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A probabilistic model for real-time quantification of building energy flexibility

Binglong Han [a], Hangxin Li [a,b,*], Shengwei Wang [a,b,*]

[a] Department of Building Environment and Energy Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong
[b] Research Institute for Smart Energy, The Hong Kong Polytechnic University, Kowloon, Hong Kong

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ABSTRACT

Buildings have great energy flexibility potential to manage supply-demand imbalance in power grids with high renewable penetration. Accurate and real-time quantification of building energy flexibility is essential not only for engaging buildings in electricity and grid service markets, but also for ensuring the reliable and optimal operation of power grids. This paper proposes a probabilistic model for rapidly quantifying the aggregated flexibility of buildings under uncertainties. An explicit equation is derived as the analytical solution of a commonly used second-order building thermodynamic model to quantify the flexibility of individual buildings, eliminating the need of time-consuming iterative and finite difference computations. A sampling-based uncertainty analysis is performed to obtain the distribution of aggregated building flexibility, considering major uncertainties comprehensively. Validation tests are conducted using 150 commercial buildings in Hong Kong. The results show that the proposed model not only quantifies the aggregated flexibility with high accuracy, but also dramatically reduces the computation time from 3605 s to 6.7 s, about 537 times faster than the existing probabilistic model solved numerically. Moreover, the proposed model is 8 times faster than the archetype-based model and achieves significantly higher accuracy.

1. Introduction

The adoption of renewable generation contributes to the decarbonization of electrical power systems but also poses great challenges to maintaining the supply-demand power balance [1]. Furthermore, more frequent extreme weather events, such as heat waves, increase the power system peak demand [2]. To address these challenges, power systems require more energy flexibility for maintaining system reliability. Flexibility has been traditionally provided by conventional generators [3]. However, as these generators are gradually phased out, procuring flexibility solely from the supply side becomes unaffordable [4]. With advancements in grid-interactive control, buildings are emerging as a feasible resource for demand flexibility due to their large power consumption and passive thermal mass storage [5,6]. Considerable field tests and studies have demonstrated that the heating, ventilation, and air conditioning (HVAC) systems in buildings can quickly and consistently reduce their operating power in response to power grid requests [7,8]. The demand response capability enables buildings a promising and cost-effective provider of grid services compared to other resources, e.g., battery storage [9] (Fig. 1).

Accurate and real-time quantification of building energy flexibility is crucial for the reliable and optimal operation of both buildings and power grids [10]. Currently, building flexibility must have a high level of performance predictability to provide grid services that are essential for maintaining grid reliability [4]. For example, buildings must guarantee a minimum success probability of 95 % when providing operating reserve services. Failing to meet this mandatory requirement results in disqualification from providing such services [11]. Grid operators also rely on accurate quantification of building flexibility to design demand response programs, such as peak load management [12]. Furthermore, the flexibility quantification needs to be rapid enough for buildings to actively participate in real-time electricity and grid service markets. For instance, building load aggregators should quantify building flexibility and submit bids in the real-time electricity market every hour, or even every 15 min, based on the updated participation information of buildings.

In practice, the flexibility of a single building generally has minimal impact on the power grid [13]. Therefore, it is important to consider a cluster of buildings and quantify their aggregated flexibility to provide grid services [14]. The key step in aggregated flexibility quantification is

* Corresponding authors.
E-mail addresses: hangxin.li@polyu.edu.hk (H. Li), beswwang@polyu.edu.hk (S. Wang).

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developing effective models that accurately describe the dynamics of individual buildings [15]. These models can be physics-based (white box), data-driven (black box), or a combination of both (grey-box) [16]. Physics-based models consider detailed building physics but are time-consuming to develop and solve [14]. Data-driven models utilize statistical or machine learning methods, offering faster computational speed [17]. Li et al. [16] propose a semantic ontology to rapidly quantify building flexibility based on measured data. In [3], a data-driven model is proposed to control a cluster of buildings for reserve provision. However, data-driven models require extensive training data and may lack generalization outside the training dataset [18].

Grey-box models incorporate the basic principles of building physics while requiring less data for calibration, facilitating the real-world implementation. Among various grey-box models, resistance and capacitance (RC) models are widely used [19]. The first-order RC model assumes the entire building thermal mass as a single thermal capacitance, ignoring the difference between the fast dynamics of indoor air and the slow dynamics of building structural mass [20]. Despite their simple structure, these models have been proven inaccurate for flexibility quantification [21,22]. In contrast, second-order RC models characterize the dynamics of indoor air and structure mass separately, offering adequate accuracy [23-25]. In recent years, second-order RC models have been increasingly used for predicting the thermal dynamics and flexibility of buildings [26,27]. For example, in [13,22,28,29], the aggregated flexibility of buildings is quantified by modeling each building with a second-order RC model. However, these studies solve the RC model numerically based on the discrete-time state space formulation and iterative computations, which is time-consuming.

A well-established building model does not guarantee accurate prediction of actual building flexibility due to various uncertainties such as uncertainties in model inputs, inherent model bias, and potential building response failures [30-32]. These uncertainties can cause buildings to fail to achieve the expected flexibility in actual operation [33,34]. Quantifying the aggregated flexibility of a cluster of buildings under uncertainties is quite challenging due to the high computational burden. Archetype-based models have been adopted to tackle this issue [35]. In [14], buildings within a cluster are classified into different groups based on their characteristics, and the archetypes representing these groups are used to estimate the aggregated flexibility. In [36], an archetype-based probabilistic model is developed to estimate the aggregated building flexibility under uncertainties, using the first-order RC model to describe the archetypes. In [37-39], the aggregated building flexibility is estimated using higher-order RC models. However, these studies solely focus on uncertainties in building thermal properties, neglecting the significant uncertainties in building response failures due to communication and control issues.

