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Extended spectral proper orthogonal decomposition for analysis of correlated surrounding flow structures and wind load components of a building

A B S T R A C T Proper orthogonal decomposition (POD) has been used in numerous studies in wind engineering to extract key features of a building's surrounding flow field and surface pressure, the connections between which, however, remain difficult to quantify. This study combined the extended POD with spectral POD (SPOD) method into a new method called extended SPOD (ESPOD) to correlate flow structures with surface pressure. SPOD wind force spectra were defined to quantify how much each pair of velocity and pressure modes contribute to the wind force on a building. The method was validated by a case study on a typical isolated high-rise building, in which periodic coherent structures were extracted to reveal the main mechanisms of the wind forces, including the influences from approaching turbulence, wake vortices, and conical vortices. Phase synchronization, which is utilized in ESPOD, is an effective criterion for distinguishing the multiple physical mechanisms at the same frequency. Additional information provided by the correlated velocity mode helps interpret the physical meanings of the relatively less informative pressure modes. Finally, compared to velocity-based approaches, the pressure-based approach can capture the wind force fluctuations more completely, and the velocity modes are not distorted too much by the non-optimal decomposition.

1. Introduction

Modal decomposition methods are increasingly utilized in wind engineering, and, among which, proper orthogonal decomposition (POD) (Lumley, 1967, 1970) and its variants, have yielded meaningful results. POD is perhaps most used in random multi-variate analysis and is, depending on the field, known by different names (Solari et al., 2007), e. g., principal component analysis (PCA), empirical orthogonal function, and Karhunen-Loeve expansion. Model decomposition (including POD) is indispensable for fluid dynamics because it deals with flows with high Reynolds numbers (i.e., turbulent flows) that result from the superposition and interaction between flows covering a wide range of spatial and temporal scales (Berkooz et al., 1993). Therefore, a low-dimensional description of turbulence is needed to capture the dominant flow features. This need is also highlighted in many applications in wind engineering where the highly turbulent wind flows surrounding a building result in complex aerodynamic loads. POD has since been increasingly applied in wind engineering as a novel analysis tool (Bienkiewicz, 1996). Applications of POD (and POD variants) are categorized according to their object of analysis. Some studies use POD to extract dominant features directly from the velocity field that surrounds a building (hereafter referred to as the 'velocity-based' approach). The main focus of such studies is to understand the wake flow (Kikitsu et al., 2008; F. Wang et al., 2019b; Wang and Lam, 2019; Wang et al., 2014), which is highly related to a building's aerodynamic forces and responses. For example, in Wang et al. (F. Wang et al., 2019a), the energy contribution from different modes at the instants that extreme wind forces occur is studied, showing that the antisymmetric modes represent Karman shedding and generate quasi-periodic across-wind forces. However, as discussed by Zhou et al. (2022a), POD only reveals large coherent structures; this is not to say that POD cannot extract features from small flow structures, but as small structures tend to curl in small spaces, and their low energy makes those modes difficult to identify in the POD spectrum. It is in this respect that spectral POD (SPOD) (Towne et al., 2018) is more advantageous than POD. In SPOD, POD's procedures are carried out in the frequency domain, and the most energetic mode is identified at discrete frequencies, so small structures can thereby be found at their characteristic frequencies. This fundamental idea of SPOD is, in fact, not new (i. e., it has been around since the 1960s (1967)), but a recently developed algorithm (Towne et al., 2018) has breathed new life into it, and it has been used to study the pedestrian-level wind environment (Zhang et al., 2020), pollutant dispersion (Zhang et al., 2022b, 2022c), and indoor ventilation (Zhang et al., 2022a) in architectural and built environments. In these studies, the most energetic SPOD modes provide clear flow patterns that are usually linkable to known physical phenomena of various spatial and temporal scales. Such quantitative flow decomposition also makes it possible to quantify how much the flow structures contribute to the processes that are being studied. Though as a common limitation of linear methods, no POD-based methods can explicitly account for nonlinear interactions (Schmidt, 2020), these methods still help solve engineering problems, as many commonly concerned dynamic behaviors in wind engineering can still be detected by linear correlation. Another category comprises studies that apply POD on surface pressure to analyze a building's wind loads and responses (this is hereafter referred to as the 'pressure-based' approach). For example, in Tamura et al. (1999), POD is used to analyze surface pressure on both low- and high-rise buildings, and the most energetic POD modes can be related to such physical phenomena as vortex shedding and quasi-steady assumption. Kikuchi et al. (1997) show that a building's response can be calculated precisely using wind force reconstructed from selecting just a few dominant modes, indicating the efficacy of using POD to extract the essence of wind forces. As discussed by Davenport (1995), POD can be used to simplify the description of wind loads. A further study has used POD to analyze the wind loads on linked buildings (Kim et al., 2018), and spectral proper transformation (SPT), a variant of POD, has been applied to analyze the surface pressure field (Carassale, 2012; Carassale et al., 2007; Carassale and Marr´e Brunenghi, 2011; Qiu et al., 2014). SPT makes it possible to temporally trace the pressure structures generated by traveling vortices (Carassale and Marr´e Brunenghi, 2011) and describe many independent mechanisms of aerodynamic excitation at different frequencies. One thing of note is that there is no essential theoretical difference between SPT and SPOD; they only differ in their algorithms, so they should be able to extract the same physical structures from the velocity and surface pressure fields. Herein, it should be clarified that decomposition can be used for either data analysis or reduced-order modeling. These are two different routes of study with totally different objectives: The former focuses on the decomposition itself. The modes and energy spectra are observed to gain knowledge of turbulence. The latter, however, aims for efficient data compression and high reconstruction completeness but hardly considers whether the obtained modes are physically meaningful. This study focuses on the former usage of POD and its variants. However, as has been pointed out by Tamura et al. (1999), the physical meaning of the modes is the individual researchers' subjective interpretation of their results and cannot be generalized. Indeed, similar to other modal decomposition methods, POD is a mathematical procedure for correlation analysis, and it does not contain any definitions of flow features or coherent structures in the physical sense. Therefore, the coherent structures extracted through POD can be non-physical, and their interpretation can be erroneous (Carassale and Marr´e Brunenghi, 2011); the same should also be true for POD variants. Notably, this problem tends to plague the pressure-based rather than velocity-based approaches (note, however, that there are fewer velocity-based studies reporting on non-physical coherent structures, and dominant velocity modes seldom depict non-physical structures (Zhang et al., 2022b)). There are two possible reasons for this: (i) POD extracts dominant modes by maximizing their energies (i.e., energy optimality). Therefore, as a criterion for extracting the modes, the definition of the total energy should have a clear physical meaning.1 In this aspect, for the velocity-based approach, the total energy is the total turbulent kinetic energy (TKE) of the flow, and, for the pressure-based approach, it is the total variance in surface pressures. Clearly, the latter is more mathematical but less physical, though the quantities are the same in the statistical sense. Strictly, the pressure-based decomposition should be termed PCA rather than POD. (ii) Surface pressure is a scalar quantity and pertains only to the surface of the building, so it is less informative and more difficult to interpret than the building's surrounding flow field. Though the surface pressure modes actually show something physical, recognizing or relating them to a physical flow structure might be difficult. At this juncture, the question arises concerning whether the velocity and surface pressure modes can be correlated; in other words, whether the velocity modes can help interpret the surface pressure modes, or distinguish the physical surface pressure modes from the non-physical ones. Beyond this question, quantifying and understanding the relationship between the surrounding flow structures and surface pressures is itself another important topic: on the one hand, we hope to relate the surface pressure modes to the flow structures, which makes it possible to relate the dominant wind forces directly to the physical flow structure. This information is also beneficial for aerodynamic modifications (Tse et al., 2021). On the other hand, changes in surface pressure reflect changes in the surrounding wind field to a certain extent, and the latter is related to various problems in the wind environment. In particular, the construction of high-rise buildings needs special alerts. The amplification of wind speed around a high-rise building may result in discomfort or even danger for pedestrians (Kamei and Maruta, 1979; Lawson and Penwarden, 1975; Stathopoulos, 1985; To and Lam, 1995; Tominaga and Shirzadi, 2021; Tsang et al., 2012), and the flow field changed by the building also significantly affects the transfer of heat and air pollutants (Keshavarzian et al., 2020; Zhang et al., 2015). As high-resolution time-resolved velocity data is more difficult to measure both in the wind tunnel and in situ, some studies turn toward the surface pressure data as a basis for estimating the surrounding flow field (Jin et al., 2018; Zhang et al., 2022d) this is expected to become an option for wind environment assessment in the future. To connect the velocity and pressure modes quantitatively, an idea can be listing all velocity and surface pressure data in a single snapshot or flow realization in a single decomposition. By doing so, the corresponding dominant velocity and surface pressure components can be paired and attributed to the same mode. The velocity and pressure modes' respective energies should however be properly normalized and weighted in order for the sum of their energy to be a meaningful quantity in physics. Mathematically, the choice of weight and normalization affects the inner product in POD, and it determines the optimality and orthogonality properties of the modes (Schmidt and Colonius, 2020). One may for example refer to Schmidt and Colonius (2020) and Vogel and Coder (2022) for weighting the density, velocity, and temperature of a compressible flow. However, finding a meaningful combination between the surrounding wind field and surface pressure is difficult. On the one hand, they are defined in different spatial domains. On the other hand, while the velocity-velocity correlation (i.e., TKE and Reynolds stress) is a widely accepted measure of the fluctuation of a fluid flow, the pressure-pressure or velocity-pressure correlation is hardly a meaningful measure. Maurell et al. (2002) and Bor´ee (2003), who introduced an extended POD (EPOD) to extract correlated events from different flow fields, offer another option: the idea is to find the EPOD modes in the target field that share the mode coefficients (i.e., a temporal basis) with the POD modes decomposed from the original field. In other words, the original POD modes of one field can be "extended" to another field using this method. By doing so, stochastic motions indicated by the POD and EPOD modes are synchronized. In this method, the two fields are decomposed separately, and the weighting and normalization are no longer necessary. EPOD has also been used to correlate several other processes, e.g., wall pressure (Hoarau et al., 2006; Hosseini et al., 2015), thermal transport (Antoranz et al., 2018; Lohrasbi et al., 2021), pollutant dispersion (Wang et al., 2021), laminar separation bubble (Verdoya et al., 2021), and aerodynamic forces on bluff bodies (P. Wang et al., 2019), with coherent structures. The main objectives of this study are to propose a practical decomposition method for (i) convenient quantifying and intuitively understanding the relationship between the surrounding flow structures and surface pressures, and (ii) for identifying the main contributors to the wind force components. Considering that Fourier analysis is commonly used in wind engineering to analyze wind fluctuations (Gu and Quan, 2004; Kim et al., 2015; Zhou et al., 2022b), the data analysis procedure proposed in this paper is conducted entirely in the frequency domain. The energy peaks in wind force spectra are focused on, and the contributors to these peaks are extracted and observed. To this end, the present study begins by following Bor´ee (2003) to correlate velocity with surface pressure but goes further to expand it to SPOD, as SPOD can better depict traveling vortices in the flow field than POD (Carassale and Marr´e Brunenghi, 2011; Zhang et al., 2020) and can therefore be expected to provide better observations. But since the temporal correlations can already be assessed via SPOD's spectral analysis, the idea in the present study is phase-synchronize of the Fourier modes across the blocks in the flow realization of SPOD's batch algorithm of SPOD (Schmidt and Colonius, 2020; Schmidt and Towne, 2019). This extended SPOD (hereafter referred to as ESPOD) directly connects the surrounding flow structures with the surface pressures, which also connects the disjointed research routes of coherent structures and wind forces, as shown in Fig. 1. Secondly, the SPOD wind force spectra are defined to quantify the contribution of each pressure mode—and thus the correlated velocity mode and coherent structure—to the fluctuations in the wind force (Fig. 1). The wind force spectra can be calculated using the modes from either velocity- or pressure-based approach. A case study of an isolated, typical-shaped high-rise building is used to test the proposed analysis procedure, wherein both the velocity- and pressure-based approaches are tested and their results compared.

