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Global damping of noise or vibration fields with locally synthesized controllers

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A locally synthesized controller (LSC) is one that uses a local feedback signal in a noise or vibration field (VF) to synthesize the actuation signal. The global damping of a VF by available LSCs requires sensor–actuator collocation. This study presents a LSC for the global damping of a VF without requiring sensor–actuator collocation, which is important to noise control applications where a sensor may be placed away from an actuator to avoid the near field effects. It is proven that the LSC damps the entire VF instead of just a local feedback loop. This is different from other LSCs that may control local feedback loops without damping the VFs. A decentralized control law is presented here to extend the LSC to a decentralized damping system using multiple actuators. © *2002 Acoustical Society of America.* [DOI: 10.1121/1.1456516]

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I. INTRODUCTION

This study investigates active control of noise and vibration in the low frequency ranges. Both the noise and vibration fields will share the abbreviation ''VF,'' which stands for a vibration field, since sound is the vibration of air and a noise field can be modeled with modal theory or transfer functions like a vibration field. The scope of this study is active damping of a VF by feedback control—an important feature not possessed by feedforward controllers. While one may combine feedback and feedforward control to utilize the advantages of both strategies, concentrating on feedback control would keep a better focus on the main scope.

The well-known modal theory enables a control engineer to decouple the dynamics of a VF into m second-order differential equations when the model is properly truncated.[1] Most feedback controllers for VFs are based on modal space feedback,[2,3] though some researchers use physical state feedback[4] to avoid estimating the modal states. These controllers use global states of the VFs to synthesize the actuation signals. They are named ''globally synthesized controllers'' (GSCs) to distinguish them from the ''locally synthesized controller'' (LSC) whose actuation signal is synthesized from a local feedback signal.

It is believed that the number of actuators should equal the number of modes to be attenuated for active damping of a VF. An optimization method is available to reduce the number of actuators.[5] Recent studies showed the possibility of using one actuator to attenuate multiple modes simultaneously.[6–8] These GSCs require exact model parameters to calculate the feedback gains. The model parameters, such as mode functions, are not always available as discussed in Banks *et al.*[9] Some of the GSCs avoid the mode functions by means of the multiple spring-mass[4,7] or finite element[8] models, but still depend on other parameters such as mass and stiffness of the finite elements, etc.

LSCs, which are much less model dependent, may be needed if model parameters are not available, or not accurate enough, for VFs with irregular geometric shapes and boundary conditions. The first LSC for active damping may be traced back to Olson and May.[10] Clark and Cole[11] presented a direct rate feedback LSC for the same purpose, which requires sensor–actuator collocation. Many LSCs use high gains to suppress the feedback signals. This will drive the closed-loop poles toward zeros of the open-loop transfer functions.[12] It requires that open-loop transfer functions be minimum phase, which is true for collocated sensor–actuators but is not necessarily true otherwise. Driving the poles of a closed loop toward the open-loop zeros does not damp the VF, as explained in Sec. III.

This study presents a LSC to damp an entire VF while relaxing the minimum phase restriction. This is important for noise control in which case transfer functions are very likely nonminimum phase. The pole placement control is applied to modify eigenvalues of a VF. A designer can select closed-loop poles instead of driving them toward open-loop zeros. The LSC is based on a transfer function available by either on-line or off-line identification, and hence may be adaptive to deal with parameter drifting.[13] While structural systems tend to be fairly stable, it is not uncommon for parameters of a noise field to drift with time. Being able to adapt with respect to possible parameter drifting is an advantage of the LSC. For best focus, this paper analyzes the global damping effect of the LSC. Stability and robustness of the adaptive version are discussed in a separate paper.[13]

The implementation of active control for a VF will involve filters in the system. A popular way to analyze the filter effects is to use truncated models,[4,6–8] which is equivalent to the use of *ideal filters* to cut-off the uncontrolled modes and leave the controlled modes intact. This study takes a new approach in the LSC design. The actuator, sensor, and filter dynamics are described by transfer functions. Both the controlled and uncontrolled modes are studied analytically to address the concern of possible spillover in the presence of

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realistic filters. The LSC design procedure may be modified, when necessary, to design decentralized LSCs for active damping of a VF using multiple actuators. All features of the LSC will be explained analytically and numerically in this paper.

II. MATHEMATICAL MODEL

The dynamics of a VF, subject to the excitation of an actuator, can be described by a general second-order equation

$$(\mathbf{M}s^2+\mathbf{K})\mathbf{q}=\mathbf{b}f+\mathbf{d}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{K}\in R^{m\times m}$ are the mass and stiffness matrices; $\mathbf{q}\in R^m$ is the generalized displacement vector; $\mathbf{b}\in R^m$ the actuator location vector; $\mathbf{d}\in R^m$ the disturbance vector and f a scalar control signal. Matrices $\mathbf{M}$ and $\mathbf{K}$ are symmetric and positive definite, but not necessarily diagonal or band diagonal. The equation is expressed in the Laplace transform domain. One may derive it with different approaches, such as the finite element method, multiple spring-mass modeling, Rayleigh–Ritz approximation, or modal analysis with a proper truncation. This study uses the properties of Eq. (1) for analysis only. Unless stated specifically, model parameters are used here with assumed *existence*, but not *availability*.

If Eq. (1) is derived by modal analysis, then $\mathbf{q}$ is the modal state vector. The local feedback signal is given by

$$y=\mathbf{h}^T\mathbf{q}, \tag{2}$$

where $\mathbf{h}^T=[\varphi_1(x_s),\ldots,\varphi_m(x_s)]$ and $\varphi_i(x_s)$ is the ith eigenfunction sampled at the sensor location x_s. Similarly, the actuator location vector is $\mathbf{b}^T=[\varphi_1(x_a),\ldots,\varphi_m(x_a)]$ with $\varphi_i(x_a)$ taken at the actuator location x_a. If Eq. (1) represents a multiple spring-mass system, then $\mathbf{h}=\mathbf{e}_i$ and $\mathbf{b}=\mathbf{e}_j$ where

$$\mathbf{e}_i=[\underbrace{0\ \cdots\ 0}_{i-1\ \text{zeros}}1\underbrace{0\ \cdots\ 0}_{m-i\ \text{zeros}}]^T.$$

The underlying assumption is that the sensor and the actuator are attached to the ith and jth bodies, respectively.

