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Research Paper

The effect of topographic density variations on the geoid and orthometric heights in Hong Kong

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A R T I C L E I N F O

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A B S T R A C T

Utilizing the adopted average topographic density of 2670 kg/m^3 in the reduction of gravity anomalies introduces errors attributed to topographic density variations, which consequently affect geoid modeling accuracy. Furthermore, the mean gravity along the plumbline within the topography in the definition of Helmert orthometric heights is computed approximately by applying the Poincaré-Prey gravity reduction where the topographic density variations are disregarded. The Helmert orthometric heights of benchmarks are then affected by errors. These errors could be random or systematic depending on the specific geological setting of the region where the leveling network is physically established and/or the geoid model is determined. An example of systematic errors in orthometric heights can be given for large regions characterized by sediment or volcanic deposits, the density of which is substantially lower than the adopted topographic density used in Helmert's definition of heights. The same applies to geoid modeling errors. In this study, we investigate these errors in the Hong Kong territory, where topographic density is about 20% lower than the density of 2670 kg/m^3. We use the digital rock density model to estimate the effect of topographic density variations on the geoid and orthometric heights. Our results show that this effect on the geoid and Helmert orthometric heights reach maxima of about 2.1 and 0.5 cm, respectively. Both results provide clear evidence that rock density models are essential in physical geodesy applications involving gravimetric geoid modeling and orthometric height determination despite some criticism that could be raised regarding the reliability of these density models. However, in regions dominated by sedimentary and igneous rocks, the geological information is essential in these applications because topographic densities are substantially lower than the average density of 2670 kg/m^3, thus introducing large systematic errors in geoid and orthometric heights.

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1. Introduction

The accurate determination of orthometric (actual) heights requires knowledge of the mean gravity value along the plumbline within the topography (i.e., between the geoid and the topographic surface). Since the actual gravity inside the topography cannot be measured directly, several methods have been proposed and applied to estimate it as a mean value using measured gravity values at the topographic surface. These proposals adopt some stipulated assumptions to realistically approximate the actual gravity gradient inside the topography by considering terrain geometry and topographic density variations [1–3]. Wirth [4] and Flury et al. [5] applied the correction for (planar) terrain geometry, and Tenzer et al. [6] numerically inspected the corresponding correction for an anomalous (lateral) topographic density.

Although several attempts have been made theoretically to compute the orthometric heights accurately [7–17], Helmert's [18,19] approximation of the mean gravity inside the topography is solely used in defining orthometric heights until now. According to this approximation, the mean gravity is computed using measured surface gravity values, and the Poincaré-Prey gravity gradient is applied as a reduction method while ignoring changes in the gravity gradient caused by anomalous topographic density variations, density heterogeneities below the geoid surface, and terrain geometry.

The effect of lateral topographic density variations on gravimetric geoid modelling has been studied previously by different researchers [20–33]. According to estimates by Pagiatakis and Armenakis [23], the effect of lateral topographic density variations

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on the geoid reaches up to approximately 10 cm in the Skeena region, while only a few millimeters in the New Brunswick region characterized by moderate topographic elevations. Huang et al. [25] investigated this effect on the geoid heights in the Canadian Rocky Mountains and obtained a total topographic density correction ranging from −7.0 to 2.8 cm. Sjöberg [27] pointed out that this effect can reach ±1.5 cm for Lake Baikal (the Earth's deepest lake) and ±1.78 m for Mt. Everest. Kiamehr [28] reported maximum (absolute) values of this effect up to 22 cm in Iran. Abbak [32] conducted a similar study for Turkey and obtained maximum values up to 35 cm in mountainous regions. Lin and Li [33] concluded that the geoidal heights in Colorado can be determined to an accuracy of about 8 cm by considering the lateral density variation of the topography.

The determination of orthometric heights from leveling measurements requires the application of orthometric correction to account for the gravity information [34–36]. Computation of the orthometric correction requires height differences between successive benchmarks alongside their respective gravity measurements. The gravity measurements are then used to compute the mean gravity along the plumbline inside the topography. Hwang and Hsiao [36] presented the expression for computing the orthometric correction that considers anomalous lateral topographic density variations. Over the mountains in Taiwan, China, they obtained a millimeter-level effect of the topographic density variation on orthometric heights. After theoretically analyzing the effect of the lateral topographic density variation on orthometric heights, Tenzer and Vaníček [6] concluded that it could attain a decimeter level. Similarly, Kingdon et al. [37] computed lateral topographic density corrections at leveling benchmarks in Canada and obtained values ranging from −4.5 cm to 6.5 cm. Albarici et al. [38] computed this effect for leveling benchmarks in São Paulo and ascertained the maximum and minimum corrections to be ~9 mm and ~17 mm, with a mean of ~3 mm.

The adopted representative value for the topographic density directly affects geoidal heights defined by the Stokes' theory. Utilizing the well-known average topographic density of 2670 kg/m^3 [39] in the reduction of gravity anomalies introduces errors that consequentially affect the accuracy of geoid modeling. Furthermore, the mean actual gravity in the definition of Helmert's orthometric heights computed by applying the Poincaré-Prey gravity reduction suffers a similar fate by adopting this average density. Subsequently, the Helmert orthometric heights can either be overestimated or underestimated depending on the geological setting of the computation area. In this study, we assess the effect of topographic density variations on the geoid and orthometric heights using the new rock density model at the Hong Kong territories developed by Nsiah Ababio and Tenzer [40].

