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Decentralized local energy trading with cooperative energy routing in energy local area network

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ABSTRACT

Local energy trading is envisioned as a promising energy management approach for distributed renewable energy sources. To efficiently facilitate the novel local energy trading, the concept of energy local area network (e-LAN) has been proposed as an innovative framework for future community power grid, which aims to improve the efficiency of end-to-end power delivery by the novel technique of energy routing. Under this framework, a proper market clearing approach is required for e-LAN to manage decentralized local energy market and coordinate end-to-end energy routing paths. In this paper, a decentralized market clearing approach with cooperative energy routing is proposed for local energy market in e-LAN. Due to the nonconvexity of the original market clearing problem, the proposed approach formulates a new problem for market clearing in e-LAN, which is approximately equivalent to the original problem. The new problem consists of two subproblems, a market clearing problem and a cooperative energy routing problem. For the market clearing problem, this paper proposes a dual decomposition based approach to clear local energy market in a decentralized way so as to avoid privacy violation. The cooperative energy routing problem is solved by proposing a Nash bargaining based approach, where each trading pair can obtain more benefits from cooperation than from noncooperation. Simulation results are presented to verify the effectiveness of the proposed approach in avoiding physical constraint violation, resolving path conflicts and improving social welfare.

1. Introduction

Renewable generation has been attracting much attention due to the benefits of energy-saving and environmentally friendly that it brings [1,2]. To accommodate a large penetration of distributed renewable energy sources (RESs), the future power system is envisioned to be organized as numerous smaller-scale area grids or microgrids [3,4,5]. In line with this, a novel energy management approach, local energy trading where each prosumer can play both roles of energy producer and consumer, is proposed to maximize the utilization of RESs [6]. However, the novel local energy trading requires a flexible end-to-end power dispatching and transmission, which is challenging to be satisfied by the traditional local power grid based on the top-down electricity market.

In [7,8,9], the concept of energy local area network (e-LAN) has been proposed as an innovative and promising framework for the future microgrid or community grid. As shown in Fig. 1, an e-LAN is a local energy grid where all electrical devices, including generators, loads and energy storages, are interconnected in the form of mesh network by using energy routers (ERs). In e-LAN, electric energy is treated similar to a packet of mail tagged with information about the sender and receiver [10]. When an energy producer and an energy consumer are matched as a trading pair, the traded power can be controlled to flow from the producer to the consumer by the collaborative work of ERs, which is called energy routing [4,11,12,13]. More details about energy routing are presented in Section 2.1.

Although the flexibility of end-to-end power dispatching and transmission is significantly improved in e-LAN, it gives rise to a new problem of energy routing, which aims to find the optimal routing paths for transmitting power between trading pairs. Note that the energy routing problem in e-LAN differs from the optimal power flow (OPF) problem in traditional power grid in a number of aspects. At first, the decision variables for the two problems are different. While OPF searches for the optimum operation parameters such as voltage and injected power, energy routing finds the optimum routing paths for trading pairs. Secondly, the two problems should be solved in different manner as they exist for different market mechanisms. In the traditional electricity market, utility companies are required to bear the cost of transmission

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Nomenclature

Abbreviations

RES	Renewable energy source
ER	Energy router
PFD	Power flow direction
KKT	Karush-Kuhn-Tucker

Variables

$\mathscr{C}$	The set of consumers
i	Index of consumers
W	Welfare function
C	Cost of purchasing electricity
Ω_i	Path set of consumer i
R	Resistance
π	Electricity price
$\underline{D_i}$	Minimum demand of consumer i
$\overline{G_j}$	maximum generation of producer j
e-LAN	Energy local area network
OPF	Optimal power flow
NSO	Network system operator
MINLP	mixed integer nonlinear programming
$\mathscr{P}$	The set of producers
j	Index of producers
u	Satisfaction function
T	Transmission cost
p	Index of energy path
V	Voltage
Ψ_j	Path set of producer j
$\overline{D_i}$	Maximum demand of consumer i
$Cap_{(m,n)}$	maximum capacity of transmission line (m,n)

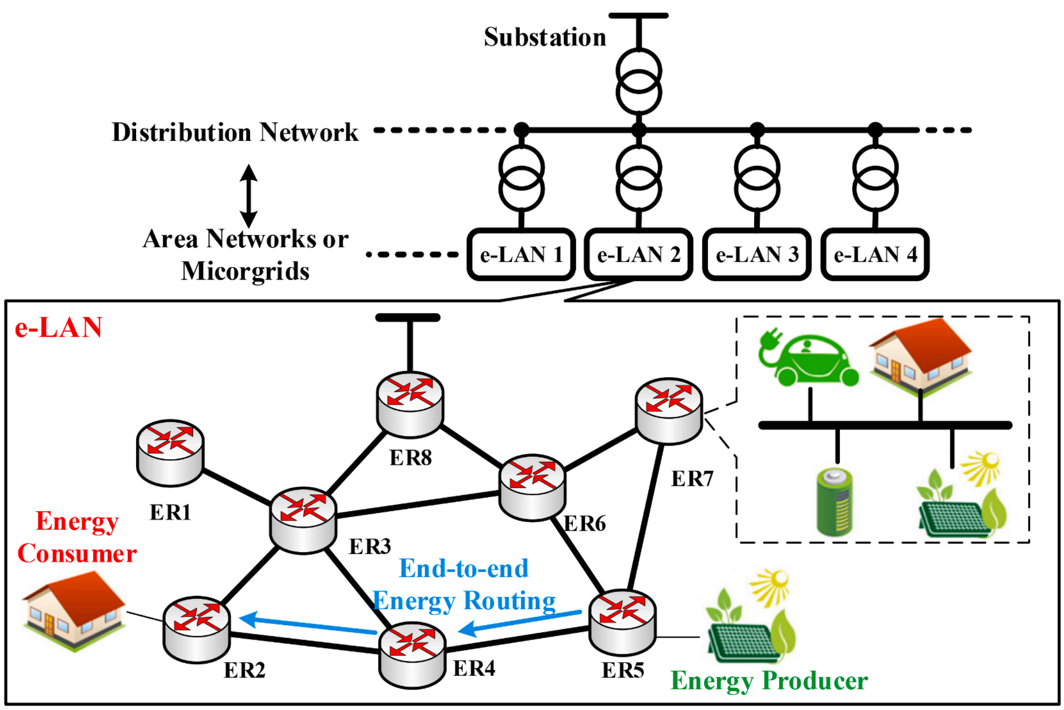

Fig. 1. Diagram of e-LAN.

loss, whereas in local energy trading the cost of transmission loss has to be borne by the trading pairs. Therefore, OPF is to directly optimize the operation efficiency of the whole power system no matter in centralized or decentralized way. However, the energy routing problem should be solved in a decentralized environment where each trading pair should be allowed to pursue their minimum-cost routing path.

Due to the difference between active energy routing and traditional passive power flow, the market clearing mechanism design for local energy trading in e-LAN becomes a challenging task. Although there exists various market clearing approaches for energy trading in micro-grid or traditional power grid [14–20], these approaches cannot deal with the problem of energy routing and thus is not appropriate for e-LAN.

In recent literature, [21] incorporates energy routing into market clearing by proposing an electrical distance driven peer matching method where each prosumer uses the shortest-path Dijkstra algorithm to find the shortest-electrical-distance path and obtain the minimum transmission cost for a potential energy transaction. It is shown by simulation results that those prosumers who considers energy routing gain more benefits from energy trading than those who ignore energy routing. Similarly, [22] present a hierarchical energy trading model in Energy Internet, which also utilizes Dijkstra algorithm to plan energy routing paths for trading pairs. In [23], a biased min-consensus-based approach is proposed for each prosumer to find the best trading partner and the optimal routing path. [24,25] investigates an energy scheduling model for vehicle-to-grid (V2G), where a reinforcement learning based algorithm is proposed to optimize the routing path for dispatching a set of electric vehicles to supply different consumers at different locations.

An important limitation of these market clearing approaches considering energy routing is that they can only allow single-path energy routing for traded power delivery. In doing so, some transmission lines having small available capacities will immediately be ignored in order to avoid congestion, leading to a low utilization rate of some

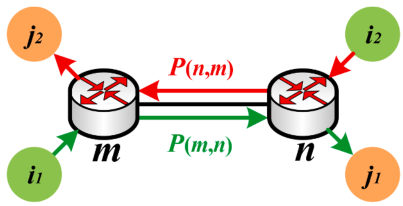

Fig. 2. An example of PFD conflict.

transmission lines. As a result, [26,27] propose a real-time transaction market based on e-LAN where traded power can be split into packets and delivered through multiple paths. By utilizing the mini-cost flow algorithm in graph theory, each trading pair can find a multi-path solution for traded power delivery. However, possible line congestion caused by decentralized energy routing among multiple trading pairs is ignored.

To bridge this gap, [28,29] present a priority ranking based market clearing method for e-LAN, where market clearing and routing management are separated into two stages. The local energy market is cleared by using a centralized matchmaking method to determine trading pairs, price and quantity. In the routing management stage, each trading pair is assigned a priority level for traded power delivery according to the traded quantity [28] or the time when the transaction is determined [29]. The available capacity of a transmission line will be assigned to trading pairs with higher priority level. In [30], a market clearing method considering network physical constraints are studied. The energy market is cleared in a decentralized way, where the global network constraints are decoupled as numerous local network constraints for prosumers. [4] proposes to eliminate line congestion in e-LAN by encouraging involved trading pairs to cooperate. The transmission cost savings obtained by cooperation is fairly assigned to trading pairs based on Shapley value.

However, all of these mentioned market clearing approaches for e-LAN ignore another important network constraint, the power flow direction (PFD) constraint, which is imposed by the physical limit that a transmission line cannot transmit power simultaneously in two opposite directions. As trading pairs plan their routing path according to their own preferences, a power or transmission line may be required to transmit power in two opposite directions simultaneously by some trading pairs, which is PFD conflict, as shown in Fig. 2. Therefore, the PFD constraint of e-LAN cannot be ignored during the process of market clearing.

It is worth noting that PFD constraint is a nonconvex set, which poses challenges on the design of market clearing approach. Due to the privacy concerns of prosumers, the local energy market in e-LAN should be cleared in a decentralized way. However, standard decentralized computation methods such as alternating direction method of multipliers (ADMM) and dual decomposition are convex optimization methods and thus cannot be directly applied on the market clearing problem in e-LAN.

Furthermore, in order not to limit energy trading in e-LAN, each PFD conflict should be resolved efficiently. A low-efficiency resolution may significantly increase transmission cost or even fail to provide feasible paths for some involved trading pairs. For example, when the PFD conflict in Fig. 2 is resolved by allowing the PFD from ER m to ER n, trading pair $\{i_2, j_2\}$ has to abandon all paths including line (m, n) and choose other lower-efficiency paths. In doing so, trading pair $\{i_2, j_2\}$ has to bear more transmission cost or even cannot find available paths. In some cases, they may receive excessive transmission cost and thus terminate their transaction. Therefore, an efficient conflict resolving method for PFD conflict should be incorporated in the market clearing approach for e-LAN.

To address the aforementioned problems, this paper proposes a market clearing approach with cooperative energy routing for local energy market in e-LAN. The proposed approach consists of two stages, a market clearing stage and a cooperative energy routing stage. The market clearing stage is to clear the local energy market in e-LAN in a decentralized fashion, where prosumers are allowed to maximize their benefits without considering PFD constraint. The cooperative energy routing stage is to efficiently eliminate all PFD conflicts among trading pairs. Compared with the previous works, the main contributions of this paper can be summarized as follows.