For vibration control, it is common to collocate the sensor with the actuator so that $\mathbf{h}=\mathbf{b}$. For noise control, however, the sensor may be placed away from the actuator to avoid the near field. Therefore $\mathbf{h}$ is not necessarily $\mathbf{b}$. The normalized version of $\mathbf{h}$ may be denoted as $\mathbf{h}_n$. Its null space is spanned by $m-1$ unit length and orthogonal basis $\{\boldsymbol{\zeta}_1\cdots\boldsymbol{\zeta}_{m-1}\}$ with $\mathbf{h}^T\boldsymbol{\zeta}_i=0$ for $1\leqslant i\leqslant m-1$. The existence of these vectors implies the existence of an orthogonal matrix $\mathbf{H}=[\mathbf{h}_n\boldsymbol{\zeta}_1\cdots\boldsymbol{\zeta}_{m-1}]$, such that $\mathbf{x}=\mathbf{H}^T\mathbf{q}$ and $\boldsymbol{\Lambda}(s)=\mathbf{H}^T(\mathbf{M}s^2+\mathbf{K})\mathbf{H}$. Therefore Eq. (1) is equivalent to

$$\boldsymbol{\Lambda}(s)\mathbf{x}=\boldsymbol{\beta}f+\boldsymbol{\eta}, \tag{3}$$

where $\boldsymbol{\beta}=\mathbf{H}^T\mathbf{b}$ and $\boldsymbol{\eta}=\mathbf{H}^T\mathbf{d}$. While Eq. (3) describes a multivariable system, the LSC is based on a single-input single-output (SISO) local plant. The feedback signal, given by Eq. (2), is actually $y=hx_1$ where $h=\|\mathbf{h}\|$. The Cramer's rule suggests

$$y=hx_1=h\,\frac{\det[\boldsymbol{\Lambda}_1(s),\boldsymbol{\beta}]f+\det[\boldsymbol{\Lambda}_1(s),\boldsymbol{\eta}]}{\det[\boldsymbol{\Lambda}(s)]}, \tag{4}$$

where $[\boldsymbol{\Lambda}_1(s),\boldsymbol{\beta}]$ and $[\boldsymbol{\Lambda}_1(s),\boldsymbol{\eta}]$ are matrices obtained by replacing the first column of $\boldsymbol{\Lambda}(s)$ with vectors $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$, respectively.

Before LSC design, a control engineer may apply a probing signal $f=F_a(s)u$ to excite the VF, where $F_a(s)$ is the actuator filter and u is a pseudorandom noise. The feedback sensor measures $\nu=F_s(s)y$ that will be used to identify the transfer function. Here $F_s(s)$ represents the sensor filter. Polynomials $\det[\boldsymbol{\Lambda}(s)]$ and $\det[\boldsymbol{\Lambda}_1(s),\boldsymbol{\beta}]$ have degrees $2m$ and $2m-2$, respectively. Many algorithms[14] are available to identify an autoregressive and moving-average plant like Eq. (4). Any one of these algorithms may be applied to obtain

$$\nu(s)=\frac{h\det[\boldsymbol{\Lambda}_1(s),\boldsymbol{\beta}]N_s(s)N_a(s)u(s)+\boldsymbol{\epsilon}}{\det[\boldsymbol{\Lambda}(s)]D_s(s)D_a(s)} \tag{5a}$$

where

$$\boldsymbol{\epsilon}=hN_s(s)D_a(s)\det[\boldsymbol{\Lambda}_1(s),\boldsymbol{\eta}] \tag{5b}$$

represents the effect of disturbance filtered by the sensor; $N_s(s)$, $N_a(s)$, $D_s(s)$, and $D_a(s)$ are the numerators and denominators of filter transfer functions $F_s(s)$ and $F_a(s)$, respectively.

Both Eqs. (4) and (5) are valid models of a SISO local plant for a specific actuator and feedback sensor loop. The latter includes actuator and sensor dynamics $f=F_a(s)u$ and $v=F_s(s)y$. Model parameters of a VF are not necessarily available, but the numerator and denominator of Eq. (5) can be identified from measurement data. These polynomials may share some common roots. Therefore one may express

$$h\det[\boldsymbol{\Lambda}_1(s),\boldsymbol{\beta}]N_s(s)N_a(s)=N(s)C(s), \tag{6a}$$

$$\det[\boldsymbol{\Lambda}(s)]D_s(s)D_a(s)=D(s)C(s), \tag{6b}$$

where $C(s)$ is a common divisor, and $N(s)$ and $D(s)$ are coprime factors for the numerator and denominator of Eq. (5a).

Equation (6) enables one to express Eq. (5a) as

$$\nu(s)=\frac{N(s)}{D(s)}u(s)+\frac{\boldsymbol{\epsilon}(s)}{D(s)C(s)}.$$

The coprime pair $N(s)$ and $D(s)$ makes a controllable feedback loop transfer function,[14] which is identified when $\boldsymbol{\epsilon}=0$ and denoted by

$$\frac{\nu(s)}{u(s)}=\frac{N(s)}{D(s)}=T(s).$$

Common divisor $C(s)$, according to control theory,[14] contains poles of uncontrollable modes to be "canceled out" in $T(s)$. It is important to know which modes are controllable by $T(s)$ in order to select sensor–actuator locations. If the roots of $D(s)$ are distinct, then the partial fraction expansion theory[15] implies

$$T(s)=\frac{N(s)}{D(s)}=M(s)+\sum_{k=1}^{n}\frac{c_k}{s^2+\omega_k^2}, \tag{7a}$$

where $M(s)$ contains terms contributed by poles of actuator–sensor filters and ω_k is the kth resonant frequency of the controllable modes. If $D(s)$ has a pair of second-order roots $\pm j\omega_n$, then one has another partial fraction expansion

$$T(s)=\frac{N(s)}{D(s)}=M(s)+\sum_{k=1}^{n-1}\frac{c_k}{s^2+\omega_k^2}+\frac{sc_{n1}+c_{n2}}{(s^2+\omega_n^2)^2}. \quad (7b)$$

Similar expansions can be derived for the cases where $D(s)$ has one or more high-order roots. In any events, the right-hand side of Eq. (7) contains resonant peaks of controllable modes. These peaks are visible via an off-line fast Fourier transform analyzer connected to both ends of the feedback loop. A count of the resonant peaks helps one to identify the modes controllable by a specific loop from the actuator to the sensor.

