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A multi-product and multi-period supply chain network design problem with price-sensitive demand and incremental quantity discount

Hongtao Hu[a], Shuyuan Guo[b], Lu Zhen[c,*], Shuaian Wang[d], Yingying Bian[e]

[a]*School of Logistics Engineering, Shanghai Maritime University, Shanghai, China*
[b]*Institute of Logistics Science & Engineering, Shanghai Maritime University, Shanghai, China*
[c]*School of Management, Shanghai University, Shanghai, China*
[d]*Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Hong Kong*
[e]*School of Economics & Management, Shanghai Maritime University, Shanghai, China*

*Corresponding author.

Email addresses: hthu@shmtu.edu.cn (Hongtao Hu), guo.shuyuan@foxmail.com (Shuyuan Guo), lzhen@shu.edu.cn (Lu Zhen), hans.wang@polyu.edu.hk (Shuaian Wang), yingyingbian@foxmail.com (Yingying Bian)

Abstract

Demand is an important factor in supply chain network design (SCND). Among the many factors that affect demand, the impact of price cannot be underestimated.This study, for the first time, provides enterprises with direct multi-period pricing and demand decision solutions in SCND by simultaneously considering the impact of price-sensitive demand and incremental quantity discounts on the network. In this way, a mixed-integer nonlinear programming model is established based on the relationships between price and demand functions to maximize the overall profit of the supply chain. Due to the nonlinear characteristics of the model, this study proves the existence of the optimal solution of the problem and the feasibility of implementing the following two algorithms by analyzing the nature of the problem. The first algorithm is based on the second-order cone programming (SOCP) method, and the second algorithm is based on the outer-approximation method. The results of numerical experiments at different scales show that the SOCP method can obtain the optimal solution for small and medium-scale experiments. In contrast, the outer-approximation method can find approximate optimal solutions within a reasonable time for large-scale experiments. The results confirm that considering incremental quantity discount can significantly enhance the profitability of supply chain networks in long-term planning. Finally, some future research directions in the field of SCND are discussed.

Keyword: supply chain network design; price-sensitive demand; incremental quantity discount; outer-approximation; second-order cone programming

1. Introduction

In the supply chain network design (SCND) problems, a multitude of factors must be taken into consideration, including the selection of factory facilities, establishment of warehouses, transportation of materials, and more. However, within this array of factors, the pivotal role of customer demand cannot be overstated (Ardjmand et al., 2016). This is because demand directly restricts sales volume, which in turn controls the quantities of goods produced and transported. In the context of a demand-driven market operating model, customer demand dictates the entirety of material flow within the supply chain network, as well as the corresponding facility siting. Therefore, demand drives production activities and imposes an essential influence on the profit of the entire supply chain network.

While numerous scholars acknowledge the paramount role of demand in SCND, in traditional SCND problems, demand is often treated as a given parameter. In reality, demand is influenced by various factors, including market dynamics, governmental policies, and environmental conditions. Among these factors, the demand for the majority of products is contingent upon price decisions (Hanjoul et al., 1990), thus constituting price-sensitive demand. Although the profit for per unit of each product increases when the price rises, the customer demand decreases, and consequently the total profit does not necessarily improve. An example is the food supply chain. According to the report released by the Food and Agriculture Organization of the United Nations in November 2022, the palm oil price was affected by the COVID-19 pandemic and extreme weather in recent years. As a result, the top palm oil consumers in India market are shifting to purchase relatively lower priced soybean oil and even sunflower oil. When the demand depends on the price of the commodity, inventory and selling price should be jointly decided, and finding the optimal pricing strategy is one of the main strategies to obtain the maximum profit (Ghoreishi et al., 2015). Therefore, it is of both theoretical and practical significance to consider the impact of price decisions on consumer demands in supply chain network research.

