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Orthogonal adaptation for multichannel feedforward control

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In active noise control, it is desired to generate destructive interference by model-independent control. This is possible for single-channel systems to which a recently proposed method, called orthogonal adaptation, is applicable. In this study, the new method is extended to multichannel systems. An important issue is how to optimize a feedforward controller in the minimum H_2 norm sense. In practice, secondary paths of some multichannel systems may be nonminimum phase. It is a difficult problem to design H_2 feedforward controllers for multichannel systems with nonminimum phase secondary paths. The problem is solved analytically here with the best achievable, a practical and an economical solution. A recursive least squares algorithm is presented for online identification of multiple paths without persistent excitations. These solutions make it possible to implement noninvasive mode-independent controllers for multichannel systems. Experiment results are presented to verify the analytical results. © *2006 Acoustical Society of America.* [DOI: 10.1121/1.2358012]

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I. INTRODUCTION

Feedforward control is a popular method for active noise control[1,2] (ANC) when the primary signal is either available or recoverable as the reference signal. Secondary sources are excited to generate destructive interference in noise fields. Analytically, eigenfunctions of noise fields are the best tools for ANC design. Practically, eigenfunctions are not available accurately. Take a one-dimensional (1D) duct for example, its eigenfunctions depend on the impedance of duct ends, which is approximated by zero/infinity for an open/closed end. If the impedance of duct ends changes from ∞-∞ to ∞-0, the kth eigenfunction of the duct changes from $\cos[(k\pi/L)x]$ to $\cos\{[(2k-1)\pi/2L]x\}$ where L is the duct length.[3] Since impedance of an open/closed duct end is not really zero/infinity, it is not possible to obtain accurate eigenfunctions for 1D ducts, let alone three-dimensional noise fields whose eigenfunctions depend on more unknown parameters.

In a practical approach, error sensors are placed in a noise field to anchor a designated quiet zone. Transfer functions from secondary sources to error sensors become the minimum information for an ANC system. The filtered-x least mean squares (FxLMS) is a popular tool for controller adaptation, whose stability depends on the accuracy of path models. A system may be unstable if phase errors in a model exceed 90°.[4–6] Since transfer functions in noise fields may drift due to variation of environmental or boundary conditions, many ANC systems apply online modeling to keep path models close to true transfer functions. These are called model independent ANC (MIANC) systems for ease of reference. Proposed here is a new MIANC system.

In most ANC systems, path models are finite impulse response (FIR) filters with many parameters. Accurate estimation of model parameters requires "persistent excitations"[7]—the invasive injection of probing signals into actuation signals.[8] Some researchers try to regulate the magnitudes of invasive signals,[9,10] others investigate noninvasive modeling of secondary. paths.[11–13]

Recently, a method called orthogonal adaptation has been proposed for single-channel systems to implement noninvasive MIANC systems.[14] It is extended here to multichannel systems. A major difficulty is how to force the regression vector as orthogonal as possible to the online models. For multichannel systems, it is more difficult to meet the orthogonal requirement if the secondary path is nonminimum phase (NMP). The problem is solved here with the best achievable, a practical and an economical solution. A recursive least squares (RLS) algorithm is proposed for online modeling of multichannel systems. Experimental results are presented to verify the extended algorithm when applied to multichannel systems.

II. BACKGROUND INFORMATION

In an ANC system, transfer functions from primary and secondary sources to error sensors are represented by $P(z)$ and $\boldsymbol{S}(z)$ as the *p*rimary *t*ransfer vector and *s*econdary transfer matrix. The actuation signal is generated by $a(z) = C(z)r(z)$ where $C(z)$ is the controller transfer vector and $r(z)$ is the reference signal. The error signals are given by

$$e(z) = P(z)r(z) + \boldsymbol{S}(z)a(z) = [P(z) + \boldsymbol{S}(z)C(z)]r(z). \tag{1}$$

when $r(z)$ is broadband noise, $e(z)=0$ requires an ideal controller

$$C(z) = -\boldsymbol{S}^{-1}(z)P(z). \tag{2}$$

Equation (2) is stable if $\boldsymbol{S}(z)$ is minimum phase (MP), which requires a nonsingular $\boldsymbol{S}(z)$ for all $|z| \geqslant 1$. Otherwise, the secondary path is NMP, Eq. (2) is unstable and an achievable objective is to minimize

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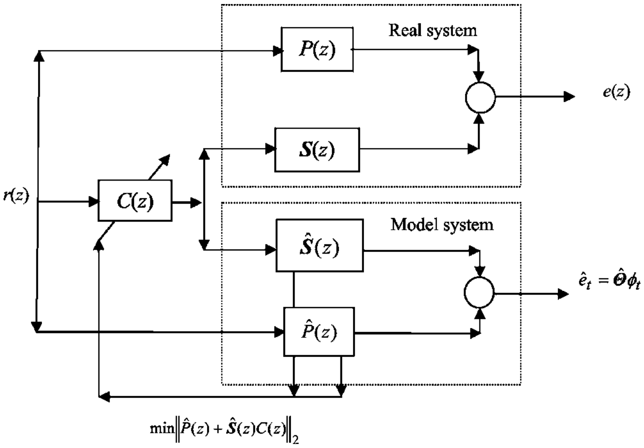

FIG. 1. Block diagram of the proposed ANC system.

$$\|e(z)\|_2 = \|P(z) + S(z)C(z)\|_2. \tag{3}$$

The proposed system achieves this objective with a block diagram shown in Fig. 1.

