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Dynamic Flexible Allocation of Slots in Container Line Transport

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Abstract: Due to the imbalance between supply and demand, liner container transportation often faces the problem of low slot utilization, which will occur in the shipping process, such as dry container demand exceeding the available dry slots and reefer slots not being fully utilized. This makes it important and challenging to maintain a balance between the actual demand and the limited number of slots allocated for liner container transport. Therefore, this study proposes a flexible allocation method: expanding the types of containers that can be loaded in the same slot. This method is suitable for handling each dynamic arrival container booking request by shipping enterprises, making decisions to accept or reject, and flexibly allocating shipping slots. In order to maximize the total revenue generated by accepting container booking requests during the entire booking acceptance cycle, we establish a dynamic programming model for the flexible allocation of slots. For model solving, we use the Q-learning reinforcement learning algorithm. Compared with traditional heuristic algorithms, this algorithm can improve solving efficiency and facilitate decision-making at the operational level of shipping enterprises. In terms of model performance, examples of different scales are used for comparison and training; the results are compared with the model without flexible allocation, and it is proved that the model proposed in this paper can obtain higher returns than the model without flexible allocation. The results show that the model and Q-learning algorithm can help enterprises solve the problem of the flexible allocation of shipping slots, and thus, this research has practical significance.

Keywords: dynamic container flexible slot allocation; dynamic programming; Q-learning algorithm

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1. Introduction

At present, the container transportation market is affected by multiple factors such as the global economic environment, trade policy, and seasonal demand, and shipping enterprises are faced with many challenges [1,2]. In order to adapt to the demand fluctuations under different market conditions, effectively manage the capacity, and enable enterprises to achieve sustainable and green development, flexible allocation of shipping slots has become one of the important strategies to achieve this goal [3–5]. From an enterprise perspective, the flexible allocation of shipping slots can help shipping companies better control transportation costs and maximize profits. This strategy not only improves market responsiveness but also optimizes resource allocation and supports effective revenue management. Therefore, it is of great significance to study the flexible allocation of shipping slots. Through an in-depth study of this problem, shipping companies can remain competitive in a complex and changing market environment and thus, achieve sustainable development.

Container liner transport refers to a mode of operation in which shipping enterprises provide container cargo transport services for most shippers according to fixed shipping schedules and fixed ports of call on fixed routes, and charge freight according to a "container freight rate". Due to this basic characteristic of "five fixations" in liner transportation, routes, segments, shipping slots, arrival times, etc. need to be formulated in advance, and will not be changed after actual operation. Typically, the liner slot planning on a certain route will forecast the future demand according to historical data, but due to the mismatch between the forecast and the actual situation, the utilization rate is not as good as expected, resulting in a deficiency of corporate revenue.

In the past, companies were unable to solve this problem because, in the traditional operation process, the shipping enterprise will receive the shipping slot reservation request submitted by the shipper to the container liner company and allocate it. The request will contain information about the containers required for booking, including, the number of containers (e.g., 5, 10), the type of container (dry or refrigerated), the size of the container (e.g., 20 feet or 40 feet), the names of the departure port and destination port, etc. Like the type of container, the liner slot can also be divided into two types: dry slots and reefer (refrigerated) slots. The traditional slot allocation rule is to load dry containers in the dry slots, and only load refrigerated containers in the reefer slots, which can provide power to retain a constant temperature. This traditional allocation method cannot alleviate the mismatch between the forecast and the actual situation, and there will be a problem that one type of slot is full and the utilization rate of another type of slot is low, resulting in a waste of slots.

Thus, the purpose of this study is to dynamically and flexibly allocate dynamically arrived shipping orders according to the loading characteristics of different shipping slots of container liners, to achieve the research purpose of improving the loading rate of liner routes, and thus improve the profits of liner enterprises. In addition, flexible allocation is an important strategy to reduce the mismatch between the forecast and the actual situation. We established a dynamic planning model for flexible shipping slot orders and adjusted dynamic order-taking operations to solve the problem of revenue reduction caused by mismatching.

Thus, one contribution of this study is that, based on the traditional distribution method, this study added dynamic flexible slot allocation into the model. The flexibility is that when dry container orders arrive but the dry slot is full, the reefer slots can be loaded with dry containers to improve slot usage. However, renting and selling reefer slots at the price of dry slots will reduce the profits obtained from reefer slots. Therefore, it is necessary to evaluate the possible future income and the income of accepting current orders according to historical experience and dynamically allocate orders to ultimately ensure the profits of enterprises and improve the utilization rate of slots, which is of great significance to the revenue management of liner container transportation enterprises.

In addition to the problem of shipping slot allocation, the solution of the model is also a problem, which is the result of two main factors. First, with the increase in the number of booking orders, manual processing is becoming more and more difficult, and it is difficult for manual workers to compare the possible future earnings with the current earnings based on historical data to determine whether to accept the current orders. In addition, the booking of shipping slots needs to consider factors such as route, segment, type of shipping slot, and quantity, which will make the difficulty of data processing increase exponentially, resulting in dimensional difficulties. Shipping enterprises hope to use machine learning and online learning algorithms for dynamic planning of shipping slot allocation in the future, which will help enterprises flexibly use shipping slots and efficiently allocate shipping slots by using models that are more in line with the actual situation and more intelligent and optimized decision-making mechanisms. Based on this, in this study, the dynamic programming model based on flexible allocation is solved by the use of a Q-learning algorithm, which has great advantages in solving mathematical models faced with dimensional difficulties and processing large amounts of data and can effectively improve the effect and practicability of the model, which is another contribution of this research.

The rest of this study is organized as follows: We review relevant studies in Section 2 to identify differences between this study and previous studies, highlighting the contribution of this study. In Section 3, we elaborate on the flexible dynamic programming model for shipping slot allocation, and the algorithm for solving the model is introduced in Section 4. In Section 5, extensive case experiments are conducted to evaluate the research methods proposed in this study and to complete the case analysis. Finally, the future research direction is summarized and prospected in Section 6.

2. Literature Review

Revenue management is about predicting customer behavior at the micro level, to optimize products and services and improve actual revenue for the enterprise. There are not many studies in this field, and relevant transport enterprises have also experimented with this approach for a long time, e.g., air transport, which was the first transport enterprise to study revenue management. In order to improve their competitiveness, airlines have conducted management research on revenue forecasting, seat reservation, pricing, and other aspects [6,7]. Strauss et al. collated a wealth of previous research on revenue management, analyzed and summarized the latest advances in demand modeling and commodity portfolio optimization, and discussed the relationship with dynamic pricing [8]. Klein et al. also reviewed the literature on revenue management, but they focused more on the applicability of revenue management and clarified the indispensability of future revenue management in different industries, which is of great research significance [9].

However, due to the differences between air transport and container liner transport, the revenue management strategy of the former cannot be directly applied to the liner transport industry: (i) Air transport is generally a one-stop type of transport, without intermediate stops, but liner transport is closed-loop transport, and the destination port of some goods can also be the starting port of other goods; thus, there will be differences in the model. (ii) Container liner traffic usually involves a complex network of multiple ports and routes, while air freight networks may be relatively simple, making the factors to consider more complex when formulating a liner strategy. (iii) There are significant differences between container liner traffic and air freight in terms of transportation mode, cargo type, transportation time, transportation capacity, etc. Container transport has more types of goods with mainly low value, long transport time, and large volume, and more considerations will lead to dimensional disaster in calculations. Therefore, many scholars have conducted research on container transportation to improve the benefits of transportation. The problem of container revenue management is mainly studied in two main directions: route, capacity, and other resource management and pricing adjustment.

In terms of resource management, some scholars have conducted in-depth studies from the perspectives of ship, container, slot, and resource seasonality. In view of shipping line resources, Fagerholt investigated the optimization problem of determining the optimal fleet of vessels in actual liner shipping operations, specifically focusing on the types of vessels and the quantity of each type [10]. The liner shipping problem is a variant of the multi-trip vehicle routing problem, which involves determining the weekly routes for the selected vessels. So Fagerholt proposed a solution approach consisting of three stages. Lai X et al. proposed a two-stage robust optimization model to solve the problems of fleet deployment and shipping revenue management in liner shipping networks under demand uncertainty [11]. A precise algorithm based on column generation and constraint generation was designed to represent the randomness of the requirements through a probabilistic uncertainty set, and the M-tightening technique was used to accelerate convergence. Lu H A et al. studied the problem of slot allocation planning by container shipping lines to meet the estimated seasonal liner service demand [12]. Considering the factors that affect the planning, a quantitative model of optimal allocation of ship slots was established, and a practical example of East-Asian short-haul routes was studied. Han et al. studied container resource management [13]. In one planning horizon, a stochastic dynamic program model was constructed to study the number of empty containers decision problem for multiple schedules with random demand. Through the planning of empty container resources, the profits of the carrier could be maximized. Similarly, our study focuses on slot resources. Conversely, revenue can also be improved through pricing adjustment. Ceryan et al. considered the problem of optimal management of different surplus stocks under multiple products [14]. By studying the alternative mechanism of price and capacity, they were able to introduce alternative products to alleviate the problem of the remaining capacity of shipping slots. In addition to spare capacity, pricing is also related to transportation time. Based on the background that transportation time is affected by port congestion, Wang proposed a freight pricing method related to transportation time. The freight rate function considering the effect of port congestion on transportation time directly reflects the relationship between pricing and time [15]. Meng et al. summarized the research progress of revenue management in liner transportation and indicated that due to the serious imbalance between supply and demand in the container transportation market in the future, liner transportation needs to find a suitable revenue management model to improve corporate revenue [16].