2. Theory

This section summarizes the expressions used to compute the effect of topographic density variations on the geoid and orthometric heights.

2.1. The effect of anomalous topographic density on the geoid

According to the KTH method, the geoid height (N) is computed as follows [41]:

$$N = \tilde{N} + \delta N_c^T + \delta N_{\mathrm{DWC}} + \delta N_c^A + \delta N^e \tag{1}$$

where $\tilde{N}$ is the approximate geoid height, δN_c^T is the combined topographic effect on the geoid height, δN_{DWC} is the downward continuation effect, δN_c^A is the combined atmospheric correction on the geoid height, and δN^e is the ellipsoidal correction.

Considering the additive corrections, the effects of the lateral density variations can be estimated using the combined topographic and downward continuation corrections in Eq. (1) while disregarding the combined atmospheric and ellipsoidal corrections, both unaffected by the topographic density variations. We then write

$$N = \tilde{N} + \delta N_c^T + \delta N_{\mathrm{DWC}} \tag{2}$$

Sjöberg [27] presented a simplified approach to computing the total effect of lateral topographic density variations on the geoid in one formula rather than computing its effect on the combined topographic and downward continuation corrections separately. This is a realistic possibility as their long-wavelength contributions cancel out. The correction for the combined topographic effect is then directly proportional to the anomalous topographic density $\Delta\rho$ and the height H of the computation point [42].

$$\delta N_{\Delta\rho} \approx -\frac{2\pi G \Delta\rho}{\gamma} H^2 \tag{3}$$

where G is the gravitational constant, and γ is the normal gravity at the reference surface. The anomalous topographic density $\Delta\rho$ in Eq. (3) is defined with respect to the mean topographic density ρ_0 of 2670 kg/m^3, so that

$$\Delta\rho = \rho - \rho_0 \tag{4}$$

2.2. The effect of anomalous topographic density on orthometric heights

The orthometric height H^O is defined by Heiskanen and Moritz [43]:

$$H^O = \frac{C}{\overline{g}} \tag{5}$$

where C denotes the geopotential number for a computation point, and $\overline{g}$ is the mean actual gravity along the plumbline inside the topography.

The orthometric height of a levelling benchmark is practically determined from adjusted values of the orthometric height differences $\Delta H^O_{i,i+1}$ that are computed by applying the orthometric corrections $OC_{i,i+1}$ to levelled height differences $\Delta H_{i,i+1}$. We then write

$$H_J^O = \sum_{i=0}^{J-1}\left(H_{i+1}^O - H_i^O\right) = \sum_{i=0}^{J-1}\Delta H^O_{i,i+1} = \sum_{i=0}^{J-1}\left(\Delta H_{i,i+1} + OC_{i,i+1}\right) \tag{6}$$

where H_i and H_{i+1} denotes the elevations of (two consecutive) benchmarks i and $i+1$ along a leveling line.

The orthometric correction $OC_{i,i+1}$ between two successive benchmarks i and $i+1$ in Eq. (6) is defined by Heiskanen and Moritz [43]:

$$OC_{i,i+1} = \sum_k \frac{g_k - \gamma_0}{\gamma_0}\,\delta H_k + \frac{\overline{g}_i - \gamma_0}{\gamma_0} H_i - \frac{\overline{g}_{i+1} - \gamma_0}{\gamma_0} H_{i+1} \tag{7}$$

where the values of the mean gravity for benchmarks i and $i+1$ are denoted by $\overline{g}_i$ and $\overline{g}_{i+1}$ respectively; the differences in the leveled heights are denoted by δH_k, with k representing the number of levelling setups in the segment between benchmarks i and $i+1$; i.e., $\Delta H_{i,i+1} = \sum_k \delta H_k$, and g_k is the corresponding surface gravity values. The normal gravity γ_0 at the reference ellipsoid in Eq. (7) is a constant value computed for the same geodetic latitude.

Here, we consider the orthometric correction formula proposed by Hwang and Hsiao [36]:

$$OC_{i,i+1} = \frac{1}{\overline{g}_{i+1}}\left(\frac{g_i + g_{i+1}}{2} - \overline{g}_{i+1}\right)\Delta h_{i,i+1} + H_i\left(\frac{\overline{g}_i}{\overline{g}_{i+1}} - 1\right) \quad (8)$$

where the surface gravity measurements at points i and $i+1$ are denoted by g_i and g_{i+1}. Regarding the assumptions used in deducing Eq (8), only the gravity values at points i and $i+1$ are required. Subsequently, their mean gravity values along their individual plumblines are computed according to the Poincaré-Prey gravity reduction. In this method, the vertical gravity gradient and topographic density are held constant with consideration given to the topographic effect of the Bouguer plate only. Note that the effect of atmospheric density on orthometric heights is completely negligible [44].