2. ESPOD and wind force spectra

2.1. SPOD base on batch algorithm

SPOD is an eigen-decomposition of the cross-spectral density matrix. All newly developed methods in the current study are based on an SPOD batch algorithm (Schmidt and Colonius, 2020; Schmidt and Towne, 2019), in which the cross-spectral density matrix is estimated via Welch's method. This section provides brief introductions to the concepts and quantities used in the current study, while one is recommended to refer to the original SPOD papers for a more comprehensive view of this sophisticated method and its delicately designed algorithm. For a generalized zero-mean quantity $\mathbf{Q}(\mathbf{x},t)$, where $\mathbf{x}$ is the spatial coordinates and t denotes time, the snapshots are divided into blocks of specific ranges of t, and the blocks may overlap. Fourier transform is then performed on each block, and the resulting batch data can be used as independent Fourier realizations to estimate converged cross-spectra. After the eigen-decomposition of the cross-spectral density matrix, SPOD modes are obtained. The l-th Fourier realization (l = 1, 2, …, Nb) can be decomposed into the modes as: where $\hat{\mathbf{Q}}^{(l)}_f$ is the temporal Fourier transform of the l-th block $\mathbf{Q}^{(l)}$, f denotes the frequency, and n = 1, 2, …, Nm is the mode number. $\mathbf{\Phi}_{f,n}(\mathbf{x})$ is the n-th SPOD mode at the frequency f, and $a_{f,n}^{(l)}$ is the corresponding mode coefficient for the l-th block. Notably, the form of Eq. (1) is similar to the decomposition of POD with the following two differences: (i) SPOD accepts the samples in the frequency domain, while POD accepts the samples in the time domain; (ii) The position of time (indexed by t) in POD is replaced by the blocks (indexed by l) in SPOD. Each SPOD mode is indexed by a frequency and a mode number. The number of the modes Nm at each discrete frequency can be no larger than the number of realizations Nb, and its value depends on the rank of the cross-spectral density matrix. In practice, however, Nb indicates the number of statistically stationary time series that can be obtained from the data, and this number (which is usually from 10 to 100) can hardly be larger than the degrees of freedom of a flow with a high Reynolds number. Therefore, the cross-spectral density matrix usually has full rank, and Nm and Nb are equal. A tradition of POD-based methods is to assume orthogonality of the modes in the decomposition. The current study expresses this assumption as follows: $\langle\mathbf{\Phi}_{f,n}(\mathbf{x}),\mathbf{\Phi}_{f,m}(\mathbf{x})\rangle=\int_\Omega\mathbf{\Phi}^\dagger_{f,m}(\mathbf{x})\mathbf{\Phi}_{f,n}(\mathbf{x})d\mathbf{x} \int_\Omega d\mathbf{x}$ $=\{0,\ n\neq m\ 1,\ n = m,$ (2) where $\langle\cdot,\cdot\rangle$ denotes the spatial inner product between the two functions. Ω is the range of the sampled spatial points $\mathbf{x}$, which can either be two-dimensional plane or three-dimensional space, and $\cdot^\dagger$ denotes the Hermitian transpose. With the orthogonal modes, the mode coefficients can be computed as: $a_{f,n}^{(l)}=\langle\hat{\mathbf{Q}}^{(l)}_f(\mathbf{x}),\mathbf{\Phi}_{f,n}(\mathbf{x})\rangle$. (3) The mode coefficients also become orthogonal, such that: $\frac{1}{TN_b}\sum_{l=1}^{N_b} a_{f,n}^{(l)}a^{*(l)}_{f,m}=\{0,\ n\neq m\ \lambda_{f,n},\ n = m,$ (4) Where T is the temporal length of each block, and $\cdot^*$ indicates a complex conjugate. $\lambda_{f,n}$ is the mode energy, which is also the eigenvalue of the cross-spectral density matrix. It can also be proved via Parseval's theorem that the sum of the mode energy is equal to the energy of the original time series of $\mathbf{Q}$, that is: $\overline{\mathbf{Q}^\dagger\mathbf{Q}}=\int_{-\infty}^{+\infty}(\sum_{n=1}^{N_m}\lambda_{f,n})df$, (5) where the overbar denotes the average over both space and time. For example, if $\mathbf{Q}$ is the velocity field, then $\overline{\mathbf{Q}^\dagger\mathbf{Q}} = 2k$ denotes the spatial-average TKE, and thus, $\lambda_{f,n}$ is the velocity spectra. If $\mathbf{Q}$ is the surface pressure, $\overline{\mathbf{Q}^\dagger\mathbf{Q}} = \overline{p^2}$ denotes the spatial-average temporal-mean square of the surface pressure, and $\lambda_{f,n}$ is the pressure spectra. *2.2. ESPOD* In the current study, we let the quantity $\mathbf{Q}$ become the velocity field around a building or the building surface pressure whose means are subtracted before the analysis. ESPOD's aim is to share the mode coefficients between the decompositions of the velocity realizations $\hat{\mathbf{u}}^{(l)}_f$ and surface pressure realizations $\hat{p}^{(l)}_f$. This target can be expressed as: $\hat{\mathbf{u}}^{(l)}_f(\mathbf{x}_u) = \sum_{n=1}^{N_m} a_{f,n}^{(l)}\boldsymbol{\varphi}_{f,n}(\mathbf{x}_u)$ $\hat{p}^{(l)}_f(\mathbf{x}_p) = \sum N_m$

$\sum_{n=1} a_{f,n}(l)\psi_{f,n}(\mathbf{x}_p)$, (6) where $\mathbf{x}_u$ and $\mathbf{x}_p$ are spatial coordinates where the velocities and surface pressures have been sampled, and $\boldsymbol{\varphi}_{f,n}(\mathbf{x}_u)$ and $\psi_{f,n}(\mathbf{x}_p)$ are a pair of velocity and pressure modes. Note that the mode coefficients contain information on phase (akin to Fourier coefficients), so the shared mode coefficients $a_{f,n}$ indicates the phase synchronization between the velocity and pressure modes across the blocks of flow realizations. If the decompositions indicated by Eq. (6) are accomplished, the two-point cross-spectrum between the velocity and surface pressure at any two arbitrary spatial points $\mathbf{x}_u$ and $\mathbf{x}_p$ can be written as: $\mathbf{S}_f(\mathbf{x}_u, \mathbf{x}_p) = \frac{1}{TN_b}\sum_{l=1}^{N_b} \hat{\mathbf{u}}^{(l)}_f(\mathbf{x}_u)\hat{p}^{(l)*}_f(\mathbf{x}_p) = \sum_{n=1}^{N_m} \lambda_{f,n}\boldsymbol{\varphi}_{f,n}(\mathbf{x}_u)\psi^*_{f,n}(\mathbf{x}_p)$. (7) In the deduction of Eq. (7), orthogonality of the mode coefficients has been applied. The summation on the right-hand side of Eq. (7) contains no cross products between modes with different mode indices. This indicates that only flow motion corresponding to the velocity mode $\boldsymbol{\varphi}_{f,n}$ correlates to the pressure mode $\psi_{f,n}$ (Borée, 2003), and vice versa. Unfortunately, the two decompositions in Eq. (6) cannot both be SPOD. Orthogonality in POD ensures energy optimality in the decomposition (Berkooz et al., 1993), which makes the resulting decomposition unique. Optimizing the energy of both velocity and pressure is an over-constraint on a single set of mode coefficients indicated by Eq. (6). Therefore, the strategy of EPOD provided by Borée (2003) is followed: we either decompose velocity or surface pressure by SPOD, and the modes of the other is obtained by the given mode coefficient. Though the latter non-SPOD modes (hereafter called ESPOD modes) are