It is assumed by many researchers that $P(z)$ and $S(z)$ are approximated by FIR filters with negligible errors, which is adopted in this study. Let $P=[P_0 P_1, \ldots, P_m]$ and $S=[S_0 S_1, \ldots, S_m]$ denote coefficients of $P(z)$ and $S(z)$ respectively, the time-domain version of Eq. (1) is a discrete-time convolution

$$e_t = \sum_{k=0}^{m} P_k r_{t-k} - \sum_{k=0}^{m} S_k a_{t-k}, \tag{4}$$

where e_t, r_t, and a_t, denote respective samples of $e(z), r(z)$, and $a(z)$. Introducing coefficient matrix $\Theta=[P\ S]$ and regression vector $\phi_t=[r_t r_{t-1}, \ldots, r_{t-m} a_t a_{t-1}, \ldots, a_{t-m}]^T$, one may rewrite Eq. (4) to

$$e_t = \Theta\phi_t. \tag{5}$$

Since the regression vector ϕ_t contains actuation signal a_t, controllers are designed to regulate ϕ_t and minimize objective function

$$J_t = \sum e_t^T e_t = \sum \phi_t^T \Theta^T \Theta \phi_t, \tag{6}$$

where the summation is over a sliding time window.

Like other MIANC systems, the proposed one minimizes J_t, when $\Theta=[P\ S]$ is not available. Online estimates of $P(z)$ and $S(z)$ are denoted as $\hat{P}(z)$ and $\hat{S}(z)$, which are obtained by minimizing estimation error

$$\begin{aligned}\varepsilon(z) &= e(z) - \hat{P}(z)r(z) - \hat{S}(z)a(z) \\ &= \Delta P(z)r(z) + \Delta S(z)a(z).\end{aligned} \tag{7}$$

Here $\Delta P(z)=P(z)-\hat{P}(z)$ and $\Delta S(z)=S(z)-\hat{S}(z)$ are model errors. If $\hat{\Theta}=[\hat{P}\ \hat{S}]$ denotes online estimate of $\Theta=[P\ S]$, then $\hat{P}=[\hat{P}_0 \hat{P}_1 \cdots \hat{P}_m]$ and $\hat{S}=[\hat{S}_0 \hat{S}_1 \cdots \hat{S}_m]$ contain coefficient matrices of $\hat{P}(z)$ and $\hat{S}(z)$, respectively. The time-domain version of Eq. (7) is

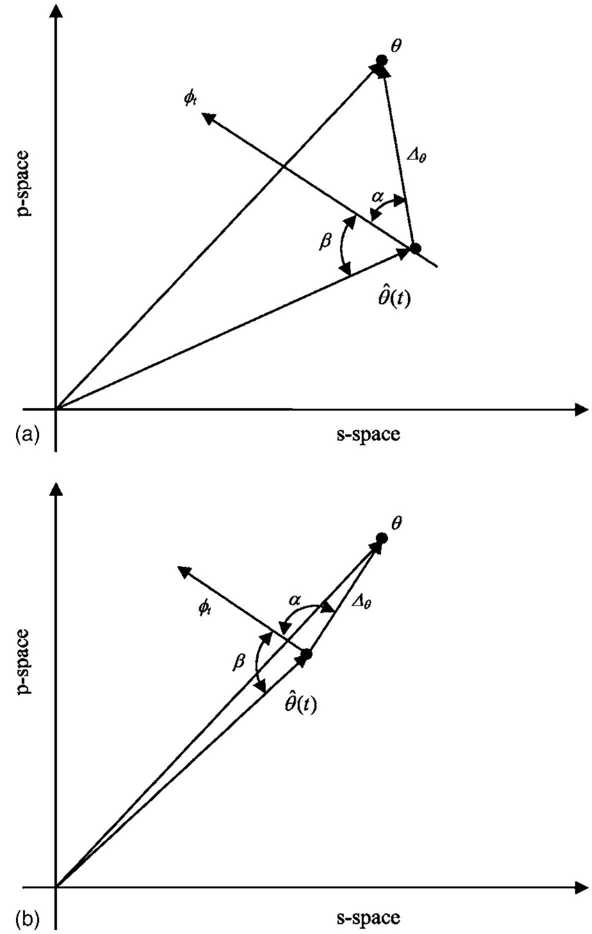

FIG. 2. Illustration of orthogonal adaptation: (a) adapting and (b) converging.

$$\varepsilon_t = e_t - \hat{\Theta}\phi_t = \Delta\Theta\phi_t, \tag{8}$$

where $\Delta\Theta = \Theta - \hat{\Theta}$. The system performs two online tasks. One is a modeling task to drive $\varepsilon_t \to 0$ and the other is an optimizing task to force $\hat{e}_t = \hat{\Theta}\phi_t \approx 0$. Here $\hat{e}_t$ is the error signal of the model system in Fig. 1. Similar to the derivation from Eq. (1) to Eq. (5), one can see that $\hat{e}_t = \hat{\Theta}\phi_t$ is equivalent to

$$\hat{e}(z) = \hat{P}(z)r(z) + \hat{S}(z)a(z) = [\hat{P}(z) + \hat{S}(z)C(z)]r(z). \tag{9}$$

The optimization task solves $C(z)$ by minimizing $\|\hat{P}(z) + \hat{S}(z)C(z)\|_2$.