In the integrated optimization of the supply chain network, it avoids the independent decision-making of the supply chain members and only optimizes their own goals, but develops a coordination mechanism to coordinate the decisions of the chain members to achieve high or optimal chain performance. Among various coordination mechanisms, quantity discounts have been proven to be successful coordination mechanisms within supply chains (Hanh et al., 2022). Discount policies offered by suppliers can harmonize the supply chain and generate higher or even maximum supply chain profits in a price sensitive environment (Hsieh et al., 2010). Quantity discounts are an effective mechanism for coordinating decentralized supply chains, reducing inventory-related costs within the supply chain and potentially attracting more demand (Ke & Bookbinder, 2018). From the buyer's point of view, quantity discounts can calculate the optimal order quantity that minimizes the buyer's total associated costs. However, switching to the supplier's point of view, quantity discounts may be used to attract buyers to increase the order quantity and improve the supplier's profit.

In recent years, many scholars have also paid attention to the impact of price-sensitive demand and quantity discount on SCND (Ahmadi-Javid & Ghandali, 2014; Fattahi et al., 2015; Ghoreishi et al., 2015; Huang et al., 2018). However, there is still room for further exploration in the current research in terms of practicality. The first is that price and demand are not treated simultaneously as variables in network design. Ahmadi-Javid & Hoseinpour (2015a) studied for the first time the profit maximization problem in the location selection inventory problem considering price-sensitive demand, but only using price as a decision variable. Although Mogale et al. (2022) regarded price and demand as decision variables at the same time, its price is not a multi-period dynamic decision, and quantity discount is not considered. Second, there are fewer studies considering both price-sensitive demand and quantity discount, especially incremental quantity discount. Previous studies have mostly used price and demand as parameters, or only one of them as variables. However, in the price-sensitive demand relationship, the uncertainty of the two can truly reflect the needs of customers in different pricing. Therefore, our paper selects product price and demand as decision variables, while considering the impact of incremental quantity discount on the entire supply chain. This approach provides a more instructive decision plan for SCND problems.

This paper studies an SCND with price-sensitive demand, and establishes a three-echelon, multi-product, and multi-period supply chain network structure consisting of multiple manufacturers, multiple warehouses, and a single customer group. The impact of incremental quantity discount on network profitability is considered at the suppliers. In addition to the decision variables (location and production decisions as well as inventory and transportation decisions) that are considered in traditional SCND problems, this study also classifies prices as decision variables and establishes a mixed-integer nonlinear programming (MINLP) model with the objective to maximize the profit of the supply chain network. The model takes into account constraints such as production, transportation, and inventory capacity of the companies in the supply chain, as well as the flow balance between production and transportation. Furthermore, the constraints of price-demand relationships are added. Consequently, the established model can more accurately reflect changes of the demand in response to different price decisions. Based on this, we would like to answer the following questions:

(1) Which manufacturers and warehouses should be open to producing and storing products in SCND problems?

(2) How much is the volume of finished goods produced, stored, and transported between network-level facilities? Which discount should upstream manufacturers choose when purchasing raw materials and how much raw materials are purchased at different discount levels?

(3) What is the impact of prices and demand for different products and incremental quantity discount on network profits in different periods?

The remainder of the paper is structured as follows: Section 2 reviews the literature related to this study. Section 3 describes the incremental quantity discounts used in this paper and constructs an SCND model that takes into account price-sensitive demand. Section 4 uses the SOCP and the outer-approximation method to address the nonlinearity of the model. Section 5 verifies the validity and feasibility of the algorithms by computational experiments. This section also analyzes the model without considering discounts to further specify the advantages of the model proposed in this paper. Finally, Section 6 presents relevant conclusions and future research directions.

2. Literature review

Problems of traditional SCND have been studied in depth over the past few decades. Several studies, such as Peng et al. (2011), De Matta & Miller (2015), and Umpfenbach et al. (2018), focused on strategic planning decisions that include capacity and supplier selection decisions of SCND. Georgiadis et al. (2011), Ardjmand et al. (2016), Behzadi et al. (2017), Li et al. (2017), Pervin et al. (2018), and Tang et al. (2019) considered tactical planning decisions including transportation, production, and inventory decisions of SCND. Due to the complexity of supply chains in real-world applications, Badri et al. (2013), Zokaee et al. (2017), Mönch et al. (2018), and Sagaert et al. (2019) integrated strategic and tactical planning decisions in SCND problems.