In view of the resource of liner slots, the research on container liner shipping slot allocation mainly focuses on the following areas: (1) Forecasting based on historical data. By referring to the related field of aviation industry forecast research [17,18]. Zhao et al., focusing on the loss of revenue and low utilization rate of ship slots caused by booking cancellations, established a data-driven model of container booking based on time-to-event modeling [19]. Based on historical data, they increased the level of vulnerability to capture the forecast characteristics of different regional markets, to better help companies complete the work of shipping slot allocation. (2) Demand uncertainty. Liu J et al. analyzed the port congestion of the maritime supply chain during the COVID-19 epidemic based on demand uncertainty, focusing on the port system and countermeasures [20]. By combining SEIR models, a container port congestion assessment model was established to study and evaluate the effect of different management measures. Chargui K et al. studied berth allocation and dock crane allocation and scheduling issues that take into account energy price changes, showing the benefits of integrating energy price changes [21]. A robust formula was proposed to optimize the worst case to deal with the uncertainty, and an exact decomposition algorithm and four new strengthening programs were designed. In the face of demand uncertainty, Xiang X et al. proposed a robust optimization method to solve the problems of route fleet deployment and empty container repositioning [22]. (3) Cooperation in slot allocation based on shipping alliances. Michael et al. applied the classical frequency-based transit allocation method to container slot allocation and proposed a global shipping container allocation model that could support the assessment of the vulnerability of the supply chain to shipping disruption [23]. There are studies to optimize liner transportation decisions by integrating dynamic slot allocation and inventory management [24]. In order to simulate the problem of full and empty shipping containers, the concept of multi-segment revenue management was adopted. The model considered market differences, service diversity, and other scenarios that cannot be predicted. Some scholars systematically reviewed liner alliance management, and discussed alliance establishment, partner selection, cooperation mechanism design, collaborative planning, and performance evaluation [25]. (4) Price and profit. Bell et al. proposed a cost-based container allocation model that included both full and empty containers and took into account port and route capacity constraints. A linear programming procedure suitable for this model was proposed and analyzed to help shipping companies, port authorities, cargo owners, and third-party service providers optimize resource allocation [26]. (5) Optimize according to the type and requirements of the shipper. Liang J et al. studied the distribution of capacity between contract and spot shippers in the liner shipping industry with the aim of maximizing total freight revenue and maintaining the market loyalty of contract shippers [27]. By formalizing the problem into a stochastic linear programming model and applying Blackwell's reachability theorem, a near-optimal strategy was designed to guide slot allocation decisions.

To sum up, although there have been many studies that have considered the impact on the allocation of shipping slots, the current allocation of booking requests for container liners is still mostly based on the First-Come First-Served (FCFS) strategy or the empirical judgment of customer value [16]. Without dynamic allocation of shipping slots, it is impossible to maximize the cargo capacity of ships and maximize profits, thus affecting the profits of enterprises. Therefore, based on the attributes of shipping slots, dynamic flexible transport allocation can improve the utilization rate of shipping slots, which is a problem rarely studied in shipping enterprises at present.

Since there are few studies on flexible allocation in shipping at present, this paper refers to the studies on flexible product production and flexible production in other industries for transformation and then carries out the optimization modeling of operation research from the perspective of flexibility. The first step is the study of flexible products and flexible production in other industries. Due to product diversification, large-scale ordering, and other phenomena, the importance of flexible production in manufacturing enterprises is increasing [28]. In the face of a dynamic market, flexible production is also very important to adapt to the dynamic market and improve customer satisfaction. Therefore, Saene et al. built a structural equation model (SEM) to evaluate manufacturing flexibility and its impact on customer satisfaction [29]. Gallego and Phillips defined a flexible product as a menu of two or more alternative products (usually alternative products) offered by a constrained supplier using a sale or booking process [30]. The concept of flexible products was introduced, and the management conditions and algorithms of a single flexible product composed of two specific products were derived. Secondly, in terms of model optimization, Petrick et al. discussed how to use mathematical models to dynamically control the revenue management of flexible products in highly uncertain demand markets and studied its practical application scenarios [31]. Petrick et al. proposed several different revenue management models and control mechanisms to deal with the revenue maximization problem of flexible products [32]. Koch et al., based on the dynamic programming model of Gallego et al., utilized the Fourier–Motzkin elimination method to construct a revenue management dynamic programming model with flexible product and customer choice, which achieved better revenue performance and improved enterprise income [33]. While there is a connection between these studies, which all consider flexible products, and the current study, the differences between them are significant and are listed below.

Firstly, methodologically, shipping optimization generally adopts mathematical modeling and optimization methods to evaluate and optimize the flexibility of products or shipping slot allocation, to achieve more effective resource utilization and revenue management [16]. Secondly, the characteristics of the subjects are different. The study of flexible products in the manufacturing industry mainly focuses on the diversity and customization of products and technologies, such as the flexibility of product mixes and production processes [34]. Flexible seat allocation in railway transportation is based on the idea of flexible rolling stock combinations, while the study of flexible shipping slots of container liners pays more attention to the dynamic adjustment of shipping slot allocation and the flexible use of ship resources to adapt to the changes of different routes and seasonal demands [35]. Third, the research focus is different. In the past, transportation problems have been solved with the minimum cost as the goal [26]; however, the flexible shipping slot problem considered in this paper will be studied with the goal of maximizing corporate profits.

Based on the flexible product approach of other industries and the relevant concepts of revenue management, this study makes the following innovations: Shipping planning can be divided into three levels: strategic, tactical, and operational [36]. Most of the existing optimization behaviors focus mainly on the first two layers [37]. In this study, dynamic shipping slot allocation is carried out according to real-time orders, and dynamic adjustment is carried out in the operation layer to maximize the enterprise's income. Unlike the previous practice that reefer slots can only store refrigerated boxes and dry slots only store dry containers, this study fully allocates and utilizes slots according to the attributes of the slot and shipping capacity [38]. Similarly, unlike traditional model-solving algorithms, this study uses machine learning algorithms to solve dynamic programming problems faster [39]. To sum up, the purpose of this study is to optimize the allocation of container slots, help shipping enterprises make better dynamic and flexible allocation decisions, and thus improve the earnings of liner enterprises, which has certain research significance (Table 1).

Table 1. Relevant Previous Studies.

Category				Reference
Container Revenue Management	Resource Management	Shipping Resource		Fagerholt K (1999) [10] "Optimal fleet design in a ship routing problem"
		Container Resource		Han G, Pu X, He Z, et al. (2018) [13] "Integrated planning and allocation: A stochastic dynamic programming approach in container transportation"
		Resource Seasonality		Lu H A, Chu C W, Che P Y (2010) [12] "Seasonal slot allocation planning for a container liner shipping service"
		Slot Resource	Forecasting	Zhao H, Meng Q, Wang Y (2020) [19] "Probability estimation model for the cancellation of container slot booking in long-haul transports of intercontinental liner shipping services"
			Demand Uncertainty	Liu J, Wang X, Chen J (2023) [20] "Port congestion under the COVID-19 pandemic: The simulation-based countermeasures" Chargui K, Zouadi T, Sreedharan V R, et al. (2023) [21] "A novel robust exact decomposition algorithm for berth and quay crane allocation and scheduling problem considering uncertainty and energy efficiency" Xiang X, Xu X, Liu C, et al. (2024) [22] "Liner fleet deployment and empty container repositioning under demand uncertainty: A robust optimization approach"
			Shipping Alliances	Bell M G H, Liu X, Angeloudis P, et al. (2011) [23] "A frequency-based maritime container assignment model" Chen J, Ye J, Zhuang C, et al. (2022) [24] "Liner shipping alliance management: Overview and future research directions" Mehrzadegan E, Ghandehari M, Ketabi S (2022) [25] "A joint dynamic inventory-slot allocation model for liner shipping using revenue management concepts"
			Price and Profit	Bell M G H, Liu X, Rioult J, et al. (2013) [26] "A cost-based maritime container assignment model"
			Shipper Type and Requirements	Liang J, Li L, Zheng J, et al. (2023) [27] "Service-oriented container slot allocation policy under stochastic demand"
			Slot Attribute	This study
	Price Management	Relationship between Remaining Slots and Price		Ceryan O, Sahin O, Duenyas I (2013) [14] "Dynamic pricing of substitutable products in the presence of capacity flexibility"
		Transit Time		Wang T, Tian X, Wang Y (2020) [15] "Container slot allocation and dynamic pricing of time-sensitive cargoes considering port congestion and uncertain demand"