Let $\theta^T=[p^T\ S^T]$ denote one row of Θ, then $\hat{\theta}^T=[\hat{p}^T\ \hat{s}^T]$ and $\Delta_\theta^T=[\Delta p^T, \Delta s^T]$ are the corresponding rows of $\hat{\Theta}$ and $\Delta\Theta$, respectively. Objectives $\varepsilon_t=\Delta\Theta\phi_t \to 0$ and $\hat{e}_t=\hat{\Theta}\phi_t \approx 0$ are equivalent to $\Delta_\theta^T\phi_t \to 0$ and $|\hat{\theta}^T\phi_t| \approx 0$ for all rows. Effects of the two tasks are illustrated in Fig. 2 where the two axes represent the p- and s spaces of $\theta^T=[p^T\ s^T]$. Driving $\Delta_\theta^T\phi_t \to 0$ is equivalent to driving $\alpha \to 90°$ while forcing $|\hat{\theta}^T\phi_t| \approx 0$ is equivalent to forcing $\beta \approx 90°$. When the controller starts, $\alpha \neq 90°$ and $\beta \neq 90°$ in Fig. 2(a). One may find respective algorithms, such that $\Delta_\theta^T\phi_t \to 0$ and $|\hat{\theta}^T\phi_t| \approx 0$ are achieved, respectively. If the system converges to α=90° and β=90°, ϕ_t is orthogonal to Δ_θ, $\hat{\theta}$ and θ simultaneously and J_t is minimized in view of Eq. (6).

Analytically, one may understand the joined effects of the two online tasks via

$$\|e_t\| = \|\varepsilon_t + \hat{\mathbf{\Theta}}\phi_t\| = \|\varepsilon_t + \hat{e}_t\| \leq \|\varepsilon_t\| + \|\hat{e}_t\|, \tag{10}$$

which is obtained by adding $\hat{e}_t = \hat{\mathbf{\Theta}}\phi_t$ to both sides of Eq. (8). On the left hand side of Eq. (10), e_t, represents the ANC error. On the right hand side, $\varepsilon_t = \Delta\mathbf{\Theta}\phi_t$ is the estimation error, and $\hat{e}_t$ is the error of the model system in Fig. 1. If $\Delta_\theta^T\phi_t \to 0$ and $|\hat{\theta}^T\phi_t| \approx 0$ for all components of $\varepsilon_t = \Delta\mathbf{\Theta}\phi_t$ and $\hat{\mathbf{\Theta}}\phi_t$, then $\|\varepsilon_t\| \to 0$ and $\|\hat{e}_t\| \approx 0$ imply $\|e_t\| \approx 0$ even $\Delta_\theta \neq 0$.

III. CONTROLLER OPTIMIZATION

It may not be difficult to minimize $\|\hat{e}_t\|$ if $\hat{\mathbf{S}}(z)$ is MP in every step of estimation. One can substitute $C(z) = -\hat{\mathbf{S}}^{-1}(z)\hat{P}(z)$ into Eq. (9) such that $[\hat{P}(z) - \hat{\mathbf{S}}(z)\hat{\mathbf{S}}^{-1}(z)\hat{P}(z)]r(z)=0$ and equivalently $\hat{e}_t=0$. Unfortunately, $\hat{\mathbf{S}}^{-1}(z)$ is unstable if $\hat{\mathbf{S}}(z)$ is NMP, in which case a stable $C(z)$ is sought to minimize $\|\hat{P}(z)+\hat{\mathbf{S}}(z)C(z)\|_2$. This is a difficult problem for multichannel systems.

A. Best achievable solution

For systems with equal inputs and outputs, the inverse of $\hat{\mathbf{S}}(z)$ has the form of

$$\hat{\mathbf{S}}^{-1}(z) = \hat{D}^{-1}(z)\hat{\mathbf{S}}_a(z), \tag{11}$$

where $\hat{D}(z)$ and $\hat{\mathbf{S}}_a(z)$ are the determinant and adjoint of $\hat{\mathbf{S}}(z)$, respectively. Singularities of $\hat{\mathbf{S}}(z)$ depend on roots of $\hat{D}(z)$. Let r_i denote the ith root of $\hat{D}(z)$, then $\hat{\mathbf{S}}(z)$ is NMP if there exist some $\{r_i\}$ such that $|r_i| > 1$.

The optimal controller is related to $\hat{D}(z) = D_m(z)D_n(z)$, where $D_m(z)$ and $D_n(z)$ are the MP and NMP parts of $\hat{D}(z)$, respectively. One may obtain a mirror polynomial $R_n(z)$ by using coefficients of $D_n(z)$ in the reversed order. It can be shown[15] that $W_a(z) = D_n(z)/R_n(z)$ is a stable all-pass filter. It then follows that

$$\hat{D}(z) = D_n(z)D_m(z) = \frac{D_n(z)}{R_n(n)}R_n(z)D_m(z) = W_a(z)W_m(z), \tag{12}$$

where $W_m(z) = R_n(z)D_m(z)$ is a MP filter. The optimal controller has the form of

$$C_o(z) = \hat{\mathbf{S}}_a(z)G(z). \tag{13}$$

Using Eqs. (11) and (12), one can see that $\hat{\mathbf{S}}_a(z)$ and $\hat{\mathbf{S}}(z)$ are related to each other by

$$\hat{\mathbf{S}}(z)\hat{\mathbf{S}}_a(z) = \hat{D}(z)\boldsymbol{I} = W_a(z)W_m(z)\boldsymbol{I}, \tag{14}$$

where $\boldsymbol{I}$ is an identity matrix. Substituting Eqs. (13) and (14) into $\|[\hat{P}(z)+\hat{\mathbf{S}}(z)C_o(z)]\|_2$, one obtains

$$\|\hat{P}(z) + \hat{\mathbf{S}}(z)C_o(z)\|_2 = \|\hat{P}(z) + W_a(z)W_m(z)G(z)\|_2. \tag{15}$$

