Nonlinear Viscoelasticity and Viscoplasticity Characteristics of Virgin and Modified Asphalt Binders

Hui Li, Ph.D.
1, School of Transportation, Southeast University, Nanjing, 211189, China.
2, Department of Civil and Environmental Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
Email: huili94@seu.edu.cn

Jian Ling
College of Civil Engineering and Architecture
Zhejiang University
866 Yuhangtang Road,
Hangzhou 310058, Zhejiang, China
Email: 22012303@zju.edu.cn

Zhen Leng, Ph.D.
Department of Civil and Environmental Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
Email: zhen.leng@polyu.edu.hk

Yuqing Zhang, Ph.D.
School of Transportation
Southeast University
Nanjing, 211189, China.
Email: zhangyuqing@seu.edu.cn

Xue Luo, Ph.D.
College of Civil Engineering and Architecture
Zhejiang University
866 Yuhangtang Road,
Hangzhou 310058, Zhejiang, China
Phone: (86) 571-88206542
Email: xueluo@zju.edu.cn

Abstract

Many engineering materials have coupled nonlinear viscoelasticity and viscoplasticity, which are affected by complex thermo-mechanical loadings. This study aims to address the challenge of accurately separating the viscoplasticity and the nonlinear viscoelasticity and formulate the viscoplasticity by considering the effects of temperatures and loading levels. First, the nonlinear viscoelastic constitutive equation is adopted to accurately separate the viscoplasticity and the nonlinear viscoelasticity. Then, a kinetics-based viscoplastic model and a new viscoplastic activation energy indicator are proposed to consider the effects of the temperature and loading level on the viscoplasticity. As typical nonlinear viscoelastic viscoplastic materials commonly used in pavement engineering, asphalt binders are selected to demonstrate the principles in this study. It was found that the proportion of the viscoplastic strain is larger than the nonlinear viscoelastic strain for virgin asphalt binders (VB) and it increases with the temperature, while high-viscosity modified asphalt binders (HVB) and rubber asphalt binders (RB) are opposite. The logarithm of the viscoplastic strain rate is linearly increased with the reciprocal of the temperature, and the viscoplastic strain rates at different temperatures are correlated and can be predicted based on the established viscoplastic strain kinetics model. The viscoplastic activation energy indicator can characterize the viscoplastic deformation resistance for nonlinear viscoelastic viscoplastic materials, and the order of the viscoplastic deformation resistances of the three binders is: VB<HVB<RB, based on this indicator.

Keywords: viscoplasticity; nonlinear viscoelasticity; kinetics-based viscoplastic model; viscoplastic activation energy

1 Introduction

Many engineering materials have coupled nonlinear viscoelasticity and viscoplasticity, which are affected by complex thermo-mechanical loadings, and the viscoplasticity will affect the service performance of the structures where these materials are used. Hence, it is necessary to investigate and model the viscoplasticity by considering the influence of the temperature and loading level on engineering materials. A representative nonlinear viscoelastic viscoplastic material, asphalt binder, widely used in pavement engineering, is selected for investigation in this study. Permanent deformation in the asphalt layer is one of the main diseases of asphalt pavements, which is mainly caused by the viscoplasticity of asphalt materials at a high temperature and overloading. The permanent deformation, also known as rutting, of asphalt pavements, can shorten the service life of asphalt pavements, compromise ride quality, and more seriously, bring safety hazards. It should be pointed out that the mechanical properties of asphalt binders play an important role in the rutting performance of asphalt mixtures and pavements (D'Angelo 2009; Tabatabaee & Tabatabaee 2010; Laukkanen et al., 2015; Walubita et al., 2022; Singh & Kumar., 2022).

Many indicators have been developed to indicate the viscoplastic performance of asphalt binders, such as rutting factor $G^*/\sin\delta$ (G^* is shear modulus, δ is phase angle) (Kennedy et al.,1994), zero shear viscosity (Sybilski, D. 1996), low shear viscosity (Morea et al., 2011) and the viscous component of the creep stiffness (Bahia et al., 2001). These indicators have been successfully applied to different types of asphalt binders (Lu et al., 2023; Shirzad et al., 2023; Padhan et al., 2020; Leng et al., 2018a; 2018b). However, these indicators have a limitation that they were established by only considering the linear viscoelastic property of asphalt binders, while many asphalt binders may have an obvious nonlinear viscoelastic characteristic, and the impact of nonlinear viscoelasticity of asphalt binder cannot be negligible (Tsantilis et al. 2021; Delgadillo et al., 2012; Masad et al. 2009; Nivitha et al., 2018; Sun et al., 2020).

Non-recoverable creep compliance is an indicator which can characterize the viscoplastic deformation of asphalt binders (D'Angelo et al., 2009). It should be noted that the non-recoverable creep compliance indicator includes the influence of the nonlinear viscoelasticity of asphalt binders. The standard non-recoverable creep compliance indicator can be determined through the creep and

recovery test (AASHTO 2014), which consists of two stress levels (0.1 and 3.2kPa). Each stress level has 10 load-and-recovery cycles, and in each cycle, creep is applied for 1.0 second followed by 9.0 seconds rest period. The indicator can be determined by the strain at the end of the recovery phase, i.e., using this strain as the non-recoverable strain of the asphalt binders. However, the accumulation creep strains of the asphalt binder cannot be recovered in a short time, and Delgadillo et al., 2012 reported that the strain recovery rate is 98% for the modified asphalt binder after 1,000 s recovery when the test temperature is 46℃. In practice, the nonlinear viscoelastic strain of asphalt binders cannot be completely recovered in a finite time. Therefore, it is not feasible to obtain the precise non-recoverable creep compliance for all asphalt binders based on the test results of the creep and recovery tests.

Schapery proposed a nonlinear viscoelastic constitutive equation by irreversible thermodynamic descriptions of the state for nonlinear viscoelastic materials (Schapery 1966; 1969). The problem of the nonlinear viscoelasticity of asphalt binders cannot be completely recovered in a finite time in the creep and recovery phase can be solved according to Schapery's pioneering work. Some researchers employed Schapery's nonlinear viscoelastic constitutive equation to obtain the nonlinear viscoelastic and non-recoverable viscoplastic strain for asphalt binders. Masad et al. (2009) separated the recoverable (nonlinear viscoelastic) strain from the non-recoverable (viscoplastic) strain and determined the nonlinear viscoelastic parameters for the asphalt binders based on Schapery's nonlinear viscoelastic constitutive equation. Sadeq et al. (2016) analyzed the linear and nonlinear viscoelastic and viscoplastic strain of the asphalt binders with warm mix asphalt additives according to the research of Masad et al. (2009). Liu et al. (2021) analyzed the steady-state strain response under the stress level of 0.1 and 3.2 kPa by Schapery's nonlinear viscoelastic constitutive equation and analyzed the test results of the asphalt binders with improved accuracy. In these studies, the non-recoverable creep compliance of asphalt binders was redefined by the viscoplastic strain obtained from Schapery's nonlinear viscoelastic constitutive equation. These studies have great significance to accurately characterize the viscoplastic performance of asphalt binders, but the effect of temperature on the asphalt binders is not considered in these studies.

The mechanical behavior of nonlinear viscoelastic viscoplastic materials is closely related to

125 temperature and loading level. It will gradually soften as the temperature increases, and large
126 amounts of energy is needed to drive viscoplastic deformation. On the contrary, when the materials
127 become stiff with the reduction of the temperature, the energy required to produce viscoplastic
128 deformation is reduced. Besides, the evolution rate of viscoplastic deformation is certainly affected
129 by the temperature. The high loading level will also increase the viscoplastic deformation of the
130 materials, while the low loading level is the opposite. Therefore, the temperatures and loading levels
131 are very significant factors for viscoplasticity. To address this issue, the objective of this study is to
132 investigate the viscoplasticity coupled with nonlinear viscoelasticity and propose a new evaluation
133 indicator of the viscoplasticity that considers the effects of the temperatures and loading levels for
134 the nonlinear viscoelastic viscoplastic materials, and asphalt binders (typical nonlinear viscoelastic
135 viscoplastic materials) are selected to demonstrate the principles.

136 To achieve the objective of this study, the following research tasks were conducted. First, a
137 nonlinear viscoelastic viscoplastic constitutive model is established, and a kinetics-based viscoplastic
138 model and a new viscoplastic activation energy indicator for nonlinear viscoelastic viscoplastic
139 materials were proposed. Then, three types of asphalt binders are selected to demonstrate the
140 principles. Finally, analysis was conducted on the mechanical model parameters (Prony series
141 coefficients and nonlinear viscoelastic parameters), the nonlinear viscoelasticity and viscoplasticity,
142 the kinetics-based viscoplastic model and viscoplastic activation energy indicator.

143

144 **2 Mechanical and kinetics modeling for nonlinear viscoelasticity and viscoplasticity**

145 Boltzmann's superposition principle is widely used to describe the linear viscoelastic constitutive
146 relationship of materials with a time-dependent response (Brinson & Brinson 2008), and a general
147 three-dimensional linear viscoelastic constitutive model may be written as follows:

148 $$\varepsilon_{ij}^{ve} = D_{ijkl}(0)\sigma_{ij} + \int_{0^-}^{t} \Delta D_{ijkl}(t-\tau)\frac{\partial \sigma_{ij}}{\partial \tau} d\tau \quad (1)$$

149 where ε_{ij}^{ve} and σ_{ij} are linear viscoelastic strain tensor and stress tensor of the materials,
150 respectively; $D_{ijkl}(0)$ is instantaneous compliance tensor; ΔD_{ijkl} is transient compliance

component tensor; and τ is the arbitrary time between 0 and *t*.

Considering that some engineering materials show obvious nonlinear viscoelastic characteristics, Schapery (1966, 1969) developed a nonlinear viscoelastic constitutive equation for the nonlinear viscoelastic materials which show the nonlinear viscoelastic characteristics based on irreversible thermodynamic descriptions of the state, and the three-dimensional nonlinear viscoelastic constitutive equation can be expressed by:

$$\varepsilon_{ij}^{nve} = g_0 D_{ijkl}(0)\sigma_{ij} + g_1 \int_{0^-}^{t} \Delta D_{ijkl}(\psi - \psi') \frac{\partial g_2 \sigma_{ij}}{\partial \tau} d\tau \tag{2}$$

where ε_{ij}^{nve} is nonlinear viscoelastic strain tensor of the materials; g_0, g_1 and g_2 are nonlinear viscoelastic parameters of the materials, which due to third and higher-order dependence of Gibb's free energy on the applied stress; ψ and ψ' are reduced-time, which are defined by:

$$\psi = \psi(t) = \int_{0^-}^{t} dt / a_\sigma(\sigma_{ij}) \tag{3}$$

$$\psi' = \psi(\tau) = \int_{0^-}^{\tau} dt / a_\sigma(\sigma_{ij}) \tag{4}$$

in which a_σ is shift factor. In general, the shift factor results from similar higher-order effects of entropy production and free energy.

It should be noted that the nonlinear viscoelastic constitutive equation for nonlinear viscoelastic materials degrades to the linear viscoelastic constitutive equation when these nonlinear viscoelastic parameters are equal to one, i.e., $g_0 = g_1 = g_2 = a_\sigma = 1$. In this study, creep and recovery tests are performed and two-step stress (loading step and unloading step) are contained. To more generally model the nonlinear viscoelastic characteristics, general two-step stress tensor is selected to study the nonlinear viscoelastic constitutive equation for nonlinear viscoelastic materials, and two-step stress tensor can be expressed by:

$$\sigma_{ij}(t) = \sigma_p H(t) I_{ij} - (\sigma_p - \sigma_q) H(t - t_p) I_{ij} \tag{5}$$

where σ_p and σ_q are magnitudes of two arbitrary constant stresses tensor $\sigma_p I_{ij}$ and $\sigma_q I_{ij}$, respectively; I_{ij} is unit second order tensor; t_p represents the moment when the $\sigma_p I_{ij}$ is

unloading; $H(t)$ is Heaviside unit-step function, which can be defined as below：

$$H\left(t-t_q\right)=\begin{cases}0 & t\le t_q\\ 1 & t>t_q\end{cases} \tag{6}$$

For two constant stress tensors, the nonlinear viscoelastic parameters are also constant at respective loading time. Substituting Eqs. (5) and (6) into the nonlinear viscoelastic constitutive equation (Eq. (2)), the strain response of the nonlinear viscoelastic materials can be expressed as below:

$$\begin{aligned}\varepsilon_{ij}^{nve}&=g_0^p\sigma_p H(t)D_{ijkl}(0)I_{ij}+\left(g_0^q\sigma_q-g_0^p\sigma_p\right)H\left(t-t_p\right)D_{ijkl}(0)I_{ij}\\&+g_1\int_{0^-}^{t}\Delta D_{ijkl}(\psi-\psi')\frac{d}{d\tau}\left(g_2^p\sigma_p H(\tau)I_{ij}+\left(g_2^q\sigma_q-g_2^p\sigma_p\right)H\left(\tau-t_p\right)I_{ij}\right)d\tau\end{aligned} \tag{7}$$

where g_0^p, g_1^p, g_2^p and a_σ^p are nonlinear viscoelastic parameters of $\sigma_p I_{ij}$; g_0^q, g_1^q, g_2^q and a_σ^q are nonlinear viscoelastic parameters of $\sigma_q I_{ij}$.