From Eq. (8), the effect of anomalous topographic density on the orthometric correction is obtained in the following form [36]:

$$\delta OC_{i,i+1} = \frac{\partial OC_{i,i+1}}{\partial \overline{g}_i}\delta\overline{g}_i + \frac{\partial OC_{i,i+1}}{\partial \overline{g}_{i+1}}\delta\overline{g}_{i+1}$$
$$= \frac{2\pi G}{\overline{g}_{i+1}^2}\left[\frac{g_i + g_{i+1}}{2}(H_{i+1} - H_i)H_{i+1}\Delta\rho_{i+1} + \overline{g}_i H_i H_{i+1}\Delta\rho_{i+1} - \overline{g}_{i+1}H_i^2\Delta\rho_i\right] \quad (9)$$

The application of this correction to the Helmert orthometric height yields

$$H_J^O = \sum_{i=0}^{J-1}\left(H_{i+1}^O - H_i^O\right) = \sum_{i=0}^{J-1}\Delta H_{i,i+1}^O = \sum_{i=0}^{J-1}\left(\Delta H_{i,i+1} + OC_{i,i+1} + \delta OC_{i,i+1}\right) \quad (10)$$

2.3. Mean gravity

The computation of the mean gravity along the plumbline within the topography according to the Poincaré-Prey gravity reduction is defined by Heiskanen and Moritz [43]:

$$\overline{g} = g - \left(\frac{1}{2}\frac{\partial g}{\partial h} + 2\pi G\rho\right)H \quad (11)$$

where g is the gravity value at the topographic surface, ρ is the topographic density, H is the computation point height, and $\partial g/\partial h$ is the free-air gravity gradient. The free-air gravity gradient can be separated into the normal gravity gradient and gravity anomaly gradient as follows:

$$\frac{\partial g}{\partial h} = \frac{\partial \gamma}{\partial h} + \frac{\partial \Delta g}{\partial H} \quad (12)$$

where the normal gravity and gravity anomaly are denoted by γ and Δg, respectively. Disregarding the gravity anomaly gradient and adopting the mean topographic density of 2670 kg/m^3 and the normal gravity gradient of -0.3086 mGal/m in Eq. (12), the mean gravity can be computed approximately from

$$\overline{g} = g + 0.0424H \quad (13)$$

From Eq (11), the effect of anomalous topographic density on the mean gravity becomes

$$\delta\overline{g} = -2\pi G\Delta\rho H \quad (14)$$

3. Input data acquisition

This section describes data used to compute the effect of topographic density variations on the geoid and orthometric heights.

3.1. Detailed rock density model

Ideally, a 3D digital density model will be advantageous in precisely defining the effect of the density of topographic masses on orthometric heights and geoid models. Nevertheless, its development has not been explored to a more detailed extent as the 2D models adopt geological maps in their development [45,46]. Regardless, the density models developed from geological maps have been adopted successfully in estimating the effect of topographic density variations on orthometric heights and the geoid [28,36]. The lack of detailed density information inside the topography in 2D topographic density models does not render it trifling. The surface density data is enough to substantially improve the accuracy of orthometric heights and geoid models, especially in mountainous regions. This provides results befitting a particular geographical area rather than adopting an unrealistic constant density value.

To estimate the contribution of lateral anomalous topographic density, we used the detailed rock density model (Fig. 1) for the Hong Kong territories prepared by Nsiah Ababio and Tenzer [40] based on a geological map by attributing average density values to main rock types. The density model is compiled on a 2 arc-second grid. As seen in Fig. 1, the rock densities in Hong Kong vary from 2101 to 2681 kg/m^3, with an average density of 2303 kg/m^3 and a standard deviation of 223 kg/m^3. This relatively low density is explained by the dominance of volcanic and sedimentary rocks over the region. The Hong Kong territories is roughly covered by 50% of volcanic rocks, which are mainly made up of sequential bulks of tuff (a highly porous rock with a relatively low density). Sedimentary rocks are also characterized by densities lower than the average topographic density of 2670 kg/m^3.

3.2. Topographic model

In this study, we used the 5 m resolution digital terrain model (HK_DTM_5 m) developed for the Hong Kong territory and produced by the Lands Department of the Hong Kong government (www.landsd.gov.hk/en/spatial-data/open-data/kf_dtm.html). This detailed terrain model was employed in conjunction with the detailed density model to estimate the effect of lateral density variations on geoidal heights over the territory to the best resolution available. The HK_DTM_5 m covers the full extent of the territory and has a maximum elevation of 952 m, a minimum of -20 m, an average of 46.893 m and a standard deviation of 103.6 m (see Fig. 2). We further applied the correction for topographic density variations to the newly developed geoid model for Hong Kong (HKGEOID-2022). In this case, we used the 1×1 arc-second SRTM DEM (digital terrain model). This was to ensure the consistency in the heights used in modeling the geoidal heights and computing the associated lateral topographic density effects on the geoid.

3.3. Vertical Control Network 2022

The official vertical geodetic datum currently adopted is the Hong Kong Principal Datum (HKPD). The benchmarks in the HKPD are