(1) The local energy trading in an e-LAN is formulated as a two-stage social welfare maximization problem, which consists of two subproblems, to address the nonconvexity of PFD constraint and facilitate decentralized market clearing.
(2) A dual decomposition based approach is proposed to clear local energy market in a decentralized manner, where the global social welfare maximization problem is decomposed into a series of personal welfare maximization problems for prosumers. Each prosumer can formulate its trading strategy and plan its routing paths without revealing the important private information to others.

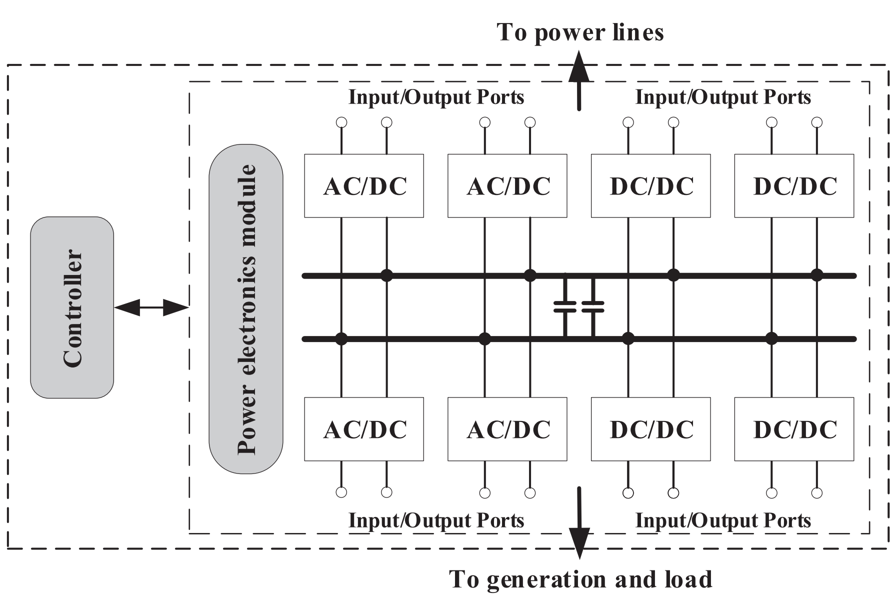

Fig. 3. Basic structure of energy router [7,12].

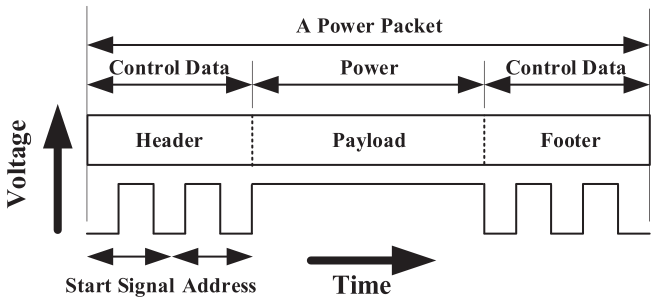

Fig. 4. Structure of power packet [10].

(3) A Nash bargaining based approach is proposed to eliminate PFD conflict efficiently and achieve cooperative energy routing. It is rigorously proved that each trading pair can obtains less transmission cost from cooperation than from noncooperation.

The rest of the paper is organized as follows. Section 2 presents a detailed description about energy routers and energy routing in e-LAN. Section 3 describes the system model of local energy market in e-LAN and formulates the market clearing problem. Section 4 proposes a market clearing approach with cooperative energy routing. The effectiveness of the proposed approach is verified by numerical simulations in Section 5. Conclusions are drawn in Section 6.

2. Energy router and energy routing

As mentioned before, the end-to-end traded power delivery in e-LAN is realized by the technique of energy routing, which is different from the passive power flow in traditional power grid. In this section, a detailed description about ER and energy routing is presented.

The typical structure of ERs is shown in Fig. 3 [7,12], where an ER is composed of a controller and a power electronics module. The power electronics module is a multi-port power converter system where each port is essentially a bidirectional AC/DC or DC/DC converter that can output or receive power. Energy generation, storage devices and loads with different terminal characteristics are interfaced to a common internal dc bus via suitably designed power converters, and exchange energy with one another via the dc bus.

Unlike the power flow in traditional grid, electrical energy in e-LAN is routed based on the power packet [8,9,10]. As shown in Fig. 4, a power packet consists of a header, the payload and a footer. The payload is the transmitted power from the power source to the load. According to [10], the amount of transmitted power can be regulated by changing the length of the payload, which is achieved through pulse width modulation. The header and footer are regarded as information tags attached to the payload. The header includes some information such as the address

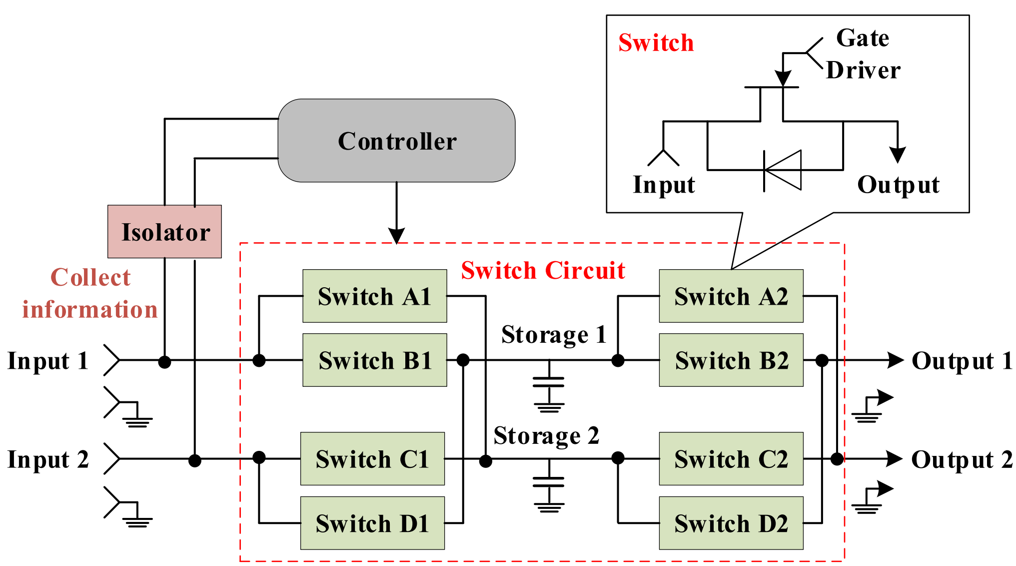

Fig. 5. Diagram of power forwarding by a simplified ER.

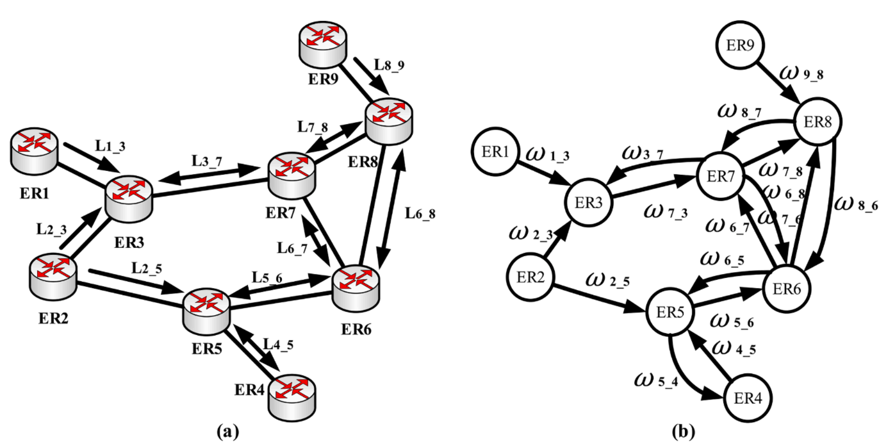

Fig. 6. (a) An example e-LAN. (b) Graph model of the e-LAN shown in Fig. 6 (a).

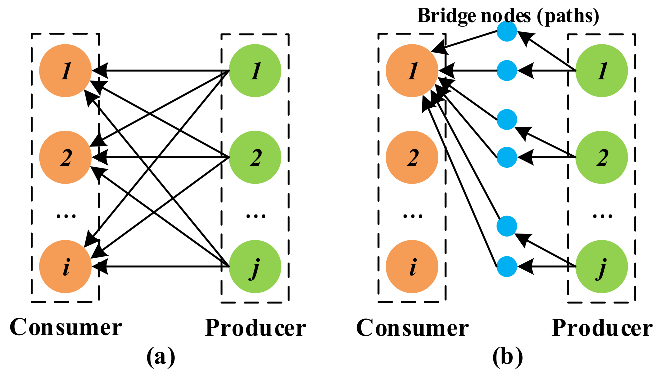

Fig. 7. (a) local energy market proposed in [6,17]. (b) local energy market in e-LAN.

of source and load and the footer contains a mark at the end of packet. In doing so, the power and information can be transmitted together, which avoids contradiction between the quantity of shared power and its information.

Energy routing in e-LAN is realized by using multiple ERs to forward the transmitted power from the power source to the power load. Fig. 5 shows a simplified ER realized using simple switches instead of power converters for illustrating the idea. The main function of the switches is to reconfigure the connection between an input port and an output port for realizing omni-directional power flow. When the transmitted power is input to an ER, the controller reads the tag information of power packets on the lines through an isolator. Then, the controller outputs signals for ON/OFF regulation to the switches on the switch circuit via the gate drivers. The former switches, i.e., A1, B1, C1, and D1 in Fig. 5, guide the received power packet to the specified storage area according to its tag information. The storage areas are separated by source to avoid any loss of origin information of power. The latter switches, i.e., A2, B2, C2, and D2, select the forwarding destination of the power packet, i.e., the objective load or the other router. In addition, the latter switches must also reproduce power packet information. They attach the information tag to the power payload based on the original tag information given by the controller.

3. System model

An e-LAN can be visualized as a community-scale power network through which a small group of prosumers are interconnected. Each prosumer can play both roles of producer and consumer. It is assumed that each prosumer in the e-LAN is equipped with some forms of energy generation and storage devices and local loads. When a prosumer experiences power deficiency or surplus, it can directly trade with other prosumers through local energy market in the e-LAN.

Due to its mesh network topology, an e-LAN is commonly modeled as a graph for analysis. Fig. 6 shows an example of a 9-node e-LAN represented as a directional network graph G. Each node i denotes an ER, and the edges of the graph denote the transmission lines that interconnect adjacent energy routers. The direction of the edges denotes the direction of power flow on the transmission lines and the edge weights $w_{(i,j)}$ denote the transmission losses resulting from the transmission line resistances.

Based on the graph model, this section establishes the welfare functions of consumers and producers and formulates the objective optimization problem. Since the market clearing model of a single trading period can be extended to multiple trading periods by adding time-coupled constraints, this paper focuses on the market design of a single trading period to more explicitly demonstrate the performance of the proposed model.

3.1. Welfare function

Energy transactions in traditional local energy market [6,17] can be represented by the bipartite graph in Fig. 7(a), where each consumer $i \in \mathscr{C}$ can purchase energy from multiple producers at the same time. When energy routing is considered, the local energy market in e-LAN is shown in Fig. 7(b), where bridge nodes denote routing or transmission paths. Each bridge node also represents a possible energy transaction between the two prosumers connected to it. Thus, each prosumer in the local energy market formulates its strategy by evaluating the profits gained from each bridge node connected to itself.