Eq. (7) can be divided into two parts, and the form of Eq. (7) can be written as follows:

$$\begin{aligned}\varepsilon_{ij}^{nve}&=g_0^p\sigma_p H(t)D_{ijkl}(0)I_{ij}+\left(g_0^q\sigma_q-g_0^p\sigma_p\right)H\left(t-t_p\right)D_{ijkl}(0)I_{ij}\\&+g_1^p\int_{0^-}^{t_p}\Delta D_{ijkl}(\psi-\psi')\left(g_2^p\sigma_p I_{ij}\frac{dH(\tau)}{d\tau}+\left(g_2^q\sigma_q-g_2^p\sigma_p\right)I_{ij}\frac{dH\left(\tau-t_p\right)}{d\tau}\right)d\tau\\&+g_1^q\int_{t_p}^{t}\Delta D_{ijkl}(\psi-\psi')\left(g_2^p\sigma_p I_{ij}\frac{dH(\tau)}{d\tau}+\left(g_2^q\sigma_q-g_2^p\sigma_p\right)I_{ij}\frac{dH\left(\tau-t_p\right)}{d\tau}\right)d\tau\end{aligned} \tag{8}$$

where $\frac{dH(\tau)}{d\tau}=\delta(\tau)$, $\frac{dH\left(\tau-t_p\right)}{d\tau}=\delta\left(\tau-t_p\right)$; and $\delta\left(\tau-t_p\right)$ is Dirac delta function, which has the following expression:

$$\begin{cases}\delta\left(\tau-t_p\right)=0 & t\ne t_p\\ \int_{-\infty}^{+\infty}\delta\left(\tau-t_p\right)d\tau=1\end{cases} \tag{9}$$

Ignoring the term whose value is zero, the expression can be simplified as follows:

$$\begin{aligned}\varepsilon_{ij}^{nve}&=g_0^p\sigma_p H(t)D_{ijkl}(0)I_{ij}+\left(g_0^q\sigma_q-g_0^p\sigma_p\right)H\left(t-t_p\right)D_{ijkl}(0)I_{ij}\\&+g_1^p\int_{0^-}^{t_p}\Delta D_{ijkl}(\psi-\psi')g_2^p\sigma_p I_{ij}\delta(\tau)d\tau+g_1^q\int_{t_p}^{t}\Delta D_{ijkl}(\psi-\psi')\left(g_2^q\sigma_q-g_2^p\sigma_p\right)I_{ij}\delta\left(\tau-t_p\right)d\tau\end{aligned} \tag{10}$$

When $0\le t\le t_p$, only the constant stress tensor $\sigma_p I_{ij}$ is applied for the materials. Eq. (10) can be

written as:

$$\varepsilon_{ij}^{nve} = g_0^p \sigma_p H(t) D_{ijkl}(0) I_{ij} + g_1^p \int_{0^-}^{t_p} \Delta D_{ijkl}(\psi - \psi') g_2^p \sigma_p I_{ij} \delta(\tau) d\tau \tag{11}$$

It can be seen from Eq. (11) that the values of $\delta(\tau)$ are zero except for $\tau = 0$. Therefore, the reduced time can be calculated by the following expressions:

$$\psi = \psi(t) = \int_{0^-}^{t} dt / a_\sigma(\sigma_{ij} = \sigma_p I_{ij}) = \frac{t}{a_\sigma^p} \tag{12}$$

$$\psi' = \psi(\tau) = \int_{0}^{\tau} dt / a_\sigma(\sigma_{ij} = \sigma_p I_{ij}) = \frac{\tau}{a_\sigma^p} = 0 \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (10), the following simplified expression of strain can be obtained:

$$\varepsilon_{ij}^{nve} = \left[g_0^p \sigma_p D_{ijkl}(0) I_{ij} + g_1^p g_2^p \Delta D_{ijkl}\left(\frac{t}{a_\sigma^p}\right) \sigma_p I_{ij} \right] H(t), \quad 0 \le t \le t_p \tag{14}$$

Similarly, when $t > t_p$, the constant stress tensor $\sigma_q I_{ij}$ is applied to the materials, Eq. (10) can be rewritten as:

$$\begin{aligned} \varepsilon_{ij}^{nve} &= g_0^q \sigma_q H(t - t_p) D_{ijkl}(0) I_{ij} + g_1^q \int_{0^-}^{t_p} \Delta D_{ijkl}(\psi - \psi') g_2^p \sigma_p I_{ij} \delta(\tau) d\tau \\ &+ g_1^q \int_{t_p}^{t} \Delta D_{ijkl}(\psi - \psi')(g_2^q \sigma_q - g_2^p \sigma_p) I_{ij} \delta(\tau - t_p) d\tau \end{aligned} \tag{15}$$

Eq. (15) contains two integral terms: the first integral term is caused by the constant stress tensor $\sigma_p I_{ij}$ at $t = 0$ and the second integral term is affected by the constant stress tensor $\sigma_q I_{ij}$ at $t = t_p$. Thus, the reduce-time of the first integral term can be determined as follows:

$$\psi = \psi(t) = \int_{0^-}^{t_p} dt / a_\sigma(\sigma_{ij} = \sigma_p I_{ij}) + \int_{t_p}^{t} dt / a_\sigma(\sigma_{ij} = \sigma_q I_{ij}) = \frac{t_p}{a_\sigma^p} + \frac{t - t_p}{a_\sigma^q} \tag{16}$$

$$\psi' = \psi(\tau) = \int_{0}^{\tau} dt / a_\sigma(\sigma_{ij} = \sigma_p I_{ij}) = \frac{\tau}{a_\sigma} = 0 \tag{17}$$

In addition, the reduce-time of the second integral term can be calculated by:

$$\psi = \psi(t) = \int_{t_p}^{t} dt / a_\sigma\left(\sigma_{ij} = \sigma_q I_{ij}\right) = \frac{t - t_p}{a_\sigma^q} \tag{18}$$

$$\psi' = \psi(\tau) = \int_{t_p}^{\tau} dt / a_\sigma\left(\sigma_{ij} = \sigma_q I_{ij}\right) = \frac{\tau - t_p}{a_\sigma^q} = 0 \tag{19}$$

Finally, substituting Eqs. (16), (17), (18), (19) into Eq. (15), the nonlinear viscoelastic strain of the nonlinear viscoelastic materials can be present by:

$$\varepsilon_{ij}^{nve} = g_0^q \sigma_q D_{ijkl}(0) I_{ij} + g_1^q g_2^p \sigma_p \Delta D_{ijkl}\left(\frac{t_p}{a_\sigma^p} + \frac{t - t_p}{a_\sigma^q}\right) I_{ij} + g_1^q\left(g_2^q \sigma_q - g_2^p \sigma_p\right) \Delta D_{ijkl}\left(\frac{t - t_p}{a_\sigma^q}\right) I_{ij}, \quad t > t_p \tag{20}$$

Therefore, when the general two-step stress tensor is applied to nonlinear viscoelastic materials, the nonlinear viscoelastic constitutive equation which is appliable to any two-step stress can be obtained. For a special case of the general two-step stress, i.e., $\sigma_p = \sigma_0$ and $\sigma_q = 0$, substituting $\sigma_p = \sigma_0$, $\sigma_q = 0$ and $g_0^q = g_1^q = g_2^q = a_\sigma^q = 1$ into Eq. (20), the following nonlinear viscoelastic constitutive equation can be obtained:

$$\varepsilon_{ij}^{nve} = g_0^p \sigma_p D_{ijkl}(0) I_{ij} + g_1^p g_2^p \sigma_p \Delta D_{ijkl}\left(\frac{t}{a_\sigma^p}\right) I_{ij}, \quad 0 \le t \le t_p \tag{21}$$

$$\varepsilon_{ij}^{nve} = g_2^p \sigma_p \Delta D_{ijkl}\left(\frac{t_p}{a_\sigma^p} + t - t_p\right) I_{ij} - g_2^p \sigma_p \Delta D_{ijkl}\left(t - t_p\right) I_{ij}, \quad t > t_p \tag{22}$$

In addition, the viscoplastic strain will appear under the high stress for the nonlinear viscoelastic viscoplastic materials. The viscoplastic strain accumulates over time and is non-recoverable. Thus, a nonlinear viscoelastic constitutive equation that contains viscoplasticity can be obtained by adding the viscoplastic strain into Eqs. (21) and (22), which can be expressed by the following equations:

$$\varepsilon_{ij} = \varepsilon_{ij}^{nve} + \varepsilon_{ij}^{vp}(t) = g_0^p \sigma_p D_{ijkl}(0) I_{ij} + g_1^p g_2^p \sigma_p \Delta D_{ijkl}\left(\frac{t}{a_\sigma^p}\right) I_{ij} + \varepsilon_{ij}^{vp}(t), \quad 0 \le t \le t_p \tag{23}$$

$$\varepsilon_{ij} = \varepsilon_{ij}^{nve} + \varepsilon_{ij}^{vp}(t_p) = g_2^p \sigma_0 \Delta D_{ijkl}\left(\frac{t_p}{a_\sigma^p} + t - t_p\right) I_{ij} - g_2^p \sigma_0 \Delta D_{ijkl}\left(t - t_p\right) I_{ij} + \varepsilon_{ij}^{vp}(t_p), \quad t > t_p \tag{24}$$

where ε_{ij} is total strain tensor of the materials; and $\varepsilon_{ij}^{vp}(t)$, $\varepsilon_{ij}^{vp}(t_p)$ are the viscoplastic strain tensor of the materials at loading time *t* and t_p.

The next step is to simulate the viscoplastic deformation process of the nonlinear viscoelastic viscoplastic materials. In this study, the viscoplastic strain of the materials is modeled by the Perzyna-type viscoplastic model (Perzyna, 1966):

$$\dot{\varepsilon}_{ij}^{vp} = \Gamma\langle\Phi(f)\rangle^{N}\frac{\partial g}{\partial \sigma_{ij}} \tag{25}$$

where $\dot{\varepsilon}_{ij}^{vp}$ is the viscoplastic strain rate; Γ is the viscosity-related parameter; N is viscoplastic rate-dependent exponent; f is viscoplastic yield surface function; g is viscoplastic plastic potential function; The McCauley brackets in Eq. (25) imply that:

$$\langle\Phi(f)\rangle = if\left[\Phi(f) \le 0, 0, f/1Pa\right] \tag{26}$$

However, only loading time and stress level are considered in Perzyna-type viscoplastic model. Temperature, one of the most important factors affecting viscoplastic strain, is not considered in this model. To consider the effect of the temperature on the viscoplasticity, a kinetics-based model will be established to characterize the viscoplasticity for the nonlinear viscoelastic viscoplastic materials. In kinetics theory, a rate constant is used to measure the time required for any physical or chemical process to reach an equilibrium state. The minimum energy that promotes the development of a certain physical or chemical process of the system is called the activation energy of the process, which has different activation energy for different materials and different physical or chemical processes. The activation energy of a certain physical or chemical process is only related to the material. The Arrhenius equation is widely used to establish the relationship between rate constant and activation energy for a certain physical or chemical process, which can be expressed as follows (Bamford and Tipper, 1969):

$$k = A_0 e^{-\frac{E_a}{RT}} \tag{27}$$

where k is the rate constant of a certain physical or chemical process, A_0 is the pre-exponential factor of the process; E_a is the activation energy of the process; R is the ideal gas constant; and T is the absolute temperature of the process.

Kinetics theory has been used to characterize the fatigue damage, heal process of the asphalt materials (Luo et al., 2020; Li et al., 2021). For example, Li et al. (2021) established a relationship

between fatigue cracking growth rate and temperature for asphalt binders based on the kinetics theory. Furthermore, the kinetics theory has also been used to correlate the modulus change rate with the temperature, and an aging prediction model has been established to characterize the aging process of the in-service asphalt pavement (Luo et al., 2015; 2017; 2018). In this study, the viscoplastic deformation process of the nonlinear viscoelastic viscoplastic materials is temperature- and stress-dependent, and the Arrhenius equation in kinetics theory is employed to introduce the temperature effect for the Perzyna-type viscoplastic model, thus Eq. (27) can be modified as:

$$\dot{\varepsilon}_{ij}^{vpT} = A_{vp} e^{-\frac{E_{avp}}{RT}} = \Gamma_0 \langle \Phi(f) \rangle^N \frac{\partial g}{\partial \sigma_{ij}} e^{-\frac{E_{avp}}{RT}} \tag{28}$$

where $\dot{\varepsilon}_{ij}^{vpT}$ is the viscoplastic strain rate considering the effects of temperatures and stress; A_{vp} is the pre-exponential factor, which equals to the stress-dependent viscoplastic strain rate $\Gamma_0 \langle \Phi(f) \rangle^N \frac{\partial g}{\partial \sigma_{ij}}$ at a reference temperature; and E_{avp} is the viscoplastic activation energy of the nonlinear viscoelastic viscoplastic materials.

Taking the logarithm of both sides of Eq. (28) yields:

$$\ln \dot{\varepsilon}_{ij}^{vpT} = \ln \left(\Gamma_0 \langle \Phi(f) \rangle^N \frac{\partial g}{\partial \sigma_{ij}} \right) - \frac{E_{avp}}{RT} \tag{29}$$

Finally, a kinetics-based viscoplastic model and a viscoplastic activation energy indicator for characterizing the viscoplasticity for the nonlinear viscoelastic viscoplastic materials can be accurately determined based on Eqs. (23), (24), (25), (28) and (29) by considering the influence of the temperature and loading level. According to Eq. (29), the greater the viscoplastic activation energy, the smaller the viscoplastic strain rate, i.e., the greater the energy required for producing the same viscoplastic strain rate. Therefore, this indicator can be used to evaluate the viscoplastic deformation resistance of the nonlinear viscoelastic viscoplastic materials.

