Received: 9 September 2024 | Revised: 6 October 2024 | Accepted: 19 October 2024

DOI: 10.1002/col.22962

RESEARCH ARTICLE

COLOR RESEARCH AND APPLICATION WILEY

# Specification of just-noticeable color difference for 2° and 10° stimuli using different color matching functions

**Mengjing Zhao** | **Minchen Wei**

Color Imaging and Metaverse Research Center, The Hong Kong Polytechnic University, Hung Hom, Hong Kong

**Correspondence**
Minchen Wei, Color Imaging and Metaverse Research Center, The Hong Kong Polytechnic University, Hung Hom, Hong Kong.
Email: minchen.wei@polyu.edu.hk

**Funding information**
Research Grants Council, University Grants Committee, Grant/Award Number: PolyU R5006-21

**Abstract**

Just-noticeable color difference (JNCD) is important in color specification and characterization. The commonly referenced specification of JNCD (i.e., 0.004 or 0.0033 $u'v'$ unit) is thought to originate from the MacAdam ellipses, which were derived using 2° color stimuli and characterized using the CIE 1931 2° color matching functions (CMFs). However, there is no universally agreed or clear definition of JNCD. Also, such a specification is widely used in various ways based on an assumption that it is applicable regardless of the actual size of the stimuli and CMFs. In this study, an experiment using a constant stimuli method was carried out. The human observers evaluated a series pairs of test and reference stimuli, with a field of view (FOV) of 2° or 10°. The chromaticities of the test stimuli were carefully calibrated using four standard CIE CMFs (i.e., CIE 1931 2°, 1964 10°, 2006 2°, and 10° CMFs). The results suggested that the widely used specification of JNCD seems to be derived based on the one standard deviation ellipses, the use of these four CMFs has little effect on the specification, and the JNCD value for stimuli with an FOV of 10° is 0.0025–0.0027 $u'v'$ unit depending on the CMFs.

**KEYWORDS**

color matching functions, color specification

## 1 | INTRODUCTION

Color characterization and specification are important in a wide range of applications, with the stimuli having the same tristimulus values or chromaticities considered to have matched appearances under the same viewing condition. The tristimulus values and chromaticities are calculated using the spectral power distribution (SPD) of the stimulus and the color matching functions (CMFs). The CIE 1931 2° CMFs is the most widely used set, which was derived from two color matching experiments with the color stimuli having a field of view (FOV) around 2° by Wright[1,2] and Guild.[3] In order to characterize the color matches for stimuli with a larger FOV, the CIE 1964 10° CMFs were developed.[4] In 2006, a technical committee in CIE developed a model based on physiology, which led to two new sets of CMFs (i.e., the CIE 2006 2° and 10° CMFs) and also allowed the calculations of CMFs for different FOVs and ages.[5,6]

Just-noticeable color difference (JNCD), commonly specified using color difference or chromaticity difference value, characterizes the threshold below which the color difference between two stimuli is not perceivable. It is widely used to define specifications for product design, engineering, and manufacturing. The most commonly used JNCD value is thought to originate from the 25 MacAdam ellipses in the CIE 1931 2° chromaticity diagram, which were derived from an experiment with the stimuli occupying an FOV of 2° in 1942.[7] The ellipses are plotted with the size being one standard deviation of the color matching results, and it is claimed that JNCD is about three times the standard deviation, which implies that a JNCD is equivalent to 0.0033 unit of $\Delta u'v'$. No clear definition of JNCD, however, has been set in the past.

MacAdam ellipses and the concept of JNCD have been widely used in color science research. For example, the uniformity of the color spaces is commonly compared based on the shape of the ellipses, with a better uniformity making the ellipses closer to circles. The performance of CMFs is also evaluated using JNCD, comparing the color or chromaticity differences calculated using different CMFs for stimuli with matched color appearances to the JNCD value. For instance, Csuti and Schanda[8] compared the chromaticity differences calculated using three different sets of CMFs for matched color stimuli with an FOV of 2° × 3°. Li et al.[9] carried out color matching experiments with different FOVs and setups, and compared the performance of various CMFs based on the calculated chromaticity differences and one JNCD value. Shi and Luo[10] carried out a color matching experiment using LCD and OLED displays with the stimuli size of 4°, and used the calculated color difference to compare the performance of different CMFs.

The analyses and findings in these studies were based on two assumptions. Firstly, it was assumed that the chromaticity differences or color differences calculated using the different CMFs were directly comparable. This was also assumed in IES TM-30-20[11] when calculating the color differences of color evaluation samples under the sources. Some efforts have been made to mitigate the possible failure of the first assumption when comparing the performance of different CMFs, Wu et al.[12] scaled the calculated chromaticity differences using the chromaticity difference between the D65 and D70 illuminants using the corresponding CMFs. In contrast, Asano et al.[13] proposed to perform a simulation of color matching between the reference and test stimuli using a certain set of CMFs, and then calculate the chromaticity difference using the CIE 1931 2° CMFs. Second, it was assumed that the JNCD value derived using 2° stimuli was applicable to those with other sizes. This, however, has never been investigated in the past.

With the above in mind, this study was designed to test the following hypotheses. Firstly, whether the JNCD values for stimuli with FOVs of 2° and 10° are comparable when characterized using the CMFs with the corresponding FOV. Secondly, whether the performance of the different CMFs can be directly compared regardless of the FOVs. It is worthwhile to mention that the stimuli in this study were produced using the same display, which helped to isolate the effect of FOVs from the spectral compositions.

## 2 | FURTHER ANALYSES OF THE EXPERIMENT DATA IN WU AND WEI

As mentioned above, the color or chromaticity differences calculated using the different CMFs have different magnitudes.[14] To illustrate this, the chromaticities of a series of blackbody radiators having a CCT from 2000 to 10 000 K with an interval of 100 K were calculated using the four standard CMFs (i.e., the CIE 1931 2°, 1964 10°, 2006 2°, and 10° CMFs). The chromaticity differences $\Delta u'v'$ between the two illuminants having the adjacent CCT values were then calculated using the four sets of CMFs and scaled with those calculated using the CIE 1931 2° CMFs, as shown in Figure 1. It can be observed that the four CMFs indeed have different scales, which also depend on the chromaticities.