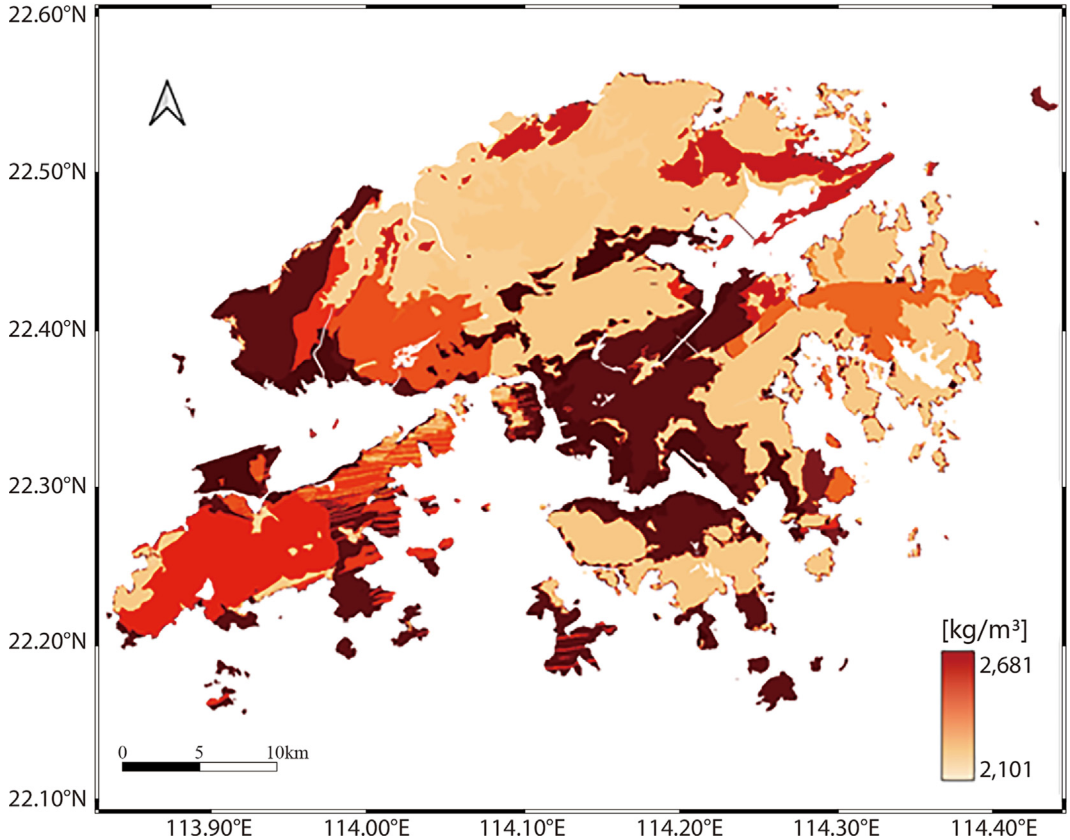

Fig. 1. The digital rock density model of Hong Kong [40].

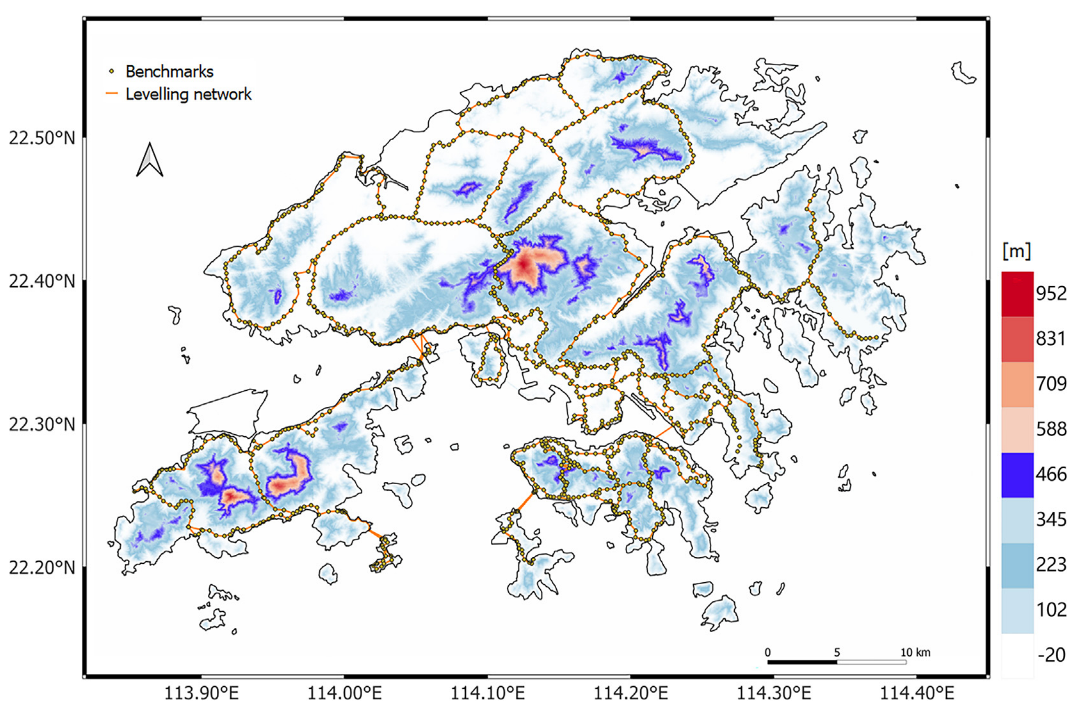

Fig. 2. The configuration of the VCN2022 at the Hong Kong territories and topographic elevations from the HK_DTM_5 m.

classified and arranged in loops, with their heights determined from spirit leveling without factoring in gravity. Nsiah Ababio and Tenzer [47] reduced the systematic errors due to the lack of gravity in the height definition using available terrestrial and marine gravity data. They computed and applied orthometric corrections to successive benchmarks considering their defined arrangement in the loops. The Vertical Control Network 2022 was presented as the final solution after the application of the orthometric corrections and subsequent adjustment. The coverage and configuration of the benchmarks in the VCN22 are illustrated in Fig. 2. There are 1069 VCN22 benchmarks with elevations below 500 m. In this study, we used the heights of the benchmarks in the VCN22, which have been corrected for systematic errors due to gravity. This study further explores the errors due to disregarding topographic density variations.

