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Improving robustness of active noise control in ducts

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A robust active noise controller (ANC) is proposed here for finite ducts. While the $\mathbf{H}_\infty$ control theory provides theoretical ground and numerical algorithms to design robust controllers, it is important for an engineer to design and formulate a robust controller so that the objective is more achievable and the $\mathbf{H}_\infty$ constraints less restrictive without sacrificing robustness. A new robust ANC is designed this way with an extra actuator to improve achievable performance and introduce more degrees of freedom to controller parameters. The new strategy relaxes $\mathbf{H}_\infty$ constraints without sacrificing robustness and enables the ANC to tolerate a wide variety of errors and uncertainties including truncation errors between a finite model and an infinite field. Theoretical analysis, numerical examples, and experimental results are presented to demonstrate the improved performance of the proposed ANC when subject to a certain level of uncertainties in a duct. [DOI: 10.1121/1.1579005]

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I. INTRODUCTION

ANC has been proven by many researchers as a viable method[1,2] for noise suppression in low frequency ranges where traditional passive noise control devices become massy, bulky, or less effective. Many ANC schemes synthesize feedforward signals to suppress noise at measured locations. A feedforward ANC usually depends on an accurate secondary path model to adjust its control transfer function.[3–5] Feedback ANC schemes, if properly designed, are able to introduce active damping to sound fields.[6] Hybrid feedforward and feedback ANC is an active research topic.[7–9] Most of the ANC schemes are based on an assumption that acoustical paths are linear time invariant.

In reality, acoustical paths, while linear, are not necessarily time invariant. Online variance of acoustical paths could affect stability of an otherwise stable ANC. Design of robust ANC schemes attracts attentions of many researchers.[10–12] A common practice of robust ANC design is to fit an ANC scheme into the standard four-port framework of robust control theory[13,14] and solve a robust controller subject to a set of $\mathbf{H}_\infty$ constraints. The $\mathbf{H}_\infty$ constraints for robust ANC's are commonly in terms of error bounds of transfer functions, with more weighting outside the control band than the in-band region.[12]

Since parameters of $\mathbf{H}_\infty$ controllers are usually numerical solutions of objective functions subject to certain constraints,[15,16] it is difficult to predict the performance of a robust controller when one designs the structure and formulates the control objective and $\mathbf{H}_\infty$ constraints. Some general design principles would be (1) specifying a control objective as achievable as possible; (2) allowing more DOF in controller parameters; and (3) relaxing, wherever possible, the $\mathbf{H}_\infty$ constraints without sacrificing robustness. This article will show that there is room to improve a robust ANC by adopting the above principles.

The first improvement is related to nonminimum phase (NMP) transfer functions from actuators to usually uncollocated sensors in ANC applications. These transfer functions do not have stable inverses. The use of pseudoinverses prevents "perfect" ANC cancellation. It may complicate the "waterbed effect" in some robust feedback ANC.[17] While there is no complete solution to the NMP problem for conventional ANC structures, perfect ANC cancellation is analytically possible by an extra actuator.[18] More importantly, the extra actuator introduces more DOF to improve robustness as to be shown here. This is a new feature of the proposed robust ANC and it does improve performance as verified by experiment and numerical results.

It is well known that transfer functions in lightly damped enclosures are rational with numerator and denominator polynomials. There are many sophisticated algorithms identifying rational transfer functions with solid bounds on the parameter errors.[19] For the same level of uncertainties in a lightly damped finite duct, it can be shown, analytically and experimentally, that the $\mathbf{H}_\infty$ constraints are less restrictive if expressed in terms of error bounds of numerators and denominators instead of error bounds of transfer functions. A new formulation is derived for robust ANC in finite ducts. The resultant ANC has an improved experimental performance. This is another improvement over existing robust ANC schemes.

For a hybrid ANC, it is important to design a finite dimensional feedback part to control an infinite dimensional acoustical field without spillover. There are important results on spillover effects and truncation errors.[20–22] This study improves the available results by including actuator/sensor dynamics in the model and tolerating other errors, such as online variance of path parameters. The new ANC achieves this objective using error polynomials to represent a wide variety of uncertainties whose effects may be within the frequency range of interest. This is a more practical method for a control engineer with little background in acoustics to design

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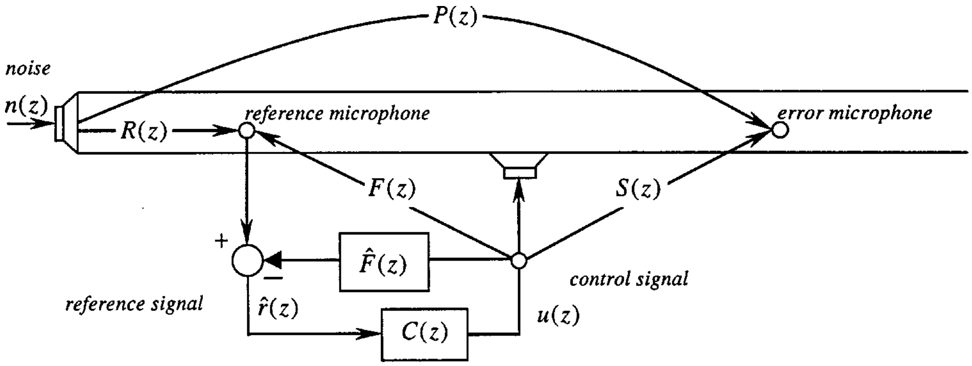

FIG. 1. A typical feedforward ANC system.

ANC. Its advantages are explained here with experimental and numerical verifications.