The above equation may be rewritten to

$$\begin{aligned}\|\hat{P}(z) + \hat{\mathbf{S}}(z)C_o(z)\|_2 &= \|W_a\|_2\|W_a^{-1}\hat{P} + W_mG\|_2 \\ &= \|W_a^{-1}(z)\hat{P}(z) + W_m(z)G(z)\|_2,\end{aligned} \tag{16}$$

where $\|W_a(z)\|_2=1$ since it is an all-pass filter. The next step is to apply the long-division and obtain

$$W_a^{-1}(z)\hat{P}(z) = \frac{R_n(z)\hat{P}(z)}{D_n(z)} = \frac{P_r(z)}{D_n(z)} + P_q(z), \tag{17}$$

where $P_q(z)$ and $P_r(z)$ are the quotient and remainder polynomial vectors. The two parts on the right hand side of Eq. (17) are orthogonal to each other in the H_2 norm sense, such that Eq. (16) may be expressed as

$$\begin{aligned}\|\hat{P}(z) + \hat{\mathbf{S}}(z)C_0(z)\|_2 &= \|W_a^{-1}(z)\hat{P}(z) + W_m(z)G(z)\|_2 \\ &= \left\|\frac{P_r(z)}{D_n(z)}\right\|_2 + \|P_q(z) + W_m(z)G(z)\|_2.\end{aligned} \tag{18}$$

When $D_n(z)$ is the NMP part of $\hat{D}(z)$, $P_r(z)/D_n(z)$ is unstable and cannot be cancelled by any stable feedforward controller. The best achievable result is $\|P_q(z) + W_m(z)G(z)\|_2=0$ by controller

$$G(z) = -W_m^{-1}(z)P_q(z) = -\frac{P_q(z)}{R_n(z)D_m(z)} \quad \text{or}$$
$$C_o(z) = -\frac{\hat{\mathbf{S}}_a(z)P_q(z)}{R_n(z)D_m(z)}. \tag{19}$$

Any other feedforward controller only increases Eq. (18) if it is not given by Eq. (19).

A key step in the above.derivations is the use of Eqs. (13) and (14). To the best of author's knowledge, this is the first reported method to find a H_2 feedforward controller for multichannel systems with square and NMP secondary paths. It is not perfect since Eqs. (13) and (14) are applicable only if $\hat{\mathbf{S}}(z)$ is square. When there are different inputs and outputs, a suboptimal controller is solvable by methods presented in the next two subsections.

B. Practical solution

One may also design a practical and suboptimal controller $C(z)$ to minimize

$$\|\hat{P}(z) + \hat{\mathbf{S}}(z)C(z)\|_2 = \|F(z)\|_2, \tag{20}$$

where $C(z)$ and $F(z)$ are FIR filters. The impulse response of Eq. (20) is given by

$$F=\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ \vdots \\ F_{2m-1} \end{bmatrix} = \begin{bmatrix} \hat{P}_1 \\ \hat{P}_2 \\ \vdots \\ \hat{P}_m \end{bmatrix} + \begin{bmatrix} \hat{S}_1 & & \\ \hat{S}_2 & \hat{S}_1 & \\ \vdots & \hat{S}_2 & \ddots & \\ \hat{S}_m & \vdots & \ddots & \hat{S}_1 \\ & \hat{S}_m & \vdots & \hat{S}_2 \\ & & \ddots & \vdots \\ & & & \hat{S}_m \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{bmatrix} = \hat{P} + \mathbf{\Gamma}_s C, \tag{21}$$

where $F^T=[F_1,\ldots,F_{2m-1}]$ and $C^T=[C_1,\ldots,C_m]$ are coefficient vectors of $F(z)$ and $C(z)$ respectively, coefficient matrices of $\hat{S}(z)$ are used to construct matrix

$$\mathbf{\Gamma}_s = \begin{bmatrix} \hat{\boldsymbol{S}}_1 & & \\ \hat{\boldsymbol{S}}_2 & \hat{\boldsymbol{S}}_1 & \\ \vdots & \hat{\boldsymbol{S}}_2 & \ddots & \\ \hat{\boldsymbol{S}}_m & \vdots & \ddots & \hat{\boldsymbol{S}}_1 \\ & \hat{\boldsymbol{S}}_m & \vdots & \hat{\boldsymbol{S}}_2 \\ & & \ddots & \vdots \\ & & & \hat{\boldsymbol{S}}_m \end{bmatrix}. \tag{22}$$