not orthogonal, the field can be reconstructed by these modes via Eq. (6). In this section, we introduce ESPOD using the pressure-based SPOD as an example (i.e., tracing the route marked out by the blue arrows in Fig. 1). The other approach can also be implemented similarly. With the mode coefficient $a_{f,n}$ and mode energy $\lambda_{f,n}$ obtained from the SPOD of the surface pressure, the ESPOD velocity modes can be computed as: $\boldsymbol{\varphi}_{f,n}(\mathbf{x}_u) = \sum_{l=1}^{N_b} \left(\frac{a^*_{f,n}(l)}{N_m T \lambda_{f,n}} \right) \hat{\mathbf{u}}^{(l)}_f(\mathbf{x}_u)$. (8) Eq. (8) is an adaptation of the definition of the extended EPOD modes in Borée (2003). Eq. (8) can also be proved by substituting $\hat{\mathbf{u}}^{(l)}_f(\mathbf{x}_u)$ with the expression in Eq. (6) and using the orthogonality property of Eq. (4). While in EPOD the ensemble average is calculated over snapshots at multiple time instants, and in ESPOD it is over Fourier realizations from multiple blocks (Towne et al., 2018). Other mathematical derivations are the same as Borée (2003). These velocity modes are used to reconstruct the velocity realizations in Eq. (6). This raises the question that whether the velocity realizations can be reconstructed with no residuals. The answer is: yes in the application of SPOD to a high-Reynolds-number flow. As discussed in Section 2.1, in practice, N_b is usually not large, and the cross-spectral density matrix at each discrete frequency usually has full rank. Therefore, if all of the mode coefficients are used ($N_m = N_b$), an N_b-dimensional hyperspace can be expanded, in which the N_b velocity realizations can be fully represented. This proves that a zero-residual reconstruction in Eq. (6) is achievable under the premise that all of the modes are used. One may notice that there may exist a residual term that is not contributed by the EPOD modes in Borée (2003). This may happen in some applications of POD, but it is hardly the case in SPOD of a high-Reynolds-number flow. A detailed discussion is provided in Appendix. The TKE can also be reconstructed, though it differs slightly from Eq. (5) and becomes $2k = \int_{-\infty}^{+\infty} \underbrace{\sum_{n=1}^{N_m} \lambda_{f,n}\langle \boldsymbol{\varphi}_{f,n}(\mathbf{x}_u), \boldsymbol{\varphi}_{f,n}(\mathbf{x}_u)\rangle}_{\text{ESPOD mode energy}} df = \int_{-\infty}^{+\infty} \sum_{n=1}^{N_m} \tilde{\lambda}_{f,n} df$. (9) Eq. (9) shows the energy budget of the ESPOD modes, which is almost identical to that in Borée (2003) and differs only in the disappearance of the residual term. When Eq. (9) is compared to Eq. (5), the additional term $\langle \boldsymbol{\varphi}_{f,n}(\mathbf{x}_u), \boldsymbol{\varphi}_{f,n}(\mathbf{x}_u)\rangle$, which should be unity in SPOD, is now not unity and is involved in the ESPOD mode energy $\tilde{\lambda}_{f,n}$. This is because the ESPOD velocity modes are no longer orthogonal. All other properties of $\tilde{\lambda}_{f,n}$ are the same as the SPOD energy $\lambda_{f,n}$. The deductions are similar when the red route in Fig. 1 is followed. Whether to choose the velocity- or the pressure-based approach depends on whether we wish to follow the mathematical assumption of velocity-or pressure-mode orthogonality, and this affects energy optimality in the decomposition. While there still lacks a widely accepted mathematical definition for coherent turbulent structures, both approaches seem reasonable. In practice, the choice of approach should depend on the purpose of the downstream applications. For example, when POD or SPOD is applied to model the equivalent static wind loads on building (Chen and Zhou, 2007), orthogonality between the pressure modes makes it possible to directly express the total response as a linear combination of the individual response components contributed by pressure modes without having to consider the complexity in the equivalent load's dependence on the individual response. Further analysis on the ESPOD velocity modes can provide information on the causes of the loads. However, when focusing on the surrounding turbulent coherent structures and their possible effects on the building's wind load, such as Wang et al. (2019b), it is more meaningful to identify the turbulent coherent structures in terms of TKE than the mean square of surface pressure. The results of the two approaches are given in Section 4. *2.3. Wind force decomposition and spectra* Wind force spectra quantify the contributions of the modes on wind forces. For the wind force, this study only considers the base bending moments of the building; the other forces, such as shear forces and local forces at various levels of the building, can be similarly deduced. The base bending moment vector, comprising the *x*-, *y*-, and *z*-components, is the cross product between the position and force vectors. Assuming that the origin of the coordinate system is at the bottom center of the building, the moment vector **M** can be expressed as: $\mathbf{M}=\begin{bmatrix} M_x & M_y & M_z \end{bmatrix}^T=\int_{\Omega_p} \mathbf{x}_p \times (-p\mathbf{n})ds=\int_{\Omega_p} p\mathbf{W}ds$, (10) where **n** is the normal to the building surface Ω_p. $\mathbf{W} = -\mathbf{x}_p \times \mathbf{n}$ is the weight vector, which is a function of the coordinates $\mathbf{x}_p$ and is the only part that needs to be changed when other aerodynamic forces, such as cross-wind and along-wind shear forces, are the analysis target. Notably, the surface pressure *p* defined in this section is pre-processed such that it has zero mean, so the moment in Eq. (10) also has zero mean. In addition, note that the shear stresses acting on the building surfaces are neglected when computing wind forces, because they are generally non-dominant (Leonardi and Castro, 2010; Perry et al., 1969) and non-critical for structural design (Tamura et al., 2001). It is thus a common practice to neglect the shear stresses in wind engineering, e.g., Kim et al. (2015) and Thordal et al. (2020). Subsequently, we can quantify the contributions of the pressure modes on the wind forces using the following quantity: $\mathbf{\Theta}_{f,n} =\lambda_{f,n} \left| \int_{\Omega_p} \psi_{f,n}\mathbf{W}ds \right|^2$, (11) where $|\cdot|$ denotes the modulus of a complex number, and the pressure mode can be either SPOD or ESPOD. It can be proved that the sum of $\mathbf{\Theta}_{f,n}$ equals the mean square of the moment, i.e., $\begin{bmatrix} \overline{M_x^2} & \overline{M_y^2} & \overline{M_z^2} \end{bmatrix}^T = \int_{-\infty}^{+\infty} \sum_{n=1}^{N_m} \mathbf{\Theta}_{f,n}df = \int_{-\infty}^{+\infty} \sum_{n=1}^{N_m} \begin{bmatrix} \Theta^{(x)}_{f,n} & \Theta^{(y)}_{f,n} & \Theta^{(z)}_{f,n} \end{bmatrix}^T df$. (12) It follows that SPOD wind force spectra can be provided by the three components in vector $\mathbf{\Theta}_{f,n}$.

