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Consensus of Multi-Agent Systems with One-Sided Lipschitz Nonlinearity Via Nonidentical Double Event-Triggered Control Subject to Deception Attacks

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Abstract

As for the multi-agent systems (MASs) with time-varying switching subject to deception attacks, the leader-following consensus problem is studied in this article. The one-sided Lipschitz (OSL) condition is utilized for the nonlinear functions, which makes the results more general and relaxed than those obtained by Lipschitz condition. The nonidentical double event-triggering mechanisms (ETMs) are adopted for only a fraction of agents, and each agent transmits the data according to its own necessity. Semi-Markov process modeling with time-varying switching probability is adopted for switching topology and deception attacks occurring in transmission channel are considered. By using the cumulative distribution function (CDF) and the linear matrix inequality (LMI) technology, sufficient conditions for MASs to achieve consensus in mean square are obtained. An effective algorithm is presented to obtain the event-based control gains. The merits of the proposed control scheme are demonstrated via a simulation example.

Keywords: Deception attack, Nonidentical double event-triggering mechanisms, One-sided Lipschitz, Semi-Markov switching topology, Partial-nodes-based control.

1. Introduction

Consensus control for nonlinear MASs has been widely investigated for a long time since its broad applications in transportation networks, smart grid, distributed sensor networks, and the formation of vehicles, robots, satellites [1–6]. The leader-following consensus agreement requires all followers to trace the trajectory of the leader by using information exchange between leader and followers. Meanwhile, several control schemes have been utilized for the consensus of MASs, such as impulsive control [7], adaptive control [8], state feedback control[9, 10], output feedback control [11] and event-triggered control [12–15].

Nonlinearity is an indispensable part in many control systems. Note that the Lipschitz condition is used for nonlinearity and usually applies to small Lipschitz constant, which are conservative and not generous. [16–18]. The OSL condition was presented to solve the problem, which incorporates the Lipschitz condition as one of its special case. In [19, 20], the consensus control for OSL nonlinear MASs was considered, and H_∞ consensus for OSL nonlinear systems with fixed topology was studied in [21]. However, the

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authors in them do not consider the effects of attacks and the time-varying topologies generated by external disturbances. Signals are generally subject to deception attacks on transmission channels, which makes it difficult to detect and seriously affects the integrity of data. Recently, numerous results about the consensus or synchronization of nonlinear systems under deception attacks have been presented [22–25]. Especially, the synchronization of the Lipschitz MASs subject to deception attacks had been studied in [23], and the authors in [25] considered the consensus problem of MASs with actuator faults and malicious deception attacks. Nevertheless, deception attacks, as the current hot topic, are rarely considered for applications in OSL nonlinear systems with time-varying switching. Hence, the OSL nonlinear MASs with switching topology under attacks require further investigate, this is the first motivation of this paper.

There are a lot of studies on MASs have focused on fixed topology [26–28]. Nevertheless, random effects in the environment often cause the change of communication topology among agents. The phenomena of time-varying switching in MASs are generally modeled as Markov or semi-Markov processes, which have been extensively studied in [29–31]. Compared with the conventional Markov process that the dwell time distribution function (DTDF) is an exponential distribution, the DTDF of semi-Markov process is extended to more general probability distribution, like the Wei-Bull distribution [32] or phase-type distribution [33]. For instance, the exponential synchronization of nonlinear semi-Markov switched system was considered in [34, 35]. The impulsive control for the OSL nonlinear semi-Markov switched MASs was studied in [36]. The main difficulty in dealing with the semi-Markov switching of MASs is the time-varying transition rate, which makes it difficult to obtain the control gains.

For reducing unnecessary data transmission, event-triggering mechanisms (ETMs) , as one of the most convincing control strategies, have been widely utilized in consensus of MASs [15, 37]. ETMs only transmit information when triggering error exceed the prescribed threshold, which reduces the cost of communication. Nowadays, growing number of researchers are concerned with designing ETMs from the sensor-controller (S-C) channel [38, 39]. Nevertheless, the cost of transmission in controller-actuator (C-A) channel should not be overlooked. The double ETMs in both S-C and C-A channels were designed to investigate the synchronization of nonlinear system in [40, 41]. The essence of the double ETMs is to transmit the triggering instants of the ETM in S-C channel to the ETM in C-A channel, then transmit the control signals to the actuator at the triggering instants of the ETM in C-A channel, that is, the ETM for C-A channel is designed on the basis of that for S-C channel. A natural question is: Can we design double ETMs for MASs to achieve consensus with each double ETMs nonidentical for different node? This is an interesting problem.

As far as we know, the ETMs for each agent are identical in most existing results on transmission mechanism of MASs [22, 42, 43]. However, the communication information is generally transmitted asynchronously according to the agent's actual requirement, hence it is more practical to consider the nonidentical triggering instants for different agents in MASs. An effective way is to design nonidentical double ETMs for MASs to lighten communication resource consumption, which will be solved in this paper.

In practice, owing to the overwhelming number of the agents in leader-follower MASs, it is not practical to control the MASs by adding feedback from leader to all followers. For a variety of reasons, including communication constraints and physical limitations, the so-called partial-nodes-based control (PNBC) is critic for only partial followers to obtain feedback from the leader. Obviously, if PNBC can be designed with nonidentical double ETMs for controlled nodes, the control scheme is not only practical but also effective. This challenging problem will be solve in the current paper.

To sum up, this article employs PNBC with nonidentical double ETMs for the cosnensus of the OSL nonlinear semi-Markov switched MASs under deception attacks. The notable contributions are threefold:

(1) The considered model is general with relaxed restrictions. The nonlinear term is only required to

satisfy OSL condition and the signals subject to deception attacks in the process of transmission.

(2) Novel nonidentical double ETMs are designed, where the set of the triggering instants generated by ETM for C-A channel is a subset of the triggering instants generated by ETM for S-C channel and each agent samples and transmits information according to its own necessity, and hence the designed ETMs can save communication resources as much as possible. Moreover, event-triggered controller (ETC) with semi-Markov switching is designed with only fractional node connected with the leader for the reason of reduce control cost.

(3) Sufficient conditions for expression by LMI are obtained for the MASs with OSL nonlinearity. The exclusion of Zeno behavior is proved and a design algorithm of event-based control gains is given.

The organization of this work is arranged as below. Section 2 presents the concept of the switched MASs and some preliminaries. Section 3 establishes the consensus criteria in mean square of MASs by strict mathematical proof. A numerical simulation is presented in Section 4 to verify the feasibility of the theoretical results. Finally, the thesis is summarized in Section 5.

Notation. $\mathbb{N}^+$ is the set of nonnegative integers, $\mathbb{R}^n$ represents the Euclidean space, $\mathbb{R}^{n\times m}$ denotes the $n\times m$-dimension matrices. I_n indicates the n-dimension unit matrix, 0 means the number zero or a zero matrix of appropriate dimension. $\mathbb{E}(\cdot)$ refers to the mathematical expectation, and $\otimes$ stands for the Kronecker product. The Euclidean norm in $\mathbb{R}^n$ is represented as $\|\cdot\|$, diag$\{\dots\}$ denotes a block diagonal matrix, $A = (a_{ij})_{m\times m}$ stands for a $m \times m$-dimension real matrix, col$\{\dots\}$ represents a column vector. A real symmetric matrix has its minimum and maximum eigenvalues represented by $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$, respectively. $A < 0(A > 0)$ means that A is real symmetric negative (positive) definite. The symbol * is used to indicate the corresponding transposed block matrix, and the symbol T indicates matrix transposition.

2. Preliminaries

2.1. Graph theory

The exchange of information among agents are represented by a directed graph $\mathcal{G} = (V, \mathcal{E})$. The elements of $V = \{1, 2, \dots, N\}$ symbolizes the nodes set, and the elements of $\mathcal{E} \subseteq V\times V$ represents the edge set. Agents are symbolized by nodes in the digraph, and the adjacency matrix is represented as $\mathcal{A}_{\mathcal{G}} = [a_{ij}]_{N\times N}$, where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ii} = 0$. $L = [l_{ij}]_{N\times N}$ indicates the Laplacian matrix of the digraph, where $l_{ii} = \Sigma_{j=1,j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The communication graph between a leader and N followers is denoted by a directed graph $\bar{\mathcal{G}}$, where $\bar{\mathcal{G}}$ consists of a leader node, directed edges from leader to followers and a graph $\mathcal{G}$. The diagonal matrix $D = \mathrm{diag}\{d_1, d_2, \dots, d_N\}$ represents the information interaction between leader and followers, where $d_i > 0$ means that the leader can send information to the follower i.

