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Reynolds number effects on the flow structure behind two side-by-side cylinders

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The wake structure of two side-by-side cylinders was experimentally investigated using the laser-induced fluorescence flow visualization, particle image velocimetry and hot-wire techniques. The investigation was focused on the asymmetrical flow regime, i.e., $T/d = 1.2-1.6$, where T is the center-to-center cylinder spacing and d is the cylinder diameter. Experiments were conducted in both the water tunnel and the wind tunnel at a Reynolds number (Re) range of $150-14\,300$. It has been found that, as Re increases, the flow structure behind the cylinders may change from one single vortex street to two streets with one narrow and one wide for the same T/d . The one-street flow structure is dominated by one frequency $f_0^* = f_0 d / U_\infty \approx 0.09$, where f_0 is the dominant frequency and U_∞ is the free-stream velocity. On the other hand, two frequencies, $f_0^* \approx 0.3$ and 0.09 , characterized the two-street flow structure. These are associated with the narrow and wide street, respectively. It is further observed that the critical Re , at which the transition from single to two streets occurs, increases as T/d decreases. The present finding clarifies previous scattered reports for $1.2 < T/d < 1.5$: the detection of one dominant frequency by some but two by others. © 2003 American Institute of Physics. [DOI: 10.1063/1.1561614]

I. INTRODUCTION

The two side-by-side circular cylinder wake has attracted considerable interest in the past because of its practical significance in many branches of engineering. As a result, our understanding of this flow has been greatly improved. It is now well established that when the ratio of the cylinder center-to-center spacing T to the diameter d is beyond 2, two distinct and coupled vortex streets have been observed.¹ For $1.5 < T/d < 2.0$, the base pressure is different between the cylinders. The gap flow between the cylinders is deflected toward the cylinder with a lower base pressure, resulting in one narrow wake. Meanwhile, one wide wake develops behind the neighboring cylinder. The deflected gap flow is bistable and randomly changes over from one side to the other.^{2,3} The time scale for the changeover is several orders of magnitude longer than that of vortex shedding and of the instability of the separated shear flows.⁴ At $T/d < 1.2$, the two cylinders behave like one structure, generating a single vortex street.⁵ Peschard and Le Gal⁶ further found that the Reynolds number may affect the way the two streets were coupled; the two streets could be in-phase locking, asymmetric locking, quasiperiodic, and phase-opposition locking.

However, many aspects remain to be clarified, in particular, in the asymmetrical flow regime, i.e., $T/d < 2.0$. For example, literature data indicate a lack of consistency in the previous reported vortex frequencies for the range $1.2 < T/d < 1.5$. Ishigai *et al.*² measured two dominant frequen-

cies in the range $T/d = 1.25-1.5$, namely $f_0^* = f_0 d / U_\infty \approx 0.1$ and 0.3 , respectively, where f_0 is the dominant frequency and U_∞ is the free-stream velocity. The two frequencies were also detected by Spivac⁷ in the range, $T/d = 1.5-2.0$. Spivac further detected a frequency at 0.2 , which could be interpreted as the second harmonic of 0.1 . However, he failed to detect the frequency $f_0^* = 0.3$ at $T/d \approx 1.25$. Kim and Durbin⁴ measured two frequencies, near 0.1 and 0.3 , respectively, for $T/d = 1.5-2.0$, but detected only one, $f_0^* = 0.1$, for $T/d < 1.5$. These results bring forward the following questions. Are these scattered observations related to the effect of initial conditions, especially the Reynolds number? Does the effect depend on T/d ? How is the flow structure affected? These questions motivate the present investigation.

In the present work we aim to study the effect of Re on the flow structure behind two side-by-side cylinders and to investigate the possible relationship between the flow structure and the scattered observation of the dominant frequencies. The flow structures were measured using the laser-induced fluorescence (LIF) flow visualization technique and the particle image velocimetry (PIV) method. The qualitative data are examined along with the quantitative information of the dominant frequencies determined from the measurements of two hot wires.

