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Enhanced elastic beam model with BADS integrated for settlement assessment of immersed tunnels

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Abstract

Excessive settlement may induce structural damage and water leakage in immersed tunnels, seriously threatening the tunnels' safety. However, making accurate assessment of the settlement in immersed tunnels is difficult due to the incomplete knowledge of the geotechnical parameters and the inadequacy of the model itself. This paper proposes an effective method to accurately assess the settlement in immersed tunnels. An enhanced beam on elastic foundation model (E-BEFM) is developed for the settlement assessment, with the Bayesian adaptive direct search algorithm adopted to estimate unknown model parameters based on previous observations. The proposed method is applied to a field case of the Hong Kong–Zhuhai–Macao immersed tunnel. The original BEFM is used for comparison to highlight the better assessment performance of E-BEFM, particularly for joints' differential settlement. Results show that the proposed method can provide accurate predictions of the total settlement, angular distortion (a representation of tubes' relatively differential settlement), and joints' differential settlement, which consequently supports the associated maintenance decision-making and potential risk prevention for immersed tunnels in service.

Keywords: Immersed tunnel; Settlement; Beam on elastic foundation model; Bayesian adaptive direct search; Hong Kong–Zhuhai–Macao tunnel

1 Introduction

With the continuous development of construction technologies that overcome challenging conditions and minimize risks, immersed tunnels are received increasing applications worldwide (Hu et al., 2015; Olsen et al., 2022). Currently, more than 150 immersed tunnels have been built in the world (Zhang & Broere, 2023). Immersed tunnels comprise a series of tubes connected by immersion joints, which are sensitive to the longitudinal settlement. Several problems, such as concrete cracking, damage to joints, and leakage, are likely to arise as excessive settlement develops, which may significantly interfere with the normal operation of an immersed tunnel (Xie et al., 2014; Zhang & Broere, 2019). Therefore, accurate predictions of the settlement are vital for ensuring the safety of immersed tunnels.

Several methods have been developed to assess the settlement of immersed tunnels, including one-dimensional consolidation or compression analysis (Shao, 2003; Wei et al., 2014), finite element modeling (FEM) using commercial software (Ding et al., 2013; Xie et al., 2014; Zhao et al., 2018), and beam on elastic foundation model (BEFM) (Wei & Lu, 2018; Tang et al., 2022). However, although a careful geotechnical analysis can be conducted, making accurate assessment of the settlement in immersed tunnels is difficult due to the incomplete knowledge of the geotechnical parameters and the inadequacy of the model itself (Grantz, 2001; Zhao et al., 2015; Jin et al., 2019; Tao et al., 2022). Advantageously, the detailed field observations of geotechnical structures provide not only a better

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understanding of their mechanical responses but also useful information for improving the assessment accuracy in geotechnical analyses (Zhao et al., 2015; Jin et al., 2018, 2019; Zhang Yin, Jin, 2022, Zhang Yin, Jin, Yang, et al., 2022; Tao et al., 2022). The use of back analysis methods makes it realistic to obtain appropriate values for geotechnical parameters by properly extracting information from observations, particularly for parameters that are difficult to determine through laboratory and field tests (Jin, Yin, Wu, & Zhou, 2018; Jin, Yin, Zhou, & Huang, 2019; Tang, He, & Zhou, 2023; Yin & Jin, 2019; Zhou, Yin, & Yuen, 2021b). To promote the settlement assessment accuracy in immersed tunnels, a novel BEFM (O-BEFM) was developed recently and combined with a Bayesian inverse framework to estimate the settlement (Tang et al., 2022). This method was effective for predicting the total settlement and angular distortion (a representation of tubes' relatively differential settlement). However, the proposed BEFM was not adequate in providing a satisfactory assessment of the joints' differential settlement, although the model parameters were effectively identified. In practice, the stiffness of joints is designed to be much smaller than that of the tube body to attract large deformation at these locations to mitigate the stress concentrations of tubes induced by the differential settlement (Song et al., 2018; Lin et al., 2019; Zhang & Broere, 2023). Accordingly, the joints become the "weakness" of immersed tunnels to some extent, and the joints' differential settlement needs to be effectively assessed to prevent settlement-induced joint damage in immersed tunnels (Zhao et al., 2018). Along this way, an enhanced BEFM (E-BEFM) would be desirable to realize more accurate settlement assessments.

In addition to the assessment model, the assessment accuracy also heavily depends on the back analysis methods adopted (Zhao et al., 2015; Jin et al., 2018, 2019). Recently, a random search optimization algorithm called Bayesian adaptive direct search (BADS) has been developed, and its performance in parameter identification has been demonstrated by both benchmark tests and practical engineering problems (Acerbi & Ma, 2017; Zhang et al., 2021; Feng et al., 2022). In detail, this algorithm is a hybrid optimization algorithm that combines Bayesian optimization (BO) and mesh adaptive direct search (MADS) (Acerbi & Ma, 2017), which features a strong fitting ability and high computational efficiency. Thus, it will be attractive if this powerful algorithm can be applied to identify parameters of the E-BEFM based on observations.