2.2. System description

Consider a nonlinear MAS with N nodes as below:

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \tilde{u}_i^{\hat{s}(t)}(t), \tag{1}$$

where $x_i(t)$, $f(x_i(t)) \in \mathbb{R}^n$ represent the state vector and the nonlinear function of the ith follower, respectively, $i = 1, \dots, N$. $\tilde{u}_i^{\hat{s}(t)}(t)$ is a switched control input with $\hat{s}(t)$ being the switching signal to be defined later and $A, B \in \mathbb{R}^{n\times n}$ are known constant matrices.

Consider the N agents in (1) as followers. The system model of the leader is depicted as

$$\dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)), \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ represents the state of leader.

The interaction topology among agents $i = 0, 1, \ldots, N$ is assumed to comply with semi-Markov switching and is denoted by $\bar{\mathcal{G}}(\hat{s}(t))$. Let $\{\hat{s}(t), t > 0\}$ be a continuum time semi-Markov process with right continuous trajectory taking values in a finite state space $S = \{1, 2, \ldots, s\}$ with generator $\Pi(h) := [\lambda_{ab}(h)]_{N\times N}$ matrix given by

$$Pr\{\hat{s}(t+h) = b|\hat{s}(t) = a\} = \begin{cases} \lambda_{ab}(h)h + o(h), a \neq b, \\ 1 + \lambda_{ab}(h)h + o(h), a = b, \end{cases} \tag{3}$$

where $h > 0$, $\lim_{h\to 0} \dfrac{o(h)}{h} = 0$, $\lambda_{ab}(h)$ is the time-varying transition rate from mode a to mode b, and $\lambda_{aa}(h) = -\sum_{b=1,b\neq a}^{s} \lambda_{ab}(h)$. The transition rate $\lambda_{ab}(h)$ is limited by constants $\overline{\lambda}_{ab}$ and $\underline{\lambda}_{ab}$, *i.e.*, $\underline{\lambda}_{ab} \leq \lambda_{ab}(h) \leq \overline{\lambda}_{ab}$ for any $h > 0$. Notice that the transition rate matrices $[\overline{\lambda}_{ab}]_{N\times N}$ and $[\underline{\lambda}_{ab}]_{N\times N}$ have zero row sum.

Our goal is to achieve the consensus between the followers (1) and leader (2) by designing event-based controller. However, the control signal may be intercepted by deception attacks during transmission. In the case of successful attack, the control signal will be replaced by the attack signal. Hence, the control signal of the ith node that may be affected by the attack signal is denoted as

$$\tilde{u}_i^{\hat{s}(t)}(t) = \theta_i(t)\xi_i(t) + (1 - \theta_i(t))u_i^{\hat{s}(t)}(t), \tag{4}$$

where $\xi_i(t)$ indicates the attack signal. The probability of successful attack $\theta_i(t)$ can be modeled as a Bernoulli distribution, defined as below:

$$\begin{cases} \Pr\{\theta_i(t) = 1\} = \bar{\theta}, \\ \Pr\{\theta_i(t) = 0\} = 1 - \bar{\theta}, \end{cases} \tag{5}$$

where $\bar{\theta}$ is a constant that refers to the mathematical expectation of success of the attack.

This paper takes a nonidentical double event-triggered strategy, where the ETM of S-C channel is defined as event triggering mechanism 1 (ETM1) and the ETM of C-A channel is denoted by event triggering mechanism 2 (ETM2). Let t_k^i and $\tilde{t}_k^i$ denote the triggering instants of the ith agent of the ETM1 and ETM2, t_{k+1}^i and $\tilde{t}_{k+1}^i$ are the next triggering instants of the ETM1 and ETM2, respectively. The triggering sequences $\{t_k^i\}$ and $\{\tilde{t}_k^i\}$ satisfy $\{\tilde{t}_k^i\} \subset \{t_k^i\}$. The event-triggering conditions for the ith agent of the ETM1 and ETM2 are devised as

$$\begin{aligned} t_{k+1}^i = &\inf\{t > t_k^i \mid \delta_i^T(t)\Theta_i\delta_i(t) \\ &> \varepsilon_i e_i^T(t)\Theta_i e_i(t)\}, \end{aligned} \tag{6}$$

$$\begin{aligned} \tilde{t}_{k+1}^i = &\inf\{t > \tilde{t}_k^i \mid \tilde{\delta}_i^T(t)\tilde{\Theta}_i\tilde{\delta}_i(t) \\ &> \tilde{\varepsilon}_i e_i^T(t_k^i)\tilde{\Theta}_i e_i(t_k^i)\}, \end{aligned} \tag{7}$$

where $t_0^i = \tilde{t}_0^i = 0$, $k \in \mathbb{N}_+$, $\delta_i(t) = e_i(t_k^i) - e_i(t)$, $e_i(t) = x_i(t) - x_0(t)$, $\tilde{\delta}_i(t) = e_i(\tilde{t}_k^i) - e_i(t_k^i)$, $i = 1, 2, \ldots, N$. ε_i and $\tilde{\varepsilon}_i$ are tunable parameters satisfying $\varepsilon_i \in (0, 1)$ and $\tilde{\varepsilon}_i \in (0, 1)$, Θ_i and $\tilde{\Theta}_i$ are known matrices.

Remark 1. Since the ETM2 in (7) is defined on the basis of the ETM1 in (6), the ETMs (6) and (7) are called double ETMs. The double ETMs can save communication resources as much as possible since they include the ETMs in both C-A and S-C channels. Additionally, the triggering instants of each node are determined by the conditions (6) and (7), so each agent has different triggering instants and transmits information according to the actual situations of the agent. However, the model in [15, 37] does not consider the ETM from C-A channel, and the triggering weight matrix is identical, which may lead to conservative results. This paper considers consensus of MASs (1) and (2) by designing nonidentical double ETMs, which is practical and has lower conservativeness. Note that the triggering instants of ETM2 (7) is based on ETM1 (6), therefore only the Zeno behavior of ETM1 requires to exclude.

Remark 2. Note that it is difficult to get accurate switching information of semi-Markov process, hence only mode-independent ETMs (6) and (7) are designed. Mode-independent control method has the advantage that all modes require only one controller. This control method is effective when switching information is inaccessible, while the mode-dependent ETMs presented in [44, 45] require the controller always gets accurate switching information and only applicable to switchings with obvious dwell time.

Considering the nonidentical double ETMs (6) and (7), the event-based controller of the ith agent is represented by

$$u_i^{\hat{s}(t)}(t) = -K_i^{\hat{s}(t)}[\sum_{j=1}^{N} l_{ij}^{\hat{s}(t)} F_{\hat{s}(t)} e_j(\tilde{t}_k^j) + d_i^{\hat{s}(t)} F_{\hat{s}(t)} e_i(\tilde{t}_k^i)], \tag{8}$$

where $F_{\hat{s}(t)} \in \mathbb{R}^{n\times n}$ is the coupling matrix and $K_i^{\hat{s}(t)} \in \mathbb{R}^{n\times n}$ is the control gain matrix.

Considering the attacked controller (4) and the event-triggered control (8), when $\hat{s}(t) = a \in S$, the error system of MASs suffering from attacks with semi-Markov switching can be written as

$$\begin{aligned} \dot{e}_i(t) =& Ae_i(t) + B\phi(e_i(t)) + \theta_i(t)\xi_i(t) \\ &- (1-\theta_i(t))K_i^a F_a[\sum_{j=1}^{N} l_{ij}^a(\tilde{\delta}_j(t) + \delta_j(t) + e_j(t)) + d_i^a(\tilde{\delta}_i(t) + \delta_i(t) + e_i(t))], \end{aligned} \tag{9}$$

where $\phi(e_i(t)) = f(e_i(t) + x_0(t)) - f(x_0(t)) \in \mathbb{R}^n$.