Table 1. *Cont.*

Category				Reference
Flexibility	Profession	Manufacturing Industry		Scherrer-Rathje M, Deflorin P, Anand G (2014) [28] "Manufacturing flexibility through outsourcing: effects of contingencies" Sáenz M J, Knoppen D, Tachizawa E M (2018) [29] "Building manufacturing flexibility with strategic suppliers and contingent effect of product dynamism on customer satisfaction" Gallego G, Phillips R (2004) [30] "Revenue management of flexible products" Mishra R, K. Pundir A, Ganapathy L (2014) [34] "Assessment of manufacturing flexibility: a review of research and conceptual framework"
		Transportation	Rail Transport	Yan Z, Li X, Zhang Q, et al. (2020) [35] "Seat allocation model for high-speed railway passenger transportation based on flexible train composition"
			Liner transport	This study
	Model and Algorithm	Heuristic Algorithm		Meng Q, Zhao H, Wang Y (2019) [16] "Revenue management for container liner shipping services: Critical review and future research directions" Petrick A, Gönsch J, Steinhardt C, et al. (2010) [31] "Dynamic control mechanisms for revenue management with flexible products" Petrick A, Steinhardt C, Gönsch J, et al. (2012) [32] "Using flexible products to cope with demand uncertainty in revenue management" Koch S, Gönsch J, Steinhardt C (2017) [33] "Dynamic programming decomposition for choice-based revenue management with flexible products"
		Machine Learning		This study
	Research Objective	Minimum Cost		Bell M G H, Liu X, Rioult J, et al. (2013) [26] "A cost-based maritime container assignment model"
		Maximum Benefit		This study

3. Problem Description and Model

In Section 3.1, the background of dynamic flexible container slot allocation is described in detail, and the influence of flexible slot allocation of dry and reefer slots on liner transport service income is illustrated by an example at the operational level. Then, in Section 3.2, a mathematical model is used to describe the problem logic. This problem is modeled as a dynamic programming model in Section 3.3.

3.1. Shipping Revenue Management

Assume that a container liner company has a route already in operation for a reasonable allocation of shipping slots and improvement of operating income, as shown in Figure 1. The route consists of four ports, with two adjacent ports forming a segment, comprising four segments: port 1 $\rightarrow$ port 2, port 2 $\rightarrow$ port 3, port 3 $\rightarrow$ port 4, and port 4 $\rightarrow$ port 1. The liner transport route starts at port 1, travels to port 4, and then returns to port 1 along the route, circulating the transport. The freight rate is impacted by different segments and different container types. There are 2000 standard container slots (TEU) on the line, which can transport 1500 dry containers and 500 plug-in refrigerated containers. Accordingly, since these 500 reefer slots require additional power supply, their freight rates will be higher than ordinary dry slots, which can bring higher profits to the shipping company. Therefore, when dry slots are sold out and reefer slots are not fully booked, there are still dry containers scheduled to arrive, and the decision on whether to sell reefer slots for loading dry containers is of great importance to the revenue management of enterprises. This study hopes to improve the operating efficiency of the enterprise and increase its income by improving the utilization rate of slots through the dynamic allocation of different types of slots on different route segments.

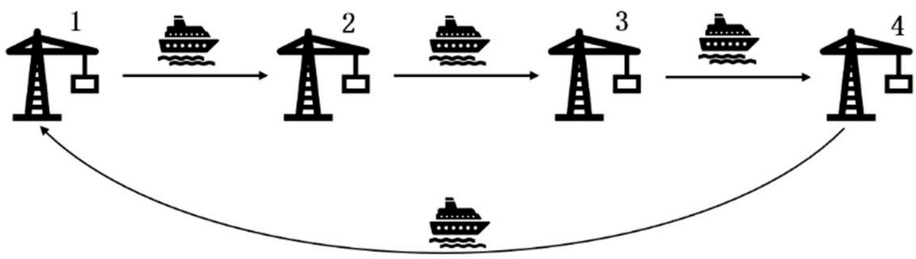

Figure 1. Liner routes with four ports of call. (The different numbers represent the serial number of the ports.).

3.2. Problem Description

Consider a container liner route consisting of M ports, such that $\mathbf{M} = \{1, \ldots, i, j, \ldots, M\}$, where i and j indicate the port serial number. The route of a container liner can form a loop between ports, which can be represented by the sequence $\{1 \rightarrow \cdots \rightarrow i \rightarrow \cdots \rightarrow M \rightarrow \cdots \rightarrow j \rightarrow \cdots \rightarrow 1\}$. In liner transportation, fixed routes, fixed ports, fixed shipping dates, and relatively fixed rates are observed, and any detour will cause a loss of income while wasting time. Therefore, in order to avoid the detour problem of port 4 $\rightarrow$ port 3 in the liner route loop, a set $\mathbf{P} = \{(i,j) : i,j \in \mathbf{M}\}$ is established, which includes all available port pairs in the route. Therefore, $P = |\mathbf{P}|$ is used to represent the number of all available port pairs in the route. The route between two adjacent ports is denoted as one segment, and all segments on the route are represented by the set $\mathbf{L} = \{1, \ldots, l, \ldots, L\}$. A membership index $\rho_l^{(i,j)}$ is introduced that takes a value of 1 for the route segments between the port of origin and the port of destination, and 0 otherwise.

In this study, four types of containers are included in the orders: 20-foot dry containers (20′ D), 40-foot dry containers (40′ D), 20-foot reefer containers (20′ R), and 40-foot reefer containers (40′ R). $\mathbf{K} = \{D, R\}$ and $\mathbf{F} = \{20, 40\}$ are thus, th set of container types and sizes. In liner shipping, a slot is a standard container (TEU) unit, and a 40-foot container occupies two TEU slots.

Next, we define and divide the time phases of the DP model. The remaining time for the container liner to accept orders before shipment is denoted as T, which is divided into multiple equal time periods t. The customer order information will be received successively, the scheduled voyage length is a random variable, and the probability of customer arrival satisfies the time homogeneity. The set of stages contained in the time period T is represented by $\mathbf{T} = \{1, \ldots, t, \ldots, T\}$. One customer, at most, can arrive in each time period, subject to a certain arrival probability.

In Figure 2, we show the decision-making process of dynamic planning for flexible slot allocation. In stage t, the presence of customers is first determined, followed by the decision on whether to accept the order based on the remaining quantity of dry and reefer slots. The remaining slot quantity is adjusted according to the decision ready for the next stage, and this process continues until all the slots are booked or the predetermined time ends.