Based on the tripartite graph, we define $\mathscr{C}$ as the set of consumers and $\mathscr{P}$ as the set of producers in a given trading period. The welfare function W_i of consumer i is given by (1), which is composed of the following four items: (i) u_i – satisfaction derived from consuming all purchased electricity [6], (ii) C_i – cost of purchasing electricity through the p th path, (iii) T_i - transmission cost associated with the p th path, and (iv) C_{i-g} – cost of purchasing electricity from the grid. Note that in each trading period the consumer of a transaction is assumed to bear the transmission cost of the transaction. Due to intermittency and fluctuation of renewable generation, each prosumer cannot always be a producer or a consumer. Therefore, it is fair to always require consumers to bear the transmission cost.

$$W_i = u_i - C_i - T_i - C_{i-g} \tag{1}$$

$$u_i = \beta_i\left(\sum\nolimits_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g}\right) - \frac{\theta_i}{2}\left(\sum\nolimits_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g}\right)^2 \tag{2}$$

$$C_i = \sum\nolimits_{p=1}^{|\Omega_i|} \pi_{i-p} P_{i-p} \tag{3}$$

$$T_i = \sum\nolimits_{p=1}^{|\Omega_i|} \sum_{(m,n)\in l_p} \pi_l \left(L_{l-(m,n)} + L_{ER-(m,n)}\right) \tag{4}$$

$$C_{i-g} = \pi_{b-g} P_{i-g} \tag{5}$$

where Ω_i denotes the set of all routing or transmission paths between consumer i and all producers in $\mathscr{P}$. β_i and θ_i in (2) are the preference parameters of consumer i which characterize its satisfaction derived from consuming electricity. P_{i-p} is the amount of power transmitted through the p th path. P_{i-g} is the amount of electricity purchased from the grid. π_{i-p} is the electricity price of the producer associated with the p th path. In (4), (m,n) denotes the power line (m,n) connecteing ER m and ER n, l_p denotes the set of all lines on the p th path, π_l is the unit price of the compensating power for the transmission loss in each transmission line. It is assumed that during the process of power transmission, the e-LAN will compensate for the power losses in the transmission lines and ERs, i.e., transmission loss, to ensure that the agreed amount of traded power is received by the consumers. To compensate for the transmission loss, each pair will be charged a transmission cost. $L_{l-(m,n)}$ is the conduction loss in line (m,n) expressed by (6), and $L_{ER-(m,n)}$ is the power conversion losses in ER m and ER n calculated by (7). π_{b-g} denotes the price of purchasing power from the grid.

$$L_{l-(m,n)} = \frac{R_{(m,n)}}{V_{(m,n)}^2}\left[\left(P_{i-p} + P_{ex(m,n)}\right)^2 - P_{ex(m,n)}^2\right] \tag{6}$$

$$L_{ER-(m,n)} = [(1 - \eta_{out-m}) + (1 - \eta_{in-n})]P_{i-p} \tag{7}$$

where $R_{(m,n)}$ and $V_{(m,n)}$ is the resistance and voltage of line (m, n), respectively. $P_{ex(m,n)}$ is the existing power on line (m,n) before P_{i-p} is added to it. η_{out-m} and η_{in-n} is the efficiency of the output port of ER m and the input port of ER n, respectively.

The welfare function W_j of producer j consists of two items: (i) ϕ_j - revenue obtained in local energy market, (ii) ϕ_{j-g} – revenue generated

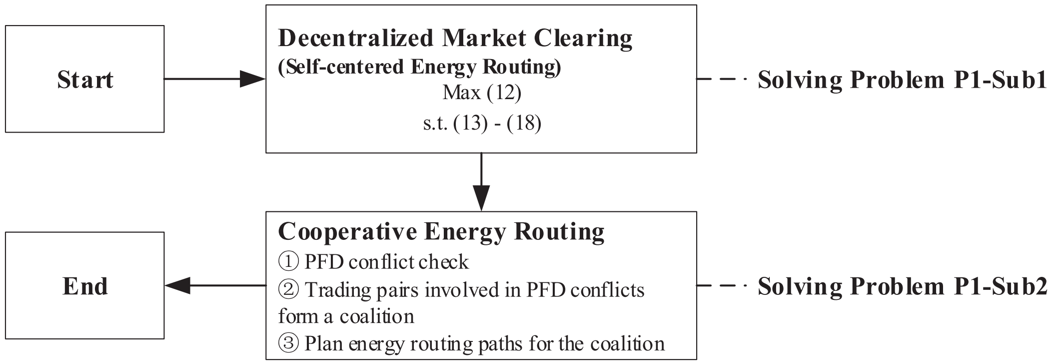

Fig. 8. Framework of proposed approach.

from selling energy to the grid, and (iii) u_j - cost function of producer j [3,30], which are calculated by (8)-(11).

$$W_j = \phi_j + \phi_{j-g} - u_j \tag{8}$$

$$\phi_j = \pi_j \sum_{j=1}^{|\Psi_j|} P_{j-p} \tag{9}$$

$$\phi_{j-g} = \pi_{s-g} P_{j-g} \tag{10}$$

$$u_j = \alpha_j \left(\sum_{j=1}^{|\Psi_j|} P_{j-p} + P_{j-g} \right) + \beta_j \left(\sum_{j=1}^{|\Psi_j|} P_{j-p} + P_{j-g} \right)^2 \tag{11}$$

where α_j and β_j are the parameters of cost function. Ψ_j is the set of all paths from producer j to all consumers. π_j is the price of producer j and π_{s-g} is the price of selling power to the grid. P_{j-p} denotes the amount of power transmitted through the p th path supplied by producer j and P_{j-g} is the amount of power sold to the grid.

3.2. Problem formulation

The aim of market clearing is to maximize the social welfare in e-LAN. Hence, the market clearing problem is usually formulated as Problem P0.

Problem P0: Original market clearing Problem

$$\max \left(\sum_{i=1}^{|\mathscr{C}|} W_i + \sum_{j=1}^{|\mathscr{P}|} W_j \right)$$

$$\triangleq \max \left(\sum_{i=1}^{|\mathscr{C}|} \widehat{W}_i + \sum_{j=1}^{|\mathscr{P}|} \widehat{W}_j \right) \tag{12}$$

s.t. (1) – (11) (13)

$$\underline{D_i} \le \sum_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g} \le \overline{D_i}, \forall i \in \mathscr{C} \tag{14}$$

$$0 \le \sum_{j=1}^{|\Psi_j|} P_{j-p} + P_{j-g} \le \overline{G_j}, \forall j \in \mathscr{P} \tag{15}$$

$$\sum_{i=1}^{|\mathscr{C}|} \sum_{p=1}^{|\Omega_{(i,j)}|} P_{i-p} = P_{j-p2p}, \forall j \in \mathscr{P} \tag{16}$$

$$P_{total}^{(m,n)} = \sum_{i=1}^{|\mathscr{C}|} \sum_{p=1}^{|\Omega_i^{(m,n)}|} P_{i-p} + P_{ex(m,n)}, \forall l_{(m,n)} \in \Xi \tag{17}$$

$$P_{total}^{(m,n)} \le Cap_{(m,n)}, \forall l_{(m,n)} \in \Xi \tag{18}$$

$$P_{total}^{(m,n)} * P_{total}^{(n,m)} = 0, \forall l_{(m,n)} \in \Xi \tag{19}$$

where $\widehat{W}_i = W_i + C_i$ and $\widehat{W}_j = W_j - \phi_j$. In (14), $\underline{D_i}$ and $\overline{D_i}$ denote the minimum and maximum demand of consumer i, respectively. $\overline{G_j}$ in (15) denotes the maximum generation of producer j. In (16), $\Omega_{(i,j)}$ denotes the set of all routing paths between consumer i and producer j, and P_{j-p2p} denotes the amount of power that producer j sells in local energy market. Note that $P_{j-p2p} = \sum_{j=1}^{|\Psi_j|} P_{j-p}$. In (17), $P_{total}^{(m,n)}$ denotes the total amount of power transmitted through transmission line (m,n), $\Omega_i^{(m,n)}$ denotes the set of consumer i's all routing paths to all producers incorporating transmission line (m,n), and $l_{(m,n)}$ denotes the transmission line (m,n). (18) is the line capacity constraint, where $Cap_{(m,n)}$ is the maximum capacity of line (m,n) and Ξ is the set of all transmission lines in e-LAN. (19) is the PFD constraint for each transmission line in e-LAN, which imposes the physical limit that a transmission line cannot transmit power simultaneously in opposite directions.

Due to the privacy concerns of prosumers, Problem P0 should be solved in a decentralized fashion. However, it is found that the PFD constraint (19) is a nonconvex set as it is the union set of $\{P_{total}^{(m,n)} = 0\}$ and $\{P_{total}^{(m,n)} = 0\}$. Hence, Problem P0 is a nonconvex problem. Those standard decentralized computation methods such as alternating direction method of multipliers (ADMM) and dual decomposition cannot be directly applied on Problem P0.

Therefore, this paper considers a new problem, which is approximately equivalent to Problem P0, for market clearing in e-LAN, as defined by Problem P1. The Problem P1 is obtained by separating Problem P0 into two independent subproblems, Problem P1-Sub1 and Problem P1-Sub2, as defined as follows. In doing so, market clearing can be performed in a decentralized manner by using a decentralized computation method to solve Problem P1-Sub1. Hence, prosumers can formulate their trading strategy and plan their routing paths without exposing their important private information to others. As Problem P1-Sub1 ignores PFD constraint, PFD conflicts may occur in the path solution of Problem P1-Sub1, Problem P1-Sub2 is to eliminate these PFD conflicts to ensure a normal operation of e-LAN.

Problem P1: New market clearing Problem

① Problem P1-Sub1: Market clearing ignoring the PFD constraint (19)

$$\max(12) \tag{20}$$

s.t. (13) –(18) (21)

② Problem P1-Sub2: Eliminating PFD conflicts existing in the path solution of Problem P1-Sub1.

Compared with Problem P0, Problem P1 possesses significant advantages in achieving decentralized market clearing. However, Problem P1 can be considered for market clearing only if the solution of Problem P1 can always bring no less social welfare than that of Problem P0. Therefore, Section 4 proposes a new solution method for Problem P1, which can always produce a no less social welfare than that of Problem P0. The detailed descriptions are presented as follows.

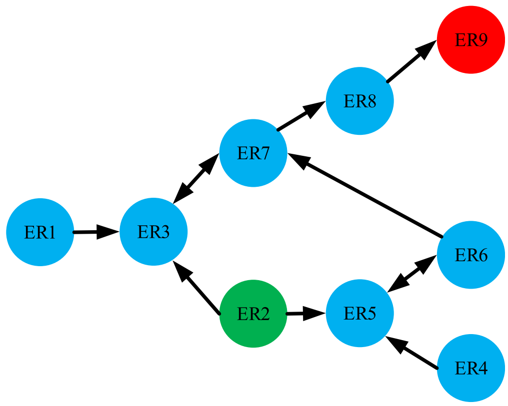

Fig. 9. Depth-first-search tree of the e-LAN shown in Fig. 6(a).

4. Market clearing with cooperative energy routing

4.1. Framework of proposed approach

For the Problem P1, this paper proposes a decentralized market clearing approach with cooperative energy routing, as shown in Fig. 8. The proposed approach is composed of two stages, a market clearing stage and a cooperative energy routing stage. The market clearing stage is to solve Problem P1-Sub1 in a decentralized manner. In this stage, each prosumer plans routing path independent of others, which is called self-centered energy routing. The detailed description about the decentralized market clearing can be found in Section 4.2. When the decentralized market clearing stage finds the optimal solution for Problem P1-Sub1, the routing paths obtained by self-centered energy routing is submitted to the network system operator (NSO) in the e-LAN for PFD conflict check. If there exists no PFD conflict, all transactions are approved by NSO. Otherwise, those trading pairs involved in PFD conflicts are encouraged to participate in cooperative energy routing. More details about the cooperative energy routing are presented in Section 4.3.