Denote $e(t) = [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T$, $\mathcal{A} = I_N \otimes A$, $\mathcal{B} = I_N \otimes B$, $\tilde{\delta}(t) = [\tilde{\delta}_1^T(t), \tilde{\delta}_2^T(t), \dots, \tilde{\delta}_N^T(t)]^T$, $\delta(t) = [\delta_1^T(t), \delta_2^T(t), \dots, \delta_N^T(t)]^T$, $\xi(t) = [\xi_1^T(t), \xi_2^T(t), \dots, \xi_N^T(t)]^T$, $c(t) = \mathrm{diag}\{\theta_1(t), \theta_2(t), \dots, \theta_N(t)\}$, $D_a = \mathrm{diag}\{d_1^a, d_2^a, \dots, d_N^a\}$, $H_a = L_a + D_a$, $\mathcal{K}_a = \mathrm{diag}\{K_1^a, K_2^a, \dots, K_N^a\}$, $C(t) = c(t) \otimes I_n$, $\mathcal{H}_a = H_a \otimes I_n$, $\mathcal{F}_a = I_N \otimes F_a$, $\Phi(e(t)) = [\phi^T(e_1(t)), \phi^T(e_2(t)), \dots, \phi^T(e_N(t))]^T$. Then the leader-follower error system (9) can be rewritten in the form of Kronecker product as below:

$$\begin{aligned} \dot{e}(t) =& \mathcal{A}e(t) + \mathcal{B}\Phi(e(t)) + C(t)\xi(t) \\ &- (I_{Nn} - C(t))\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a(e(t) + \delta(t) + \tilde{\delta}(t)). \end{aligned} \tag{10}$$

Before studying the asymptotic consensus, the subsequent lemmas, definitions, and assumptions are introduced.

Assumption 1. The nonlinear function $f(x_i(t))$ satisfy the OSL and quadratically inner-bounded (QIB) conditions, *i.e.*, there are scalars η, α, β such that, for $\forall x_u, x_v \in \mathbb{R}^n$, there hold

$$\begin{cases} \langle f(x_u(t)) - f(x_v(t)), x_u - x_v\rangle \le \eta\|x_u - x_v\|^2, \\ (f(x_u(t)) - f(x_v(t)))^T(f(x_u(t)) - f(x_v(t))) \le \beta\langle x_u - x_v, f(x_u(t)) - f(x_v(t))\rangle + \alpha\|x_u - x_v\|^2. \end{cases} \tag{11}$$

Assumption 2. The unknown attack $\xi_i(t)$ is bounded with $\|\xi_i(t)\| \leq \|G_i e_i(t)\|, i = 1, \ldots, N$, where G_i is a known matrix with $G_i^T G_i > 0$.

Assumption 3. All topologies of MASs (1)–(2) have a spanning tree with (2) as the root.

Lemma 1. *For matrix* $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix}$ *with* $M_{11} = M_{11}^T, M_{22} = M_{22}^T$, *the statements below are equivalent:*

1) $M < 0$,
2) $M_{22} < 0, M_{11} - M_{12}M_{22}^{-1}M_{12}^T < 0$,
3) $M_{11} < 0, M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0$.

Lemma 2. *[22]. For any given* $a, b \in \mathbb{R}^n$, *matrices* $W > 0$, H *and* K *of appropriate dimensions, there holds*

$$2a^T HSb \leq a^T HWH^T a + b^T K^T W^{-1} Kb. \tag{12}$$

3. Main results

Several sufficient conditions on the consensus of MASs (1)-(2) with semi-Markov switching are given in this section. We first consider the influence of deception attacks (4) and the event-based controller (8), then the PNBC with only fractional node connected with the leader is also studied. Meanwhile, the control gains $\mathcal{K}_a$, $a \in S$ are obtained.

Denote $G = \mathrm{diag}\{G_1^T G_1, G_2^T G_2, \ldots, G_N^T G_N\}$, $\varepsilon = \mathrm{diag}\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N\} \otimes I_n$, $\tilde{\varepsilon} = \mathrm{diag}\{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_N\} \otimes I_n$, $\Theta = \mathrm{diag}\{\Theta_1, \Theta_2, \ldots, \Theta_N\}$, $\tilde{\Theta} = \mathrm{diag}\{\tilde{\Theta}_1, \tilde{\Theta}_2, \ldots, \tilde{\Theta}_N\}$. Following is our first result.

Theorem 1. *Suppose that Assumptions 1–3 are satisfied. For given positive scalars* $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5$, *if there exist matrix* $P_a > 0$, *matrix* $\mathcal{K}_a$, $a \in S$, *scalars* η, α, β *such that*

$$\begin{pmatrix} \prod & \zeta_{1a} & \zeta_{2a} & \zeta_{3a} & P_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1} I_{Nn} \end{pmatrix} < 0, \tag{13}$$

then, under controllers (4) and (8), the MASs (1)–(2) achieve consensus in mean-square sense, where
$\prod = \sum_{b=1}^{s} \lambda_{ab}(h) P_b + \hat{\Gamma}_a$,
$\hat{\Gamma}(a) = P_a\mathcal{A} + \mathcal{A}^T P_a - (1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a - (1-\bar{\theta})(\mathcal{F}_a)^T(\mathcal{H}_a)^T(\mathcal{K}_a)^T P_a + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$
$+ \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G$,
$\zeta_{1a} = -(1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$,
$\zeta_{2a} = -(1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a$,
$\zeta_{3a} = P_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn}$,
$\zeta_4 = -\epsilon_4\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$.

PROOF. When $\hat{s}(t) = a \in S$, choose the following Lyapunov function

$$V(e(t), \hat{s}(t), t) = e^T(t)P_a e(t). \tag{14}$$

The weak infinitesimal operator is represented by the symbol $\mathbb{L}$, then

$$\mathbb{L}V(e(t),\hat{s}(t),t)=\lim_{l\to+0}\frac{1}{l}[\{V(e(t+l),\hat{s}(t+l),t+l)|e(t),\hat{s}(t)=a\}-V(e(t),\hat{s}(t),t)]. \tag{15}$$

For $\hat{s}(t)=a\in S$, employing the law of conditional probability and the cumulative distribution function (CDF), there holds

$$\begin{aligned}
&\mathbb{E}\{\mathbb{L}V(e(t),\hat{s}(t),t)\}\\
&=\lim_{l\to+0}\frac{1}{l}[\mathbb{E}\{\sum_{b=1,b\neq a}^{s}Pr\{\hat{s}(t+l)=b|\hat{s}(t)=a\}e^T(t+l)P_be(t+l)\\
&+Pr\{\hat{s}(t+l)=a|\hat{s}(t)=a\}e^T(t+l)P_ae(t+l)\}-e^T(t)P_ae(t)]\\
&=\lim_{l\to+0}\frac{1}{l}[\mathbb{E}\{\sum_{b=1,b\neq a}^{s}\frac{q_{ab}(J_a(h+l)-J_a(h))}{1-J_a(h)}e^T(t+l)P_be(t+l)\\
&+\frac{1-J_a(h+l)}{1-J_a(h)}e^T(t+l)P_ae(t+l)\}-e^T(t)P_ae(t)],
\end{aligned} \tag{16}$$

where $J_a(h)$ denotes the CDF that depends on step size h when the topology of MASs switches to mode a, and q_{ab} stands the probability intensity of the mode from a to b.

Consider that l is an infinitesimal positive number, the Taylor expansion of $e(t+l)$ is approximately

$$e(t+l)=e(t)+\dot{e}(t)l+o(l), \tag{17}$$

where $o(l)$ is an infinitesimal component of l and $\lim_{l\to+0}\frac{o(l)}{l}=0$.