For active control of a VF, controllability depends on the actuator location while observability depends on the sensor location. Both GSCs and LSCs face the problem of selecting actuator and sensor locations. A controllability or observability check of a GSC requires exact model parameters. For a LSC, such a check is much easier, thanks to the hint of Eq. (7). The first step is to use a probing device to select the feedback sensor location, where the sensor should be able to measure a selected or maximum number of resonant peaks excited by the disturbance. Empirically, this is a spot in a VF where vibration or noise is most significant. The actuator should then collocate with the feedback sensor wherever possible. Otherwise, one should select an actuator location near the feedback sensor such that it excites a selected or maximum number of resonant peaks measurable by the feedback sensor. This is possible by trial and error without knowing exact model parameters of the VF.

Physically, the forming of a resonance depends on two conditions: (1) there exists a path for the vibration energy to bounce in a VF; and (2) the phase condition of a round trip along an energy path is in favor of energy accumulation at a resonant frequency. Vibration energy distribution along the bouncing paths corresponds to mode functions of the modal theory. The above-mentioned experimental procedure enables a control engineer to place the actuator and feedback sensor in the intersection of energy paths for multiple resonant peaks. This is physically equivalent to a controllability check for a GSC that requires model parameters. If a LSC absorbs vibration energy at these resonant frequencies in the selected location, it will weaken the resonant conditions of the corresponding energy paths. This implies global damping to the VF. Section III will explain analytically how this is possible.

III. LSC DESIGN AND ANALYSIS

The pole-placement control strategy may be applied to design the LSC. A prototype polynomial $P(s)$ is used to prescribe the closed-loop poles. The control law is given by

$$u=\frac{R(s)}{S(s)}\nu, \quad (8)$$

where polynomials $R(s)$ and $S(s)$ are solved from a Bezout equation[14]

$$P(s)=D(s)S(s)+N(s)R(s). \quad (9a)$$

One may denote $P(s)=p_0s^{4n-3}+p_1s^{4n-4}\cdots+p_{4n-4}s+p_{4n-3}$ and apply similar notations to $D(s)$, $S(s)$, $N(s)$, and $R(s)$ whose degrees are $2n$, $2n-3$, $2n-2$, and $2n-1$, respectively. Then Eq. (9a) has a matrix-vector expression

$$\begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_{4n-4} \\ p_{4n-3} \end{bmatrix} = \underbrace{\begin{bmatrix} d_0 & \cdots & 0 \\ \vdots & \ddots & 0 \\ d_{2n} & \vdots & d_0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & d_{2n} \end{bmatrix}}_{2n-2}\!\!\!\underbrace{\begin{matrix} n_0 & \cdots & 0 \\ \vdots & \ddots & 0 \\ n_{2n-2} & \vdots & n_0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & n_{2n-2} \end{matrix}}_{2n} \begin{bmatrix} s_0 \\ \vdots \\ s_{2n-3} \\ r_0 \\ \vdots \\ r_{2n-1} \end{bmatrix} \quad (9b)$$

where the matrix, known as the Sylvester resultant,[14] is nonsingular since $N(s)$ and $D(s)$ are coprime. Coefficients of $S(s)$ and $R(s)$ can be solved from Eq. (9b) for the controller transfer function (8). Substituting Eqs. (8) and (9a) into Eq. (5a), one can express the closed-loop SISO system as

$$\nu=\frac{S(s)}{C(s)P(s)}\epsilon,$$

where the prototype polynomial $P(s)$ becomes a part of the closed-loop denominator. Its roots are placed as closed-loop poles of the local SISO system.

The irreducible part of Eq. (6b) may be expressed as $D(s)=D_a(s)D_s(s)\Pi_k(s^2+\omega_k^2)$, where $D_s(s)$ and $D_a(s)$ are the filter denominators and $\Pi_k(s^2+\omega_k^2)$ represents the resonant poles. One may use $P(s)=D_a(s)D_s(s)\Pi_k(s^2+\zeta_k s+\omega_k^2)$ to damp the resonant peaks and keep the filter denominators $D_s(s)$ and $D_a(s)$ in the closed loop. In view of Eq. (5b), one may write

$$\nu=\frac{S(z)\det[\mathbf{\Lambda}_1(s),\boldsymbol{\eta}]}{C(s)\Pi_k(s^2+\zeta_k s+\omega_k^2)}F_s(s).$$

Although the roots of $C(s)$, which correspond to uncontrollable modes, remain poles of the closed-loop SISO system, these components will be blocked by a filter $F_s(s)$—the numerator $N_s(s)$ in Eq. (5b) will combine with the denominator $D_s(s)$ in $P(s)$ to form $F_s(s)$ and remove any signals in its stop band.

The LSC design does not require exact model parameters. Only the numerator and denominator of Eq. (5) are involved that are available by identification. While actuator and sensor dynamics are taken into explicit account in terms of $N_s(s)$, $N_a(s)$, $D_s(s)$, and $D_a(s)$, these are hidden in the numerator $N(s)$ and denominator $D(s)$ of the local transfer function. There is no restriction on the types of the sensor and actuator filters, as long as they are represented by stable transfer functions whose exact forms need not be known.