II. EXPERIMENTAL DETAILS

A. Laser-induced fluorescence (LIF) flow visualization in a water tunnel

The LIF measurements were carried out in a water tunnel with a square working section ($150\text{ mm} \times 150\text{ mm}$) of 0.5 m long. A regulator valve controls the flow speed and the

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maximum velocity attained in the working section is about 0.32 m/s. The working section is made up of four 20 mm thick Perspex panels. Further details of the water tunnel can be found in Zhang and Zhou.⁸ Two side-by-side acrylic circular tubes with an identical diameter $d = 10$ mm were horizontally mounted 0.20 m downstream of the exit plane of the tunnel contraction and placed symmetrically to the midplane of the working section. They spanned the full width of the tunnel. The resulting blockage was 13.3%. Dye (Rhodamine 6G 99% which has a faint red color and will become metallic green when excited by a laser) was introduced through injection pinholes located at the midspan of each cylinder at $\pm 90^\circ$ from the forward stagnation point. A thin laser sheet, which was generated by laser beam sweeping, provided illumination vertically over $0 \leq x/d \leq 10$ at the midplane of the working section. A Spectra-Physics Stabilite 2017 Argon Ion laser with a maximum power output of 4 W was used to generate the laser beam and a professional digital video camera recorder (JVC GY-DV500E), set perpendicular to the laser sheet, was used to record the dye-marked vortex streets at a framing rate of 25 frames per second. Experiments were carried out for $T/d = 1.2$ – 1.6 and at a Reynolds number $Re (= U_\infty d / \nu) = 150$ – 1000 .

The LIF measurement was also conducted in the wind tunnel using the PIV system. See Sec. II B for more details.

B. Wind tunnel experiments

Both PIV and hot-wire experiments were carried out in a closed circuit wind tunnel to obtain both qualitative and quantitative data and also to cover a large range of Re . The wind tunnel has a square working section ($0.6 \text{ m} \times 0.6 \text{ m}$) of 2.4 m in length. The working section walls were made of optical glass to provide the best quality in flow visualization and PIV measurements. The wind speed in the working section can be adjusted from about 0.3 to 50 m/s. The wake was generated by two brass cylinders ($d = 12.7$ mm) arranged side-by-side (Fig. 1). The cylinders were installed horizontally in the midplane and spanned the full width of the working section. They were located 20 cm downstream of the exit plane of the contraction. This resulted in a maximum blockage of about 4.2% and an aspect ratio of 47. In the free-stream, the longitudinal turbulence intensity was measured to be approximately 0.4%.

1. PIV measurements

A Dantec standard PIV 2100 system was used. Flow was seeded by the smoke, generated from paraffin oil, of a particle size around $1 \mu\text{m}$ in diameter. The flow illumination in the plane of mean shear was provided by two Newwave standard laser sources of a wavelength of 532 nm, each with a maximum energy output of 120 mJ. Single-exposed digital particle images were taken using one CCD camera (HiSense type 13, gain $\times 4$, double frames, 1280×1024 pixels). Synchronization of image taking with flow illumination was provided by the Dantec FlowMap Processor (PIV2100 type). Each image covered an area of $75 \text{ mm} \times 60 \text{ mm}$ of the flow field, i.e., $x/d = 0.8d \sim 6.8d$ and $y/d = -2.5d \sim +2.5d$ presently. The horizontal image magnification was about 0.06 mm/pixel. Each pulse lasted for $0.01 \mu\text{s}$. The interval be-