II. ANALYTICAL GROUNDS OF THE PROPOSED ANC

Figure 1 plots a typical feedforward ANC system in a finite duct, with a primary source, a secondary source, an upstream reference sensor, and a downstream error sensor. The acoustical paths are modeled by $P(z)$, $R(z)$, $F(z)$, and $S(z)$ for the primary, reference, acoustical feedback, and secondary paths, respectively. The reference signal $r(z)$ is mixed with acoustical feedback when the ANC is active. A common way to cancel acoustical feedback is to subtract $\hat{F}(z)u(z)$ from the measured signal, where $\hat{F}(z)$ is an identified version of $F(z)$. As the result, one can express the error signal $e(z)$ as

$$e(z)=\left[P(z)+\frac{S(z)C(z)R(z)}{1-\Delta F(z)C(z)}\right]n(z), \tag{1}$$

where $\Delta F(z)=F(z)-\hat{F}(z)$ is the model error of acoustical feedback path. The ANC system is stable if $\|\Delta F(z)C(z)\|_\infty<1$ by the small game theory. A typical approach for robust ANC design[10–12] is to fit $\hat{P}(z)$, $\hat{R}(z)$, and $\hat{S}(z)$ into the four-port framework of the $\mathbf{H}_\infty$ control theory, and solve for a stable $C(z)$ that minimizes $\|\hat{P}(z)+\hat{S}(z)C(z)\hat{R}(z)\|$ subject to $\|\Delta_F C(z)\|_\infty<1$ if the model error $\Delta F(z)$ could be bounded by Δ_F.

A problem with the above approach is the NMP distortion caused by transfer functions from actuators to usually uncollocated sensors in ANC applications. This means an unstable $S^{-1}(z)$ and makes it impossible to solve $\|P(z)+S(z)C(z)R(z)\|=0$ with a stable $C(z)$ even without any constraints. If subject to $\|\Delta_F C(z)\|_\infty<1$, $\|P(z)+S(z)C(z)R(z)\|$ is definitely non-zero and no smaller than what it could be without constraints. It means a sacrifice of performance for robustness when the best achievable performance is an uncertainty in the first place.

It is desirable to improve robustness and performance simultaneously. A possible way is to modify the ANC structure by adding an extra secondary source. According to the modal theory, path transfer functions in a resonant sound field are rational functions with a common denominator,[18] namely

$$P(z)=\frac{n_p(z)}{d(z)}, \quad \mathbf{S}(z)=\frac{1}{d(z)}[n_{s1}(z) \quad n_{s2}(z)],$$

and (2a)

$$R(z)=\frac{n_{r1}(z)}{d(z)}.$$

The degrees of polynomials $d(z)$, $n_p(z)$, $n_{s1}(z)$, $n_{s2}(z)$, and $n_r(z)$ are $2m$ and $2m-1$ if the model is truncated to the first m modes. A properly designed diagonal transfer matrix $\mathbf{C}(z)=\mathrm{Diag}[c_1(z),c_2(z)]$ will translate objective function $\|P(z)+\mathbf{S}(z)\mathbf{C}(z)R(z)\|$ into

$$\|n_p(z)R^{-1}(z)+n_{s1}(z)c_1(z)+n_{s2}(z)c_2(z)\|$$
$$=0 \text{ or a minimum value.} \tag{2b}$$

This is a Bezout equation[13,14] solvable if $n_{s2}(z)$ does not share roots with $n_{s1}(z)$, which is not a problem for most ANC applications.[18] There exists scalar polynomials $c_1(z)$ and $c_2(z)$, with a finite degree $2m$ and bounded coefficients, such that Eq. (2b) equals zero.[18]

For robust ANC, the solution of Eq. (2b) is subject to $\|\Delta_F\mathbf{C}(z)\|_\infty<1$ that means a tradeoff between performance and robustness. The objective function may not necessarily be zero if subject to the $\mathbf{H}_\infty$ constraint. However, since Eq. (2b) has a zero solution in the unconstrained case, it is reasonable to expect its constrained solution be better than what is achievable by the constrained solution of a traditional ANC, as to be shown in this paper. It will also be made clear that the extra actuator improves ANC robustness by allowing more DOF in the controller parameters.

Another way to improve ANC performance is to reduce Δ_F when an ANC is subject to the same environment conditions. Being a bound of uncertainties, Δ_F should bound two types of possible errors. The initial identification error and the potential online variance of $F(z)$. For a rational path transfer function, the input-output relation can be expressed by an autoregressive and moving average (ARMA) model

$$y_t-d_1y_{t-1}-d_2y_{t-2}-\cdots d_py_{t-p}$$
$$=n_1u_{t-d}+n_2u_{t-d-1}+\cdots n_{p-1}u_{t-d-p+1},$$

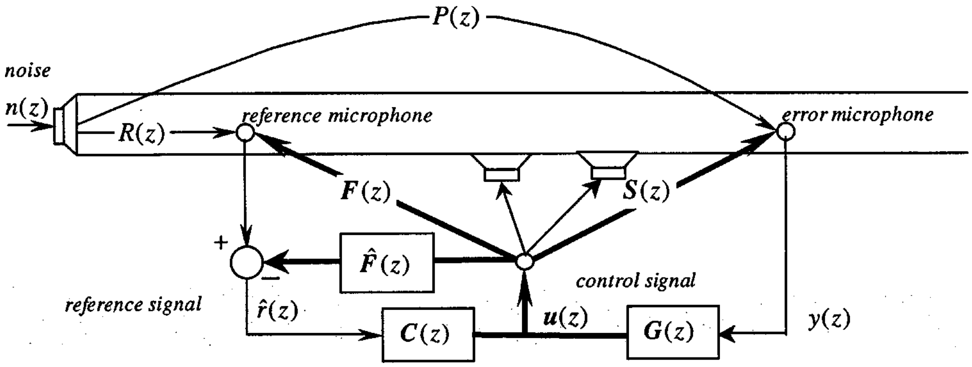

FIG. 2. The proposed ANC system.