Attention is now directed to the global effect of the LSC. One may wonder how the LSC introduces global damping to a VF. Consider, for example, a duct with cross section much smaller than the wavelengths of the noise source. This is a one-dimensional noise field. Its resonant is due to an impedance mismatch in both ends, which allows acoustic energy to bounce around and accumulate. If the impedance of either end of the duct matches the characteristic imped ance of the duct, then the resonant peaks can be attenuated. This means global damping is possible by a local (passive) control.

The LSC has been shown able to damp the resonant peaks at a local point. Since this is the intersection of multiple energy paths, local damping weakens the resonant conditions and prevents global energy accumulation. Its global effect is investigated here analytically. In view of $f = F_a(s)u$ and $v = F_s(s)y$, Eq. (8) implies

$$f = -G(s)y,$$

where

$$G(s) = \frac{R(s)N_s(s)N_a(s)}{S(s)D_s(s)D_a(s)}. \tag{10}$$

The actuator and sensor dynamics have been included in the controller transfer function $G(s)$ explicitly. Substituting Eq. (10) into Eq. (3), one obtains

$$\mathbf{\Lambda}^*(s)\mathbf{x} = [\mathbf{\Lambda}(s) + hG(s)\boldsymbol{\beta}\mathbf{e}_1^T]\mathbf{x} = \boldsymbol{\eta}, \tag{11}$$

where $y = \mathbf{h}^T\mathbf{q} = hx_1 = h\mathbf{e}_1{}^T\mathbf{x}$ has been substituted. The modification introduced by $hG(s)\boldsymbol{\beta}\mathbf{e}_1^T$ only changes the first column of $\mathbf{\Lambda}(s)$. Hence

$$\mathbf{\Lambda}^*(s) = \begin{bmatrix} \alpha_{11}(s) + hG(s)\beta_1 & \alpha_{12}(s) & \cdots & \alpha_{1m}(s) \\ \alpha_{21}(s) + hG(s)\beta_2 & \alpha_{22}(s) & \cdots & \alpha_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1}(s) + hG(s)\beta_m & \alpha_{m2}(s) & \cdots & \alpha_{mm}(s) \end{bmatrix}, \tag{12}$$

where β_k is the kth element of vector $\boldsymbol{\beta}$.

The global effect of the LSC is reflected in det$[\mathbf{\Lambda}^*(s)]$, which is the characteristic equation of the closed-loop VF. One may express det$[\mathbf{\Lambda}^*(s)]$ with respect to the first column of $\mathbf{\Lambda}^*(s)$. It reads

$$\det[\mathbf{\Lambda}^*(s)] = \sum_{k=1}^{m} (-1)^{k+1}[\alpha_{k1}(s) + hG(s)\beta_k]\det[\mathbf{\Lambda}_{1k}(s)],$$

where $\mathbf{\Lambda}_{1k}(s)$ is a matrix obtained by removing the first column and the kth row from either $\mathbf{\Lambda}(s)$ or $\mathbf{\Lambda}^*(s)$. The above-given expression suggests

$$\begin{aligned} \det[\mathbf{\Lambda}^*(s)] = \det[\mathbf{\Lambda}(s)] + hG(s)\det[\mathbf{\Lambda}_1(s), \boldsymbol{\beta}] &= \frac{\det[\mathbf{\Lambda}(s)]D_s(s)D_a(s)S(s) + h\det[\mathbf{\Lambda}_1(s), \boldsymbol{\beta}]N_s(s)N_a(s)R(s)}{D_s(s)D_a(s)S(s)} \\ &= \frac{C(s)[D(s)S(s) + N(s)R(s)]}{D_s(s)D_a(s)S(s)} = \frac{C(s)P(s)}{D_s(s)D_a(s)S(s)}, \end{aligned} \tag{13}$$

where Eqs. (10), (6), and (9a) have been substituted sequentially in deriving the last equation.

One may understand the global damping of the LSC by comparing det$[\mathbf{\Lambda}(s)]$ and det$[\mathbf{\Lambda}^*(s)]$. These are, respectively, open- and closed-loop characteristic equations of the VF. Roots of these polynomials are eigenvalues of the VF before and after the application of the LSC. Therefore damping ratios of det$[\mathbf{\Gamma}(s)]$ and det$[\mathbf{\Gamma}^*(s)]$ are, respectively, damping ratios of the VF before and after the application of the LSC. It is not difficult to see, from Eqs. (6b) and (13), that

$$\det[\mathbf{\Lambda}(s)] = \frac{C(s)D(s)}{D_s(s)D_a(s)},$$

$$\det[\mathbf{\Lambda}^*(s)] = \frac{C(s)P(s)}{D_s(s)D_a(s)S(s)}.$$

Roots of $P(s)$ replace roots of $D(s)$ to be the closed-loop eigenvalues of the VF. With $P(s)$ preselectable, a control engineer may select the damping ratio of $P(s)$ to tune the global damping ratio for the closed-loop VF. This is, of course, limited to the controllable modes. Roots of the common divisor $C(s)$ correspond to those modes uncontrollable by $T(s)$. Damping ratios of $C(s)$ remain unchanged due to the physical constraint of actuator–sensor locations.