4.2. Decentralized market clearing

In this subsection, a decentralized computation approach is proposed to clear local energy market in a decentralized fashion. As Problem Sub-P1 is a quadratic programming problem with all constraints being affine, the duality is always zeros [31]. Hence, Problem P1-Sub1 holds strong duality. The technique of dual decomposition can be used decompose Problem P1-Sub1 into a series of independent subproblems. The Lagrangian of Problem P1-Sub1 is given in (22) by relaxing constraints (16), (18). Constraints (14) and (15) are not included as they are local constraints for prosumers.

$$\xi(\mathbf{P_i},\mathbf{P_j},\boldsymbol{\pi},\boldsymbol{\gamma}) = \sum_{i=1}^{|\mathscr{C}|}\widehat{W}_i + \sum_{j=1}^{|\mathscr{P}|}\widehat{W}_j + \sum_{j=1}^{|\mathscr{P}|}\pi_j\left(P_{j-p2p} - \sum_{i=1}^{|\mathscr{C}|}\sum_{p=1}^{|\Omega_{(i,j)}|}P_{i-p}\right) + \sum_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}\left(Cap_{(m,n)} - \sum_{i=1}^{|\mathscr{C}|}\sum_{p=1}^{|\Omega_i^{(m,n)}|}P_{i-p} - P_{ex(m,n)}\right) \tag{22}$$

where π_j is the Lagrange multiplier for the constraint (16) of producer j. From an economic viewpoint, π_j can be viewed as the electricity price of producer j to maintain its supply and demand to it. $\gamma_{(m,n)}$ is the Lagrange multiplier for the capacity constraint of power line (m,n).

The dual problem of (22) is defined as

$$\min_{\boldsymbol{\pi},\boldsymbol{\gamma}}\sup_{\mathbf{P_i},\mathbf{P_j}}\xi(\mathbf{P_i},\mathbf{P_j},\boldsymbol{\pi},\boldsymbol{\gamma})$$
$$= \min_{\boldsymbol{\pi},\boldsymbol{\gamma}}\left[\sum_{i=1}^{|\mathscr{C}|}\varsigma_i(\boldsymbol{\pi},\boldsymbol{\gamma}) + \sum_{j=1}^{|\mathscr{P}|}\upsilon_j(\boldsymbol{\pi}) + \sum_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}\left(Cap_{(m,n)} - P_{ex(m,n)}\right)\right] \tag{23}$$

where $\varsigma_i(\boldsymbol{\pi},\boldsymbol{\gamma})$ and $\upsilon_j(\boldsymbol{\pi})$ are the subproblems to be solved by consumer i and producer j, respectively.

Based on the sub-gradient projection method, dual problem (23) can be solved by the following distributed update.

Subproblem for consumer ***i***:

$$\varsigma_i\left(\boldsymbol{\pi}^{(k)},\boldsymbol{\gamma}^{(k)}\right) \triangleq \max_{\mathbf{P_i}}\left[\widehat{W}_i - \sum_{j=1}^{|\mathscr{P}|}\pi_j^{(k)}\sum_{p=1}^{|\Omega_{(i,j)}|}P_{i-p}\right) - \sum_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}^{(k)}\sum_{p=1}^{|\Omega_i^{(m,n)}|}P_{i-p}]$$
$$= \max_{\mathbf{P_i}}\left[W_i - \sum_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}^{(k)}\sum_{p=1}^{|\Omega_i^{(m,n)}|}P_{i-p}\right] \tag{24}$$

s.t. (14) (25)

where $\pi_j^{(k)}$ and $\gamma_{(m,n)}^{(k)}$ denote variables π_j and $\gamma_{(m,n)}$ at the k th iteration. $\gamma_{(m,n)}\sum_{p=1}^{|\Omega_i^{(m,n)}|}P_{i-p}$ can be interpreted as a penalty fee imposed on the trading pairs for using line (m, n) to avoid congestion. When $\sum_{i=1}^{|\mathscr{C}|}\sum_{p=1}^{|\Omega_i^{(m,n)}|}P_{i-p} + P_{ex(m,n)} < Cap_{(m,n)}$, $\gamma_{(m,n)} = 0$.

Before solving the subproblem, consumer i needs to construct the path set $\Omega_{(i,j)}$, which is the problem to find all available paths between source node j and sink node i. This can be solved by using depth-first-search based algorithm in graph theory [32]. For instance, Fig. 9 depicts the depth first-search tree of the e-LAN shown in Fig. 6(a). The two paths between node ER2 and node ER9 can be found as $2\rightarrow3\rightarrow7\rightarrow8\rightarrow9$ and $2\rightarrow5\rightarrow6\rightarrow7\rightarrow8\rightarrow9$ by the depth-first-search based algorithm. More details about depth-first-search based algorithm can be found in [32].

It is found that the subproblem for consumer i is a convex problem. Therefore, the optimal solution of the subproblem can be obtained by using Karush-Kuhn-Tucker (KKT) conditions, as given by (26) and (27), where $\widehat{\xi}$ denotes the Lagrangian of the subproblem for consumer i, $\underline{\lambda_i}$ and $\overline{\lambda_i}$ are Lagrange multipliers.

$$\widehat{\xi}(\mathbf{P_i},\boldsymbol{\lambda}) = W_i - \sum_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}^{(k)}\sum_{p=1}^{|\Omega_i^{(m,n)}|}P_{i-p} + \underline{\lambda_i}\left(\underline{D_i} - \sum_{p=1}^{|\Omega_i|}P_{i-p} - P_{i-g}\right) + \overline{\lambda_i}\left(\sum_{p=1}^{|\Omega_i|}P_{i-p} + P_{i-g} - \overline{D_i}\right) \tag{26}$$

$$\begin{cases} \dfrac{\partial\widehat{\xi}}{\partial P_{i-p}} = 0 \\ \underline{\lambda_i}\left(\underline{D_i} - \sum_{p=1}^{|\Omega_i|}P_{i-p} - P_{i-g}\right) = 0 \\ \overline{\lambda_i}\left(\sum_{p=1}^{|\Omega_i|}P_{i-p} + P_{i-g} - \overline{D_i}\right) = 0 \end{cases} \tag{27}$$

Subproblem for producer ***j***:

$$\upsilon_j\left(\boldsymbol{\pi}^{(k)}\right) \triangleq \max_{\mathbf{P_j}}\left[\widehat{W}_j + \pi_j^{(k)}P_{j-p2p}\right]$$
$$= \max_{\mathbf{P_j}}W_j \tag{28}$$

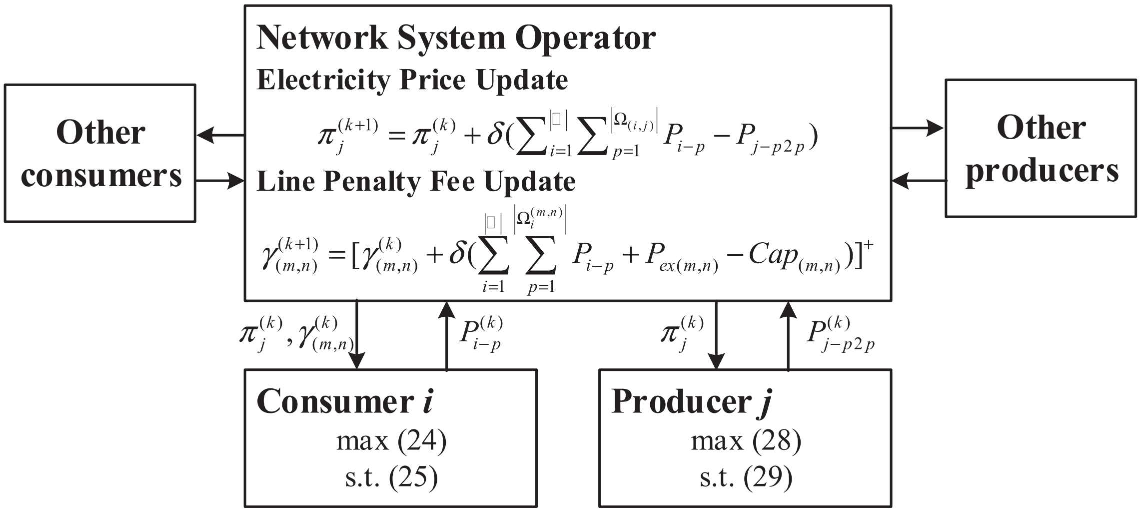

Fig. 10. Information exchange between network system operator and prosumers.

s.t. (15) (29)

The subproblem for producer j can be solved by using the same method, KKT conditions. The solution is given by (30) and (31), where $\overline{\overline{\xi}}$ is the Lagrangian and μ is the Lagrange multiplier.

$$\overline{\overline{\xi}}(\mathbf{P_j},\mu) = W_j + \mu\left(\overline{G_j} - P_{j-p2p} - P_{j-g}\right) \tag{30}$$

$$\begin{cases} \dfrac{\partial\overline{\overline{\xi}}}{\partial P_j} = 0 \\ \mu\left(\overline{G_j} - P_{j-p2p} - P_{j-g}\right) = 0 \end{cases} \tag{31}$$

Network system operator updates $\boldsymbol{\pi},\boldsymbol{\gamma}$:

$$\pi_j^{(k+1)} = \pi_j^{(k)} + \delta\left(\sum_{i=1}^{|\mathscr{C}|}\sum_{p=1}^{|\Omega_{(i,j)}|} P_{i-p}^{(k)} - P_{j-p2p}^{(k)}\right) \tag{32}$$

For each transmission line in Ξ,

$$\gamma_{(m,n)}^{(k+1)} = \left[\gamma_{(m,n)}^{(k)} + \delta\left(\sum_{i=1}^{|\mathscr{C}|}\sum_{p=1}^{|\Omega_i^{(m,n)}|} P_{i-p} + P_{ex(m,n)} - Cap_{(m,n)}\right)\right]^+, \forall l_{(m,n)} \in \Xi \tag{33}$$

where δ is the step size. To ensure convergence of the distributed update, δ should be less than $2/L$, where L is the Lipschitz constant subject to [33]

$$\|\nabla\Theta(O_1) - \nabla\Theta(O_2)\| \leq L\|O_1 - O_2\| \tag{34}$$

where $\Theta = \sup_{\mathbf{P_i},\mathbf{P_j}} \xi\left(\mathbf{P_i},\mathbf{P_j},\boldsymbol{\pi},\boldsymbol{\gamma}\right)$ and $O = (\pi,\gamma)$ is the single Lagrange multiplier.

The implementation of the decentralized update requires iterations between the network system operator and prosumers, which can be achieved by information exchange in Fig. 10. The decentralized optimization is executed by Algorithm 1. It is notable that the proposed decentralized approach for Problem P1-Sub1 is scalable in the number of participants where all participants can solve their own optimization problems in parallel.

Algorithm 1 Decentralized Optimization Algorithm

Initialize: Graph G for e-LAN, $k = 1$

```
repeat
    NSO Broadcasts electricity prices {π_j^(k), ∀j ∈ 𝒫} and penalty prices
    {γ_(m,n)^(k), ∀l_(m,n) ∈ Ξ} to each consumer i in 𝒞 and each producer j in 𝒫.
    for each consumer i in 𝒞 (in parallel) do
        Updating its power demand P_(i-p)^(k) at each path of Ω_i according to (26) and (27);
        Submitting its power demand P_(i-p)^(k) at each path of Ω_i to NSO;
    end for
    for each producer j in 𝒫 (in parallel) do
        Updating its power supply according to (30) and (31)
        Submitting its power supply P_(j-p2p)^(k) to NSO;
    end for
    NSO updates the electricity prices {π_j^(k+1), ∀j ∈ 𝒫} and penalty prices
    {γ_(m,n)^(k+1), ∀l_(m,n) ∈ Ξ} according to (32) and (33).
    k = k + 1;
Until |π_j^(k+1) − π_j^(k)| ⟨ ϵ, ∀j ∈ 𝒫 and |γ_(m,n)^(k+1) − γ_(m,n)^(k)| ⟨ ϵ, ∀l_(m,n) ∈ Ξ
```

4.3. Cooperative energy routing

When the decentralized market clearing stage converges, the proposed approach enters the cooperative energy routing stage. Firstly, NSO performs PFD conflict check for the obtained solution of Problem P1-Sub1. As it is possible that there already exists power transmission at some power lines in e-LAN, NSO does not only detect the PFD conflicts between trading pairs but also the PFD conflicts between trading pairs and the existing source-load pairs in e-LAN. Trading pairs or source-load pairs involved in PFD conflicts are encouraged to form a grand coalition to resolve these PFD conflicts, which is called cooperative energy routing. To ensure that each trading pair or source-load pair is willing to cooperate, the designed cooperative energy routing method should satisfy the following requirements.