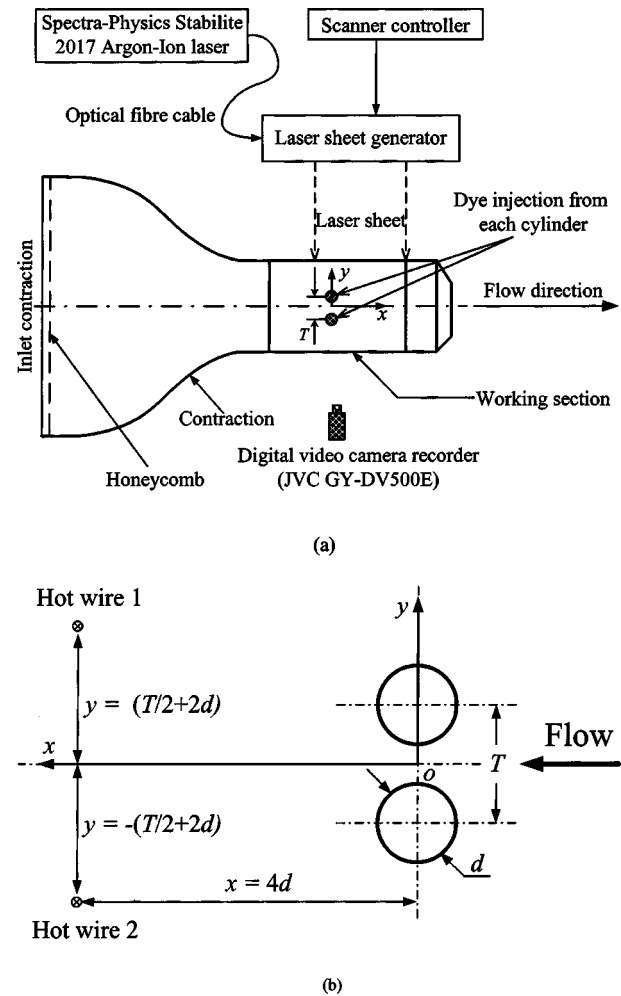


FIG. 1. Experimental setup. (a) Laser-illuminated flow visualization in the water tunnel. (b) Schematic arrangement of hot wires in the wind tunnel.

tween two successive pulses was typically $50 \mu\text{s}$. Thus, a particle would only travel 0.05 mm (0.83 pixels or $0.004d$) at $U_\infty = 1.0$ m/s. To minimize reflection noise, the cylinder surface and the working section wall hit by the laser-illuminated sheet were painted black. Furthermore, an optical filter was used to allow only the green wavelength (532 nm) generated by a laser to pass. The raw vector field of particle displacement was computed using a cross-correlation algorithm,⁹ which was built into correlator units in the Dantec FlowMap processor. The interrogation area consisted of 64 pixels ($\approx 0.3d$) with 25% overlap in the horizontal and vertical directions, respectively. Accordingly, the in-plane velocity vector field included 26×21 vectors, giving 26×21 of vorticity points. The spatial resolution for vorticity estimate was about $0.23d$. The experiment was carried out for $T/d = 1.3$ and $Re = 350$ – 680 .

Flow visualization was conducted in the wind tunnel using the visualization function of the PIV system in order to investigate the flow structure at Re higher than that investigated in the water tunnel. The experimental setup was exactly the same as the PIV measurements. The camera was operated in the single frame mode. A wide-angle lens was used to increase the view field up to $14.1d \times 10.7d$. The recording interval between a pair of successive images was

0.2 s. The experiment was carried out for $T/d=1.2$ to 1.6 and $Re=300$ to 1500. More than 100 images were collected for each Re .

2. Hot-wire measurements

The vortex shedding frequencies of two cylinders were measured using two hot wires in the wind tunnel. Two single hot wires were placed symmetrically at $x/d \approx 4$ and $y/d \approx \pm(T/2d + 2)$, respectively [Fig. 1(b)], to monitor simultaneously the instantaneous flow velocity behind each cylinder. Constant-temperature circuits were used for the operation of the hot wires. The experiments were carried out for $T/d = 1.2-1.6$ and $Re=300-14\,300$. Signals from the circuits were offset, amplified, and then digitized using a 16 channel (12 bit) A/D board and a personal computer at a sampling frequency $f_{\text{sampling}} = 3.5$ kHz per channel. The duration of each record was about 40 s.

III. EFFECTS OF T/D AND Re ON DOMINANT FREQUENCIES

As noted in the Introduction, previous reports of the number of dominant frequencies for $T/d=1.2-1.5$ were not consistent.^{2,4,7} Zhou *et al.*¹⁰ speculated that a possible transition occurred between $T/d=1.2-1.5$ from the regime of a wide and a narrow vortex street to that of a single vortex street. During transition, the gap flow was probably deflected, as evidenced by different base pressures associated with the two cylinders.⁵ However, gap vortices might or might not be generated, depending on the initial conditions or experimental setup, thus giving rise to inconsistent observations. The present measurement indeed reconfirmed the occurrence of one dominant frequency at $f_0^* = f_0 d / U_\infty \approx 0.09$ or two at $f_0^* \approx 0.09$ and 0.3 , respectively, for $1.25 < T/d < 1.6$. It was further found that the number of dominant frequencies was dependent on Re as well as T/d .

At $T/d=1.3$ and $Re=300$, the power spectra of both hot-wire signals (Fig. 2) display one single peak at $f_0^* \approx 0.09$. As Re is increased to 500, the u spectrum from hot wire 1 showed one more peak at $f_0^* \approx 0.3$, whereas the peak in the u spectrum from hot wire 2 remains at $f_0^* \approx 0.09$. This suggests that the flow structure or patterns may be quite different between the low and high Reynolds numbers, which is verified by the flow visualization data (Sec. IV). More discussion will be provided in Sec. IV on the occurrence of two dominant frequencies detected from hot wire 1. Similar observations were made for $1.25 < T/d < 1.6$, i.e., the detected dominant frequency changes from one to two as Re increases. However, at $T/d \leq 1.25$, one dominant frequency only was detected, as illustrated in Fig. 3, which shows one single peak at $f_0^* \approx 0.09$ in the u -power spectra of both hot wires ($T/d=1.25$), even though Re has been increased to 14 000. This could imply a flow structure different from that at $T/d > 1.25$, as confirmed in Sec. IV. Note that a minor peak occurs at $f_0^* \approx 0.18$ in Fig. 3(a), which is likely to be the second harmonic of $f_0^* \approx 0.09$.