We did not consider such a difference in Wu and Wei,[14] but we considered such a difference by scaling the chromaticity differences calculated using the different

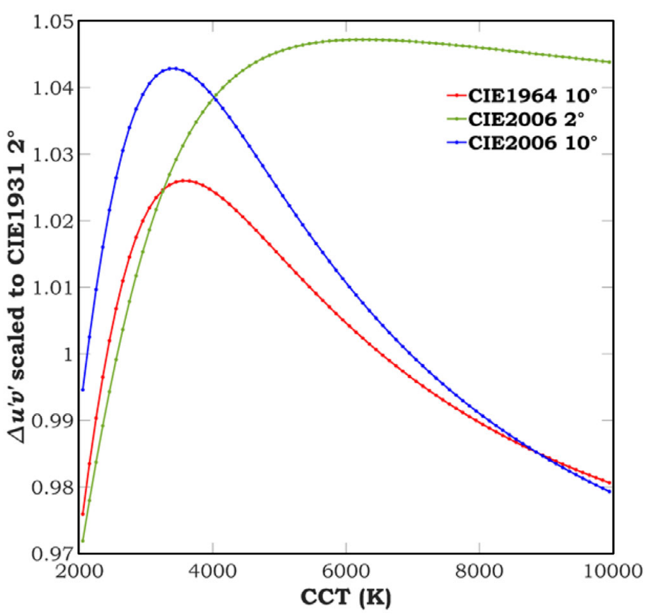


**FIGURE 1** Illustration of the different scales of chromaticity differences calculated using the four standard CMFs, with the chromaticity differences $\Delta u'v'$ between two blackbody radiators with a CCT difference of 100 K calculated using the CIE 1931 2°, 1964 10°, 2006 2°, and 10° CMFs and scaled to those calculated using the CIE 1931 2° CMFs.

CMFs to that between the D65 and D70 illuminants using the corresponding CMFs. Asano et al.,[13] however, proposed using the following method to compare the performance of the different CMFs. In order to evaluate the performance of a certain set of CMFs, a simulation of the color matching is performed using this set of CMFs, by adjusting the intensities of the primaries of the test stimulus to have the same tristimulus values of the reference stimulus. The SPD of the test stimulus derived from the intensities of the primaries and the SPD of the test stimuli adjusted by the observers were then used to calculate the chromaticity difference using the CIE 1931 2° CMFs.

Thus, we performed further analyses on the data we collected in Wu and Wei[14] using such a method, with the calculated chromaticity differences shown in Figure 2. This can be directly compared to the original analysis (i.e., Figure 8 in Reference 14), with the scatter plot showing the direct comparison in Figure 3. The

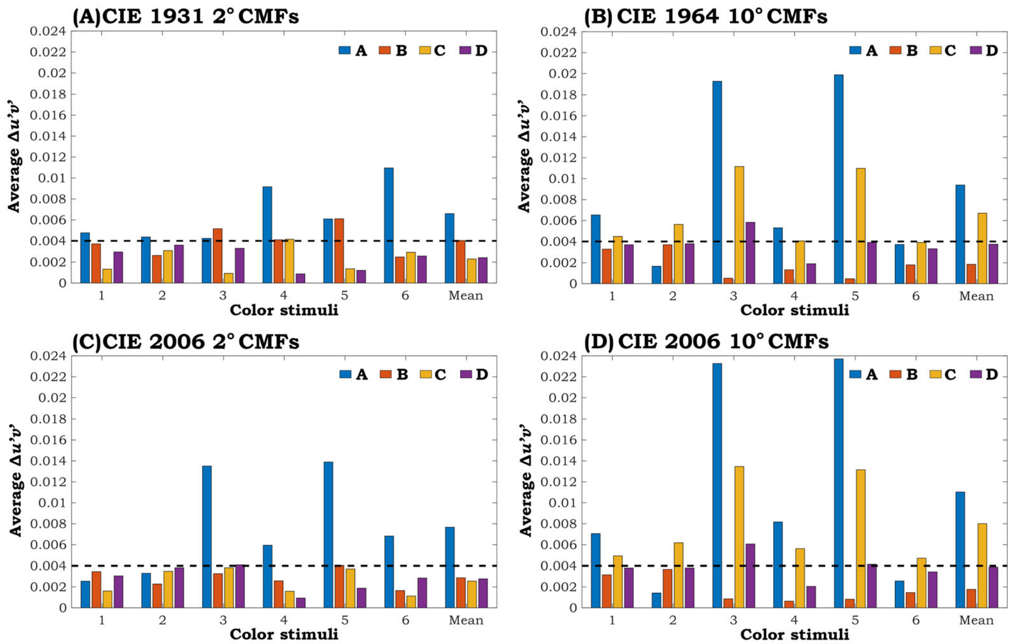


FIGURE 2 Chromaticity differences $\Delta u'v'$ calculated using the CIE 1931 2° CMFs between the average chromaticities of the test stimulus adjusted by the observers and the test stimulus, which was derived using the four CMFs through simulations to produce the same tristimulus values as the reference stimulus. (A) CIE 1931 2° CMFs; (B) CIE 1964 10° CMFs; (C) CIE 2006 2° CMFs; (D) CIE 2006 10° CMFs. This figure can be directly compared with Figure 8 in Reference 14, and the JNCD values of 0.004 can be directly applied to all the results here.