where u_t and y_t are the input and output signals of a path model. There are many available algorithms[19] to identify parameters $\{\hat{d}_i\}_{i=1}^{p}$ and $\{\hat{n}_i\}_{i=1}^{p-1}$. Identification errors and online variance of a transfer function may be described by $\Delta d(z)$ and $\Delta n(z)$, respectively.

In theory, acoustical path transfer functions in a duct have infinite degrees. The identified transfer functions, however, can only have numerators and denominators with finite degrees. Uncertainty polynomials $\Delta d(z)$ and $\Delta n(z)$ also describe the effect of model truncation in the design of a robust ANC scheme, especially when the ANC includes a feedback control part. Expressing $\mathbf{H}_\infty$ constraints in terms of error polynomials enables an ANC to tolerate truncation error between a finite model and an infinite field. While the exact forms of $\Delta d(z)$ and $\Delta n(z)$ are not known, it is possible to estimate solid bounds on these polynomials.[19] For a rational transfer function $F(z)=n_f(z)/d(z)$, the error polynomials imply $\Delta F(z)=n_f(z)\Delta d(z)-d(z)\Delta n_f(z)/d^2(z)$ and

$$\Delta_F \propto \left\| \frac{1}{d^2(z)} \right\|_\infty (|\Delta d|+|\Delta n_f|) \tag{3}$$

as a conservative bound for $\Delta F(z)$. Similar bounds can be derived for all paths including secondary paths.

The same level of identification errors $\Delta d(z)$ and $\Delta n(z)$ may have different impact to $\Delta F(z)$ depending on $\|1/d(z)\|_\infty$, as suggested by Eq. (3). For an acoustical field, $d(z)=0$ is the system characteristic equation. This means large $\|\Delta F(z)\|$ or $\|\Delta S(z)\|$ for lightly damped ducts when both $|\Delta d(z)|$ and $|\Delta n(z)|$ are small. If a robust controller can be formulated to tolerate $|\Delta d(z)|$ and $|\Delta n(z)|$ instead of $\|\Delta F(z)\|$ or $\|\Delta S(z)\|$, then the robust constraints are less restrictive to the main objective function like Eq. (2b). This idea leads to a new ANC design strategy in the present study.

If the cross section of a duct is small enough and the duct length is equivalent to a propagation dely of k_l sampling intervals, then it is not difficult to see

$$d(z)=1-\gamma_o\gamma_u(z)\gamma_d(z)z^{-2k_l},$$

where $0<\gamma_o\approx 1$ is the round-trip attenuation, $\gamma_u(z)$ and $\gamma_d(z)$ represent, respectively, the up- and down-stream reflections of the duct. The resonant condition is characterized by a round-trip phase condition $\gamma_o\gamma_u(e^{-j\omega_i}) \times\gamma_d(e^{-j\omega_i})e^{-2jk_l\omega_i}=\gamma_i$ with positive real numbers γ_i and $1\leqslant i\leqslant k_l$. The ith resonant peak is proportional to $1/(1-\gamma_i)$. Let $\gamma=\max\{\gamma_i\}$, then $\|1/d(z)\|_\infty\propto 1/(1-\gamma)$. An attempt to reduce Δ_F is linked to the attempt to reduce γ, as suggested by Eq. (3). Since a properly designed feedback controller is able to introduce active damping to a sound field,[6] the proposed ANC includes a pole placement controller. Its objective is to reduce $\|1/d(z)\|_\infty\propto 1/1-\gamma$ by shifting all roots of $d(z)$ into a disc with radius $\gamma_m<\gamma$.

III. ANC DESIGN PROCEDURES

The proposed ANC structure is depicted in Fig. 2, where thick lines represent the paths of vector signals. The actuation signal is a vector synthesized by

$$\mathbf{u}(z)=\mathbf{C}(z)\hat{\mathbf{r}}(z)+\mathbf{G}(z)y(z), \tag{4}$$

where $y(z)$ represents the feedback signal from the error sensor, or optionally plus the signal from the reference sensor. For the sake of simplicity, only the error signal is used here as the feedback. The ANC design can be carried out in two steps. The first step is design of a robust feedback gain $\mathbf{G}(z)$ and the second one is the design of a robust feedforward gain $\mathbf{C}(z)$.