4. Results and discussion

This section presents the results and discusses its implications on the vertical geodetic control network in Hong Kong.

4.1. Geoid heights

To evaluate the effect of lateral density variations on the recently developed gravimetric geoid model for Hong Kong, HKGEOID-2022 [48], the lateral density model was resampled to a 1 × 1 arc-minute grid of the geoid model. To maintain consistency, heights from the 1 arc-second SRTM DEM, previously used in the modeling of HKGEOID-2022, were again used to compute the effect of anomalous topographic density according to Eq. (3). The effect varies between −0.1 and 18.6 mm in Hong Kong, with an average of 0.6 mm and a standard deviation of 1.5 mm. The result is graphically illustrated in Fig. 3. It is evident that maxima of this effect are found at locations with the largest topographic elevations where the effect of anomalous topographic density variations is significantly magnified by elevated topography (i.e., defined in Eq. (3) as a function of height). In contrast, minima are seen over lowlands where this effect remains very small (in the absolute sense) even if anomalous topographic density values are large.

Furthermore, we computed the effect of lateral density variations on the geoid of a 2 arc-second grid to find the maximum value of this effect in Hong Kong territories. For this purpose, we used the detailed HK_DTM_5 m topographic model with a 5 m resolution. According to this model, the maximum height in Hong Kong is 952.0 m. As seen in Fig. 4, the maximum of this effect coincides with the highest elevations. For this detailed resolution, this effect varies from −0.1 to 20.1 mm, with a mean of 0.56 mm and a standard deviation of 1.4 mm. The minimum and maximum values thus slightly exceed those obtained using a less detailed digital elevation model employed in the determination of HKGEOID-2022. Although dependent on the topographic density variations (Figs. 3 and 4), the effect of the anomalous lateral density is highly spatially correlated with the topographic elevations. This is particularly the case for Hong Kong, where large values of anomalous topographic density variations have a systematic character but with relatively small spatial variations (see the next paragraph).

In Hong Kong territories, the topographic density values are typically lower than the adopted constant density value of 2670 kg/m^3. Therefore, the anomalous density is mostly negative (see Fig. 5) ranging from -569 to 11 kg/m^3 with an average and standard deviation of −334.4 and 234.7 kg/m^3, respectively. Hence, the effect of anomalous topographic density is generally systematically positive.

The evaluation of the geoid model after applying the anomalous topographic density correction that could theoretically improve the accuracy is an ideal situation. Checking the accuracy of the corrected geoid model with respect to the GNSS-leveling points in the computation area provides the most definite way of accessing the impact of this correction. Nonetheless, only 16 VCN22 leveling benchmarks have accurately determined ellipsoidal heights by GNSS measurements, and most of these points are located in places with elevations below 100 m [48]. Considering the nature of the correction, that is, errors increasing with respect to elevation, it is obvious that the maximum effects are along mountain chains where GNSS-leveling benchmarks are completely missing. This factor significantly restricts our ability to assess the impact of applying this correction on the improvement of accuracy. Kiamehr [28] experienced the same issue in his accuracy assessment. To accurately assess the improvement of the geoid model, after the consideration of the variations in topographic density, benchmarks with accurate ellipsoidal heights over the elevated regions are required.

4.2. Mean gravity and orthometric corrections of VCN22

Expressions summarized in Section 2 were used to compute the effect of anomalous topographic density on orthometric heights of VCN22 leveling benchmarks. Since gravity measurements along leveling lines in Hong Kong were not conducted, Nsiah Ababio and Tenzer [47] used interpolated surface gravity values at leveling benchmarks, prepared from marine and land gravity data to compute the orthometric correction. In this study, we used the orthometric corrections and gravity data presented by Nsiah Ababio and Tenzer [47]. Leveled height differences and interpolated surface gravity values (along leveling lines) were used to compute the mean gravity from the surface gravity at leveling benchmarks by applying the Poincaré-Prey gravity reduction. The computation of the mean

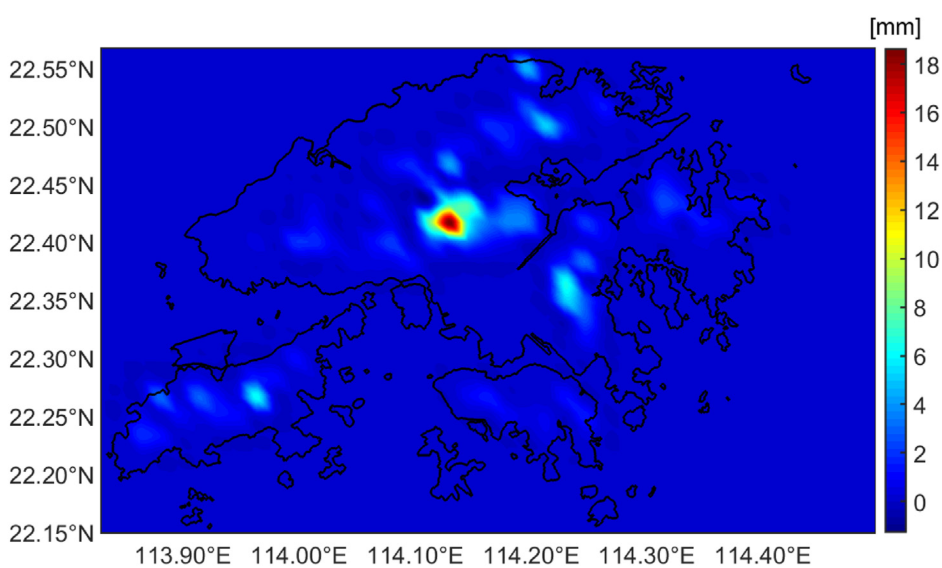

Fig. 3. Effect of anomalous topographic density on geoidal heights over Hong Kong on a 1 arc minute grid.

