JULY 01 2006

Orthogonal adaptation for active noise control

Jing Yuan

J. Acoust. Soc. Am. 120, 204–210 (2006)
https://doi.org/10.1121/1.2202908

Orthogonal adaptation for active noise control

Jing Yuan[a)]
Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hunghom, Kowloon, Hong Kong

(Received 12 January 2006; revised 7 April 2006; accepted 17 April 2006)

Many active noise control (ANC) systems apply the filtered-x least mean squares (FXLMS) algorithm for controller adaptation. The accuracy of path models is an important issue in these systems. Since parameter drifting in a noise field may cause model error between the secondary path and its prestored model in an ANC system, some ANC systems employ two adaptive processes for path modeling and controller adaptation respectively. In this paper, a new ANC system is proposed with adaptive path modeling and nonadaptive controller design. The proposed ANC system is noninvasive without persistent excitations. It avoids the slow convergence and inevitable estimation errors in controller adaptation. A rigorous analysis is presented to prove that the new ANC system will converge to an optimal one in the minimum H_2 norm sense. Experimental results are presented to verify the performance of the proposed ANC system.
 [DOI: 10.1121/1.2202908]

PACS number(s): 43.50.Ki, 43.50.Gf [KA] Pages: 204–210

I. INTRODUCTION

An important principle of active noise control (ANC) is to create quiet zones by generating destructive interference at sensed locations. Since magnitude distributions of sound fields are related to the wavelengths of sound signals, the range of destructive interference is proportional to the wavelengths of sound signals. This makes ANC attractive for controlling low-frequency noise.

The filtered-x least mean squares (FXLMS) algorithm is a popular tool for controller adaptation.[1,2] The algorithm is stable[3–5] if the phase error in the secondary path model is less than 90°. In many applications, environmental or boundary conditions of noise fields are not necessarily constant. There may be parameter drifting in noise fields. Online modeling of the secondary path is adopted in many ANC systems to keep the model as close to the real path as possible.

In most ANC systems, path models are finite impulse response (FIR) filters with many parameters. Accurate estimation of model parameters requires "persistent excitations"[6]—the invasive injection of probing signals into the ANC system.[7] Since the probing signals cannot be cancelled by the controller, some researchers investigate how to regulate the magnitudes of probing signals according to ANC performance,[8,9] others focus on noninvasive identification of secondary paths.[10–12]. An ANC system may perturb controller parameters to estimate the secondary path accurately. The FXLMS is used by existing noninvasive ANC systems for controller adaptation.

In this paper, a new ANC system is proposed with a single adaptation process for noninvasive path modeling. A major difference between the proposed and existing noninvasive ANC systems is the controller that is designed by the orthogonal principle in the new system. The new method only requires the estimated models to share a normal vector with the true paths. Path modeling is easier without persistent excitations. Stability and convergence of the new ANC system is proven via the Lyapunov approach.

With the controller designed by the orthogonal principle, the new ANC system avoids the slow convergence and estimation errors in controller adaptation. It will converge to an optimal ANC system in the minimum H_2 norm sense. The new ANC system uses the least squares (LS) algorithm that tends to converge faster than LMS counterparts.[13] Experimental results are presented to demonstrate the performance of the new ANC system.

II. PROBLEM STATEMENT

A typical ANC system may be described by a block diagram in Fig. 1(a), where $P(z)$, $S(z)$, $F(z)$, and $R(z)$ are the *p*rimary, *s*econdary, *f*eedback, and *r*eference paths, respectively. Path models and signals in Fig. 1 are in the Z-transform domain. Many researchers assume that path models can be approximated by FIR filters with negligible errors (assumption A1). This study is also based on such an assumption. If the primary source $n(z)$ is not available to the controller, a signal $x(z)$ is measured in the noise field to recover the reference $r(z)$. The controller must cancel feedback signals in $x(z)$ to ensure a stable closed-loop, which requires an estimated model $\hat{F}(z)$ to approximate $F(z)$.

A. Controller structure and objective

It is shown in Fig. 1(a) that $x(z)=R(z)n(z)+F(z)a(z)$. The reference is recovered in Fig. 1(b) as $r(z)=x(z)-\hat{F}(z)a(z)$; here $a(z)$ is the actuation signal. Eliminating $a(z)$ from the above three equations, one obtains

$$r(z) = \frac{R(z)}{1+\Delta F(z)G(z)}n(z) = \tilde{R}(z)n(z), \qquad (1)$$

where $\Delta F(z)=\hat{F}(z)-F(z)$ is the model error of the feedback path. The closed-loop is stable if $\|\Delta F(z)G(z)\|_\infty < 1$, which is

[a)]Electronic mail: mmjyuan@polyu.edu.hk

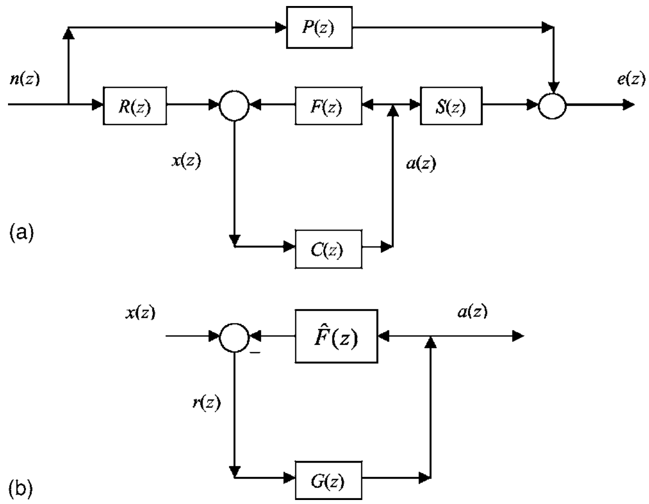

FIG. 1. (a) Block diagram of a typical ANC system with acoustic feedback. (b) Block diagram of ANC controller $C(z)$.

