The maximum profits of the cash refund policy and the coupon refund policy are \$564,550 and \$588,000, respectively. Therefore, the port will take the coupon refund policy. The booking fee is \$4,000 and the value of the coupon is also \$4,000 with a shelf life of 12 days. Table 2 also indicates that the returned cash is equal to the booking fee in the optimal cash refund policy, which means all the booing fees should be returned to the shipping company if its ships do not arrive at the port on time. The booking fee in the optimal coupon refund policy is higher than that in the optimal cash refund policy and shipping company 3 will not make an appointment under the coupon refund policy. That is, the port loses shipping company 3 as a customer of the booking system but increases the overall revenue by increasing the booking fee. Moreover, as the shelf life of the coupon is limited, the shipping company will revisit the port before the coupon expires, which will undoubtedly help the port to form long-term cooperation with the shipping company.

In summary, the basic results indicate that the coupon refund policy outperforms the cash refund policy in terms of increasing profit and user (i.e., shipping company) stickiness. The next section will check the robustness of this finding through sensitivity analyses.

5.3 Sensitivity analyses

We first test the impacts of the size of shipping companies by keeping the values of the other parameters unchanged. The total market size (i.e., the total number of ships) remains the same and we only change the market share of the three shipping companies. The results are shown in Table 3. When the three shipping companies are no longer evenly matched and shipping company 2 dominates the market, the total profit of the port increases in both policies. As the optimal cash refund policy improves the booking fee, shipping company 3 will no longer book. The shelf life of the coupon stays the same, i.e., 12 days, in the three cases. As $\widetilde{d_1} \sim \mathrm{U}(8, 12)$ and $\widetilde{d_2} \sim \mathrm{U}(14, 16)$, shipping company 1 will definitely use the coupon and shipping company 2 will definitely not use the coupon. However, as shipping company 2 owns a high delay cost, it prefers to make a reservation even knowing the coupon is useless to it. Thus, by taking advantage of this feature of shipping company 2, the port can set a high booking fee and a short coupon expiry period while guaranteeing the participation of shipping company 2. This is why the booking fee is always \$4,000 and the shelf life of the coupon is always 12 days. Moreover, as shipping company 2 will never use the coupon, the coupon refund policy outperforms the cash refund policy even when the booking fee of the two policies is the same.

We next analyze the impacts of the probability that ships arrive at the port on time. We set t_2 to values with an interval of 0.1 between 0 and 1 and then observe the maximum profit and shipping company 2's booking decision. The results are shown in Table 4. When $t_2 < 0.5$, i.e, the probability that ships belonging to shipping company 2 arrive at the port on time is less than 0.5, shipping company 2 will not make an appointment in both policies. And the profit of both policies is the same (\$353,500). When $t_2 = 0.5$, shipping company 2 will book under the optimal coupon refund policy and the shelf life of the coupon is 16 days, indicating that shipping companies 1 and 2 will definitely use the coupon and shipping company 3 will use the coupon with the probability of 0.15. The booking fee of the refund policy also decreases

Table 2 Basic results

Policy	Profit ($)	Booking fee ($)	Cash returned ($)	Value of coupon ($)	Shelf life of coupon(days)	Booking companies
Cash refund	564,550	3,500	3,500	–	–	1,2,3
Coupon refund	588,000	4,000	–	4,000	12	1,2

Table 3 Sensitivity analysis: the impacts of the size of shipping companies

Policy	Profit ($)	Booking fee ($)	Cash returned ($)	Value of coupon ($)	Shelf life of coupon(days)	Booking companies
$N_1 = 1000$, $N_2 = 1000$, $N_3 = 1000$ (basic results)						
Cash refund	564,550	3,500	3,500	–	–	1,2,3
Coupon refund	588,000	4,000	–	4,000	12	1,2
$N_1 = 500$, $N_2 = 2000$, $N_3 = 500$						
Cash refund	642,400	4,000	4,000	–	–	1,2
Coupon refund	696,000	4,000	–	4,000	12	1,2
$N_1 = 250$, $N_2 = 2500$, $N_3 = 250$						
Cash refund	683,000	4,000	4,000	–	–	1,2
Coupon refund	750,000	4,000	–	4,000	12	1,2