(a): It should avoid violating prosumers' privacy.

(b): It should be able to minimize the total transmission cost of the grand coalition.

(c): Each trading pair should obtain less transmission cost from cooperative energy routing than from the path solution of Problem P1-

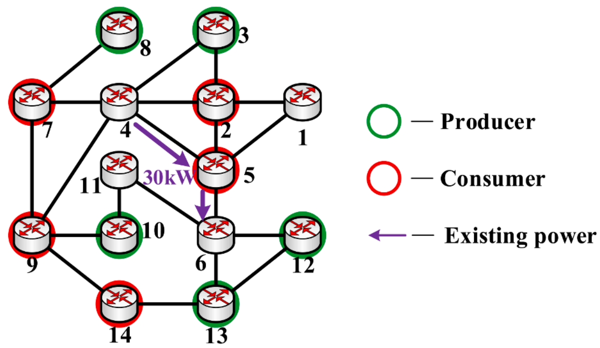

Fig. 11. An e-LAN modified from IEEE 14-bus system [22].

Sub1, i.e., self-centered energy routing. Also, each existing source-load pair should receive less transmission cost from cooperation than from noncooperation.

Therefore, this paper proposes a Nash bargaining based method to plan paths for cooperative energy routing. Mathematically, Problem P1-Sub2 is formulated as follows. It is notable that the cooperative energy routing differs from the self-centered energy routing in the market clearing stage. In the proposed approach, each consumer can receive power from all producers and all existing power sources in the coalition and each producer can send power to all consumer and all existing power loads in the coalition, whereas each producer or consumer in self-centered energy routing can only send or receive power to or from its trading partner.

Problem P1-Sub2: Cooperative energy routing problem

$$\max \prod_{\{i,j\}\in\Gamma}\left[T_{ij}^{(s)} - T_{ij}^{(c)}\right]\prod_{\{x,y\}\in \mathrm{E}}\left[T_{xy}^{(ex)} - T_{xy}^{(c)}\right]$$

$$= \max \prod_{\{i,j\}\in\Gamma}\left[T_{ij}^{(s)} - \left(T_{ij} + \Delta T_{ij}\right)\right]\prod_{\{x,y\}\in \mathrm{E}}\left[T_{xy}^{(ex)} - \left(T_{xy} + \Delta T_{xy}\right)\right] \tag{35}$$

s.t. (4), (6), (7), $T_{ij}^{(s)} > T_{ij}^{(c)}, \forall\{i,j\}\in\Gamma$ and $T_{xy}^{(ex)} > T_{xy}^{(c)}, \forall\{x,y\}\in\mathrm{E}$ (36)

$$P_{j-p2p}^{(s)} = \sum_{i=1}^{|\Gamma_{\mathscr{C}}|}\sum_{p=1}^{|\Omega_{(i,j)}|}P_{i-p} + \sum_{x=1}^{|E_x|}\sum_{p=1}^{|\Omega_{(x,j)}|}P_{x-p}, \forall j\in\Gamma_{\mathscr{P}} \tag{37}$$

$$P_{y}^{(ex)} = \sum_{i=1}^{|\Gamma_{\mathscr{C}}|}\sum_{p=1}^{|\Omega_{(i,y)}|}P_{i-p} + \sum_{x=1}^{|\mathrm{E}_x|}\sum_{p=1}^{|\Omega_{(x,y)}|}P_{x-p}, \forall y\in \mathrm{E}_y \tag{38}$$

$$P_{total}^{(m,n)} = \sum_{i=1}^{|\Gamma_{\mathscr{C}}|}\sum_{p=1}^{|\Omega_{i}^{(m,n)}|}P_{i-p} + \sum_{d=1}^{|\mathrm{E}_d|}\sum_{p=1}^{|\Omega_{d}^{(m,n)}|}P_{d-p} + P_{(m,n)}^{(s)}, \forall l_{(m,n)}\in\Xi \tag{39}$$

$$P_{total}^{(m,n)} \leq Cap_{(m,n)}, \forall l_{(m,n)}\in\Xi \tag{40}$$

$$P_{total}^{(m,n)} * P_{total}^{(n,m)} = 0, \forall l_{(m,n)}\in\Xi \tag{41}$$

where Γ is the set of all trading pairs in the grand coalition. $T_{ij}^{(s)}$ is the transmission cost of trading pair $\{i,j\}$ in the path solution of Problem P1-Sub1. In other words, $T_{ij}^{(s)}$ is the transmission cost of trading pair $\{i,j\}$ in the self-centered routing stage. $T_{ij}^{(c)}$ is the final transmission cost of trading pair $\{i,j\}$ obtained from the cooperative energy routing stage. E is the set of all existing source-load pairs in the coalition. x and y denotes the index of the existing power sources and loads in the coalition, respectively. $T_{xy}^{(ex)}$ is the transmission cost of the existing source-load pair $\{x,y\}$ and $T_{xy}^{(c)}$ is the final transmission cost of the existing source-load pair $\{x,y\}$ obtained from the cooperative energy routing stage. $T_{ij}^{(c)}$ consists of two parts, transmission cost T_{ij} of trading pair $\{i,j\}$ in the path solution of Problem P1-Sub2 and assigned transmission cost compensation or charge ΔT_{ij}. Similarly, $T_{xy}^{(c)}$ is composed of transmission cost T_{xy} of source-load pair $\{x,y\}$ in the path solution of Problem P1-Sub2 and assigned transmission cost compensation or charge ΔT_{xy}. In (37), $P_{j-p2p}^{(s)}$ is the amount of power that producer j sells in the solution of Problem P1-Sub1, $\Gamma_{\mathscr{C}}$ is the set of all consumers in the coalition, E_x is the set of existing power loads in e-LAN, x is the index for power sources and $\Gamma_{\mathscr{P}}$ is the set of all producers in the coalition. In (38), $P_y^{(ex)}$ is the amount of power that the existing power source y sends and E_y denotes the set of the existing power sources in the coalition. $P_{(m,n)}^{(s)}$ in (39) denotes the amount of power transmitted through line (m,n) by those trading pairs who do not have any PFD conflicts.

Note that the proposed Nash bargaining based method can satisfy the three requirements above.

For requirement (a), the proposed Nash bargaining based method does not require any information of prosumers' utility function. Therefore, the privacy concerns of prosumers can be eliminated.

For requirement (b), although its objective function is (35), the optimal solution of Problem P1-Sub2 minimizes the total transmission cost of the grand coalition, as proved by Theorem 1.

Theorem 1. *The solution of Problem P1-Sub2 minimizes the total transmission cost of the grand coalition, which is given by* $\left(\sum_{\{i,j\}\in\Gamma}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(c)}\right)$.

Proof: The proof is given in the Appendix A.

For requirement (c), the proposed Nash bargaining based method can bring each trading pair or source-load pair less transmission cost compared with noncooperation. Mathematically, it is required that $T_{ij}^{(s)} > T_{ij}^{(c)}$ for $\{i,j\}\in\Gamma$ and $T_{xy}^{(ex)} > T_{xy}^{(c)}, \forall\{x,y\}\in\mathrm{E}$ in the optimal solution of Problem P1-Sub2. Actually, $T_{ij}^{(s)} > T_{ij}^{(c)}$ for $\{i,j\}\in\Gamma$ and $T_{xy}^{(ex)} > T_{xy}^{(c)}, \forall\{x,y\}\in\mathrm{E}$ belong to the constraint (36). Therefore, the requirement (c) can be satisfied when Problem P1-Sub2 has solution, i.e., is solvable.

Theorem 2. *Problem P1-Sub2 is solvable.*

Proof: The proof is given in the Appendix B.

Due to the nonconvexity of the PFD constraint (41), Problem P1-Sub2 appears to be nonconvex. However, as each consumer can receive power from all producers and power sources in the coalition and each producer can send power to all consumer and loads in the coalition, constraint (41) is found to be an inactive constraint and will not impose any limitation on the optimal solution of the objective function (35). The relevant proof is provided in Theorem 3.

Theorem 3. *The nonconvex PFD constraint (41) is an inactive constraint and will not impose any limitation on the optimal solution of objective function (35).*

Proof: The proof is given in the Appendix C.

With Theorem 3, it is clear that Problem P1-Sub2 is actually a convex problem. Therefore, it can readily be solved by using standard convex optimization methods.

4.4. Comparison between Problem P0 and Problem P1

As this paper uses a new problem, Problem P1, to replace the original problem, Problem P0, for market clearing, this subsection is to demonstrate the feasibility of the replacement by comparing the social welfare of the two problems. In Problem P1, there are two subproblems, Problem P1-Sub1 and Problem P1-Sub2. We define the social welfares of Problem P1, Problem P1-Sub1 and Problem P0 as $SW^{P1-Sub1}$, SW^{P1} and SW^{P0}, respectively. It is found that $SW^{P1-Sub1} \geq SW^{P0}$, as Problem P1-Sub1 has the same objective function but less constraints compared with Problem P0. When the path solution of Problem P1-Sub1 has no PFD conflicts,

Table 1
Parameters of prosumers.

Consumers	β_i(HKD/kWh)	θ_i(HKD/kWh)	$\overline{D_i}$(kW)	$\underline{D_i}$ (kW)
$\mathscr{C}_2$	2.065	0.0028	56	78
$\mathscr{C}_5$	1.92	0.001	44	67
$\mathscr{C}_7$	2.13	0.0038	61	88
$\mathscr{C}_9$	2.15	0.0053	52	75
$\mathscr{C}_{14}$	2.09	0.0025	55	87
Producers	α_j(HKD/kWh)	β_j(HKD/kWh)	$\overline{G_j}$(kW)	
$\mathscr{P}_3$	0.72	0.0056	85	
$\mathscr{P}_8$	0.81	0.0064	62	
$\mathscr{P}_{10}$	0.78	0.0071	73	
$\mathscr{P}_{12}$	0.80	0.01	58	
$\mathscr{P}_{13}$	0.69	0.0082	66	

Table 2
Market outcomes obtained by the market clearing stage.

	$\mathscr{P}_3$	$\mathscr{P}_8$	$\mathscr{P}_{10}$	$\mathscr{P}_{12}$	$\mathscr{P}_{13}$	Grid
π_j	1.609	1.679	1.803	1.607	1.703	–
$\mathscr{C}_2$	71.5	0	0	0	0	0
$\mathscr{C}_5$	8.5	0	0	35.5	0	0
$\mathscr{C}_7$	0	62	0	0	0	0
$\mathscr{C}_9$	0	0	33.5	0	23.2	0
$\mathscr{C}_{14}$	0	0	38.5	5.0	38.1	0
Grid	0	0	0	0	0	–

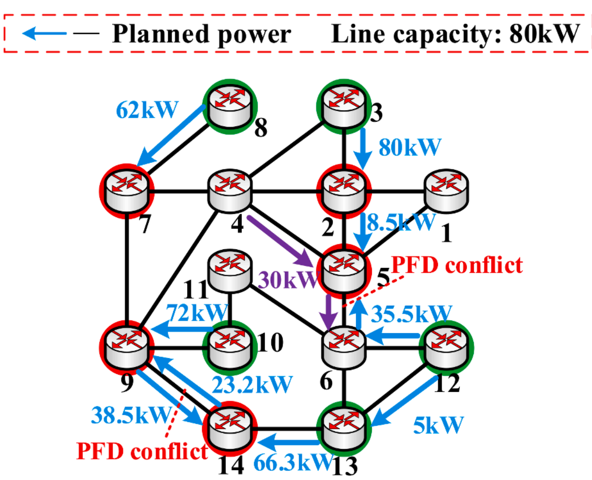

Fig. 12. Energy routing paths obtained by the self-centered energy routing or market clearing stage.

