AUGUST 01 2009

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J. Acoust. Soc. Am. 126, 564–567 (2009)
https://doi.org/10.1121/1.3158917

Design of a non-traditional dynamic vibration absorber (L)

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(Received 20 August 2008; revised 15 March 2009; accepted 26 May 2009)

A non-traditional dynamic vibration absorber is proposed for the minimization of maximum vibration velocity response of a vibrating structure. Unlike the traditional damped absorber configuration, the proposed absorber has a linear viscous damper connecting the absorber mass directly to the ground instead of the main mass. Optimum parameters of the proposed absorber are derived based on the fixed-point theory for minimizing the maximum vibration velocity response of a single-degree-of-freedom system under harmonic excitation. The extent of reduction in maximum vibration velocity response of the primary system when using the traditional dynamic absorber is compared with that using the proposed one. Under the optimum tuning condition of the absorbers, it is proved analytically that the proposed absorber provides a greater reduction in maximum vibration velocity response of the primary system than the traditional absorber. © *2009 Acoustical Society of America.* [DOI: 10.1121/1.3158917]

PACS number(s): 43.40.Tm [JGM] Pages: 564–567

I. INTRODUCTION

Dynamic vibration absorber (DVA) is a passive resonator mounted on a primary system. At a pre-tuned frequency, the resonator absorbs vibration energy and contributes a sharp valley to the frequency response of the primary system. However, peaks still remain at other frequencies in the frequency response of the primary system. Therefore DVA is normally used for passive control of narrowband vibration.[1–4]

The traditional damped vibration absorber has a damper added between the absorber mass m and the primary mass M as shown in Fig. 1(a) in order to limit the vibration amplitude when it experiences lower resonance during system start-up and shut down. However, it is not possible to eliminate steady-state vibrations of the original mass after damping is added in the auxiliary mass-spring system. Optimization of the frequency and damping parameters of the traditional damped vibration absorber for minimizing the maximum amplitude response based on the fixed-point theory was well documented in the textbooks of Den Hartog[1] and Korenev and Reznikov.[2]

The traditional DVA as illustrated in Fig. 1(a) provides a cheap and convenient solution for suppressing vibration displacement amplitude of a single-degree-of-freedom system under harmonic excitation. A damped DVA in non-traditional form as shown in Fig. 1(b) has been proved by Ren[5] and Wong and Cheung[6] to be more effective than the traditional absorber in minimizing the maximum vibration displacement response of the primary system under force and ground motion excitation, respectively. In some cases where the dynamic absorber is used for isolating vibrations in vehicle suspension system,[7] reducing vibration energy of structures,[8] and reducing sound radiation from a vibrating surface,[9–13] the vibration absorber need to be designed such that the maximum vibration velocity in the whole spectrum of the primary system needs to be minimized. The optimum tuning frequency and damping of a vibration absorber are different from those values reported by Ren[5] if the maximum velocity amplitude response is required to be minimized. While the optimum tuning frequency and damping of the traditional vibration absorber for minimizing the maximum velocity response have been reported,[14,15] those of the proposed absorber cannot be found in the literature. In this paper, the tuning frequency and damping of this damped DVA of non-traditional form have been derived for minimizing maximum vibration velocity response of a single-degree-of-freedom system. The derivation of the formulas for the optimum tuning frequency and damping of the absorber was based on the fixed-point theory of Den Hartog.[1] It is proved in Sec. III that the proposed absorber provides a greater reduction in maximum velocity amplitude response of the primary system under harmonic force excitation than the traditional absorber.

II. THE TRADITIONAL DAMPED DVA

A schematic diagram of a traditional damped DVA attached to an undamped mass-spring system under sinusoidal excitation ($f=Fe^{j\omega t}$) is shown in Fig. 1(a). This vibration model is called model A in the following discussion. The non-dimensional velocity amplitude response of the primary mass M may be written as[15]

$$G_{\mathrm{A}} = \left| \frac{\dot{X}_1}{\omega_n X_{\mathrm{st}}} \right|_{\mathrm{A}} = \sqrt{\frac{(2\zeta\beta r^2)^2 + r^2(\beta^2 - r^2)^2}{(1 - r^2 + \mu r^2)^2(2\zeta\beta r)^2 + [(1 - r^2)(\beta^2 - r^2) - \mu\beta^2 r^2]^2}}, \tag{1}$$

where $X_{\mathrm{st}}=F/K$, $\omega_n=\sqrt{K/M}$, $\omega_a=\sqrt{k/m}$, $r=\omega/\omega_n$, $\beta=\omega_a/\omega_n$, and $\zeta=c/2\sqrt{mk}$.