3 Materials and test methods

3.1 Materials

To validate the feasibility of the theoretical process above and the applicability of the new indicator for nonlinear viscoelastic viscoplastic materials, asphalt binders (typical nonlinear viscoelastic viscoplastic materials) were taken as examples of analysis. Three types of asphalt binders supplied by Hebei Transportation Investment Group Corporation in China were selected, including a virgin asphalt binder, a high-viscosity modified asphalt binder and a rubber asphalt binder, which were denoted as VB, HVB and RB, respectively. VB does not have any additives, HVB contains styrene–butadiene–styrene and other polymer additives, and RB has 30% of crumb rubber. The basic properties (penetration, softening point and ductility) of these binders were obtained in accordance with the Chinese specification, as listed in Table 1.

Table 1. Basic properties of asphalt binders

Properties	Unit	Virgin asphalt binder	High-viscosity modified asphalt binder	Rubber asphalt binder
Penetration at 25°C	0.1mm	25	75	50
Softening point	°C	71.7	91.0	87.8
Ductility (5 °C)	cm	>50	>20	>10

3.2 Sample preparation

In this study, the Discovery Hybrid Rheometer (DHR) from TA Instruments was adopted to perform laboratory tests for three types of asphalt binders. 25 mm diameter parallel plates and a 1 mm gap were used. The main steps for preparing the asphalt binder samples include: (1) heat the asphalt binders by putting them into the oven; (2) pour the hot flow-state asphalt binders into the silicon rubber mould, and cool the asphalt binder sample to room temperature for some time; (3) take out the asphalt binder sample by flexing the silicon rubber mould; (4) adhere the asphalt binder sample to the lower parallel plate and press the top surface of the sample gently; (5) turn off the environmental chamber of the instrument and raise the temperature to soften the asphalt binder sample; (6) lower the upper parallel plate to the trim gap and then lock the rotating lever of the instrument; (7) trim the

300 needless asphalt binder by using the heated trim tool and lower the upper parallel plate to the target
301 gap; and (8) accurately control the temperature of asphalt binder samples by liquid nitrogen and
302 environmental chamber before starting the laboratory tests. Thus, the preparation of the asphalt
303 binder sample was completed.
304

305 **3.3 Test methods**

306 To investigate the viscoplasticity of the asphalt binder by accurately separating it from nonlinear
307 viscoelasticity and propose a new evaluation indicator of the viscoplasticity that considers the
308 influence of temperature, creep and recovery tests under a low-stress level and a high-stress level
309 were performed on three types of asphalt binders (VB, HVB and RB).
310

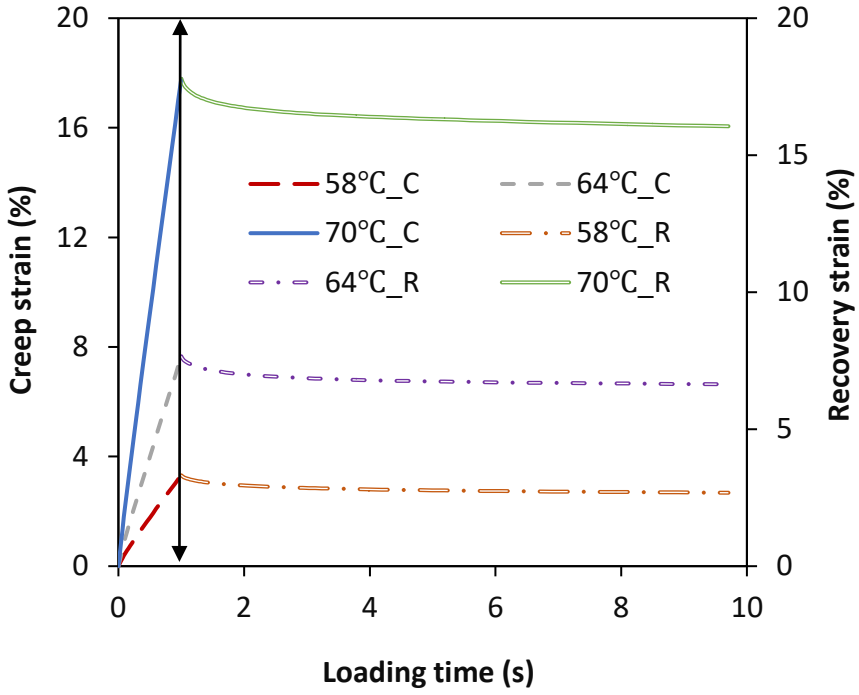
311 **3.3.1 Creep and recovery test under a low-stress level**

312 Some studies indicated that the linear viscoelastic information of the asphalt binders can be
313 determined by the recovery stain at the creep stress of 0.1 kPa (Shirodkar, et al., 2012; Liu et al.,
314 2021). The linear viscoelastic mechanical properties of asphalt binders were needed to determine the
315 nonlinear viscoelastic parameters. Hence, the creep stress level of the creep and recovery test under a
316 low-stress level was selected as 0.1kPa in this study, and the selected testing temperatures were 58°C，
317 64°C and 70°C for three types of asphalt binders (VB, HVB and RB). Taking the test results of VB
318 as an example, Figure 1a shows the typical creep and recovery test results of the VB at 0.1kPa and
319 three temperatures. In the legend, "_C" stands for creep strain and "_R" stands for recovery strain of
320 the asphalt binders. It can be seen from Figure 1a that the loading time is 1s and the creep strain
321 increases with the loading time during the creep phase, and the recovery time is 9s and the recovery
322 strain decreases with the recovery time during the recovery phase. Besides, the creep strain and
323 recovery strain increase with the increase of the temperature.

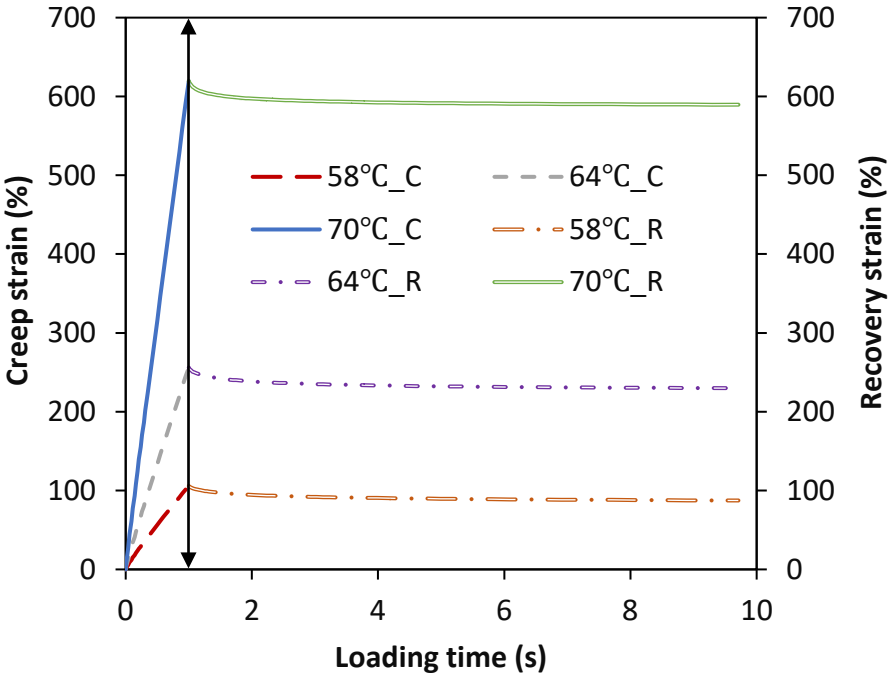
324 **3.3.2 Creep and recovery test under a high-stress level**

325 To investigate the nonlinear viscoelasticity and viscoplasticity of asphalt binders, the creep and
326 recovery tests under a high-stress level were performed. The selected high-stress level was 3.2kPa.

To make sure the mechanical properties of the asphalt binder determined under the low-stress level were applied to those under the high-stress level, the same testing temperatures (i.e., 58℃，64℃ and 70℃) were applied to the tests under the high-stress level. Similarly, taking the test results of VB as an example, Figure 1b plots the typical creep and recovery test results of the VB at 3.2kPa and different temperatures. The relationship trends of creep strain and recovery strain are the same as Figure 1a, but the high-stress level can increase energy dissipation of the asphalt binders and induce larger strain.



(a)



(b)

Figure 1. Typical creep and recovery test results of the VB at: (a) 0.1kPa; (b) 3.2kPa

4 Results and discussion

4.1 Mechanical model parameters

According to Eqs. (23) and (24), the total strain consists of nonlinear viscoelastic and viscoplastic strain. Before separating the viscoplastic strain and the nonlinear viscoelastic strain for three types of asphalt binders in this study, mechanical model parameters need to be first determined. There are two aspects: (1) determine the creep compliance based on the test results of the recovery test under the low-stress level (0.1kPa); and (2) obtain the nonlinear viscoelastic parameters according to the test results of the creep and recovery test under the high-stress level (3.2kPa).

4.1.1 Determination of the creep compliances

When the one-dimensional constant stress is applied to the asphalt binder under linear viscoelastic condition, the creep compliance of the binder can be calculated by:

$$D(t)=\frac{\varepsilon(t)}{\sigma_p} \tag{30}$$

The Prony series model is widely used to analyze the creep compliance of asphalt materials (Luo, et al., 2020; Zhang, et al., 2016). In this study, the creep compliances of three types of asphalt binders are also modeled by the Prony series model:

$$D(t) = D_0 + \Delta D = D_0 + \sum_{i=1}^{M} D_i \left(1 - \exp\left(-t / \lambda_i\right)\right) \tag{31}$$

where $\Delta D = D(t) - D_0$ represents the transient compliance; D_i is components of the creep compliance; λ_i is components of the retardation time; M is the total number of the Maxwell elements.

Furthermore, some studies show that the linear viscoelastic properties of the asphalt binder can be determined by the recovery strain under the creep stress of 0.1 kPa (Shirodkar, et al., 2012; Liu et al. 2021). Under the linear viscoelastic condition, the nonlinear viscoelastic parameters g_0^q, g_1^q, g_2^q, a_σ^q are equal to one. Therefore, the recoverable linear viscoelastic strain in the recovery phase of the asphalt binder can be calculated based on Eqs. (23) and (24), and it can be expressed as:

$$\Delta\varepsilon_{ve} = \left[\Delta D\left(t_p\right) - \Delta D(t) + \Delta D\left(t - t_p\right)\right]\sigma_p \tag{32}$$

where $\Delta\varepsilon_{ve}$ is the recoverable viscoelastic strain in the recovery phase of the asphalt binder.

According to the test results of the creep and recovery test, it can be found that the initial components of the creep compliance D_0 are zero for three types of asphalt binders. Then, the other Prony series coefficients D_i and λ_i can be determined by Eq. (32) and the test results in the recovery phase of the asphalt binder based on the "Solve" tool in Excel. Taking the VB as an example, Figure 2 shows the test results and model values of the recoverable viscoelastic strain of VB at three temperatures (58℃，64℃ and 70℃). It can be seen that the model values can match the test data well at three temperatures for the VB, and the same findings apply to HVB and RB. It indicates that the Prony series can be used to model the creep compliance of the asphalt binders. The Prony series coefficients of the creep compliance of VB, HVB and RB at 64℃ are shown in Table 2.