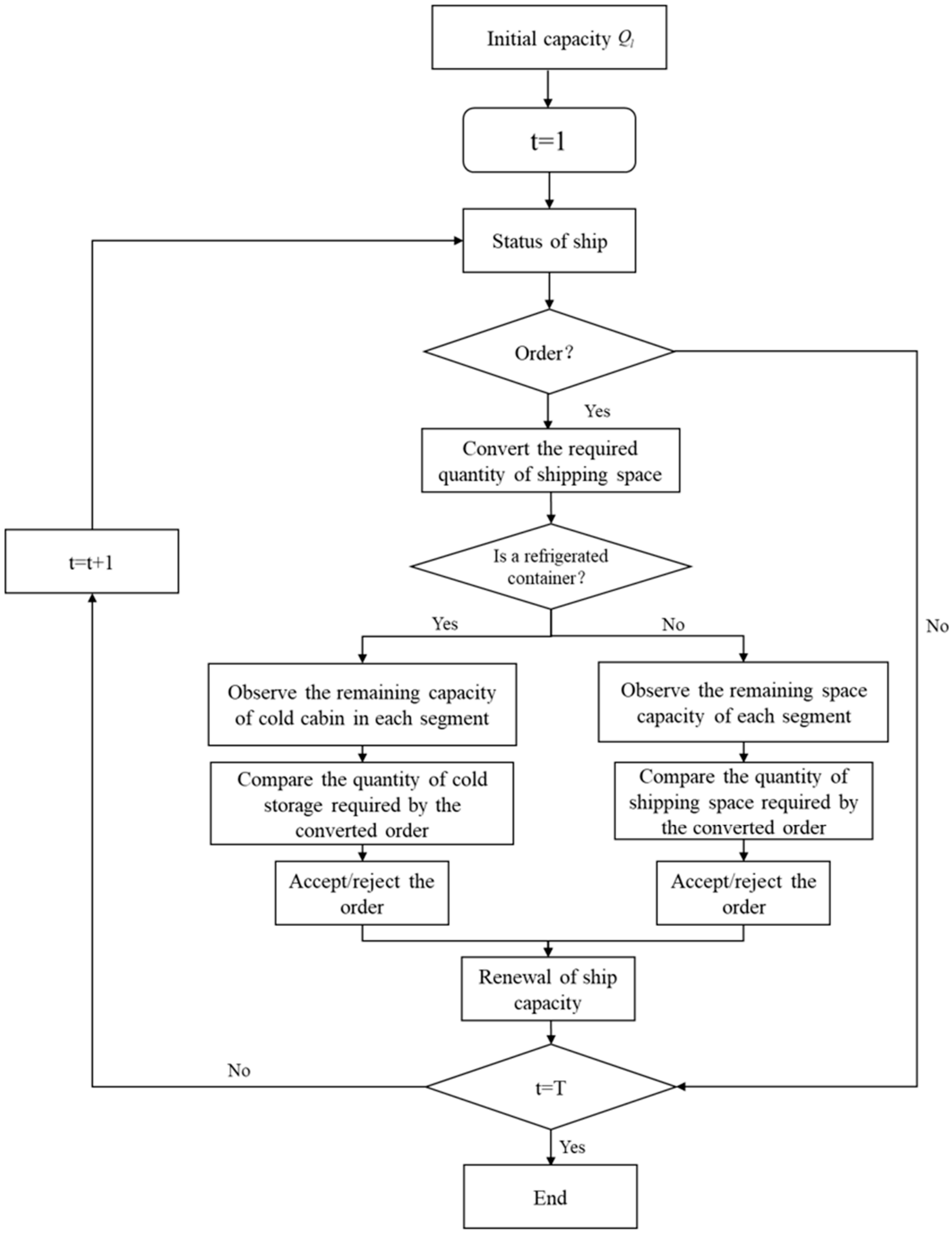

Figure 2. Dynamic allocation decision flow chart.

3.3. Dynamic Programming Model

Before establishing the model, we first introduce the parameter symbols used in the process (Table 2).

Table 2. Model Symbols.

Set	
$\mathbf{K}$	The set of container types, $\mathbf{K} = \{D, R\}$, D stands for dry container, R stands for reefer container
$\mathbf{F}$	The set of container sizes, $\mathbf{F} = \{20, 40\}$, representing 20 feet and 40 feet
$\mathbf{L}$	The set of legs involved in a line shipping route indexed by l, $\mathbf{L} = \{1, \ldots, l, \ldots, L\}$
$\mathbf{M}$	The set of ports called at on a liner shipping route indexed by i and j, $\mathbf{M} = \{1, \ldots, i, j, \ldots, M\}$
$\mathbf{N}$	The set of the number of containers needed by a container slot booking request indexed by n, $\mathbf{N} = \{1, \ldots, n, \ldots, N\}$
$\mathbf{P}$	The set of the non-detoured port pairs on a liner shipping route, $\mathbf{P} = \{(i, j) : i, j \in \mathbf{M}\}$
$\mathbf{T}$	The set of time periods during which the liner accepts the order, $\mathbf{T} = \{1, \ldots, t, \ldots, T\}$
Parameter	
Q_l	Segment l total initial available slots
$\mathrm{m}_{\mathrm{kf}}^{(i,j)}$	Initial request: transport m containers of type k and size f from Port i to Port j
$\mathrm{n}_{\mathrm{k}}^{(i,j)}$	Modified request: n standard containers of type k to be shipped on OD port pair (i, j), $\mathrm{n}_{\mathrm{k}}^{(i,j)} = \mathrm{m}_{\mathrm{kf}}^{(i,j)} \times 40/\mathrm{f}$
$\lambda^{\mathrm{t}}\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$	The probability of request $\mathrm{n}_{\mathrm{k}}^{(i,j)}$ arrives within booking period t, $\left(1 - \lambda^{\mathrm{t}}\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)\right)$ means no request at period t
$s_{kl}^{(i,j)}$	The number of type k slots required in the segment l during the transportation from Port i to Port j
$r_{k}^{(i,j)}$	The cost of container type k from Port i to Port j
$R\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$	The revenue of shipping a request $\mathrm{n}_{\mathrm{k}}^{(i,j)}$, $R\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) = u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) \times \mathrm{n}_{\mathrm{k}}^{(i,j)} \times r_{n}^{(i,j)}$
x_{kt}^{l}	The number of remaining slots of type k on the flight segment of stage t
Decision variable	
$u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$	1 if the order of stage t is accepted; Otherwise, it is 0

Assume that during the booking period of a container liner company, the customer request $\mathrm{m}_{\mathrm{kf}}^{(i,j)}$ is received at stage t, that is, the customer requires the liner container carrier to transport n containers of type k and 20/40 feet from origin port i to destination port j without detour. First, the number of containers in the request is converted into the number of containers: $\mathrm{n}_{\mathrm{k}}^{(i,j)} = \mathrm{m}_{\mathrm{kf}}^{(i,j)} \times 40/\mathrm{f}$. $\mathrm{n}_{\mathrm{k}}^{(i,j)}$ is subsequently used to represent the demand of the request.

After receiving the request, the liner company decides whether to accept it. Firstly, it is necessary to convert the number of containers in the customer request into the actual number of shipping slots accepted in the liner order. The transformed order information is used as an auxiliary request. An L-dimensional row vector $s\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$ is used to describe the shipping slot demand of the request $\mathrm{n}_{\mathrm{k}}^{(i,j)}$ on L segments of the same route, which can be expressed specifically as $s\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) = \left(s_l\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)\right)$, where $s_l\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$ describes the shipping slot quantity required by the original order $\mathrm{n}_{\mathrm{k}}^{(i,j)}$ on segment L, which is converted by using the membership index $\rho_l^{(i,j)}$:

$$s_l\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) = n\rho_l^{(i,j)},\ \forall (i,j) \in \mathbf{P},\ l \in \mathbf{L},\ k \in \mathbf{K},\ n \in \mathbf{N} \tag{1}$$

After conversion, the acceptance of the request needs to meet the condition $s_l\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) \leq Q_l$ to ensure that the number of shipping slots required for the request does not exceed the initial total number of available shipping slots for the segment; otherwise, the request is rejected.

Next, we decide on the current available capacity of different types of shipping slots on all route segments of the request path to determine whether the request can be accepted. When the remaining capacity on all segments of the route meets the request, the request is accepted, i.e., $u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) = 1$; otherwise, it is rejected and the value is 0.

The state variable is set as x_t^l, and x_t^l is used to represent the total number of remaining slots on segment l at stage t. The L-dimensional row vector $X_t \equiv \left(x_t^l\right)$ is used to describe the capacity status of different segments of slots at the beginning of stage t, and is called the state vector of stage t. In order to distinguish the difference between the remaining capacity of dry slots and reefer slots, x_{kt}^l is used to represent the number of container type k slots remaining on segment l at stage t, while meeting $x_t^l = x_{Dt}^l + x_{Rt}^l$. In addition, let $Q \equiv (Q_l)$ be the L-dimensional initial state vector, where $Q_l = x_1^l$, $l \in L$ represents that the remaining capacity of segment l is equal to Q_l when the initial $t = 1$. All the possible state variables x_t^l of the t phase form the state slot of the t phase, namely the vector X_t. In order to accept a pre-booked request for stage t, the following constraints (2) must also be met, indicating that there are sufficient slots remaining for stage t to accept the arrival order:

$$X_t \geq u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)s\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right),\ \forall \mathrm{t} \in \mathbf{T},\ (i,j) \in \mathbf{P},\ k \in \mathbf{K},\ n \in \mathbf{N} \tag{2}$$

The vector $u^t \equiv \left(u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)\right)$ is used to represent the decisions that may be made during the t phase. The feasible decision set for any stage t is $U(X_t) = \left\{u^t \in \{0,1\}^{KPN} : \mathrm{X}_t \geq u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)s\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right),\ \forall \mathrm{k} \in \mathbf{K},\ (i,j) \in \mathbf{P},\ n \in \mathbf{N}\right\}$, which means that if a request is accepted for that period, the available slots for that period must meet the number of slots required by the order. In terms of benefits, if a request arrives at stage t and the constraint is accepted, then benefits $u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)R\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$ will be brought, where $R\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) = r_k^{(i,j)}n$.