According to Parserval's theorem, minimizing $\|\hat{P}(z)+\hat{S}(z)C(z)\|_2=\|F(z)\|_2$ is equivalent to minimizing $\|F\|^2$. The objective function is

$$\begin{aligned} F^TF &= (\hat{P}+\mathbf{\Gamma}_sC)^T(\hat{P}+\mathbf{\Gamma}_sC) \\ &= \hat{P}^T\hat{P}+\hat{P}^T\mathbf{\Gamma}_sC + C^T\mathbf{\Gamma}_s^T\hat{P}+C^T\mathbf{\Psi}_sC, \end{aligned} \tag{23}$$

where $\mathbf{\Psi}_s=\mathbf{\Gamma}_s^T\mathbf{\Gamma}_s$ is the autocorrelation matrix of the impulse response of $\hat{S}(z)$. One may introduce $\mathbf{\Xi}=\mathbf{\Gamma}_s^T\hat{P}$ and substitute $C^T\mathbf{\Gamma}_s^T\hat{P}=C^T\mathbf{\Psi}_s\mathbf{\Psi}_s^{-1}\mathbf{\Xi}$ into Eq. (23). This leads to

$$F^TF = \hat{P}^T\hat{P} - \mathbf{\Xi}^T\mathbf{\Psi}_s^{-1}\mathbf{\Xi} + (\mathbf{\Psi}_sC+\mathbf{\Xi})^T\mathbf{\Psi}_s^{-1}(\mathbf{\Psi}_sC+\mathbf{\Xi}), \tag{24}$$

where parameter vector C only affects $(\mathbf{\Psi}_sC+\mathbf{\Xi})^T\mathbf{\Psi}_s^{-1}(\mathbf{\Psi}_sC+\mathbf{\Xi})$.

In this solution, $C(z)$ minimizes F^TF if $\mathbf{\Psi}_sC=-\mathbf{\Xi}$ is solvable or the rank of $\mathbf{\Gamma}_s$ equals its row size. This is not a problem for single-channel systems. For multichannel systems, however, special care has to be taken to satisfy the rank requirement. The resultant controller is suboptimal if compared with $C_o(z)$ in Eq. (19), but it is more practical and less computationally expensive.

Since $\mathbf{\Gamma}_s$ depends on coefficient matrices of $\hat{S}(z)$, its rank could change as the system identifies $\hat{S}(z)$. There may be instants when $\mathbf{\Gamma}_s$ is rank defective and $\mathbf{\Psi}_s=\mathbf{\Gamma}_s^T\mathbf{\Gamma}_s$ is singular. A pseudoinverse of $\mathbf{\Psi}_s$ may be used to minimize $\mathbf{\Psi}_sC+\mathbf{\Xi}\approx 0$ in such cases. Substituting into Eq. (24), one can see that F^TF is not minimized to the full extent when $\mathbf{\Psi}_sC+\mathbf{\Xi}\approx 0$. While a rank defective $\mathbf{\Gamma}_s$ degrades ANC performance, it does not affect system stability as long as $C(z)$ is bounded.

C. Economic solution

In many ANC applications, $P(z)$ and $\boldsymbol{S}(z)$ are approximated by FIR filters with large numbers of coefficients. The dimension of $\mathbf{\Psi}_s$ is $(l\times m)^2$ if a multichannel system has l actuators and the degree of $C(z)$ is m. Calculation of $\mathbf{\Psi}_s$ and $\mathbf{\Xi}$ requires additional work. Online solution of $\mathbf{\Psi}_sC=-\mathbf{\Xi}$ is very expensive.

Alternatively, one may consider a positive definite function $O=0.5F^TF$ and a recursive algorithm that updates C and minimizes O. The time derivative of O is given by

$$\dot{O} = F^T\mathbf{\Gamma}_s\dot{C}, \tag{25}$$

where Eq. (21) is used to link $\dot{F}$ to $\dot{C}$. The above equation suggests a very simple way to modify C. It is given by

$$\dot{C} = -\mu\mathbf{\Gamma}_s^TF \quad \text{or} \quad C_{t+1}=C_t-\mu\mathbf{\Gamma}_s^TF\delta t, \tag{26}$$

where μ is a small positive constant and δt is the sampling interval. Combining Eqs. (25) and (26), one obtains $\dot{O}=-\mu F^T\mathbf{\Gamma}_s\mathbf{\Gamma}_s^TF\leqslant 0$. Therefore $O=0.5F^TF$ will be minimized by the recursive use of Eq. (26).

The advantage of this solution is to avoid online inverse of $\mathbf{\Psi}_s$. The method is not able to minimize $O=0.5F^TF$ in every step of identification, though it reduces the computational cost. Mathematically, this solution is equivalent to the practical one upon the convergence of Eq. (26). Therefore the same rank condition is preferred. In case $\mathbf{\Gamma}_s$ is rank defective, $\mathbf{\Psi}_s$ is singular and there does not exist a suboptimal C for $\mathbf{\Psi}_sC=-\mathbf{\Xi}$ no matter what method is used to solve $C(z)$. In view of Eq. (24), F^TF cannot be minimized to the full extent as C_t converges to a finite vector C but $\mathbf{\Psi}_sC+\mathbf{\Xi}\neq 0$. System stability, however, will not be affected by the rank of $\mathbf{\Gamma}_s$ as long as $C(z)$ is bounded.