satisfied in most ANC systems. Many researchers assume $\Delta F(z)=0$ (assumption A2) and use $\tilde{R}(z)=R(z)$ in the analysis. The present study also adopts such an assumption since the presence of $\Delta F(z)\neq 0$ only causes unnecessary distractions without affecting the analytical results, as to be verified by experimental results.

In Fig. 1(b), the actuation signal is $a(z)=G(z)R(z)n(z)$ after substitution of $r(z)=R(z)n(z)$ by assumption A2. As a result, the error signal in Fig. 1(a) can be expressed as

$$e(z)=P(z)n(z)+S(z)a(z)=[P(z)+S(z)G(z)R(z)]n(z). \tag{2}$$

The ANC objective is $e(z)=0$ for broadband noise control. The above equation has an ideal solution

$$G(z)=\frac{-P(z)}{S(z)R(z)}. \tag{3}$$

Practically, the ideal controller may not be stable due to possible nonminimum phase (NMP) roots in $S(z)$ or $R(z)$.

For distributed-parameter systems to which the modal theory is applicable, only path transfer functions between collocated sources and sensors are surely minimum phase (MP). It means a MP $R(z)$ if the reference sensor is substantially collocated with the primary source. To avoid the near-field effects, however, the error sensor is usually placed far away from the secondary source and $S(z)$ is NMP without a stable inverse. A practically achievable objective of the ANC system is the minimization of

$$\|P(z)+S(z)G(z)R(z)\|_2 \tag{4}$$

in the H_2 norm sense. The proposed ANC system is intended to achieve this objective.

B. Online path modeling

When the ANC system recovers the reference signal $r(z)=R(z)n(z)$ to synthesize the actuation signal $a(z)=G(z)r(z)$ and estimate the path models, Eq. (2) may be rewritten as

$$e(z)=H(z)r(z)+S(z)a(z)=[H(z)+S(z)G(z)]r(z), \tag{5}$$

where an equivalent primary path $H(z)=P(z)/R(z)$ is introduced as if the noise field was caused by $r(z)$. If the reference sensor is substantially collocated with the primary source, $R(z)$ is MP with a stable inverse. It is assumed that the equivalent primary path $H(z)$ can be approximated by a FIR filter with negligible errors (assumption A3). The ANC objective is modified from the minimization Eq. (4) to the minimization of

$$\|H(z)+S(z)G(z)\|_2. \tag{6}$$

This is a practical approach due to the unavailability of $n(z)$. The proposed ANC system is able to minimize Eq. (4) if $n(z)$ is available.

The ANC system estimates the equivalent primary path $H(z)$ and the secondary path $S(z)$ simultaneously by minimizing estimation error

$$\begin{aligned}\|\varepsilon(z)\|_2&=\|e(z)-\hat{H}(z)r(z)-\hat{S}(z)a(z)\|_2\\&=\|\Delta H(z)r(z)+\Delta S(z)a(z)\|_2,\end{aligned} \tag{7}$$

where $\hat{H}(z)$ and $\hat{S}(z)$ are online models of $H(z)$ and $S(z)$, respectively. The model errors are $\Delta H(z)=H(z)-\hat{H}(z)$ and $\Delta S(z)=S(z)-\hat{S}(z)$. Let $\hat{h}=[\hat{h}_1\cdots\hat{h}_m]$, $h=[h_1\cdots h_m]$, $\hat{s}=[\hat{s}_1\ldots\hat{s}_m]$ and $s=[s_1\cdots s_m]$ denote coefficient vectors of $\hat{H}(z)$, $H(z)$, $\hat{S}(z)$, and $S(z)$, respectively. The time domain version of Eq. (7) would be

$$\begin{aligned}\sum\varepsilon_t^2&=\sum\left(e_t-\sum_{k=1}^{m}\hat{h}_k r_{t-k}-\sum_{k=1}^{m}\hat{s}_k a_{t-k}\right)^2\\&=\sum\left(\sum_{k=1}^{m}\Delta h_k r_{t-k}+\sum_{k=1}^{m}\Delta s_k a_{t-k}\right)^2,\end{aligned} \tag{8}$$

where $\{\Delta h_k=h_k-\hat{h}_k\}$ and $\{\Delta s_k=s_k-\hat{s}_k\}$. The summation of error squares is carried out in a sliding time window. Equation (8) may be written as

$$\begin{aligned}\sum\varepsilon_t^2&=\sum\left(e_t-\sum_{k=1}^{m}\hat{h}_k\psi_k(t)-\sum_{k=1}^{m}\hat{s}_k\varphi_k(t)\right)^2\\&=\sum\left(\sum_{k=1}^{m}\Delta h_k\psi_k(t)+\sum_{k=1}^{m}\Delta s_k\varphi_k(t)\right)^2\end{aligned} \tag{9}$$

by introducing $\psi_k(t)=r_{t-k}$ and $\varphi_k(t)=a_{t-k}$.