Figure 4 summarizes the dominant frequencies detected at various T/d . The present data are essentially in agreement with those reported in the literature, which are also included

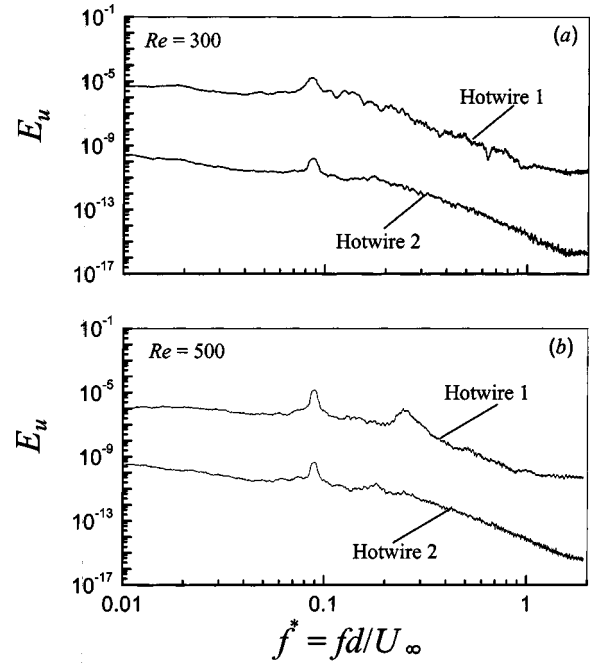


FIG. 2. Typical power spectra of the hot wire signal u at $T/d=1.3$. The two hot wires were placed at $x=4d$ and $y=\pm(T/2+2d)$ [see Fig. 1(b)].

in the figure. Apparently, one single frequency occurs at $f_0^* \approx 0.1$ for $T/d < 1.25$; it shifts to $f_0^* \approx 0.2$ for $T/d > 2.0$. As T/d is between 1.6 and 2.0, two frequencies occur. For $1.25 \leq T/d \leq 1.6$, one or two frequencies are possible, perhaps depending on Re .

Note that using one single hot wire Bearman and Wadcock³ observed one dominant frequency at $f_0^* \approx 0.1$ for $1 < T/d < 1.5$. Zhang and Zhou⁸ measured the flow up to $x/d=10$ behind three side-by-side cylinders ($T/d=1.5$, Re

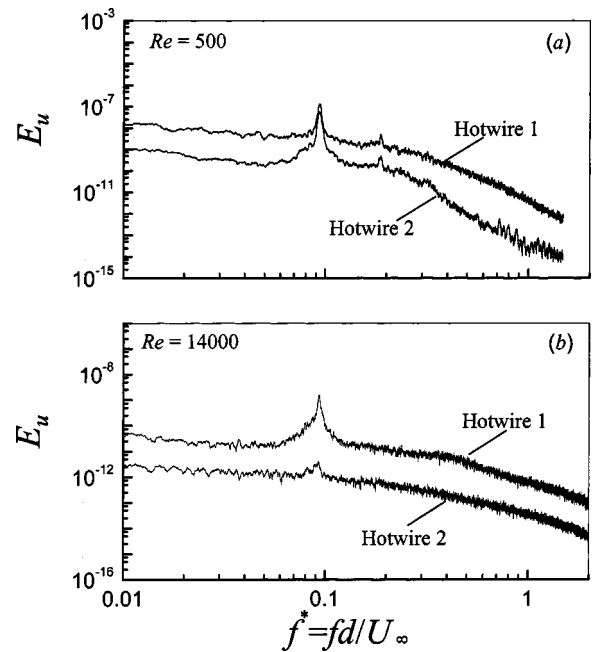


FIG. 3. Typical power spectra of the hot wire signal u at $T/d=1.25$. The two hot wires were placed at $x=4d$ and $y=\pm(T/2+2d)$ [see Fig. 1(b)].