An analytical implication of Eq. (13) is that the LSC introduces global damping to the controlled modes without affecting the uncontrolled modes. It manages to avoid spillover by taking account of $N_s(s)$, $N_a(s)$, $D_s(s)$, and $D_a(s)$ in its local design stage. To the best knowledge of the author, this is the first attempt to investigate both controlled and uncontrolled modes analytically and realistically in a controller design for a distributed system. It is different from a GSC design where the uncontrolled modes are usually ignored in calculating the GSC gain matrix.[4,6–8]

Other LSCs do not necessarily damp the VF globally. Take the high-gain LSC for example, its modal models are $(s^2\mathbf{I} + \mathbf{K})\mathbf{q} = \mathbf{d}$ and $(s^2\mathbf{I} + \mathbf{K} + g\mathbf{b}\mathbf{h}^T)\mathbf{q} = \mathbf{d}$, respectively, for the open- and closed-loop of a VF when the open-loop damping is negligible. Here $\mathbf{K}$ is diagonal with elements $\{\omega_i^2\}$; g is the LSC gain. In order for the closed loop to be stable, $\mathbf{K} + g\mathbf{b}\mathbf{h}^T$ must be positive definite, which is true for collocated sensor–actuator ($\mathbf{h} = \mathbf{b}$) but not necessarily true otherwise. This is consistent with the conclusion of the local transfer function approach.[12] Let $\{\lambda_i\}$ be the eigenvalues of $\mathbf{K} + g\mathbf{b}\mathbf{h}^T$, then it is not difficult to see det$(s^2\mathbf{I} + \mathbf{K} + g\mathbf{b}\mathbf{h}^T) = \Pi(s^2 + \lambda_i)$. Comparing with det$(s^2\mathbf{I} + \mathbf{K}) = \Pi(s^2 + \omega_i^2)$, one can see that a high-gain LSC does not damp the VF, though it suppresses the sensed signals[12] in a local feedback loop.

The analysis is based on the continuous model (1) in the Laplace transform domain. Practically, Eq. (5) is usually identified as a discrete-time model in the Z-transform domain.[14] A strictly rigorous approach would be converting the VF model to its discrete-time version and reaching the same conclusion. That may introduce unnecessary distractions since the discrete-time model is slightly more complicated and the conversion may introduce new symbols. Since the continuous-time model is physically and mathematically equivalent to its discrete-time version and the conclusion is the same, using the continuous model may keep a better focus here. Practically, it is recommended to use the discrete-time model to design the LSC. When the number of modes to be damped is large and the resonant frequencies span a wide range, the roots of $N(s)$ and $D(s)$ would spread along the imaginary axis in a wide range if one works with a continuous model. With a discrete-time model, however, the imaginary axis of the s plane is mapped to the unit circle in the z plane. The roots of $N(z)$ and $D(z)$ should be near the unit circle. As a result, the condition number of the Sylvester resultant can be improved and numerical errors can be reduced.

IV. DECENTRALIZED DESIGN OF LSCs

A VF may need active damping by multiple actuators mounted in different locations, probably because several modes share the same resonant frequency or the actuators have limited power. Cross coupling of the actuation signals is inevitable, which causes difficulties in controller design. For active control of a VF by multiple actuators, currently available methods depend on the exact model parameters to deal with the problem of cross coupling between actuators and feedback sensors. These are centralized design methods. This paper presents a simple decentralized design procedure for active damping of a VF by multiple actuators.

The idea is to add a LSC to a VF that has been controlled by a stable active controller. The existing controller may be designed with any available schemes in the literature, including the LSC scheme presented earlier. The dynamic model of the VF, subject to the existing controller and the additional LSC, may be expressed as

$$[\mathbf{M}s^2+\mathbf{K}+\mathbf{G}(s)]\mathbf{q}=\mathbf{b}f+\mathbf{d}, \tag{14}$$

where $\mathbf{G}(s)$ represents the gain of the existing controller. Other symbols remain the same as in Eq. (1). Since the existing controller may be designed by a scheme different from LSC, $\mathbf{G}(s)$ is not necessarily available. However, it is important to assume that the existing controller is *stable* such that roots of $\det[\mathbf{M}s^2+\mathbf{K}+\mathbf{G}(s)]$ are all in the negative half of the s plane.

The normalized version of $\mathbf{h}$ is still denoted as $\mathbf{h_n}$. Its null space is still spanned by $m-1$ unit length and orthogonal basis $\{\boldsymbol{\zeta}_1\cdots\boldsymbol{\zeta}_{m-1}\}$ with $\mathbf{h}^T\boldsymbol{\zeta}_i=0$ for $1\leqslant i\leqslant m-1$. The existence of these vectors implies the existence of an orthogonal matrix $\mathbf{H}=[\mathbf{h}_n\boldsymbol{\zeta}_1\cdots\boldsymbol{\zeta}_{m-1}]$, such that $\mathbf{x}=\mathbf{H}^T\mathbf{q}$ and $\Gamma(s)=\mathbf{H}^T[\mathbf{M}s^2+\mathbf{K}+\mathbf{G}(s)]\mathbf{H}$. Therefore Eq. (14) is equivalent to

$$\boldsymbol{\Gamma}(s)\mathbf{x}=\boldsymbol{\beta}f+\boldsymbol{\eta}, \tag{15}$$

where $\boldsymbol{\beta}=\mathbf{H}^T\mathbf{b}$ and $\boldsymbol{\eta}=\mathbf{H}^T\mathbf{d}$ remains the same as in Eq. (3). The feedback signal may be expressed as

$$y=hx_1=h\,\frac{\det[\boldsymbol{\Gamma}_1(s),\boldsymbol{\beta}]f+\det[\boldsymbol{\Gamma}_1(s),\boldsymbol{\eta}]}{\det[\boldsymbol{\Gamma}(s)]}, \tag{16}$$

where $[\boldsymbol{\Gamma}_1(s),\boldsymbol{\beta}]$ and $[\boldsymbol{\Gamma}_1(s),\boldsymbol{\eta}]$ are matrices obtained by replacing the first column of $\boldsymbol{\Gamma}(s)$ with vectors $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$, respectively. Parameters of Eqs. (14)–(16) are not necessarily available.

The LSC design procedure, presented in Sec. II, may be repeated here. A control engineer uses a probing signal $f=F_a(s)u$ to excite the VF while the feedback sensor measures $v=F_s(s)y$, which will be used to identify the transfer function. Only this time, the existing controller must be *turned on* so that a local plant model

$$\nu(s)=\frac{h\det[\boldsymbol{\Gamma}_1(s),\boldsymbol{\beta}]N_s(s)N_a(s)u(s)+\boldsymbol{\epsilon}}{\det[\boldsymbol{\Gamma}(s)]D_s(s)D_a(s)} \tag{17}$$

can be identified from measurement data. Although Eq. (17) looks similar to Eq. (5), it is different because potential coupling effects of the existing controller are identified by Eq. (17). The additional LSC depends on Eq. (17) to avoid cross coupling with the existing controller. While identifying Eq. (17), a control engineer may adjust the actuator and sensor locations to excite and measure selected modes. These should be modes not controlled by the existing controller.