This study aims to develop a feasible method to accurately predict the settlement in immersed tunnels, not only for the total settlement and angular distortion but also for the joints' differential settlement. An E-BEFM is established as the settlement assessment model based on the O-BEFM. Additionally, the BADS algorithm is used to estimate unknown model parameters in the E-BEFM using observations. The remainder of this paper is organized as follows. Section 2 presents the methodology by briefly introducing the proposed E-BEFM and the BADS algorithm. In Section 3, the performances of the O-BEFM and E-BEFM are demonstrated and systematically compared in the field case study of the Hong Kong–Zhuhai–Macao (HZM) immersed tunnel. Finally, Section 4 presents the conclusions.

2 Methodology

2.1 E-BEFM

As stated earlier, the E-BEFM is developed to assess the settlement of immersed tunnels. As this model is established by increasing the model complexity of the O-BEFM, the basics of the O-BEFM will be briefly introduced first. In the O-BEFM, an immersed tunnel is regarded as a series of beams joined together (Fig. 1(a)). The soil–structure interactions are simulated via a series of closely spaced independent springs, and the stiffness of these springs is the foundation modulus ($k_1 - k_{N+1}$). This model employs the linearly varying foundation moduli to consider the continuous variations of soil stiffness. The foundation moduli on both sides of the joints are assumed to be the same as the longitudinal length of the joints is much smaller compared to that of the tubes. Additionally, vertical springs are adopted to simulate the shear joints between adjacent tubes, and the stiffness of these vertical springs is the shear stiffness (k_s).

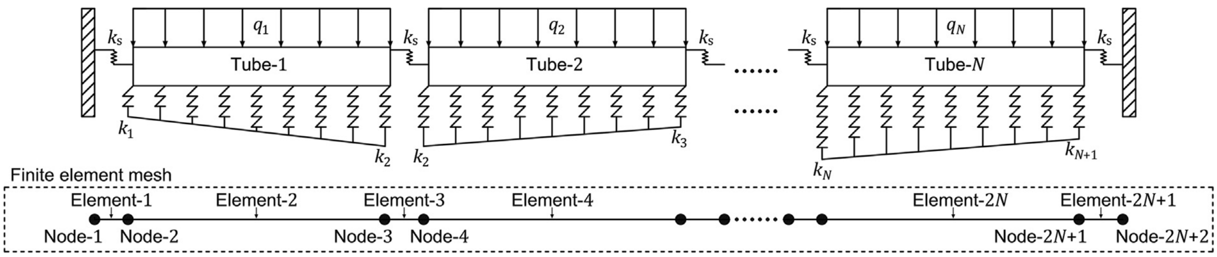
The model assumptions of the E-BEFM basically follow those of the O-BEFM, whereas the difference lies in the fact that independent foundation modulus parameters are assigned to the two sides of joints in the E-BEFM (Fig. 1(b)). Thus, the E-BEFM allows different foundation moduli on both sides of joints to better accommodate the differential settlement at joints. Additionally, this assumption enables the E-BEFM to better account for the complex variations of foundation modulus in immersed tunnels, especially for those tunnels located in highly variable soft soil layers or adopting various foundation treatment measures (Hu et al., 2018; Tang et al., 2022).

The governing equation of the E-BEFM is expressed as follows (Reddy, 2006; Tang et al., 2022, 2023):

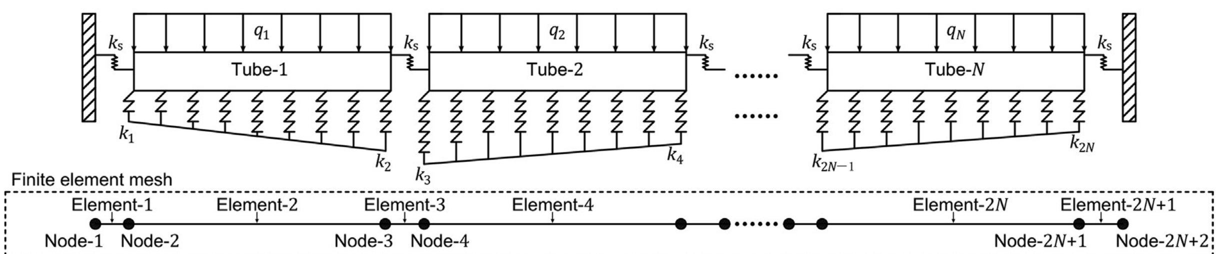
$$EI\frac{\mathrm{d}^4w}{\mathrm{d}x^4} = (q - kw)b, \tag{1}$$

where EI denotes the bending stiffness of the tubes; x is the coordinate along the tunnel alignment; w is the corresponding settlement at x; q is the load above the tunnel; k is the foundation modulus; and b is the tunnel width.