IV. CONVERGENCE OF PATH MODELING

In the previous section, a rank condition on $\mathbf{\Gamma}_s$ is preferred by the optimization task. According to Eq. (22), $\mathbf{\Gamma}_s$ consists of coefficient matrices of $\hat{S}(z)$ and $\hat{S}(z)$ is an estimate of $\boldsymbol{S}(z)$. If $P(z)$ and $\boldsymbol{S}(z)$ were available accurately, the optimization should be applied to $P(z)$ and $\boldsymbol{S}(z)$. The rank condition basically depends on $\boldsymbol{S}(z)$ and system configuration such as locations of actuators and error sensors. This is actually a preferred condition for all multichannel ANC systems. It has not been discussed in the literature since most ANC systems update controllers by FxLMS. A controller obtained by FxLMS is not necessarily optimal. In view of Eq. (24), a multichannel feedforward controller is not optimal in the minimum H_2 norm sense if the rank condition is not satisfied, in which case satisfying the rank condition is a new hint for system improvement.

Here $\boldsymbol{S}(z)$ and $\hat{S}(z)$ are said to satisfy the rank condition respectively, if $\mathbf{\Gamma}_s$ satisfies the rank condition when coefficient matrices of $\boldsymbol{S}(z)$ and $\hat{S}(z)$ are substituted in Eq. (22), respectively. There are two possible cases when $\mathbf{\Gamma}_s$ does not satisfy the rank condition: (1) $\boldsymbol{S}(z)$ does not satisfy the rank condition and $\hat{S}(z)$ converges to $\boldsymbol{S}(z)$; and (2) $\boldsymbol{S}(z)$ satisfies the rank condition but $\hat{S}(z)$ does not converge to $\boldsymbol{S}(z)$. One must adjust system configuration such that $\boldsymbol{S}(z)$ satisfies the rank condition in the first place. It is then possible to identify $\hat{S}(z)$ that satisfies the rank condition at each step of estimation. This requires a very complicated algorithm[16] and is an additional restriction to online modeling.

In this section, a simple RLS algorithm is proposed to drive $\varepsilon_t = \Delta\boldsymbol{\Theta}\phi_t \to 0$ and satisfy the rank requirement on $\boldsymbol{\Gamma}_s$. The algorithm has a matrix form

$$\hat{\boldsymbol{\Theta}}(t+1) = \hat{\boldsymbol{\Theta}}(t) + \frac{\varepsilon_t \phi^T}{\phi^T \phi}. \tag{27}$$

One may use positive definite function $V(t) = \mathrm{Tr}\{\Delta\boldsymbol{\Theta}^T(t)\Delta\boldsymbol{\Theta}(t)\}$ to analyze the convergence of Eq. (27). Similar to identity $a^2 - b^2 = (a-b)(a+b)$, it can be verified that

$$V(t+1) - V(t) = \mathrm{Tr}\{ \times[\Delta\boldsymbol{\Theta}(t+1) - \Delta\boldsymbol{\Theta}(t)]^T[\Delta\boldsymbol{\Theta}(t+1) + \Delta\boldsymbol{\Theta}(t)]\}. \tag{28}$$

with the help of Eq. (27) and $\Delta\boldsymbol{\Theta} = \boldsymbol{\Theta} - \hat{\boldsymbol{\Theta}}$, one can obtain

$$\Delta\boldsymbol{\Theta}(t+1) - \Delta\boldsymbol{\Theta}(t) = \hat{\boldsymbol{\Theta}}(t) - \hat{\boldsymbol{\Theta}}(t+1) = \varepsilon_t \phi^T(t)/\phi^T\phi \tag{29}$$

and

$$\Delta\boldsymbol{\Theta}(t+1) + \Delta\boldsymbol{\Theta}(t) = 2\boldsymbol{\Theta} - \hat{\boldsymbol{\Theta}}(t) - \hat{\boldsymbol{\Theta}}(t+1) = 2\Delta\boldsymbol{\Theta}(t) - \varepsilon_t \phi^T(t)/\phi^T\phi. \tag{30}$$

The next step is to substitute Eqs. (29) and (30) into Eq. (28), which leads to

$$V(t+1) - V(t) = \mathrm{Tr}\left\{ \frac{-\phi\varepsilon_t^T}{\phi^T\phi}\left[2\Delta\boldsymbol{\Theta}(t) - \frac{\varepsilon_t\phi^T}{\phi^T\phi}\right]\right\}. \tag{31}$$

Substituting $\mathrm{Tr}\{\phi\varepsilon_t^T\Delta\boldsymbol{\Theta}\} = \varepsilon_t^T\Delta\boldsymbol{\Theta}\phi = \varepsilon_t^T\varepsilon_t$ and $\mathrm{Tr}\{\phi\varepsilon_t^T\varepsilon_t\phi^T\} = \varepsilon_t^T\varepsilon_t\mathrm{Tr}\{\phi\phi^T\}\varepsilon_t^T\varepsilon_t\phi^T\phi$ into Eq. (31), one can finish the derivation with

$$V(t+1) - V(t) = -\frac{\varepsilon_t^T\varepsilon_t}{\phi^T\phi} \leqslant 0. \tag{32}$$

It indicates monotonous decrease of $V(t) = \mathrm{Tr}\{\Delta\boldsymbol{\Theta}^T(t)\Delta\boldsymbol{\Theta}(t)\}$ until $\varepsilon_t \to 0$.