Although archetype-based models can serve as tools for preliminary planning purposes, they may lack adequacy and accuracy for control applications in highly diverse building clusters where buildings have distinct characteristics [13]. To address this issue, the first-order RC model is transformed into an equivalent virtual battery model, so that the flexibility of individual buildings can be quantified efficiently [19]. In [40,41], the aggregated flexibility of buildings is quantified by modelling each building as a virtual battery, considering uncertainties in model inputs and response failures. But these models are reformulated as complex optimization problems, which are difficult to solve. In [42], a probabilistic model is developed to directly quantify the aggregated flexibility of demand-side resources, but this model ignores the uncertainties in model inputs. More importantly, these studies rely on the first-order RC model that ignores the dynamic interaction between the indoor air and building mass, lacking sufficient accuracy.

Based on the above literature, it can be observed that there is still a lack of an accurate and computationally efficient model for quantifying the aggregated flexibility of buildings under uncertainties. Existing probabilistic models are either archetype-based, which inadequately capture the distinct characteristics of diverse buildings, or developed based on the first-order RC model that has been proven inadequately accurate. Although several studies quantify the aggregated flexibility by characterizing each building using a second-order RC model, these models are solved numerically, which are computationally intensive when the uncertainties are considered. Furthermore, none of these studies consider the major uncertainties in flexibility quantification comprehensively.

This paper proposes a novel probabilistic model for real-time quantification of aggregated building flexibility under uncertainties. A commonly used second-order RC model is used to describe building dynamics, and the sampling-based uncertainty analysis is used to comprehensively capture the effect of major uncertainties, including model inputs, model bias and building response failures. Unlike existing models that rely on a few archetypes and are solved numerically, the proposed model applies an explicit equation to analytically quantify the flexibility of each building in the cluster while maintaining computational feasibility.

This paper has three major original contributions, including:

- To our best knowledge, this study represents the first attempt to achieve real-time flexibility quantification of building clusters in a bottom-up approach, while comprehensively considering major uncertainties in flexibility quantification and the second-order building thermal dynamics.
- An explicit equation is derived to analytically quantify the flexibility of individual buildings, based on a typical second-order building thermodynamic model. It eliminates the need of iterative and finite difference computations, greatly reducing the computational time.
- The superior accuracy and computational efficiency of the proposed model is validated. The model can quickly provide probability-capacity curves for aggregated flexibility in real-time applications. This not only facilitates the active participation of building energy flexibility in real-time electricity and grid service markets, but also

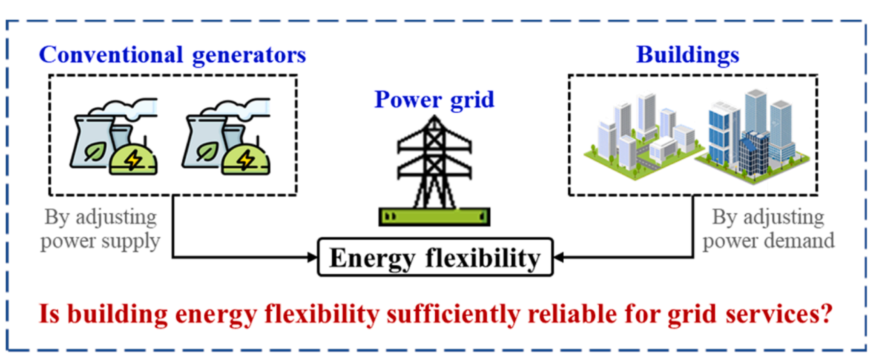

Fig. 1. Utilizing building energy flexibility to provide grid services.

facilitates the reliable and optimal scheduling and dispatch of smart grids.

The rest of this paper is organized as follows. Section 2 introduces the proposed probabilistic model. Section 3 presents the validation test arrangement. Section 4 presents the test results to validate the performance of the proposed model. Section 5 presents the discussion. Section 6 draws the conclusions.

2. Proposed probabilistic model

The proposed probabilistic model for quantifying the aggregated energy flexibility of a cluster of buildings under uncertainties is outlined in Fig. 2. This model combines a straightforward equation to directly quantify the flexibility of each individual building with a sampling-based uncertainty analysis to obtain the distribution of their aggregated flexibility, as detailed below.

2.1. Second-order building thermodynamic model

Modelling individual buildings is the basis for quantifying their aggregated flexibility. In this study, HVAC systems are considered to provide flexibility, since they are the major power consumers in buildings and can effectively utilize the passive thermal mass storage of buildings. Unlike the first-order RC models used in [36,40,41], the second-order RC model accounts for the dynamic interactions of indoor air and building internal mass, resulting in significantly improved accuracy [13,43]. This study leverages a commonly used second-order RC model to characterize the thermal dynamics of buildings and determine the operating power of HVAC systems, as illustrated in Fig. 3. The model is suitable for buildings in cooling-dominated regions, which usually have light outer walls but relatively heavy internal mass. If the building thermal mass is dominated by outer walls, other second-order RC models would be more appropriate, such as that adopted by [28]. Note that a deterministic RC model is used in this study, since we focus on flexibility quantification based on given building parameters. For real applications, a stochastic RC model that use stochastic differential equations can be applied for more robust parameter estimation based on measured data [44,45].