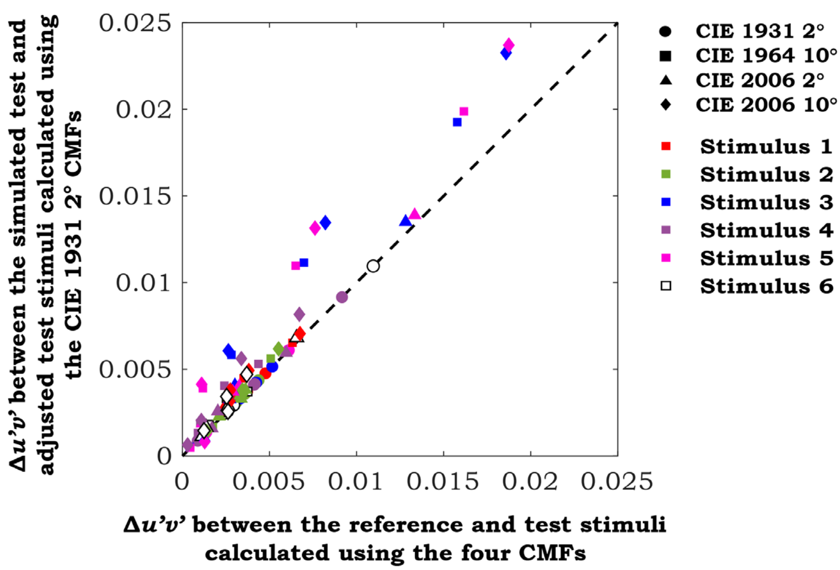


FIGURE 3 Comparisons of the chromaticity differences between the test and reference stimuli calculated using the four CMFs directly, as in Reference 14, versus those between the test stimulus, which was derived using the four CMFs through simulations, and the test stimuli adjusted by the observers using the CIE 1931 2° CMFs.

discrepancies can be easily observed, especially if the JNCD value of 0.004 is used to evaluate the performance of the different CMFs, though the stimuli used in the experiment had an FOV around 20.2°. Such an analysis clearly suggests the necessity to investigate whether the same JNCD value can be used with different CMFs.

# 3 | EXPERIMENT METHOD

## 3.1 | Apparatus and setup

The experiment was carried out using an 8-bit EIZO display (CS2371), with dimensions of 60.0 cm (length) × 33.5 cm (width) and a resolution of 2560 × 1440. During the experiment, the observer was seated with his or her chin fixed on a rest, which helped to ensure the similar viewing geometry experienced by all the observers. The reference and test stimuli were shown vertically, with the reference stimulus presented at the top and the test stimulus presented at the bottom. The thickness of the dividing line was set to around 0.04° FOV. The background of the display was set to black. The sizes of the stimuli were designed to produce two FOVs (i.e., 2° × 2° and 10° × 10°). Figure 4 shows the experiment GUI viewed by the observer during the experiment.

## 3.2 | Color stimuli

The reference stimuli included five different color centers (i.e., white, red, yellow, green, and blue), with the chromaticities listed in Table 1. In particular, the chromaticities of the white stimulus were the same as the D65 chromaticities, and the chromaticities of the other four were selected based on the CIE recommendation.

In order to investigate the magnitude of the JNCD, the experiment was carried out using the constant stimuli method. The observer viewed a series pairs of reference and test stimuli for each color center, and judged whether the test and reference stimuli in each pair had a perceived color difference. The test stimuli were designed to have the chromaticities uniformly distributed around the corresponding reference stimulus in the CIE 1976 $u'v'$ chromaticity diagram, with an interval of 0.0015 $u'v'$ unit and a range of ±0.01 $u'v'$ unit from the chromaticities of the reference stimulus, so that the chromaticities of the test stimuli formed a grid of 14 × 14.

When calibrating the display using the Gain-Offset-Gamma model and the stimuli, we used the four standard CMFs (i.e., CIE 1931 2°, 1964 10°, 2006 2°, and 2006 10° CMFs) to produce four sets of test stimuli, since we aimed to investigate the JNCD using these CMFs. However, it was found that the stimuli calibrated using the CIE 2006 2° CMFs can make the chromaticities uniformly distributed around the reference stimuli when calculated using the different CMFs, with the grids of the chromaticities globally shifted. Therefore, we only used the set of the 196 test stimuli and the five reference stimuli, with a luminance of 80 cd/m$^2$, calibrated using the CIE 2006 2° CMFs. The SPDs of the stimuli were measured, with Figure 5 showing the chromaticities and Figure 6 showing the boxplots of the luminance calculated using the four CMFs. The luminance values were generally within ±10%.

## 3.3 | Observers

Nine observers (five males and four females) between 25 and 29 years of age completed the experiment (mean = 27, SD = 1.45). All observers had normal color vision, as tested using the Ishihara Color Vision Test.

## 3.4 | Experiment procedure

Upon arrival, the observer completed an information record form and the Ishihara Color Vision Test. Then the

**TABLE 1** Chromaticities of the five reference stimuli calculated using the CIE 1964 10° CMFs.

| Chromaticity | White | Red | Yellow | Green | Blue |
|---|---|---|---|---|---|
| $u'$ | 0.198 | 0.315 | 0.209 | 0.147 | 0.174 |
| $v'$ | 0.470 | 0.519 | 0.538 | 0.506 | 0.384 |

**FIGURE 4** Experiment GUI viewed by the observer during the experiment. The display was placed so that the stimuli occupied an FOV of 2° × 2° and 10° × 10°. Left: 2° × 2°; Right: 10° × 10°.

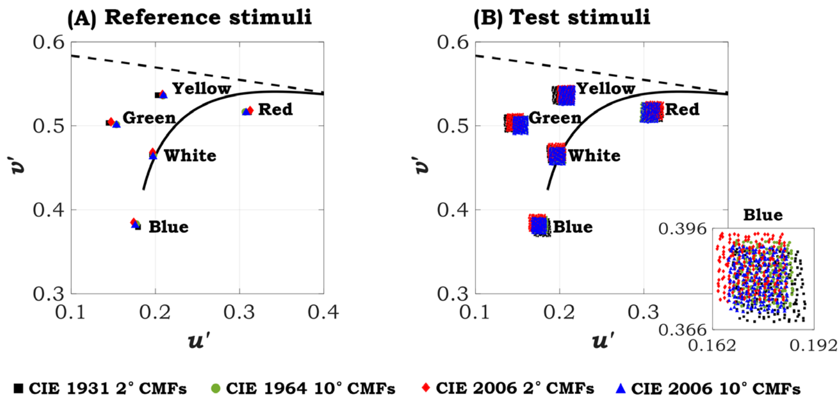


**FIGURE 5** Chromaticities of the stimuli calculated using the four CMFs in the CIE 1976 $u'v'$ chromaticity diagram. (A) Reference stimuli; (B) test stimuli.