Using the fixed-point theory,[1] the optimum tuning frequency and damping of the absorber leading to minimum vibration amplitude at resonance are written as[15]

$$\beta_{\mathrm{opt\_A}} = \frac{1}{1+\mu}\sqrt{\frac{2+\mu}{2}}, \tag{2a}$$

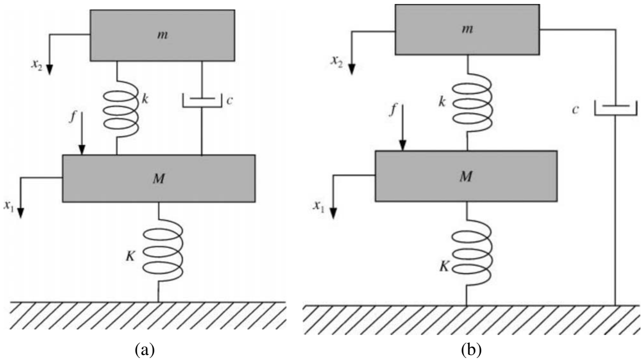

FIG. 1. A damped DVA as an auxiliary mass-spring-damper (m-k-c) system attached to a primary system (M-K): (a) traditional design of the absorber (Ref. 1) and (b) the proposed design of the absorber.

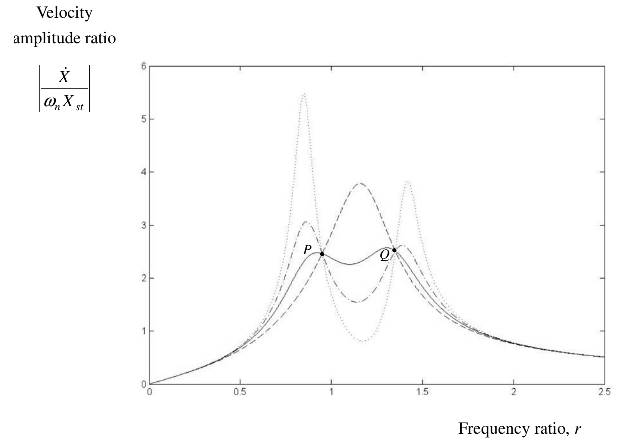

FIG. 2. Velocity amplitude ratio of the primary system of model B at different mass ratios m/M.

and

$$\zeta_{\text{opt\_A}} = \frac{1}{4(2+\mu)}\sqrt{\frac{\mu(24+24\mu+5\mu^2)}{1+\mu}}. \qquad (2b)$$

An approximate value of the velocity amplitude ratio at resonance based on the fixed-point theory is written as

$$G_{\text{A\_opt}} = \sqrt{\frac{2+\mu}{\mu(1+\mu)}}. \qquad (3)$$

Equation (3) above shows that the theoretical limit of the velocity amplitude ratio at resonance is zero when the mass ratio approaches infinity.

III. A VARIANT FORM OF THE DAMPED DVA

A variant form of the damped DVA as shown in Fig. 1(b) is called model B in the following discussion. In Fig. 1(b), the motions of the primary system and the DVA are governed by the following equations:

$$M\ddot{x}_1 = -k(x_1 - x_2) - Kx_1,$$

$$m\ddot{x}_2 = -k(x_2 - x_1) - c\dot{x}_2. \qquad (4)$$

The normalized amplitude of the steady-state response of the primary mass can be derived as

$$G_{\text{B}} = \left|\frac{\dot{X}}{\omega_n X_{\text{st}}}\right| = \sqrt{\frac{(2\zeta\beta r^2)^2 + r^2(\beta^2 - r^2)^2}{(1+\mu\beta^2 - r^2)^2(2\zeta\beta r)^2 + [(1-r^2)(\beta^2-r^2) - \mu\beta^2 r^2]^2}}. \qquad (5)$$

Equation (5) may be rewritten as

$$G_{\text{B}} = \sqrt{\frac{A\zeta^2 + B}{C\zeta^2 + D}}, \qquad (6)$$

where $A=4\beta^2 r^4$, $B=r^2(\beta^2-r^2)^2$, $C=4\beta^2 r^2(1+\mu\beta^2-r^2)^2$, and $D=[(1-r^2)(\beta^2-r^2)-\mu\beta^2 r^2]^2$.