The numerator and denominator of Eq. (17) may share some common roots. Therefore one should express

$$h\det[\boldsymbol{\Gamma}_1(s),\boldsymbol{\beta}]N_s(s)N_a(s)=N_e(s)C_e(s), \tag{18a}$$

$$\det[\boldsymbol{\Gamma}(s)]D_s(s)D_a(s)=D_e(s)C_e(s), \tag{18b}$$

where the roots of common divisor $C_e(s)$ correspond to vibration modes not controllable or observable by the additional LSC. $N_e(s)$ and $D_e(s)$ are coprime polynomials. The control law of the additional LSC is given by

$$u=-\frac{R_e(s)}{S_e(s)}\,\nu, \tag{19}$$

where polynomials $R_e(s)$ and $S_e(s)$ are solved from a different Bezout equation

$$P_e(s)=D_e(s)S_e(s)+N_e(s)R_e(s), \tag{20}$$

where $P_e(s)$ is a prototype polynomial assigned to the additional LSC. Substituting Eqs. (19) and (20) into Eq. (17), one can express the closed-loop local system as

$$\nu=\frac{S_e(s)}{C_e(s)P_e(s)}\,\epsilon,$$

where $P_e(s)$ becomes a part of the closed-loop denominator. Similar to the arguments of Sec. III, Eq. (19) implies

$$f=-G_e(s)y,$$

where

$$G_e(s)=\frac{R_e(s)N_s(s)N_a(s)}{S_e(s)D_s(s)D_a(s)}. \tag{21}$$

Substituting Eq. (21) into Eq. (15), one obtains

$$\boldsymbol{\Gamma}^*(s)\mathbf{x}=[\boldsymbol{\Gamma}(s)+hG_e(s)\boldsymbol{\beta}e_1^T]\mathbf{x}=\boldsymbol{\eta},$$

where

$$\mathbf{\Gamma}^*(s)=\begin{bmatrix}\gamma_{11}(s)+hG(s)\beta_1 & \gamma_{12}(s) & \cdots & \gamma_{1m}(s)\\ \gamma_{21}(s)+hG(s)\beta_2 & \gamma_{22}(s) & \cdots & \gamma_{2m}(s)\\ \vdots & \vdots & \ddots & \vdots\\ \gamma_{m1}(s)+hG(s)\beta_m & \gamma_{m2}(s) & \cdots & \gamma_{mm}(s)\end{bmatrix}.$$

One may derive, similar to the derivation of Sec. III,

$$\det[\mathbf{\Gamma}^*(s)]=\det[\mathbf{\Gamma}(s)]+hG(s)\det[\mathbf{\Gamma}_1(s),\boldsymbol{\beta}]$$

$$=\frac{C_e(s)P_e(s)}{D_s(s)D_a(s)S_e(s)},$$

where $P_e(s)$ has been placed as a part of the closed-loop characteristic equation. This implies global damping to the selected modes not controlled by the existing controller.

The above mentioned derivation can be generalized to the case where $\mathbf{G}(s)$ represents the feedback gains of several existing controllers. This makes it possible to add LSCs to a VF one by one for active damping using multiple actuators. Each time a new LSC is added to the VF, its actuator and feedback sensor location should be chosen in such a way that the local plant is controllable and observable with respect to a set of modes uncontrolled by the previous controllers.

V. A NUMERICAL EXAMPLE

A numerical example is presented here to demonstrate how global damping is possible by a LSC. The VF is a one-dimensional cantilever beam satisfying the Bernoulli–Euler equation

$$EI\frac{\partial^4 w}{\partial x^4}+\rho\frac{\partial^2 w}{\partial t^2}=d(x,t)+f(x,t),$$

where w is the vibration displacement; x and t are spatial and temporal variables; ρ is the mass density; E is the Young's modulus of elasticity, and I the moment of inertia of the cross-sectional area with respect to the neutral axis. The disturbance and the LSC control forces are denoted by $d(x,t)$ and $f(x,t)$, respectively. The eigenfunctions of the beam are given by

$$\varphi_i(x)=[\cosh(\lambda_i+x)-\cos(\lambda_i x)]$$
$$-\mu_i[\sinh(\lambda_i x)-\sin(\lambda_i x)],$$

where

$$\mu_i=\frac{\cos(\lambda_i L)+\cosh(\lambda_i L)}{\sin(\lambda_i L)+\sinh(\lambda_i L)}$$

depends on λ_i— the ith root of $\cos(\lambda_i L)\cosh(\lambda_i L)+1=0$. The eigenvalue λ_i also determines the ith mode frequency $\omega_i=(\lambda_i L)^2\sqrt{EI/\rho L^4}$, where $L=1$ and $\sqrt{EI/\rho L^4}=0.05$ are assumed. Such a choice does not lose generality since implementation of the LSC is carried out with a discrete-time model in the Z-transform domain. One can always select the sampling frequency such that the resonant frequencies of the modes are properly spaced in the unit circle. This demonstration uses a continuous model just to be consistent with the previous sections.