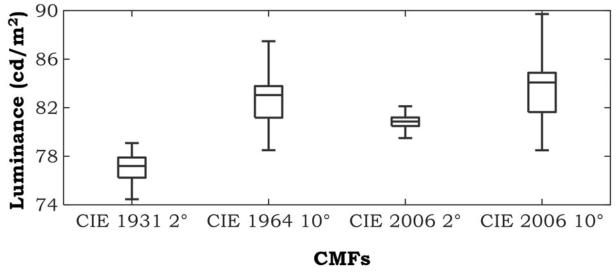


**FIGURE 6** Boxplots of the luminance of the test stimuli calculated using the four CMFs.

experimenter explained the task to the observer, and the observer was escorted to the display with his or her chin fixed on the rest. The illumination in the laboratory was then switched off, and the observer completed a 5-min dark adaptation. A pair of stimuli was shown on the display and the observer judged whether the two stimuli had a color difference, which was a forced choice, by pressing one of the two keys on a keyboard to record the judgment. After that, the next pair of stimuli were shown. When making the judgment, the observer was allowed to take as much time as he or she needed, but the observers generally made the decision immediately. The order of the pairs of the stimuli presented to each observer were randomized. In order to evaluate the intra-observer variations, five of the 196 pairs of the stimuli were presented twice. Thus, each observer made 2010 judgments (i.e., 201 pairs × 5 color centers × 2 FOVs), which took around 1 hour. Before the experiment, the display was warmed up for an hour.

# 4 | RESULTS AND DISCUSSIONS

## 4.1 | Intra- and inter-observer variations

The intra-observer variations were characterized based on the repeated judgments made by the observers, with the repeated judgments made on 50 pairs of stimuli by each observer and a total of 450 repeated judgments. Among these 450 repeated judgments, 380 judgments were the same (~ 84.4%). The inter-observer variations were characterized based on the difference between the judgments made by each observer and judgments made by an average observer (i.e., the judgments made by more than half of the observers). In other words, if a pair of the stimuli was judged by more than half of the observers to have no color difference, this pair was considered to have no difference, which was compared to the judgment made by each observer. For each observer, 82.49% to 96.52% of the judgments, with a mean of 92.80%, were the same as those made by an average observer. This clearly suggested the consistent judgments made by the observers.

## 4.2 | Derivation of JNCD ellipses

The experiment data collected through the constant stimuli method can be used to derive the JNCD ellipses. As mentioned above, however, the concept of JNCD is not clearly defined or agreed, which has been clearly stated in CIE TN 001:2014.[15] Different studies specified the JNCD in different ways, including one standard deviation,[16] three times the standard deviation,[7] 95% confidence,[17] and 50% probability.[18]

Here, we followed the method in MacAdam[19] and Wyszecki[20] to fit the standard deviation ellipses and the 95% confidence ellipses, with the chromaticities of the test stimuli that were judged to have no color difference to the reference stimuli used as the input. With the Gaussian distribution, one and three standard deviation ellipses correspond to 39.35% and 98.89% confidence level; 95% confidence level corresponds to 2.45 SD, with Figure 7 showing the relationship among the three ellipses. (Note: 50% confidence level corresponds to 1.18 SD). It can be observed that these three types of ellipses have different sizes, but the same shape and orientation.

For the 50% probability ellipse, a bivariate Gaussian model was fitted using the chromaticities of all the test stimuli and the percentage of the judgments that the test and reference stimuli had no color difference using Equations (1–3), with the 50% probability used to derive the ellipse, as illustrated in Figure 8.

$$W(u',v') = a_1 + a_2 \times \exp\left\{-\frac{1}{2} \times d^2(u',v')\right\}; \quad (1)$$

$$d^2(u',v') = (X - X_c)^T \times \sum\nolimits^{-1} \times (X - X_c); \quad (2)$$

$$X = \begin{pmatrix} u' \\ v' \end{pmatrix}; X_c = \begin{pmatrix} u'_{center} \\ v'_{center} \end{pmatrix}; \quad (3)$$

where $\Sigma^{-1}$ is the inverse of the covariance matrix, $X$ and $Xc$ are the chromaticities of the stimulus and the estimated center, respectively, $a_1$ and $a_2$ are the parameters that need to be optimized to fit the distribution.

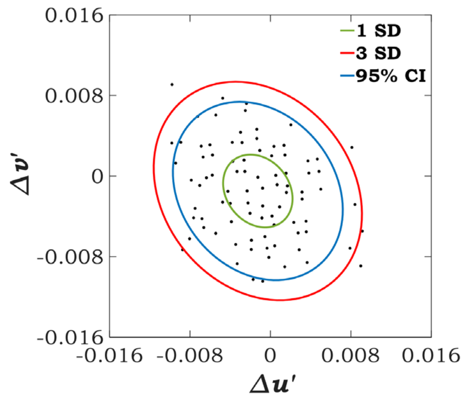


FIGURE 7 Illustration of one and three standard deviation ellipses, and 95% confidence ellipse for the same sets of input. With Gaussian distribution, one and three standard deviation ellipses correspond to 39.35% and 98.89% confidence level; 95% confidence level corresponds to 2.45 SD. The data points shown in the figure are the chromaticities of the stimuli that were judged to have no color difference to the reference stimulus.

In total, 160 ellipses were fitted (i.e., 5 color centers × 2 FOVs × 4 CMFs × 4 types of ellipses). Since the one standard deviation ellipses, three standard deviation ellipses, and 95% confidence ellipses have the same shape and orientation, as illustrated in Figure 7, only the one SD ellipses are shown in Figure 9. Figure 10 shows the 50% probability ellipses. It can be seen that the ellipses derived using the different CMFs had very similar shapes and sizes for each color center and FOV. Thus, Tables 2 and 3 only summarize the parameters of the ellipses for the five color centers and the two FOVs using the CIE 1931 2° CMFs, including the lengths of the semi-major and minor axes (i.e., $a$ and $b$), the size of the ellipse (i.e., $\sqrt{ab}$), the orientation of the ellipse $\theta$, and the shape of the ellipse as characterized using $b/a$. Figure 11 shows the fitted ellipses together with the MacAdam ellipses in the CIE 1976 $u'v'$ chromatcity diagram using the CIE 1931 2° CMFs, providing a better illustration of the size of the JNCD values in this study.