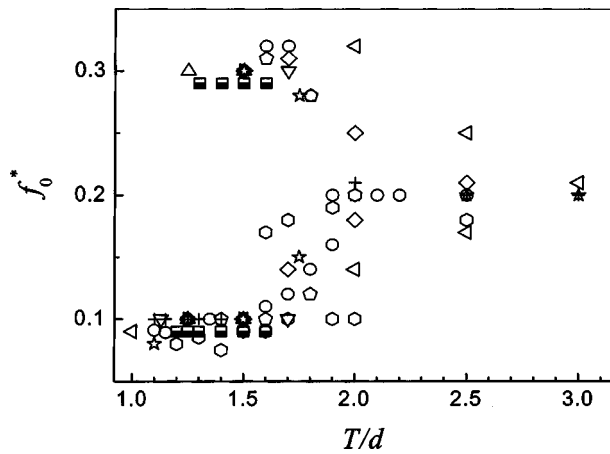


FIG. 4. The dependence of dominant frequencies on spacing between two side-by-side cylinders in a cross-flow. +, Bearman and Wadcock (Ref. 3), $Re=25\,000$; ▷, Ishigai *et al.* (Ref. 2), $Re=1500$; ☆, Kamemoto (Ref. 17), $Re=30\,000$; ◇, Kim and Durbin (Ref. 4), $Re=3300$; ○, Kiya *et al.* (Ref. 15), $Re=20000$; △, Quadflieg (Ref. 18), $Re=30\,000$; ◊, Spivak (Ref. 7), $Re=28\,000$; ◁, Sumner *et al.* (Ref. 5), $Re=1200$ – 2200 ; ◊, Williamson (Ref. 19), $Re=55$, 100 and 200; ▽, Zhou *et al.* (Ref. 13), $Re=120$ and 3500; ■, present data, $Re=300$ to 14 300.

=5800) using both hot wire and laser-induced fluorescence flow visualization techniques. They observed a wide wake behind the central cylinder and a narrow wake on each side of the wide wake. It was found that the vortical structures in the narrow wakes, which were shed from the side cylinder, interacted with the wide wake and could not be detected at $x/d \approx 5$ by hot wires. On the other hand, the vortical structures in the wide wake started to roll up at $x/d \approx 5$ ($Re > 450$). They were very weak initially but grew in strength with increasing x/d , resembling those in a screen near-wake, which were ascribed to the shear layer instability of the developing wake.^{11,12} The two-cylinder case may bear a resemblance to that of three cylinders. This is supported by the Zhou *et al.* observation¹³ at $T/d=1.7$ that the two cross-stream vortices in the narrow wake amalgamated with the gap vortex in the wide wake, forming a single street downstream. The vortices seen in the wide wake may be generated from the shear layer instability in the developing wake, whereas those shed from the cylinder in the narrow wake probably disappear at $x/d \approx 5$. Note that Bearman and Wadcock³ placed their hot wire at $x/d \approx 6$ and the flow Re was 2.5×10^4 . It is well known that the vortex formation length behind an isolated cylinder reduces as Re increases.¹⁴ This is probably also true for the flow behind two side-by-side cylinders, implying that the opposite-sign vortices in the narrow wake may amalgamate with the gap vortex in the wide wake at a location closer to the cylinders for a large Re . Therefore, the narrow wake in the Bearman and Wadcock flow could have vanished before $x/d \approx 6$, thus leaving only the dominant frequency in the wide wake to be detected.

Figure 5 presents the dependence of single- or two-frequency detection on Re and T/d . Again, the data available in the literature are included. It is evident that the number of dominant frequencies is independent of Re , provided that $T/d < 1.25$ or $T/d > 1.6$; one frequency occurs for $T/d < 1.25$ but two for $T/d > 1.6$ (but < 2.0). Between T/d