Equation (5) is calculated with mass ratio $\mu=0.2$ at four different damping ratios, and the results are plotted in Fig. 2. It can be observed that there are stationary points P and Q at which the response G_{B} is independent of the damping of the absorber. At the stationary or fixed points P and Q, we may write

$$\frac{A}{C} = \frac{B}{D}, \qquad (7)$$

i.e.,

$$\frac{1}{(1+\mu\beta^2 - r^2)^2} = \frac{(\beta^2 - r^2)^2}{[(1-r^2)(\beta^2-r^2) - \mu\beta^2 r^2]^2} \qquad (8a)$$

or

$$2r^4 - 2(1+\mu\beta^2+\beta^2)r^2 + 2\beta^2 + \mu\beta^4 = 0. \qquad (8b)$$

The solutions of Eq. (8b) are

$$r_{1,2}^2 = \frac{1+(1+\mu)\beta^2 \mp \sqrt{1+2(\mu-1)\beta^2+(1+\mu^2)\beta^4}}{2}. \qquad (9)$$

The sum and the product of the solutions of the equations are

$$r_1{}^2 + r_2{}^2 = 1 + \mu\beta^2 + \beta^2,$$

$$r_1{}^2 r_2{}^2 = \frac{2\beta^2 + \mu\beta^4}{2}. \qquad (10)$$

Consider Eq. (5) and let $\zeta \to \infty$. The height of the fixed points P and Q can be found as

$$G_{\text{B}} = \sqrt{\frac{r^2}{(1+\mu\beta^2 - r^2)^2}}. \qquad (11)$$

Following the procedures used by Den Hartog,[1] the optimum value of β is obtained by $G_{\text{B}}(r_1)=G_{\text{B}}(r_2)$:

$$\frac{r_1}{1+\mu\beta^2 - r_1{}^2} = \frac{-r_2}{1+\mu\beta^2 - r_2{}^2}, \qquad (12)$$

leading to

$$\beta_{\text{opt\_B}} = \sqrt{\frac{1-\sqrt{1-2\mu}}{\mu\sqrt{1-2\mu}}}. \tag{13}$$

Substituting r_1 and $\beta_{\text{opt\_B}}$ into Eq. (11) gives the height of the fixed points:

$$G_B(r_1) = G_B(r_2) = G_{\text{B\_opt}} = \sqrt{\frac{\mu\sqrt{1-2\mu}}{1-\mu-\sqrt{1-2\mu}}}. \tag{14}$$

Equation (14) above shows that the velocity amplitude ratio $G_{\text{B\_opt}}$ of model B with optimum damping and tuning frequency at resonance is zero when the mass ratio is 0.5 while model A can achieve zero velocity amplitude ratio only when the mass ratio approaches infinity.

Brock's[16] approach is applied to find the optimum damping ratio of the absorber in the following. The curve of G_B is required to pass horizontally through the fixed point P which becomes the highest point on the curve of G_B. Consider this curve passing through a point P' of ordinate $G_{\text{B\_opt}}$ and abscissa $r^2 = r_1^2 + \delta$ with δ approaching zero. Rewriting Eq. (5) as

$$\zeta^2 = \frac{r^2(\beta^2 - r^2)^2 - G_B^2[(1-r^2)(\beta^2-r^2) - \mu\beta^2 r^2]^2}{(2\beta r)^2[G_B^2(1+\mu\beta^2 - r^2)^2 - r^2]} \tag{15}$$

and substituting $r^2 = r_1^2 + \delta$ and $G_B^2 = \mu\sqrt{1-2\mu}/1-\mu-\sqrt{1-2\mu}$ into the above equation, we would have a result in the form

$$\zeta^2 = \frac{A_0 + A_1\delta + A_2\delta^2 + \cdots}{B_0 + B_1\delta + B_2\delta^2 + \cdots}. \tag{16}$$