Table 4 Sensitivity analysis: the impacts of t_2

Policy	Profit ($)	Booking fee ($)	Cash returned ($)	Value of coupon ($)	Shelf life of coupon(days)	Booking companies
$t_2 = 0$						
Cash refund	353,500	3,500	3,500	–	–	1,3
Coupon refund	353,500	3,500	–	3,500	30	1,3
$t_2 = 0.1$						
Cash refund	353,500	3,500	3,500	–	–	1,3
Coupon refund	353,500	3,500	–	3,500	30	1,3
$t_2 = 0.2$						
Cash refund	353,500	3,500	3,500	–	–	1,3
Coupon refund	353,500	3,500	–	3,500	30	1,3
$t_2 = 0.3$						
Cash refund	353,500	3,500	3,500	–	–	1,3
Coupon refund	353,500	3,496	–	2,432	30	1,3
$t_2 = 0.4$						
Cash refund	353,500	3,500	3,500	–	–	1,3
Coupon refund	353,500	3,200	–	2,500	30	1,3
$t_2 = 0.5$						
Cash refund	353,500	3,500	3,500	–	–	1,3
Coupon refund	355,375	2,500	–	2,500	16	1,2,3
$t_2 = 0.6$						
Cash refund	423,600	3,000	3,000	–	–	1,2,3
Coupon refund	426,100	2,840	–	2,600	30	1,2,3
$t_2 = 0.7$						

Table 4 continued

Policy	Profit ($)	Booking fee ($)	Cash returned ($)	Value of coupon ($)	Shelf life of coupon(days)	Booking companies
Cash refund	517,650	3,500	3,500	–	–	1,2,3
Coupon refund	517,650	3,500	–	3,500	30	1,2,3
$t_2 = 0.8$						
Cash refund	541,100	3,500	3,500	–	–	1,2,3
Coupon refund	541,100	3,500	–	3,500	30	1,2,3
$t_2 = 0.9$ (basic results)						
Cash refund	564,550	3,500	3,500	–	–	1,2,3
Coupon refund	588,000	4,000	–	4,000	12	1,2
$t_2 = 1.0$						
Cash refund	588,000	3,500	3,500	–	–	1,2,3
Coupon refund	588,000	3,500	–	3,500	30	1,2,3

to \$2,500 as it makes all companies involved. When $0.5 < t_2 \leq 0.8$ and $t = 1.0$, the three shipping companies will all participate in the reservation in both policies. Table 4 shows that the profit of the coupon refund policy is at least as good as the cash refund policy.

We then change the expected value of the random variable $\widetilde{q}$ to 3 and 8, respectively. The results are shown in Table 5. As the waiting time increases, the total profit and booking fee of the two policies increase. And these results are in line with practice because the longer waiting time will make shipping companies pay more for the delay and thus shipping companies prefer to make an appointment. Therefore, the port can take advantage of the trend to increase the booking fee. Shipping company 3 will not book in the coupon refund policy because the average cost of booking is higher than the waiting cost faced by direct arrival. The shelf life of the coupon indicates that the coupon is useless for shipping company 2. But the delay cost is high for shipping company 2, which leads to its booking decision. Moreover, the port also takes advantage of the high delay cost of shipping company 2 to improve the booking fee and increase its own profit. The results in Table 5 also indicate that more congested ports, i.e. more prosperous ports, own higher initiative as shipping companies prefer to book in advance. The coupon refund policy still outperforms the cash refund policy in terms of profit in the three cases in Table 5.