To aid the stability analysis, one may express model errors $\Delta H(z)$ and $\Delta S(z)$ in terms of coefficient vectors $\Delta_h^T=[\Delta h_1,\ldots,\Delta h_m]$, $\Delta_s^T=[\Delta s_1,\ldots,\Delta s_m]$ and $\Delta_\theta^T=[\Delta_h^T,\Delta_s^T]$. Similarly, the signal samples are represented by regression vectors $\psi^T(t)=[\psi_1(t),\ldots,\psi_m(t)]$, $\varphi^T(t)=[\varphi_1(t),\ldots,\varphi_m(t)]$, and $\phi^T(t)=[\psi^T(t),\varphi^T(t)]$. As a result, Eq. (9) may be seen as the square sum of a linear projection $\varepsilon_t=\phi^T(t)\Delta_\theta$. The convergence of $\varepsilon_t\to 0$ does not necessarily mean $\Delta_h\to 0$ or $\Delta_s\to 0$. For this reason, some researchers propose to perturb the ANC controller and hence perturb $a(z)$ and $\varphi(t)$. The projection of Δ_θ on a time-varying vector $\phi(t)$ becomes the projection on linearly independent vectors if $\varphi(t)$ is perturbed properly. This is exactly the idea of persistent excitation.[6]

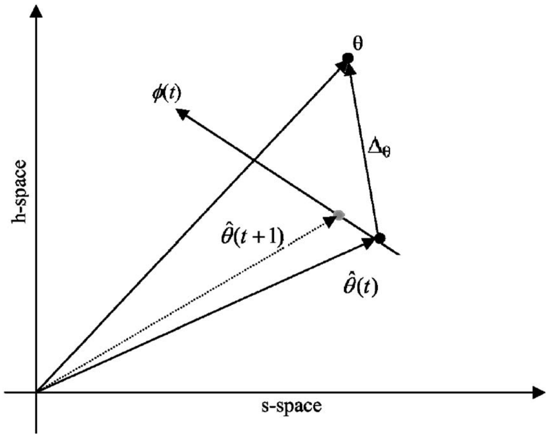

FIG. 2. Illustration of the orthogonal adaptation algorithm.

Identification accuracy depends analytically on the level of persistent excitation to the regression vector[14,15] whose elements are samples of $a(z)$ in an ANC system. Since controller perturbation is equivalent to persistent excitation of $a(z)$, it is still an invasive method. Only the persistent excitation is generated by a different method.

A significant difference between the proposed and existing noninvasive ANC systems is the absence of persistent excitations in the new system. Optimal ANC is possible if Eq. (9) is minimized. The remaining part of the paper is intended to explain how this is possible.

C. Orthogonal adaptation

A visual illustration of the proposed method is presented in Fig. 2, before a rigorous analysis in the next section. The vertical and horizontal axes of Fig. 2 represent, respectively, the h space and s space. These are vector spaces of the coefficients of $H(z)$ and $S(z)$, respectively.

By assumption A1, the true path transfer functions $H(z)$ and $S(z)$ are represented by coefficient vector $\theta^T=[h^T,s^T]$ in Fig. 2. Similarly, path models $\hat{H}(z)$ and $\hat{S}(z)$ are represented by coefficient vector $\hat{\theta}^T(t)=[\hat{h}^T,\hat{s}^T]$. It is reasonable to assume a constant θ since the drift of acoustical parameters is significantly slower than the adaptation rate. The purpose of Fig. 2 is to show that (*i*) $\hat{\theta}(t+1)$ will be closer to θ than $\hat{\theta}(t)$ and (*ii*) optimal ANC is possible without persistent excitations.

An important feature of the proposed ANC system is to design a controller $G(z)$ that forces $\phi(t)$ as orthogonal to $\hat{\theta}(t)$ as possible in every step of the adaptation. To keep a better focus, detailed design of $G(z)$ will be delayed to Sec. IV. The effect of this principle is illustrated here. Since perfectly orthogonal between $\phi(t)$ and $\hat{\theta}(t)$ may not be possible, $\phi(t)$ is not perfectly orthogonal to $\hat{\theta}(t)$ in Fig. 2. The estimation error is rewritten as

$$\varepsilon_t=\sum_{k=1}^{m}\Delta h_k\psi_k(t)+\sum_{k=1}^{m}\Delta s_k\varphi_k(t)=\phi^T(t)\Delta_\theta. \quad (10)$$

Online path modeling is updated by

$$\hat{\theta}(t+1)=\hat{\theta}(t)+\kappa\varepsilon_t\phi(t), \quad (11)$$

where $\kappa>0$ is a positive constant. Since $\hat{\theta}(t+1)$ is updated by adding a fraction of $\phi(t)$ to $\hat{\theta}(t)$, it will bias to a new point in Fig. 2, depending on the sign of $\varepsilon_t=\phi^T(t)\Delta_\theta$.