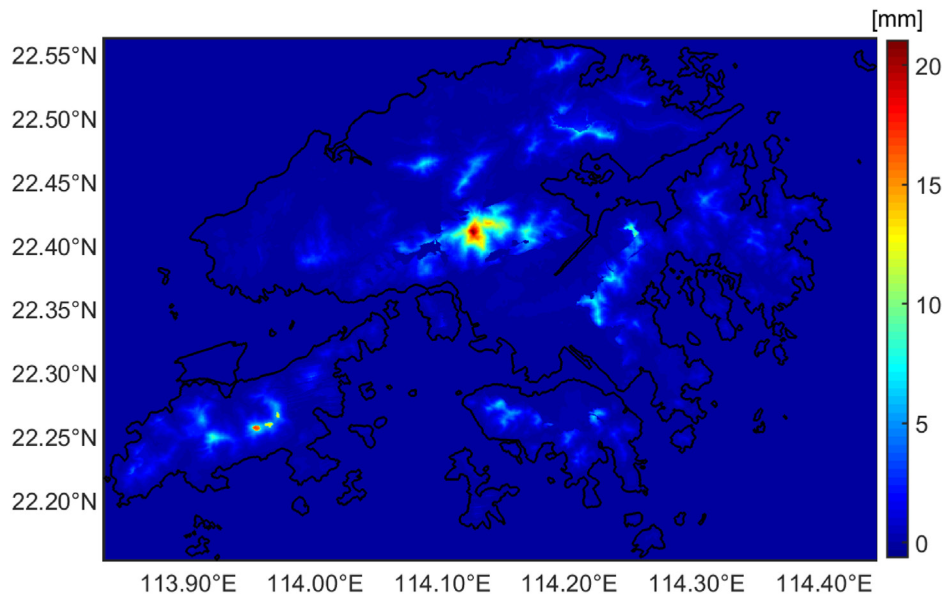

Fig. 4. Changes in geoidal heights over Hong Kong due to adopting the actual density variations over a 2 arc second grid.

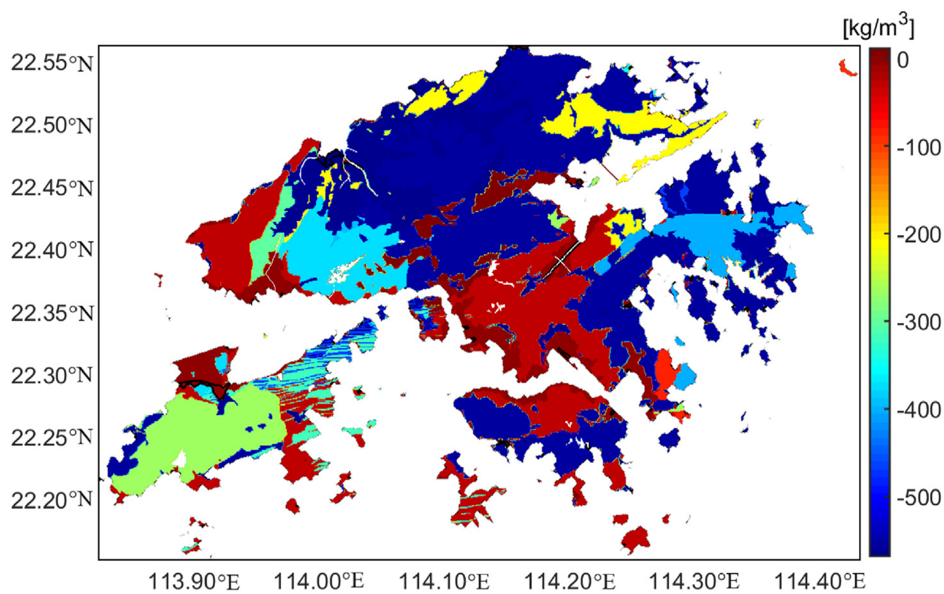

Fig. 5. Anomalous lateral density variation over Hong Kong on a 2 arc second grid.

gravity is thus realized pointwise for each benchmark. Therefore, the effect of the anomalous lateral topographic density for each benchmark can be estimated using their unique topographic density value. The effect of anomalous topographic density on the mean gravity is linearly correlated with anomalous density values and the elevation. Foroughi and Tenzer [15] demonstrated that only a partial improvement can be achieved when considering the terrain geometry by incorporating the mean planar terrain correction to the Poincaré-Prey gravity gradient. Therefore, making it relevant to evaluate the effect of the anomalous lateral density. The heights of computation points were obtained from the elevation of the benchmarks, whereas the anomalous topographic density was interpolated from the rock density model of Nsiah Ababio and Tenzer [40]. The largest negative anomalous topographic density due to the variations in geological setting of the computation area is -569 kg/m^3, which is around 22% of the constant density value of 2670 kg/m^3. The rock density values throughout Hong Kong are generally lower than the constant density value, making the mean gravity values mostly underestimated. These variations together with the elevations in Hong Kong result in a maximum effect of approximately 11 mGal on the mean actual gravity at locations of leveling benchmarks. The effect is particularly small considering the moderate nature of the terrain, as the minimum effect is -0.14 mGal with an average and standard deviation of 0.67 and 1.49 mGal, respectively (see Table 1). The individual effects of anomalous topographic density at leveling benchmarks are shown in Fig. 6, together with the scatter plot to show the trend of the errors with respect to the heights of leveling benchmarks.