The governing equations of the second-order RC model are shown in Eqs. (1) and (2). Where, t is the time. R, C and T refer to thermal resistance, thermal capacitance, and temperature respectively. The superscripts, i.e., *in, m,* and *out*, denote the indoor air, building structure mass, and outdoor air respectively. Q^{in} and Q^{m} are the heat gains of indoor air and building structure mass respectively. P^{ac} and COP are the operating power and the overall coefficient of performance of the HVAC system

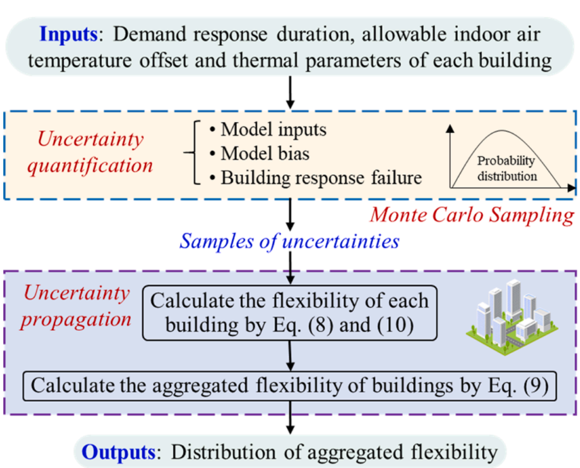

Fig. 2. Outline of the proposed probabilistic model.

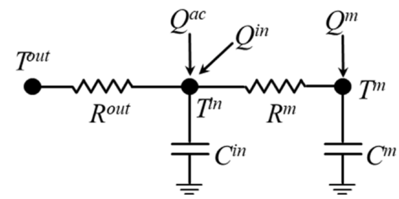

Fig. 3. Illustration of the second-order building RC model.

respectively.

$$C^{in}\frac{dT^{in}}{dt} = \frac{T^{out} - T^{in}}{R^{out}} + \frac{T^{m} - T^{in}}{R^{m}} + Q^{in} - COP \cdot P^{ac} \tag{1}$$

$$C^{m}\frac{dT^{m}}{dt} = \frac{T^{in} - T^{m}}{R^{m}} + Q^{m} \tag{2}$$

A single second-order RC model may not capture the thermal dynamics of individual thermal zones of a complex multi-zone building. However, a “single-zone equivalent” second-order model would sufficiently capture the volume-averaged zone temperature and the total cooling load of these zones, as demonstrated in [46–48]. For a building with highly diverse thermal zones, multiple second-order models can be applied to model the representative zones in the building. In this case, the last term of Eq. (1), i.e., $COP \cdot P^{ac}$, can be replaced by the cooling loads of individual zones. Also, the impact of varied COP during demand response can be considered by using Eq. (1) to compute cooling load. This study focuses on the aggregated flexibility of a large number of buildings, a constant COP is therefore assumed for each HVAC system during demand response.

2.2. Numerical method of quantifying flexibility of individual buildings

In this study, the energy flexibility of a building is characterized by its maximum load reduction capacity during a demand response event while considering the allowable maximum offset of indoor air temperature [4]. Several common assumptions are made, including: (i) the indoor air temperature remains at its baseline value in the normal operation scenario (i.e. without the demand response event); (ii) the HVAC operating power is consistently reduced by a certain amount during the demand response period to provide qualified grid services [49]; (iii) the building heat gains are not affected by the implementation of demand response.

The objective is to quantify the load reduction ΔP^{ac} of a building for a given demand response duration Δt and an indoor air temperature offset ΔT^{in}. For convenience, we denote the start time of the demand response period as t_0 and the end time as t_1. This problem is typically solved numerically by transforming the RC model into the discrete-time state space formulation, as shown in Eq. (3). The indoor air temperature at time t_1 can be determined using the forward difference method, as commonly used in the implementation of model predictive control [23]. Iterative computations (e.g., binary search) or complex optimization are then employed to find the appropriate ΔP^{ac} that corresponds to the desired ΔT^{in}. More details can be found in [6,50].

$$x(j+1) = Ax(j) + B \cdot P^{ac}(j) + C\omega(j) \tag{3}$$

where, $x=[T^{in}, T^{m}]^{T}$ is the state vector. j is the time step. $\omega=[T^{out}, Q^{in}, Q^{m}]^{T}$ is the disturbance vector. The disturbances are the outdoor aim temperature and the heat gains of the indoor air and building structure mass, which are independent. Matrices A, B, and C can be easily derived from Eqs. (1) and (2).

2.3. Analytical equation for directly quantifying flexibility of individual buildings

The numerical solution method discussed above is computationally intensive when conducting the sampling-based uncertainty analysis for a large number of buildings. To mitigate this issue, existing studies reduce the problem complexity by using a few building archetypes to represent the entire building cluster. However, this approach inadequately considers the distinct characteristics of diverse buildings, compromising the accuracy [35].

To address this limitation, we derive the analytical solution of the second-order RC model during a demand response event. For this purpose, Eqs. (1) and (2) are expressed by a second-order differential equation, as shown in Eq. (4), and solved as shown in Eq. (5) [13].

$$a\frac{d^2T^{in}}{dt^2}+b\frac{dT^{in}}{dt}+cT^{in}=d \tag{4}$$

$$T^{in}(t)=A_1e^{r_1t}+A_2e^{r_2t}+d/c \tag{5}$$

where, $a=C^{in}C^mR^m$, $b=\left(1+\frac{R^m}{R^{out}}\right)C^m+C^{in}$, $c=\frac{1}{R^{out}}$, $d=\frac{T^{out}}{R^{out}}+Q^m+Q^{in}-COP{\cdot}P^{ac}$, $r_1=\frac{-b+\sqrt{b^2-4ac}}{2a}$, $r_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$, $A_1=\frac{\frac{dT^{in}}{dt}(0)+\left(\frac{d}{c}-T^{in}(0)\right)r_2}{r_1-r_2}$, $A_2=\frac{\frac{dT^{in}}{dt}(0)+\left(\frac{d}{c}-T^{in}(0)\right)r_1}{r_2-r_1}$.