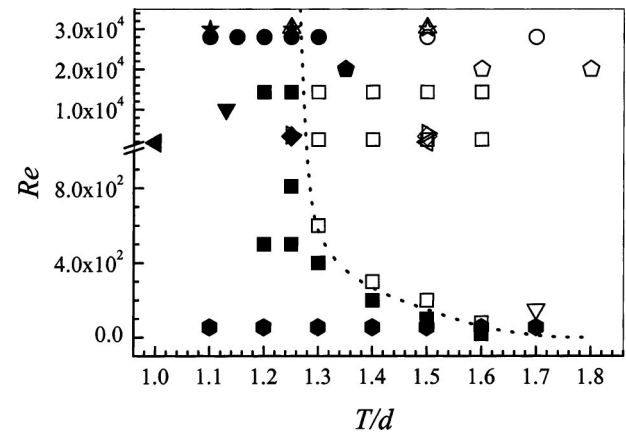


FIG. 5. Effect of the Reynolds number on the number of dominant frequencies. The solid and open symbols represent the detection of one and two dominant frequencies, respectively; the dotted line is indicative only of the boundary between them. ▷, Ishigai *et al.* (Ref. 2); ★ and ☆, Kamemoto (Ref. 17); ◇ and ◊, Kim and Durbin (Ref. 4); ● and ○, Kiya *et al.* (Ref. 15); △, Quadflieg (Ref. 18); ◊ and ◊, Spivak (Ref. 7); ◁ and ◁, Sumner *et al.* (Ref. 5); ● and ○, Williamson (Ref. 19); ▽ and ▽, Zhou *et al.* (Ref. 13); ■ and □, present data.

=1.25 and 1.6, the number of dominant frequencies depends on Re as well as T/d . The flow is dominated by one frequency for relatively low Re but two, as Re is beyond a critical value. As T/d reduces, the transition from one dominant frequency to two occurs at a higher Re . It is pertinent to point out that the critical Re , where “transition” occurs could be affected by the initial conditions, such as boundary conditions, wind tunnel, experimental setup, etc., other than Re . This is reflected in the slight deviation of some data (e.g., Ref. 7) from the indicative transition line in Fig. 5. Kiya *et al.*¹⁵ detected two dominant frequencies at $f_0^* \approx 0.1$ and 0.3, respectively, for $T/d > 1.35$, but failed to detect the frequency at $f_0^* \approx 0.3$ (the narrow street) for $T/d=1.35$. There could be two possible causes for the discrepancy between their data and the indicative transition line, that is, there was a difference in the initial conditions or their hot wire happened to be in the wide wake region. They used one single hot-wire and did not specify the wire location for the side-by-side cylinder arrangement in their paper. Previous reports^{8,13} indicated that the detection of the frequency at $f_0^* \approx 0.3$ was quite sensitive to the hot wire location.

IV. FLOW STRUCTURES

The dependence of dominant frequencies on both T/d and Re suggests a variation in the flow structure with these parameters, and it is important to clarify the correspondence between the typical flow structure or patterns and the dominant frequencies (Fig. 5). Figure 6 presents the typical flow structure at $T/d=1.4$ from the LIF measurement in the water tunnel. At $Re=150$, the gap flow between the cylinders is biased downward, resulting in one single vortex street behind the upper cylinder. A small closed near-wake region occurs behind the lower cylinder, without shedding any vortices [Fig. 6(a)]. This flow structure is rather stable. The same structure is observed for $Re < 300$. As Re reaches 300, the gap flow between the cylinders is deflected toward the upper

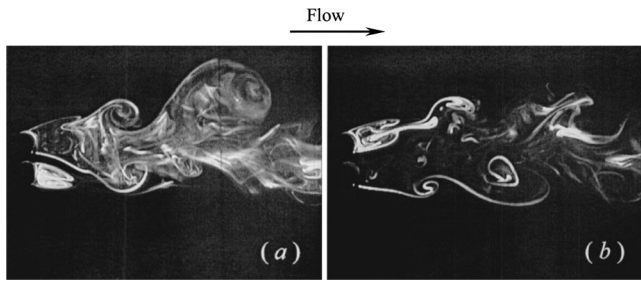


FIG. 6. Typical flow patterns at $T/d=1.4$ measured in the water tunnel: (a) a one-street flow structure, $Re=150$; (b) a two-street flow structure, $Re=300$. Flow is left to right.

cylinder, forming one narrow street behind the upper cylinder and one wide street behind the lower cylinder [Fig. 6(b)]. The gap flow may change the direction of deflection and is bistable. This flow structure remains unchanged with further increases in Re .

The same observation was made in the wind tunnel using the flow visualization function of the PIV system. Figure 7 shows photographs taken at $T/d=1.3$. The flow at $Re=450$ [Fig. 7(a)] displays one single vortex street, with a closed near-wake region identifiable behind the lower cylinder. At $Re=1000$, the flow [Fig. 7(b)] resembles that in Fig. 6(b), displaying two streets, one narrow and one wide, behind the cylinders. The simultaneous hot wire measurements indicate one dominant frequency at $f_0^* \approx 0.09$ for the former and two at $f_0^* \approx 0.09$ and 0.3 , respectively, for the latter.