A simple attempt to meet the rank condition on $\boldsymbol{\Gamma}_s$ is a proper choice of initial guess for $\hat{S}(z)$. When the secondary path is square, a possible initial guess of $\hat{S}(z)$ would be an identity matrix. When there are different inputs and outputs, one may initialize $\hat{S}(z)$ with pseudorandom values such that the initial rank of $\boldsymbol{\Gamma}_s$ equals its row size. This is an effective way to avoid heavy computations and maintain good ANC performance. In simulations, the ANC performance degrades drastically if the initial guess of $\hat{S}(z)$ does not meet the rank condition while other conditions remain unchanged.

V. COMPARISON WITH OTHER MIANC SYSTEMS

Model independent by online modeling is the common feature of all MIANC systems including the proposed one. While there are slight differences in identification algorithms, the computational cost of online modeling is almost the same. The main difference is how to design the controller and how to evaluate performance, which are the subjects of comparison.

Most MIANC systems need another task for controller adaptation by FxLMS, which requires manageable computations in one sample but takes many samples for $C(z)$ to converge after the convergence of $\hat{S}(z)$ in online modeling. Since FxLMS is an estimation algorithm, there are inevitable estimation errors in $C(z)$. System stability is a problem if the phase error of $\hat{S}(z)$ exceeds 90°. After FxLMS drives the convergence of $C(z)$, it is difficult to see if $C(z)$ is optimal or not. In case $C(z)$ is not optimal, it is not clear how to improve system performance.

The proposed system optimizes $C(z)$ without adaptation. If $\hat{S}(z)$ is square, the best achievable solution is proposed. If $\hat{S}(z)$ is not square, two suboptimal solutions are proposed. Advantages of the proposed method are weak points of FxLMS: (a) The methods are stable and optimal even $\hat{S}(z)$ does not converge to $\boldsymbol{S}(z)$; (b) it is possible to achieve the best achievable performance; (c) if hardware is fast enough, $C(z)$ can be solved by minimizing $\|\hat{P}(z) + \hat{\boldsymbol{S}}(z)C(z)\|_2$ immediately after the convergence of $\hat{P}(z)$ and $\hat{S}(z)$; (d) there are no estimation errors in $C(z)$; and (e) the controller is optimal in the minimum H_2 norm sense if a rank condition on $\hat{S}(z)$ is satisfied.

With available hardware, the first two solutions are too computationally expensive to be done in every sample interval. The economic solution requires roughly m times the storage and computations required by FxLMS, where m is the degree of $C(z)$. These methods are no match to FxLMS in terms of computational cost in each sample interval. Advantages of the proposed system will eventually become attractive when faster optimization algorithms are available or computers are faster and less expensive to offset drawbacks of heavy online computations.

VI. IMPLEMENTATION AND VERIFICATION

An experiment was conducted to test orthogonal adaptation for multichannel systems. The controller was implemented in a dSPACE 1103 board. The degrees of $C(z)$, $\hat{P}(z)$, and $\hat{\boldsymbol{S}}(z)$ were $m = 300$. The primary source and secondary sources were 6 in. speakers. The online tasks were implemented using Eqs. (26) and (27) with initial guesses $\hat{P}(z) = 0$, $\hat{\boldsymbol{S}}(z) = \boldsymbol{I}$ and $C(z) = [1, 1, 1]^T$. The system sampling rate was 1538 samples/s and all signals were low-pass filtered with cutoff frequency 500 Hz. In the experiment, $r(z)$ was broadband noise. While it is possible to recover $r(z)$ from a measured signal, it would take additional computer time for online modeling of feedback paths and cancellation of acoustical feedbacks. To save computer time for the main focus, $r(z)$ was directly available to the controller. Figure 3 shows the experiment configuration.

The experiment was conducted in an anechoic chamber. The controller was turned off first to collect signals from the

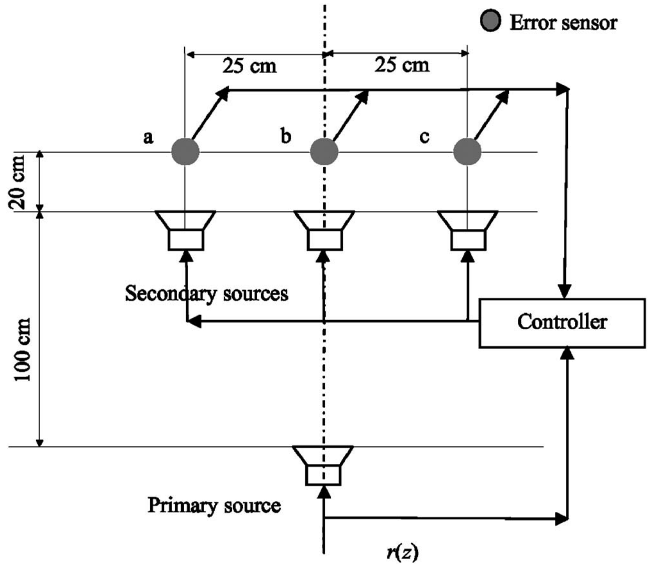

FIG. 3. Sensor/actuator configuration in the experiment.

error sensors. Power spectral densities (PSDs) of the signals were normalize by the PSD of $r(z)$ as the comparison references, which are shown as the gray curves in Figs. 4(a)–4(c). The ANC was then turned on for a while before taking signals from the error sensors. The black curves in Figs. 4(a)–4(c) represent normalized PSDs of the error signals collected after the ANC was active. Significant noise reduction is observed in the entire frequency range of interest. It is verified by the experiment that orthogonal adaptation is an effective approach for model-independent feedforward control of multichannel systems.