### 4.3 | Effect of CMFs, FOVs, and color centers

Figure 12 shows the average $\sqrt{ab}$ of the five color centers calculated using the four CMFs for the two FOVs. It can be seen that the four CMFs did not introduce large differences in the size of the ellipses. Importantly, the ellipses for the 2° FOV were consistently greater, with a scale around 1.4, than those for the 10° FOV, suggesting that the observers were more sensitive to the chromaticity differences when viewing 10° stimuli. The orientations of the ellipses for the two FOVs, however, were generally similar, as shown in Figures 9 and 10.

Figures 9 and 10, together with Tables 2 and 3, show that the ellipses for the five color centers had some differences, in terms of the size, shape, and orientation. In particular, the $\sqrt{ab}$ values for the blue center were always

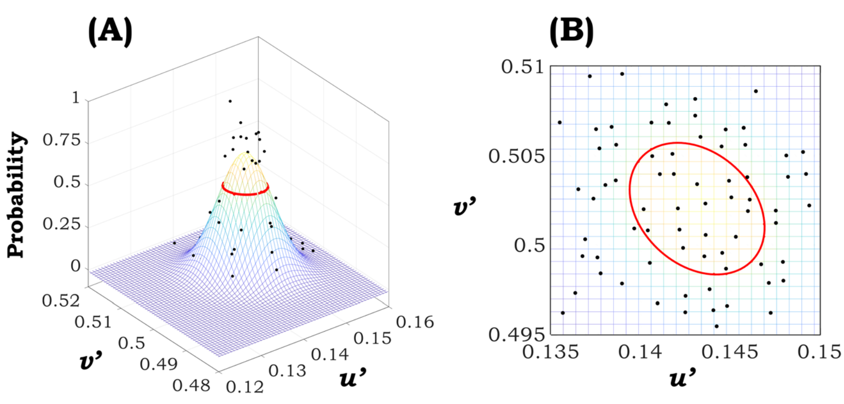


FIGURE 8 Illustration of the derivation of the 50% probability ellipses fitted using a bivariate Gaussian model. (A) Three-dimensional bivariate Gaussian distribution fitted using the experiment data; (B) Derivation of the 50% probability ellipse.

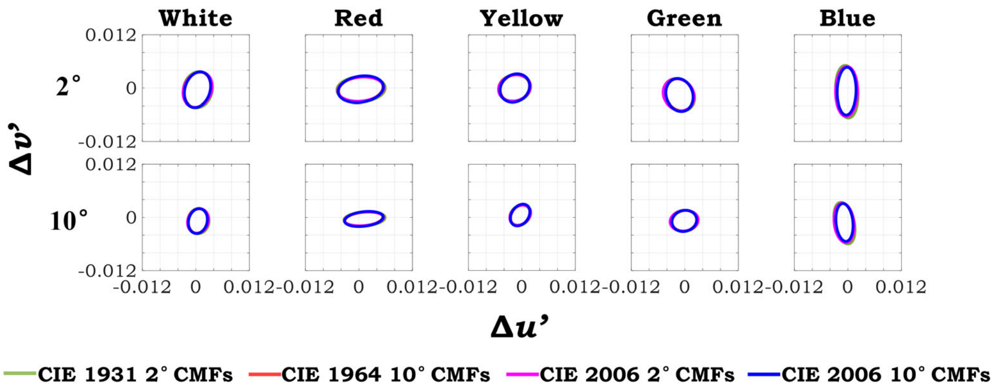


**FIGURE 9** One standard deviation ellipses derived using the four CMFs for the five color centers and two FOVs in the CIE 1976 $u'v'$ chromaticity diagram, with the chromaticities of the reference stimulus shifted to the origin.

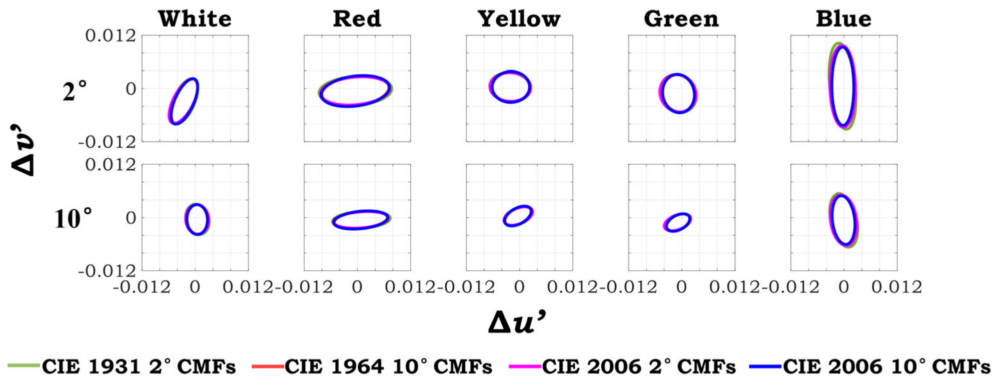


**FIGURE 10** 50% probability ellipses were derived using the four CMFs for the five color centers and two FOVs in the CIE 1976 $u'v'$ chromaticity diagram, with the chromaticities of the reference stimulus shifted to the origin.