The PIV data provide some quantitative information on the flow structure. The isocontours of the spanwise vorticity $\omega^* = \omega d / U_\infty$ (Fig. 8) at $T/d=1.3$, derived from the PIV measurement of the velocity field, reconfirm the LIF results, i.e., as Re increases, the flow changes from a one-street structure to a two-street structure. The PIV measurement further indicates that for the one-street structure [Fig. 8(a)] the vorticity concentration associated with the closed near-wake region is quite comparable to that of the vortex street. In the case of two streets [Fig. 8(b)], the maximum vorticity concentration in the narrow street (upper) is comparable with that in the wide wake (lower). The maximum vorticity concentration associated with the inner shear layer, near $y/d=0$, of the wide wake is 1.4 , appreciably smaller than that (2.0) of the narrow wake or the two outer shear layers toward the free-stream. This may suggest a weak gap vortex in the

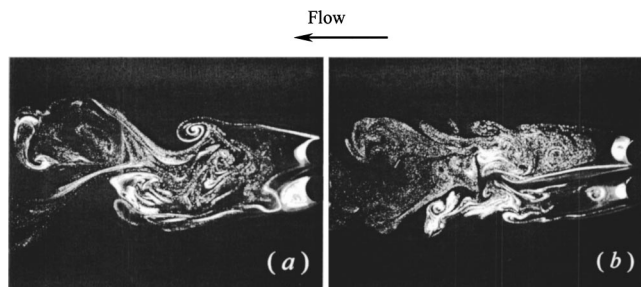


FIG. 7. Typical flow patterns at $T/d=1.3$ measured in the wind tunnel: (a) a one-street flow structure, $Re=450$; (b) two-street flow structure, $Re=1000$. Flow is right to left.

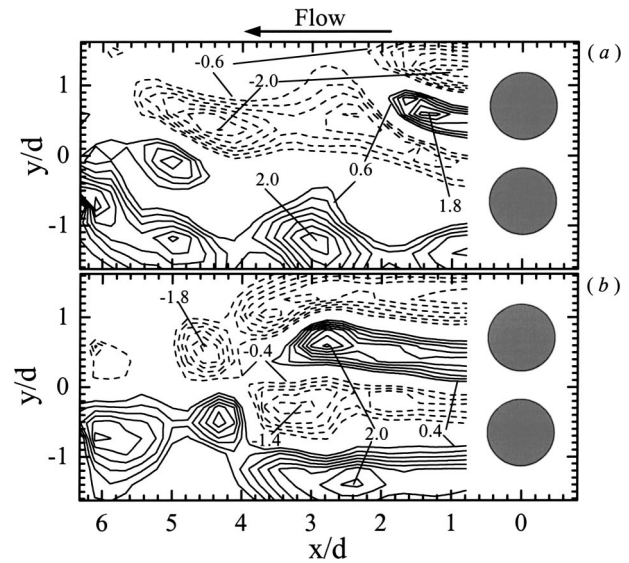


FIG. 8. The PIV measurement of instantaneous vorticity contours $\omega^* = \omega d / U_\infty$, the contour increment $= 0.2$, $T/d=1.3$. (a) One-street flow structure, $Re=350$; (b) two-street flow structure, $Re=680$. Flow is right to left.

wide wake. As a matter of fact, flow visualization data indicate that, frequently, the gap vortex in the wide wake could not be seen [e.g., Fig. 6(b)].

A similar observation was made for $T/d=1.5$ and 1.6 (not shown), that is, one single vortex street occurs for relatively low Re , the structure turning into one narrow and one wide vortex street as Re increases. Furthermore, the former was dominated by one frequency, while the latter by two. For $1.6 < T/d < 2.0$, the one-street structure was not observed, irrespective of the Re value. At $T/d=1.2$ or smaller, however, the flow structure (Fig. 9) is totally different. The closed near-wake region is absent. Vortices shed from the outer side only of the two cylinders, forming one single vortex street. At such a small T/d , the gap bleeding, without the presence of the gap vortex, though, is identifiable. It is deflected upward at the higher Re [Fig. 9(b)] but swerving around the upper cylinder at lower Re [Fig. 9(a)]. One dominant frequency at $f_0^* \approx 0.09$ was detected across the wake. The present observation is consistent with previous reports.^{5,13}

It is well known that a steady separation or closed near-wake is associated with an isolated circular cylinder in the Re range $4-5 < Re < 30-48$, and the periodic vortex shedding occurs for $Re \geq 50$.¹⁶ The Zhou *et al.* measurements¹³ indi-