Like other MIANC systems, the proposed one only minimizes error signals. Due to hardware limit, the experimental system only had three error sensors. Its bandwidth was half the sampling frequency with the shortest wavelength of 48.23 cm. The range of the quiet zone was limited by two main facts: (1) an error sensor anchors a quiet region whose range is a small fraction of shortest wavelength; and (2) any combination of three anchors is in a two-dimensional plane. A larger quiet zone could be possible if it is sheltered by many closely spaced anchors to prevent noise from penetrating between anchors. It is not yet possible to test the proposed methods in such a large scale with our available hardware and programming skills.

A further study is in the way to find faster algorithms for minimizing $\|\hat{e}_t\|$, or program available ones in a more efficient way. A possible attempt is to combine online economic solution with offline best solution to improve performance

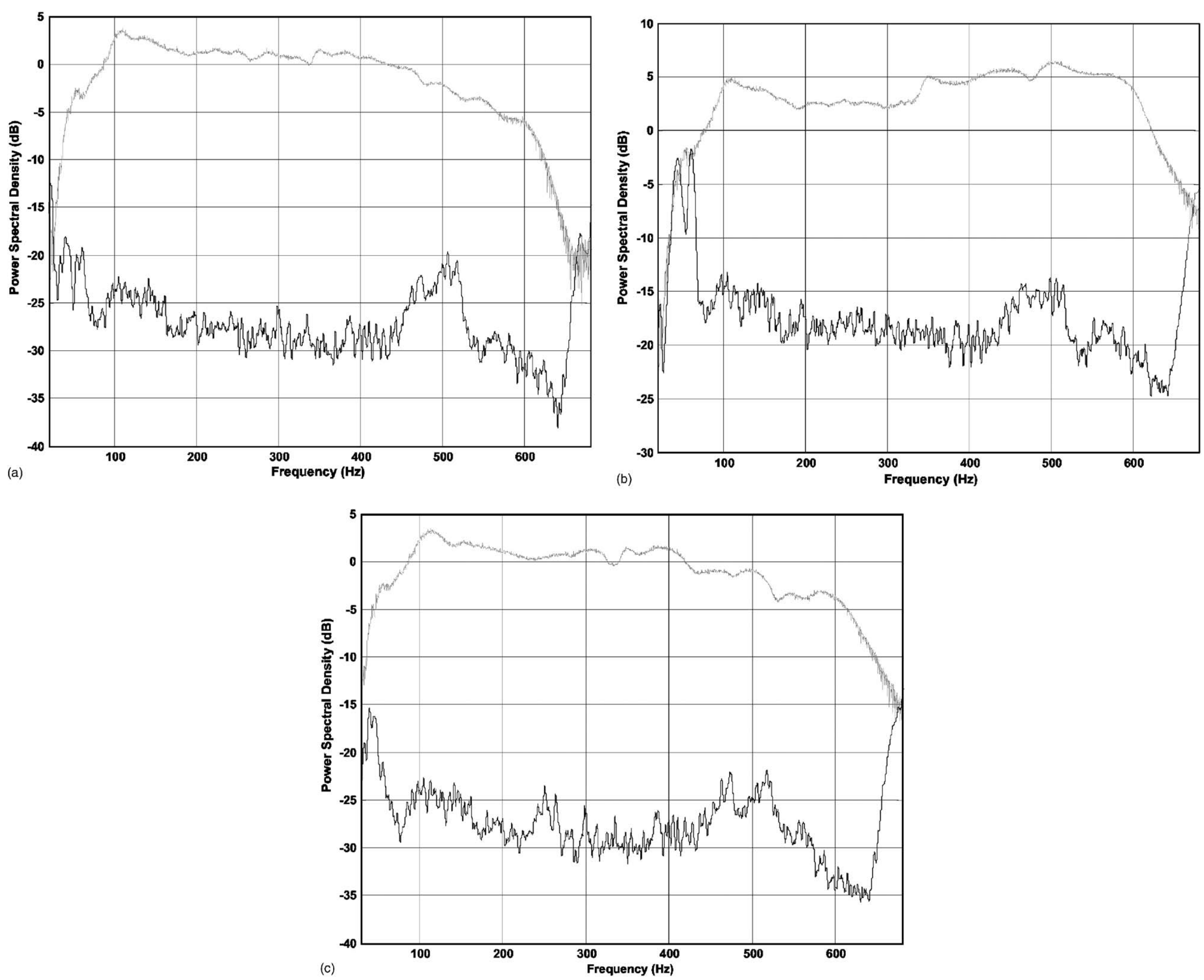

FIG. 4. Normalized PSDs of uncontrolled (gray) and controlled (black) noise, measured by error sensors (a), (b), and (c).

without heavy online computations. It is reasonable to expect more inputs/outputs in the near future when faster hardware or algorithms are available.

VII. CONCLUSION

Orthogonal adaptation is extended to multichannel MIANC systems in this study. An important issue is online optimization of a feedforward controller. This problem is solved here with the best achievable, a practical and an economical solution. A simple RLS algorithm is presented for online modeling of multichannel systems. Experimental results are presented to verify the performance of the extended algorithm. In the present stage, only the economical solution is implemented in experiments with limited inputs/outputs. As a result of rapid advance of technologies, it will eventually become possible to implement the best solution with more inputs/outputs when faster hardware is available.

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