Since G_B is independent of the damping at the fixed point P, Eq. (16) would assume the indeterminate form 0/0 if $\delta = 0$ leading to $A_0 = B_0 = 0$. As δ is a very small number, we may neglect the higher order terms, and the desired result is given by

$$\zeta^2 = \frac{A_1}{B_1}, \tag{17}$$

where

$$A_1 = (r_1^2 - \beta^2)(3r_1^2 - \beta^2) - 2G^2((1-r_1^2)(\beta^2 - r_1^2) - \mu\beta^2 r_1^2)(-1 + 2r_1^2 - \beta^2(1+\mu)), \tag{18}$$

$$B_1 = 4\beta^2 r_1^2(G^2(-2 - 2\mu\beta^2 + 2r_1^2) - 1) + 4G^2(1 + \mu\beta^2 - r_1^2)^2 - 4r_1^2. \tag{19}$$

Now substituting r_1^2 of Eqs. (5), (8a), (8b), and (9) into Eq. (11), we may write

$$\zeta_{r_1}^2 = \frac{(-2+2\alpha+(10-8\alpha)\mu - 7\mu^2)\gamma + 4 - 4\alpha - 4\mu + (5\alpha-7)\mu^2 + 3\mu^3}{\beta^2\mu\alpha(4-4\alpha+4\mu)\gamma + \beta^2\mu\alpha(8-8\alpha-8\mu\alpha-12\mu^2)}, \tag{20}$$

where $\alpha = \sqrt{1-2\mu}$ and $\gamma = \sqrt{2-3\mu^2-(2\mu+2)\sqrt{1-2\mu}}$.

By a similar procedure with r_2^2 of Eq. (9), we obtain

$$\zeta_{r_2}^2 = \frac{(2-2\alpha-(10-8\alpha)\mu + 7\mu^2)\gamma + 4 - 4\alpha - 4\mu + (5\alpha-7)\mu^2 + 3\mu^3}{-\beta^2\mu\alpha(4-4\alpha+4\mu)\gamma + \beta^2\mu\alpha(8-8\alpha-8\mu\alpha-12\mu^2)}. \tag{21}$$

As suggested by Brock,[16] a convenient average value of the damping ratio is selected as the optimum damping ratio which is written as

$$\zeta_{\text{opt\_B}} = \sqrt{\frac{A\sqrt{1-2\mu}+B}{8\mu^2\sqrt{1-2\mu}(-7\mu^2-2\mu+4+(3\mu^2-2\mu-4)\sqrt{1-2\mu})}}, \tag{22}$$

where $A = 16 - 48\mu - 12\mu^2 + 84\mu^3 + 3\mu^4 - 15\mu^5$ and $B = -16 + 64\mu - 28\mu^2 - 112\mu^3 + 61\mu^4 + 38\mu^5$.

The velocity amplitude ratios of the primary system for models A and B under optimum tuning and damping with mass ratio $\mu = 0.2$ were calculated using Eqs. (1), (2a), (2b), (5), (13), and (22), and the results are plotted in Fig. 3. The maximum non-dimensional velocity amplitude of mass M of model B was 18% smaller than that of model A.

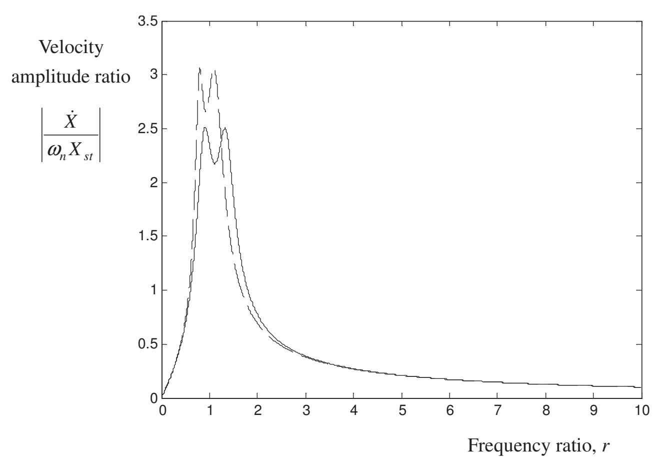

FIG. 3. Comparison of the velocity amplitude ratio between the vibration of the primary system for model A (-----) and model B (solid line) at mass ratio $m/M = 0.2$.

