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Energy harvesting and energy conversion in an electromechanical coupling acoustic black hole beam

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Abstract. Acoustic black hole (ABH) shows unique and attractive features of energy focusing when the flexural wave propagates along a structure with a variable power-law thickness profile, which are found to be conducive to effective energy harvesting. In this paper, an improved electromechanical ABH model is established based on Timoshenko beam theory, which considers the high-frequency shear and rotational effects of the beam, as well as its coupling with PZT coating and other additional elements like damping layers. External electrical modules including both linear and nonlinear circuits can also be easily integrated into the system to form a fully coupled electromechanical model. The proposed model is then used to analyze typical ABH-specific features such as energy focusing and energy harvesting. Numerical results demonstrate the excellent energy harvesting performance and confirm that installing PZT on the ABH beam warrants higher electrical power than the uniform beam. Moreover, studies explore the relationship between the electromechanical coupling and the energy harvesting efficiency, and different methods to enhance the electromechanical coupling are also investigated. Finally, experimental results are presented to demonstrate the feasibility of ABH beam in energy harvesting.

Introduction

Flexural waves propagate along a thin-walled structure and interact with the surrounding medium, causing structural vibrations and noise. Manipulating waves by tailoring the physical or geometrical properties of a structure is the foundation for the vibration and noise control and has drawn increasing attention in the research community [1-3]. The concept of Acoustic Black Hole (ABH) was proposed based on this principle, which is a new wave manipulation method and has aroused intense interest since its inception [4, 5].

ABH shows unique and attractive features of energy focusing when flexural waves propagate along a structure with a tailed thickness profile following a reduced power law [6-8]. The local phase velocity and the group velocity of the flexural wave gradually decrease as the thickness decreases [9, 10]. In an ideal scenario, wave reflection is annulled when the thickness becomes zero, and this causes a high energy concentration. In contrast to traditional approaches, in ABH only a small amount of damping material can effectively dissipate concentrated energy [11, 12]. In addition to energy dissipation, strong energy concentration in ABH portion also offers new opportunities for effective energy harvesting [13-15].

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However, existing studies on ABH-based energy harvesting are still limited to focus on phenomenon analyses of specific structures through FEM simulations. There is a lack of basic theoretical investigations and flexible simulation tools to guide system design and optimization. Moreover, the physical process of energy harvesting involves numerous parameters of the electromechanical coupling system, including those of the host structures, piezoelectric materials and external circuits, posing a challenge to parameter optimization design. To sum up, a flexible simulation tool is needed 1) to solve the specific requirements of ABH-based energy harvesting to guide the design and optimization of the coupled system; and 2) to better understand and quantify the energy conversion and electromechanical coupling in the system.

Motivated by the above, this paper proposes an electromechanical semi-analytical ABH model based on Timoshenko beam theory, which considers the high-frequency shear and rotational effects. PZT patches and other additional elements like damping layers can be coated in this model, and further, external circuit modules can also be easily integrated into the system to form a fully coupled electromechanical model. The proposed model is then used to investigate a few important issues pertinent to energy harvesting. Numerical results prove the better energy harvesting performance of ABH beam than the uniform beam. Studies explore the relationship between electromechanical coupling and energy harvesting efficiency, and different methods to enhance the electromechanical coupling are also investigated. Finally, experimental results are presented to demonstrate the feasibility of ABH beam in energy harvesting.

Theoretical Model and Formulation

Figure 1 shows a beam with flexural vibration while a point force $f_{ext}(t)$ is excited at x_f. It is symmetrical with respect to the mid-line, the width is constant b. This beam is composed of an uniform part from 0 to x_u (the thickness is constant $2h_u$) and an ABH part from x_u to l (the thickness $2h_b$ is variable following a power-law profile), $h_b(x)=\beta(L-x)^m$. L is the total length when the thickness can ideally be decreased to 0 without truncation. Besides, single or multiple piezoelectric patches with constant thickness h_p can be symmetrically laid on the two sides of the beam. Fig.1 shows an example using a resistance (R) resonant circuit for the purpose of energy harvesting, while other circuits can also be connected. Both ends of the beam are elastically supported by two springs, the stiffness can be adjusted to simulate different boundary conditions [16, 17].