Since there is currently no reliable way to obtain the values of the foundation modulus $k_1 - k_{2N}$ and joints' shear stiffness k_s, these parameters are taken as unknowns and identified by the inverse analysis using observations. For an immersed tunnel with N tubes, the number of the unknown foundation modulus parameters is $2N$, and one additional unknown parameter should be assigned for the shear stiffness of joints. Thus, the total number of uncertain model parameters is $2N + 1$.



(a)



(b)

Fig. 1. Schematic of the BEFM for the settlement estimations of immersed tunnels: (a) O-BEFM proposed by Tang et al. (2022), and (b) E-BEFM proposed in this study.

A finite element solving algorithm is developed for the E-BEFM to compute the total settlement w. An immersed tunnel containing N tubes can be partitioned into $2N + 1$ beam elements. N beam elements are generated for tubes, while $N + 1$ beam elements are generated for shear joints. Details about this solving algorithm can be found in Tang et al. (2022).

Notably, the angular distortion δ and joints' differential settlement w_J are important references for judging the potential damage risks of immersed tunnels (Shao, 2003; Zhao et al., 2018; Tang et al., 2022; Zhang & Broere, 2023). Here, the angular distortion δ is a representation of the tubes' relatively differential settlement, which is defined as the ratio of the differential settlement w_T between two reference points on the tubes to their distance L ($\delta = w_T/L$) (Shao, 2003; Tang et al., 2022), as illustrated by Fig. 2.

2.2 BADS

Compared with the O-BEFM, more model parameters need to be determined from back analysis for the E-BEFM. Thus, the calculations can involve extremely high-dimensional optimization problems when considering a large-scale immersed tunnel with numerous tubes, and a powerful inverse algorithm is required for the calculations. The BADS algorithm is such an optimization algorithm and is employed herein to determine unknown model parameters. As noted previously, the BADS algorithm is a hybridization of BO and MADS (Acerbi & Ma, 2017). Thus, this section briefly reviews the basics of BO and MADS before introducing the BADS algorithm.

The BO is an iterative process for determining the extrema (minimum or maximum) of objective functions, especially when the functions are computationally expen-

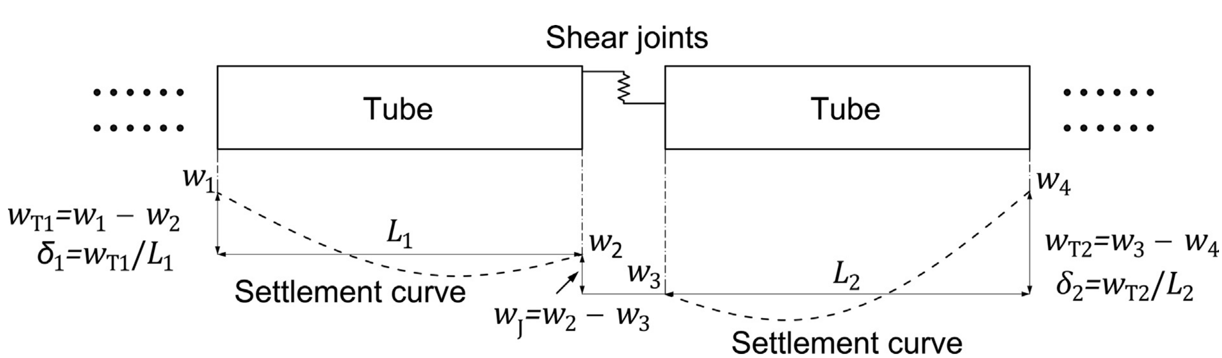


Fig. 2. Schematic of the angular distortion δ and joints' differential settlement w_J.

sive to evaluate (Frazier, 2018). BO is very good at obtaining the global extreme with a minimum number of trials and has been successfully applied in geotechnical problems (Kim, Kwon, Pham, Oh, & Choi, 2022; Li, Liu, Xiao, Zhou, & Armaghani, 2022; Zhang, Hu, Liu, & Tan, 2020; Zhou, Yin, & Yuen, 2021a). It typically comprises two steps. First, a surrogate function is established to approximate the objective function by combining the priors with the observations by applying the Bayes' theorem (Snoek et al., 2012). The surrogate function is usually obtained via the Gaussian Process (GP), wherein Gaussian priors are used due to their flexibility and tractability. Second, an acquisition function is applied to construct a utility function from the model posterior, which serves as guidance for the next sampling location (Snoek et al., 2012; Zhang et al., 2020). The next sampled point is where the acquisition function is maximized. The objective function observation at the next sampling point is then evaluated, and the new observation is augmented into GP to update the posterior of the surrogate function. This iterative search process is repeated until an optimal value is obtained (Zhang et al., 2020).