3. A case study of an isolated building

3.1. Case description and simulation

This study uses a typical high-rise building as a case study. The time history data of the surrounding wind field and surface pressure was obtained by a validated large-eddy simulation (LES) conducted by Zhang et al. (2022d). The geometry of the building was set according to Case A in the guidebook issued by the Architectural Institute of Japan (AIJ) (Architectural Institute of Japan, 2016). The building had a square cross section with its height twice as long as its width (details can be found on AIJ's website: https://www.aij.or.jp/jpn/publish/cfdguide/index_e. htm. Fig. 2 shows the simulation domain. In this LES, the building's width *b* was set to be 0.1 m. The building was placed in a wind tunnel 1.2 m wide and 1 m high. The inlet and outlet of the tunnel were 0.6 m and 1.55 m away from the center of the building, respectively. The Reynolds number, calculated using the building's width and the mean wind speed at the top of the building at the inlet, was 31,000. The flow field was calculated via LES using the open-source CFD code OpenFOAM v8 (The OpenFOAM Foundation Ltd, 2020), by which the filtered Navier–Stokes equations for incompressible flow were solved. The wall-adapting local eddy-viscosity model (Nicoud and Ducros, 1999) was utilized to model the subgrid-scale turbulence. The pressure implicit with splitting order (Issa, 1986) was applied as the pressure–velocity calculation procedure for the continuity and momentum equations. The time-history velocity data input at the inlet boundary was provided by Okaze et al. (2021), who ran an additional LES to reproduce a turbulent flow through the roughness blocks and spires in the wind tunnel. The generated wind was assumed to be in roughness category IV specified by *AIJ recommendations for load on building* (Architectural Institute of Japan, 2015), and the profile of the mean wind should follow the power law with $\alpha = 0.27$. The turbulent flow data is available on the AIJ website (https://www.aij.or.jp/jpn/publish/ cfdguide/index.htm). The mean wind speed at the building height at the inlet was approximately 4.7