A. Design procedures for the feedback part

For robust pole placement, one may only work on $\mathbf{S}(z)$ without being distracted by other transfer functions (matrices) in the ANC system. Robust pole placement for a path model $\mathbf{S}(z)$ is similar to robust stabilization of a plant $\mathbf{S}(z)$ subject to the same set of $\mathbf{H}_\infty$ constraints. Being a multiple-input–multiple-output (MIMO) transfer matrix, $\mathbf{S}(z)$ has a left- and a right-coprime factorization,[13,14] which are identified from offline data as

$$\mathbf{S}(z)=\frac{1}{\hat{d}(z)}[\hat{n}_{s1}(z)\quad \hat{n}_{s2}(z)]=\hat{d}^{-1}(z)\hat{\mathbf{N}}_s(z) \tag{5a}$$

and

$$\hat{\mathbf{S}}(z)=[\hat{n}^*_{s1}(z)\quad \hat{n}^*_{s2}(z)]\hat{\mathbf{D}}^{*-1}(z)=\hat{\mathbf{N}}^*_s(z)\hat{\mathbf{D}}^{*-1}(z), \tag{5b}$$

where $\hat{\mathbf{D}}^*(z)$ is a 2×2 polynomial matrix. The coprime factorizations relate to each other by[13,14]

$$\hat{d}(z)\hat{\mathbf{N}}^*_s(z)=\hat{\mathbf{N}}_s(z)\hat{\mathbf{D}}^*(z). \tag{6}$$

There are many sophisticated algorithms to identify these transfer matrices with sufficient accuracy.[19]

As stated previously, robust pole placement is very similar to robust stabilization. There exists a feedback controller[13,14]

$$\mathbf{G}_o(z)=\frac{-1}{a_o(z)}\begin{bmatrix}b_{o1}(z)\\ b_{o1}(z)\end{bmatrix}=\frac{-1}{a_o(z)}\mathbf{B}_o(z)$$

whose elements are scalar polynomials, with finite degrees, solved from a Bezout equation

$$\hat{d}(z)a_o(z)+\hat{n}_{s1}(z)b_{o1}(z)+\hat{n}_{s2}(z)b_{o2}(z)=d_p(z). \quad (7)$$

This controller is able to force the closed-loop characteristic equation to $d_p(z)=0$, but is not necessarily robust with respect to the uncertainties.

An important feature of the proposed ANC is more DOF in its parameters, thanks to the extra actuator. There exists an arbitrary finite-degree polynomial matrix $\boldsymbol{\chi}(z)=[\chi_1(z)\ \chi_2(z)]^T$ to parametrize a family of infinitely many controllers[13,14]

$$\mathbf{G}(z)=\frac{-1}{a_o(z)+\hat{\mathbf{N}}_s^*(z)\boldsymbol{\chi}(z)}[\mathbf{B}_o(z)-\hat{\mathbf{D}}^*(z)\boldsymbol{\chi}(z)]$$

$$=\frac{-1}{a(z)}\mathbf{B}(z). \quad (8)$$

All controllers described by Eq. (8) will place the closed-loop poles as roots of $d_p(z)=0$.[13,14] An ANC designer has ample freedom to choose the most robust controller from the above family.

Due to initial identification errors, environmental variance, and model truncation, the real secondary path transfer matrix is given by $\mathbf{S}(z)=d^{-1}(z)\mathbf{N}_s(z)$ that deviates from Eq. (5a) by

$$d(z)=\hat{d}(z)+\Delta d(z) \quad (9a)$$

and

$$\mathbf{N}_s(z)=\hat{\mathbf{N}}_s(z)+\Delta\mathbf{N}_s(z)$$

$$=[\hat{n}_{s1}(z)+\Delta n_{s1}(z) \quad \hat{n}_{s1}(z)+\Delta n_{s1}(z)]. \quad (9b)$$

When the feedback controller is switched on, the contribution of the secondary sources may be described by

$$\mathbf{S}_c(z)\mathbf{C}(z)\hat{\mathbf{r}}(z)=\frac{1}{1-\mathbf{S}(z)\mathbf{G}(z)}\mathbf{S}(z)\mathbf{C}(z)\hat{\mathbf{r}}(z)$$

in view of Eq. (4). The closed-loop secondary path has a transfer matrix

$$\mathbf{S}_c(z)=\frac{1}{1-\mathbf{S}(z)\mathbf{G}(z)}\mathbf{S}(z)$$

$$=\frac{a(z)}{d_p(z)+\Delta\, da(z)+\Delta N_s B(z)}\mathbf{N}_s(z) \quad (10)$$

upon substitution of Eqs. (5)–(9). If the uncertainties $\Delta d(z)$, $\Delta n_{s1}(z)$, and $\Delta n_{s2}(z)$ are bounded respectively by weighting functions Δ_d, Δ_{s1}, and Δ_{s2}, then one would like to find a controller from the family of Eq. (8) such that (i) roots of $d_p(z)=0$ are in a disc with radius 1; and (ii) the following constraint is satisfied:

$$\left\|\frac{\Delta_d a(z)+\Delta_{s1}b_1(z)+\Delta_{s2}b_2(z)}{d_p(z)}\right\|_\infty<1. \quad (11)$$

If such a controller exists, then the closed-loop poles will remain stable as long as the uncertainties remain bounded by their respective weighting functions.

There are available algorithms[15,16] to solve a robust controller from the family described by Eq. (8). For ANC applications, the open-loop poles of $\mathbf{S}(z)$ are always in a disc with radius $\gamma<1$. The objective of a feedback controller is to place these poles in a smaller disc with radius $\gamma_m<\gamma$. The design conditions are modified to (i) roots of $d_p(z)=0$ are in a disc with radius $\gamma_m<\gamma$, and (ii) the left-hand side of Eq. (11) is smaller than $\gamma_m<\gamma$ with γ_m as small as possible. In view of Eq. (3), weighting functions Δ_d, Δ_{s1}, and Δ_{s2} are generally much smaller than $|\Delta\mathbf{S}(z)|$ or Δ_F, especially for lightly damped ducts. The closed-loop poles will remain inside a disc with radius $\gamma_m<\gamma$ as long as uncertainties $\Delta d(z)$, $\Delta n_{s1}(z)$, and $\Delta n_{s2}(z)$ remain bounded by weighting functions Δ_d, Δ_{s1}, and Δ_{s2}. Robust pole placement design is solvable by available numerical algorithms.[15,16] A experimental and numerical example will be presented to demonstrate the advantages of the proposed feedback control part.