In the present form of Fig. 2, $\phi^T(t)\Delta_\theta>0$ and $\hat{\theta}(t+1)$ will bias in the same direction of ϕ to be closer to θ. If the direction of ϕ flips by 180°, then $\phi^T(t)\Delta_\theta<0$ and $\hat{\theta}(t+1)$ will bias in the opposite direction of ϕ to be closer to θ. In the next section, a rigorous stability analysis will prove the monotonous reduction of $\|\Delta_\theta\|_2$ until ε_t converges to zero. For the new ANC system, $\varepsilon_t\to 0$ is good enough to ensure stable ANC performance. The key measure is to force $\phi^T(t)\hat{\theta}(t)=0$ for all t. This condition and $\phi^T(t)\Delta_\theta=0$ mean $\phi^T(t)\theta=\sum_{k=1}^m h_k r_{t-k}+\sum_{k=1}^m s_k a_{t-k}=0$. As a result, the proposed ANC system converges to an optimal ANC controller such that $\|H(z)r(z)+S(z)a(z)\|_2=0$. A rigorous proof is presented in the next section.

III. STABILITY ANALYSIS

Since the proposed ANC system involves a single adaptation process, there are no coupling effects between adaptation processes. When the orthogonal condition $\phi^T(t)\hat{\theta}(t)\approx 0$ is enforced by the nonadaptive method of Sec. IV, the present section will focus on the adaptation process when analyzing the stability of the overall system.

A. Convergence of estimation error

All recursive identification algorithms depend on the regression vector $\phi(t)$ to update the parameter vector. The difference between algorithms is the multiplier to $\phi(t)$. Here a simple LS algorithm is adopted with a scalar multiplier $\kappa=1/\|\phi(t)\|^2$. As a result, Eq. (11) becomes

$$\hat{\theta}(t+1)=\hat{\theta}(t)+\frac{\varepsilon_t}{\|\phi(t)\|^2}\phi(t). \quad (12)$$

The convergence of Eq. (12) may be analyzed with the help of a positive definite function $V(t)=\Delta_\theta^T(t)\Delta_\theta(t)$. It can be verified that

$$V(t+1)-V(t)=[\Delta_\theta(t+1)-\Delta_\theta(t)]^T[\Delta_\theta(t+1)+\Delta_\theta(t)]. \quad (13)$$

Using Eq. (12) and $\Delta_\theta=\theta-\hat{\theta}$, one can write

$$\Delta_\theta(t+1)-\Delta_\theta(t)=\hat{\theta}(t)-\hat{\theta}(t+1)=-\varepsilon_t\phi(t)/\|\phi(t)\|^2, \quad (14)$$

and

$$\begin{aligned}\Delta_\theta(t+1)+\Delta_\theta(t)&=2\theta-\hat{\theta}(t)-\hat{\theta}(t+1)\\&=2\Delta_\theta(t)-\varepsilon_t\phi(t)/\|\phi(t)\|^2. \quad (15)\end{aligned}$$

Combining Eqs. (13)–(15), one obtains

$$V(t+1)-V(t)=\frac{-\varepsilon_t}{\|\phi(t)\|^2}\phi^T(t)\left[2\Delta_\theta(t)-\frac{\varepsilon_t}{\|\phi(t)\|^2}\phi(t)\right]. \quad (16)$$

The above equation becomes

$$V(t+1)-V(t)=-\frac{\varepsilon_t^2}{\|\phi(t)\|^2}\leqslant 0 \quad (17)$$

by substitution of $\varepsilon_t=\phi^T(t)\Delta_\theta$. Therefore $V(t)=\Delta_\theta^T(t)\Delta_\theta(t)$ decreases monotonously until $\varepsilon_t\to 0$.

B. Divide and conquer

The performance of the ANC system is judged by the error signal $e(z)$. The time domain expression of this signal is given by

$$e_t=\sum_{k=1}^{m}h_k r_{t-k}+\sum_{k=1}^{m}s_k a_{t-k}=\phi^T(t)\theta. \quad (18)$$

If one subtracts

$$\phi^T(t)\hat{\theta}(t)=\sum_{k=1}^{m}\hat{h}_k r_{t-k}+\sum_{k=1}^{m}\hat{s}_k a_{t-k} \quad (19)$$

from Eq. (18) and then adds it back, the result would be

$$e_t=\varepsilon_t+\sum_{k=1}^{m}\hat{h}_k r_{t-k}+\sum_{k=1}^{m}\hat{s}_k a_{t-k}=\varepsilon_t+\phi^T(t)\hat{\theta}(t). \quad (20)$$

The error signal is now divided into two parts to be conquered by two respective methods.

The first part of e_t is ε_t. It is conquered by online path modeling. In the previous subsection, a simple LS algorithm is proven able to drive the convergence of $\varepsilon_t\to 0$. There are many other LS or LMS algorithms suitable for this purpose. The difference is the convergence rate and computational load.[6]

The second part of e_t is $\phi^T(t)\hat{\theta}(t)$. Several methods will be discussed to force $|\phi^T(t)\hat{\theta}(t)|\approx 0$ in every step of the adaptation. The convolution of Eq. (19) implies the equivalence between $|\phi^T(t)\hat{\theta}(t)|\approx 0$ and $\|\hat{H}(z)+\hat{S}(z)G(z)\|_2\approx 0$ with coefficients of $\hat{H}(z)$ and $\hat{S}(z)$ taken from $\hat{\theta}(t)$.