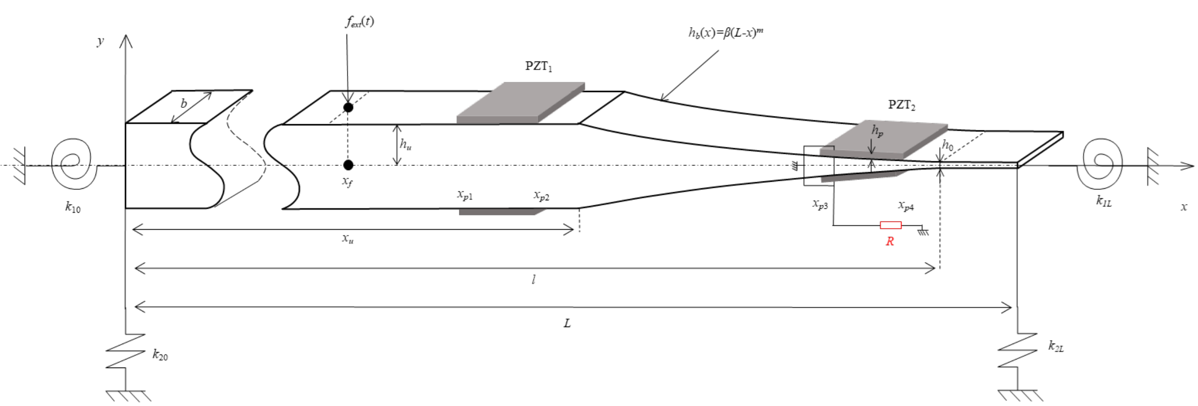


Figure 1. An ABH beam connected with multiple piezoelectric patches and a resistance resonant circuit.

In this paper, an electromechanical coupling ABH model based on Timoshenko beam with PZT patches and the external circuit is proposed through Rayleigh-Ritz method [18]. The out-of-plane displacement $w(x, t)$ and the rotation angle $\theta(x, t)$ of the beam are decomposed into a set of assumed shape functions [19], whose corresponding temporal coordinates (in two unknown vectors $\mathbf{a}$(t) and $\mathbf{b}$(t)). And the kinetic energy, potential energy, electrical load and work done by $\mathbf{f}_{ext}$(t) can be mathematically expressed to form the Lagrangian of the coupled system. This electromechanical ABH beam with a pure resistance circuit can be written as the matrix form:

$$\begin{bmatrix} \boldsymbol{M}_{a1}+\boldsymbol{M}_{a2} & \boldsymbol{M}_{b1} & 0 \\ \boldsymbol{M}_{b1} & \boldsymbol{M}_{b2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\boldsymbol{a}}(t) \\ \ddot{\boldsymbol{b}}(t) \\ \ddot{q}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & R \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{a}}(t) \\ \dot{\boldsymbol{b}}(t) \\ \dot{q}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{a1}+\boldsymbol{K}_{a2}+\boldsymbol{K}_{a3} & \boldsymbol{K}_{b1}+\boldsymbol{K}_{b2} & -{C_{eq}}^{-1}\boldsymbol{\Theta}_1 \\ \boldsymbol{K}_{b1}+\boldsymbol{K}_{b2} & \boldsymbol{K}_{b3}+\boldsymbol{K}_{b4}+\boldsymbol{K}_{b5} & -{C_{eq}}^{-1}\boldsymbol{\Theta}_2 \\ -{C_{eq}}^{-1}{\boldsymbol{\Theta}_1}^{\boldsymbol{T}} & -{C_{eq}}^{-1}{\boldsymbol{\Theta}_2}^{\boldsymbol{T}} & {C_{eq}}^{-1} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{a}(t) \\ \boldsymbol{b}(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{ext}(t) \\ \boldsymbol{0} \\ 0 \end{bmatrix}. \quad (1)$$

The linear frequency response curve (FRC) of this forced vibration system and the harvested electrical charge can be obtained by solving Eq. (1) directly. For free vibration, setting the force vector $f_{ext}(t)$ to zero leads to the corresponding eigenvalue equation.