B. What is new in the proposed ANC

As stated in the introduction, many researchers have taken advantage of the potential of hybrid ANC schemes to enhance performance. A possible approach of hybrid and robust ANC design was proposed to fit relevant transfer functions into a standard four-port frame work of $\mathbf{H}_\infty$ control theory.[17] The controller minimizes selective signals in the closed-loop. It is expected that minimization of those signals improves robustness with respect to model uncertainties. Other robust ANC schemes are designed with similar strategies.

The proposed ANC is designed with a different strategy in that (1) the objective of the feedback part is pole placement instead of direct noise suppression; and (2) the $\mathbf{H}_\infty$ constraints are specified with respect to error polynomials. This enables an ANC to tolerate small error polynomials $\Delta d(z)$, $\Delta n_{s1}(z)$, and $\Delta n_{s2}(z)$ that could mean a significant $\Delta S(z)$ in lightly damped ducts in view of Eq. (3). The new strategy and structure enable the feedback controller to place closed-loop poles inside a small disk as long as $\Delta d(z)$, $\Delta n_{s1}(z)$, and $\Delta n_{s2}(z)$ are bounded by weighting functions Δ_d, Δ_{s1}, and Δ_{s2}. This control part will reduce uncertainties in all transfer functions, as to be demonstrated by numerical examples in the Sec. IV.

When a finite dimensional feedback controller, like the proposed one, is applied to an infinite dimensional system like a duct, the problem of model truncation and spillover becomes important. There are methods or stability conditions to deal with spillover or ensure stability with respect to truncation errors.[20–22] The actuator and sensor dynamics, which play important roles in ANC control, are absent in those models. Inevitable errors in parameters of model matrices or online variance of plant parameters are not considered in Refs. 20–22 that focus on flexible structure with relatively invariant plants. For ANC applications, however, errors caused by online variance of path parameters may be equally, if not more, threatening to ANC stability since their effects may be within the frequency band of interest.

On the other hand, bounds on error polynomials are conservative enough to include the effects of a wide variety of errors in transfer functions identified from input–output data.[19] The actuator/sensor dynamics are integrated in these transfer functions. The proposed feedback controller is subject to a set of $\mathbf{H}_\infty$ constraints given by Eq. (11) that is in terms of bounds on error polynomials. It enables the new ANC to tolerate a wide variety of errors including model truncation errors and errors within the frequency band of interest due to online variation of path parameters. In Sec. IV, the proposed ANC will be shown to work well in a finite duct. It also tolerates a set of conservative parameter variations in simulations. These data show that the new design strategy makes the $\mathbf{H}_\infty$ constraints less restrictive without sacrificing robustness. It is an important feature for robust ANC schemes.

C. Design procedures for the feedforward part

Before designing the feedforward part, the primary source should be shut-off and the feedback part of the ANC should be activated with a control signal

$$\mathbf{u}(z)=\mathbf{G}(z)y(z)+\mathbf{v}(z),$$

where $\mathbf{v}(z)$ is substituted by a pseudorandom noise signal. By measuring sound signals from all the sensors, one should be able to identify $\hat{\mathbf{S}}_c(z)$ and $\hat{\mathbf{F}}_c(z)$, respectively. The subscript "c" indicates that the transfer matrices are "closed-loop" ones when the feedback part of the ANC is switched on. Alternatively, one may shut-off $\mathbf{v}(z)$ and turn on the primary source to identify $\hat{P}_c(z)$ and $\hat{\mathbf{R}}_c(z)$ with $\mathbf{u}(z)=\mathbf{G}(z)y(z)$ active. These transfer functions (matrices) are the design basis for the feedforward part of the proposed ANC scheme.

Starting from this step, $\mathbf{u}(z)$ becomes invisible to the design of $\mathbf{v}(z)=\mathbf{C}(z)r(z)$ though $\mathbf{u}(z)$ remains active and its effects are included in transfer functions/matrices identified by the above process. One may focus on $n(z)$ as the exogenous input and $\mathbf{v}(z)$ as the control input. The proposed ANC structure fits well into the framework of a four-port system for the $\mathbf{H}_\infty$ control theory, with a transfer matrix

$$\hat{\mathbf{T}}_c(z)=\begin{bmatrix}\hat{P}_c(z) & \hat{\mathbf{S}}_c(z)\\ \hat{R}_c(z) & \hat{\mathbf{F}}_c(z)\end{bmatrix}.$$

The feedforward control part is synthesized by $\mathbf{v}(z)=\mathbf{C}(z)r(z)$. Its objective is given in Eq. (2b). In this case, one should substitute scalar polynomials $d_c(z)$, $n_{cp}(z)$, $n_{cs1}(z)$, $n_{cs2}(z)$, and $n_{cr}(z)$ as elements of transfer matrices

$$P_c(z)=\frac{n_{cp}(z)}{d_c(z)},\quad \mathbf{S}_c(z)=\frac{1}{d_c(z)}[n_{cs1}(z)\ \ n_{cs2}(z)],$$

and

$$R_c(z)=\frac{n_{cr}(z)}{d_c(z)}$$

to replace those given by Eq. (2a). The corresponding $\mathbf{H}_\infty$ constraint is $\|\Delta_F C(z)\|_\infty<1$. This problem is solvable by the available $\mathbf{H}_\infty$ controller design algorithms.[15,16]

IV. EXPERIMENTAL AND NUMERICAL VERIFICATION

The proposed ANC is tested in an experimental duct with a length of 2 m and a cross section of 11×15 cm^2. The primary source locates at one end of the duct while the two secondary sources are, respectively, 80 cm and 120 cm away from the primary source. The reference sensor is 20 cm away from the primary source and the error sensor 40 cm away from the outlet. The ANC system has a sampling frequency of 2.4 kHz. The dynamics of actuators, sensors, amplifiers, and anti-alias filters are integrated in the transfer functions without any separate treatment.