The MADS is a directional direct search method proposed by Audet and Dennis (2006). It includes two stages: the search stage and the poll stage. The search stage can be regarded as a process of global search in the entire parameter space for identifying the feasible region containing the local optimum, whereas the poll stage is a process of local search in the neighborhood of the best current solution to precisely find the optimal point (Audet & Dennis, 2006; Liu, 2018). In the search stage, the parameter space of variables is meshed with an initial mesh size. Moreover, the target values of a finite number of points at the current mesh are evaluated by a provided search strategy to determine a feasible solution for improving the objective function (Audet & Dennis, 2006; Zhang et al., 2020; Feng et al., 2022). If a point with an improved objective value is found, the grid center will be moved to this location and the search stage will be repeated. The poll stage is performed if the search stage fails to improve the objective function value. The search in each direction is performed with an iteration-dependent mesh until an improvement is achieved or all directions have been tried. Notably, this search is limited to a local region that is controlled by the poll size (Audet & Dennis, 2006; Liu, 2018). If the poll stage succeeds in improving the objective function, then the mesh center will be moved to the improvement point and a larger mesh size and poll size will be adopted in the next poll stage. Otherwise, a reduced mesh size and poll size will be used. The algorithm proceeds until a prescribed stopping criterion is met.

The idea behind the BADS algorithm is to utilize the surrogate models constructed in the BO process to assist in generating candidate solutions in the search and poll stages of the MADS (Zhang et al., 2020). Briefly, the BADS algorithm alternates between a series of fast and local BO steps (search stage) and a systematic and slow exploration of the mesh grid (poll stage) (Audet & Dennis, 2006). The parameter space can be effectively explored in the search stage, and an adequate surrogate model of the objective function can be constructed using GP. In the poll stage, the information provided by GP approximation is beneficial for choosing a suitable set of polling directions and polling order to improve the success rate of sample point selection and reduce the number of iterations. In contrast, the information about the local shape of the objective function can be collected in the poll stage to build an improved surrogate for the next search stage. Briefly, the two stages complement each other in such an alternation, enabling the BADS algorithm to deal with various optimization problems efficiently and robustly. Considering a D dimension minimization problem $\boldsymbol{\theta}^* = \arg\min f(\boldsymbol{\theta})(\boldsymbol{\theta} \in \mathbb{R}^D)$, the workflow of the BADS algorithm can be described as follows (Audet & Dennis, 2006; Zhang et al., 2020):

(1) The algorithm starts from a starting point z_0, and the initial mesh size (S_0^{mesh}) and poll size (S_0^{poll}) are set to 2^{-10} and 1.0, respectively.
(2) In the k th iteration, a fast approximate optimization of the chosen acquisition function in the neighborhood of the incumbent $\boldsymbol{\theta}_k$ is performed to generate the candidate solution $\boldsymbol{\theta}_{\text{search}}$ (Hansen et al., 2003).
(3) $f(\boldsymbol{\theta}_{\text{search}})$ is evaluated, and if the improvement is sufficient, then the algorithm terminates. If the search stage is not successful, the iteration skips to the poll stage.
(4) In the poll stage, a set of polling directions $\boldsymbol{D}_k$ is generated and proportionally rescaled to the current GP length scale (Audet & Dennis, 2006). Additionally, the order of points evaluated in the poll set is based on the ranking given by the acquisition function.
(5) If the poll stage succeeds in sufficiently improving the objective function, the incumbent is updated and the BADS algorithm switches to a new iteration with mesh and poll sizes multiplied by τ. τ is the adjustment coefficient of the mesh and poll sizes, which has a default value of 2 and can be adjusted in subsequent calculations. Otherwise, the incumbent remains unchanged and the BADS algorithm switches to a new iteration with the mesh and poll sizes divided by τ. These steps are repeated until a preset maximum number of iterations is reached, the poll size becomes extremely small, or no sufficient improvement in the objective function is observed after a prescribed number of continuous iterations.