Numerical Analyses

An electromechanical cantilever ABH beam is numerically investigated, its geometrical and material parameters are shown in Table 1. The cantilever beam can be simulated by setting k_{10} and k_{20} to 10^{12} for the uniform end, and k_{1L} and k_{2L} assigning to 0 at the ABH portion as detailed in [20]. The ABH beam is excited by a harmonic point force with amplitude of 1N at x_f=100mm on the uniform portion.

Table 1. Geometrical and material parameters of the beam, PZT and external circuit.

Geometrical parameters	**Material parameters**
Beam $m = 2$ $\beta = 0.1$ b = 50mm x_u = 250mm l = 450mm L = 500mm h_u = 6.25mm h_0 = 0.5mm PZT x_{p1} = 400mm x_{p2} = 450mm h_p = 0.5mm	Beam Density: ρ_b = 7800kg/m^3 Damping loss factor: η_b = 0.005 Elasticity modulus: E_b = 210GPa PZT Density: ρ_p = 7600kg/m^3 Damping loss factor: η_p = 0 Elasticity modulus: E_p = 132GPa Piezoelectric stress constant: e = -3C/m^3 Dielectric constant: $\varepsilon^s = 2.8\times10^{-9}$F/m External circuit Resistance: R = 1000Ω

(1) Energy focusing of ABH

Firstly, in order to illustrate the potential advantages from energy harvesting perspective, the modal shapes of some modes of the ABH beam without any external module are shown in Figure 2. They are beneficial to understand the baseline system dynamics and the subsequent ABH phenomenon and the corresponding energy focusing feature. It can be seen that the ABH portion (the grey regions), particularly near the tip, oscillates very strongly, the vibration amplitudes are much greater than the uniform part, especially the higher order modes. This indicates a strong concentration of vibration energy around the tip, which is a typical ABH feature and facilitates energy harvesting.

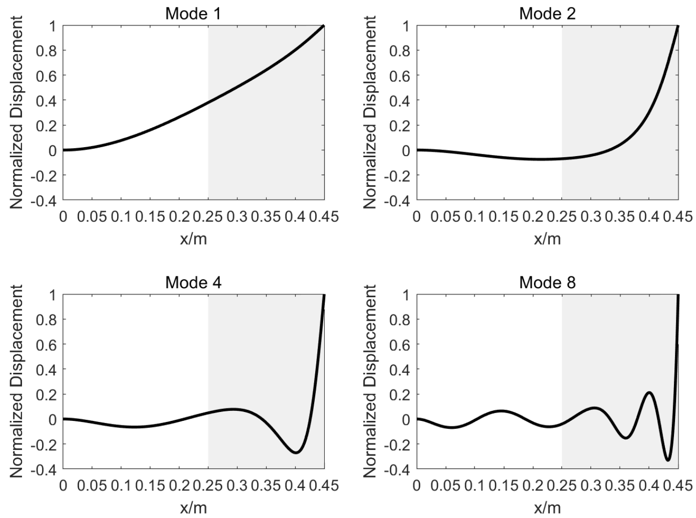


Figure 2. Mode shapes of modes 1, 2, 4 and 8 without PZT.

(2) Energy harvesting and energy conversion
Having demonstrated the unique advantages of ABH beam on energy focusing, PZT patches are added to the beam to harvest the electrical power. The harvested output power of each PZT patch can be calculated as $P = V^2/R$, so the total harvested output power is $P_{out} = 2P$. The harvested electrical power has also been compared in Figure 3 with that when the beam is uniform all the way. From the comparison it can be seen that the energy harvesting efficiency of ABH beam is much higher than the uniform beam in a wide band range, demonstrating the advantages of ABH beam in energy harvesting.