**TABLE 2** Parameters of the JNCD ellipses derived using the CIE 1931 2° CMFs for the five color centers and the 2° FOV.

| Parameters | | $a$ | $b$ | $\sqrt{ab}$ | $\theta$ (°) | $b/a$ |
|---|---|---|---|---|---|---|
| One standard deviation ellipse | White | 0.0041 | 0.0032 | 0.0036 | 78.4278 | 0.7904 |
| | Red | 0.0054 | 0.0028 | 0.0039 | 3.386 | 0.5229 |
| | Yellow | 0.0036 | 0.003 | 0.0033 | 9.8129 | 0.8296 |
| | Green | 0.0039 | 0.0031 | 0.0035 | 128.8429 | 0.8020 |
| | Blue | 0.0059 | 0.0025 | 0.0038 | 94.2391 | 0.4197 |
| Three standard deviations ellipse | White | 0.0122 | 0.0097 | 0.0109 | 78.4278 | 0.7904 |
| | Red | 0.0162 | 0.0085 | 0.0117 | 3.386 | 0.5229 |
| | Yellow | 0.0108 | 0.0089 | 0.0098 | 9.8129 | 0.8296 |
| | Green | 0.0117 | 0.0094 | 0.0105 | 128.8429 | 0.8020 |
| | Blue | 0.0177 | 0.0074 | 0.0115 | 94.2391 | 0.4197 |
| 95% confidence ellipse | White | 0.0100 | 0.0079 | 0.0089 | 78.4278 | 0.7904 |
| | Red | 0.0132 | 0.0069 | 0.0096 | 3.386 | 0.5229 |
| | Yellow | 0.0088 | 0.0073 | 0.0080 | 9.8129 | 0.8296 |
| | Green | 0.0095 | 0.0077 | 0.0085 | 128.8429 | 0.802 |
| | Blue | 0.0145 | 0.0061 | 0.0094 | 94.2391 | 0.4197 |
| 50% probability ellipse | White | 0.0054 | 0.0023 | 0.0036 | 67.4805 | 0.4316 |
| | Red | 0.0082 | 0.0033 | 0.0052 | 3.9944 | 0.3994 |
| | Yellow | 0.0046 | 0.0032 | 0.0038 | 173.8979 | 0.6845 |
| | Green | 0.0044 | 0.0037 | 0.0041 | 126.8177 | 0.8474 |
| | Blue | 0.0096 | 0.0029 | 0.0052 | 95.8659 | 0.2997 |

**TABLE 3** Parameters of the JNCD ellipses derived using the CIE 1931 2° CMFs for the five color centers and the 10° FOV.

| Parameters | | $a$ | $b$ | $\sqrt{ab}$ | $\theta$ (°) | $b/a$ |
|---|---|---|---|---|---|---|
| One standard deviation ellipse | White | 0.0028 | 0.0024 | 0.0026 | 85.3652 | 0.8358 |
| | Red | 0.0046 | 0.0016 | 0.0027 | 4.1724 | 0.3442 |
| | Yellow | 0.0025 | 0.0021 | 0.0023 | 43.8048 | 0.8196 |
| | Green | 0.0031 | 0.0023 | 0.0027 | 3.8961 | 0.7579 |
| | Blue | 0.0048 | 0.0022 | 0.0032 | 101.8703 | 0.4621 |
| Three standard deviations ellipse | White | 0.0085 | 0.0071 | 0.0078 | 85.3652 | 0.8358 |
| | Red | 0.0138 | 0.0048 | 0.0081 | 4.1724 | 0.3442 |
| | Yellow | 0.0076 | 0.0062 | 0.0068 | 43.8048 | 0.8196 |
| | Green | 0.0092 | 0.0070 | 0.0080 | 3.8961 | 0.7579 |
| | Blue | 0.0143 | 0.0066 | 0.0097 | 101.8703 | 0.4621 |
| 95% confidence ellipse | White | 0.0069 | 0.0058 | 0.0063 | 85.3652 | 0.8358 |
| | Red | 0.0113 | 0.0039 | 0.0066 | 4.1724 | 0.3442 |
| | Yellow | 0.0062 | 0.0051 | 0.0056 | 43.8048 | 0.8196 |
| | Green | 0.0075 | 0.0057 | 0.0066 | 3.8961 | 0.7579 |
| | Blue | 0.0117 | 0.0054 | 0.0079 | 101.8703 | 0.4621 |
| 50% probability ellipse | White | 0.0034 | 0.0025 | 0.0029 | 104.3852 | 0.7384 |
| | Red | 0.0065 | 0.0019 | 0.0035 | 3.5515 | 0.2902 |
| | Yellow | 0.0034 | 0.0018 | 0.0024 | 24.0658 | 0.5258 |
| | Green | 0.0030 | 0.0016 | 0.0022 | 22.397 | 0.5488 |
| | Blue | 0.0060 | 0.0029 | 0.0042 | 103.0117 | 0.4730 |

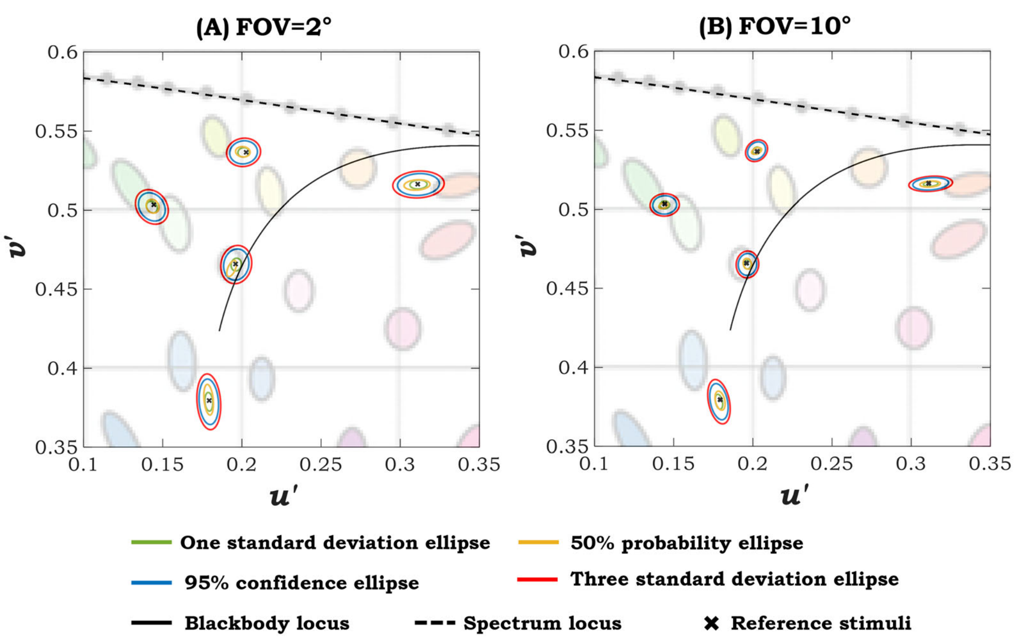


**FIGURE 11** Fitted ellipses based on the experiment methods using the CIE 1931 2° CMFs in the CIE 1976 $u'v'$ chromaticity diagram, together with the MacAdam ellipses. (A) Experiment results with an FOV of 2°; (B) Experiment results with an FOV of 10°.