C. Co-plane requirement versus persistent excitation

Since $\varepsilon_t=\phi^T\Delta_\theta$ is an inner product, its convergence to zero does not necessarily mean $\Delta_\theta\to 0$. However, if $|\phi^T(t)\hat{\theta}(t)|\approx 0$ for all t, $\varepsilon_t=\phi^T\Delta_\theta\to 0$ implies the co-plane relation of θ and $\hat{\theta}$. This is the combined result of two methods employed by the proposed ANC system to conquer the two parts of Eq. (20). In Fig. 2, one can see that forcing $|\phi^T(t)\hat{\theta}(t)|\approx 0$ and driving $\varepsilon_t=\phi^T\Delta_\theta\to 0$ is very cooperative. It is therefore possible to achieve optimal ANC performance without persistent excitations.

IV. OPTIMAL H_2 NORM CONTROLLER

The key feature of the proposed system is how to design a controller such that $\phi(t)$ is as orthogonal to $\hat{\theta}$ as possible. This is not a problem if $\hat{S}(z)$ is MP in every step of the adaptation process. The controller has a simple form $G(z)=-\hat{H}(z)/\hat{S}(z)$, and $\hat{\theta}$ is perfectly orthogonal to $\phi(t)$, because $\phi^T(t)\hat{\theta}=0$ is equivalent to

$$\hat{H}(z)r(z)+\hat{S}(z)a(z)=\left[\hat{H}(z)-\hat{S}(z)\frac{\hat{H}(z)}{\hat{S}(z)}\right]r(z)=0. \quad (21)$$

In practice, the error sensor is not collocated with the secondary source to avoid the near-field effects. $S(z)$ and $\hat{S}(z)$ are very likely to be NMP. Design methods must be studied to find $G(z)$ such that $\hat{\theta}$ is as orthogonal to $\phi(t)$ as possible, which is the focus of the present section. This is equivalent to minimizing $\|\hat{H}(z)+\hat{S}(z)G(z)\|_2$ by an optimal controller $G(z)$.

A. IIR solution

When $\hat{S}(z)$ is NMP, it is possible to achieve optimal performance in the minimum H_2 norm sense using an infinite impulse response (IIR) filter. The first step is the factorization of $\hat{S}(z)=S_m(z)S_n(z)$ where $S_m(z)$ and $S_n(z)$ are the MP and NMP parts of $\hat{S}(z)$, respectively. If $S_n(z)=\Sigma_{i=0}^{d}s_{ni}z^{-i}$, then $\tilde{S}_n(z)=\Sigma_{i=0}^{d}s_{n(d-i)}z^{-i}$ can be obtained by using coefficients of $S_n(z)$ in the reversed order. Since roots of $S_n(z)$ are $\{|r_i|>1\}$, $\tilde{S}_n^{-1}(z)$ is stable because roots of $\tilde{S}_n(z)$ are $\{|1/r_i|<1\}$.[16] One may write

$$\hat{S}(z)=S_n(z)S_m(z)=\frac{S_n(z)}{\tilde{S}_n(z)}\tilde{S}_n(z)S_m(z), \quad (22)$$

where $|\tilde{S}_n(e^{j\omega})|=|e^{-dj\omega}S_n(e^{-j\omega})|=|S_n(e^{j\omega})|$ for all ω[16] and $S_n(z)/\tilde{S}_n(z)$ is a stable all-pass filter.

Equation (22) implies the factorization of $\hat{S}(z)=F_a(z)F_m(z)$ into the product of an all-pass filter $F_a(z)=S_n(z)/\tilde{S}_n(z)$ and a MP filter $F_m(z)=\tilde{S}_n(z)S_m(z)$. The objective is to minimize $\|[\hat{H}(z)+\hat{S}(z)G(z)]r(z)\|_2$, which is equivalent to the minimization of

$$\|\hat{H}+\hat{S}G\|_2=\|F_a(z)\|_2\|F_a^{-1}\hat{H}+F_mG\|_2=\|F_a^{-1}\hat{H}+F_mG\|_2, \quad (23)$$

where $\|F_a(z)\|_2=1$ since it is an all-pass filter. One may apply the long-division to obtain

$$F_a^{-1}(z)\hat{H}(z)=\frac{\tilde{S}_n(z)\hat{H}(z)}{S_n(z)}=\frac{H_r(z)}{S_n(z)}+H_q(z), \quad (24)$$

where $H_q(z)$ and $H_r(z)$ are, respectively, the quotient and remainder polynomials. The two parts on the right-hand side of Eq. (24) are orthogonal to each other in the H_2 norm sense.

Since $H_r(z)/S_n(z)$ is the unstable part of Eq. (24), it cannot be cancelled by any stable feedforward controller. Optimal control, in the minimum H_2 norm sense, is the cancellation of the stable part of $F_a^{-1}(z)\hat{H}(z)$ by a controller in the form of

$$G(z) = -F_m^{-1}(z)H_q(z) = -\frac{H_q(z)}{\tilde{S}_n(z)S_m(z)}. \quad (25)$$

Substituting Eqs. (24) and (25) into Eq. (23), one minimizes $\|\hat{H}+\hat{S}G\|_2=\|H_r/\tilde{S}_n\|_2$ in the H_2 norm sense. Any other feedforward controller only increases $\|\hat{H}+\hat{S}G\|_2$ if it replaces $G(z)$. This method requires online root-finding and heavy computations. It is desired to design a suboptimal controller in a less expensive way, which will be discussed in the next subsection.