Consider the characteristics of the CDF, one derives

$$\lim_{l\to+0}\frac{q_{ab}(J_a(h+l)-J_a(h))}{(1-J_a(h))l}=\lambda_{ab}(\tau),\ \lim_{l\to+0}\frac{1-J_a(h+l)}{1-J_a(h)}=1,\ \lim_{l\to+0}\frac{J_a(h+l)-J_a(h)}{1-J_a(h)}=0. \tag{18}$$

As can be deduced from (16)–(18) that

$$\begin{aligned}
&\mathbb{E}\{\mathbb{L}V(e(t),\hat{s}(t),t)\}\\
&=\lim_{l\to+0}\frac{1}{l}[\mathbb{E}\{\sum_{b=1,b\neq a}^{s}\frac{q_{ab}(J_a(h+l)-J_a(h))}{1-J_a(h)}\\
&(e(t)+\dot{e}(t)l+o(l))^TP_b(e(t)+\dot{e}(t)l+o(l))+\frac{1-J_a(h+l)}{1-J_a(h)}\\
&(e(t)+\dot{e}(t)l+o(l))^TP_a(e(t)+\dot{e}(t)l+o(l))-e^T(t)P_ae(t)]\\
&=\sum_{b=1,b\neq a}^{s}\lambda_{ab}(h)e^T(t)P_be(t)+2\dot{e}^T(t)P_ae(t)-\lambda_a(h)e^T(t)P_ae(t)\\
&=\sum_{b=1}^{s}\lambda_{ab}(h)e^T(t)P_be(t)+2e^T(t)P_a\mathcal{A}e(t)+2e^T(t)P_a\mathcal{B}\Phi(e(t))\\
&+2\bar{\theta}e^T(t)P_a\xi(t)-2(1-\bar{\theta})e^T(t)P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a(e(t)+\delta(t)+\tilde{\delta}(t)).
\end{aligned} \tag{19}$$

For any constants $\epsilon_1 > 0, \epsilon_2 > 0$, one has from the QIB and OSL conditions in Assumption 1 that

$$\epsilon_1 \eta e^T(t)e(t) \geq \epsilon_1 e^T(t)\Phi(e(t)), \tag{20}$$

$$\epsilon_2 \alpha e^T(t)e(t) + \epsilon_2 \beta e^T(t)\Phi(e(t)) \geq \epsilon_2 \Phi^T(e(t))\Phi(e(t)). \tag{21}$$

For $\epsilon_3 > 0$, from Lemma 2, one has

$$\begin{aligned} 2e^T(t)P_a\xi(t) \leq& \epsilon_3^{-1}e^T(t)P_aP_ae(t) + \epsilon_3\xi^T(t)\xi(t) \\ \leq& \epsilon_3^{-1}e^T(t)P_aP_ae(t) + \epsilon_3 e^T(t)Ge(t). \end{aligned} \tag{22}$$

Besides, for $\epsilon_4 > 0$ and $\epsilon_5 > 0$, the event-trigger conditions in (6) and (7) imply that

$$\epsilon_4 e^T(t)\varepsilon\Theta e(t) - \epsilon_4\delta^T(t)\Theta\delta(t) \geq 0, \tag{23}$$

$$\epsilon_5(e(t)+\delta(t))^T\tilde{\varepsilon}\tilde{\Theta}(e(t)+\delta(t)) - \epsilon_5\tilde{\delta}^T(t)\tilde{\Theta}\tilde{\delta}(t) \geq 0. \tag{24}$$

Therefore, one has from (19)–(24) that

$$\begin{aligned} &\mathbb{E}\{\mathbb{L}V(e(t),\hat{s}(t),t)\} \\ \leq& e^T[\sum_{b=1}^{s}\lambda_{ab}(h)P_b + P_a\mathcal{A} + \mathcal{A}^TP_a - (1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta} \\ &-(1-\bar{\theta})(\mathcal{F}_a)^T(\mathcal{H}_a)^T(\mathcal{K}_a)^TP_a + \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G + \epsilon_3^{-1}\bar{\theta}P_aP_a]e(t) \\ &+e^T(t)[P_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn}]\Phi(e(t)) + \Phi^T(e(t))[\mathcal{B}^TP_a - 0.5\epsilon_1 I_{Nn} \\ &+0.5\epsilon_2\beta I_{Nn}]e(t) - \epsilon_2\Phi^T(e(t))\Phi(e(t)) - e^T(t)[(1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a - \epsilon_5\tilde{\varepsilon}\tilde{\Theta}]\delta(t) \\ &-\delta^T(t)[(1-\bar{\theta})(\mathcal{F}_a)^T(\mathcal{H}_a)^T(\mathcal{K}_a)^TP_a - \epsilon_5\tilde{\varepsilon}\tilde{\Theta}]e(t) - \delta^T(t)(\epsilon_4\Theta - \epsilon_5\tilde{\varepsilon}\tilde{\Theta})\delta(t) \\ &-e^T(t)[(1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a]\tilde{\delta}(t) - \tilde{\delta}^T(t)[(1-\bar{\theta})(\mathcal{F}_a)^T(\mathcal{H}_a)^T(\mathcal{K}_a)^TP_a]e(t) \\ &-\tilde{\delta}^T(t)\epsilon_5\tilde{\Theta}\tilde{\delta}(t) \\ =& Z^T(t)\Omega_a Z(t), \end{aligned} \tag{25}$$

where $Z(t) = \mathrm{col}\{e(t), \delta(t), \tilde{\delta}(t), \Phi(e(t))\}$,

$$\Omega_a = \begin{pmatrix} \sum_{b=1}^{s}\lambda_{ab}(h)P_b + \Gamma_a & \zeta_{1a} & \zeta_{2a} & \zeta_{3a} \\ * & \zeta_4 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 \\ * & * & * & -\epsilon_2 I_{Nn} \end{pmatrix},$$

$\Gamma_a = P_a\mathcal{A} + \mathcal{A}^TP_a - (1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a - (1-\bar{\theta})(\mathcal{F}_a)^T(\mathcal{H}_a)^T(\mathcal{K}_a)^TP_a + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta} + \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G + \epsilon_3^{-1}\bar{\theta}P_aP_a$, $\zeta_{1a} = -(1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$, $\zeta_{2a} = -(1-\bar{\theta})P_a\mathcal{K}_a\mathcal{H}_a\mathcal{F}_a$, $\zeta_{3a} = P_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn}$, $\zeta_4 = -\epsilon_4\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$.

From Lemma 1, we can infer that the inequality (13) is equivalent to $\Omega_a < 0$. Reminding the inequality (25), a positive scalar c_1 exists such that

$$\mathbb{E}\{\mathbb{L}V(e(t),a,t)\} \leq -c_1\mathbb{E}\{\|Z(t)\|^2\} \leq -c_1\mathbb{E}\{\|e(t)\|^2\}. \tag{26}$$

From Itô̂s formula, there holds

$$\mathbb{E}\{V(e(t),a,t)\} - \mathbb{E}\{V(e(0),\hat{s}(0),0)\} = \mathbb{E}\{\int_0^t \mathbb{L}V(e(s),a,s)ds\}. \tag{27}$$

From the inequalities (25)–(27), there is a positive scalar c_2 such that

$$\begin{aligned} c_2\mathbb{E}\{\|e(t)\|^2\} \leq & \mathbb{E}\{V(e(t),a,t)\} \\ = & \mathbb{E}\{V(e(0),\hat{s}(0),0)\} + \mathbb{E}\{\int_0^t \mathbb{L}V(e(s),a,s)ds\} \\ \leq & \mathbb{E}\{V(e(0),\hat{s}(0),0)\} - c_1\int_0^t \mathbb{E}\{\|e(s)\|^2\}ds, \end{aligned} \tag{28}$$

from which one obtains $\lim_{t\to\infty} \mathbb{E}\{\|e(t)\|^2\} = 0$. Hence the MASs (1)–(2) achieve asymptotic consensus in mean square.

Next, the exclusion of Zeno phenomenon has been proved, *i.e.*, the lower bound of the event-triggering intervals determined by (6) and (7) are greater than a known positive number.

Consider the inequality (6), for $t_k^i < t < t_{k+1}^i$, when $\hat{s}(t) = a \in S$, there holds

$$\begin{aligned} \dot{\delta}_i(t) = & -Ae_i(t) - B\phi(e_i(t)) - \theta_i(t)\xi_i(t) + (1-\theta_i(t)) \\ & K_i^aF_a[\Sigma_{j=1}^N l_{ij}^a(\tilde{\delta}_j(t) + \delta_j(t) + e_j(t)) + d_i^a(\tilde{\delta}_i(t) + \delta_i(t) + e_i(t))]. \end{aligned} \tag{29}$$

It follows from the Theorem 1 that $e_i(t), \delta_i(t), \tilde{\delta}_i(t)$ are upper bounded, *i.e.*, there are nonnegative scalars $e_M, \delta_M, \tilde{\delta}_M$ such that $\|e_i(t)\| \leq e_M$, $\|\delta_i(t)\| \leq \delta_M$, $\|\tilde{\delta}_i(t)\| \leq \tilde{\delta}_M$, $i = 1,2,\ldots,N$. From Assumption 2, one has $\|\xi_i(t)\| \leq \|G_ie_i(t)\| \leq \|G_i\|e_M$.