Problem P1-Sub2 is invalid. As a result, $SW^{P1} = SW^{P1-Sub1} = SW^{P0}$. Otherwise, when there exists PFD conflicts in the path solution of Problem P1-Sub1, $SW^{P1-Sub1} > SW^{P0}$. However, the path solution of Problem P1-Sub1 cannot be physically achieved. Hence, we have Problem P1-Sub2, which will be solved by the proposed Nash bargaining based approach. According to Theorem 2 and Theorem 3, we can find that the path solution of Problem P1-Sub2 can avoid PFD conflicts and always have less transmission cost than that of Problem P1-Sub1. Consequently, the final solution of Problem P1 is better than that of Problem P1-Sub1, i.e., $SW^{P1} > SW^{P1-Sub1} > SW^{P0}$. Therefore, $SW^{P1} \geq SW^{P0}$ always holds. In other words, using Problem P1 to replace Problem P0 for market clearing in local energy trading is feasible.

5. Simulation results and analysis

In this section, extensive simulations are performed to demonstrate the advantages of the proposed approach in avoiding physical constraint violation, resolving path conflicts and improving social welfare.

All simulations are performed based on an e-LAN in [22], which is modified from IEEE 14-bus system, as shown in Fig. 11. The conversion efficiency of ER is taken from [0.9,0.99] and the transmission voltage between ERs is 400Vdc [7]. The physical parameters of transmission lines in the e-LAN can be found in [22]. For the local energy market, it is assumed that there are five producers and five consumers in the e-LAN. Their basic information is listed in Table 1. In Hong Kong, the price of purchasing electricity from the power grid π_{b-g} is 2.0 HKD/kWh, and the price of selling electricity to the power grid is assumed to be 1.0 HKD/kWh. The step size δ for price adjustment is taken as 0.001. In addition, it is assumed that there already exists a source-load pair {4,6} in the e-LAN, which transmits 30 kW power through the path 4 → 5 → 6.

5.1. Physical constraint and path conflict

This subsection is to evaluate the performance of the proposed approach in avoiding physical constraint violation and resolving path conflicts. As the proposed approach consists of a market clearing stage and a cooperative energy routing stage, the performance evaluation on the proposed approach is performed by analyzing the simulation results of the two stages, respectively.

Table 2 give the market outcome obtained by the proposed market clearing stage. The corresponding energy routing paths planned by the market clearing stage or self-centered energy routing stage are depicted in Fig. 12. As the market clearing stage eliminates line congestion by using Lagrange multipliers as penalty factors and impose them on trading pairs using congested power lines, each power or transmission line in e-LAN is free from line congestion. However, PFD conflicts occur in both power lines (9,14) and (5,6), which are between trading pairs $\{\mathscr{C}_9, \mathscr{P}_{13}\}$ and $\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$ and trading pair $\{\mathscr{C}_5, \mathscr{P}_{12}\}$ and an existing source-load pair {6,4}, respectively.

In view of these two PFD conflicts, the cooperative energy routing stage is activated to resolve these PFD conflicts and ensure that these involved transactions can be physically achieved. To demonstrate the advantages of the proposed cooperative energy routing method, comparative simulations are performed between the proposed method and other existing conflict-resolving methods on resolving these two PFD conflicts shown in Fig. 12. Actually, there already exists various conflict-resolving methods, which can be used to eliminate PFD conflict. The following three commonly used conflict-resolving methods are selected for the comparative simulation.

- Method 1: Imposing penalty fee on conflicting lines [29,30].
- Method 2: Priority ranking method [28].
- Method 3: Traditional cooperation [4].
- Method 4: Proposed cooperative energy routing.

For ease of reading, the PFD conflict between trading pairs $\{\mathscr{C}_9, \mathscr{P}_{13}\}$ and $\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$ is defined as PFD conflict I, and the PFD conflict between trading pair $\{\mathscr{C}_5, \mathscr{P}_{12}\}$ and an existing source-load pair {6,4} is defined as PFD conflict II.

PFD conflict I: It occurs in power line (9,14). When Method 1 is used to resolve the PFD conflict, penalty fee is imposed on trading pairs $\{\mathscr{C}_9, \mathscr{P}_{13}\}$ and $\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$ for using power line (9,14). As the penalty fee gradually increases, trading pair $\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$ has to abandon the power line (9,14) and choose another path 10 → 11 → 6 → 13 → 14, as shown in Fig. 13(a). In order to depict the resolution more clearly, it is notable that the paths of other trading pairs, which has no line overlapping with the obtained resolution, are not shown in Fig. 13(a). The following figures adopt the similar way. However, congestion occurs in power line (13,14). In addition, there exists no other available paths for trading pair $\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$ so that they have to terminate their transaction. Fig. 13(b) depicts the resolution of Method 2 for the PFD conflict, where trading pair $\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$ is assigned priority to use power line (9,14) as they have larger traded quantity than trading pair $\{\mathscr{C}_9, \mathscr{P}_{13}\}$. Although trading pair $\{\mathscr{C}_9, \mathscr{P}_{13}\}$ plan to use another path 13 → 6 → 11 → 10 → 9, power

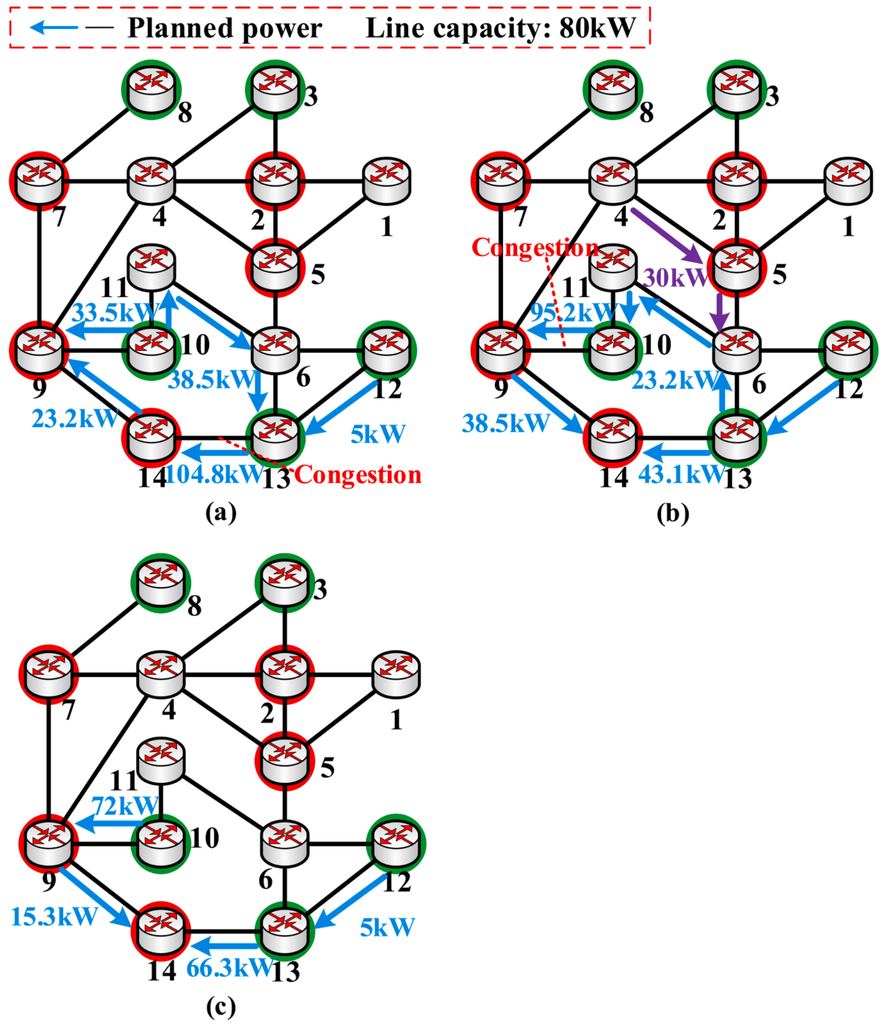

Fig. 13. Resolutions for PFD conflict I obtained by (a) Method 1, (b) Method 2 and Method 3, and (c) Method 4.

Table 3
Transmission costs of trading pairs in the resolution of the four methods for PFD conflict I.

	Method 1 / HK $	Method 2 / HK $	Method 3 / HK $	Method 4 / HK $
$\{\mathscr{C}_9, \mathscr{P}_{13}\}$	2.16	–	–	1.56
$\{\mathscr{C}_{14}, \mathscr{P}_{10}\}$	–	2.36	2.36	1.76

Table 4
Amount of power that consumers $\mathscr{C}_9$ and $\mathscr{C}_{14}$ purchase from the power grid in the four methods.

	Method 1 / kW	Method 2 / kW	Method 3 / kW	Method 4 / kW
$\mathscr{C}_9$	0	18.5	18.5	0
$\mathscr{C}_{14}$	36.9	0	0	0

line (10,9) is congested. In Method 3, traditional cooperative energy routing is utilized to eliminate the PFD conflict. As trading pairs in traditional cooperative energy routing are required to strictly observe the rule of end-to-end power transmission, $\mathscr{C}_9$ and $\mathscr{C}_{14}$ are only allowed to receive power from $\mathscr{P}_{13}$ and $\mathscr{P}_{10}$, respectively. To minimize the total transmission cost, Method 3 obtains the same resolution with Method 2 for the PFD conflict, as shown in Fig. 13(b). Fig. 13(c) shows the resolution obtained by the proposed cooperative energy routing method, Method 4. It is found that consumer $\mathscr{C}_9$ receives its all power demand (56.7 kW) from producer $\mathscr{P}_{10}$ and consumer $\mathscr{C}_{14}$ receives 15.3 kW power from producer $\mathscr{P}_{10}$ and 61.3 kW power from producer $\mathscr{P}_{13}$. As a result, PFD conflict I is resolved by using the flexible resolution of Method 4.

Table 3 lists the transmission cost of trading pairs in the resolutions shown in Fig. 13. Those trading pairs whose paths cannot be physically realized have no transmission cost and thus are assigned '–'. It can be seen from Table 3 that Method 4 can provide a resolution with less transmission cost for each trading pair compared with the remaining three methods. As some trading pairs cannot find available paths for traded power delivery, they have to terminate their transactions. Considering that the purchased power in local energy market cannot satisfy consumers' minimum demand, they may need to trade with the power grid. Table 4 shows the amount of power that consumers purchase from the power grid in the four methods. Compared with consumers in Method 1, Method 2 and Method 3, consumers in Method 4 can buy enough power from local energy trading to meet their demand and thus do not need to make transactions with the power grid. This indicates that Method 4 possesses significant advantages in promoting energy sharing between prosumers and reducing their dependence on the power grid compared with Method 1, Method 2 and Method 3.