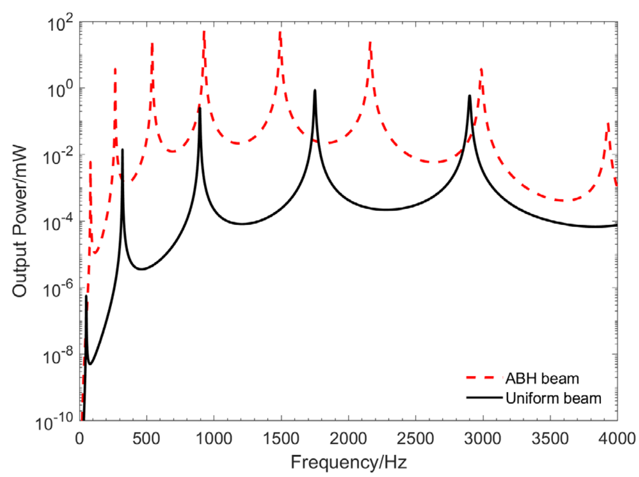


Figure 3. Comparison of harvested electrical power between ABH beam and uniform beam.

As an important indicator of energy recovery efficiency, in addition to the harvested output power, we also introduce an energy conversion rate, δ, which is defined as the ratio of the harvested output power from PZTs to the input mechanical power [20]:

$$\delta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{{}^1/_2\, Re\{F \cdot v^*\}}. \quad (2)$$

Note that, due to energy conservation, P_{in} includes both the harvested output power from PZTs and the dissipated power by the mechanical system, which is related to the amplitude of the external force and the velocity of the excitation point. The comparison of energy conversion rate between the ABH beam and the uniform beam is shown in Figure 4.

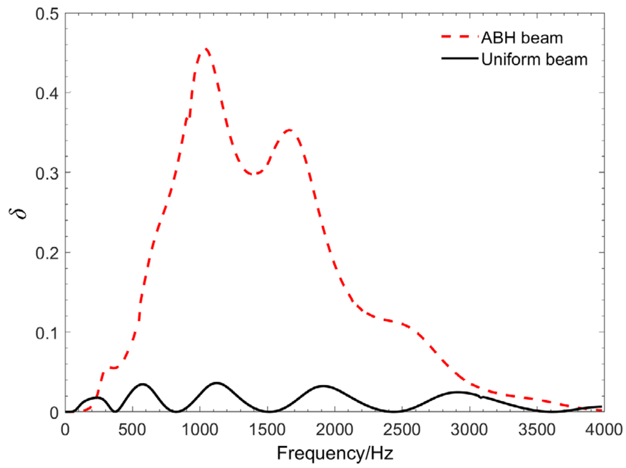


Figure 4. Comparison of energy conversion rate between ABH beam and uniform beam.

As shown in Figure 4, by comparing the energy conversion rate between the two different beams, the obvious advantages of the ABH beam in energy harvesting can be better seen. We can immediately conclude that the uniform beam shows no advantages over ABH beam in a fairly wide frequency range, when PZT patches are placed on the same position for the uniform beam and ABH beam, highly consistent with the conclusion obtained from the output power. It is precisely because of the energy focusing effect of ABH structures that energy from the uniform portion is trapped by the ABH portion, resulting in a high energy focusing around the ABH tip, and then harvest by the PZTs. It means that when the ABH effect occurs, broadband energy harvesting based on ABH structure shows great potential, including energy conversion in the high frequency range.

(3) Energy conversion and electromechanical coupling

There are many parameters in the system affecting the energy harvesting efficiency and the most fundamental factor is the strength of electromechanical coupling. Therefore, designing an efficient system is a complex problem that requires a clear understanding the role of electromechanical coupling in the energy conversion process, and to take a good quantification of the electromechanical coupling into consideration. In order to establish its possible relationship between the energy conversion rate and electromechanical coupling, the electromechanical coupling factor k is introduced, which is defined as [21, 22]:

$$k^2 = \frac{{\omega^2}_{oc} - {\omega^2}_{sc}}{{\omega^2}_{sc}}. \tag{3}$$

where ω_{oc} and ω_{sc} are the angular natural frequencies of the coupled system when PZT is open-circuited and short-circuited, respectively. They can be obtained easily, including by solving corresponding eigenvalue problems through this established model and by experimental tests. The relationship between the electromechanical coupling factor k and the energy conversion rate δ corresponding to the ABH beam are shown in Figure 5.