In view of the OSL and QIB conditions, there holds $\|\phi(e_i(t))\| \leq |\alpha + \beta\eta|\|e_i(t)\| \leq |\alpha + \beta\eta|e_M$.

Therefore, it is obtained from above analysis that

$$\begin{aligned} \|\dot{\delta}_i(t)\| \leq & \|A\|e_M + \|B\||\alpha + \beta\eta|e_M + \|G_i\|e_M \\ & + \|K_i^a\|\|F_a\|[\sum_{j=1}^N |l_{ij}^a|(e_M + \delta_M + \tilde{\delta}_M) + d_i^a(\|\delta_i(t)\| + e_M + \tilde{\delta}_M)] \\ = & \varrho^a\|\delta_i(t)\| + \omega^a, \end{aligned} \tag{30}$$

where $\varrho^a = \|K_i^a\|\|F_a\|d_i^a, \omega^a = (\|A\| + \|B\||\alpha + \beta\eta| + \|G_i\|)e_M + \|K_i^a\|\|F_a\|[\sum_{j=1}^N |l_{ij}^a|(e_M + \delta_M + \tilde{\delta}_M) + \|K_i^a\|\|F_a\|d_i^a(e_M + \tilde{\delta}_M)$.

Consider two cases: $d_i^a \neq 0$ and $d_i^a = 0$.

Case I: When $d_i^a \neq 0$, it means the ith agent is interacting with the leader. Due to $\|\delta_i(t_k^i)\| = 0$, one has

$$\|\delta_i(t)\| \leq \frac{\omega^a}{\varrho^a}[\exp(\varrho^a(t - t_k^i)) - 1].$$

Solving the above inequality yields that

$$t - t_k^i \geq \ln(1 + \varrho^a\|\delta_i(t)\|/\omega^a)/\varrho^a. \tag{31}$$

In view of the ETM1 (6), the next event of the ith agent will be triggered when

$$\delta_i(t_{k+1}^{i-})^T\Theta_i\delta_i(t_{k+1}^{i-}) = \varepsilon_i e_i(t_{k+1}^{i})^T\Theta_i e_i(t_{k+1}^{i}), \tag{32}$$

where $\delta_i(t_{k+1}^{i-}) = \lim_{t\to t_{k+1}^{i-}} \delta_i(t)$. Thus

$$\|\delta_i(t_{k+1}^{i-})\| \geq w_i\|e_i(t_{k+1}^{i})\|, a \in S, \tag{33}$$

where $w_i = \sqrt{\varepsilon_i\lambda_{\min}(\Theta_i)/\lambda_{\max}(\Theta_i)}$.

Combining (29)–(33), one has

$$t_{k+1}^{i} - t_k^i \geq \frac{1}{\varrho^a}\ln(1 + \varrho^a w_i\|e_i(t_{k+1}^{i})\|/\omega^a) > 0. \tag{34}$$

Case II: When $d_i^a = 0$, it means the follower i has no connection to the leader. Hence, $\varrho^a = 0$ and $\|\dot{\delta}_i(t)\| \leq \omega^a$. There is a small scalar $\iota(t_k^i) > 0$ such that $\|\delta_i(t)\| \leq \omega^a(t - t_k^i) - \iota(t_k^i)$. Therefore,

$$t_{k+1}^{i} - t_k^i \geq (w_i\|e_i(t_{k+1}^{i})\| + \iota(t_k^i))/\omega^a > 0. \tag{35}$$

One can easily derive from (34) and (35) that the event-triggering intervals determined by ETM1 (6) is lower bounded by a positive number. The proof is completed.

Remark 3. One of the main difficulties is how to design the event-based consensus agreement. The purpose of this protocol is to capture the time-varying switching dynamics of MASs while reducing unnecessary data transmission. In comparison with conventional control method in MASs which only depends on the current coupling states [23, 46], the proposed consensus protocol (8) consider time-varying switching and state feedback at the event-triggering instants, which can obtain better performance.

Note that the result of Theorem 1 is related to the transition rate, which is depends on the step size h. Unlike semi-Markov process, the transition rate of Markov process is a constant. For this reason, one obtains the following corollary from Theorem 1.

Corollary 1. *Suppose that Assumptions 1–3 are satisfied. For given positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$, ϵ_5, if there exist matrix $P_a > 0$, matrix $\mathcal{K}_a$, $a \in S$, scalars η, α, β such that*

$$\begin{pmatrix} \prod_1 & \zeta_{1a} & \zeta_{2a} & \zeta_{3a} & P_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1} I_{Nn} \end{pmatrix} < 0, \tag{36}$$

then, under controllers (4) and (8), the MASs (1)–(2) with Markov switching topologies achieve consensus in mean-square sense, where $\hat{\Gamma}_a, \zeta_{1a}, \zeta_{2a}, \zeta_{3a}, \zeta_4$ have same definitions in Theorem 1 and $\prod_1 = \sum_{b=1}^{s}\lambda_{ab}P_b + \hat{\Gamma}_a$.

Proof. The proof procedure is the same as Theorem 1 for $\lambda_{ab}(h) = \lambda_{ab}$. The detailed proof is elided.

As the sufficient conditions (13) in Theorem 1 contains the nonlinear term $\lambda_{ab}(h)$, it is difficult to verify the validity of the model by solving inequality (13). Therefore the following corollary is presented to overcome this difficulty.

Corollary 2. *Suppose that Assumptions 1–3 are satisfied. For given positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$, ϵ_5, if there exist matrix $P_a = \text{diag}\{P_1^a, P_2^a, \ldots, P_N^a\} > 0$, matrix $Y_a = \text{diag}\{Y_1^a, Y_2^a, \ldots, Y_N^a\}$, $a \in S$, scalars η, α, β such that*

$$\begin{pmatrix} \overline{\prod} & \zeta_{1a} & \zeta_{2a} & \zeta_{3a} & P_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1}I_{Nn} \end{pmatrix} < 0, \tag{37}$$

$$\begin{pmatrix} \underline{\prod} & \zeta_{1a} & \zeta_{2a} & \zeta_{3a} & P_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1}I_{Nn} \end{pmatrix} < 0, \tag{38}$$

then, under controllers (4) and (8), MASs (1)–(2) achieve consensus in mean-square sense. Meanwhile, the control gains are given by $\mathcal{K}_a = P_a^{-1}Y_a$, where
$\overline{\prod} = \sum_{b=1}^{s} \overline{\lambda}_{ab}P_b + \hat{\Gamma}_a$, $\underline{\prod} = \sum_{b=1}^{s} \underline{\lambda}_{ab}P_b + \hat{\Gamma}_a$,
$\hat{\Gamma}_a = P_a\mathcal{A} + \mathcal{A}^TP_a - (1-\bar{\theta})Y_a\mathcal{H}_a\mathcal{F}_a - (1-\bar{\theta})(\mathcal{F}_a)^T(\mathcal{H}_a)^TY_a^T + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta} + \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G$,
$\zeta_{1a} = -(1-\bar{\theta})Y_a\mathcal{H}_a\mathcal{F}_a + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$, $\zeta_{2a} = -(1-\bar{\theta})Y_a\mathcal{H}_a\mathcal{F}_a$,
$\zeta_{3a} = P_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn}$,
$\zeta_4 = -\epsilon_4\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$.

Proof. When the conditions of Theorem 1 holds, the MASs with transition rate $\lambda_{ab}(h)$ achieve consensus in mean-square sense. For a particular h , $\lambda_{ab}(h)$ can be written as the linear combination $\lambda_{ab}(h) = \omega_1\overline{\lambda}_{ab} + \omega_2\underline{\lambda}_{ab}$, where $\omega_1 + \omega_2 = 1, \omega_1, \omega_2 > 0$. Multiplying (37) by ω_1 and (38) by ω_2, one has

$$\begin{pmatrix} \omega_1\overline{\prod} + \omega_2\underline{\prod} & \zeta_{1a} & \zeta_{2a} & \zeta_{3a} & P_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1}I_{Nn} \end{pmatrix} < 0, \tag{39}$$

by tuning ω_1 and ω_2, all possible $\lambda_{ab}(h) \in [\underline{\lambda}_{ab}, \overline{\lambda}_{ab}]$. So the proof is completed.