5.4 Experiments of relaxing modeling assumptions

5.4.1 Computational performance of introducing virtual shipping companies

As we introduce virtual shipping companies in relaxing Assumption 1, experiments of testing the computational efficiency are needed. We randomly initialize the required parameters and set $|I|$=50, 100, 200, 300, and 500. For each setting of $|I|$, 10 instances are randomly generated. The average computational times for solving the two refund policies are reported in Table 6. The largest instance with $|I|$=500 can be solved in less than 16 min on average, which indicates that our methods can handle the problem size up with 500 virtual shipping companies and thus are efficient enough for industry applications where non-heterogeneous ships are to be accommodated.

5.4.2 Experiments on limited available berths and virtual ports

We further test the revised models for relaxing Assumption 2 under the virtual ports setting (together with the relaxation of Assumption 3). Suppose that the booking system of the focal port sets a three-stage charging strategy based on three different booking lead times: booking one week in advance, booking three days in advance, and booking within three days. Thus, we introduce three virtual ports (Virtual ports 1, 2, and 3) for these three different lead times, respectively. Moreover, we take into account berth availability and the uncertainty in estimating on-time arrival probability. We still adopt the three instances generated in the basic experiment in Sect. 5.2. We set ρ_T to 1, 0.8, and 0.5, which indicates that if a shipping company books within three days, there is a 50% chance of not being able to book an available berth. And ϕ_T is set to 0.3, 0.5, and 0.9, which means that if a shipping company books more than one week in advance, the accuracy of its estimation of the on-time arrival probability is only 0.3.

Table 7 reports optimal solutions for the cash refund policy of the three virtual ports. The piecewise booking fee under the cash refund policy is shown in Fig. 1a. We find that all three shipping companies will choose to book within three days and thus the total profit all

Table 5 Sensitivity analysis: the impacts of waiting time

Policy	Profit ($)	Booking fee ($)	Cash returned ($)	Value of coupon ($)	Shelf life of coupon(days)	Booking companies
$\widetilde{q} \sim \mathrm{N}(3, 0.5^2)$						
Cash refund	338,730	2,100	2,100	–	–	1,2,3
Coupon refund	352,800	2,400	–	2,400	12	1,2
$\widetilde{q} \sim \mathrm{N}(5, 0.5^2)$ (basic results)						
Cash refund	564,550	3,500	3,500	–	–	1,2,3
Coupon refund	588,000	4,000	–	4,000	12	1,2
$\widetilde{q} \sim \mathrm{N}(8, 0.5^2)$						
Cash refund	903,280	5,600	5,600	–	–	1,2,3
Coupon refund	940,800	6,400	–	6,400	12	1,2

Table 6 Average CPU times(s) for different setting of $|I|$

Policy	$\|I\|$				
	50	100	200	300	500
Cash refund	0.143	0.246	0.546	0.742	2.363
Coupon refund	2.198	15.109	48.957	84.411	922.092

Table 7 Optimal solutions for cash refund policy after relaxing assumptions

Port	Profit ($)	Booking fee ($)	Cash returned ($)	Booking companies
Virtual port 1	0	4,500	4,500	–
Virtual port 2	0	2,025	0	–
Virtual port 3	508,095	3,500	3,500	1,2,3