An important feature of the proposed ANC is robust reduction of resonant peaks in the path transfer functions via feedback control. The experiment verifies such effect in a wide frequency range as shown in Fig. 3. The feedback control reduces the magnitudes of the resonant peaks of all path transfer functions inside the duct. This reduces Δ_F and $\|\Delta S(z)\|$ in view of Eq. (3). Another feature of the proposed ANC is the use of Eq. (2b) to deal with the NMP secondary path transfer functions for broadband cancellation. The cancellation effect of the proposed ANC is illustrated in Fig. 4 where the noise is suppressed in a wide frequency range, as shown by the gray-dashed and black-solid curves, respectively.

The experimental result demonstrates the ANC performance in the presence of model truncation and identification errors. The ANC is also able to tolerate online variance of path parameters. A numerical study is conducted to demonstrate this feature. A duct with a length of $L=3.4$ m is simulated with the first 10 lightly damped modes in the model. The damping ratios are chosen in such a way that the open-loop poles have magnitudes 0.994 when the transfer functions are converted into discrete-time models. In the simulation, the primary source locates at $x_p=0$; the secondary sources locate at $x_{s1}=0.5$ L and $x_{s2}=0.6$ L, respectively. The reference sensor and the error sensor are placed at $x_f=0.2$ L and $x_e=0.7$ L, respectively.

One way to model the online variance is to let the coefficients of $\Delta d(z)$ and $\Delta n(z)$ be zero-mean random numbers with $\sigma=0.05$ for a transfer function with denominator $d(z)$ and numerator $n(z)$. This test may not be realistic, yet it helps to assess the robustness of the ANC whose $\mathbf{H}_\infty$ constraints are set conservatively to tolerate uncertainties including the variance used here. In order to assess the impact of $\Delta d(z)$ and $\Delta n(z)$ generated this way, a Matlab script is written to compute and plot the frequency responses of a set of $\Delta F_1(z)$'s given by

$$\Delta F_1(z)=\frac{n_{f1}(z)}{d(z)}-\frac{n_{f1}(z)+\Delta n_{f1}(z)}{d(z)+\Delta d(z)}.$$

Each member of this set represents a possible distortion caused by a particular version of the variance. The results are

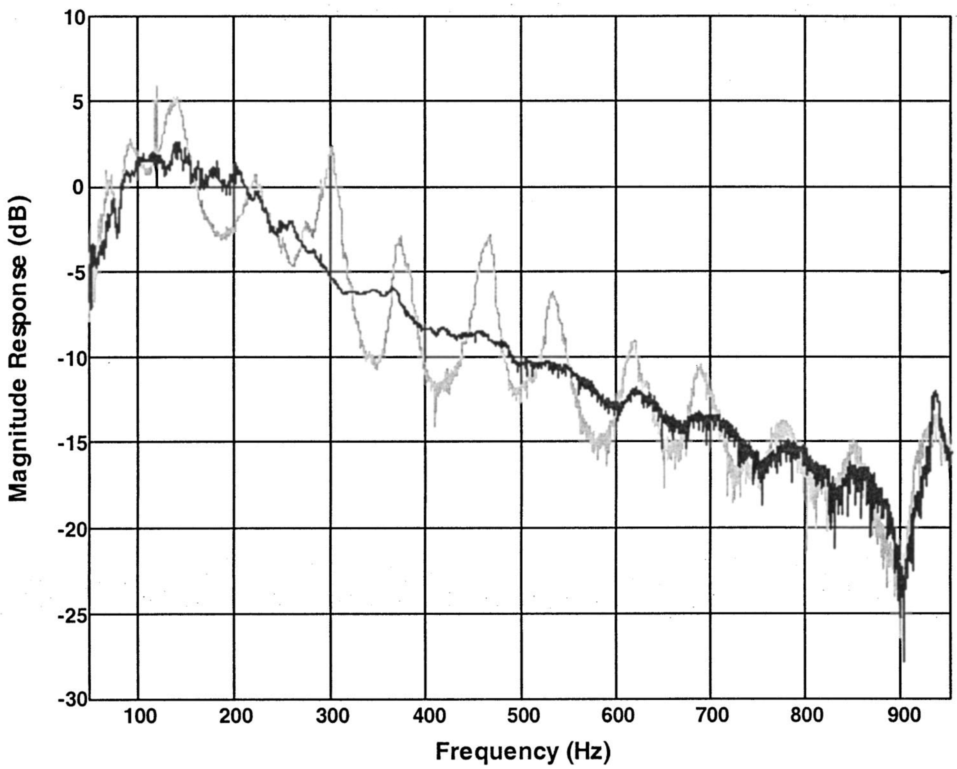

FIG. 3. Magnitude responses of a path from a secondary source to the error sensor with (black-solid) and without (gray-dashed) feedback damping.

shown in Fig. 5 where the lightly damped resonant causes a very small $\|d(z)\|_\infty$ that magnifies the impact of $\Delta d(z)$ and $\Delta n(z)$ to $\|\Delta F_1(z)\|_\infty$.