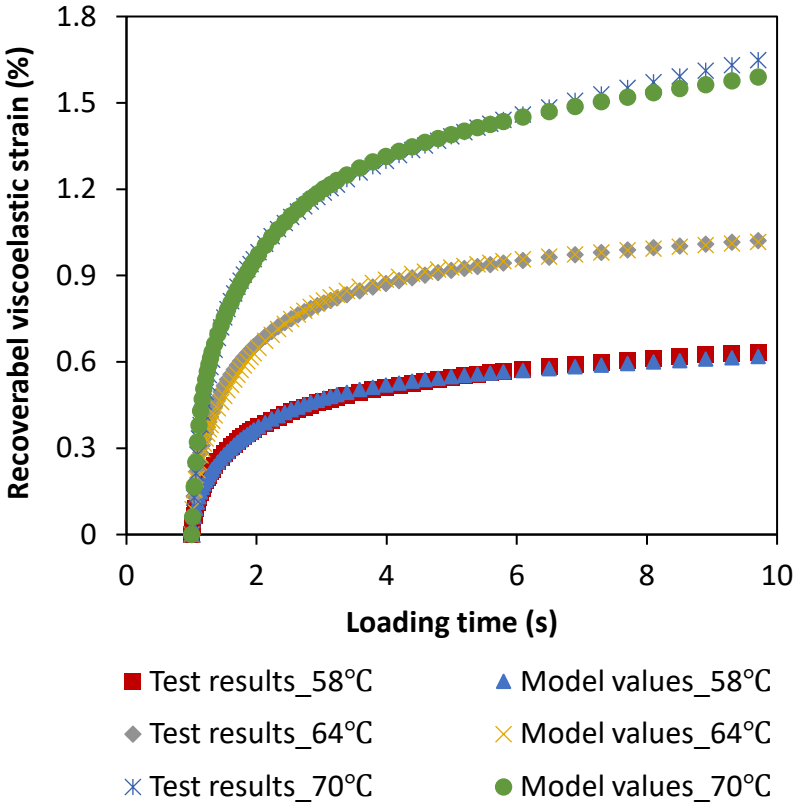


Figure 2. Test results and model values of the recoverable viscoelastic strain of VB at three temperatures (58℃，64℃ and 70℃)

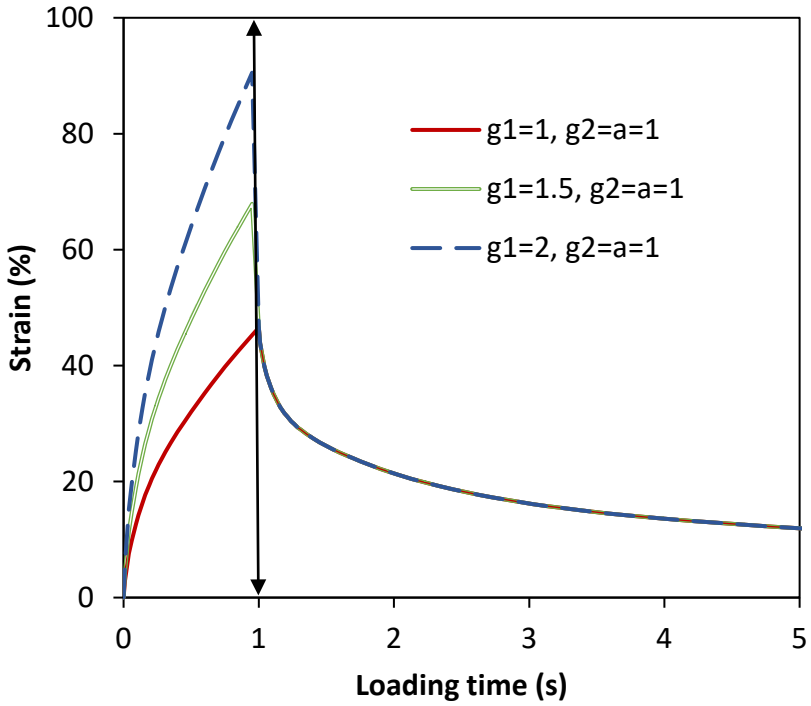
Table 2. Prony series coefficients of creep compliance of asphalt binders at 64℃

Materials		VB	HVB	RB
Components of creep compliance (1/kPa)	D_1	0.27022	0.31474	0.40391
	D_2	0.32163	2.19249	0.89889
	D_3	0.09651	0.53762	0.14015
	D_4	0.01373	0.21691	0.05142
	D_5	0.00899	0.03895	0.01474
Components of retardation time (s)	λ_1	100	100	100
	λ_2	10	10	10
	λ_3	1	1	1
	λ_4	0.1	0.1	0.1
	λ_5	0.01	0.01	0.01

4.1.2 Determination of the nonlinear viscoelastic parameters

To have a better understanding of the effects of the nonlinear viscoelastic parameters, the curves of

384 the strain vs loading time are plotted with some different values of nonlinear viscoelastic parameters.
385 Figure 3 presents the effects of different nonlinear parameters on the creep and recovery strain
386 response. Figure 3a shows the strain responses with different nonlinear viscoelastic parameters, g1. It
387 shows that g1 influences the creep strain response, and the creep strain increases with the increase of
388 g1. However, the recovery strain responses keep unchanged with the increase of g1. Figure 3b
389 presents the strain responses with different nonlinear viscoelastic parameters, g2. It can be seen that
390 the creep and recovery strain responses increase with the increase of g2. Figure 3c illustrates the
391 strain responses with different nonlinear viscoelastic parameters, a_σ. The creep strain and recovery
392 strain responses are both affected by a_σ, and decrease with the increase of a_σ. Besides, to more
393 clearly show the effect of the nonlinear viscoelastic parameters on the strain responses, Figure 3d
394 presents the change percentage of the strain responses when the nonlinear viscoelastic parameter is
395 1.5 ($g1 = g2 = a_\sigma = 1.5$). The following conclusions can be obtained from Figure 3d: (1) the nonlinear
396 viscoelastic parameter g1 only affects the creep strain response of the materials, while the nonlinear
397 viscoelastic parameter g2 and a_σ both influence the creep and recovery strain response of the
398 materials; (2) the nonlinear viscoelastic parameter g1 and g2 have the same influence to the creep
399 strain response, and the effect values keep unchanged with the loading time; (3) the nonlinear
400 viscoelastic parameter a_σ decreases the creep and recovery strain responses, and the change
401 percentage of the strain first increases, and then decreases with the loading time in the creep phase,
402 and continuously decreases with the loading time in the recovery phase; and (4) the effect of
403 nonlinear viscoelastic parameter g1 and g2 on the strain response is more significant than that of the
404 nonlinear viscoelastic parameter a_σ.



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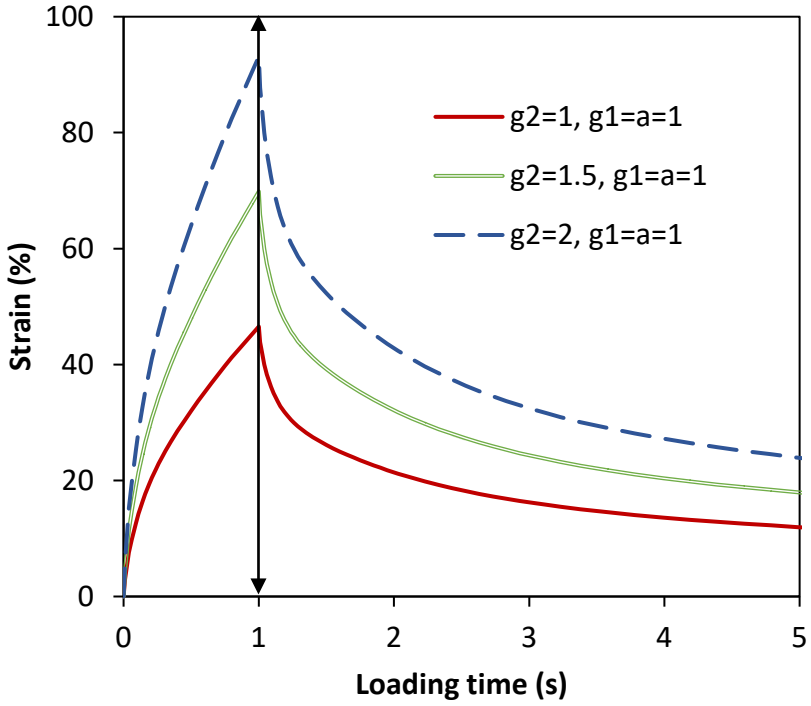

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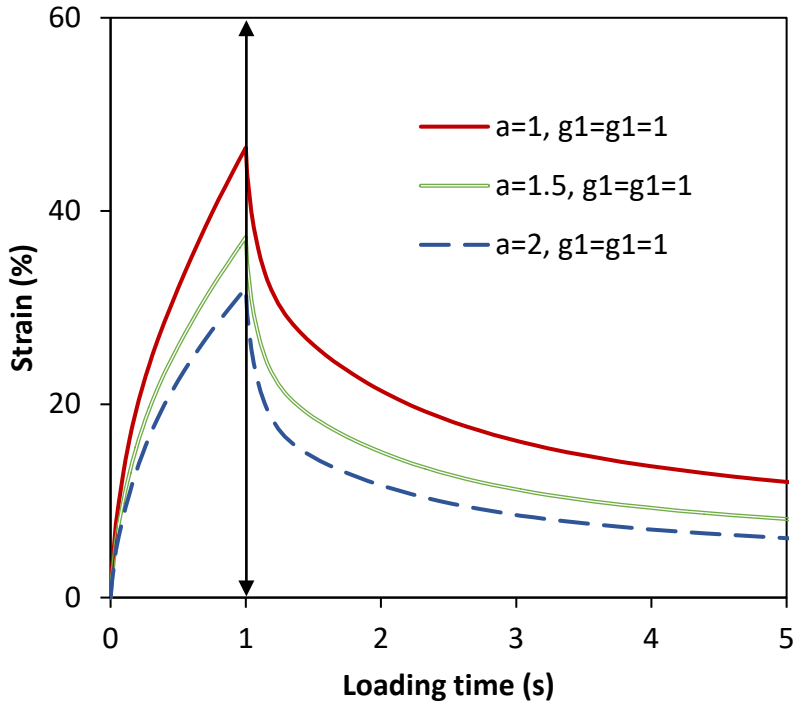

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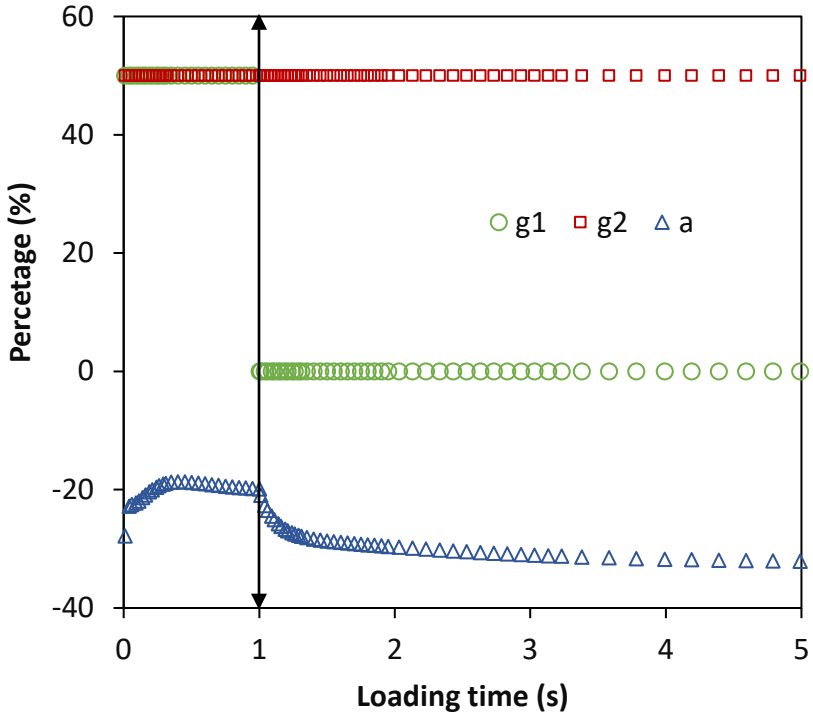
410 (c)



411

412 (d)

413 **Figure 3.** Strain responses at different nonlinear viscoelastic parameters and change percentage of

414 the strain responses when $g1 = g2 = a_\sigma = 1.5$: (a) at different nonlinear viscoelastic parameters g1; (b)

415 at different nonlinear viscoelastic parameters g2; (c) at different nonlinear viscoelastic parameters
416 a_σ; (d) change percentage of the strain response when $g1 = g2 = a_\sigma = 1.5$
417
418 Hence, the nonlinear viscoelasticity of the materials is affected by these three nonlinear viscoelastic
419 parameters together. In this study, the creep and recovery test under the high-stress level of 3.2kPa
420 are performed on the asphalt binders, and the nonlinear viscoelastic characteristics are displayed. As
421 introduced above, Eqs. (23) and (24) can be used to analyze the test results of the creep and recovery
422 test for asphalt binders. It is noted that the expressions of strains are different when $0 \le t \le t_p$ and
423 $t > t_p$. When $0 \le t \le t_p$, the asphalt binder is in a creep phase. Thus, the strain at $t = t_p^-$ can be
424 determined as:

425 $$\varepsilon\left(t_p^-\right) = g_1^p g_2^p \Delta D(\frac{t_p^-}{a_\sigma^p}) + \varepsilon^{vp}\left(t_p^-, \sigma_0\right) \tag{33}$$

426 According to Eq. (33), the nonlinear viscoelastic strain can be obtained as:

427 $$\varepsilon\left(t_p^-\right) - \varepsilon^{vp}\left(t_p^-, \sigma_0\right) = g_1^p g_2^p \sigma_0 \Delta D(\frac{t_p^-}{a_\sigma^p}) \tag{34}$$

428 When $t = t_p^+$, the constant stress σ_0 is removed, the nonlinear viscoelastic strain will gradually
429 recover with the increase of recovery time. In addition, the viscoplastic strain is non-recoverable in
430 the recovery phase, thus the strain at $t = t_p^+$ can be calculated by:

431 $$\varepsilon\left(t_p^+\right) = g_2^p \sigma_0 \Delta D\left(\frac{t_p}{a_\sigma^p} + t_p^+ - t_p\right) - g_2^p \sigma_0 \Delta D\left(t_p^+ - t_p\right) + \varepsilon^{vp}\left(t_p^+, 0\right) \tag{35}$$

432 The viscoplastic strain is the same at $t = t_p^-$ and $t = t_p^+$, that is $\varepsilon^{vp}\left(t_p^-, \sigma_0\right) = \varepsilon^{vp}\left(t_p^+, 0\right)$, and
433 combining Eqs. (33) and (35), the following equation can be obtained:

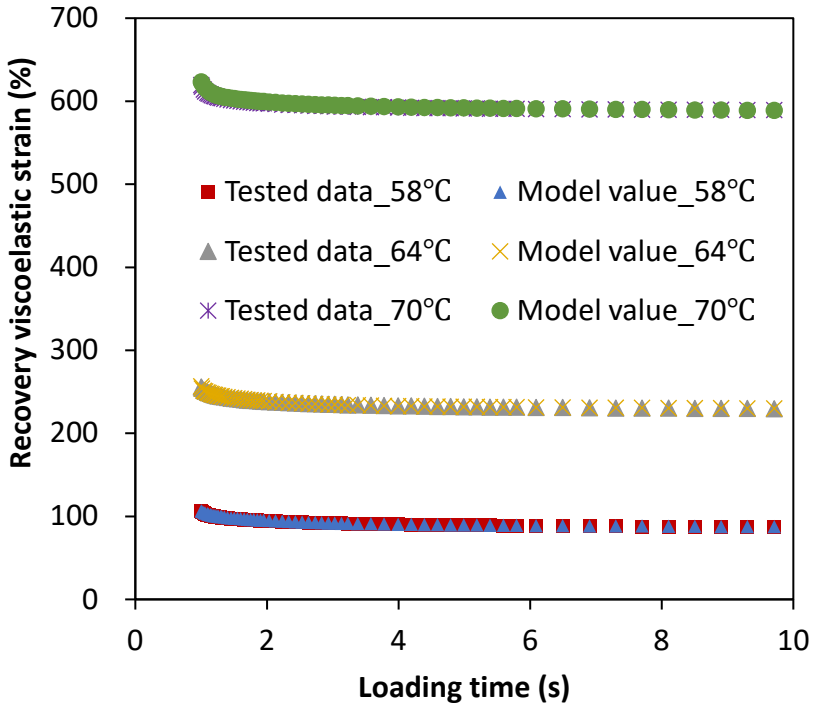
434 $$\varepsilon\left(t_p^-\right) - \varepsilon\left(t_p^+\right) = g_2^p \sigma_0 \left[g_1^p \Delta D(\frac{t_p^-}{a_\sigma^p}) - \Delta D\left(\frac{t_p}{a_\sigma^p} + t_p^+ - t_p\right)\right] + \sigma_0 \Delta D\left(t_p^+ - t_p\right) = g_2^p \sigma_0 \Delta D(\frac{t_p}{a_\sigma^p})\left(g_1^p - 1\right) \tag{36}$$

435 For the asphalt binders, the creep strain at t_p^- and the recovery strain at t_p^+ are the same, i.e.,
436 $\varepsilon\left(t_p^-\right) = \varepsilon\left(t_p^+\right)$. Thus, solving for g_1^p based on Eq. (36), i.e., $g_1^p = 1$. This is an important conclusion

437 that the nonlinear parameter g_1^p of asphalt binders is always equal to one. Hence, the nonlinear
438 parameters g_0^p and g_1^p in nonlinear viscoelastic constitutive equation will not affect the nonlinear
439 viscoelastic behavior of the asphalt binders. Next, other nonlinear parameters g_2^p and a_σ^p for the
440 asphalt binders can be determined. Combining Eq. (24) in the recovery phase and Eq. (33) in the
441 creep phase of the asphalt binders, the expression of the nonlinear viscoelastic strain during the
442 recovery phase can be formulated as:

443 $$\varepsilon(t)-\varepsilon(t_p^-)=g_2^p\sigma_0\Delta D\left(\frac{t_p}{a_\sigma^p}+t-t_p\right)-g_2^p\sigma_0\Delta D(t-t_p)-g_1^pg_2^p\Delta D(\frac{t_p^-}{a_\sigma^p}) \quad (37)$$

444 Solving the nonlinear parameters g_2^p and a_σ^p in Eq. (37) by fitting the test data in the recovery
445 phase of the asphalt binders. Figure 4 shows the test data and model values in the recovery phase
446 under the stress level of 3.2kPa at three temperatures (58℃，64℃ and 70℃) of VB, HVB and RB. It
447 can be seen from Figure 4 that the test data can match the model value calculated by Eq. (37) for all
448 binders at the three temperatures. Table 3 presents nonlinear viscoelastic parameters at the three
449 temperatures of the three types of asphalt binders.



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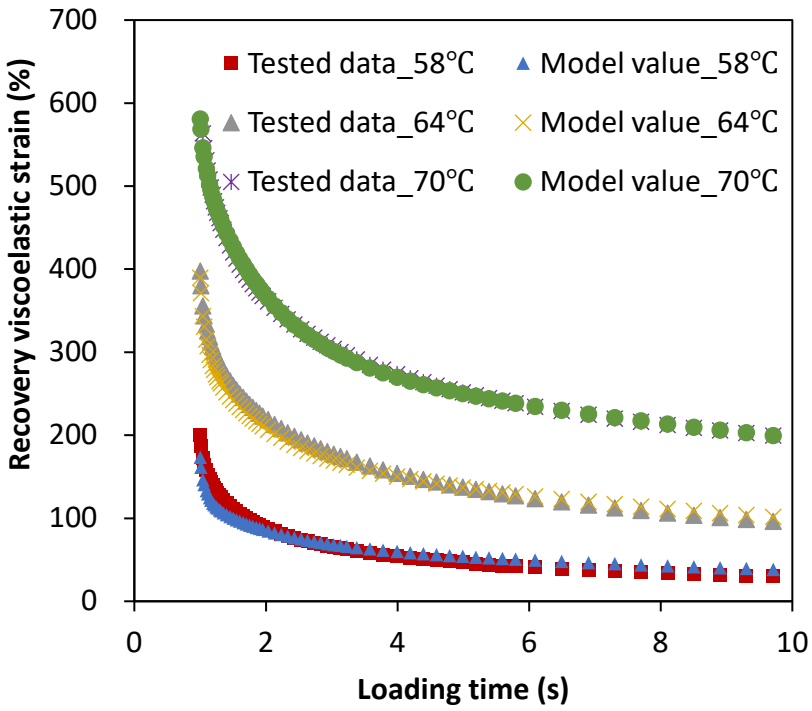

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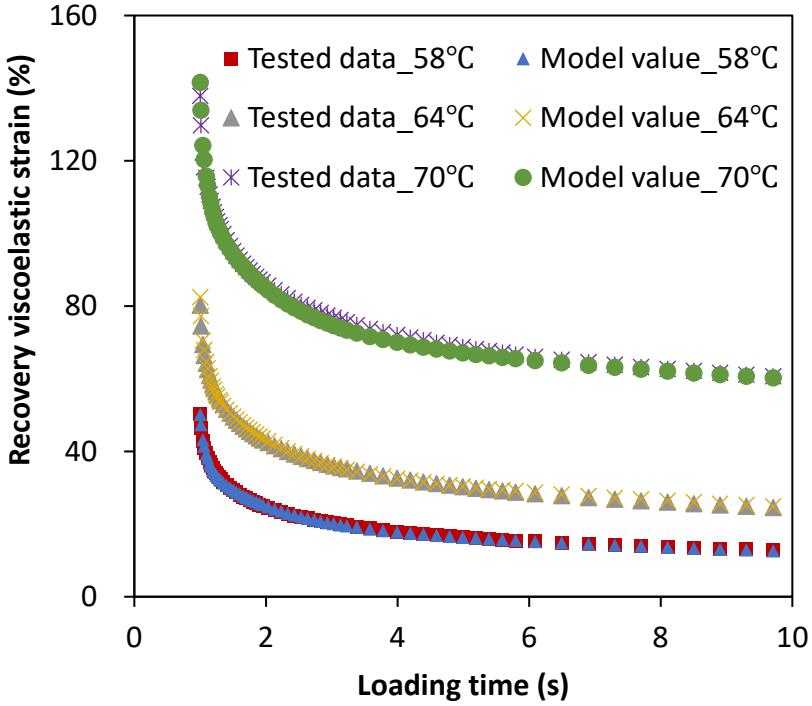

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455 (c)

456 **Figure 4.** Test data and model values in the recovery phase under the stress level of 3.2kPa at three

457 temperatures (58°C，64°C and 70°C) of three types of asphalt binders (VB, HVB and RB): (a) VB; (b)

458 HVB; (c) RB

459

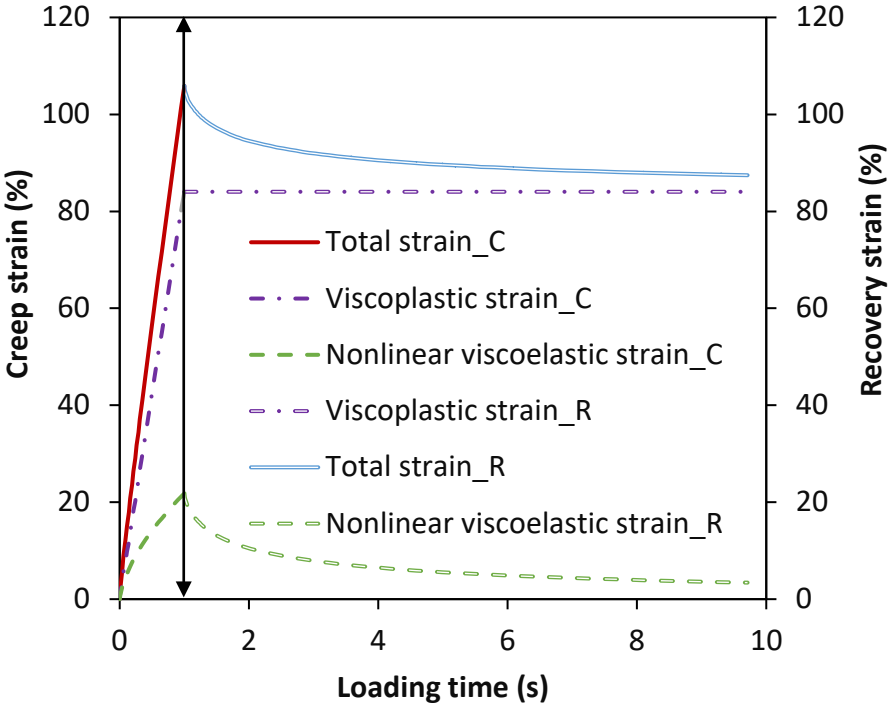
460 **Table 3.** Nonlinear viscoelastic parameters at three temperatures of three types of asphalt binders

Materials	VB			HVB			RB		
Parameters	g_1^p	g_2^p	a_σ^p	g_1^p	g_2^p	a_σ^p	g_1^p	g_2^p	a_σ^p
58℃	1	1.023	1.115	1	1.025	1.202	1	1.070	1.241
64℃	1	1.140	1.850	1	1.177	1.196	1	1.081	1.537
70℃	1	1.059	2.338	1	0.678	0.412	1	1.087	1.503

461

462 **4.2 Nonlinear viscoelasticity and viscoplasticity**

463 When the mechanical model parameters are determined, the nonlinear viscoelastic strain and
464 viscoplastic strain can be separated based on the test results. Specifically, by substituting the
465 mechanical model parameters into Eq. (21), the nonlinear viscoelastic strain of the asphalt binder can
466 be determined. Then, combining the nonlinear viscoelastic parameters and Eq. (23), the viscoplastic
467 strain can be accurately separated. Taking the analysis results of 58℃ as an example, Figure 5
468 presents the nonlinear viscoelastic strain, viscoplastic strain and total strain in the creep and recovery
469 phase of VB, HVB and RB at 58℃. In the legend, "_C" stands for the strain response in the creep
470 phase, and "_R" stands for the strain response in the recovery phase. For example, "Total strain_C"
471 refers to the total creep strain and "Total strain_R" refers to the total recovery strain of the asphalt
472 binders. It can be seen from Figure 5 that the total creep strain consists of nonlinear viscoelastic
473 strain and viscoplastic strain in the creep and recovery phase. Hence, the nonlinear viscoelastic strain
474 and viscoplastic strain can be clearly separated based on the analysis in this study.