By considering time t_0 as the initial time, the indoor air temperature at time t_1 can be expressed by Eq. (6), which is a function of the HVAC operating power P^{ac} and the duration Δt from the initial time.

$$T^{in}(t_1)=A_1e^{r_1\Delta t}+A_2e^{r_2\Delta t}+d/c=\varphi(\Delta t,P^{ac}) \tag{6}$$

Due to the implementation of demand response, the indoor air temperature is increased by ΔT^{in}, while the HVAC operating power is reduced by ΔP^{ac} from its normal value $P^{ac,base}$ for a duration of Δt. In contrast, the indoor air temperature remains at the baseline value in the normal operation scenario. The indoor air temperature offset ΔT^{in} at time t_1 is determined by Eq. (7).

$$\Delta T^{in}=T^{in,flex}(t_1)-T^{in,nor}(t_1)=\varphi\left(\Delta t,P^{ac,\ base}-\Delta P^{ac}\right)-\varphi\left(\Delta t,P^{ac,\ base}\right) \tag{7}$$

The load reduction ΔP^{ac} is quantified using Eq. (8), by combining Eqs. (5)-(7). It is a function of the indoor air temperature offset ΔT^{in}, the demand response duration Δt, and the building parameters. This finding aligns with the numerical simulations results reported in [6,50]. Note that the analytical solutions of other second-order RC models can be derived in a similar way.

$$\Delta P^{ac}=\frac{(r_1-r_2)C^{in}\Delta T^{in}}{\left[e^{r_1\Delta t}-e^{r_2\Delta t}+C^{in}R^{out}\left(r_1-r_2+r_2e^{r_1\Delta t}-r_1e^{r_2\Delta t}\right)\right]\cdot COP} \tag{8}$$

Eq. (8) enables direct quantification of the flexibility of individual buildings, eliminating the need of time-consuming iterative and forward difference computations. This makes it feasible to perform sampling-based uncertainty analysis while considering the characteristics of each building in a building cluster. Note that constant operating conditions (e.g., T^{out}) of individual buildings are assumed when deriving Eq. (5). However, the analytical solution is still valid under time-varying boundary conditions, as demonstrated in Section 5.1.

2.4. Quantification of the aggregated flexibility of buildings

Eq. (8) characterizes the magnitude of building load reduction ΔP^{ac}. In practice, individual buildings may fail to respond to power grid requests in real-time operation due to various reasons, such as the random behavior of building occupants, grid-building communication failures due to malicious cyberattacks, and malfunction of building control systems [42]. Besides, the second-order RC model is only an approximation of the real system, due to missing physics, overlooked input variables, numerical approximations, and incorrect hypotheses [51]. These two factors should also be considered when quantifying building energy flexibility.

The aggregated energy flexibility of a cluster of buildings (EF^{agr}) is quantified using Eq. (9). It is determined by the sum of flexibilities of all individual buildings in the cluster [39]. This bottom-up approach takes into account the characteristics of diverse buildings more effectively than the archetype-based approach [13]. It can be seen that the building response failure and the model bias are considered in Eq. (10), enabling a more accurate and realistic flexibility quantification [33].

$$EF^{agr}=\sum_{k=1}^{N^{bui}}EF_k^{bui} \tag{9}$$

$$EF_k^{bui}=\omega_k\Delta P^{ac}(x_k)+e_k \tag{10}$$

$$f(\omega_k)=\begin{cases}p_k^{bui}, & \omega_k=0\\ 1-p_k^{bui}, & \omega_k=1\end{cases} \tag{11}$$

where, k refers to the index of individual buildings. N^{bui} is the number of buildings in the cluster. The binary variable, ω_k, represents the response activation state of the building, which is modeled as a Bernoulli distribution as shown in Eq. (11) [41]. A value of 0 indicates a failed response, and a value of 1 indicates a successful response. ${p^{bui}}_k$ is the response failure rate of the building, which can be derived from empirical or historical data [7,42]. x_k and e_k are the inputs and bias of the building RC model [21,33].

2.5. Sampling-based uncertainty analysis

The above deterministic model can provide a deterministic output for the aggregated building flexibility based on a given set of inputs. To account for the impact of uncertainties, a typical sampling-based uncertainty analysis method, i.e., Monte Carlo simulation, is adopted to obtain the distribution of the aggregated building flexibility [32].

The uncertainty analysis consists of four procedures, as illustrated in Fig. 2. *First*, the major uncertain parameters are quantified, including the inputs and bias of the building RC model, and the response failure rates of buildings. Each uncertain parameter is assigned an appropriate probability distribution based on the available data. *Second*, Latin Hypercube Sampling is used to generate a set of samples [32]. Each sample represents a possible realization of these uncertain parameters, which are sampled randomly from their respective probability distributions. *Third*, the uncertain parameters in each sample are imported into the deterministic quantification model to propagate the combined effect of uncertainties. *Finally*, the values of aggregated flexibility across all samples are collected. The distribution of aggregated building flexibility is obtained.