2.3 Flowchart

By combining the E-BEFM and BADS algorithm, the flowchart of the proposed method for the settlement assessment of immersed tunnels is as follows (Fig. 3):

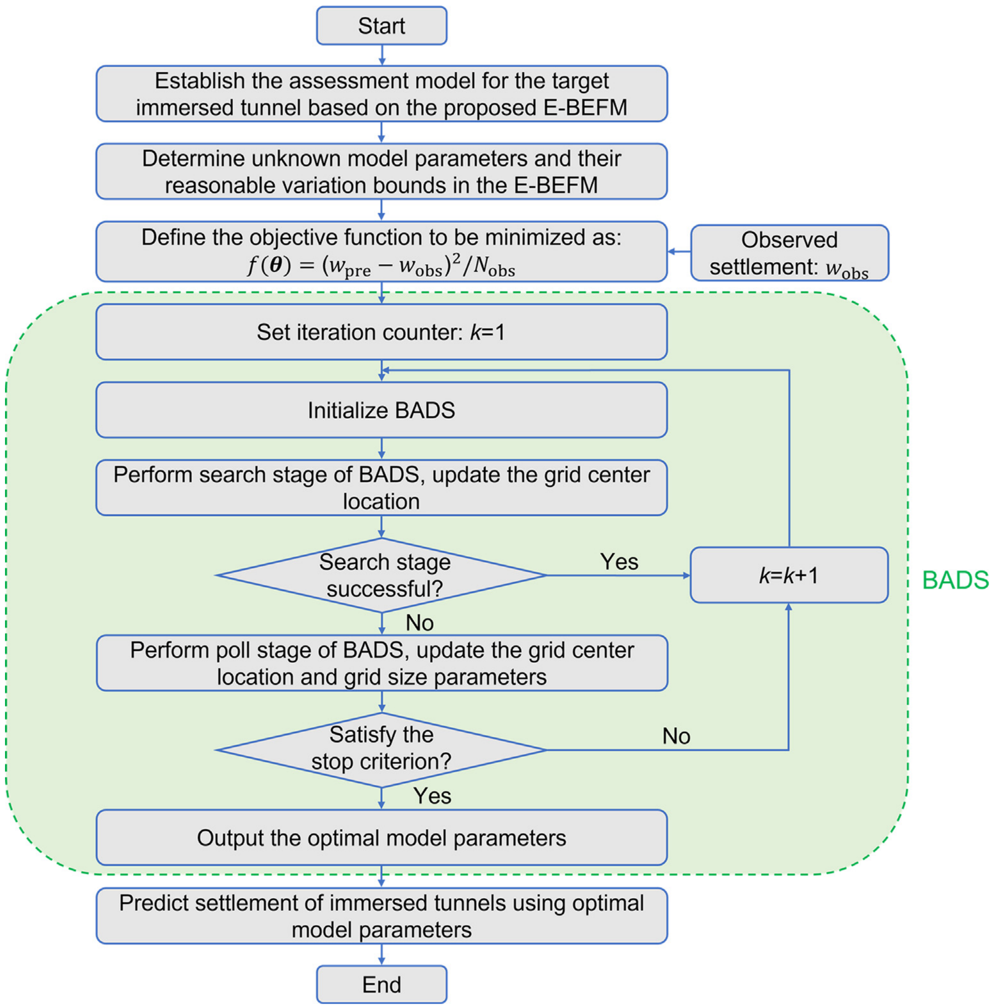


Fig. 3. Flowchart of the proposed method.

(1) Collect the necessary engineering data, such as tunnel geometry and structural parameters. Establish the assessment model for the immersed tunnels based on the proposed E-BEFM.
(2) Acquire the reasonable prior variation bounds of unknown model parameters, i.e., foundation modulus and joints' shear stiffness.
(3) Define the objective function to be minimized as follows:

$$f(\boldsymbol{\theta}) = (w_{\text{pre}} - w_{\text{obs}})^2/N_{\text{obs}}, \tag{2}$$

where $\boldsymbol{\theta}$ is the vector of unknown parameters; w_{obs} is the observed settlement; w_{pre} is the corresponding predicted settlement at observed points; and N_{obs} denotes the total number of observed points.

(4) Execute the BADS algorithm to minimize the objective function to obtain the optimal values of unknown model parameters.
(5) Perform forward calculation to obtain the total settlement, angular distortion, and joints' differential settlement of immersed tunnels using the E-BEFM with optimal model parameters.

3 Field case study of HZM tunnel

The HZM linkage is one of China's largest infrastructure projects, which connects Hong Kong and Zhuhai/Macao and crosses the Pearl River Estuary in southern China (Hu et al., 2018; Song et al., 2018; Yu et al., 2018). The HZM immersed tunnel is a part of the HZM linkage, which comprises 33 underwater tubes and 2 buried sections, including 35 tubes in total (China Communications Construction Co., Ltd., 2012). Table 1 lists the length of each tube. The total length of the HZM tunnel reaches 6087.3 m. Figure 4 shows the geological and foundation profile of the immersed tunnel (China Communications Construction Co., Ltd., 2012). Most tubes rest on soft layers. Additionally, several treatment measures such as prestressed high-strength concrete (PHC) pipe piles, high-pressure jet grouting piles, and sand compaction piles (SCP) with varied replacement rates were

Table 1
Lengths of tubes for the HZM tunnel.