Further considering the dynamic planning problem of reefer slots and dry slots, container type k slots can be divided into D and R, where D represents the dry slots and R represents the reefer slots. Then, the state vector X_t represents the total remaining capacity of t stage, which is composed of the remaining capacity vectors of the two types of slots, $X_t \equiv \left(x_{D,t}^l, x_{R,\mathrm{t}}^l\right)$, where $x_{D,t}^l$ and $x_{R,\mathrm{t}}^l$ represent the state variables of dry and reefer slots at stage t, respectively. The change in the status variable is affected by the quantity and type of the request slot:

(1) If a request arrives at stage t and the demand is for reefer containers, only reefer slots can be used to fulfill the request, and dry slots cannot be used, the conditions $x_{R,\mathrm{t}}^l \geq s\left(\mathrm{n}_R^{(i,j)}\right)$ must be met. The dry slot state variable of the next $t+1$ stage is $x_{D,t+1}^l = x_{D,t}^l$, which remains unchanged, while, the reefer slot status variable in the next stage is limited by capacity and the number of slots required by the request. The state transfer equation is $x_{R,\mathrm{t+1}}^l = \max\left\{x_{R,\mathrm{t}}^l - u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right), 0\right\}$, which means: if $x_{R,\mathrm{t}}^l > u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right)$, the remaining capacity of reefer slots meets the demand of the request and the request can be accepted, then the status variable changes to $x_{R,\mathrm{t+1}}^l = x_{R,\mathrm{t}}^l - u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right)$; conversely, if $x_{R,\mathrm{t}}^l < u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right)$, the remaining capacity of the reefer slots is less than the demand of the request, thus, the request cannot be accepted, $u^t\left(\mathrm{n}_R^{(i,j)}\right) = 0$ and $u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right)$ are 0, then $x_{R,\mathrm{t+1}}^l = x_{R,\mathrm{t}}^l - u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right) = x_{R,\mathrm{t}}^l$; alternatively, if $x_{R,\mathrm{t}}^l = u^t\left(\mathrm{n}_R^{(i,j)}\right)s\left(\mathrm{n}_R^{(i,j)}\right)$, the remaining capacity of the reefer slots is equal to the demand of the request, and the request is accepted, then $x_{R,\mathrm{t+1}}^l = 0$. Finally, the state variable is maximized.

(2) If a request arrives at stage t and the demand is for a dry slot, either a dry slot or a reefer slot can be used under the condition of $X_t \geq s\left(\mathrm{n}_D^{(i,j)}\right)$. However, the use of reefer slots to load dry container cargo will affect the return of reefer slots, and the priority should be allocated to dry slots in this case. When the remaining capacity of dry slots is greater than the number of slots required for accepting the request, that is, when $x_{D,\mathrm{t}}^l \geq u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right)$, the dry slot status variable in the next stage is $x_{D,\mathrm{t}+1}^l = x_{D,\mathrm{t}}^l - u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right)$, and the reefer slot status variable in the next stage is maintained, i.e., $x_{R,\mathrm{t}+1}^l = x_{R,\mathrm{t}}^l$. When the remaining capacity of dry slots is less than the number of slots required for accepting the request, but the remaining demand can accept the request if the cargo is transported by reefer slots, that is, when $x_{D,\mathrm{t}}^l < u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right)$, $x_{D,\mathrm{t}}^l + x_{R,\mathrm{t}}^l \geq u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right)$, the dry slot status variable in the next stage is $x_{D,\mathrm{t}+1}^l = 0$, and the reefer slot status variable in the next stage is $x_{R,\mathrm{t}+1}^l = x_{R,\mathrm{t}}^l - \left[u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right) - x_{D,\mathrm{t}}^l\right]$.

Next, $f(t, X_t)$ is further set to represent the maximum expected return value in the given state X_t of stage t, which becomes the value function of stage t. The termination conditions of dynamic programming are: $f(t+1, X_{t+1}) = 0$ and $f(T, 0) = 0$. We set the initial value function of stage t as $f(t, X_t)$, then, during the scheduled period, a request $\mathrm{n}_\mathrm{k}^{(i,j)}$ arrives at stage t, and the optimal decision needs to consider the type of shipping slot required by the request:

$$u^t\left(\mathrm{n}_\mathrm{k}^{(i,j)}\right) = \begin{cases} 1,\ \text{if } \left(x_{R,\mathrm{t}}^l\right) \geq s\left(\mathrm{n}_R^{(i,j)}\right) \text{ and } R\left(\mathrm{n}_R^{(i,j)}\right) \geq f(t{+}1, X_t) - f(t{+}1, X_{t+1}) \\ 1,\ \text{if } X_t \geq s\left(\mathrm{n}_D^{(i,j)}\right) \text{ and } R\left(\mathrm{n}_D^{(i,j)}\right) \geq f(t{+}1, X_t) - f(t{+}1, X_{t+1}) \\ \qquad 0,\ \ \textit{otherwise} \end{cases} \tag{3}$$

where $R\left(\mathrm{n}_\mathrm{k}^{(i,j)}\right) \geq f(t{+}1, X_t) - f(t{+}1, X_{t+1})$ indicates that the benefit of accepting the request meets or exceeds the opportunity cost of the slot to be consumed; otherwise, the request should be denied.

Finally, the Bellman equation of the DP model is as follows:

$$f(t, X_t) = \max_{u^t \in U(X^t)} \left\{ \begin{array}{c} \sum\limits_{k\in K} \sum\limits_{(i,j)\in P} \sum\limits_{n\in N} \lambda^\mathrm{t}\left(\mathrm{n}_\mathrm{k}^{(i,j)}\right)\left[u^t\left(\mathrm{n}_k^{(i,j)}\right)R\left(\mathrm{n}_\mathrm{k}^{(i,j)}\right) + f(t+1, X_{t+1})\right] \\ + \left[1 - \sum\limits_{k\in K} \sum\limits_{(i,j)\in P} \sum\limits_{n\in N} \lambda^\mathrm{t}\left(\mathrm{n}_\mathrm{k}^{(i,j)}\right)\right] f(t+1, X_t) \end{array} \right\},\ \forall t \in \mathbf{T},\ X_t \in X_t \tag{4}$$

4. Solution Method

Due to the dimension disaster of the DP model, if it is directly solved, it takes a long time to solve and it is difficult to obtain a feasible solution, which cannot meet the needs of shipping enterprises for dynamic order processing. Reinforcement learning algorithms have a good performance in dealing with these problems. Therefore, in our study, we adopt the Q-learning reinforcement learning algorithm to solve this problem.

This algorithm is based on value iteration, which is often used to solve Markov problems (MDP). The Q-learning algorithm is based on the idea of dynamic programming. By learning a value function (called a Q-value function), it guides the agent to make decisions in the environment to obtain the maximum cumulative reward, that is, the goal of the maximum return in the whole shipping cycle that this study hopes to obtain. The specific steps are as follows (Algorithm 1):

Algorithm 1. Q-learning Algorithm

Input: Entire booking horizon set, port set, OD port pair set, freight, quantity of dry and reefer slot
Output: Accept predetermined requests and total revenue value
Step 1. Given parameters γ and reward matrix r
Step 2. Initialize
Step 3. For each episode:
Choose an initial state s at random.
Using the selected behavior a, the next state 8 is obtained.
Calculate Q: $Q(s,a) = Q(s,a) + \alpha \times [r + \gamma \times \max(Q(s',a')) - Q(s,a)]$
s = s′
until s is terminal

Next, the Q-learning algorithm is used to solve examples of different types and scales. We then compare the Q-learning algorithm with other solving algorithms to verify its performance. The detailed results are in Section 5.

5. Computational Investigations

5.1. Experimental Study Description

This study refers to the liner transport route of a well-known shipping company in the Asia–Pacific region and sets the background parameters of the example in detail to evaluate the effectiveness of the model for solving the problem and the performance of the solving algorithm. In this example route, the docking sequence is: Ningbo → Shanghai → Sihanoukville → Bangkok → Laem Chabang → Qinzhou → Ningbo. The actual route is shown in Figure 3. Table 3 provides the arrival and departure times at the four ports in the route, based on actual sailing times.

Figure 3. Schematic diagram of liner routes. (From: https://www.maersk.com.cn/, accessed on 1 April 2024).