The probability that buildings can successfully achieve a specific committed flexibility capacity (EF^{com}) in actual operation can be determined using Eq. (12) [52]. Where, p^{agr} is the distribution of the aggregated building flexibility.

$$P(EF^{agr}\geq EF^{com})=\int_{EF^{com}}^{+\infty}p^{agr} \tag{12}$$

By varying the demand response duration, a set of probability-capacity curves can be obtained. These curves serve as a valuable tool for effectively designing demand response programs [42]. By setting the success probability to a desired confidence level (e.g., 99.9 %), building energy flexibility can be leveraged in a reliable manner, similar to conventional generators.

3. Validation test arrangement

3.1. Description of the tested building cluster

In this study, a building cluster consisting of 150 large-sized commercial buildings is chosen to provide demand-side flexibility. Commercial buildings are considered due to their feasibility of implementing advanced grid-interactive control using existing building automation systems. The building cluster is generated based on a prototype commercial building in Hong Kong, following the method adopted by [13, 28]. The design parameters and normal usage pattern (e.g., occupancy profile) of the prototype building can be found in [6,8]. The overall COP of HVAC system of prototype building is assumed to be constant and equal to 3. With simulation data from the software TRNSYS, the parameters of the prototype building are estimated for characterizing its second-order thermal dynamics. For the building cluster, the parameters (e.g., thermal resistance and COP) of each building are randomly sampled from uniform distributions, with $a \pm 20$ % variation range around the parameters of the prototype building. This large wide variation range is chosen to ensure the diversity among individual buildings. Note that the randomization process for generating the building cluster is conducted only once. After randomization, each building has unique thermal parameters, HVAC system, and normal usage pattern. The validation tests are conducted using the weather data on a typical summer day in Hong Kong, when the building HVAC systems are in operation and can provide energy flexibility [8]. The maximum allowable indoor air temperature offset for each building is assumed to be 2 °C during the demand response period. Individual buildings are assumed to have independent controls, enabling them to respond to grid requests independently.

3.2. Distributions of the major uncertainties

Building energy flexibility quantification involves various uncertainties. Based on the previous studies on sensitivity analysis of building parameters [37,39], the following uncertain inputs of the second-order building RC model are considered, including: the thermal resistance between the indoor air and building structure mass, and the actual indoor air temperature offset during the demand response. These uncertainties are assumed to follow normal distributions with mean values corresponding to their deterministic values. In practical operation, buildings may fail or reject to respond to grid requests due to communication and control issues. The uncertainty in response failures is quantified using the response failure rate, which is assumed to follow a binomial distribution based on data from a real-world demonstration project in Germany [7]. The bias of the second-order RC model is assumed to follow a normal distribution with a zero mean and a standard deviation that is 5 % of the deterministic predicted value [33]. Note that when quantifying the flexibility of a single building and a small-scale building cluster, the uncertainties in measurements of power and temperature should be considered. Table 1 provides detailed information on the distributions of major uncertain parameters.

Table 1
Major uncertain parameters and their distributions.

Group	Parameter	Probability distribution
Building design	Thermal resistance	Normal (R_{det},0.05 R_{det})
Building operation	Indoor air temperature offset	Normal ($\Delta T^{\mathrm{in}}_{\mathrm{det}}$,0.25)
Response state	Response failure rate	Binomial (N^{bui}, 0.1)
Model bias	RC model bias	Normal (0, 0.05 P_{det})

Note: Normal (μ, σ) refers to a normal distribution with mean μ and standard deviation σ.

3.3. Overview of the tested models

Four different models are tested and compared to verify the effectiveness and advantages of the proposed model in quantifying aggregated flexibility of buildings, as listed below.

- *Deterministic model*: This model does not consider uncertainties in flexibility quantification [13]. It quantifies the aggregated flexibility by modeling each individual building.
- *Proposed model*: This model considers the major uncertainties in flexibility quantification. It quantifies the aggregated flexibility by modeling each individual building, using the analytical equation introduced in Section 2.3.
- *Probabilistic numerical model*: This model is extended based on the deterministic model by considering the major uncertainties. It quantifies the aggregated flexibility by modeling each individual building, using the numerical solution method described in Section 2.2.
- *Archetype-based model*: This model considers the major uncertainties. It estimates the aggregated flexibility by classifying the building cluster into 5 groups (i.e., archetypes) [36,39]. The flexibility of each archetype is quantified using the numerical solution method.

4. Performance evaluation of proposed probabilistic model

This section presents the flexibility quantification results of the proposed probabilistic model. The accuracy and computational efficiency of the proposed model are demonstrated and compared with the deterministic model and the probabilistic models respectively. All flexibility quantification models are implemented in Matlab on a PC with an eight-core Intel Core i7 CPU. The number of samples for Monte Carlo simulation is chosen as 5000 to ensure convergence.

4.1. Flexibility quantification results of the proposed model

Distribution of the aggregated building flexibility

Fig. 4 shows the distribution and cumulative distribution function (CDF) of the aggregated building flexibility of the building cluster during a 1-hour demand response event, as quantified by the proposed probabilistic model. The aggregated flexibility capacity ranges from about 160 MW to 210 MW under uncertainties. In contrast, the deterministic model, which overlooks uncertainties, gives a single output of 183 MW, which corresponds to 47.5 % of the CDF. This indicates a high probability (i.e., 52.5 %) of overestimating the aggregated flexibility when using the deterministic model, posing a significant risk to power grid operation if such flexibility capacity is committed by buildings. Therefore, it is necessary to consider and quantify the impact of uncertainties on building energy flexibility.