Tube	BW	E1, E2	E3–E26, E31	E27, E28	E29, E30	E32, E33	BE
Length (m)	193	112.5	180	157.5	177	135	230.7

Note: E1–E33 denote the tubes from the west artificial island to the east artificial island. BW and BE denote the buried sections at the west and east ends, respectively.

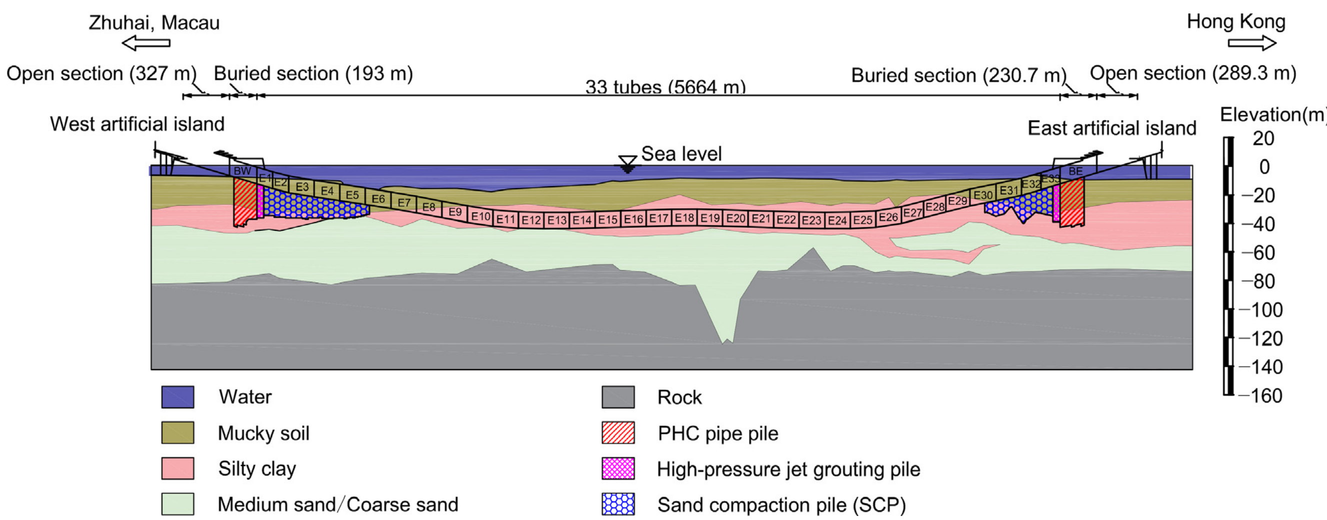


Fig. 4. Geological and foundation profile of the HZM tunnel.

used for the tubes that were close to the artificial islands to prevent the significant settlement in the HZM tunnel (Hu et al., 2018; Tang et al., 2022). This further complicated the variations in foundation stiffness along the tunnel alignment. The risk of structural damage due to the differential settlement is high for the HZM tunnel due to high and uneven loads caused by back siltation accumulations and future channel dredging, uneven foundation stiffness, and the combination of the two (Lin et al., 2019). Additional construction details of the HZM tunnel can be found in Hu et al. (2015, 2018), Song et al. (2018), Wang et al. (2018), and Yu et al. (2018).

3.1 Parameter settings

The settlement assessment model for the HZM tunnel is established based on the E-BEFM, as illustrated in Section 2. The bending stiffness is 1.05×10^{11} kN·m^2 and the tunnel width is 37.95 m. Totally, 71 beam elements are generated, with 35 beam elements for tubes and 36 beam elements for joints. The total number of unknown parameters is 71, including the foundation modulus $k_1 - k_{70}$ and shear stiffness k_s. Table 2 summarizes the bounds of these unknown parameters, where the corresponding variation ranges are determined based on site investigations, lab tests, and engineering experience (Li, 2013; Steenfelt et al., 2013; Lu, 2018; Song et al., 2018). To demonstrate the performance of the proposed E-BEFM, the O-BEFM is also established for comparisons.

Table 2
Bounds of unknown parameters.

Bounds	$k_1 - k_{70}$(kPa/m)	k_s(kN/m)
Lower bound	1.00×10^2	1.00×10^5
Upper bound	5.00×10^3	1.20×10^7

The total number of unknown parameters in the O-BEFM is 37 (Fig. 1(a)), and the corresponding bounds of these unknown parameters are the same as those of the E-BEFM (Tang et al., 2022).

In this study, the six most recent sets of data available, namely settlements and corresponding loads observed from July 2019 to November 2020, are used for analysis. The observation points are located at both ends of each tube, so there are a total of 70 observation points for 35 tubes. The latest two sets of measurement data (data from September 2020 and November 2020) are employed as test sets to verify the effectiveness of the developed assessment model, and the remaining four sets of data (data from July 2019 to April 2020) are employed as training sets. For convenience, the data from September 2020 and November 2020 are denoted as test set-1 and test set-2 herein.