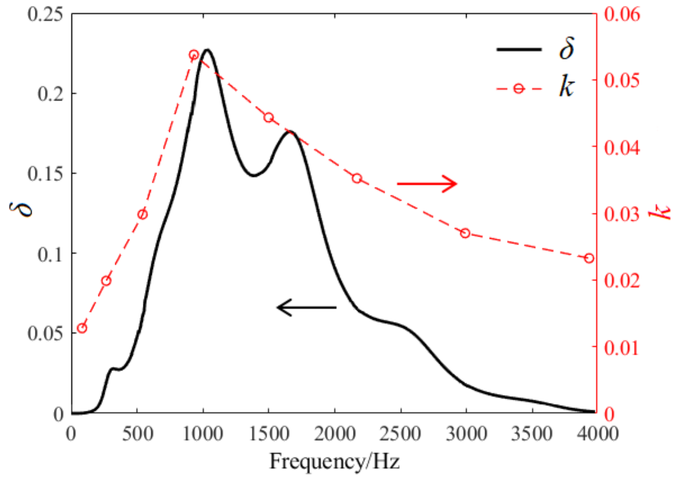


Figure 5. Comparison of energy conversion rate and electromechanical coupling factor.

Figure 5 shows that electromechanical coupling factor and energy conversion rate have very consistent trends with respect to the frequency. It means that the system with stronger electromechanical coupling can obtain the higher energy harvesting efficiency. Therefore, the electromechanical coupling factor k can be used as an intrinsic and simple indicator of the energy harvesting efficiency to guide the system design.

There are many different methods to enhance the electromechanical coupling effect, including designing external circuits, laying multiple sets of piezoelectric patches and so on. Figure 6(a) and 6(b) show the effect of adding negative capacitance in external circuit and using two sets of PZTs on the enhancement of electromechanical coupling. The results show the effectiveness of these methods, especially by adding the negative capacitance (Figure 6(a)), the electromechanical coupling effect can be greatly increased, and the design of the external circuit is relatively more flexible and convenient.

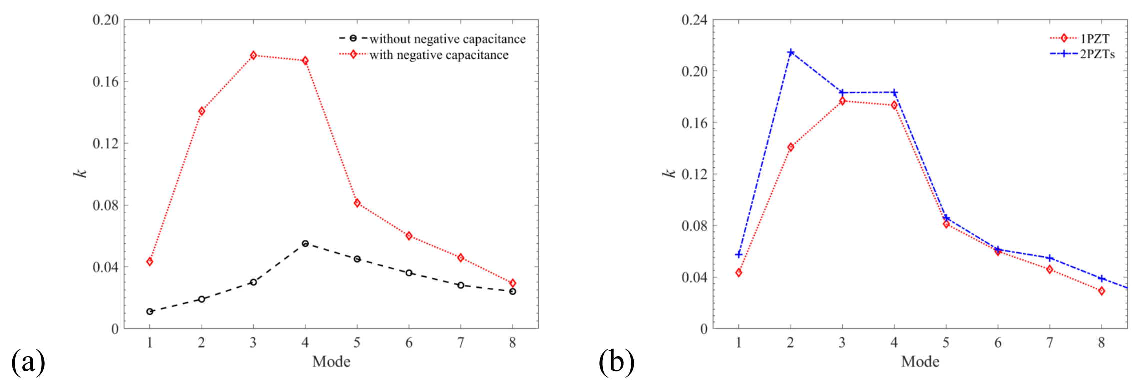


Figure 6. Effect of different methods on electromechanical coupling factor: (a) negative capacitance; (b) two sets of PZTs.

Experimental Results

The phenomena of ABH phenomenon and corresponding energy harvesting effect have been analyzed and discussed in the previous section. Here, experiments were conducted to further validate the energy focusing phenomenon of ABH effect and the advantage of energy harvesting. The test sample is shown in Figure 7. The uniform end of the beam is clamped by a fixed bracket and the ABH end is set free. The beam is excited at x_f=0.1m, measured from the clamped end on the uniform portion, by an electromagnetic shaker using a sweep signal from 5Hz to 1000Hz. A laser vibrometer (NLV-2500) is used to measure the point velocity of the beam at different positions (every 1cm) on the ABH beam for the modal shape test. Figure 8 shows the mode shapes for the first four structural modes of the ABH beam without PZTs, the energy focusing phenomenon has been validated in experiments.