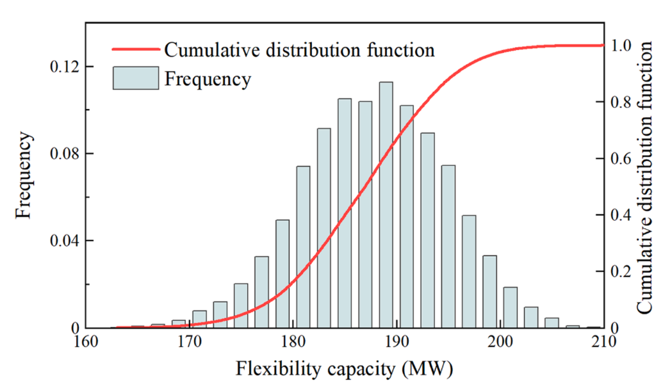

Fig. 4. Distribution of the aggregated flexibility of buildings.

Probability-capacity curves of the aggregated building flexibility

Based on the quantified distribution of aggregated building flexibility, the probability-capacity curves are generated for different demand response durations, as shown in Fig. 5. It can be observed that for a given demand response duration, a higher committed flexibility capacity generally corresponds to a lower probability of successfully achieving the committed flexibility in actual operation. On the other hand, for the same desired success probability, buildings can provide more energy flexibility with shorter demand response durations, which aligns with the findings in [37,42]. Therefore, the demand response duration should be carefully determined to optimize the coordination of building energy flexibility and power grids.

The flexibility provided by buildings achieves a very high probability of success (i.e., 99.99 %) when the committed capacity is below a certain threshold (e.g., 160 MW for a response duration of 1 hour). The success probability is even higher than that of most conventional generators (i.e., 99.9 %) [53]. This can be attributed to the aggregating effect of multiple buildings, which offers three significant benefits. *First*, it mitigates the impact of uncertainties from individual buildings on their aggregated flexibility [37]. *Second*, even if a subset of buildings cannot provide adequate flexibility, the other buildings can compensate for this temporal shortage [14]. *Third*, the flexibility committed by buildings can be flexibly adjusted as needed, unlike conventional generators with fixed and large-rated capacities. Therefore, buildings can serve as a reliable complement to conventional generators for providing grid services in smart grids.

4.2. Accuracy and scalability of the proposed model

The accuracy of the proposed probabilistic model is verified by comparing its output with that of the numerical model commonly used in the literature, as introduced in Section 2.2. For this purpose, the time step of the numerical model is set to 1 second to obtain an accurate solution. However, using such a small time step increases the computational intensity of the numerical model when applied to the 150 buildings. To simplify the comparison, a random sample of uncertainties is utilized. It is found that the difference between the aggregated flexibility obtained from the two models, using this sample, is less than 0.01 %. This negligible difference confirms the accuracy of the proposed model and the correctness of the analytical equation derived from the second-order RC model.

The scalability of the proposed model is demonstrated by applying it to quantify the aggregated flexibility of building clusters consisting of different number of buildings (i.e., 10, 100, 500, 1000, and 2000). Fig. 6 illustrates the computational time measured under different number of buildings. It can be seen that even when dealing with 2000 buildings, the proposed model only takes about 140 s to give a solution, confirming its scalability. This means that even if we use ten second-order RC models for modelling the 150 buildings, the computation time of the proposed model is still very fast (<2 min). This computational time can satisfy the application requirements of power grid real-time dispatch and engaging building flexibility into real-time electricity market, which usually have a time interval of 15 min. Furthermore, the measured computational time exhibits a linear growth trend, which aligns with the theoretical trend inferred from the bottom-up structure of the proposed model.

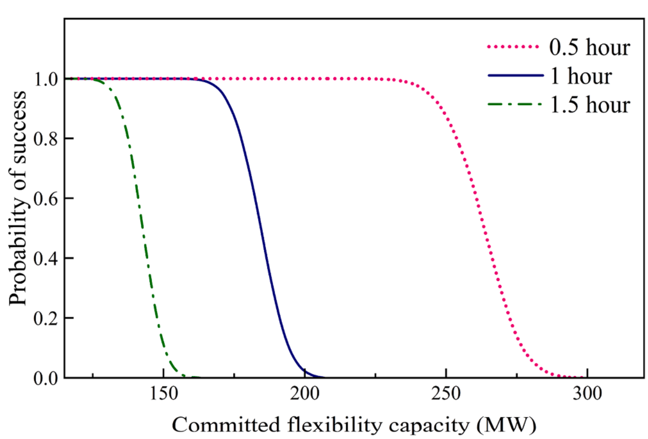

Fig. 5. Probability-capacity curves for different demand response durations.

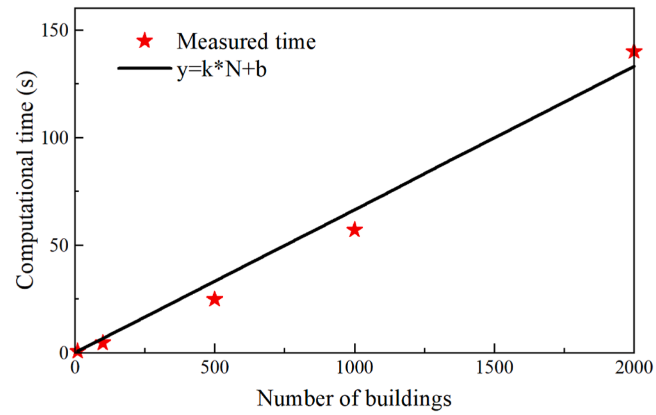

Fig. 6. Computational time of the proposed model under different number of buildings.

4.3. Comparison with existing probabilistic models

The superior computational accuracy and efficiency of the proposed probabilistic model is confirmed by comparing it with two existing probabilistic models, i.e., the probabilistic numerical model and the archetype-based model. The demand response duration is set to 1 hour for comparison.