m/s, which was chosen as the reference wind speed (U_H) in this study. Please refer to Zhang et al. (2022d) for other details on the boundary conditions, grid resolution, and solver settings. *3.2. Data sampling and validation* The LES was run for 120 s in total, but sampled only for the last 90 s to remove any influence from initial conditions. The time step was adjusted automatically to be within the range of 1.6–2.5 × 10^{-4} s to keep the Courant's number lower than 1 during the first 30 s (without sampling), but was held constant at 1.6 × 10^{-4} s during the subsequent 90 s. The preparation time of 30 s is approximately equal to 1400 b/U_H and 65 times as long as the flow passes through the tunnel. Therefore, the flow field was considered fully developed in the last 90 s. The velocity was sampled at all cell centers within the spatial range of $-1.5b < x < 3.5b$, $-2b < y < 2b$, and $0 < z < 3.5b$ (see Fig. 2), where approximately 0.33 million cells were included. The surface pressure was also sampled at the center of the cell faces on the building (9600 cell faces included in total). The sampling frequency was 625 Hz, and 56,250 snapshots were obtained in LES. Fig. 3 provides basic statistics for the sampled velocity and surface pressure fields, in which surface pressures are expressed in the form of pressure coefficients defined by $C_p = p/(0.5\rho U_H^2)$. As this is a classical case study of wake flow and building aerodynamical features, similar mean velocity and TKE fields (Okaze et al., 2021; F. Wang et al., 2019b; Yoshie et al., 2007) and mean and standard deviation of the surface pressure fields in (Thordal et al., 2020; Zhou et al., 2021) can be found in previous studies. Notably, the velocity and pressure fields on the two sides and near the windward edges of the building have large variances. This is expected to be highly related to the variation of wind load components, especially the cross-wind and torsional ones. This is one of the main concerns in the flow decomposition analysis. The simulation has been partially validated by Zhang et al. (2022d) by comparing the LES results with the experimental data, while the current study provides more such comparisons for comprehensive validation. For the surrounding velocity field, Zhang et al. (2022d) provided the mean and standard deviation of the streamwise velocity only on the plane of $z = 0.125b$ and compared them with the experimental data provided by AIJ, while further comparisons on the planes of $z = 1.25b$ and $y = 0$ are provided in Fig. 4. For the surface pressure, the experimental data for the same building geometry and approaching flow conditions can be found in a database provided by Tokyo Polytechnic University (TPU) (available at http://wind.arch.t-kougei.ac.jp/sys tem/eng/contents/code/tpu). Please refer to Zhang et al. (2022d) for the comparison of the mean and standard deviation of the surface pressure coefficients between the current LES and TPU data. Moreover, the power spectral densities (PSDs) of the three components of the base bending moments are compared in Fig. 5. All the above show that the simulation and experimental results are in good agreement. The LES results are in high accuracy and thus reliable in all the above aspects.

4. Results and discussion

4.1. Results of the pressure-based decomposition

In the pressure-based approach, the pressure modes are extracted by SPOD, and the velocity modes are extracted by ESPOD according to phase synchronization with the pressure modes across the flow realizations (blue route in Fig. 1). The tasks in this case are to interpret the physical meanings of the surface pressure modes by the corresponding velocity modes and quantify how much the surrounding flow structures contribute to the fluctuation of the base bending moment. In the implementation of SPOD, the cross-spectral density matrix was estimated via Welch's method (Welch, 1967). The sampled time history data was divided into 29 (= N_b) blocks with 50% overlap, which means that the maximum number of available modes at each discrete frequency (N_m) is 29. A Hamming window was imposed on the Fourier transform to avoid any adverse effect from spectral leakage, as caused by the truncation of the data. Fig. 6 provides the SPOD spectra of the surface pressure field, velocity field, and three components of the base bending moment including cross-wind moment M_x, along-wind moment M_y, and torsional moment M_z. Each discrete point in a spectrum corresponds to a pair of modes associated with a pair of periodic motions within the surface pressure and velocity fields. For example, the modes at the circled points are provided in Figs. 7–11. The vertical axes of the spectra in

Fig. 6 indicate contributions of the periodic motions to the mean variance of surface pressure, mean TKE, and variances of the moment components, and these quantities are also used to normalize the spectra. For the wind force spectra, the Fourier spectra (i.e., the PSDs in Fig. 5) are also provided in Fig. 6, which are calculated directly from the time history data of the moment components by Welch's method (Welch, 1967) with the same division scheme. The Fourier spectra show the total energy at each discrete frequency. Though the modes in Figs. 7–11 are presented as statical images of their real (Re) and imaginary (Im) parts, they should be understood as periodic motions which are temporally sinusoidal waves with continuous phase shifts (see Videos 1–5). One cycle of the periodic motion is approximately Re → –Im → –Re → Im → Re, which corresponds to the phase shifting from 0 to 2π with the interval of $\pi/2$. Supplementary video related to this article can be found at https://doi.org/10.1016/j.jweia.2023.105512 In addition, the modes depict periodic motions that only exist in mean-subtracted turbulence flow fields. Therefore, the flow motions depicted by the modes may not match our usual perceptions of flow fields, and it may be helpful to add the mean flow back onto the modes before interpreting its physical meaning, i.e., considering (mean ± αRe) or (mean ± αIm), where α is a positive coefficient that relates to the total variance of the velocity or surface pressure (Zhang et al., 2022a). For example, in the velocity mode in Fig. 8, the opposite directions of the flow on the two sides of the building do not necessarily indicate a reverse flow against the mean flow. A more reasonable interpretation is that the reconstructed flow is stronger than the mean flow on one side but is weaker on the other side, and the extent depends on the mode coefficient or energy.