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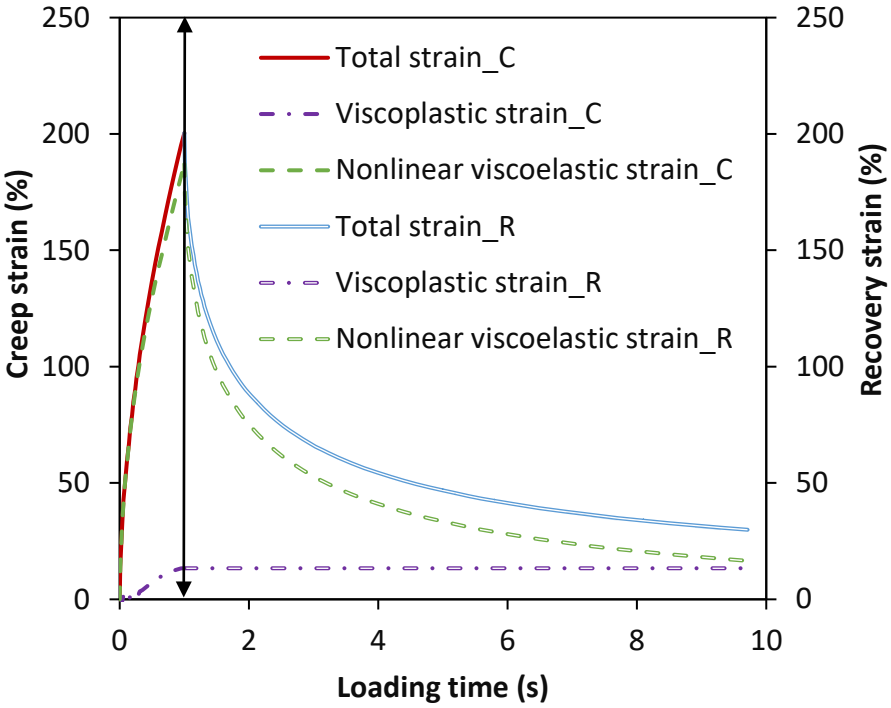

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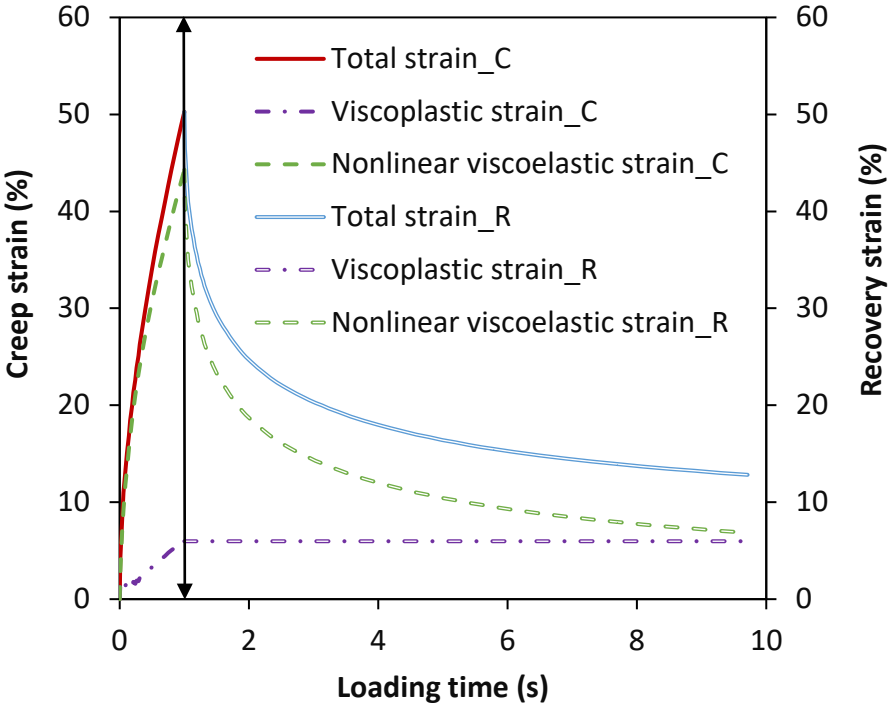
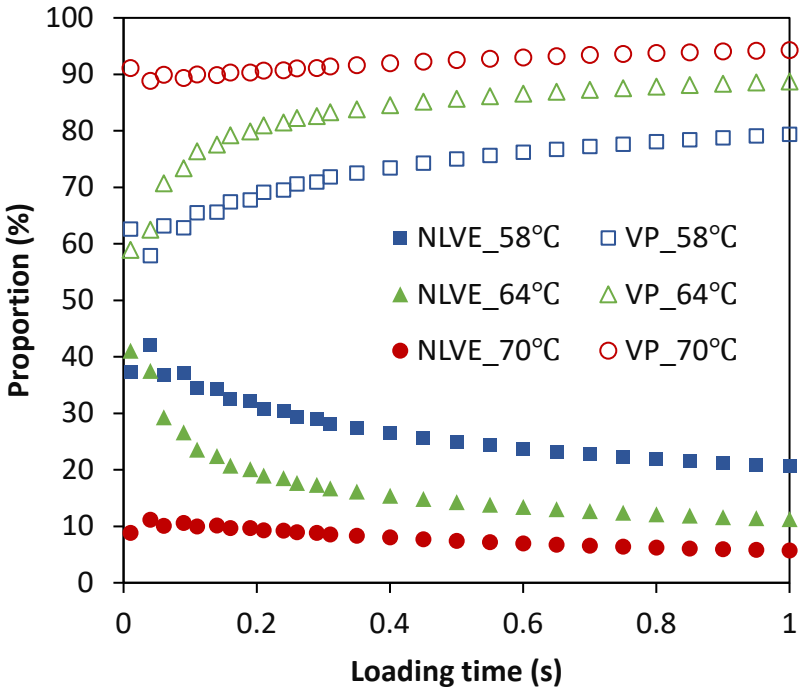


(c)

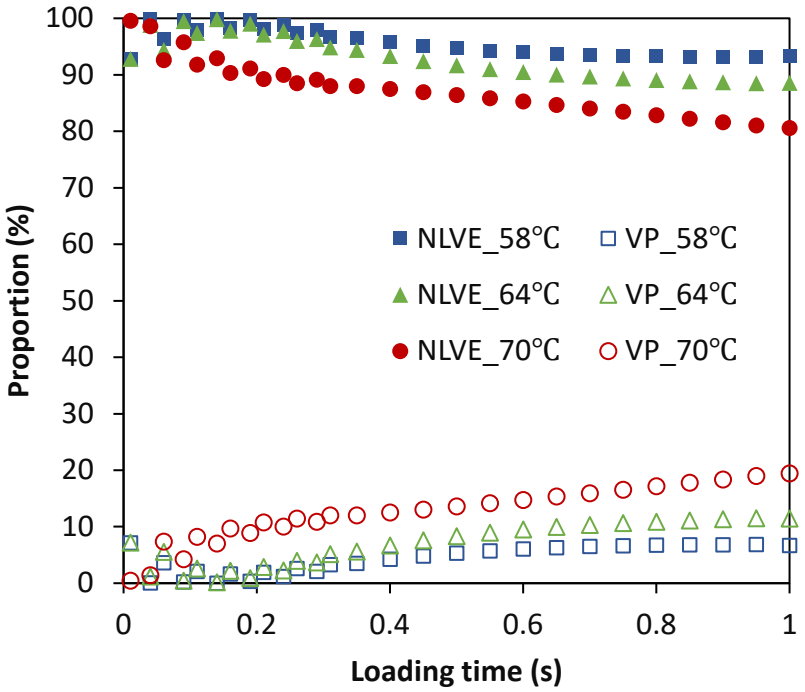
Figure 5. Nonlinear viscoelastic, viscoplastic strain and total strain in creep and recovery phase of three types of asphalt binders (VB, HVB and RB) at 58℃: (a) VB; (b) HVB; (c) RB

Figure 6 plots the proportions of the nonlinear viscoelastic strain, the viscoplastic strain of three types of asphalt binders (VB, HVB and RB) at three temperatures (58℃, 64℃ and 70℃). In the legend, "NLVE" stands for the nonlinear viscoelasticity and "VP" stands for the viscoplasticity of the asphalt binders. It can be observed from Figure 6 that the proportion of the viscoplastic strain is larger than the nonlinear viscoelastic strain for VB (the proportion exceeds 60%), while it's opposite for HVB and RB. This is because additives can improve the viscoplastic deformation resistance of the modified asphalt binders compared with the virgin asphalt binders without any additives. Besides, the proportion of the viscoplastic strain increases with the increase of the temperature for VB, HVB and RB, while the nonlinear viscoelastic strain is opposite. This is because the asphalt binders will gradually soften as the temperature increases, and more energy drives viscoplastic deformation of the asphalt binders compared with that at the lower temperature. In addition, the proportion of viscoplastic strain increases with the loading time for VB, HVB and RB at the three temperatures,

while the proportion of the nonlinear viscoelastic strain is opposite. It indicates that the viscoplastic strain is accumulated more quickly than the nonlinear viscoelastic strain of asphalt binders.



(a)



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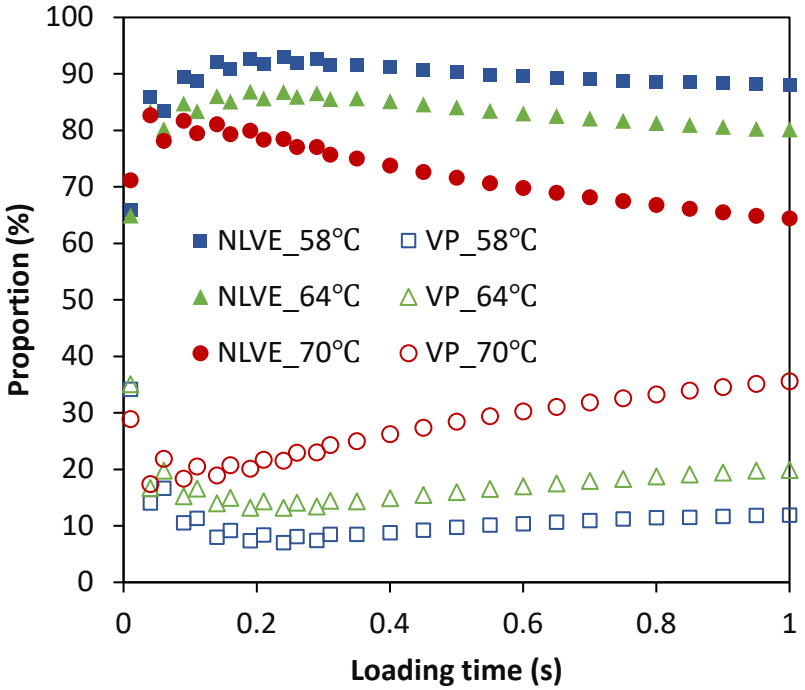


(c)

Figure 6. Proportions of the nonlinear viscoelastic strain, the viscoplastic strain of three types of asphalt binders (VB, HVB and RB) at three temperatures (58℃, 64℃ and 70℃): (a) VB; (b) HVB; (c) RB

In addition, the non-recoverable creep compliance J_{nr} has been widely used to evaluate the viscoplastic deformation of asphalt binders (D'Angelo et al., 2009). In the standard calculation method, J_{nr} is calculated by the residual strain in the recovery phase and applied creep stress. However, the residual strain in the last loading time contains viscoplastic strain and not fully recovered nonlinear viscoelastic strain. Therefore, the standard calculation method for J_{nr} is inaccurate because the nonlinear viscoelastic strain is not fully recovered. In this study, the viscoplastic strain and nonlinear viscoelastic strain are separated. Hence, the viscoplastic strain can be determined for three types of asphalt binders accurately. To improve the calculation accuracy of the non-recoverable creep compliance, J_{nr} can be redefined as:

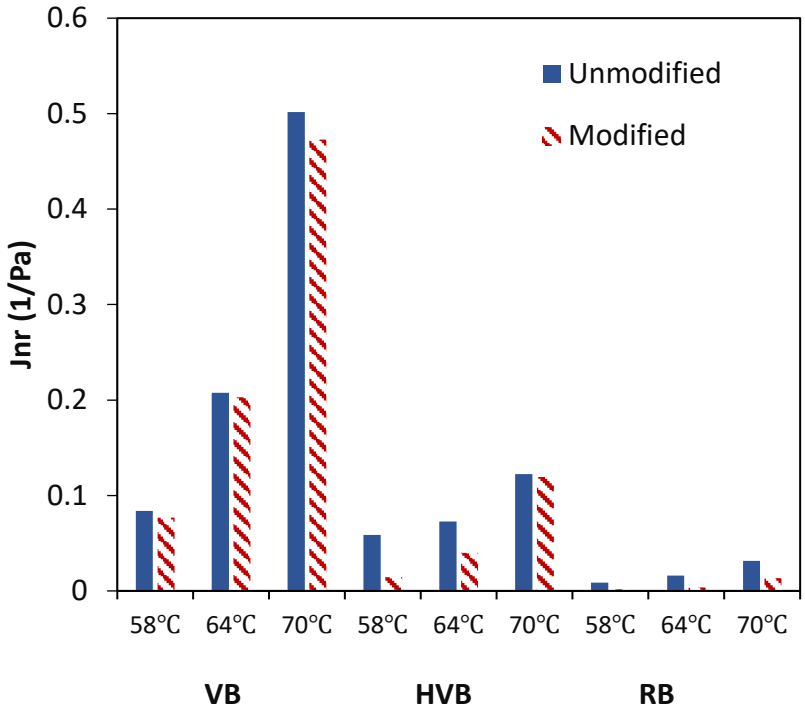
$$J_{nr}^{m} = \frac{\varepsilon_{vp}}{\sigma_{p}} \tag{38}$$

where J_{nr}^{m} is modified non-recoverable creep compliance of the asphalt binders in the creep and

519 recovery test.

520 Figure 7 presents the comparison between unmodified J_{nr} and modified J_{nr}^{m} at three temperatures
521 (58℃，64℃ and 70℃), two creep stress levels (0.1kPa and 3.2kPa), as well as their difference
522 percentage. The following findings can be obtained: (1) the unmodified J_{nr} is larger than the
523 modified J_{nr}^{m} indicator at any temperatures, creep stress (0.1kPa and 3.2kPa) for all binders; (2) the
524 J_{nr}^{m} indicator increases with the increase of the temperature, and creep stress level for all binders; (3)
525 the J_{nr}^{m} indicator of VB is larger than those of HVB and RB, i.e., the order of viscoplasticity
526 deformation resistance is RB>HVB>VB; and (4) different materials, temperatures and creep stress
527 levels produce different difference percentages between unmodified J_{nr} and modified J_{nr}^{m}, the
528 difference percentage of VB is minimum (less than 10%), while the difference percentages of HVB
529 and RB are much larger (most are more than 50%).

530 The reason for the finding (1) is that the modified J_{nr}^{m} is calculated by removing the total nonlinear
531 viscoelastic strain, which is true “non-recoverable” creep compliance. The reason for finding (2) is
532 that the viscoplastic strain increases with the increase of temperature and creep stress level. The
533 reason for finding (3) is that the modified asphalt binders are strengthened by the interaction between
534 the additives and virgin asphalt binders, leading to better viscoplastic deformation resistance of RB
535 and HVB. The reason for finding (4) is that viscoplasticity is more dominant than nonlinear
536 viscoelasticity for VB, while RB and HVB are opposite, which can be observed in Figure 6.