The time step of the probabilistic numerical model is set to 1 and 5 min, respectively. The probabilistic numerical model with a 1-minute time step takes 3605.6 s to give a solution, as shown in Table 2. In contrast, the proposed model only takes 6.7 s, which is 537 times faster than the numerical model. The probability-capacity curves given by these two probabilistic models are very close, as shown in Fig. 7. Their differences in flexibility capacities at success probabilities of 0.999 and 0.95 are below 1.8 %. For the probabilistic numerical model with a 5-minute time step, the computational time is 1503.2 s, and the estimated flexibility capacity at a success probability of 0.999 is 4.4 % lower compared to that of the proposed model. Although this deviation may seem small on its own, it may significantly affect power grid scheduling. It could lead grid operators to schedule an additional large-scale conventional generator to meet the flexibility requirement, thereby increasing the operational cost of the power system. Note that these differences are attributed to the discrete-time state space formulation of the probabilistic numerical model [54], and the outputs of the proposed model have been demonstrated to be accurate in Section 4.2.

The performance of the archetype-based model, as adopted in [37,38,39], is also tested and compared. Note that the archetype-based

Table 2
Computational results of different probabilistic models.

Model	Capacity at Prob. 0.999 (MW)	Capacity at Prob. 0.95 (MW)	Calculation time (s)
Proposed	160	171	6.7
Prob. Numerical (1-min step)	162	174	3605.6
Prob. Numerical (5-min step)	153	165	1503.2
Archetype-based	141	147	54.5

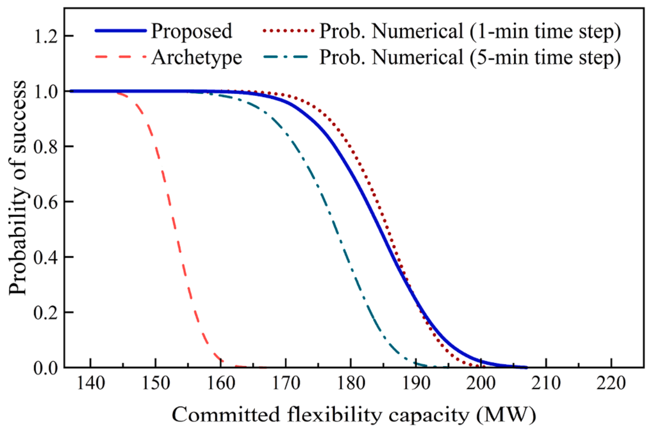

Fig. 7. Probability-capacity curves given by different probabilistic models.

model has been extended to incorporate the building response failures. In this model, the 150 buildings in the building cluster are classified into 5 groups based on their characteristics. The numerical time step is set to 5 min. As shown in Table 2, although the archetype-based model reduces the computational time by using a few building archetypes to represent the entire building cluster, it still requires 54.5 s to give a solution. This computational time is about 8 times that of the proposed model (6.7 s). More importantly, the archetype-based model generates a probability-capacity curve that significantly deviates from the correct curve, as shown in Fig. 7. It gives a flexibility capacity of 147 MW for a success probability of 0.95, which is 14 % lower than the correct value. The lower accuracy of the archetype-based model is due to its inadequate consideration of diverse characteristics of individual buildings. In summary, the proposed model outperforms existing models in terms of both accuracy and computational efficiency.

5. Discussion

5.1. Analytical solution of building thermodynamic model in demand response

The proposed model is based on the analytical solution of a building thermodynamic model in demand response condition. This solution, represented by Eq. (8), is valid even under time-varying boundary conditions (e.g., T^{out}), although constant boundary conditions are assumed when deriving Eq. (5). This is because, as observed from Eq. (5), the evolution of indoor air temperature of a building has two time constants, which only depend on building parameters, not boundary conditions. Besides, the indoor air temperature offset is determined using Eq. (7), where terms related to boundary conditions are offset in the normal operation and demand response scenarios. Therefore, the assumption of constant boundary conditions does not affect the validity of the solution.

To validate the solution, numerical simulation is conducted on a commercial building under time-varying boundary conditions. The time-varying outdoor temperature and building heat gains are generated using trigonometric functions. The HVAC power reduction is assumed to be constant during demand response, according to typical requirement of power grid operators. The demand response event is assumed to begin at 3600 s and ends at 7200 s. First, the HVAC power reduction of the building is computed using the analytical solution, considering an indoor air temperature offset of 2 °C. The HVAC operating power in the demand response scenario is obtained, as shown in Fig. 8(a). Using this operating power as input to Eqs. (1) and (2), the evolution of indoor air temperature during demand response is computed numerically. As shown in Fig. 8(b), the indoor air temperature increases from 24 °C to 26 °C at the end of demand response. This indicates the solution is valid under time-varying boundary conditions.

5.2. Real-world implementation of the proposed model

This study presents a novel probabilistic model for real-time quantification of the aggregated flexibility of buildings under uncertainties. The model can be used by load aggregators to quantify the flexibility of a building cluster and perform bidding in the real-time electricity and grid service markets, which usually requires flexibility providers to have a minimum reliability of 95 %. Also, the outputs of this model can be directly used by grid operators for optimized power grid scheduling and real-time dispatch, as widely studied in the smart grid fields.

To implement this model on a building cluster, the following steps are required. Firstly, smart devices should be installed in each building to enable effective communication and interaction between the power grid and buildings. Secondly, the parameters (e.g., thermal resistance and thermal capacitance) of each building are required. These parameters can be identified either locally by individual buildings or centrally by grid operators. Existing software tools (e.g., CTSM-R) can be employed for facilitating parameter estimation based on the measured data in each building [44]. Finally, the proposed model is applied to quantify the aggregated building flexibility based on the collected information. The probability-capacity curves of the aggregated building flexibility can then be utilized by load aggregators or grid operators.