Evaluating the effect of lateral density variations on the mean actual gravity informs us of how much the gravity along the

Table 1
Statistics of the anomalous topographic density using the Hong Kong density model with constant average topographic density (2670 kg/m^3) and its effects on the mean gravity along the plumbline.

Min	Max	Mean	STD
Anomalous density (kg/m^3)			
−569.000	11.000	−283.204	245.461
Anomalous density effect (mGal)			
−0.140	11.162	0.669	1.494

Table 2
Statistics of the errors in orthometric correction (OC) due to density variation (units: mm).

	Min	Max	Mean	STD
Errors in OC	−2.704	2.762	0.002	0.209
Cumulative errors in OC	−5.450	0.716	−0.131	0.592

plumbline deviates with respect to Helmert's definition. Subsequently, the effect is reflected in the Helmert orthometric heights. The orthometric corrections to leveled height differences of the geodetic vertical control at the Hong Kong territories are practically realized in the solution of the Vertical Control Network 2022 (VCN2022). Since the Helmert orthometric heights have been computed by applying orthometric corrections to the elevations of the benchmarks, the effect of the anomalous lateral topographic density can be estimated on the orthometric corrections by Eq. (9). As explained earlier, the topographic density values in Hong Kong are much smaller than the density 2670 kg/m^{3}used in Helmert's definition. Consequently, the orthometric corrections are generally smaller than those computed according to Helmert's theory. Individually, the orthometric corrections are small, reaching a maximum and minimum value of ±3 mm but cumulatively attaining the highest value of about 13 mm [49]. Similarly, individual effects of topographic density variations on orthometric corrections reach a maximum of 2.8 mm and a minimum of −2.7 mm. This effect is generally negligible for benchmarks located in flat areas as it is directly dependent on the height of the computation point (Table 2). Fig. 7 shows that the effect is generally close to zero, with increments appearing at the mountain chains in Kowloon, Lantau Island, and Hong Kong Island. Nevertheless, their cumulative effect on the orthometric correction reaches the highest negative value of −5.45 mm (Fig. 8), with the scatter plot showing an increasing downward trend with respect to the height. The negative effect is a result of the overestimation of the initial orthometric corrections due to the higher adopted value of the constant topographic density of 2670 kg/m^3. This finding agrees with the numerical results presented by Foroughi and Tenzer [15] and Tenzer et al. [14]. They acquired that this contribution is typically within ±2 cm globally, except for the highest mountain ranges also characterized by the complex geology, where this contribution could reach even ±20 cm. Subsequently, it can be deduced that the summation of the cumulative effect of the density variation and the cumulative orthometric correction of a leveling benchmark will ideally result in the true orthometric height of the benchmarks.

5. Conclusions

We have investigated the effect of topographic density variations on the geoid and orthometric heights in Hong Kong, where the anomalous lateral variations of topographic density are significant, varying from -569 to 11 kg/m^3. The results show that this effect on the geoid reaches up to 2.1 cm. Arguably, this result ascertains that the detailed density model should be used to improve the geoid model, especially in mountainous areas. Nevertheless, we were not able to assess the extent of improvement due to a lack of GNSS-leveling benchmarks in Hong Kong. In total, 16 GNSS-leveling benchmarks currently established in Hong Kong are located in lowlands, where the effect of anomalous topographic density is almost negligible, as this effect is a function of a square of the computation point height. Furthermore, the effect of topographic density variations was applied to the mean gravity at locations of VCN22 leveling benchmarks. The result indicated that the largest anomalous topographic density was around 22% of the constant density value and amounted to an approximate maximum effect of 11 mGal in combination with the topographic heights. Finally, the effects on orthometric corrections, which lead to the orthometric heights, were evaluated. As established, the density of the topographic masses over Hong Kong is smaller than the constant average density of 2670 kg/m^3. The orthometric corrections are then mostly overestimated. The individual effects on the orthometric corrections are notable along mountain chains with the maximum and minimum within ±2.8 mm. Cumulatively, the maximum effect of anomalous density on VCN22 orthometric heights reaches 5.45 mm. This effect is estimated for heights below 500 m. Therefore, it is expected to increase with respect to an increase in the heights of benchmarks.