When a MAS has a great number of agents, it is unrealistic to control a MAS by adding feedback from leader to all followers. The PNBC is one of the most effective synchronization strategies for MASs, where the control feedback from leader are only applicable to partial followers.

In general, reordering of the followers in MASs (1), and let the first p agents be controlled by the leader. Then the follower systems can be represented as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \tilde{u}_i^{\hat{s}(t)}(t), i = 1, 2, \ldots, p, \\ \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \bar{u}_i^{\hat{s}(t)}(t), i = p+1, p+2, \ldots, N, \end{cases} \tag{40}$$

where $\bar{u}_i^{\hat{s}(t)}(t) = \theta_i(t)\xi_i(t) - (1-\theta_i(t))K_i^{\hat{s}(t)} \sum_{j=1}^N l_{ij}^{\hat{s}(t)} F_{\hat{s}(t)} e_j(\tilde{t}_k^j)$, the control input $\tilde{u}_i^{\hat{s}(t)}(t)$ has been defined by (4) and (8).

Subtracting (2) from (40) obtains that

$$\begin{cases} \dot{e}_i(t) = Ae_i(t) + B\phi(e_i(t)) + \tilde{u}_i^{\hat{s}(t)}(t), i = 1,2,\dots,p, \\ \dot{e}_i(t) = Ae_i(t) + B\phi(e_i(t)) + \bar{u}_i^{\hat{s}(t)}(t), i = p+1, p+2,\dots,N. \end{cases} \tag{41}$$

When $\hat{s}(t) = a$, the error system (41) is written in Kronecker product form as below:

$$\begin{aligned} \dot{e}(t) =& \mathcal{A}e(t) + \mathcal{B}\Phi(e(t)) + C(t)\xi(t) \\ &- (I_{Nn} - C(t))\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a(e(t) + \delta(t) + \tilde{\delta}(t)), \end{aligned} \tag{42}$$

where $\mathcal{A} = I_N \otimes A$, $\mathcal{B} = I_N \otimes B$, $\mathbf{D}_a = \mathrm{diag}\{d_1^a, d_2^a, \dots, d_p^a, 0, \dots, 0\} \in \mathbb{R}^{N\times N}$, $\mathbf{H}_a = (\mathbf{D}_a + L_a) \otimes I_n$, $\mathcal{K}_a = \mathrm{diag}\{K_1^a, K_2^a, \dots, K_N^a\}$.

The following theorem states the sufficient conditions for the consensus in mean-square sense of the MASs (2) and (40) under PNBC.

Theorem 2. *Suppose that Assumptions 1–3 are satisfied. For given positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5$, if there exist matrix $Q_a > 0$, matrix $\mathcal{K}_a$, $a \in S$, scalars η, α, β such that*

$$\begin{pmatrix} \Xi & \tilde{\zeta}_{1a} & \tilde{\zeta}_{2a} & \tilde{\zeta}_{3a} & Q_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1} I_{Nn} \end{pmatrix} < 0, \tag{43}$$

then, under controllers (4) and (8) with PNBC, the MASs (2) and (40) achieve consensus in mean-square sense, where
$\Xi = \sum_{b=1}^s \lambda_{ab}(h)Q_b + \tilde{\Gamma}_a$,
$\tilde{\Gamma}_a = Q_a\mathcal{A} + \mathcal{A}^T Q_a - (1-\bar{\theta})Q_a\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a - (1-\bar{\theta})(\mathcal{F}_a)^T(\mathbf{H}_a)^T(\mathcal{K}_a)^T Q_a + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta} + \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G$,
$\tilde{\zeta}_{1a} = -(1-\bar{\theta})Q_a\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$,
$\tilde{\zeta}_{2a} = -(1-\bar{\theta})Q_a\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a$,
$\tilde{\zeta}_{3a} = P_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn}$,
$\zeta_4 = -\epsilon_4\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$.

PROOF. When $\hat{s}(t) = a \in S$, consider the Lyapunov function candidate

$$V(e(t), \hat{s}(t), t) = e^T(t)Q_a e(t). \tag{44}$$

Following the same steps (14)–(19) as those given in the proof of Theorem 1, there holds

$$V(e(t), \hat{s}(t), t) \leq \sum_{b=1}^s \lambda_{ab}(h)Q_b + 2\dot{e}^T(t)Q_a e(t). \tag{45}$$

Combining (20)–(24), one obtains

$$\mathbb{E}\{\mathbb{L}V(e(t), \hat{s}(t), t)\} \leq Z^T(t)\tilde{\Omega}_a Z(t). \tag{46}$$

In order to obtain the asymptotic consensus of MASs with PNBC, one requires

$$\tilde{\Omega}_a = \begin{pmatrix} \sum_{b=1}^{s} \lambda_{ab}(h) Q_b + \bar{\Gamma}(a) & \tilde{\zeta}_{1a} & \tilde{\zeta}_{2a} & \tilde{\zeta}_{3a} \\ * & \zeta_4 & 0 & 0 \\ * & 0 & -\epsilon_5 \tilde{\Theta} & 0 \\ * & * & * & -\epsilon_2 I_{Nn} \end{pmatrix} < 0,$$

where $\bar{\Gamma}_a = Q_a\mathcal{A} + \mathcal{A}^T Q_a - (1-\bar{\theta})Q_a\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a - (1-\bar{\theta})(\mathcal{F}_a)^T(\mathbf{H}_a)^T(\mathcal{K}_a)^T Q_a + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta} + \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G + \epsilon_3^{-1}\bar{\theta}Q_aQ_a$, $\tilde{\zeta}_{1a} = -(1-\bar{\theta})Q_a\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$,
$\tilde{\zeta}_{2a} = -(1-\bar{\theta})Q_a\mathcal{K}_a\mathbf{H}_a\mathcal{F}_a$, $\tilde{\zeta}_{3a} = P_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn}$, $\zeta_4 = -\epsilon_4\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}$.

From the Lemma 1, it is easy to infer that the inequality (43) is equal to $\tilde{\Omega}_a < 0$. This completes the proof.

Remark 4. In Theorem 2, it is more realistic to assumed that only fractional followers can directly track the leader. In addition, if there is no follower that can get feedback from the leader, the consensus of MASs (2) and (40) with PNBC cannot be achieved. Therefore, there exists at least a directed path from the leader to the all followers. The consensus between two multi-agent networks with PNBC was proposed in [9], but the author assume that the number of leaders is the same as the number of followers, which is not suitable for the actual situation. In fact, there are usually only a few leaders and a large number of followers, and only a few followers connected with the leader.

From Corollaries 1, 2 and Theorems 1, 2, the following corollary can be easily deduced. The proof is omitted.

Corollary 3. *Suppose that Assumptions 1–3 are satisfied. For given positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$, ϵ_5, if there exist matrix $Q_a > 0$, matrix $\mathcal{K}_a$, $a \in S$, scalars η, α, β such that*

$$\begin{pmatrix} \Xi_1 & \tilde{\zeta}_{1a} & \tilde{\zeta}_{2a} & \tilde{\zeta}_{3a} & Q_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1} I_{Nn} \end{pmatrix} < 0, \tag{47}$$

then, under controllers (4) and (8) with PNBC, the MASs (2) and (40) with Markov switching topologies achieve consensus in mean-square sense, where $\Xi_1 = \sum_{b=1}^{s} \lambda_{ab} Q_b + \bar{\Gamma}_a$, $\bar{\Gamma}_a, \tilde{\zeta}_{1a}, \tilde{\zeta}_{2a}, \tilde{\zeta}_{3a}, \zeta_4$ have same definitions in Theorem 2.