Theoretically the LSC is able to damp as many modes as the actuator and feedback sensor can excite and observe. Practically, however, most active control devices are applied to the low frequency ranges because high-order modes are absorbed more effectively with inexpensive passive methods. For this reason, the model is truncated to the first five modes with $\omega_1=0.7706$, $\omega_2=2.4982$, $\omega_3=5.2124$, $\omega_4=8.9135$ and $\omega_5=13.6015$ (rad/s). If fitted into Eq. (1), these parameters make $\mathbf{K}$ diagonal, $\mathbf{M}$ identity, and $\det(\mathbf{M}s^2+\mathbf{K})=\Pi_{k=1}^5(s^2+\omega_k^2)$. The actuator and feedback sensor are collocated near the middle of the beam such that $f(x,t)=\delta(x-0.58L)f(t)$, or $\mathbf{b}=\mathbf{h}=[\varphi_1(0.58),\ldots,\varphi_5(0.58)]^T$. A local transfer function is derivable as $T(s)=N(s)/D(s)=\Sigma_{k=1}^5\varphi_k^2(0.58)/s^2+\omega_k^2$.

A prototype polynomial $P(s)=(s+2)^6\Pi_{k=1}^5(s^2+0.08\omega_k s+\omega_k^2)$ is used to prescribe closed-loop poles for the local system, which are also closed-loop poles of the entire beam, as to be demonstrated in this example. The Sylvester resultant, defined in Eq. (9b), may be numerically ill conditioned when the resonant frequencies spread in a wide range. For this reason, a real multiple pole $(s+2)^6$ is added to $P(s)$ to reduces possible numerical errors in solving $P(s)=D(s)S(s)+N(s)R(s)$ for the LSC gain

$$G(s)=\frac{R(s)}{S(s)}.$$

If one designs the LSC with a discrete-time model, however, the real pole may not be necessary because the condition number for the Sylvester resultant will be much better.

In order to see the global damping effect, a monitor sensor could be placed anywhere along the beam to measure the vibration spectrum. A possible location is $x_m=0.98L$—near the free end of the cantilever beam. The disturbance is acting at $x_d=0.78L$, which is physically between the LSC actuator and the monitor sensor. The location vectors are $\mathbf{m}=[\varphi_1(0.98),\ldots,\varphi_5(0.98)]^T$, and $\mathbf{d}=[\varphi_1(0.78),\ldots,\varphi_5(0.78)]^T$ respectively. Let $y_m(s)$ denote the vibration signal of the monitor sensor and assume a broadband random disturbance source $d(s)=1$, then the spectra of $y_m(s)$ are given by

$$y_m(s)=\mathbf{m}^T[\mathbf{M}s^2+\mathbf{K}+\mathbf{b}\mathbf{b}^TG(s)]^{-1}\mathbf{d},$$

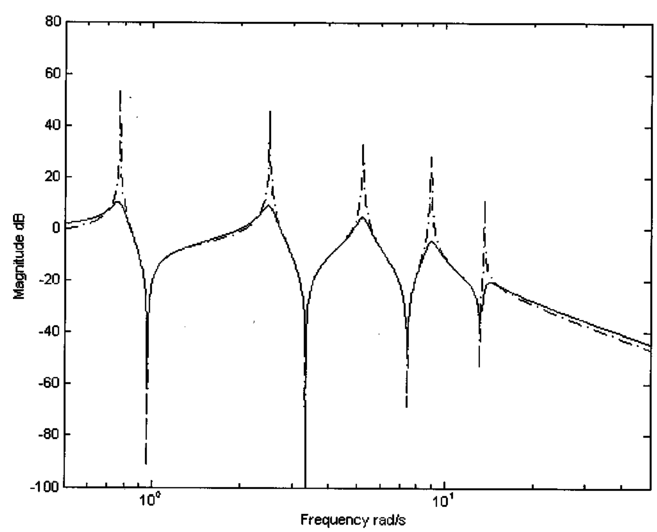

FIG. 1. Vibration spectra with (solid line) and without (dash-dot line) LSC damping, measured near the free end of the beam.

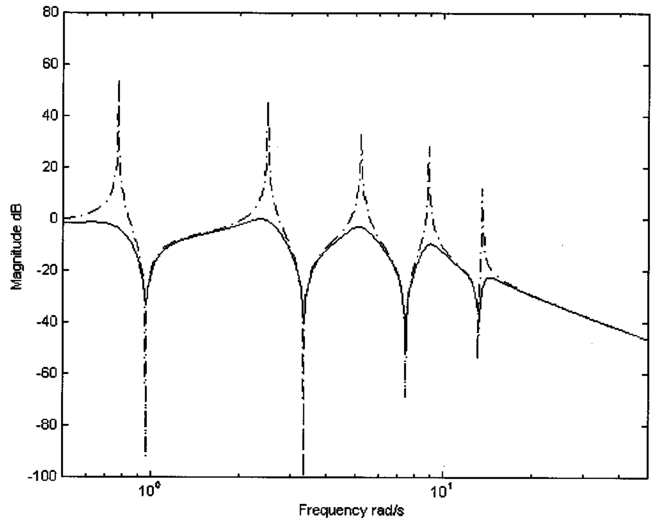

FIG. 2. Vibration spectra with (solid line) and without (dash-dot line) GSC damping, measured near the free end of the beam.

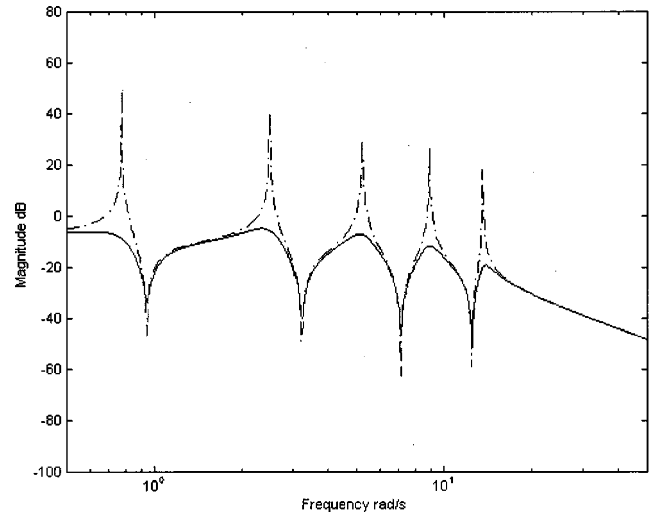

FIG. 4. Vibration spectra with (solid line) and without (dash-dot line) GSC damping, measured near the middle of the beam.

$$y_m(s)=\mathbf{m}^T[\mathbf{M}s^2+\mathbf{K}]^{-1}\mathbf{d}$$

for the cases with and without LSC damping, respectively. These spectra are plotted in Fig. 1 for comparison. Although the LSC is designed for a local system, its damping effect on the monitor signal, measured near the free end of the beam, is obvious. For the same beam, a linear quadratic optimal GSC is designed using the exact modal parameters. Figure 2 plots the global damping effects of the GSC. Generally, the performance of a GSC is better than that of a LSC at the expense of exact modal parameters. A GSC is always the first choice if exact modal parameters are available. The LSC is recommended if these parameters are not available, or not accurate enough, for VFs with irregular geometric shapes and boundary conditions.