B. FIR solution

With some sacrifice of performance, it is possible to find a FIR filter controller $G(z)$ and minimize $\|\hat{H}(z)+\hat{S}(z)G(z)\|_2 =\|Q(z)\|_2$. When $G(z)$ is a FIR filter, $Q(z)$ is also a FIR filter with coefficients given by

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ \\ \vdots \\ q_{2m-1} \end{bmatrix} = \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \vdots \\ \hat{h}_m \\ \\ \\ \end{bmatrix} + \begin{bmatrix} \hat{s}_1 & & & \\ \hat{s}_2 & \hat{s}_1 & & \\ \vdots & \hat{s}_2 & \ddots & \\ \hat{s}_m & \vdots & \ddots & \hat{s}_1 \\ & \hat{s}_m & \vdots & \hat{s}_2 \\ & & \ddots & \vdots \\ & & & \hat{s}_m \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix}$$

$$= \hat{h} + M_s g, \quad (26)$$

where $q^T=[q_1,\ldots,q_{2m-1}]$ and $g^T=[g_1,\ldots,g_m]$ are coefficient vectors of $Q(z)$ and $G(z)$, respectively.

According to Parserval's theorem, minimization of $\|\hat{H}(z)+\hat{S}(z)G(z)\|_2=\|Q(z)\|_2$ is equivalent to minimization of $\|q\|^2$. The focus is now directed to

$$q^Tq = (\hat{h}+M_sg)^T(\hat{h}+M_sg) = \hat{h}^T\hat{h} + \hat{h}^TM_sg + g^TM_s^T\hat{h} + g^TR_sg, \quad (27)$$

where $R_s=M_s^TM_s$ is the autocorrelation matrix of the impulse response of $\hat{S}(z)$. Introducing $p=M_s^T\hat{h}$, one may substitute $g^TM_s^T\hat{h}=g^TR_sR_s^{-1}p$ into Eq. (27) to write

$$q^Tq = \hat{h}^T\hat{h} - p^TR_s^{-1}p + (R_sg+p)^TR_s^{-1}(R_sg+p), \quad (28)$$

where parameter vector g only affects $(R_sg+p)^TR_s^{-1}(R_sg+p)$. The minimization of q^Tq, subject to the constraint of a FIR $G(z)$, has a unique solution $R_sg=-p$.

The structure of R_s depends on the structure of M_s. Let m_i denote the ith column of M_s. An examination of Eq. (26) will enable one to see that m_i and m_j are related to each other by $|i-j|$ vertical shifts. Let $r_s(i,j)$ denote the (i,j)th element of R_s. Then

$$r_s(i,j) = m_i^Tm_j = m_j^Tm_i = \sum_{k=1}^{m-|i-j|} \hat{s}_k\hat{s}_{k+|i-j|}. \quad (29)$$

It turns out that R_s is a Toeplitz matrix. There are many fast algorithms for the solution of $R_sg=-p$ by taking advantages of the Toeplitz structure of R_s.[17] Most of them are available commercially in the Digital Signal Processing (DSP) block sets of MATLAB. The FIR controller is suboptimal if compared with its IIR counterpart, but it is less computationally expensive.

C. Iterative FIR solution

In many ANC applications, $H(z)$ and $S(z)$ must be approximated by FIR filters with sufficiently large numbers of coefficients. The degree m may be very large so that direct solution of $R_sg=-p$ is still expensive even using the available efficient algorithms. In that case, it is possible to consider iterative minimization of q^Tq.

Consider a positive definite function $J=0.5qTq$ and an iterative algorithm that keeps modifying g to minimize J. The time derivative of J is given by

$$\dot{J} = q^TM_s\dot{g} \quad (30)$$

where Eq. (26) has been used to link $\dot{q}$ to $\dot{g}$. The above equation suggests a very simple way to modify g. It is given by

$$\dot{g} = -\mu M_s^Tq. \quad (31)$$

where μ is a small positive constant. When coded in a high-level computer language, such as MATLAB, this is equivalent to an instruction "$g=g-\mu M_s^Tq\delta t$." Combining Eqs. (30) and (31), one can see that $\dot{J}=-\mu q^TM_sM_sq\leqslant 0$, which means $J=0.5qTq$ will be minimized by the simple modification rule of Eq. (31).

The only advantage of iterative solution is the reduced computations. It comes with a further sacrifice that $J=0.5qTq$ is not minimized instantly. If adaptation speed is not a critical issue, this method may be used to reduce the cost of the ANC system.

V. EXPERIMENTAL VERIFICATION

An experiment was conducted to verify the analytical results. A feedforward ANC was implemented in a duct with a cross-sectional area of 11×14.5 cm^2. The primary source was generated by a 4-in. loudspeaker placed at the upstream end of the duct. It was excited by the pseudo-random noise $n(z)$ that was not available to the ANC system. The secondary source was a 4-in. loudspeaker placed at the midpoint of the 2-m duct. A microphone sensor was placed in front of the primary source to measure the reference signal. The error sensor was another microphone placed 0.2 m downstream from the secondary source to avoid the near field effects of the secondary source. The sampling frequency of the system was 2.5 kHz, and all signals were low-pass filtered with a cutoff frequency 950 Hz.