Corollary 4. *Suppose that Assumptions 1–3 are satisfied. For given positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$, ϵ_5, if there exist matrix $Q_a = \mathrm{diag}\{Q_1^a, Q_2^a, \ldots, Q_N^a\} > 0$, matrix $R_a = \mathrm{diag}\{R_1^a, R_2^a, \ldots, R_N^a\}$, $a \in S$, scalars η, α, β such that*

$$\begin{pmatrix} \Xi & \tilde{\zeta}_{1a} & \tilde{\zeta}_{2a} & \tilde{\zeta}_{3a} & Q_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1} I_{Nn} \end{pmatrix} < 0, \tag{48}$$

$$\begin{pmatrix} \underline{\Xi} & \tilde{\zeta}_{1a} & \tilde{\zeta}_{2a} & \tilde{\zeta}_{3a} & Q_a \\ * & \zeta_4 & 0 & 0 & 0 \\ * & 0 & -\epsilon_5\tilde{\Theta} & 0 & 0 \\ * & * & * & -\epsilon_2 I_{Nn} & 0 \\ * & * & * & * & -\epsilon_3\bar{\theta}^{-1} I_{Nn} \end{pmatrix} < 0, \tag{49}$$

then, under controllers (4) and (8), the control gains are given by $\mathcal{K}_a = Q_a^{-1}R_a$ *and the MASs (2) and (40) with PNBC achieve consensus in mean-square sense, where*
$\overline{\Xi} = \sum_{b=1}^{s} \overline{\lambda}_{ab} Q_b + \tilde{\Gamma}_a,$
$\underline{\Xi} = \sum_{b=1}^{s} \underline{\lambda}_{ab} Q_b + \tilde{\Gamma}_a,$
$\tilde{\Gamma}_a = Q_a\mathcal{A} + \mathcal{A}^T Q_a - (1-\bar{\theta})R_a\mathbf{H}_a\mathcal{F}_a - (1-\bar{\theta})(\mathcal{F}_a)^T(\mathbf{H}_a)^T R_a^T + \epsilon_4\varepsilon\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta} + \epsilon_1\eta I_{Nn} + \epsilon_2\alpha I_{Nn} + \epsilon_3\bar{\theta}G,$
$\tilde{\zeta}_{1a} = -(1-\bar{\theta})R_a\mathbf{H}_a\mathcal{F}_a + \epsilon_5\tilde{\varepsilon}\tilde{\Theta},$
$\tilde{\zeta}_{2a} = -(1-\bar{\theta})R_a\mathbf{H}_a\mathcal{F}_a,$
$\tilde{\zeta}_{3a} = Q_a\mathcal{B} - 0.5\epsilon_1 I_{Nn} + 0.5\epsilon_2\beta I_{Nn},$
$\zeta_4 = -\epsilon_4\Theta + \epsilon_5\tilde{\varepsilon}\tilde{\Theta}.$

4. Numerical examples

This section gives a numerical example to illustrates the validity of the proposed scheme in Theorem 2, and demonstrate the advantage of the nonidentical double ETMs.

The leader of MASs is depicted as

$$\dot{x}_0(t) = Ax_0(t) + Bf(x_0(t)), \tag{50}$$

where $x_0(t) = (x_{01}(t), x_{02}(t))^T$,

$$f(x_0(t)) = \begin{pmatrix} -x_{01}(t)(x_{01}^2(t) + x_{02}^2(t)) \\ -x_{02}(t)(x_{01}^2(t) + x_{02}^2(t)) \end{pmatrix}, A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Consider the follower systems with semi-Markov switching based on PNBC under deception attacks as follows.

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \tilde{u}_i^{\hat{s}(t)}(t), i = 1, \\ \dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \bar{u}_i^{\hat{s}(t)}(t), i = 2, 3, 4, \end{cases} \tag{51}$$

$$\begin{cases} \tilde{u}_i^{\hat{s}(t)}(t) = \theta_i(t)\xi_i(t) - (1-\theta_i(t))K_i^{\hat{s}(t)}[\sum_{j=1}^{N} l_{ij}^{\hat{s}(t)} F_{\hat{s}(t)} e_j(\tilde{t}_k^j) + d_i^a F_a e_i(\tilde{t}_k^i)], \\ \bar{u}_i^{\hat{s}(t)}(t) = \theta_i(t)\xi_i(t) - (1-\theta_i(t))K_i^{\hat{s}(t)} \sum_{j=1}^{N} l_{ij}^{\hat{s}(t)} F_{\hat{s}(t)} e_j(\tilde{t}_k^j), \end{cases}$$

$$\xi_i(t) = \tanh(G_i e_i(t)), i = 1, 2, 3, 4,$$

where

$$x_i(t) = \begin{pmatrix} x_{i1}(t) \\ x_{i2}(t) \end{pmatrix}, f(x_i(t)) = \begin{pmatrix} -x_{i1}(t)(x_{i1}^2(t) + x_{i2}^2(t)) \\ -x_{i2}(t)(x_{i1}^2(t) + x_{i2}^2(t)) \end{pmatrix}.$$

Let $F_a = I_2, a = 1, 2, 3$. The communication links of MASs in three modes are illustrated in Fig. 1. The Laplacian and leader adjacency matrices L_1, L_2, L_3 and $\mathbf{D_1}, \mathbf{D_2}, \mathbf{D_3}$ are assumed to be

$$L_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}, L_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}, L_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{D_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathbf{D_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathbf{D_3} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

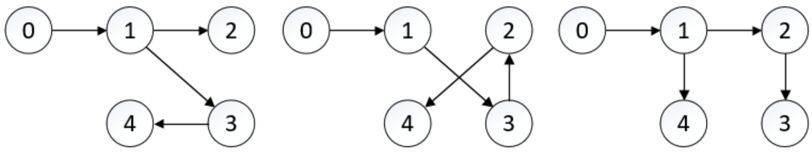


Fig. 1. Switching topologies for the MASs.

Let the uniform discretionary step size is $h = 0.01$, the transition rate of the semi-Markov jumping parameters are given as below

$$\lambda_{11}(h) \in (-90, -80), \lambda_{12}(h) \in (50, 55), \lambda_{13}(h) \in (30, 35),$$
$$\lambda_{21}(h) \in (20, 25), \lambda_{22}(h) \in (-90, -80), \lambda_{23}(h) \in (60, 65),$$
$$\lambda_{31}(h) \in (40, 42), \lambda_{32}(h) \in (40, 50), \lambda_{33}(h) \in (-92, -80).$$

Hence the lower and upper bounds of the transition rate matrices are obtained as

$$\underline{\lambda} = \begin{pmatrix} -80 & 50 & 30 \\ 20 & -80 & 60 \\ 40 & 40 & -80 \end{pmatrix}, \overline{\lambda} = \begin{pmatrix} -90 & 55 & 35 \\ 25 & -90 & 65 \\ 42 & 50 & -92 \end{pmatrix}.$$

The MASs have OSL nonlinearities, the choice of constants η, α and β determines the range of operations, and can be arbitrarily large. Same as those obtained in [47], taking $\eta = 0$, $\alpha = -99$, $\beta = -100$, the nonlinear functions satisfy Assumption 1 in the operational region $\mathcal{D} = \{x_i \in \mathbb{R}^2 : \|x_i\| \leq 5\}$.

Let $\epsilon_1 = 0.001, \epsilon_2 = 0.0015, \epsilon_3 = 0.1, \epsilon_4 = \epsilon_5 = 1$, $\bar{\theta} = 0.3$. By solving LMIs (48) and (49), Theorem 2 has feasible solutions and control gains shown in APPENDIX.

The initial values are given as $x_0(0) = (2, -1)^T, x_1(0) = (1.2, -2.4)^T, x_2(0) = (-2.7, 2.3)^T, x_3(0) = (2.8, -2.3)^T, x_4(0) = (2.6, -1.5)^T$. Taking the time-step $h = 0.01$ and $\hat{s}(0) = 1$, we have the switching signal generated by the semi-Markov switching presented in Fig. 2.

Fig. 3 are the trajectories of the consensus errors between (50) and (51) with $\tilde{u}_i^a(t) = \bar{u}_i^a(t) = 0$, $a = 1, 2, 3$, from which one can see that the consensus cannot be realized without control. The time response of the consensus error with control is represented in Fig. 3. As seen in Fig. 4, all follower agents track the leader agent by the event-based state feedback control. Fig. 5 shows the double event-triggering instants for all agents. Comparing (a) and (b) in Fig. 5, the number of triggering instants of each agent in ETM2 is much less than the number of triggering instants in ETM1, which greatly reduce the number of data transmission. In addition, the triggering instants of each agent in ETM1 and ETM2 are different, which indicates that the designed ETMs are nonidentical and less conservative results can be obtained.