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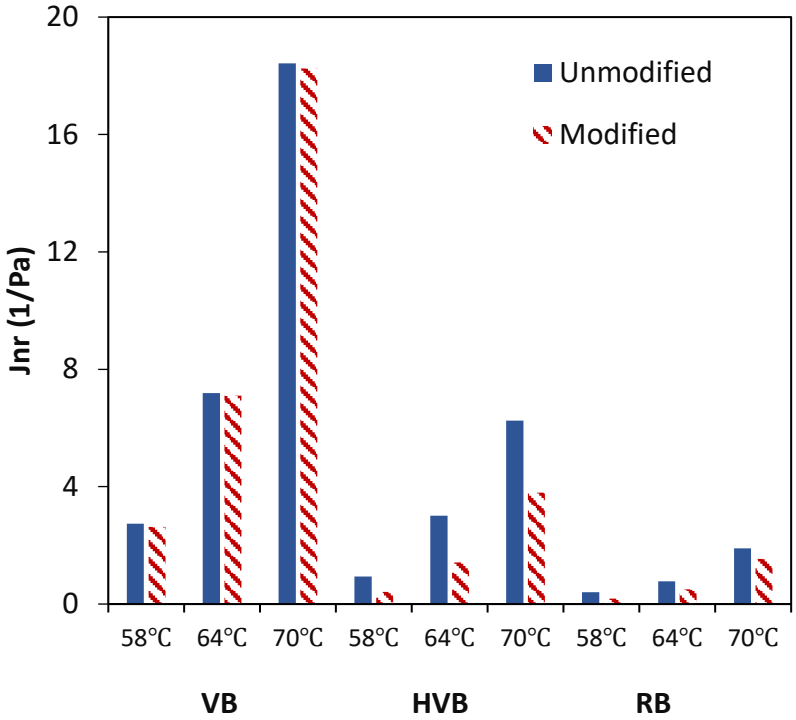

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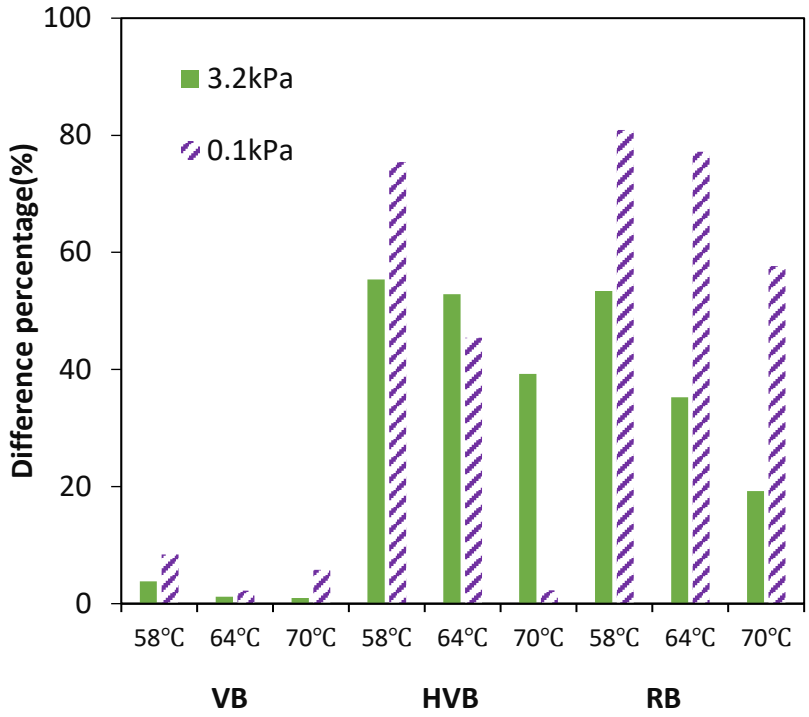


(c)

Figure 7. Comparison between unmodified and modified non-recoverable creep compliance at three temperatures (58°C，64°C and 70°C), two creep stress levels (0.1kPa and 3.2kPa) and their difference percentage for three types of asphalt binders (VB, HVB and RB): (a) at 0.1kPa; (b) at 3.2kPa; (c) difference percentage of the unmodified and modified non-recoverable creep compliance

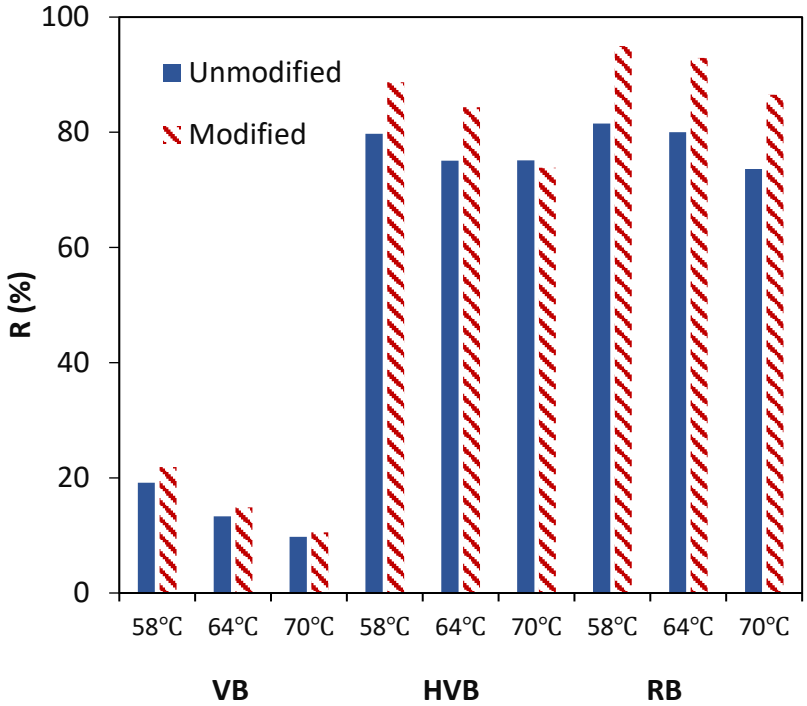
Furthermore, a percent recovery R is widely used to evaluate the recoverable ability of asphalt binders. However, the standard calculation approach of R is not accurate either, because the residual strain at the end of the recovery stage contains some parts of not fully recovered nonlinear viscoelastic strain. Similarly, to improve the calculation accuracy of the percent recovery, it can be redefined as:

$$R^m = \frac{\varepsilon^{NVVE}}{\varepsilon^{Total}} \times 100 = \frac{\varepsilon^{Total} - \varepsilon_{vp}}{\varepsilon^{Total}} \times 100 \tag{39}$$

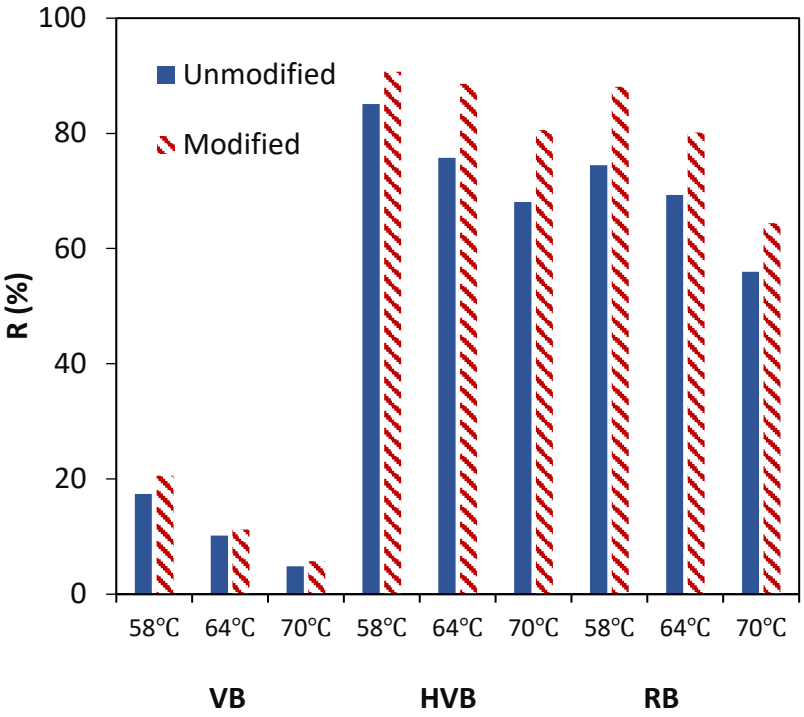
where R^m is modified percent recovery; ε^{NVVE} is the recoverable nonlinear viscoelastic strain of the asphalt binders; and ε^{Total} is total creep strain, which is equal to the creep strain at the end of creep stage or the recovery strain at the initial recovery stage of the asphalt binders.

Figure 8 presents the comparison between unmodified percent recovery R and modified percent recovery R^m at different temperatures and creep stress levels, as well as their difference percentages for three types of asphalt binders. It can be found from Figure 8 that: (1) the unmodified percent recovery R is smaller than the modified R^m at any temperatures, creep stress (0.1kPa and 3.2kPa) for all binders, and most difference percentages between unmodified and modified percent recovery are more than 10% as shown in Figure 8c; (2) the R^m value of VB is the minimum; (3) the R^m values of all binders decrease with the increase of the temperature.

The reason for finding (1) is that the modified R^m is calculated by the "true" recoverable strain, while unmodified R is calculated by the residual strain at the end of the recovery stage. The reason for finding (2) is that VB without the additives is viscoplasticity dominant, while HVB and RB strengthened by the additives are dominated by nonlinear viscoelastic characteristics. The reason for the finding (3) is that all asphalt binders will produce viscoplastic deformation more easily at high temperatures.



(a)



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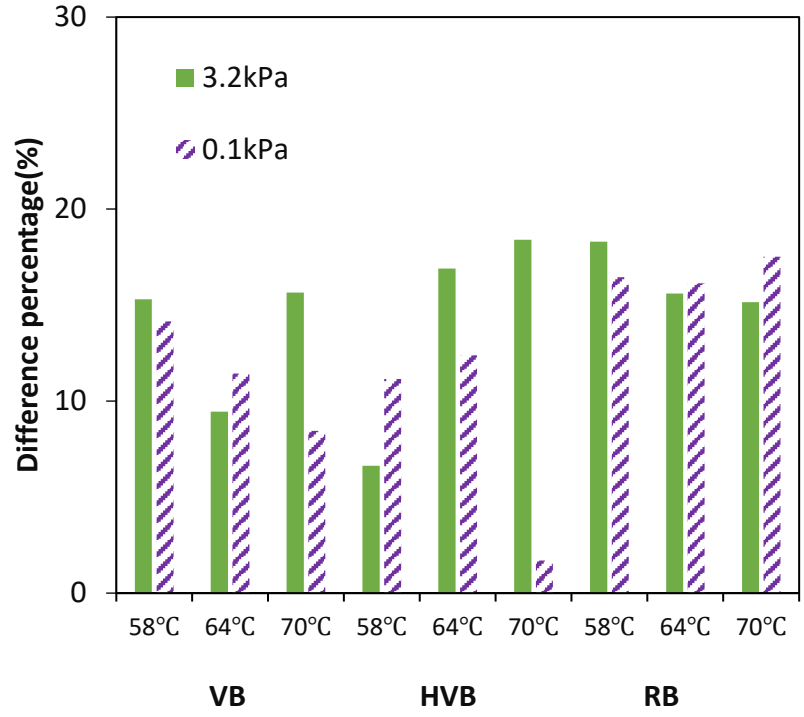
573 (b)



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575 (c)

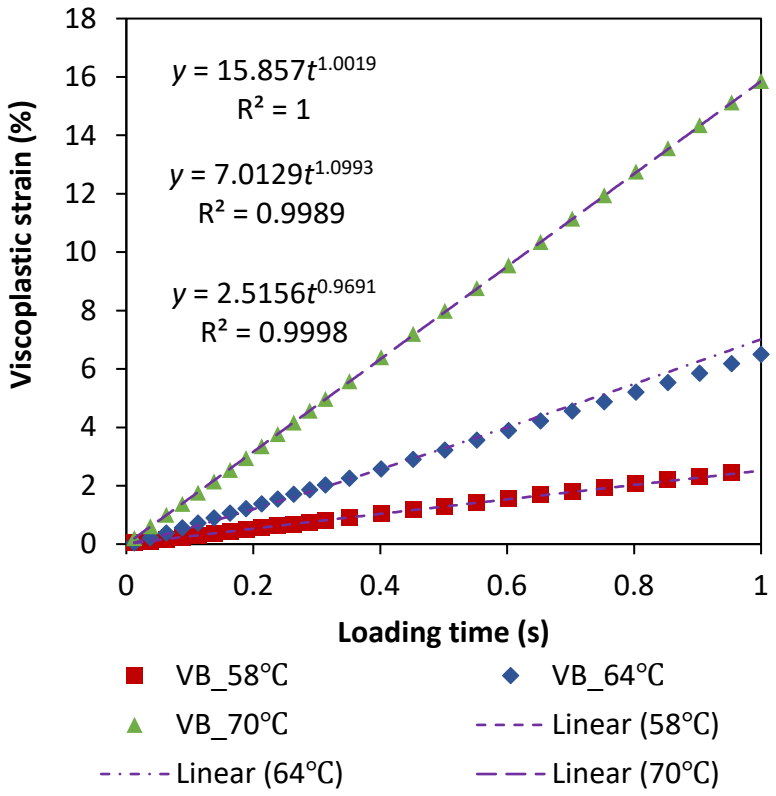
576 **Figure 8**. Comparison between unmodified and modified percent recovery at three temperatures