4.2. Mode analysis based on the pressure-based decomposition

For analytical purposes, we are more concerned with the physical meanings of the most energetic modes than the complete accounting of energy. Therefore, to grasp the main features of the current flow fields, observing the physical meanings of only one or two of the most energetic modes at several characteristic frequencies will suffice. The mode number (n) in this SPOD is determined by the contribution to the mean variance of surface pressure in the surface pressure spectra, but the most energetic modes in the surface pressure spectra do not necessarily make the greatest contributions in the other spectra. Fortunately, it seems that the modes with $n = 1$ and 2 can cover all the main features in all the spectra in Fig. 6. Subsequently, the characteristic frequencies are to be decided. First, the frequency multiplied velocity spectra in Fig. 6 exhibit a □ 2/3 slope before $fb/UH \approx 1$, which corresponds to Kolmogorov's □ 5/3 law for the inertial subrange (Pope, 2000), and the energy decays quickly after $fb/UH \approx 1$. This indicates that, at the very least, the inertial-subrange turbulence within $fb/UH < 1$ has been resolved in the current LES, and the observation should not be beyond this range. Second, Carassale et al. (2007) have found two dominant frequencies in the POD and SPT analysis for the surface pressure of a tall building: a low frequency around $fb/UH \approx 0.01$, depending on the approaching turbulence buffeting, and a frequency around $fb/UH \approx 0.1$, corresponding to the vortex shedding. Finally, the wind force spectra in Fig. 6 show energy peaks, and interpreting these energy peaks is our top priority in this study. Considering all the above, the modes at the frequencies of fb/UH = 0.02, 0.10, and 0.32 are observed in the following analysis. At low frequency ($fb/UH = 0.02$), the periodic motions of surface pressure and surrounding flow are mainly caused by turbulence in the approaching flow. Points A1 and A2 in Fig. 6 show left-right symmetric and antisymmetric periodic motions, respectively. Fig. 7 shows the modes at point A1, wherein the SPOD surface pressure mode mainly shows a pattern similar to the mean surface pressure field in Fig. 3, which may be interpreted as the consequence of the low-frequency change in the intensity of the approaching flow. More straightforwardly, it shows that the approaching flow is sometimes strong and sometimes weak, and the characteristic frequency of this fluctuation is usually lower than that of the shedding vortices. The ESPOD velocity mode extracted according to the phase synchronization with the pressure mode satisfactorily justifies this interpretation. The velocity mode also shows a flow pattern similar to the mean flow in Fig. 6. However, slight antisymmetric flow patterns can also be observed in both of the imaginary parts of velocity and pressure modes, which may be caused by the slight asymmetry of the approaching flow given at the inlet. This does not change the modes' main contributions to the aerodynamic forces. In Fig. 6, this symmetric mean-flow-like flow structure significantly affects the fluctuation of the along-wind moment, whereas its influence on the cross-wind and torsional moment fluctuations are less significant than the antisymmetric flow structure at the same frequency. Fig. 8 shows the modes at point A2. These modes exhibit an antisymmetric flow pattern caused by the antisymmetric components in the approaching flow, i.e., the approaching flow sometimes leans towards the $y > 0$ side and sometimes towards the $y < 0$ side. As mentioned, the opposite flow directions on the two sides of the building do not necessarily indicate reverse flow. When the real part of the velocity mode is linearly combined with the mean flow, it shows that the flow on the $y < 0$ side is stronger than on the $y > 0$ side. Correspondingly, the surface pressure mode exhibits a gradient color on the sidewall of the building in Fig. 8, indicating the change in the flow separation and reattachment. For example, on the $y < 0$ side, both the separation and reattachment are strengthened, and thus both positive and negative fluctuations occur on the same surface on the wall. This surface pressure pattern is the main contributor to the torsional moment fluctuation and is the minor contributor to the cross-wind moment fluctuation, as shown in Fig. 6. For the square cylinder, the frequency at $fb/UH \approx 0.10$ is known as Strouhal frequency (Sakamoto and Arie, 1983; Wang and Lam, 2019). The most energetic modes at this frequency in both velocity and pressure spectra in Fig. 6 (point B1) illustrate the same flow mechanism, that is the vortex shedding in the wake, as shown in Fig. 9. The pressure mode shows an antisymmetric distribution. Considering the periodic motion indicated by the modes (Re → –Im → –Re → Im → Re), it shows that pressure fluctuations on the two sidewalls emerge from the windward edge, travel downstream, and finally disappear on the leeward wall. As indicated also by Carassale and Marr´e Brunenghi (2011) according to a similar mode obtained by SPT method, this is a temporal tracing of the pressure structures generated by traveling vortices. The current ESPOD method provides an additional velocity mode that allows for better observation of these traveling vortices. It shows not only vortex shedding in the wake but also vortices that have separated from the windward edges. The three vortices travel downstream with the same speed and merge in the wake. For the aerodynamic forces, this flow mechanism contributes mainly to the cross-wind moment fluctuation, as shown in Fig. 6. The second energetic mode at the same frequency (point B2 in Fig. 6) shows very different mechanisms. As shown in Fig. 10, the pressure mode shows a nearly symmetric distribution, but the fluctuations on the two sidewalls behave the same as those associated with the vortex shedding. Such a surface pressure pattern can also be found in Carassale and Marr´e Brunenghi (2011), but the corresponding flow mechanism was not clearly interpreted. The ESPOD velocity mode shows a similar mechanism to the mode at point A1, and the flow structure results from the symmetric components within the approaching flow. However, at this frequency where $fb/UH = 0.10$, the decomposition tends to extract the relatively small-scale components. The vortices in the approaching flow can be observed at the upstream, and they cause a more complicated flow pattern around the building when they travel downstream. As this distribution is a nearly symmetric, it contributes to the along-wind moment fluctuation, as shown in Fig. 6. Finally, the modes at frequency $fb/UH = 0.32$ (point C in Fig. 6) show relatively small vortex shedding on the two sides of the building (Fig. 11). These are also called conical vortices (Okuda and Taniike, 1993), whose separation and reattachment characteristics are assumed to affect the surface pressure fluctuations (Unnikrishnan et al., 2017; Zhou et al., 2021). In this study, we can quantify their influence on the wind forces via the spectra in Fig. 6. Though the pressure mode in Fig. 11 shows an antisymmetric distribution, it contributes little to the cross-wind moment fluctuation, because positive and negative fluctuations on the same sidewall surface almost nullify each other. However, it is related to the trailing-edge peak pressures, which are severe suctions on the building's sidewalls, as identified by Cao et al. (2022b, 2022a) using a method called conditional POD. In addition, the current study finds it an important flow mechanism that contributes to the torsional moment fluctuation, as shown in Fig. 11. The above analysis covers the main flow structures that have the most influence on the three components of the wind forces. The remarkable points and advantages of the proposed analysis procedure are: (i) Multiple flow structures with different physical mechanisms may occur at the same frequency, and they affect wind forces in different ways. Note that the most energetic modes in SPOD do not necessarily contribute the most to all the moment components. A notable example is the mechanisms shown by the modes at points A1 and A2. The symmetric components of the approaching turbulence (A1) contribute more to the pressure and along-wind moment fluctuations, whereas the antisymmetric to the velocity, cross-wind moment, and torsional moment fluctuations. (ii) The additional information provided by the ESPOD velocity mode helps interpret the physical meanings of the SPOD pressure modes which are less informative. For example, the SPOD pressure modes at A2 and B1 seem similar in shape, and it might be difficult to distinguish the corresponding flow structures without the ESPOD velocity modes. In the current simple case of a square-section isolated building, one may obtain hints from the prior knowledge that the frequency at B1 is related to the Strouhal frequency (Sakamoto and Arie, 1983; Wang and Lam, 2019). However, there might be little information for the mode at A2, and there might be no reference for the vortex-shedding frequency from a building with complex geometry or aerodynamical interference. With the ESPOD velocity modes, the corresponding surrounding flow structures can be visualized, and this information can help research and design buildings and structures. (iii) Phase synchronization also ensures the correlation between the velocity and surface pressure within any snapshot of the periodic motions. For example, the real parts of the surface pressure modes are physically linked to the real parts of the velocity modes, and the two's imaginary parts are also linked. This facilitates the understanding of the velocity-pressure relationship, such as the effect of the separation and reattachment of the vortices on the sidewalls. *4.3. Results of the velocity-based decomposition and comparison* In the velocity-based approach, the velocity modes are extracted by SPOD, and the pressure modes are extracted by ESPOD according to phase synchronization with the velocity modes across the flow realization (red route in Fig. 1). The tasks are to identify the coherent structures according to the velocity modes and quantify their contributions to surface pressure and wind forces. A comprehensive SPOD analysis for the velocity field around a square prism with the same geometry has been provided by Zhang et al. (2021): though a different time-history dataset was used at the inlet boundary in the LES, the mean and turbulence intensity profiles were almost the same as those in the current study, and LES results were also in good agreement with the experimental data provided by AIJ. Therefore, the coherent structures identified from the velocity field are considered the same as those in Zhang et al. (2021), and the discussion in the current study focuses on energy distributions. Fig. 12 shows the SPOD spectra obtained via this approach, wherein the discrete points at the same frequencies as those in the pressure-based approach are circled. The velocity and surface pressure modes at these points are provided in Supplementary Material, as they are considerably similar to those in Figs. 7–11. The differences lie mainly in (i) the small details on the shapes of the vortices and (ii) the flow structures at non-significant locations where the modes present low values. Relatively, the velocity modes in the velocity-based approach are slightly more reasonable in the physical sense. For example, the velocity mode at point B1 in the velocity-based approach shows the merge of the three vortices more naturally than that in the pressure-based approach (Fig. 9). On the contrary, the pressure fluctuations shown in the pressure modes in the pressure-based approach are smoother and more regular. For example, the pressure mode at point B1 in Fig. 9 shows a better antisymmetric distribution than that in the velocity-based approach. This is because the velocity-based approaches. If the modes are extracted by optimizing their energy percentages in the pressure variance, they must show the most characteristic fluctuation patterns in the pressure field, while their energy percentages in the TKE can never be optimized, unless the relationship between the pressure and velocity are linear, which is never true. The energy optimality in multiple fields could not be fulfilled simultaneously (Wang et al., 2021). The consequences of the different strategies of energy optimization are negligible when interpreting the physical meanings according to the visualizations of the modes, while this issue may have some influences on the analysis of the wind forces. In this aspect, we can compare how close the energy values of the most energetic modes are to the Fourier spectra of the wind forces in Figs. 6 and 12. It can be seen that the points of the most energetic modes almost cling to the Fourier spectra in Fig. 6, but are much farther from the Fourier spectra in Fig. 12. This indicates that the pressure-based analysis can better capture the coherent structures in the surface pressure field that is most influential to the wind forces, but the velocity modes may not optimally depict the key coherent structures in the velocity field. To quantitatively analyze the consequences of the different strategies of energy optimization, Fig. 13 provides the energy percentage when reconstructing the signal using a designated number of the most energetic modes. In total, there are Nm × Nfft modes (where Nfft is the length of the signal in the discrete Fourier transform). The energy percentage for a generalized quantity **Q** is defined as: Percentage=$\Sigma S(\mathbf{Q})\ f,n\ \Delta f\ \mathbf{Q}^\dagger\mathbf{Q}$ × 100%, (13) where $S(\mathbf{Q})\ f,n$ is the spectra of the quantity **Q**. $S(\mathbf{Q})\ f,n$ is the SPOD mode energy $\lambda f,n$ or ESPOD mode energy $\tilde{\lambda} f,n$ when **Q** is the velocity or surface pressure field, and $S(\mathbf{Q})\ f,n$ is a component in the vector $\boldsymbol{\Theta} f,n$ when **Q** is a component of the wind force moment. As illustrated in Fig. 13, the discrepancies in the reconstructed energy of velocity and surface pressure fields are not large. Taking the reconstruction using 100 modes as an example, the discrepancies in the energy percentage values are only 4% and 7% between the optimal and non-optimal approaches. This partly explains why the modes obtained from the pressure- and velocity-based approaches are similar. However, the discrepancies in the reconstructed energy of both the along-wind and cross-wind moments are larger. In this perspective, the pressure-based approach might be a better choice when the wind forces are the final target of analysis, as the energy of the wind forces can be more completely captured by a designated number of modes, and the velocity modes are not distorted too much by the non-optimal decomposition.