The effect of feedback damping can be evaluated by comparing any closed-loop transfer function with its open-loop counterpart. For demonstration, $\Delta F_{c1}(z)$ is compared with $\Delta F_1(z)$ when subject to the same level of variances. Frequency responses of a set of $\Delta F_{c1}(z)$'s are plotted in Fig. 6. Each $\Delta F_{c1}(z)$ is computed subject to the same level of variance as $\Delta F_1(z)$ is. Comparing Figs. 5 and 6, one can see that $\|\Delta F_{c1}(z)\|$ is reduced significantly by the feedback control part. The weight function Δ_F can be reduced by 20 dB approximately in the entire frequency of interests.

Since the proposed ANC is subject to a set of less restrictive $\mathbf{H}_\infty$ constraints and does not have the NMP problem, it is expected to have a better cancellation result when sub-

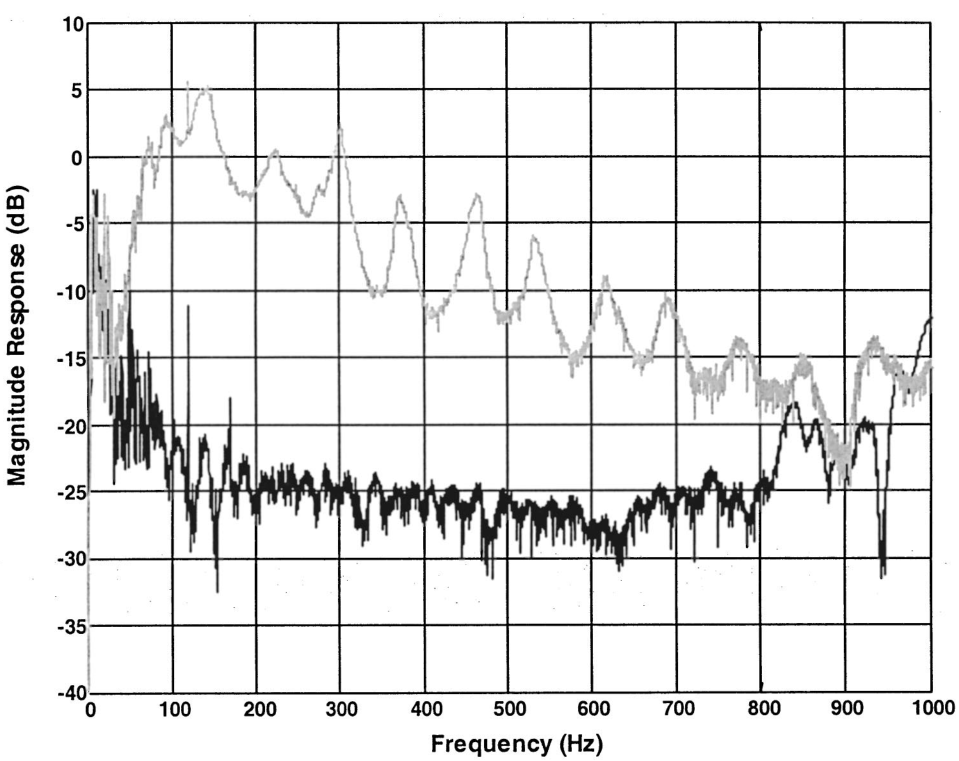

FIG. 4. Magnitude responses of the primary path to the error sensor with (black-solid) and without (gray-dashed) active control.

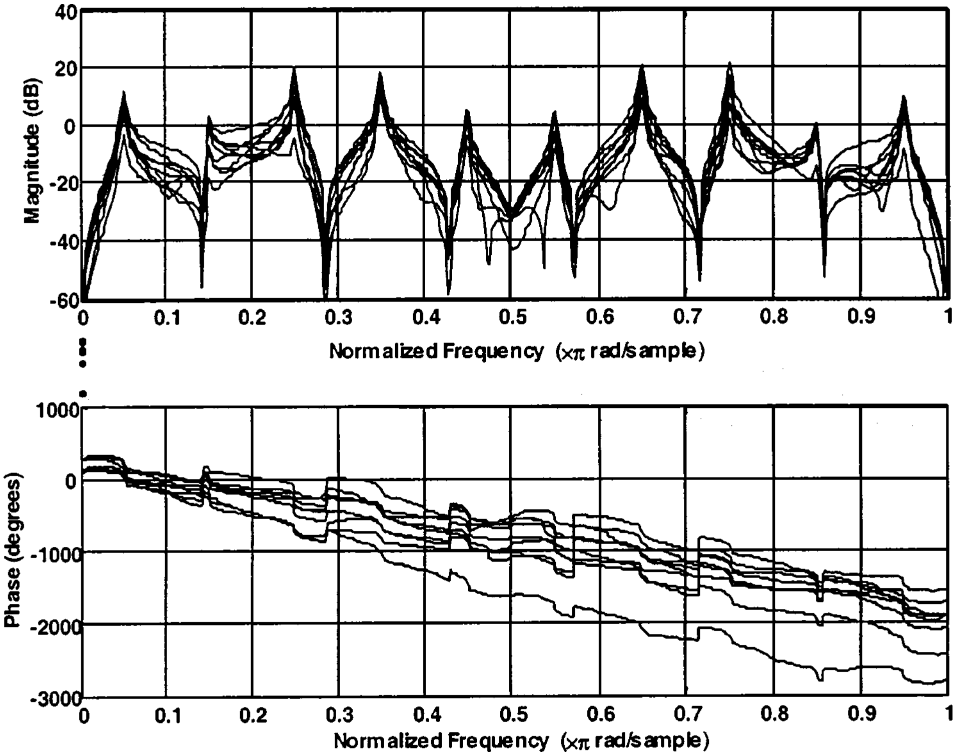

FIG. 5. Frequency responses of a set of $\Delta F_1(z)$'s (without feedback control), when coefficients of $\Delta d(z)$ and $\Delta n_{f1}(z)$ are zero-mean random numbers with $\sigma = 0.05$.