PFD conflict II: It exists in power line (5,6). Since the existing source-load pair (4,6) is previously approved for routing their power, their supply and demand should always be met in the resolution for PFD

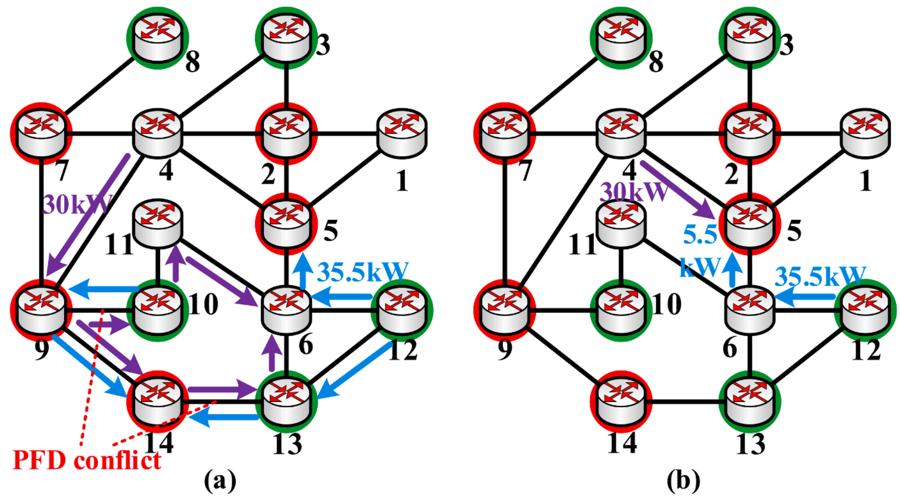

Fig. 14. Resolutions for PFD conflict II obtained by (a) Method 3 and (b) Method 4.

Table 5
Transmission costs of trading pair $\{\mathscr{C}_5, \mathscr{P}_{12}\}$ and source-load pair {4,6} in different path solutions.

	Self-centered energy routing / HK $	Cooperative energy routing / HK $
$\{\mathscr{C}_5, \mathscr{P}_{12}\}$	9.88	6.455
{4,6}	6.84	3.415

Table 6
Amount of power that consumer $\mathscr{C}_5$ purchases from the power grid in the four methods.

	Method 1 / kW	Method 2 / kW	Method 3 / kW	Method 4 / kW
$\mathscr{C}_5$	35.5	35.5	35.5	0

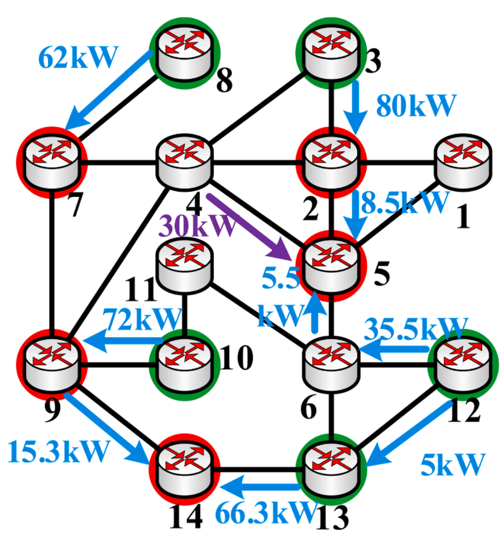

Fig. 15. Final energy routing paths of Method 4.

conflict II. Consequently, noncooperative methods such as Method 1 and Method 2 are not appliable to resolve PFD conflict II. The resolution of Method 3 is shown in Fig. 14(a), where the utilization of power line (5,6) is given to trading pair $\{\mathscr{C}_5,\mathscr{P}_{12}\}$ and the source-load pair (4,6) will use the paths $4 \rightarrow 9 \rightarrow 14 \rightarrow 13 \rightarrow 6$ and $4 \rightarrow 9 \rightarrow 10 \rightarrow 11 \rightarrow 6$. However,

Table 7
Social welfares obtained by solving different problems.

	Problem P0	Problem P1-Sub1	Problem P1
Social welfare / HK$	208.31	216.17	224.22

both two paths are going to have PFD conflicts. As a result, the path solution given by Method 3 is not feasible. Fig. 14(b) shows a feasible and efficient resolution obtained by Method 4, in which producer $\mathscr{P}_{12}$ delivers 30 kW to the load 6 and 5.5 kW to consumer $\mathscr{C}_5$ and the power source 4 sends 30 kW to consumer $\mathscr{C}_5$.

Table 5 gives the transmission costs of trading pair $\{\mathscr{C}_5,\mathscr{P}_{12}\}$ and source-load pair {4,6} in self-centered energy routing and cooperative energy routing. Note that the path obtained by self-centered energy routing is shown in Fig. 12. It is found that the proposed cooperative energy routing can not only provide a feasible solution to resolve PFD conflict II but also bring less transmission cost compared with self-centered energy routing. Additionally, Table 6 lists the amount of power that consumer $\mathscr{C}_5$ purchases from the power grid in the four methods. As Method 1, Method 2 and Method 3 cannot obtain a feasible resolution for PFD conflict II, consumer $\mathscr{C}_5$ has to buy 35.5 kW power from the power grid for its minimum demand. In comparison, Method 4 can physically realize the transaction of trading pair $\{\mathscr{C}_5,\mathscr{P}_{12}\}$ so that consumer $\mathscr{C}_5$ does not need to trade with the power grid.

The final energy routing paths obtained by Method 4 for efficiently achieving all energy transactions in Table 2 are shown in Fig. 15, where each producer or existing power source and each consumer or existing load can send and receive agreed amount of power without violating PFD constraint.

5.2. Social welfare comparison between Problem P1 and Problem P0

This subsection is to compare the social welfares obtained by solving Problem P1 and Problem P0. As Problem P0 is a mixed integer nonlinear programming (MINLP) problem, the solution of Problem P0 is obtained by using a MINLP solver 'OuterApproximation' in Matlab. Table 7 lists the social welfares in the solutions of Problem P0, Problem P1-Sub1 and Problem P1. The social welfare obtained from Problem P1-Sub1 is higher than that of Problem P0 as it ignores PFD constraint. However, PFD conflicts occur in the solution of Problem P1-Sub1. The proposed cooperative energy routing approach is used to efficiently eliminate these PFD conflicts and facilitate energy transactions in local energy market. In consequence, the transmission costs of involved trading pairs are reduced, which leads to a higher social welfare of Problem P1

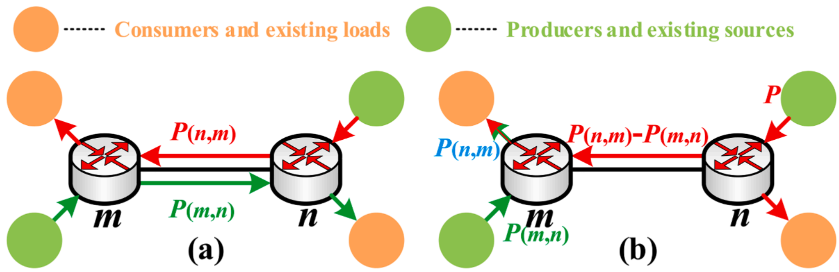

Fig. 16. Scenarios designed for the proof of theorem 2.

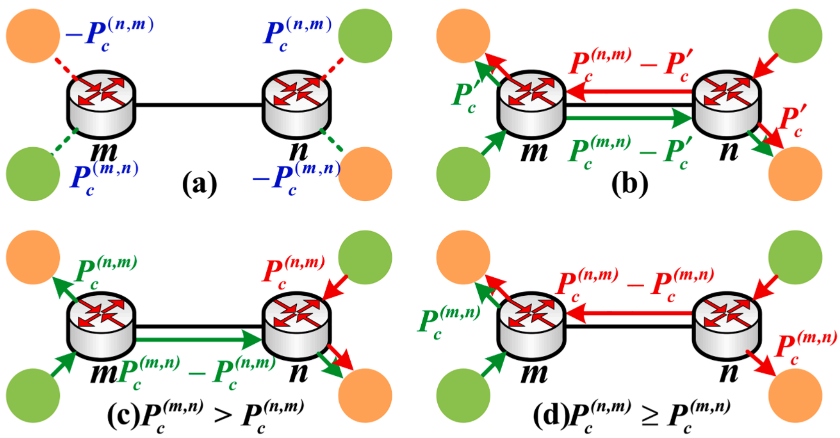

Fig. 17. Scenarios designed for the proof of theorem 3.

compared with that of Problem P1-Sub1. Therefore, Problem P1 is superior to Problem P0 for market clearing.

6. Conclusion

This paper proposes a market clearing approach with cooperative energy routing for local energy trading in e-LAN. The proposed approach formulates a two-stage problem for market clearing in e-LAN, which consists of a market clearing stage and a cooperative energy routing stage. The market clearing stage allows prosumers to pursue their benefits from the local energy market without revealing its important private information to others. In addition, the power line congestion management is considered in the market clearing stage by using Lagrange multipliers as penalty factors. The cooperative energy routing stage is to efficiently eliminate the PFD conflicts existing in the path solution from the market clearing stage and make obtained energy transactions physically feasible. A Nash bargaining based method is proposed to plan routing paths for involved trading pairs. It is rigorously proved that the proposed method can always bring less transmission costs for involved trading pairs than noncooperation. Numerical simulation results indicates that the proposed approach can clear the local energy market in e-LAN without violating physical constraints. Furthermore, compared with the existing conflict-resolving methods, the proposed approach can provide higher-efficiency and more flexible resolutions for PFD conflicts.

In the future, we plan to extend our work by considering energy transactions between e-LANs so as to further promote the energy sharing and maximize the utilization of RESs.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. .

Proof of Theorem 1:

The Theorem 1 is proved by contradiction. Consider $\{T^*_{ij}, \Delta T^*_{ij}, \{i,j\} \in \Gamma, T^*_{xy}, \Delta T^*_{xy}, \{x,y\} \in \mathrm{E}\}$ is the optimal solution of Problem P1-Sub2, we assume that $(\sum_{\{i,j\}\in\Gamma} T^*_{ij} + \sum_{\{x,y\}\in\mathrm{E}} T^*_{xy})$ does not minimize $(\sum_{\{i,j\}\in\Gamma} T^{(c)}_{ij} + \sum_{\{x,y\}\in\mathrm{E}} T^{(c)}_{xy})$. Then, there exists $(\sum_{\{i,j\}\in\Gamma} T'_{ij} + \sum_{\{x,y\}\in\mathrm{E}} T'_{xy})$ such that $(\sum_{\{i,j\}\in\Gamma} T'_{ij} + \sum_{\{x,y\}\in\mathrm{E}} T'_{xy}) < (\sum_{\{i,j\}\in\Gamma} T^*_{ij} + \sum_{\{x,y\}\in\mathrm{E}} T^*_{xy})$. Let $\varrho_{ij} = T'_{ij} - T^*_{ij}$ and $\varrho_{xy} = T'_{xy} - T^*_{xy}$, we then have

$$\sum_{\{i,j\}\in\Gamma}\varrho_{ij} + \sum_{\{x,y\}\in \mathrm{E}}\varrho_{xy} < 0 \tag{42}$$