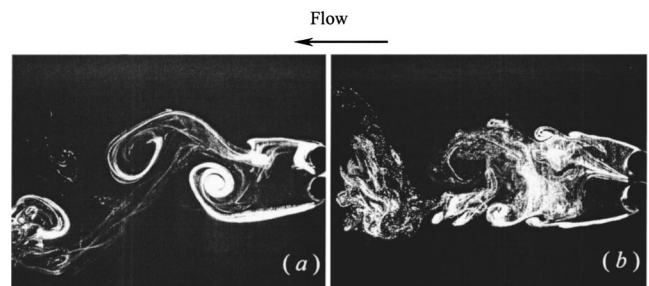


FIG. 9. Typical flow patterns at $T/d=1.2$ measured in the wind tunnel: (a) $Re=450$; (b) $Re=1500$. Flow is right to left.

cate that, as T/d reduces, the pressure between the cylinders increases. This implies a decrease in the gap flow velocity. The corresponding actual Re will be small. This may explain why a small closed wake occurs in Figs. 6(a), 7(a), and 8(a). When the gap flow velocity increases beyond the critical Reynolds number for the periodic vortex shedding, a narrow wake is generated, as observed in Figs. 6(b), 7(b), and 8(b). The interpretation is also consistent with the observation (Fig. 5) that the critical Reynolds number for the transition from the one- to the two-frequency regime increases as T/d decreases. For $T/d < 1.25$, because of the near-wall effect, the gap flow velocity is so small that the actual Re for the inner side of the cylinders cannot exceed the critical value for periodic shedding, irrespective of the value of U_∞ . As a result, vortices shed from the outer layers only of the two cylinders, thus generating a single vortex street or single dominant frequency (Figs. 3 and 9).

Peschard and Le Gal⁶ analyzed the stability of the coupled streets behind two side-by-side cylinders for $1.7 < T/d < 6.0$ and classified the flow as in-phase locking, asymmetric locking, quasiperiodic and phase opposition locking as Re increased from 90 to 150. A comparison between the present Fig. 5 and their Fig. 5 indicates that the asymmetric locking, quasiperiodic regime, and phase-opposition locking, in fact, correspond to the present two-street regime, while in-phase locking coincides with part of the one-street regime. However, there is a difference between a one-street regime and in-phase locking. The one-street regime consists of one vortex street and one small closed wake [e.g., Fig. 6(a)], which is different from in-phase locking wakes.

V. CONCLUSIONS

The near-wake of two side-by-side cylinders has been measured using the LIF, PIV, and hot wire techniques. The investigation focuses on the dominant frequencies, flow structures, and their relation in the asymmetrical flow regime, i.e., $1.0 < T/d < 1.7$. The following conclusions can be drawn.

- (1) As $T/d < 1.25$, the near-wall effect is dominant; the Re based on the gap flow velocity is so small that periodic vortex shedding cannot be induced from the inner side of a cylinder. As a result, vortices shed from the outer side only of the two cylinders, generating a single vortex street.
- (2) For $1.25 \leq T/d \leq 1.6$, two typical flow structures have been observed. At a relatively low Re , a small closed wake occurs behind one cylinder; the flow is characterized by one single vortex street, dominated by a frequency at $f_0^* \approx 0.09$. The occurrence of the small wake is probably due to the near-wall effect at small T/d . The effect causes a small gap flow velocity. The Reynolds number based on this velocity is then substantially smaller than Re , which is based on U_∞ . As U_∞ or Re increases, the Reynolds number based on the gap flow velocity may exceed a critical value. The small closed wake then ceases existing. Meanwhile, periodical vortex

shedding starts, forming one narrow vortex street, coexisting with the wide street. The narrow and wide street are associated with a dominant frequency of about 0.3 and 0.09, respectively. Apparently, the smaller the gap between the cylinders, the stronger is the near-wall effect. Consequently, the transition from the one-street to the two-street flow structure occurs at a larger Re . The present observation and interpretation help clarify the seemingly inconsistent reports of the dominant frequencies for $1.25 \leq T/d \leq 1.5$ in the literature.

- (3) For $T/d > 1.6$ but < 2.0 , the near-wall effect could be negligible, and therefore the one-street flow structure is absent. The flow consists of one narrow and one wide street throughout the Re range investigated. At $T/d > 2.0$, two predominantly antiphase streets^{6,13} occur.

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