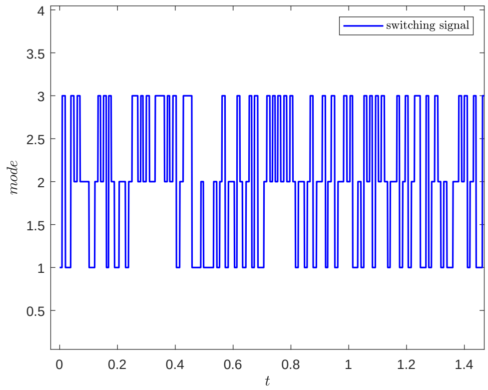
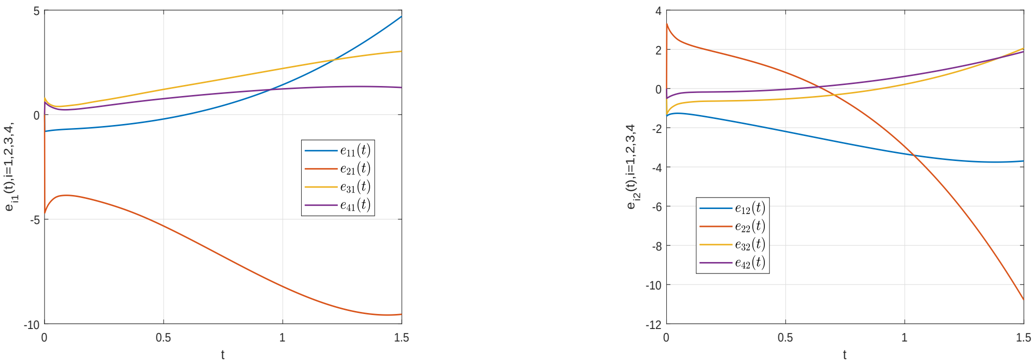
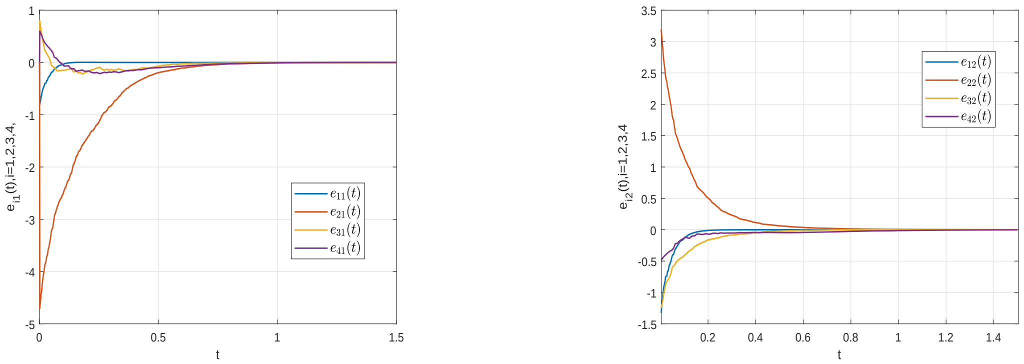


Fig. 2. Trajectory of the semi-Markov switching with $h = 0.01$ and $\hat{s}(0) = 1$.



(a) (b)

Fig. 3. Time evolutions of the consensus errors $e_i(t) = (e_{i1}(t), e_{i2}(t))$ without control input.



(a) (b)

Fig. 4. Time evolutions of the consensus errors $e_i(t) = (e_{i1}(t), e_{i2}(t))$.

5. Conclusions

The PNBC with nonidentical double ETMs for the consensus of switched MASs subject to deception attack is investigated in this article. A switching rule based on semi-Makrov process has been presented and the influence of deception attacks in transmission channel have been considered. To reduce the cost

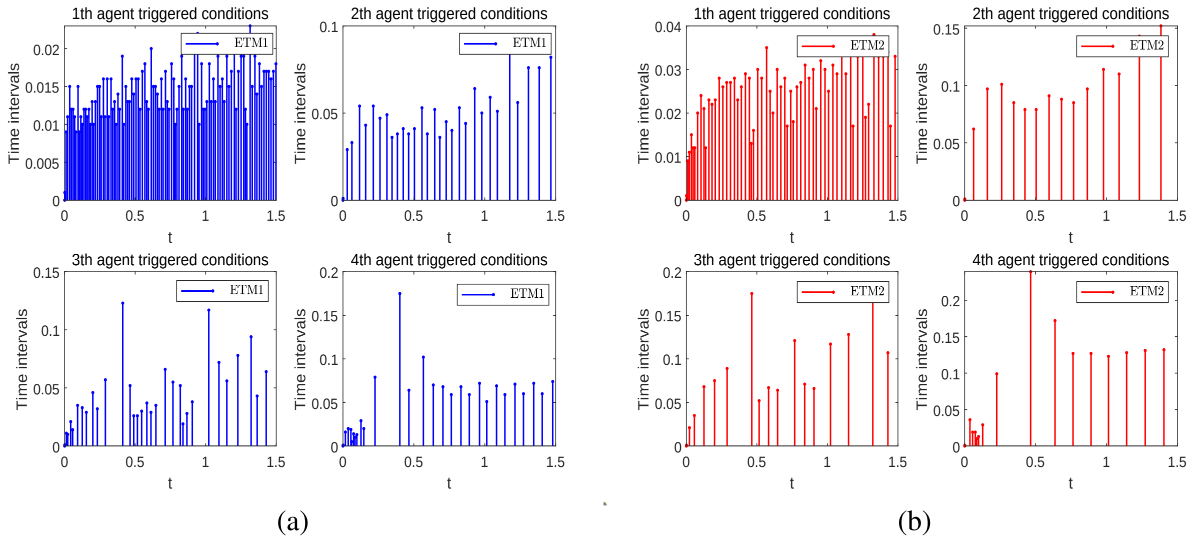


Fig. 5. (a) Triggering instants and intervals of each agent in ETM1, (b) Triggering instants and intervals of each agent in ETM2.

of communication, nonidentical double ETMs in S-C and C-A channels have been presented, in which the transmission conditions of each agent are nonidentical in order to satisfy different agent necessity. Moreover, for nonlinear function in MASs, OSL condition is considered to obtain less conservative results. Meanwhile, the PNBC that partial followers to obtain feedback from the leader has been presented. A numerical simulation has verified the viability of the proposed approach.

Appendix

$$
\begin{aligned}
K_1^1 &= \begin{pmatrix} 22.8285 & 4.5201 \\ 4.4630 & 22.8951 \end{pmatrix}, K_2^1 = \begin{pmatrix} 4.3978 & 0.3856 \\ 0.3871 & 4.3648 \end{pmatrix}, \\
K_3^1 &= \begin{pmatrix} 7.8723 & 0.9992 \\ 0.9909 & 7.9174 \end{pmatrix}, K_4^1 = \begin{pmatrix} 2.9326 & -0.0302 \\ -0.0244 & 2.8932 \end{pmatrix}, \\
K_1^2 &= \begin{pmatrix} 21.3246 & 4.8153 \\ 4.7513 & 21.4087 \end{pmatrix}, K_2^2 = \begin{pmatrix} 5.2754 & 0.4598 \\ 0.4558 & 5.2258 \end{pmatrix}, \\
K_3^2 &= \begin{pmatrix} 9.1741 & 1.0046 \\ 1.0023 & 9.2196 \end{pmatrix}, K_4^2 = \begin{pmatrix} 2.4177 & 0.0466 \\ 0.0476 & 2.3873 \end{pmatrix}, \\
K_1^3 &= \begin{pmatrix} 22.1040 & 2.9861 \\ 2.9589 & 21.9990 \end{pmatrix}, K_2^3 = \begin{pmatrix} 7.3204 & 1.9604 \\ 1.9271 & 7.3304 \end{pmatrix}, \\
K_3^3 &= \begin{pmatrix} 3.8718 & 1.0421 \\ 1.0338 & 3.9097 \end{pmatrix}, K_4^3 = \begin{pmatrix} 3.6992 & 0.1207 \\ 0.1202 & 3.6656 \end{pmatrix}.
\end{aligned}
$$

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