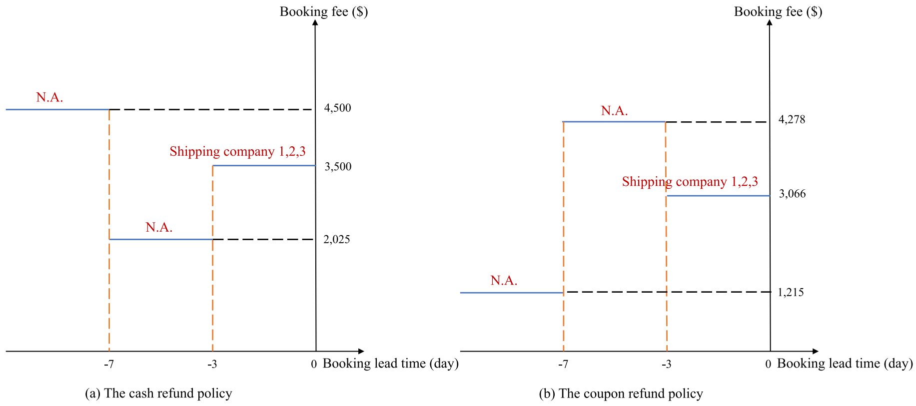

Fig. 1 The piecewise booking fee

comes from virtual port 3. Unexpectedly, virtual port 1 has the highest booking fee, which indicates that virtual port 1 is useless as no shipping company is willing to pay more when the uncertainty is higher. And booking close to the port of call also generates more benefits to the port because the port faces a lower probability of returning the booking fee. Interestingly, if we change the booking fees of virtual ports 1 and 2 to other values, say reduce the booking fees of these two virtual ports from $4,500 and $2,025 to $1,000 and $500, respectively, it would motivate some shipping companies to turn to these two virtual ports, namely change their booking lead times. However, this will reduce the total profit of the focal port. This reveals that the port's motivation of maximizing the profit also plays a critical role in making the shipping companies book berths when their ships are close to the port. If the focal port is willing to compromise its profit from this booking system and adjust its booking fees for different booking lead times, it would motivate some shipping companies to increase their booking lead times. This will allow the focal port to better plan the seaside and yard operations and manage its resources to avoid port congestion. When the port insists on profit maximization and is unwilling to adjust its booking fees, the berth reservation time window for the booking system does not need to be too long, but should not be too short as well because a too-short booking lead time will cause pressure to port operation planners and the relevant resources.

Table 8 Optimal solutions for coupon refund policy after relaxing assumptions

Port	Profit ($)	Booking fee ($)	Value of coupon ($)	Shelf life of coupon(days)	Booking companies
Virtual port 1	0	1,215	384	11	–
Virtual port 2	0	4,278	4,096	30	–
Virtual port 3	406,693	3,066	3,065	15	1,2

Optimal solutions for the coupon refund policy of the three virtual ports are shown in Table 8. We illustrate the piecewise booking fee of the coupon refund policy in Fig. 1b. Same to the cash refund policy, all booking companies choose to book in the third stage, i.e., virtual port 3. Compared to basic results in Table 2, the total profits in Tables 7 and 8 decrease because we consider the uncertainty in estimating the on-time arrival probability, which increases the probability of returning cash or coupon to shipping companies. Therefore, under the refund mechanism, both the port and shipping companies tend to accept or make reservations when the ship is about to arrive in port. However, the port should be aware of the fact that a too-short booking lead time will cause pressure to resource planning in seaside and yard operations.

In comparison between Tables 7 and 8, when we allow the piecewise format of the booking fee, the cash refund policy outperforms the coupon refund policy, mainly because of the higher booking fee and the involvement of three shipping companies. This is reasonable because in the situation of a short booking time every shipping company is confident about the on-time arrivals of their ships and coupons lose their attraction compared to refunded cash. Therefore, the port cannot make the booking fee of the coupon refund policy higher than that of the cash refund policy. This also delivers the managerial insight that the cash refund policy is recommended when a short booking lead time exists in the berth booking system.

5.5 Dicussion

With the above experimental results, we answer the proposed three RQs in Sect. 1.

Response to RQ1: The basic experimental results and sensitivity analyses all indicate that the coupon refund policy is better than the cash refund policy in terms of maximizing the port's profit. The main reason is that with the coupon refund policy, the port can set a short shelf life of the coupon and thus prevent some shipping companies from using coupons. However, these shipping companies still decide to make a reservation because of the high delay cost. Thus, our study recommends the coupon refund policy to ports if the booking fee is designed to be independent of the booking lead time. After Assumption 2 and Assumption 3 are relaxed, we find that clearly introducing a short booking lead time (say three days) in the berthing booking system that provides shipping companies with the ability to better estimate their on-time arrival probability will make coupons lose their attractiveness compared to cash refunds.