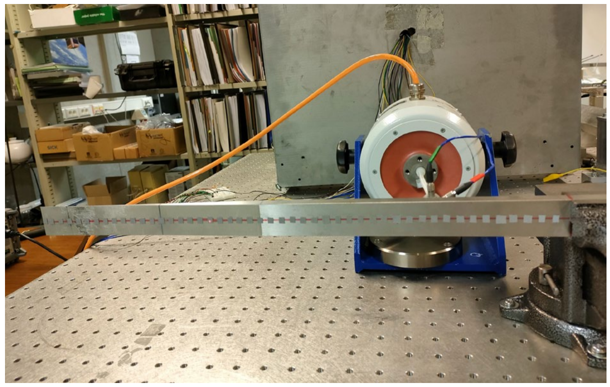


Figure 7. Experimental setup of mechanical ABH system.

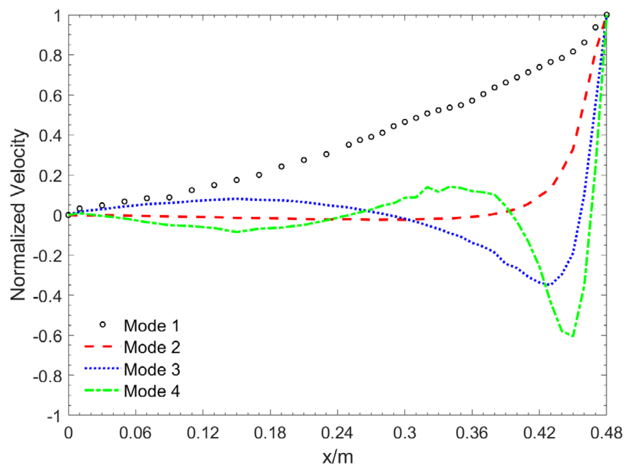


Figure 8. Experimentally measured modal shapes of the first four modes without PZTs.

The next step is to verify the advantages of ABH beam in terms of energy harvesting. An essential part in an energy harvesting circuit is rectification, which is also called AC/DC conversion. We use the most common rectifier circuit, the bridge rectifier circuit, including four rectifier diodes, to convert the AC voltage into a positive voltage that can be directly applied to the load. Although the voltage rectified by the bridge rectifier circuit can continuously supply power to the load, it remains unstable. Therefore, a smoothing capacitor is added to the rectifier circuit to form a full-wave rectifier circuit, as shown in Figure 6.9, wherein the smoothing capacitor is used to convert the rectified positive voltage signal into DC to better supply power to the load. Here, a LED is connected into the circuit to visually demonstrate the energy harvesting effect.

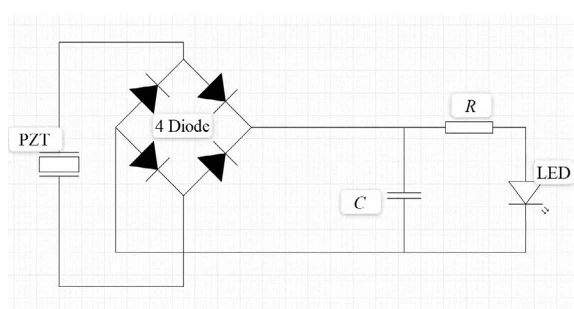
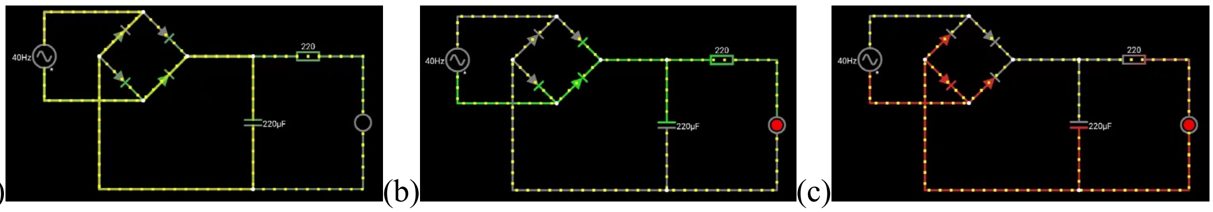


Figure 9. Schematic diagram of energy harvesting circuit in the experiment.