Table 3. Sailing Times and Departure Times between Ports of Call.

Port of Call	Transit Time	ETA	ETD
Ningbo	-	-	Day 1 (Tuesday)
Shanghai	2 DAY	Day 3 (Thursday)	Day 3 (Thursday)
Sihanoukville	8 DAY	Day 9 (Wednesday)	Day 9 (Thursday)
Bangkok	11 DAY	Day 12 (Saturday)	Day 12 (Sunday)
Laem Chabang	12 DAY	Day 13 (Sunday)	Day 13 (Monday)
Qinzhou	16 DAY	Day 17 (Thursday)	Day 17 (Friday)
Ningbo	21 DAY	Day 22 (Tuesday)	Day 22 (Tuesday)

The shipper can book voyages between any two ports, giving 20 possible voyages, as shown in Table 4.

Table 4. The 20 Possible Bookable Voyages.

Number	Port of Departure	Port of Destination	Transit Time (Days)
1	Ningbo (CNNBG)	Shanghai	2
2		Sihanoukville	8
3		Bangkok	11
4		Laem Chabang	12
5		Qinzhou	16
Number	Port of departure	Port of destination	Transit time (Days)
6	Shanghai (CNSHG)	Sihanoukville	6
7		Bangkok	9
8		Laem Chabang	10
9		Qinzhou	14
10		Ningbo	19
Number	Port of departure	Port of destination	Transit time (Days)
11	Sihanoukville (KHKOS)	Bangkok	3
12		Laem Chabang	4
13		Qinzhou	8
14		Ningbo	13
Number	Port of departure	Port of destination	Transit time (Days)
15	Bangkok (THBKK)	Laem Chabang	1
16		Qinzhou	5
17		Ningbo	10
Number	Port of departure	Port of destination	Transit time (Days)
18	Laem Chabang (THLCB)	Qinzhou	4
19		Ningbo	9
Number	Port of departure	Port of destination	Transit time (Days)
20	Qinzhou (CNQZH)	Ningbo	5

Liner transport is rich in ship types, and the ships responsible for transport on the Asia–Pacific route are generally medium-sized or large container ships. Because the focus of this study is on the effectiveness of the model and the quality of the algorithm, the selection of ship size has relatively little influence on the calculation results and the verification of the model algorithm. In addition, in the liner transport of the Asia–Pacific route, the 2000 TEU liner is the most commonly chosen ship type by shipping enterprises, so the result of choosing this ship type has strong universality. For this example, a medium-sized container ship capable of carrying 2000 20-foot standard containers (TEUs) and 21,700 deadweight tons is selected. This consists of 1500 dry slots and 500 reefer slots.

5.2. Solution Quality

5.2.1. Model Linearization

To verify the quality of the solution obtained by the Q-learning algorithm for solving the dynamic programming model, we used the CPLEX commercial solver to ensure the model's feasibility and effectiveness. Since CPLEX and other commercial solvers can only handle linear models, the dynamic programming model was first linearized.

$$\max \sum_{i=1}^{n} \sum_{j=1}^{j} \left(a_j x_{ij} + b_j y_{ij}\right) u_i \tag{5}$$

$s.t.$

$$\sum_{i=1}^{i}\sum_{j=1}^{j} x_{ij} \times u_i \leq 1500 \tag{6}$$

$$\sum_{i=1}^{i}\sum_{j=1}^{j} y_{ij} \times u_i \leq 500 \tag{7}$$

$$u_i \in \{0,1\},\ i = 1,2,\cdots,n \tag{8}$$

Firstly, x_{ij} and y_{ij} in (x_{ij}, y_{ij}), respectively, represent the quantities of dry and reefer slots required by the i request for the j segment. An order contains the information on slot demand for six segments: (x_{i1}, y_{i1}), (x_{i2}, y_{i2}), (x_{i3}, y_{i3}), (x_{i4}, y_{i4}), (x_{i5}, y_{i5}), (x_{i6}, y_{i6}). The decision variable u_i in the objective function is 0 or 1, indicating whether to accept the i reservation. And the goal is to maximize the total income. Constraint (6) means that the total number of dry slots occupied by all accepted orders cannot exceed the number of dry slots available on each segment. Constraint (7) means that the total number of reefer slots occupied by the orders accepted cannot exceed the number of reefer slots available on each segment. Constraint (8) indicates that the decision variable is 0 or 1.

5.2.2. Algorithm Quality Verification

The linearized model is solved by the CPLEX solver, and the profit value obtained is compared with the result obtained with the Q-learning algorithm. The results show that the solution obtained by CPLEX is close to that of the Q-learning algorithm, which proves the effectiveness of the algorithm. For details, see Table 5.

Table 5. Computational Results for Route Case. (Unit: CNY).

Request	100	300	500	700	900	1100	1300	1500	1700	1900
Q-learning (flexible)	344,050	979,600	1,655,350	2,397,200	2,984,450	3,570,050	4,087,850	4,912,550	5,705,950	6,161,950
CPLEX	329,500	860,100	1,476,600	2,204,850	2,685,900	3,251,400	3,747,450	4,489,250	5,067,150	5,535,500

Through comparative analysis of Table 5, it can be found that there is little difference between the results obtained by the Q-learning algorithm and the CPLEX commercial solver when solving dynamic programming models. Since the CPLEX solver has high accuracy in solving dynamic programming models, this proves that the Q-learning algorithm provides a high-quality and reliable solution.

5.3. Model Analysis

5.3.1. Inflexible Allocation Model

In order to verify the advantages of the model with flexible slot allocation over a model without flexible slot allocation, a comparison was made between the two models.

The inflexible dynamic programming model is as follows:

When an order arrives at stage t and the demand is for dry slots, under the condition of $X_t \geq s\left(\mathrm{n}_D^{(i,j)}\right)$, only dry slots will be used, and reefer slots will not be considered.

When the remaining capacity of dry slots is greater than the number of slots required to accept the order, that is, when $x_{D,\mathrm{t}}^{l} \geq u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right)$, the dry slot status variable in the next stage is $x_{D,\mathrm{t}+1}^{l} = x_{D,\mathrm{t}}^{l} - u^t\left(\mathrm{n}_D^{(i,j)}\right)s\left(\mathrm{n}_D^{(i,j)}\right)$, and the reefer slot status variable in the next stage remains unchanged, i.e., $x_{R,\mathrm{t}+1}^{l} = x_{R,\mathrm{t}}^{l}$.

The termination condition of dynamic programming remains unchanged, and an order $\mathrm{n}_{\mathrm{k}}^{(i,j)}$ arrives at stage t during the scheduled period. The optimal decision is made without considering the type of slot required by the order:

$$u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right) = \begin{cases} 1,\ \text{if } X_t \geq s\left(\mathrm{n}_k^{(i,j)}\right) \text{ and } R\left(\mathrm{n}_k^{(i,j)}\right) \geq f(t+1, X_t) - f(t+1, X_{t+1}) \\ \qquad 0, \qquad \textit{otherwise} \end{cases} \tag{9}$$

where $R\left(n_k^{(i,j)}\right) \geq f(t+1, X_t) - f(t+1, X_{t+1})$ indicates that the benefit of accepting the request meets or exceeds the opportunity cost of the slot(s) to be consumed; otherwise, the request should be denied.

The Bellman equation of the inflexible slot allocation dynamic programming model is as follows:

$$f(t, X_t) = \max_{u^t \in U(X^t)} \left\{ \begin{array}{c} \sum\limits_{k \in K} \sum\limits_{(i,j) \in P} \sum\limits_{n \in N} \lambda^t\left(n_k^{(i,j)}\right)\left[u^t\left(n_k^{(i,j)}\right) R\left(n_k^{(i,j)}\right) + f(t+1, X_{t+1})\right] \\ + \left[1 - \sum\limits_{k \in K} \sum\limits_{(i,j) \in P} \sum\limits_{n \in N} \lambda^t\left(n_k^{(i,j)}\right)\right] f(t+1, X_t) \end{array} \right\}, \forall t \in \mathbf{T}, X_t \in X_t \quad (10)$$

5.3.2. Comparison of Flexible and Inflexible Allocation

Employing identical sample order data, the optimal objective function values for both the flexible and inflexible slot allocation models were compared and the results are presented in Figure 4. The comparison revealed an increase in total revenue value with an increasing number of orders received, confirming the feasibility of both models. In addition, the results also prove that the profit of the liner enterprises considering flexible shipping slot allocation is higher than that of the liner enterprises without flexible shipping slot allocation, and the difference is large. Therefore, for enterprises, using the flexible model to allocate shipping slots in actual operations can improve earnings and bring higher profits.