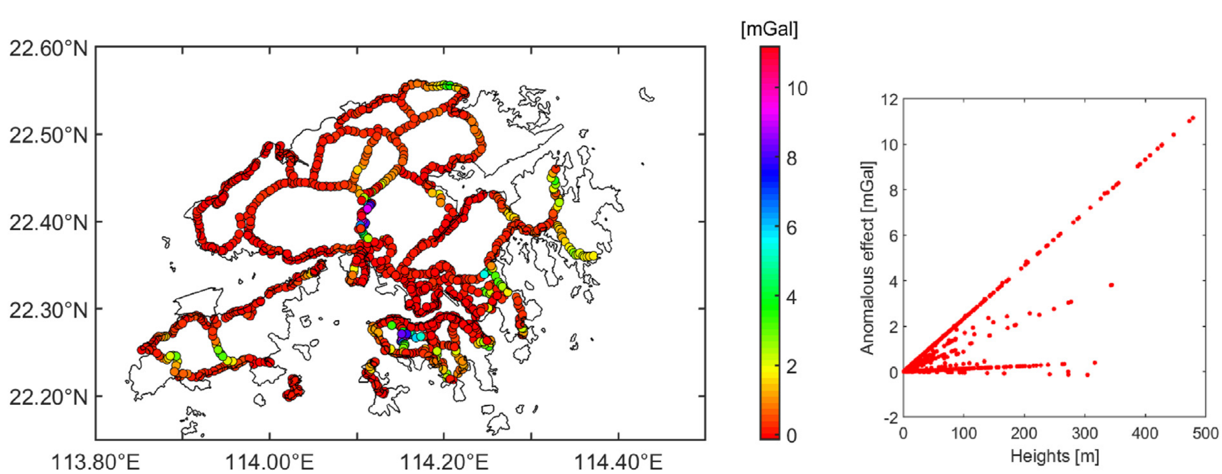

Fig. 6. The effect of anomalous topographic density on the mean gravity of the plumbline and their relationship with respect to the heights of the benchmarks in the VCN2022.

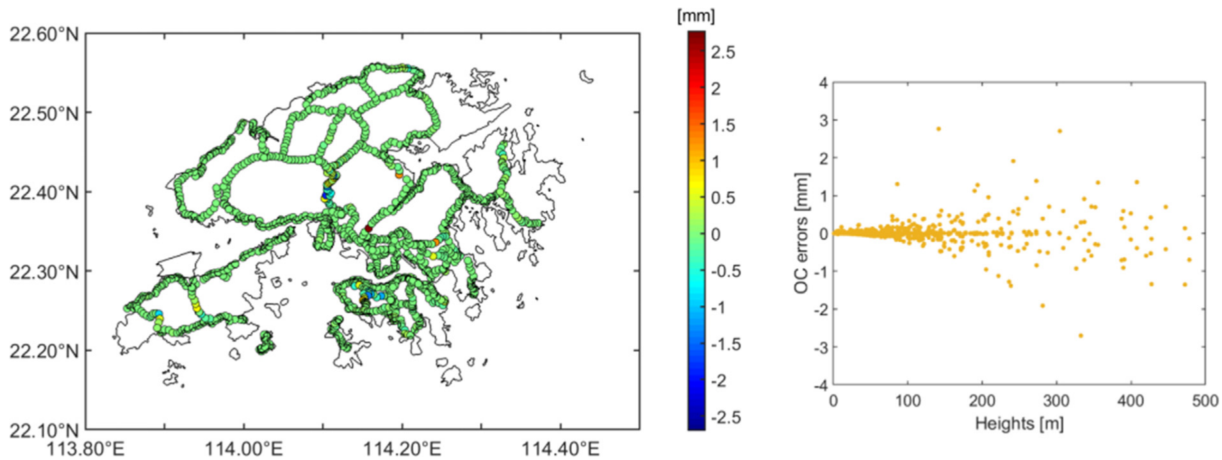

Fig. 7. Errors in orthometric correction to leveling benchmarks.

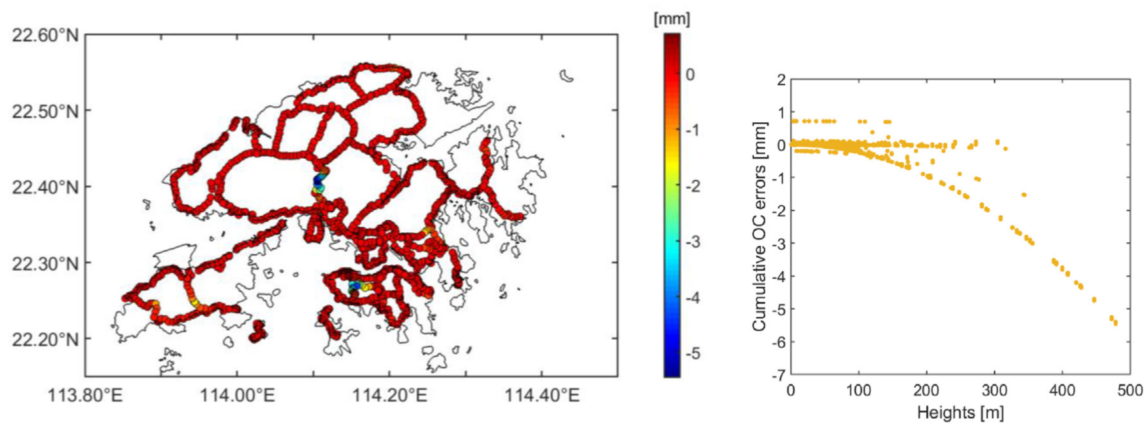

Fig. 8. Cumulative errors in orthometric correction to leveling benchmarks.

Notwithstanding, it is evident that using digital rock density models in the geoid and orthometric height determination is essential, especially in regions with elevated topography and complex geological structure.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to restrictions.

Declaration of competing interest

The authors declare that there is no conflicts of interest.

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