The total load used in this study comprises the self-weight of the tunnel body, backfill load, back siltation load, and building weight above the buried sections. The back siltation load is considered as a variable load, which varies with the back siltation thickness above the tunnel (Tang et al., 2022).

3.2 Assessment performance and comparisons

Using the BADS algorithm, the optimal values of unknown model parameters can be obtained for the established E-BEFM of the HZM tunnel. Figure 5 presents the variations of the calculated foundation moduli along the cumulative length of the tunnel alignment. The foundation modulus ranges from 1.00×10^2 to 4.56×10^3 kPa/m. Additionally, the shear stiffness of joints is computed as 4.49×10^5 kN/m.

Using these back-analyzed model parameters, settlement predictions can be made under different load conditions. Figure 6 presents the ratios of the predicted total settlement to the observed total settlement (w_{pre}/w_{obs}), with the test set-1 as an example. Notably, the performance of the O-BEFM is also presented herein for systematic comparison. Although the calculated ratios of the two models are all centralized around 1.0, the ratios of the E-BEFM are obviously closer to 1.0. To quantify the assessment performance, the coefficient of determination (R^2) for the total settlement is calculated and listed in Table 3. The R^2 values of the E-BEFM exceed 97% for both test sets, while those of the O-BEFM are around 90%. Overall, this comparison indicates that the proposed E-BEFM yields better prediction results of the total settlement compared with the O-BEFM.

Figure 7 presents the comparison of the angular distortion, where both the predicted and observed angular distortions are normalized using the maximum observed angular distortion. Unsurprisingly, the E-BEFM still performs better than the O-BEFM. The R^2 for test set-1 increases from 86.16% to 99.09%, while that for test set-2 increases from 85.49% to 98.98%.

In terms of the joints' differential settlement, which is the focus of this study, the comparison between predictions and observations is presented in Fig. 8. Notably, both pre-

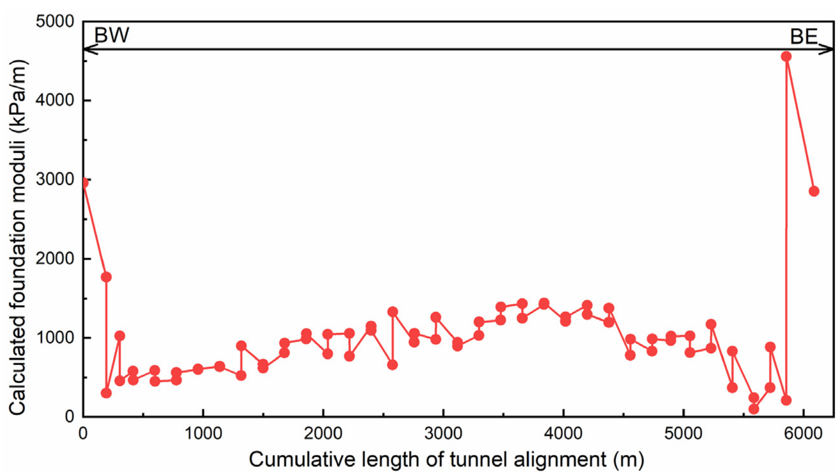


Fig. 5. Calculated foundation moduli along the cumulative length of tunnel alignment.

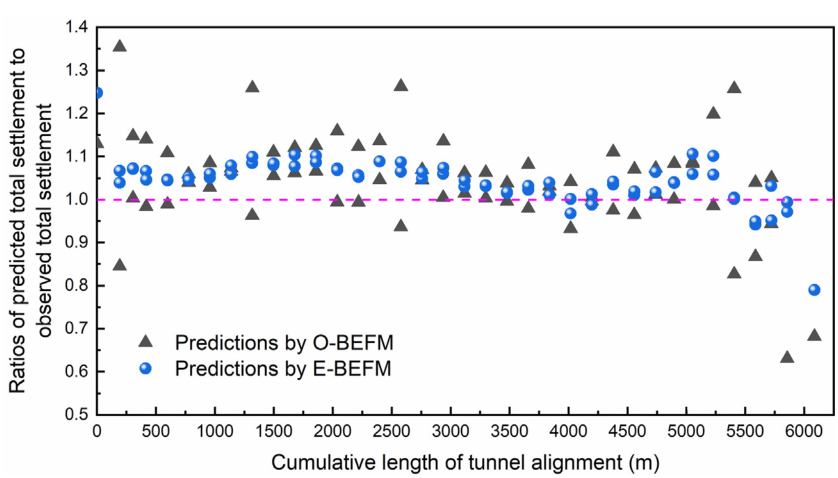


Fig. 6. Comparisons of total settlement w (Test set-1).

Table 3
Coefficient of determination (R^2) for two test sets.