Consider another feasible solution $T_{ij} = T_{ij}^{*} + \varrho_{ij}$ for $\{i,j\}\in\Gamma$, $T_{xy} = T_{xy}^{*} + \varrho_{xy}$ for $\{x,y\}\in\mathrm{E}$, $\Delta T_{ij} = \Delta T_{ij}^{*} - \varrho_{ij}$ for $\{i,j\}\in\Gamma\backslash\{i\ ,\ j\ \}$, $\Delta T_{i\ j} = \Delta T_{i\ j}^{*} - \varrho_{i\ j} + \sigma$ and $\Delta T_{xy} = \Delta T_{xy}^{*} - \varrho_{xy}$ for $\{x,y\}\in\mathrm{E}$, where $\sigma = \sum_{\{i,j\}\in\Gamma}\varrho_{ij} + \sum_{\{x,y\}\in\mathrm{E}}\varrho_{xy} < 0$. Substituting the solution into (35),

$$\prod_{\{i,j\}\in\Gamma}\left[T_{ij}^{(s)} - \left(T_{ij} + \Delta T_{ij}\right)\right]\prod_{\{x,y\}\in\mathrm{E}}\left[T_{xy}^{(ex)} - \left(T_{xy} + \Delta T_{xy}\right)\right]$$

$$= \prod_{\{i,j\}\in\Gamma\backslash\{i\ \ j\ \}}\left[T_{ij}^{(s)} - \left(T_{ij}^{*} + \varrho_{ij} + \Delta T_{ij}^{*} - \varrho_{ij}\right)\right]$$

$$*\left[T_{i\ j}^{(s)} - \left(T_{i\ j}^{*} + \varrho_{i\ j} + \Delta T_{i\ j}^{*} - \varrho_{i\ j} + \sigma\right)\right]$$

$$*\ \prod\nolimits_{\{x,y\}\in\mathrm{E}}[T_{xy}^{(ex)} - (T_{xy}^{*} + \varrho_{xy} + \Delta T_{xy}^{*} - \varrho_{xy})]$$

$$= \prod_{\{i,j\}\in\Gamma\backslash\{i\ \ j\ \}}\left[T_{ij}^{(s)} - \left(T_{ij}^{*} + \Delta T_{ij}^{*}\right)\right]*\left[T_{i\ j}^{(s)} - \left(T_{i\ j}^{*} + \Delta T_{i\ j}^{*} + \sigma\right)\right]$$

$$*\prod\nolimits_{\{x,y\}\in\mathrm{E}}[T_{xy}^{(ex)} - (T_{xy}^{*} + \Delta T_{xy}^{*})]$$

$$> \prod_{\{i,j\}\in\Gamma\backslash\{i\ \ j\ \}}\left[T_{ij}^{(s)} - \left(T_{ij}^{*} + \Delta T_{ij}^{*}\right)\right]*\left[T_{i\ j}^{(s)} - \left(T_{i\ j}^{*} + \Delta T_{i\ j}^{*}\ \right)\right]$$

$*\ \prod_{\{x,y\}\in\mathrm{E}}[T_{xy}^{(ex)} - (T_{xy}^{*} + \Delta T_{xy}^{*})]$ (43).

This contradicts the assumption that $\{T_{ij}^{*}, \Delta T_{ij}^{*}, \{i,j\}\in\Gamma, T_{xy}^{*}, \Delta T_{xy}^{*}, \{x,y\}\in\mathrm{E}\}$ is the optimal solution of Problem P1-Sub2. Therefore, the optimal solution $(\sum_{i,j\}\in\Gamma}T_{ij}^{*} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{*})$ of Problem P1-Sub2 minimizes $(\sum_{i,j\}\in\Gamma}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(c)})$.

Appendix B. .

Proof of Theorem 2:

In Problem P1-Sub2, $T_{ij}^{(s)} > T_{ij}^{(c)}$ for $\{i,j\}\in\Gamma$ and $T_{xy}^{(ex)} > T_{xy}^{(c)}$ for $\{x,y\}\in\mathrm{E}$ is equivalent to $(\sum_{\{i,j\}\in\Gamma}T_{ij}^{(s)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(s)}) > (\sum_{\{i,j\}\in\Gamma}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(c)})$. If there exists at least a trading pair $\{i,j\}$ or an existing source-load pair $\{x,y\}$ such that $T_{ij}^{(s)} = T_{ij}^{(c)}$ or $T_{xy}^{(s)} = T_{xy}^{(c)}$ when $(\sum_{\{i,j\}\in\Gamma}T_{ij}^{(s)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(s)}) > (\sum_{\{i,j\}\in\Gamma}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(c)})$. the objective function (35) equals to 0 and thus cannot be maximized. Therefore, we can prove Theorem 2 by proving that there always exists at least a solution such that $(\sum_{\{i,j\}\in\Gamma}T_{ij}^{(s)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(s)}) > (\sum_{\{i,j\}\in\Gamma}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}}T_{xy}^{(c)})$. .

This paper will prove it by contradiction. Consider a PFD conflict at transmission line (m,n) in the self-centered energy routing stage, some trading pairs and source-load pairs intend to deliver their power $P_{(m,n)}$ from ER m to ER n and other trading pairs and source-load pairs plan to transmit their power $P_{(n,m)}$ from ER n to ER m, as shown in Fig. 16(a).

Therefore, their expected transmission cost is given by

$$\sum_{\{i,j\}\in\Gamma^{(m,n)}}T_{ij}^{(s)} + \sum_{\{x,y\}\in\mathrm{E}^{(m,n)}}T_{xy}^{(s)} = T_{\to m}\left(P_{(m,n)}\right) + T_{m\to}\left(P_{(n,m)}\right) + T_{m\to n}\left(P_{(m,n)}\right)$$

$$+ T_{\to n}\left(P_{(n,m)}\right) + T_{n\to}\left(P_{(m,n)}\right) + T_{n\to m}\left(P_{(n,m)}\right) \tag{44}$$

where $\Gamma^{(m,n)}$ and $\mathrm{E}^{(m,n)}$ denotes the set of all trading pairs and all source-load pairs involved in the PFD conflict at transmission line (m,n), respectively. $T_{\to m}\left(P_{(m,n)}\right)$ denotes the transmission cost of routing $P_{(m,n)}$ power from producers at ER m side to ER m. $T_{m\to}\left(P_{(n,m)}\right)$ represents the transmission cost of routing $P_{(n,m)}$ power from ER m to consumers at ER m side. $T_{m\to n}\left(P_{(m,n)}\right)$ is the transmission cost of routing $P_{(m,n)}$ power from ER m to ER n. Other variables have the similar definition.

When $P_{(n,m)} > P_{(m,n)}$. Consider a path solution shown in Fig. 16(b) to resolve the PFD conflict at transmission line (m,n), the total transmission cost of these trading pairs and source-load pairs is calculated by

$$\sum_{\{i,j\}\in\Gamma^{(m,n)}}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}^{(m,n)}}T_{xy}^{(c)} = T_{\to m}\left(P_{(m,n)}\right) + T_{m\to}\left(P_{(n,m)}\right) + T_{\to n}\left(P_{(n,m)}\right)$$

$$+ T_{n\to}\left(P_{(m,n)}\right) + T_{n\to m}\left(P_{(n,m)} - P_{(m,n)}\right) \tag{45}$$

We can compare $(\sum_{\{i,j\}\in\Gamma^{(m,n)}}T_{ij}^{(s)} + \sum_{\{x,y\}\in\mathrm{E}^{(m,n)}}T_{xy}^{(s)})$ and $(\sum_{\{i,j\}\in\Gamma^{(m,n)}}T_{ij}^{(c)} + \sum_{\{x,y\}\in\mathrm{E}^{(m,n)}}T_{xy}^{(c)})$ by

$$\sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(s)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(s)} - \sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(c)} - \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(c)}$$

$$= T_{m\to n}\left(P_{(m,n)}\right) + T_{n\to m}\left(P_{(n,m)}\right) - T_{n\to m}\left(P_{(n,m)} - P_{(m,n)}\right) \tag{46}$$

As $T_{n\to m}\left(P_{(n,m)}\right)\rangle T_{n\to m}\left(P_{(n,m)} - P_{(m,n)}\right)$, $\left(\sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(s)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(s)}\right) > \sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(c)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(c)})$. Moreover, compared with the path solution shown in Fig. 16(a), the path solution shown in Fig. 16(b) does not impose more power at each transmission line. As a result, the path solution shown in Fig. 16(b) is free from congestion and thus feasible. We can find a similar path solution such that $\left(\sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(s)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(s)}\right) > \sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(c)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(c)})$ when $P_{(n,m)} \le P_{(m,n)}$.

For each PFD conflict, there always exists at least a feasible solution that can lead to a less transmission cost than the solution from the self-centered energy routing stage. Therefore, there always exists a feasible solution such that $\left(\sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(s)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(s)}\right) > \sum_{\{i,j\}\in\Gamma^{(m,n)}} T_{ij}^{(c)} + \sum_{\{x,y\}\in E^{(m,n)}} T_{xy}^{(c)})$. This completes the proof.

Appendix C. .

Proof of Theorem 3:

The nonconvex PFD constraint (41) can be proven to be inactive for Problem P1-Sub2 if the optimal solution of Problem P1-Sub2 does not include any PFD conflicts. This paper will prove it by contradiction.

Consider a coalition c with its power supply and demand distributed on both sides of a power line (m,n), as shown in Fig. 17(a), on the ER m side, there exists $P_c^{(n,m)}$ demand (expressed by $-P_c^{(n,m)}$) and $P_c^{(m,n)}$ supply (expressed by $P_c^{(m,n)}$). For generality, the $P_c^{(n,m)}$ demand and $P_c^{(m,n)}$ supply may come from multiple consumers and loads or multiple producers and power sources. These producers or consumers at ER m side are not necessarily the direct neighbors of ER m and may be connected to ER m through multiple lines and ERs. The situation is similar for the $P_c^{(m,n)}$ demand (expressed by $-P_c^{(m,n)}$) and $P_c^{(n,m)}$ supply distributed on the ER n side.

It is assumed that the path solution shown in Fig. 17(b), where $0 < P_c^{'} < \min(P_c^{(m,n)}, P_c^{(n,m)})$, is the optimal solution of coalition c, which implies that there exists PFD conflicts in coalition c and thus constraint (41) is an active constraint for Problem P1-Sub2. The total transmission cost of the path solution in Fig. 17(b) is defined by $T_{total}^{(b)}$, as given by (47).

$$T_{total}^{(b)} = T_{\to m}\left(P_c^{(m,n)}\right) + T_{m\to}\left(P_c^{(n,m)}\right) + T_{m\to n}\left(P_c^{(m,n)} - P_c^{'}\right)$$

$$+ T_{\to n}\left(P_c^{(n,m)}\right) + T_{n\to}\left(P_c^{(m,n)}\right) + T_{n\to m}\left(P_c^{(n,m)} - P_c^{'}\right) \tag{47}$$

If $P_c^{(m,n)} > P_c^{(n,m)}$, consider another path solution shown in Fig. 17(c) and its total transmission cost is defined as $T_{total}^{(c)}$, as given by (48).

$$T_{total}^{(c)} = T_{\to m}\left(P_c^{(m,n)}\right) + T_{m\to}\left(P_c^{(n,m)}\right) + T_{m\to n}\left(P_c^{(m,n)} - P_c^{(n,m)}\right)$$

$$+ T_{\to n}\left(P_c^{(n,m)}\right) + T_{n\to}\left(P_c^{(m,n)}\right) \tag{48}$$

We can compare $T_{total}^{(b)}$ and $T_{total}^{(c)}$ by

$$T_{total}^{(b)} - T_{total}^{(c)}$$

$$= T_{m\to n}\left(P_c^{(m,n)} - P_c^{'}\right) + T_{n\to m}\left(P_c^{(n,m)} - P_c^{'}\right) - T_{m\to n}\left(P_c^{(m,n)} - P_c^{(n,m)}\right) \tag{49}$$

Since $0 < P_c^{'} < \min(P_c^{(m,n)}, P_c^{(n,m)})$, $T_{m\to n}\left(P_c^{(m,n)} - P_c^{'}\right)\rangle T_{m\to n}\left(P_c^{(m,n)} - P_c^{(n,m)}\right)$ is obtained. Consequently, $T_{total}^{(b)}$ is always larger than $T_{total}^{(c)}$.

Similarly, we can obtain the same conclusion when $P_c^{(m,n)} \le P_c^{(n,m)}$. As a result, the path solution shown in Fig. 17(b) must not be the optimal solution for coalition c. This contradicts the assumption. In other words, PFD conflict cannot occur at line (m,n) in the solution of Problem P1-Sub2. Considering the line (m,n) is defined as a general transmission line, there exists no PFD conflict at each transmission line in the optimal solution of Problem P1-Sub2. Therefore, the nonconvex PFD constraint (41) is inactive for the objective function (35).

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