Figure 10 shows the working process of the energy harvesting circuit. Firstly, the alternating current generated by the piezoelectric patches is rectified by the bridge rectifier circuit to charge the capacitance, as shown in Figure 10(a). The capacitance then starts to discharge, which can supply power to the load to light up a LED (Figure 10(b)). Finally, the capacitance will finish discharge and the process repeats (shown as Figure 10(c)). The corresponding prototype of energy harvesting circuit used in the experiment is shown in Figure 11, which demonstrates the effect of energy harvesting: the LED light can be always on.



(a) (b) (c)

Figure 10. Working process of energy harvesting circuit: (a) capacitance is charging; (b) capacitance starts to discharge; (c) capacitance will finish discharge.

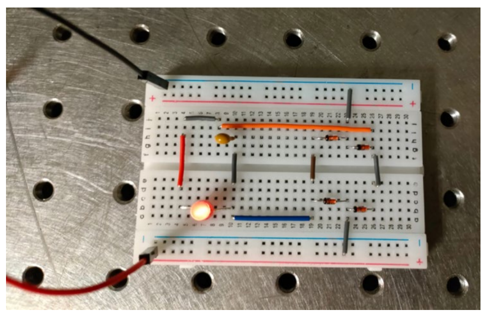


Figure 11. Experimentally measured effectiveness of energy harvesting: LED can be always on.

An oscilloscope is used to measure the voltage produced by the piezoelectric patches. Fast Fourier transform is performed on the entire voltage signal with the resultant spectrum shown in Figure 12. The output voltage can reach up to 6V, exceeding the 2V required for lighting up the LED. The electricity generated by the electromechanical ABH beam is deemed feasible as energy harvesting devices to not only power the LED light, but also for other microelectronic devices.

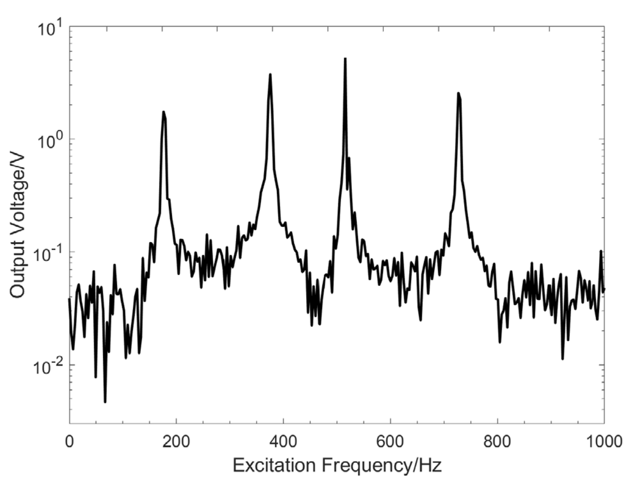


Figure 12. Experimentally measured output voltage from PZTs.

Conclusion

In this paper, a semi-analytical electromechanical ABH model is established to analyze the electromechanical coupling of Timoshenko beam coated with PZT patches, and its effects on energy harvesting and energy conversion. The unique energy focusing feature of the ABH effect makes the energy density near the ABH tip higher, favoring the PZT installation in the vicinity of its tip area and warranting better energy harvesting performance than the uniform beam. Moreover, the electromechanical coupling is shown to exhibit consistent variations with the energy harvesting performance of the system and thus is a good indicator for either evaluating the energy harvesting performance or tuning/optimizing system parameters for enhanced energy harvesting performance.

Experimental results confirm the excellent energy focus effect in the mechanical ABH beam, which offers the possibility for effective energy harvesting. After the voltage generated by the piezoelectric patches is rectified by the rectifier circuit and converted by the smoothing capacitance, the generated signal can stably supply power to a LED, with an appreciable voltage level, which holds promises for powering other microelectronic devices. As a final remark, it is relevant to note that theoretical analyses and experimental validation were used to emphasize some fundamental issues pertinent to ABH-based energy harvesting, but no effort has been made to achieve optimal energy harvesting performance. If necessary, the optimal design can be realized using the proposed simulation model.

Acknowledgements

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