the largest, as found in various past studies,[12,14] suggesting the observers were the least sensitive to the chromaticity differences in the blue region. Also, the $b/a$ values for all the five color centers were very different from 1 and the orientations of the ellipses significantly varied with the color center, suggesting the non-uniformity of

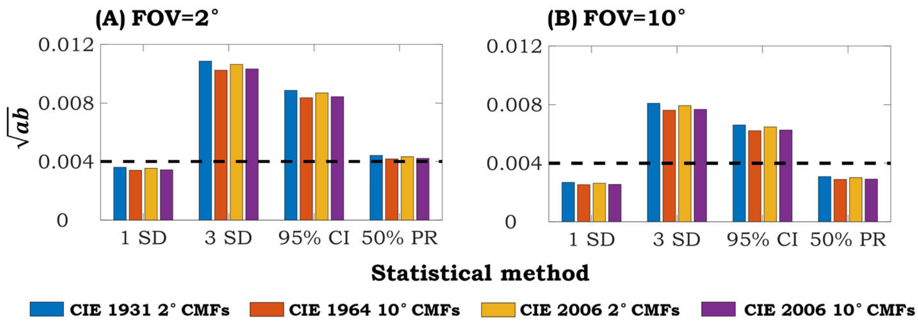


FIGURE 12 Comparison of the average $\sqrt{ab}$ values of the four types of ellipses derived using the four CMFs for the two FOVs. (A) FOV of 2°; (B) FOV of 10°. (1 SD: One standard deviation ellipse; 3 SD: Three standard deviation ellipse; 95% CI, 95% confidence ellipse; 50% PR, 50% probability ellipse).

TABLE 4 Summary of the JNCD values, in terms of the average $\sqrt{ab}$ values, of the four types of the ellipses derived using the four CIE standard CMFs for the 2° and 10° FOVs, with the value derived using the 1 SD ellipse being the closest to the common specification of JNCD.

| | | CIE 1931 2° | CIE 1964 10° | CIE 2006 2° | CIE 2006 10° |
|---|---|---|---|---|---|
| One standard deviation | 2° | 0.0036 | 0.0034 | 0.0036 | 0.0034 |
| | 10° | 0.0027 | 0.0025 | 0.0026 | 0.0026 |
| Three standard deviations | 2° | 0.0109 | 0.0103 | 0.0107 | 0.0103 |
| | 10° | 0.0081 | 0.0076 | 0.0079 | 0.0077 |
| 95% confidence interval | 2° | 0.0089 | 0.0084 | 0.0087 | 0.0084 |
| | 10° | 0.0066 | 0.0062 | 0.0065 | 0.0063 |
| 50% probability | 2° | 0.0044 | 0.0041 | 0.0043 | 0.0042 |
| | 10° | 0.0030 | 0.0029 | 0.0030 | 0.0029 |

*Note*: The values derived from the one standard deviation, as highlighted in shaded cells, seem to be simliar to the specifications that are widely used.

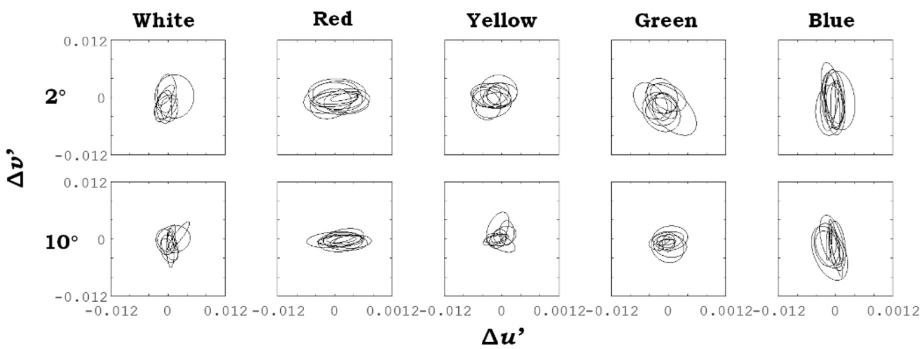


FIGURE 13 One standard deviation ellipses for each of the nine individual observers derived using the CIE 1931 2° CMFs for the five color centers and the two FOVs, with the chromaticities of the reference stimulus shifted to the origin.

the CIE 1976 $u'v'$ chromaticity diagram. The $b/a$ values for the white, yellow, and green color centers were closer to 1.

## 4.4 | Specification of JNCD values

The average sizes of the different ellipses can be used to specify the JNCD values, with Table 4 summarizing the average $\sqrt{ab}$ of the four types of the ellipses derived using the four CMFs for the two FOVs.

It seems to suggest that the widely used JNCD value (i.e., 0.004 or 0.033 $u'v'$ unit) for a stimulus with an FOV of 2° is derived based on the one standard deviation ellipses, especially the $\sqrt{ab}$ value for the white center is 0.0036, as listed in Table 2. Based on this, the JNCD value for a stimulus with an FOV of 10° should be specified as 0.0027, 0.0025, 0.0026, and 0.0026 $u'v'$ unit using each of the four CMFs. If such a specification was used, the findings and evaluations in many past studies would have been changed.