577 (58°C， 64°C and 70°C), two creep stress levels (0.1kPa and 3.2kPa) and their difference percentage

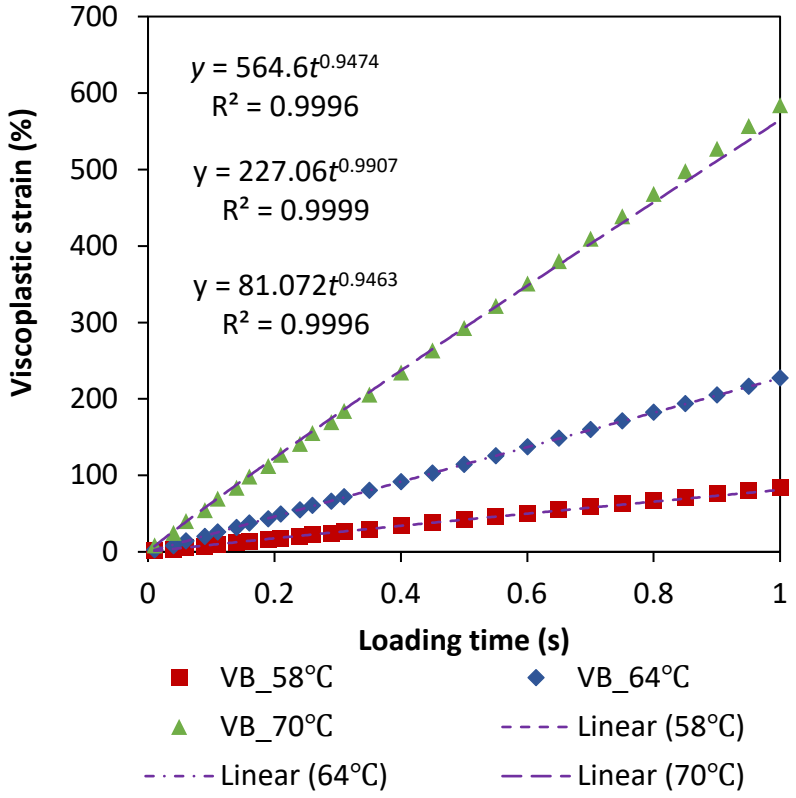
for three types of asphalt binders (VB, HVB and RB): (a) at 0.1kPa; (b) at 3.2kPa; (c) difference percentages of unmodified and modified percent recovery

4.3 Viscoplastic activation energy indicator

As can be seen from Figure 7 and Figure 8, the non-recoverable creep compliance and percent recovery are both affected by temperature and stress level. This study has also developed a temperature and stress-independent indicator (viscoplastic activation energy indicator) to characterize the viscoplasticity. Based on the separated viscoplastic strain, the viscoplastic strain rate can be obtained. Taking the VB as an example, Figure 9 shows the relationship between the viscoplastic strain and loading time of VB at two creep stress levels (0.1 and 3.2kPa) levels and three temperatures (58℃，64℃ and 70℃). The exponential term of the power function is almost equal to one, which indicates that the viscoplastic strain rate basically keep unchanged on this condition. The separated viscoplastic strain and viscoplastic strain model can match well at the two creep stress levels and three temperatures (the R^2 values are larger than 0.9).



(a)



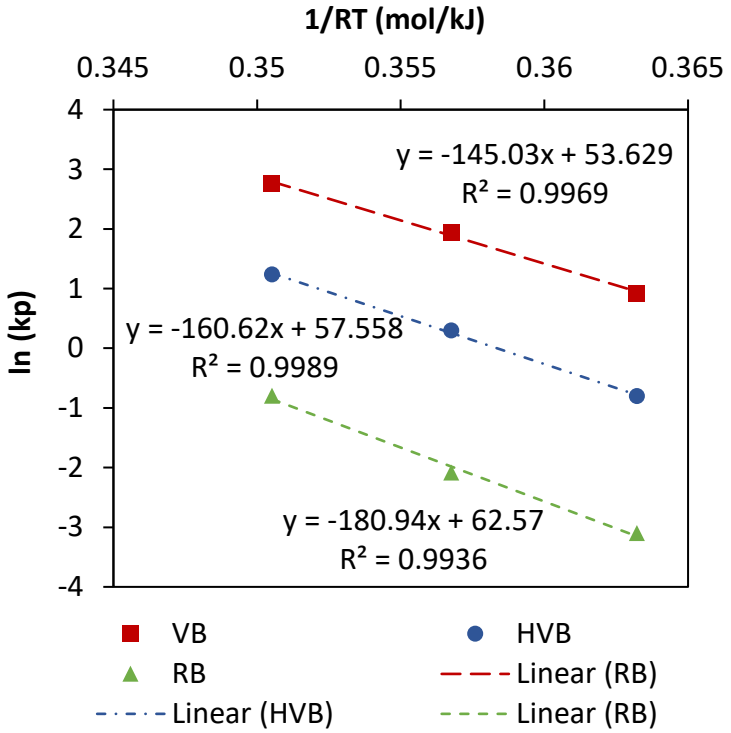
(b)

Figure 9. Relationship between the viscoplastic strain of VB and loading time at two creep stress levels (0.1 and 3.2kPa), three temperatures (58°C， 64°C and 70°C): (a) at 0.1kPa; (b) at 3.2kPa

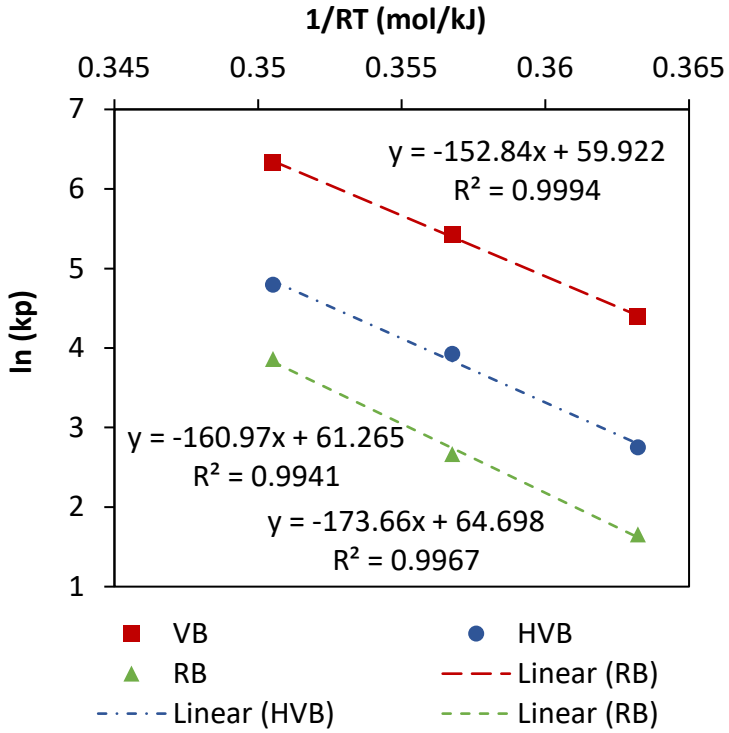
According to Eq. (29), the logarithm of the viscoplastic strain rate is linear with $1/RT$ based on the analysis of the kinetics theory. This rule can be verified by the separated viscoplastic stain at the three temperatures. Figure 10 shows the relationship between the logarithm of the viscoplastic strain rate and $1/RT$ of VB, HVB and RB at two creep stress levels (0.1 and 3.2kPa). $\ln k_{vp}$ increases with the decrease of $1/RT$, because high temperature leads to faster viscoplastic deformation of the asphalt binders. Besides, the relationships between $\ln k_{vp}$ and $1/RT$ of VB, HVB and RB are linear (the R^2 values are larger than 0.99). Hence, Eq. (29) is verified, i.e., the viscoplastic deformation process can be well modeled by the kinetic theory. In this way, the viscoplasticity of asphalt binders at different temperatures is correlated and the viscoplastic strain rate of asphalt binders at different temperatures can be predicted based on the viscoplastic strain kinetics model (Eq. (29)).

Figure 11 presents the viscoplastic activation energy of three types of asphalt binders at two creep

stress levels. It can be seen that the order of the viscoplastic activation energy of the binders is VB<HVB<RB. According to the kinetics theory, the greater the viscoplastic activation energy, the greater the energy required for viscoplastic deformation. Hence, the order of viscoplastic deformation resistance is VB<HVB<RB. Furthermore, the viscoplastic activation energy at the two stress levels (0.1kPa and 3.2kPa) are basically the same, which indicates that the viscoplastic activation energy is independent of the loading level (0.1kPa and 3.2kPa), which is consistent with the findings on the cracking activation energy of asphalt binders (Li et al., 2021). Therefore, the viscoplastic activation energy can be used as a characteristic indicator only related to the viscoplastic deformation process of the nonlinear viscoelastic viscoplastic materials, which can be used to evaluate the viscoplastic deformation resistance for the nonlinear viscoelastic viscoplastic materials.



(a)



(b)

Figure 10. Relationship between the logarithm of the viscoplastic strain rate and 1/*RT* of three types of asphalt binders (VB, HVB and RB) at two creep stress levels (0.1 and 3.2kPa): (a) at 0.1kPa; (b) at 3.2kPa

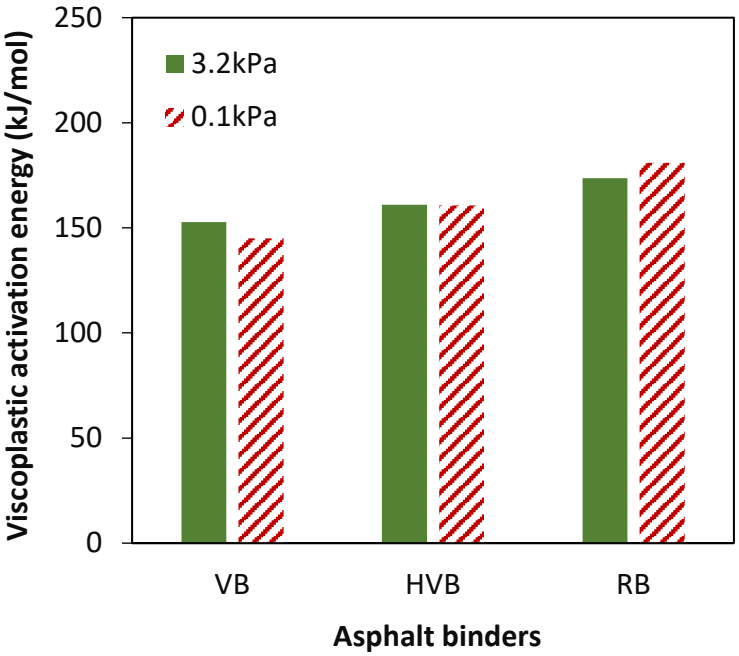


Figure 11. Viscoplastic activation energy of three types of asphalt binders (VB, HVB and RB) at two creep stress levels (0.1 and 3.2kPa)

5. Findings and future work

This study investigates the viscoplasticity coupled with the nonlinear viscoelasticity and proposes a new viscoplastic evaluation indicator that considers the influence of the temperatures and loading levels for the nonlinear viscoelastic viscoplastic materials. The following points summarize the main findings of this study:

- For any nonlinear viscoelastic material, nonlinear viscoelastic parameter g1 only affects the creep strain response, while nonlinear viscoelastic parameter g2 and a_σ both affect the creep and recovery strain response; and the effect of g1 and g2 on the strain response is more significant than a_σ. For asphalt binders, g1 is always equal to one and will not affect the nonlinear viscoelastic behavior of the asphalt binders.
- The proportion of the viscoplastic strain is larger than the nonlinear viscoelastic strain for VB (the proportion exceeds 60%), while it's opposite for HVB and RB; and the proportion of the viscoplastic strain increases with the increase of the temperature and loading time for VB, HVB and RB, while the nonlinear viscoelastic strain presents the opposite trend.
- The non-recoverable creep compliance and percent recovery are redefined based on the viscoplastic strain and nonlinear viscoelastic strain, which can accurately characterize the viscoplastic deformation and strain recovery ratio of asphalt binders comparing with the standard indicator.
- The logarithm of the viscoplastic strain rate has a linear relationship with the reciprocal of the temperature, and it increases with the decrease of the reciprocal of the temperature. The viscoplastic strain rates of asphalt binders at different temperatures are correlated, and their relationship can be predicted based on the established kinetics-based viscoplastic model.
- The viscoplastic activation energy indicator is proposed to characterize the viscoplastic deformation resistance for the nonlinear viscoelastic viscoplastic materials. The greater the viscoplastic activation energy, the greater the energy required for producing viscoplastic deformation. The order of viscoplastic deformation resistance of the asphalt binders is: VB<HVB<RB, based on the viscoplastic activation energy indicator.

In the future study, more materials and conditions will be employed to investigate the nonlinear viscoelasticity and viscoplasticity, which will provide more solid evidence to understand the mechanical behavior of the nonlinear viscoelastic viscoplastic materials.

Acknowledgments

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