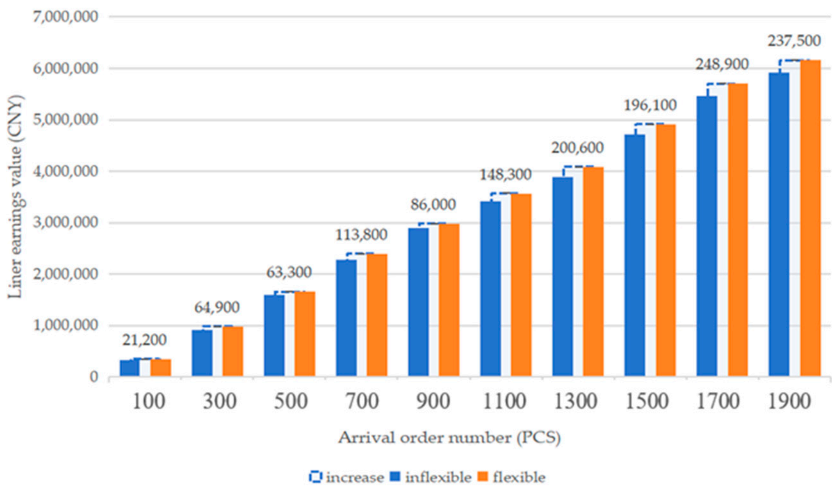

Figure 4. Revenue of different data sizes under different allocation methods.

Note: (1) For liner container transport companies, freight income can be obtained without considering flexible allocation, but it is not the best slot allocation strategy; (2) The difference in revenue is precisely because in the inflexible allocation model, the method of putting dry containers into reefer slots is not considered, and some orders that will bring revenue are rejected. After considering the flexible allocation, when a dry container consignment is scheduled to arrive but the actual number of dry slots is insufficient, it will weigh the future income and consider whether to put the dry containers in reefer slots, thus increasing the overall income, and thus, verifying the validity of our model.

5.4. Algorithm Performance

In order to compare the solving performance of the Q-learning algorithm, a dynamic programming algorithm and the Q-learning reinforcement learning algorithm are used in this study.

5.4.1. Dynamic Programming Algorithm

The dynamic programming algorithm decomposes the original problem into several overlapping sub-problems, and the solving process of each sub-problem constitutes a stage. After the calculation of one stage is completed, the dynamic programming algorithm performs the calculation of the next stage.

The solution steps of the dynamic programming algorithm are as follows (Algorithm 2):

Algorithm 2. Dynamic Programming Algorithm

Input: Entire booking horizon set, Port set, OD port pair set, Freight, quantity of dry and reefer slot
  Output: Accept predetermined requests and total revenue value
  for $t = 0$ to $T - 1$ do
    The optimal policy is calculated based on state $u^t\left(\mathrm{n}_{\mathrm{k}}^{(i,j)}\right)$
    State transition equation $f(t, X_t)$
  return optimal value

5.4.2. Performance Comparison of Different Algorithms

(1) Examples at different scales

In Figures 5 and 6, blue symbol represents the overall operation time, and orange symbol represents the solution time. The difference represents the time it takes the algorithm to generate the examples. In Figure 5, due to the long calculation time, the time to generate the example is covered in the figure. As can be seen in Figures 5 and 6, the solving time of the two algorithms will increase significantly with increasing numbers of customers. However, under the same example, the Q-learning algorithm has a faster solving speed and a greater advantage in solving efficiency.

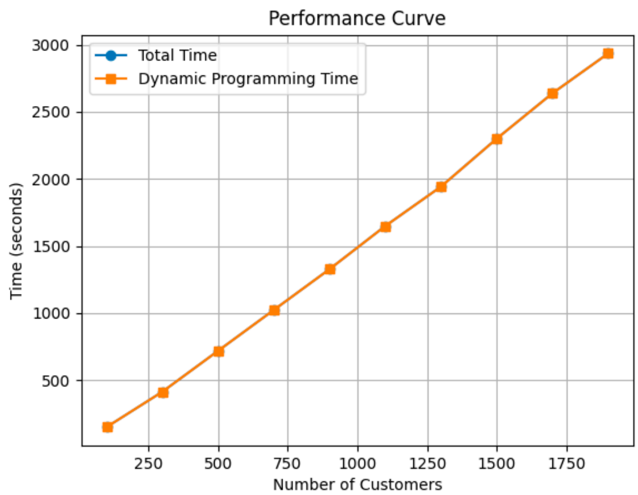

Figure 5. Operation time of dynamic programming algorithm.

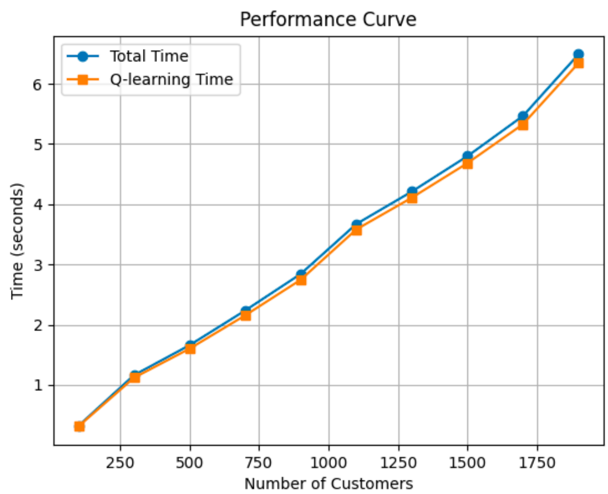

Figure 6. Operation time of Q-learning algorithm.

When comparing the performance of the algorithm, this study uses data of 10 different sizes, such as 100 orders and 300 orders. As can be seen from Figure 7, the operational efficiencies of the dynamic programming algorithm and Q-learning algorithm are quite different under the same scale of data. When utilizing dynamic programming, the original problem is divided into multiple sub-problems, with the solution being stored and reused, thus leading to a significant increase in the state space. By interacting with the environment, the Q-learning algorithm updates the Q-value function according to the immediate reward and state transition, to obtain the final optimal strategy. Rather than storing all potential states and actions, the optimal solution is achieved through the exploration and learning of the state and action space, resulting in higher operational efficiency and improved solution effectiveness. This affirms the efficiency of the Q-learning algorithm.

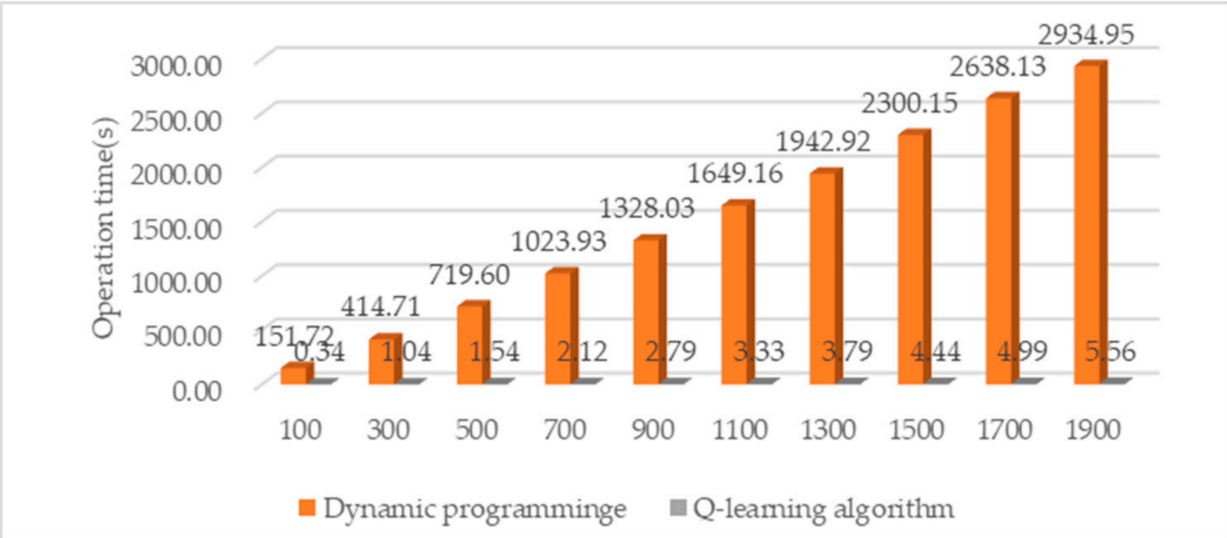

Figure 7. Comparison of the running time of the two algorithms under flexible allocation.