There are several potential challenges related to the real-time application of the proposed model for building flexibility quantification. First, the model requires each building to communicate its operational parameters to grid operators or load aggregators. This may raise concerns regarding the reliability of communication protocols and the privacy of the data being transmitted. Second, the thermal zones within a building may be highly diverse, necessitating the use of multiple RC models to represent each building. In such case, the proposed model can still provide real-time flexibility quantification due to its verified

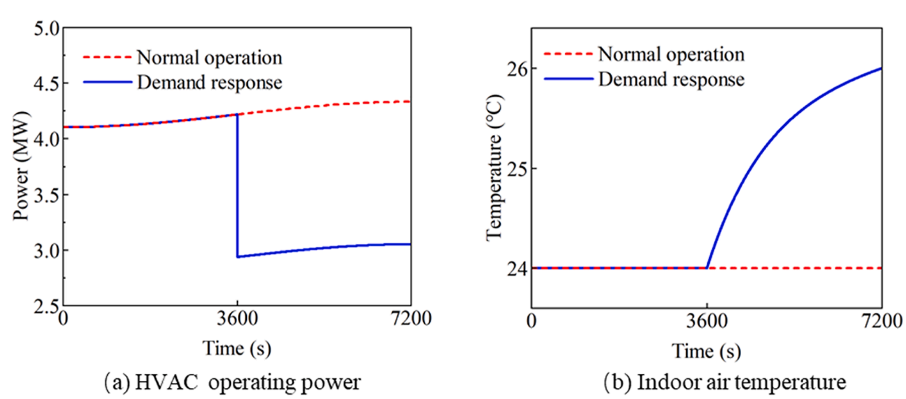

Fig. 8. HVAC operating power and indoor air temperature of a building in demand response.

scalability, but obtaining the cooling load of each thermal zone for training the RC model is challenging, because typically only the HVAC operating power is measured. Third the thermal parameters of a building may change over time. Ignoring this issue in model development can compromise the accuracy of energy flexibility quantification. To tackle this issue, the RC models should be trained adaptively in real-time application to continuously update its parameters and accommodate the time-varying conditions [55].

5.3. Limitations of this study and future research directions

This study addresses the significant computational challenges associated with real-time quantification of building energy flexibility under uncertainties. However, there are several limitations that need to be addressed in future research. First, this study aims to address the unsatisfactory computational performance of existing probabilistic models that rely on numerical solution method to characterize building thermodynamics during demand response. Therefore, it only quantifies the flexibility potential of passive thermal mass storage of buildings, without detailed modelling of HVAC systems. In practical application, the predicted load increase may not be achieved due to the rated cooling capacity of chillers, while the predicted load reduction may not be achieved due to the minimum operational level of air supply systems. Future work could consider extending the proposed model to incorporate these component-level details. Second, the proposed model relies on a second-order RC model to quantify the flexibility of individual buildings. While this model has proven effective for buildings in cooling-dominated regions, it is not suitable for buildings in cold regions with radiative heating systems. In such cases, it may be necessary to explore the analytical solutions of higher-order RC models. Third, this study focuses on the technical flexibility capacity of a building cluster. In reality, actual flexibility provision also depends on the incentives that building owners can obtain [12,56]. Characterizing the flexibility function of a building cluster under both uncertainties and incentives will be an interesting research direction.

6. Conclusion

This paper proposes a novel probabilistic model for real-time quantification of the aggregated flexibility of buildings under uncertainties. An explicit equation is derived as the analytical solution of a second-order building thermodynamic model to rapidly quantify the flexibility of individual buildings, eliminating the need of time-consuming iterative and finite difference computations. The uncertainty analysis accounts for the major uncertainties in flexibility quantification, including model inputs, model bias and building response failures. Validation tests are conducted using a building cluster consisting of 150 buildings. Based on the test results, main conclusions are drawn as follows.

- The proposed probabilistic model can provide more robust quantification of the aggregated building flexibility by considering uncertainties, compared to the deterministic model. For a demand response duration of 1 hour, the flexibility capacity estimated by the deterministic model has a high probability of 52.5 % of being overestimated.
- The proposed model outperforms the existing probabilistic models in terms of both accuracy and computational efficiency. It can accurately quantify the aggregated flexibility in 6.7 s, which is 535 times faster than the probabilistic model solved numerically. Furthermore, it is 8 times faster than the archetype-based model while offering significantly higher accuracy.
- The scalability of the proposed probabilistic model is validated. The proposed model only takes 140 s to quantify the aggregated flexibility of 2000 buildings, which can satisfy the real-time application scenario of building energy flexibility.
- The aggregated flexibility of buildings has a very high success probability (e.g., 99.99 %) when the committed capacity is below a certain threshold (e.g., 160 MW for a 1-hour response duration). This reliability level is even higher than that of conventional generators (i.e., 99.9 %). Therefore, buildings can provide reliable grid services by properly setting their committed flexibility capacity.

The proposed model can quickly and accurately generate probability-capacity curves for the aggregated flexibility of buildings in real-time applications, significantly facilitating the active participation of building energy flexibility in real-time electricity and grid service markets.

CRediT authorship contribution statement

Binglong Han: Writing – original draft, Validation, Methodology, Investigation, Conceptualization. **Hangxin Li:** Writing – review & editing, Supervision, Project administration, Formal analysis. **Shengwei Wang:** Writing – review & editing, Supervision, Project administration, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.adapen.2024.100186.

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