Settlement indicator	R^2 (%)			
	Test set-1 (September 2020)		Test set-2 (November 2020)	
	O-BEFM	E-BEFM	O-BEFM	E-BEFM
w	89.43	97.92	90.47	99.05
δ	86.16	99.09	85.49	98.98
w_J	10.21	97.95	13.66	98.06

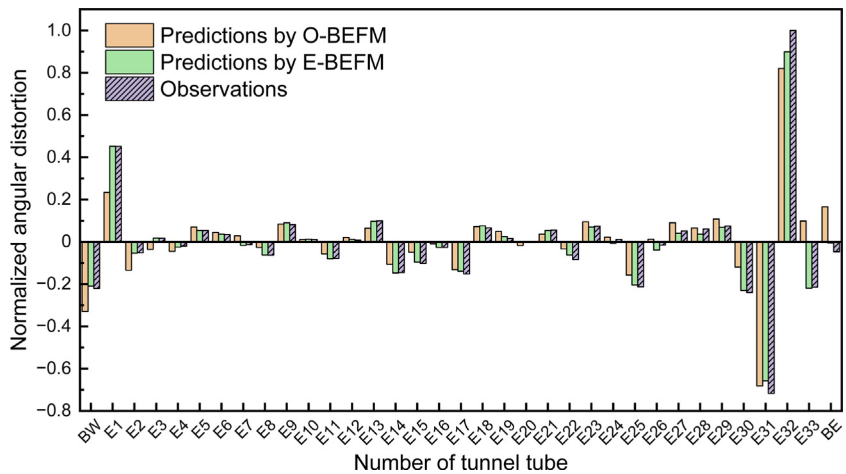


Fig. 7. Comparisons of angular distortion δ (Test set-1).

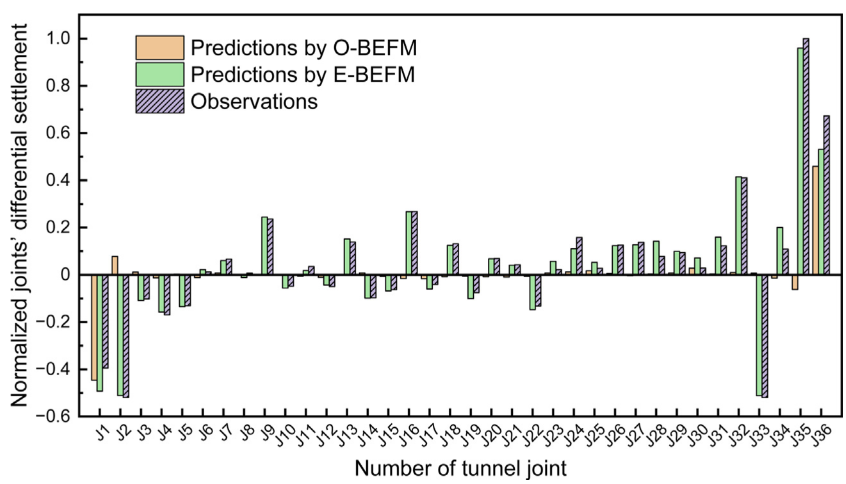


Fig. 8. Comparisons of joints' differential settlement w_J (Test set-1).

dictions and observations are normalized using the maximum observed joints' differential settlement, and J1 to J36 denote the immersion joints from tubes BW to BE. Predictions made by the E-BEFM are consistent with observations, whereas the predictions made by the O-BEFM deviate from observations. Quantitatively, the R^2 values corresponding to the O-BEFM for the two test sets are below 14% (Table 3), indicating that satisfactory assessment accuracy cannot be achieved by this model. In contrast, the R^2 values of the E-BEFM are all larger than 97%. Thus, the proposed E-BEFM is more effective than the O-BEFM in the joints' differential settlement assessments.

4 Conclusion

In this paper, an E-BEFM extended from the O-BEFM was established for settlement assessments of immersed tunnels, which allowed different foundation moduli at both sides of joints to better accommodate the differential settlement at joints. To ensure effective parameter identifications in the E-BEFM, the BADS algorithm was employed as the back analysis framework. The performances of the M−BEFM and O-BEFM were systematically compared in the field case study of the HZM immersed tunnel. Results showed that the BADS algorithm was sufficient to make good inverse estimations of model parameters. The proposed E-BEFM was able to: (1) realize the accurate assessment of joint differential settlement ($R^2 \approx 0.98$) and (2) significantly promote the assessment accuracy for the total settlement (from $R^2 \approx 0.90$ to $R^2 \approx 0.98$) and angular distortion (from $R^2 \approx 0.86$ to $R^2 \approx 0.99$) compared with the O-BEFM. Overall, this study provides a practical tool for assessing the settlement of immersed tunnels, which consequently supports the associated maintenance decision-making and potential risk prevention.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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