(2) Temporary response speed

At the actual operational level of enterprises, more cases will be encountered, i.e., during the booking period, a certain number of orders have already arrived and been accepted, and a batch of goods arriving when part of the shipping space has already been occupied. Business operators need to make quick decisions and arrange the right accommodation. Therefore, an example is set to calculate the solving time of different algorithms when a temporary order arrives, representing the performance of this method when applied to the actual operation layer. The results are shown in Table 6.

It can be seen from the results that the solving speed of the dynamic programming algorithm is closely related to the number of remaining slots. The larger the number of remaining slots, the longer the solving speed. When the number of remaining slots is smaller, the solving speed of the dynamic programming also becomes faster. In obvious contrast, the solving speed of the Q-learning algorithm has little correlation with the number of remaining shipping slots. The solving speed for different shipping slots is still relatively similar, and the solving efficiency is stable, which can ensure stable solving speed and quality under different data scales, and is more suitable for enterprise operation and decision-making.

Table 6. Different Order Arrival Processing Times.

Example 1		(Dry, Reefer)	Request 1		Solution Time (s)		Request 2		Solution Time (s)		Request 3		Solution Time (s)	
			Dry	Reefer	DP	QL	Dry	Reefer	DP	QL	Dry	Reefer	DP	QL
Leg	1	(950, 357)	8	-	0.207478	0.003000	-	-	0.207445	0.002994	-	-	0.271291	0.002984
	2	(1350, 426)	8	-			10	-			-	-		
	3	(1270, 253)	8	-			10	-			-	-		
	4	(857, 321)	-	-			-	-			-	-		
	5	(902, 334)	-	-			-	-			-	-		
	6	(857, 132)	-	-			-	-			-	18		
Example 1		**(Dry, Reefer)**	**Request 1**		**Solution time (s)**		**Request 2**		**Solution time (s)**		**Request 3**		**Solution time (s)**	
			Dry	**Reefer**	**DP**	**QL**	**Dry**	**Reefer**	**DP**	**QL**	**Dry**	**Reefer**	**DP**	**QL**
Leg	1	(153, 56)	-	-	0.002992	0.002989	-	-	0.003989	0.002991	-	-	0.002990	0.002992
	2	(452, 198)	-	-			-	-			-	11		
	3	(345, 43)	-	-			-	-			-	11		
	4	(287, 21)	-	-			-	10			-	11		
	5	(109, 34)	-	-			-	10			-	-		
	6	(57, 32)	14	-			-	-			-	-		
Example 1		**(Dry, Reefer)**	**Request 1**		**Solution time (s)**		**Request 2**		**Solution time (s)**		**Request 3**		**Solution time (s)**	
			Dry	**Reefer**	**DP**	**QL**	**Dry**	**Reefer**	**DP**	**QL**	**Dry**	**Reefer**	**DP**	**QL**
Leg	1	(5, 3)	-	-	0.000997	0.002984	-	-	0.000997	Reject	-	-	0.001996	0.002991
	2	(10, 11)	-	-			-	3			-	-		
	3	(5, 2)	-	-			-	3			-	-		
	4	(57, 21)	-	-			-	3			-	-		
	5	(90, 35)	2	-			-	-			-	6		
	6	(87, 13)	2	-			-	-			-	6		

5.5. Optimization of Slot Allocation Operation Process

Based on the advantages of the flexible allocation model and reinforcement learning algorithms, we propose the following optimization suggestions for slot allocation operations in liner shipping enterprises:

In the shipping management system, key data such as route information, port of call, arrival times, remaining slot capacity, and slot types should be set in advance. During the order-taking process, priority should be given to the type and characteristics of the containers to ensure that the slots allocated to reefer containers meet the temperature control requirements of the cargo. At the same time, while adhering to traditional allocation methods, allowing the storage of dry goods in reefer slots under specific circumstances will enhance the utilization rate of slots.

The order processing workflow consists of the following steps:

(1) Order Reception: The booking system receives order requests in real time and analyzes historical data using the reinforcement learning model to score each order, recommending whether to accept the order and allocate appropriate slots. The system will first calculate the number of standard container slots required based on the cargo dimensions. If the cargo is refrigerated, and once it is confirmed that there are sufficient reefer slots available and the calculation supports accepting the order, the system will accept the order and arrange for loading. For dry containers, the system will determine whether to accept the order and allocate slots based on the current slot usage status and future revenue forecasts. If there is available reefer slot capacity, and accepting the order can bring higher future returns for the enterprise, dry containers may be stored in reefer slots.

(2) Storage Decision: based on the calculation results and real-time slot usage status, the algorithm will assign specific storage locations for slot allocation to optimize utilization rates.

(3) Order Confirmation: the system will automatically generate a detailed slot allocation plan to ensure transparency in the decision-making process, provide order confirmation to the shipper, and ensure completion of subsequent documentation.

Finally, after each route no longer accepts orders and sails smoothly according to the slot allocation method, the slot usage data and the order fulfillment status are collected to provide feedback for the reinforcement learning model and continue to optimize the flexible slot allocation system. The slot allocation model and parameters are thus regularly adjusted and updated according to the route, and the efficiency of slot allocation is improved with the help of the reinforcement learning algorithm.

By optimizing flexible slot allocation and applying a reinforcement learning algorithm, liner companies can achieve higher profits, efficient resource utilization, and satisfactory customer service in container shipping. We hope this research can provide substantial assistance to the operations of shipping companies.

6. Conclusions

6.1. Discussion and Implications

This study aims to enhance the consideration of flexible allocation of shipping slots in liner transportation. In order to more accurately replicate the dynamic arrival of shipping orders in actual enterprise operations, we establish a dynamic programming model of flexible allocation. According to the loading characteristics of different types of slots, we adjust the way of loading containers flexibly: reefer slots can accommodate refrigerated containers or regular dry containers can be transported in an unplugged and cost-effective manner, and two 20-foot slots can be used to transport a single 40-foot container. Then, we establish a mathematical model to realize flexible shipping slot allocation. In terms of the computational solution, the reinforcement learning method is an effective tool to deal with dynamic decision problems. In this study, we use the Q-learning reinforcement learning algorithm to improve the dynamic programming model.

In the example experiment, firstly, the routes and orders are designed based on the actual liner routes of a shipping enterprise to verify the applicability of the model proposed in this study and the solving efficiency of the Q-learning algorithm is compared with that of the dynamic programming algorithm without Q-learning. Furthermore, we construct a wide range of numerical examples to further investigate the performance of our proposed solution. The findings of this study are as follows: (1) The results of the CPLEX commercial solver and Q-learning algorithm are compared to verify the quality of the solution of the algorithm; (2) The income generated in the same example can be effectively improved by the use of flexible allocation compared with inflexible allocation; (3) Compared with the dynamic programming algorithm, the solution based on the Q-learning algorithm significantly improves the solution efficiency and is more suitable for enterprise operation decision-making.

The main contributions of this paper can be summarized as follows:

(1) The incorporation of flexible slot allocation into liner voyages improves the shipping revenue of liner enterprises.

(2) We use the Q-learning reinforcement learning algorithm to improve the solving efficiency of the dynamic programming model. Due to its rapid solving speed, reinforcement learning can be applied in the actual operations of liner transportation enterprises, providing intelligent decision-making and a swift response to orders.

Finally, based on the results obtained from the optimized model and algorithm, we put forward suggestions on the operating process of shipping companies. Shipping companies can use the results of the model as a guide for slot allocation operations.

6.2. Limitations and Future Research

In the modern transport industry, the promotion of sustainable development and green shipping has become an important topic. The flexible slot allocation in this study not only improves transportation efficiency but may also play a positive role in environmental protection, for example, the use of two 20-foot slots to flexibly transport a 40-foot container and the rational use of reefer slots to improve the utilization rate of shipping space and avoid waste of resources. Future studies can further explore the balance between optimizing shipping resource allocation and environmental impact to support the promotion of green shipping concepts.

In addition, the flexible slot allocation model proposed in this study is closely related to revenue management. When there is a difference between the market order and the forecast, the shipping company can dynamically and flexibly adjust the allocation of shipping slots and timely adjust the allocation of shipping slots and cargo according to the market demand and order situation, effectively improving the resource utilization rate to improve the overall income and achieve the purpose of revenue management.

However, there are some limitations to this study. In actual liner transportation operations, there will be other needs and situations such as customer requirements for transshipment and liner enterprises' empty container transportation. So the applicability of the current model may be affected. Secondly, our research mainly focuses on the shipping slot allocation model and algorithm, and other factors such as service quality and port weather conditions are not considered in the model, which may affect transportation quality and final customer satisfaction. Future research may consider incorporating these factors into the model to enhance its comprehensiveness and practicability and improve the practical significance of the research, which will be our future work.

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