5. Discussion on nonlinear interactions

One limitation of the proposed method might be that the extracted modes are usually not one-to-one corresponding to the physical mechanisms. Notably, POD, EPOD, and Fourier analysis are all linear methods. Because of the universally existing nonlinear interactions, a physical mechanism may need multiple modes to describe. In practice, one may find out that multiple modes belong to the same physical mechanism. For example, a low-Reynolds-number wake of a cylinder needs a low-order reconstruction using several modes to relatively comprehensively describe (Noack et al., 2003). In addition, similar modes can also be found in a range of frequencies (Zhang et al., 2020), because a physical process may occur in a broadband area. Perfectly extracting a physical mechanism from a complex high-Reynolds-number flow remains a large difficulty, because a selection criterion should be made to determine whether a mode belongs to the target physical mechanism. This selection criterion needs a certain degree of knowledge of the target physical mechanism. However, recall that the purpose of modal decomposition itself is to obtain knowledge of a physical mechanism. Fortunately, completely extracting a physical mechanism is seldom a topic that needs special attention in engineering applications. Identifying “broken pieces” or “traces” of coherent structures to interpret the causes of large wind force fluctuations satisfies most wind The section provides a detailed discussion in this regard. Specifically, nonlinear interactions can affect the performance of the proposed analysis method in the following aspects. *5.1. Inter-frequency contributions* A velocity motion at a single frequency may contribute to a pressure motion at other frequencies. However, this is not considered in the current study. The inability to consider the inter-frequency contributions is the limitation of Fourier analysis. Notably, Fourier temporal modes (i.e., sinusoidal functions) are orthogonal modes, which are assumed to have no linear correlations among each other. Therefore, the very act of using Fourier analysis implies that one does not intend to consider the relations between signals of different frequencies. In the case of the current study, Fourier analysis is used inevitably, as the main target is to interpret the energy peaks in Fourier spectra. To discuss this issue in depth, the following Poisson equation of pressure for incompressible fluids is examined: $-\frac{1}{\rho}\Delta p=\nabla\cdot(\nabla\cdot\mathbf{uu})$. (14) Notice that the notations in the section differ from those in previous sections: The pressure $p = \bar{p} + p'$ and velocity $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$ in Eq. (14) have nonzeros temporal means, which are denoted by the quantities with overbars. To apply the Fourier decomposition, the temporal means of both sides of the Poisson equation are subtracted, and this results in the following equation: $-\frac{1}{\rho}\Delta p'=\underbrace{2\nabla\cdot(\nabla\cdot(\bar{\mathbf{u}}\mathbf{u}'))}_{\text{Linear}} + \underbrace{\nabla\cdot(\nabla\cdot(\mathbf{u}'\mathbf{u}' - \overline{\mathbf{u}'\mathbf{u}'}))}_{\text{Nonlinear}}$. (15) Because the velocity and pressure are stationary stochastic processes, one can regard the temporal means as constant and apply the Fourier decomposition only to the fluctuation parts p' and $\mathbf{u}'$. The two terms on the right side of Eq. (15) show the different contributions of a velocity motion to the pressure fluctuation on the left side. The first term is the the Fourier modes of pressure with the corresponding frequency. The second term is the nonlinear term resulting in a contribution to the Fourier modes of pressure with a double frequency. Because the velocity and surface pressure fields are defined in different spatial domains in the current study, the linear and nonlinear contributions are not perfectly described by Eq. (15). However, a qualitative conclusion can still be made based on Eq. (15): The inter-frequency contributions indicated by the nonlinear term are more likely to occur in a case with a higher turbulence intensity. This is perhaps why in those typical studies of Karman wake behind a cylinder, the surface pressure or lift force signal has the same dominant frequency with the velocity field, and the double-frequency energy content can hardly be observed. This indicates that the linear interaction dominates the fluid dynamics in this case. However, in the studies of wind engineering, a 10–20% turbulence intensity at the inlet is usually assumed. This might affect the performance of the current method, and it deserves further quantification in future studies. *5.2. Treatment of in-frequency nonlinear relationships* With the target frequency specified in the proposed ESPOD method, the POD/EPOD processes are the same as those processes applied to a normal time series. It is well-known that POD can only detect linear relationships, but the flow field can be perfectly reconstructed if all modes are used. Similarly, EPOD can only detect linear relationships between velocity and pressure, but the two fields can be perfectly reconstructed by each other, as shown in Fig. 13. This seems to contradict the fact that nonlinear interactions exist universally in the flow. Interestingly, even though the two datasets are random sets with no correlated contents at all, the POD/EPOD algorithm can normally extract modes and reconstruct both datasets according to the knowledge of linear algebra. However, this does not mean anything in physics, because those extracted modes and coefficients are just random numbers. For flow data, true linear relationships will be detected, as they are associated with those coherent structures. These will be structures can never be perfectly described by linear correlations. This results in the existence of some fake flow structures indicated by POD modes and some fake velocity-pressure relationships indicated by EPOD pairing. Fortunately, they seldom become the leading modes, so they can hardly be noticed in a study that aims only for data analysis. However, this will become an issue in the flow prediction. For example, it seems that a new dataset of velocity can be predicted by Eq. (6) using the extracted EPOD velocity modes obtain from the old velocity data and the new mode coefficients from the new pressure data. This is equivalent to a linear stochastic estimation (LSE), as proved by Bor´ee (2003). However, errors will occur because the fake linear relationships identified in the current dataset will not be inherited in the future realizations of the flow fields.