One may select other locations for the monitor sensor. This means different choices of $\mathbf{m}=[\varphi_1(x_m),\ldots,\varphi_5(x_m)]^T$. If substituted into the above-given equations, similar damping effects can be obtained as shown in Figs. 3 and 4. Altering the locations of the disturbance source and the monitor sensor changes the values of location vectors **m** and **d**, respectively. It does not affect the global damping effects since the eigenstructure of $\mathbf{M}s^2+\mathbf{K}+\mathbf{b}\mathbf{b}^TG(s)$ is determined by the LSC gain $G(s)$ and the location vector **b** (for the actuator and feedback sensor).

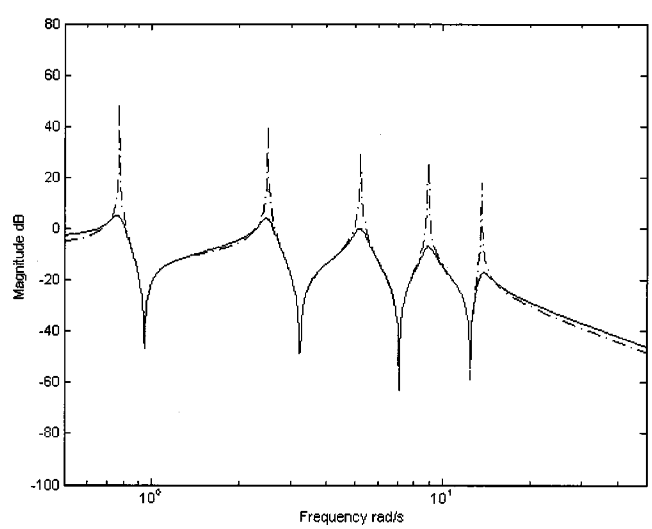

FIG. 3. Vibration spectra with (solid line) and without (dash-dot line) LSC damping, measured near the middle of the beam.

VI. CONCLUSION

This study investigates, analytically and numerically, the global damping of a VF by means of a single LSC. A positive conclusion has been reached with an easy procedure for the LSC design and implementation. The LSC has many positive features. It does not require detailed dynamic model of the VF. One may use available system identification techniques to estimate a local transfer function, check its controllability, and design the LSC. Unlike other available controllers that ignore uncontrolled modes when computing the feedback gains, the LSC design deals with both controlled and uncontrolled modes plus sensor and actuator dynamics. The design procedure allows one to design and implement, when necessary, multiple LSCs to damp the resonant peaks of a VF selectively.

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[1] L. Meirovitch, *Dynamics and Control of Structures* (Wiley, New York, 1990).

[2] J. S. Vipperman and R. L. Clark, "Multivariable feedback active structural acoustic control using adaptive piezoelectric sensoriactuators," J. Acoust. Soc. Am. **105**, 219–225 (1999).

[3] S. Griffin, C. Hansen, and B. Cazzolato, "Feedback control of structurally radiated sound into enclosed spaces using structural sensing," J. Acoust. Soc. Am. **106**, 2621–2628 (1999).

[4] K. Seto, M. Ren, and F. Doi, "Feedback vibration control of a flexible plate at audio frequencies by using a physical state-space approach," J. Acoust. Soc. Am. **103**, 924–934 (1998).

[5] R. E. Lindberg and R. W. Longman, "On the number and placement of

actuators for independent modal shape control,'' J. Guid. Control Dyn. **2**, 215–221 (1984).
[6] N. Rizet, M. Brissaud, P. Gonnard, J.-C. Bera, and M. Sunyach, ''Modal control of beam flexural vibration,'' J. Acoust. Soc. Am. **107**, 2061–2067 (2000).
[7] K. Seto, M. Ren, and F. Doi, ''Modeling and feedback structural acoustic control of a flexible plate,'' Trans. ASME, J. Vib. Acoust. **123**, 18–23 (2001).
[8] T. Samejima, ''Modifying modal characteristics of sound fields by state feedback control,'' J. Acoust. Soc. Am. **110**, 1408–1414 (2001).
[9] H. T. Banks, Z.-H. Luo, L. A. Bergman, and D. J. Inman, ''On the existence of normal modes of damped discrete-continuous systems,'' ASME Trans. J. Appl. Mech. **65**, 980–989 (1998).
[10] H. F. Olson and May, ''Electronic sound absorber,'' J. Acoust. Soc. Am. **25**, 1130–1136 (1953).
[11] R. L. Clark and D. G. Cole, ''Active damping of enclosed sound fields through direct rate feedback,'' J. Acoust. Soc. Am. **97**, 1710–1716 (1995).
[12] R. L. Clark and G. P. Gibbs, ''Analysis, testing and control of a reverberant sound field within the fuselage of a business jet,'' J. Acoust. Soc. Am. **105**, 2277–2286 (1999).
[13] J. Yuan, ''Improving an adaptive controller for non-minimum phase plants,'' Automatica (to be published).
[14] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction and Control* (Prentice–Hall, Englewood Cliffs, NJ, 1984).
[15] K. Ogata, *Modern Control Engineering* (Prentice–Hall, Englewood Cliffs, NJ, 1997).