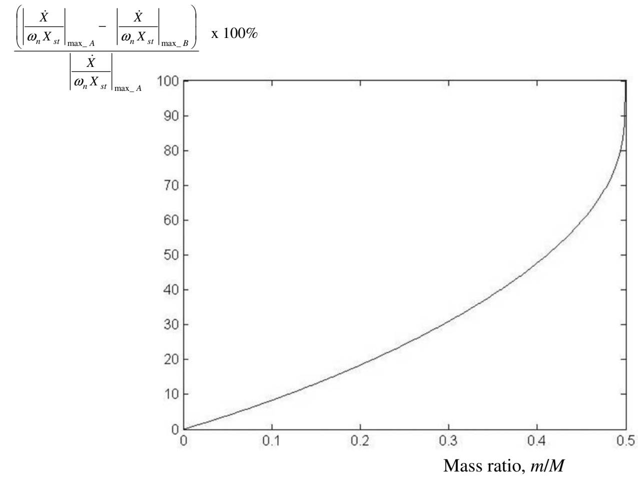

FIG. 4. Percentage reduction in vibration velocity amplitude of M of the proposed absorber relative to the traditional absorber at different mass ratios.

To prove that the proposed absorber is in general superior to the traditional absorber for vibration velocity suppression, the difference of maximum velocity amplitude of the primary system between using the proposed absorber and using the traditional absorber is written as

$$\left(\left|\frac{\dot{X}_1}{\omega_n X_{st}}\right|_{\max\_A}-\left|\frac{\dot{X}_1}{\omega_n X_{st}}\right|_{\max\_B}\right)$$
$$=\sqrt{\frac{2+\mu}{\mu(1+\mu)}}-\sqrt{\frac{\mu\sqrt{1-2\mu}}{1-\mu-\sqrt{1-2\mu}}}$$
$$=\left[\frac{2+\mu}{\mu(1+\mu)}-\frac{\mu\sqrt{1-2\mu}}{1-\mu-\sqrt{1-2\mu}}\right]\Bigg/\left[\sqrt{\frac{2+\mu}{\mu(1+\mu)}}+\sqrt{\frac{\mu\sqrt{1-2\mu}}{1-\mu-\sqrt{1-2\mu}}}\right]$$
$$=\left[\frac{2+\mu}{\mu(1+\mu)}-\frac{(1-\mu+\sqrt{1-2\mu})\mu\sqrt{1-2\mu}}{(1-\mu)^2-(1-2\mu)}\right]\Bigg/\left[\sqrt{\frac{2+\mu}{\mu(1+\mu)}}+\sqrt{\frac{(1-\mu+\sqrt{1-2\mu})\mu\sqrt{1-2\mu}}{(1-\mu)^2-(1-2\mu)}}\right]$$
$$=\left[\frac{1-\sqrt{1-2\mu}}{\mu(1+\mu)}+\frac{\mu\sqrt{1-2\mu}}{1+\mu}+2\right]\Bigg/\left[\sqrt{\frac{2+\mu}{\mu(1+\mu)}}+\sqrt{\frac{(1-\mu+\sqrt{1-2\mu})\sqrt{1-2\mu}}{\mu}}\right]>0, \quad (23)$$

where $0<\mu\leq 0.5$.

The above equation shows that the proposed absorber (model B) provides smaller maximum vibration velocity amplitude of the primary mass M excited by a harmonic force than the traditional absorber (model A) if both absorbers are optimally tuned based on the fixed-point theory. Plotted in Fig. 4 is Eq. (23) which shows the corresponding percentage reduction in velocity amplitude of the mass M of the proposed absorber relative to the traditional absorber at different mass ratios.

IV. CONCLUSION

Optimum tuning condition including the frequency and damping ratios of the proposed absorber has been derived based on the fixed-point theory. Under the optimum tuning condition of the absorbers, it is proved analytically that the proposed absorber provides a larger suppression of maximum vibration velocity response of the primary system than the traditional absorber. The comparison reveals that the maximum velocity amplitude response under the optimized condition of model B is always less than that of model A. The proposed absorber (model B) may be a better alternative absorber design than the traditional damped DVA whenever it is practical to be used. In some applications such as vibration absorption of aircraft panels,[13] the proposed absorber may not be applicable because it is impossible to connect the absorber directly to the ground via a damper. However, the proposed absorber is recommended whenever its configuration is practical for applications requiring the maximum vibration velocity response of the primary system to be minimized.

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