## 4.5 | Variations among observers

Instead of only focusing on an average observer, we also investigated the variations among the nine individual observers, with Figure 13 showing the one standard

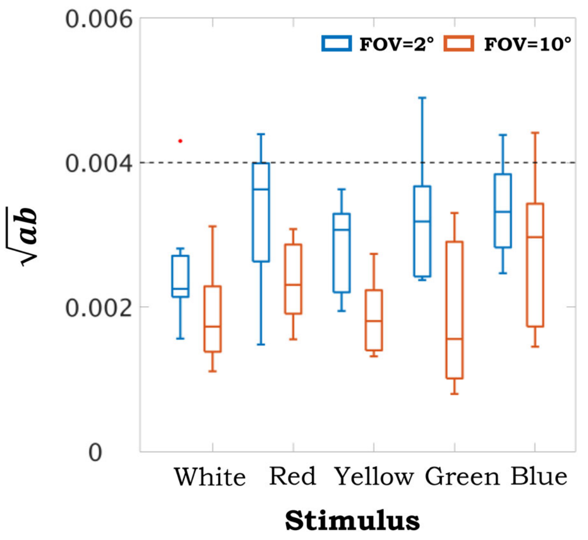


**FIGURE 14** Boxplots of the $\sqrt{ab}$ values of the ellipses shown in Figure 13.

deviation ellipses and Figure 14 showing the boxplots of the $\sqrt{ab}$ values. In general, the variations among the observers, in terms of the size and orientation of the ellipse, were larger when the FOV was 10°.

## 5 | CONCLUSION

JNCD is important in color specification and characterization, but there is no clear or agreed definition of JNCD. Also, the specification of JNCD was widely used in various past studies in different ways, without considering whether such a specification can be used with different CMFs and for stimuli with different FOVs. In this experiment, the human observers evaluated whether there was a color difference between the test and reference stimuli. The test stimuli were carefully calibrated using four CIE standard CMFs, with the chromaticities uniformly distributed around the reference stimulus. The stimuli were designed to have five color centers and two FOVs (i.e., 2° and 10°). Different methods were used to derive the ellipses for the JNCD specifications. It was found that the widely used JNCD value (i.e., 0.004 or 0.0033 $u'v'$ unit) seemed to be based on the one standard deviation ellipses. The specification of the JNCD was found not affected by using the four different CMFs. More importantly, it was found that the specification of JNCD for stimuli with an FOV of 10° is 0.0025–0.0027 $u'v'$ unit depending on the CMFs.

## AUTHOR CONTRIBUTIONS

*Conceptualization and experiment design*: Minchen Wei. *Data collection and analysis*: Mengjing Zhao. *Manuscript preparation*: Mengjing Zhao. *Manuscript revision*: Minchen Wei.


## ACKNOWLEDGMENTS

The authors would like to thank the reviewers for the comments.

## FUNDING INFORMATION

Research Grants Council, University Grants Committee (PolyU R5006-21).


## DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.


## ORCID

*Minchen Wei* https://orcid.org/0000-0002-0045-3160


## REFERENCES


[1] Wright WD. A re-determination of the trichromatic coefficients of the spectral colours. *Trans Opt Soc*. 1929;30(4):141-164.

[2] Wright W. A re-determination of the mixture curves of the spectrum. *Trans Opt Soc*. 1930;31(4):201-218.

[3] Guild J. The colorimetric properties of the spectrum. *Phil Trans R Soc Lond A*. 1931;230(681–693):149-187.

[4] CIE. Colorimetry. *CIE 15:2018*. 4th ed. CIE; 2018.

[5] CIE. Fundamental chromaticity diagram with physiological axes – part 1. *CIE 170-1:2006*. CIE; 2006.

[6] CIE. Fundamental chromaticity diagram with physiological axes – part 2: spectral luminous efficiency functions and chromaticity diagrams. *CIE 170-2:2015*. CIE; 2015.

[7] MacAdam DL. Visual sensitivities to color differences in daylight. *J Opt Soc Am A*. 1942;32(5):247-274.

[8] Csuti P, Schanda J. Colour matching experiments with RGB-LEDs. *Color Res Appl*. 2008;33(2):108-112.

[9] Li J, Hanselaer P, Smet KAG. Impact of matching field size on color matching (functions) accuracy. *Color Res Appl*. 2023;48(1):88-102.

[10] Shi K, Luo MR. Factors affecting colour matching between displays. *Opt Express*. 2022;30(15):26841-26855.

[11] IES. Method for evaluating light source color rendition. *IES TM-30-20*. IES; 2020.

[12] Wu J, Wei M, Fu Y, Cui C. Color mismatch and observer metamerism between conventional liquid crystal displays and organic light emitting diode displays. *Opt Express*. 2021;29(8):12292-12306.

[13] Asano Y, Fairchild MD, Blondé L, Morvan P. Color matching experiment for highlighting interobserver variability. *Color Res Appl*. 2015;41(5):530-539.

[14] Wu J, Wei M. Color mismatch and observer metamerism between conventional liquid crystal displays and organic light emitting diode displays, part II: adjacent stimuli with a larger field of view. *Opt Express*. 2021;29(25):41731-41744.

[15] CIE. Chromaticity difference specification for light sources. *CIE TN 001:2014*. CIE; 2014.

[16] Brown WRJ. The influence of luminance level on visual sensitivity to color differences. *J Opt Soc Am A*. 1951;41(10):684-688.

[17] Yebra A, Garcia JA, Romero J. Color discrimination data for 2 and 8 degrees and normalized ellipses. *J Opt*. 1994;25(6):231-242.

[18] Brusola F, Tortajada I, Lengua I, Jordá B, Peris-Fajarnés G. Strip-pair comparison method for building threshold color-difference model: theoretical model validation. *Opt Express*. 2020;28(14):21336-21347.

[19] MacAdam DL. Specification of small chromaticity differences. *J Opt Soc Am*. 1943;33(1):18-26.

[20] Wyszecki G. Propagation of errors in colorimetric transformations. *J Opt Soc Am*. 1959;49(4):389-393.


## AUTHOR BIOGRAPHIES


**Mengjing Zhao** is a PhD student at The Hong Kong Polytechnic University. Her research mainly focuses on color difference and color appearance.

**Minchen Wei** is a professor at The Hong Kong Polytechnic University. His research mainly focuses on color science and imaging science.



**How to cite this article:** Zhao M, Wei M. Specification of just-noticeable color difference for 2° and 10° stimuli using different color matching functions. *Color Res Appl*. 2025;50(2):161-171. doi:10.1002/col.22962