The block diagram of the experimental system is the same as a typical ANC system in Fig. 1. Since the duct is a resonant system, the downstream reflection joins the secondary signal to contaminate the measured signal $x(z)=R(z)n(z)+F(z)a(z)$. Although the reference sensor was placed in front of the primary source, $R(z)\neq 1$ due to the resonant effects. The collocation of the reference sensor and the primary source causes a MP $R(z)$. Since the error signal was not collocated with the secondary source, the secondary path $S(z)$ was NMP. In the experiment, both $F(z)$ and $S(z)$

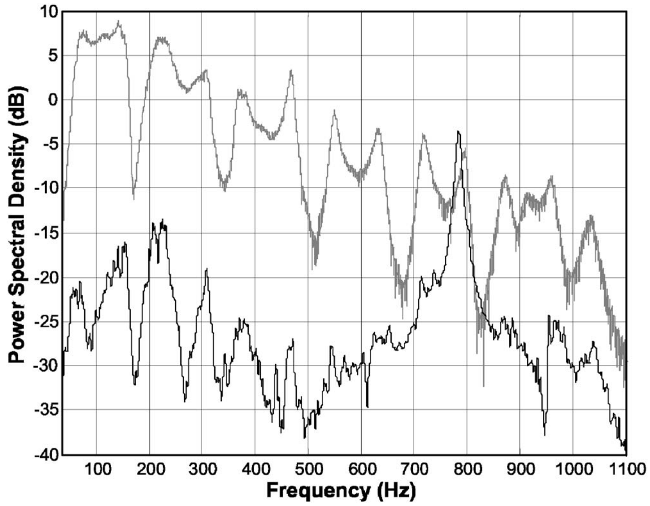

FIG. 3. Power spectral density plots of uncontrolled (gray) and controlled (black) noise.

were not available to the ANC system. A FIR controller with $m=400$ coefficients was implemented using the method of Sec. IV B. The optimal IIR controller, discussed in Sec. IV A, was not implemented due to its heavy computations for online root-finding.

The experimental results are plotted in Fig. 3 as two curves. The gray curve represents the normalized power spectral density (PSD) $|e(z)/r(z)|$ of the uncontrolled noise. It was obtained when the active controller was turned off. The black curve plots the normalized PSD $|e(z)/r(z)|$ of the controlled noise. It demonstrates significant noise suppression effects in most frequencies except a small region around 780 Hz. The ANC enhanced noise near 780 Hz instead of suppressing it. In the experiment, broadband noise was heard when the ANC system started. As the controller converged, audible noise became weaker and eventually reduced to a weak single tone noise.

The poor performance near 780 Hz was due to the NMP zero of $S(z)$. In Fig. 4, the magnitude responses of the primary and secondary paths are shown. The secondary path has zeros (dips) in a number of frequencies. When the error sensor was not too far away from the secondary source, most zeros of $S(z)$ were MP except one near 780 Hz. A NMP $S(z)$ is the problem of all ANC systems. Even if both $H(z)$ and $S(z)$ were available without errors, perfect cancellation of broadband noise would still be impossible for a single-source feedforward ANC system, since $-S^{-1}(z)H(z)$ is not stable. It is possible to design an IIR controller to match the stable part of $-S^{-1}(z)H(z)$ for optimal ANC performance. In simulations, the IIR controller neither enhances nor suppresses noise at the NMP zeros. Its FIR counterpart reduces online computations significantly with inevitable sacrifice in ANC performance that is most visible near the NMP zeros of $S(z)$. Since heavy online computations prevented the testing of the IIR controller at 2.5-kHz sampling rate, a FIR solution was tested to approximate $-S^{-1}(z)H(z)$. The experimental performance of the FIR controller is very similar to its simulation performance.

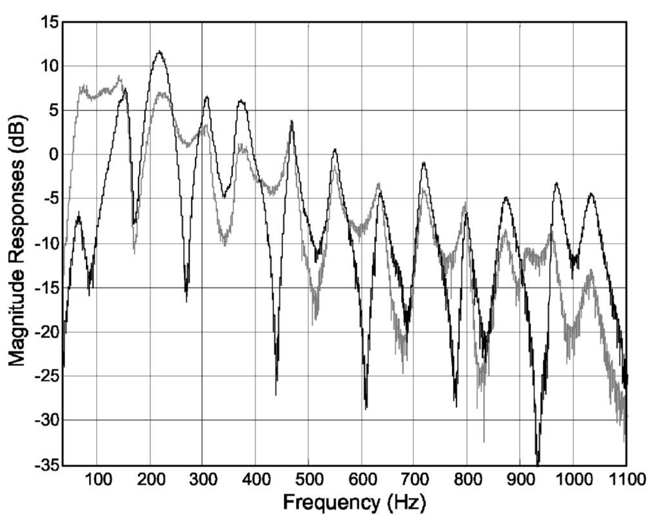

FIG. 4. Magnitude responses of primary (gray) and secondary (black) paths.