Response to RQ2: Our models can produce the optimal booking fee and returned cash for the cash refund policy, and produce the optimal booking fee, the value of the coupon, and the shelf life of the coupon for the coupon refund policy. By keying in the information related to the focal port (e.g., the distribution of waiting time) and the shipping companies (e.g., the company size, the probability of arriving on time, and the delay cost), we can investigate the changes in optimal solutions of the two policies. Thus, our study provides a decision-making tool for port authorities or terminal operators.

Response to RQ3: When the booking cost is less than the average delay cost, the shipping company will make a booking. As the port knows the delay cost and probability of arriving on time of each shipping company, it can take advantage of the high delay costs of the shipping companies to increase the booking fee for profit while the shipping companies are still willing to participate in the berth booking mechanism. Second, when the cash refund policy is adopted with piecewise booking fees according to different booking lead times, the port has to carefully consider its objective of profit maximization because this usually motivates the shipping companies to select the shortest booking lead times, which will eventually cause pressure to resource planning for seaside and yard operations. If the port proactively compromises its profit by adjusting its booking fees, it helps motivate some shipping companies to choose longer lead times and reduces the pressure of resource allocation and planning facing the port.

6 Conclusion

In this study, we propose a berth appointment/booking mechanism with a refund policy that allows shipping companies to book berths in advance via a booking system. From a macro point of view, this mechanism can alleviate port congestion, reduce emissions of ships, and help achieve environmental sustainability. From a micro perspective, this mechanism produces a win-win outcome for the port and shipping companies. The port can increase its revenue by charging booking fees; shipping companies can secure berths in advance for its ships through booking to avoid high delay costs and reduce its ships' bunker fuel consumption costs and emissions at sea through just-in-time (virtual) arrivals. By developing a bi-level optimization model and further transforming it into two mixed-integer linear programming models, we obtain the optimal booking fee and returned cash for the cash refund policy, and obtain the optimal booking fee, the value of the coupon, and the shelf life of the coupon for the coupon refund policy. We also propose the approaches of relaxing modeling assumptions, which shows the wider applicability of our methods.

With numerical experiments, we compare the maximum profits of the cash refund policy and the coupon refund policy with different parameter settings and find that the coupon refund policy is a better choice for the port if a constant booking fee independent of booking lead time is designated. Sensitivity analyses for one thing confirm the robustness of this finding and for another investigate the shipping companies' reactions according to different parameter settings. In the experiments of validating assumption relaxations, we find that the cash refund policy can bring more profit if the port can consider the booking fee as a step function of the booking lead time (that includes a short booking lead time) and we suggest that the booking time window does not need to be too long. However, the port has to carefully consider its objective of profit maximization because this usually motivates the shipping companies to select the shortest booking lead times, which will eventually cause pressure to resource planning for seaside and yard operations.

Our study takes the initiative to explore the charging and refund policies in a berth booking system, and for the first time demonstrates the industry potential and benefits of a berth appointment mechanism. Our research also provides a groundbreaking discussion on the mechanism behind the berth booking ICT system, thereby providing a theoretical basis for the implementation of the berth booking ICT system in the industry.

However, our research is not without limitations. We did not take into account the interactions between shipping companies in the model. In real-world scenarios, whether a shipping company will book berths for its ships is affected by the booking status and behaviors of other shipping companies. Future studies can build more complicated models to capture more practical considerations. As the berth appointment mechanism assists shipping companies to implement the just-in-time (virtual) arrival policy and reduce their ships' bunker fuel consumption at sea, future studies can also quantitatively investigate the savings of bunker fuel cost from the shipping companies' perspective.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical Approval This study does not contain any studies with human participants or animals performed by any of the authors.

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