6. Concluding remarks

This study has proposed a new analytical method called ESPOD (a combination of EPOD and SPOD) to identify the correlations between a building's surrounding flow structures and surface pressure, which are depicted as periodic events by the modes. ESPOD extracts modes from a different flow field according to the phase synchronization of the SPOD modes across the flow realizations. In addition, the SPOD wind force spectra were defined to quantify how much each pair of velocity and pressure modes contribute to the wind force fluctuations. A case study of a typical isolated high-rise building was conducted to validate the proposed analysis procedure. In the decompositions, we extracted periodic coherent structures from the velocity and surface pressure fields that reveal the main mechanisms of the wind forces. Such mechanisms include the flow patterns caused by the symmetric and shedding in the wake and on the two sides of the building, and conical vortices on the two sides of the building. Notably, multiple flow structures caused by different physical mechanisms were found to occur at the same frequency, and they contribute to different components of wind forces. As an analysis procedure, ESPOD is more advantageous than the original POD or SPOD for the following reasons: (i) Because multiple physical mechanisms can occur at the same frequency and contribute differently, neither frequency nor energy value cannot be used as a criterion to determine whether two events in the velocity and surface pressure fields are caused by the same physical mechanism, but this can be judged via the phase synchronization in ESPOD. (ii) Additional information provided by the corresponding velocity mode helps interpret the physical meanings of the relatively less informative pressure modes. In general, the flow separation and shedding vortices are more clearly recognized in a velocity mode than in a pressure mode. While some coherent structures can result in a similar appearance in surface pressure fluctuation, such as those in Figs. 8 and 9, they are usually distinguishable in the velocity mode. (iii) Phase synchronization also ensures a correlation between velocity and surface pressure at any snapshot within the periodic motions, which further facilitates the understanding of the velocity-pressure relationship. Finally, the pressure- and velocity-based approaches were compared in the case study. As the energy in the surface pressure field was optimized in pressure-based decomposition, the pressure modes can better capture the coherent structures within the surface pressure field that is most influential to the wind forces, but the corresponding velocity mode may not optimally depict the key coherent structures in the velocity field. On the contrary, in velocity-based decomposition, the coherent structures in the velocity field were better depicted by the velocity modes, but extracted pressure modes were less smooth and regular and were less influential to the wind forces. The energy optimality in multiple fields could not be fulfilled simultaneously. However, according to the quantification of the reconstructed energies, the pressure-based approach was considered a better choice when the wind forces were the final target of analysis, as the energy of the wind forces can be more completely captured by a designated number of modes, and the velocity modes were not distorted too much by the non-optimal decomposition. This study proposed a pure decomposition method only for analytical purposes and did not suggest any low-dimensional representation, which might be a limitation. As discussed in the case study, different coherent structures contribute to different wind force components, and the contributors are not necessarily the most energetic coherent structures of the flow. Therefore, it is suggested to analyze all the modes to obtain a comprehensive understanding of the wind forces. In some where a low-dimensional representation is required, the selection criteria imposed on the modes should be well studied with a comprehensive consideration of all the velocity, pressure, and wind force spectra. The analysis procedure proposed in this study is considered effective in identifying the correlated periodic motions in the velocity and surface pressure fields, and it facilitates the understanding of the contributions of the coherent structures to the wind forces. Further studies can be carried out to examine the wind-induced interference effects on buildings (Hu et al., 2020; Khanduri et al., 1998), wherein the flow structures are more complicated, and the surface pressure fluctuation patterns can be more difficult to interpret. The periodic motions of the surface pressure synchronized on the multiple buildings can be extracted by SPOD, and the corresponding turbulent structures around the buildings can be extracted by ESPOD to aid the interpretations of the physical mechanisms.

CRediT authorship contribution statement **Bingchao Zhang:** Conceptualization, Methodology, Software, Formal analysis, Writing – original draft. **Lei Zhou:** Conceptualization, Methodology, Writing – review & editing. **Tim K.T. Tse:** Conceptualization, Resources, Writing – review & editing, Supervision. **Liangzhu Wang:** Writing – review & editing. **Jianlei Niu:** Writing – review & editing. **Cheuk Ming Mak:** Writing – review & editing. **Declaration of competing interest** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. **Data availability** Data will be made available on request.

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Appendix. Discussion on the Integrity of Reconstruction in EPOD A previous paper on EPOD (Maurell et al., 2002) indicated that a field can be reconstructed by the modes extended from another field. Bor´ee (2003), however, introduced a residual term that cannot be correlated to any mode of the other field, resulting in the inability of perfect reconstruction. These conclusions are not contradictory, because the preconditions used in the deduction in Maurell et al. (2002) are relatively stronger: First, the coefficient matrix should be invertible, that is, it has full rank. Second, the number of available modes is equal to the number of flow realizations, and all those modes are used. With these preconditions, linear algebra will ensure that the latent dimensions of the two data matrices are equal, so they can be perfectly reconstructed by the extended modes from each other. In the applications of SPOD to a high-Reynolds-number flow, the above preconditions are satisfied automatically in most cases: First, as a high- Reynolds-number flow has infinite dimensions, the finite data matrices sampled from the flow usually have full rank. Second, the flow realizations refer to the Fourier realizations, the number of which is equal to the number of blocks in the SPOD algorithm (*N*b), and it is usually much smaller than the number of spatial sensors. In this case, the ranks of both data matrices are *N*b. Therefore, all the vectors in the two data matrices can be linearly represented by an *N*b-dimensional linear basis. If the number of spatial sensors is infinite, this linear representation becomes Eq. (6) using the functional form in Hilbert space. Herein, the set of the chosen linear basis is the SPOD mode coefficient *af*,*n*(*l*) of either velocity or pressure. In other words, a complete reconstruction shown in Eq. (6) can always be achieved when all the *N*m (= *N*b) modes at each frequency are used. However, the preconditions used by Maurell et al. (2002) might be too ideal in a usual POD analysis. The following situations are examples: (i) For a low-Reynolds-number flow, the number of coherent structures might be limited, so the rank might be a finite number that does not change with the matrix's size. In this case, only the rank matters. If the rank of field A is smaller than that of field B, then field B cannot be fully reconstructed by the modes extended from field A, resulting in a non-zero residual term in reconstruction. This probably happens when extending the modes from a simpler field to a more complex one. (ii) The data matrix has full rank, but it is too large in size. This may cause numerical problems (e.g., matrix singularity caused by round-off), resulting in the inability or inefficiency to compute all the modes from the matrix. This is also common in POD applications for fluid analysis. In this case, one prefers to use only a few most energetic modes, and this leads to low-dimensional representation. The residual part that cannot be recovered is caused by the abandonment of the modes. In the situations such as the above, the decorrelated residual term defined in Bor´ee (2003) becomes nonzero. In this regard, the EPOD definition in Bor´ee (2003) is more universal than that in Maurell et al. (2002).

Appendix A. Supplementary data Supplementary data to this article can be found online at https://doi.org/10.1016/j.jweia.2023.105512.

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