In Eq. (20), the ANC control error is divided into $e_t = \varepsilon_t + \phi^T(t)\hat{\theta}(t)$. All LS or LMS algorithms are able to drive the convergence of $\varepsilon_t \rightarrow 0$ regardless the zeros of $S(z)$. The NMP zeros of $\hat{S}(z)$ only hinder the minimization of $|\phi^T(t)\hat{\theta}(t)|$. The FIR controller is not able to minimize $|\phi^T(t)\hat{\theta}(t)|$ as well as an IIR controller, but it is stable and achieved the smallest $|\phi^T(t)\hat{\theta}(t)|$ achievable by a FIR controller in the H_2 norm sense.

The NMP problem is solvable by adding an extra secondary source. If the secondary paths are co-prime, it is possible to achieve perfect cancellation performance.[18,19] A further study is conducted to integrate the proposed method with the extra-speaker method. The main obstacle is online computations. If a controller involves the online inverse of an $m \times m$ matrix, then two controllers involve the online inverse of a $2m \times 2m$ matrix. This means substantial reduction of sampling rate and controller bandwidth. Faster methods are sought for online minimization of $|\phi^T(t)\hat{\theta}(t)|$.

VI. CONCLUSION

An orthogonal adaptation algorithm is proposed for noninvasive adaptive noise control. All available noninvasive adaptive algorithms consist of two adaptation processes: one for online path modeling and the other for controller tuning. The proposed method solves controller parameters to speed up system convergence. This is the first feature of the proposed method. Some of the noninvasive algorithms perturb controllers to avoid the probing signals. Those methods are not truly noninvasive since controller perturbation is an invasive measure. The second feature of the proposed method is the replacement of persistent excitation with the co-plane requirement. It leads to a truly noninvasive adaptive ANC system without controller perturbation or probing signals. All available noninvasive algorithms involve two adaptation processes; it is very difficult to analyze the coupling effects of the adaptation processes and derive rigorous stability proofs. For the proposed method a rigorous stability analysis is presented using the orthogonal projection method plus the Lyapunov approach. It is proven that the proposed controller will converge to an optimal one in the minimum H_2 norm sense. Experimental results are presented to demonstrate the performance of the proposed method.

[1]C. H. Hansen and S. D. Snyder, *Active Control of Noise and Vibration* (E and FN Spon, London, 1997).

[2]P. A. Nelson and S. J. Elliott, *Active Control of Sound* (Academic, London, 1992).

[3]M. A. Vaudrey, W. T. Baumann, and W. R. Saunders, "Stability and operation constraints of adaptive LMS-based feedback control," Automatica **39**, 595–605 (2003).

[4]E. Bjarnason, "Analysis of the filtered-x LMS algorithm," IEEE Trans. Speech Audio Process. **3**(3), 504–514 (1995).

[5]S. D. Snyder and C. H. Hansen, "The influence of transducer transfer functions and acoustic time delay on the LMS algorithm in active noise control systems," J. Sound Vib. **140**(3), 409–424 (1990).

[6]G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction and Control* (Prentice-Hall, Englewood Cliffs, NJ, 1984).

[7]L. J. Eriksson and M. C. Allie, "Use of random noise for on-line transducer modeling in an adaptive active attenuation system," J. Acoust. Soc. Am. **85**, 797–802 (1989).

[8]M. Zhang, H. Lan, and W. Ser, "Cross-updated active noise control system with online secondary path modeling," IEEE Trans. Speech Audio Process. **9**, 598–602 (2000).

[9]M. Zhang, H. Lan, and W. Ser, "A robust online secondary path modeling method with auxiliary noise power scheduling strategy and norm constraint manipulation," IEEE Trans. Speech Audio Process. **11**(1), 45–53 (2003).

[10]W. C. Nowlin, G. S. Guthart, and G. K. Toth, "Noninvasive system identification for multichannel broadband active noise control," J. Acoust. Soc. Am. **107**, 2049–2060 (2000).

[11]X. Qiu and C. H. Hansen, "An algorithm for active control of transformer noise with on-line cancellation path modelling based on the perturbation method," J. Sound Vib. **240**(4), 647–665 (2001).

[12]B. J. Kim and D. C. Swanson, "Linear independence method for system identification/secondary path modeling for active control," J. Acoust. Soc. Am. **118**(3), 1452–1468 (2005).

[13]B. Widrow and E. Walach, "On the statistical efficiency of the LMS algorithm with nonstationary inputs," IEEE Trans. Inf. Theory **30**(2), 211–221 (1984).

[14]I. Markovsky, J. C. Willems, P. Rapisarda, and B. L. M. De Moor, "Algorithms for deterministic balanced subspace identification," Automatica **41**, 755–766 (2005).

[15]V. Saligrama, "A convex analytic approach to system identification," IEEE Trans. Autom. Control **50**(10), 1550–1566 (2005).

[16]P. A. Regalia, *Adaptive IIR Filtering in Signal Processing and Control* (Dekker, New York, 1995).

[17]J. Makhoul, "Linear prediction: A tutorial review," Proc. IEEE **63**(4), 561–580 (1975).

[18]J. S. Vipperman and R. A. Burdisso, "Adaptive feedforward control of non-minimum phase structural systems," J. Sound Vib. **183**(3), 369–382 (1995).

[19]M. Miyoshi and Y. Kaneda, "Active control of broad-band random noise in a reverberant 3-dimensional space," Noise Control Eng. J. **36**(2), 85–90 (1991).