ject to the same level of variances. This is verified by the numerical results shown in Fig. 7. The gray-dashed curve represents the spectrum of a noise without ANC control. The black-solid curves represent the spectra of a set of noise signals with ANC control and subject to a small level of variances in the duct. The ANC is shown able to suppress noise in a wide frequency range subject to the online variance of path parameters.

V. CONCLUSION

A new robust ANC scheme is proposed for finite ducts. It uses an extra actuator to achieve "perfect cancellation" of a broadband noise and introduce extra DOF in robust ANC design. Another advantage of the proposed ANC is a new design strategy for its feedback control part. The new structure and the new strategy enable the ANC to tolerate a cer-

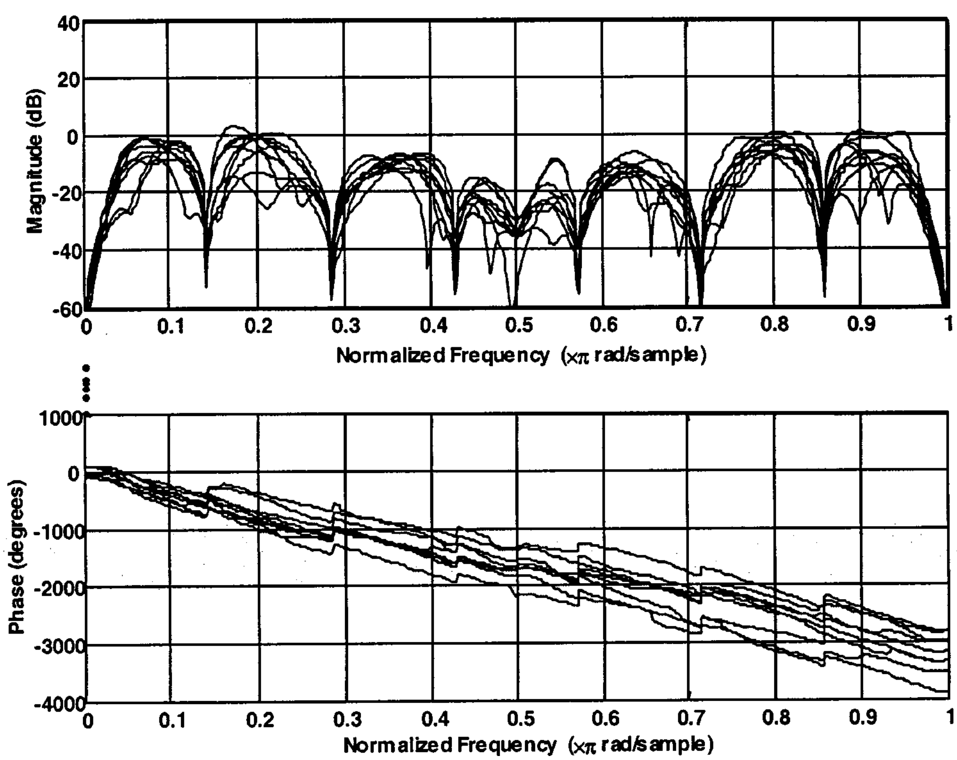

FIG. 6. Frequency responses of a set of $\Delta F_{c1}(z)$'s (with feedback control), when coefficients of $\Delta d(z)$ and $\Delta n_{f1}(z)$ are zero-mean random numbers with $\sigma = 0.05$.

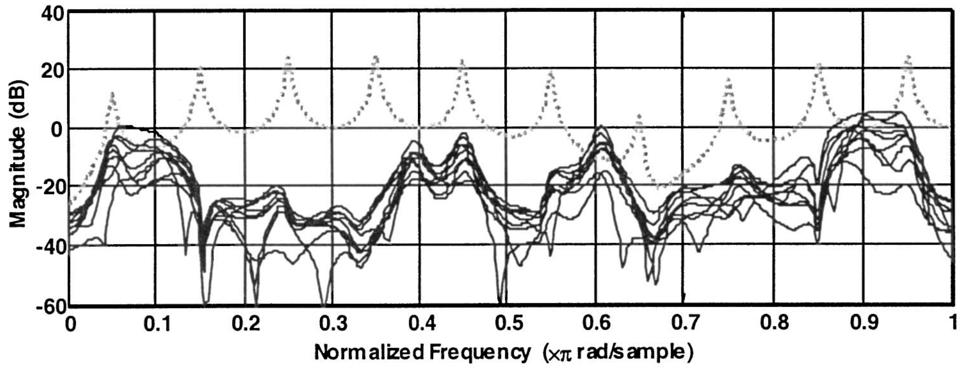

FIG. 7. Cancellation performance of the proposed ANC subject to a small level of variances: a gray dashed curve represents uncontrolled noise spectrum and the thin-black curves represent controlled noise spectra.

tain level of uncertainties and truncation errors to keep closed-loop poles in a disc with a small radius. This reduces the ANC sensitivity to uncertainties in path transfer functions significantly. The proposed ANC system has an improved performance and robustness, verified numerically and experimentally.

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