In recent years, these SCND problems with integrated tactical and strategic planning decisions have been extended in several directions. For example, reverse logistics and closed-loop supply chains (Özceylan & Paksoy, 2013; Keyvanshokooh et al., 2016), finance management (Nickel et al., 2012; Temocin et al., 2018; Kılıç & Uğur, 2023), pricing strategy (Azevedo et al., 2018), smart warehouse operations(Zhen & Li, 2022), and supply chain resilience(Chen et al., 2023). Fattahi et al. (2017) considered a multi-period SCND with the delivery lead-time of products. Mohammed & Wang (2017) and Babazadeh et al. (2017) designed a three-echelon supply chain considering the environmental impacts of both activities and facilities. Margolis et al. (2018) developed a multi-objective model with production capacities of supply nodes for each commodity and other research, considering the impact of demand on SCND problems. Gupta et al. (2021) proposed a fuzzy multi-objective linear program for integrated food procurement, storage and distribution problems under cost, flexibility, and quality considerations. Song et al. (2022) studied the influences of uncertainty brought about by technological innovation on the supply chain in terms of supply and demand, shortage penalty, and budget, and proposed a three-level model of the anti-epidemic supply chain.

The research about SCND, which considers the impact of demand, can be divided into three categories: production is equal to demand, production is allowed to exceed demand, and production is not allowed to exceed demand. The total customer demands were satisfied in research of Pan & Nagi (2013), Baud-Lavigne et al. (2016), and Cheng & Tang (2018). A number of papers also provide surveys of SCND problems that allow the production to exceed demand (see Sahling & Kayser, 2016). Other studies do not allow production to exceed demand. For example, Guo & Li (2014) ensured that the total expected ordering quantity from all selected suppliers does not exceed the total expected demand and guaranteed that there was no extra inventory cost.

In addition, several studies pointed out that pricing greatly impacts demand. Cachon & Harker (2002) indicated that price cuts would increase demand, thus reducing the average cost per unit of demand. Hsieh et al. (2010) reported that pricing should be an effective tool to maximize revenue when considering price-sensitive demand. Later, scholars also considered the relationship between price and demand in studies of supply chain network problems. For example, Ahmadi-Javid & Hoseinpour (2015b) studied an SCND problem with price-sensitive demands and inventory-capacity constraints, and presented a location-inventory-pricing model. Wang et al. (2016) investigated the bullwhip effect on inventory under different information sharing settings based on price-sensitive demand. Ke & Bookbinder (2018) developed a tri-level programming approach to coordinate decisions of three supply chain members on discount policies, for the case when demand is sensitive to price changes. However, most scholars used demand and price as parameters when building models, rather than as decision variables for direct decision making.

Another related line of research is the study of the SCND literature in the presence of quantity discount. In most of the literature, studies on SCND under quantity discounts usually assume that the supply chain network is single-echelon and that the problem includes only suppliers and customers (Burke et al., 2008). Quantity discounting problems are usually classified into incremental and all-units quantity discount. Tamjidzad & Mirmohammadi (2018) discussed a multi-item inventory system with a discrete stochastic demand stream under continuous review, where there is a limited resource and all-units quantity discount. Yang et al. (2019) proposed a deterministic model to minimize the cost of service, provided that each lane being served by carriers and quantity discount are taken into account. Jackson & Munson (2019) developed effective solutions for a constrained, multi-product lot-sizing problem with a common replenishment cycle, a flexible common resource capacity, and all-units quantity discounts.

While Hsieh et al. (2010) and Ke & Bookbinder (2018) focused on price-sensitive demand and quantity discounts, their research primarily delved into tactical-level decisions and did not encompass multi-level strategic decisions. In studies similar to our present work, Ahmadi & Ghasemi (2022) employed a fuzzy inference system to forecast demand and considered the impact of energy efficiency and environmental factors on hotel pricing to determine the optimal pricing under competitive conditions. Momenitabar et al. (2022) optimized strategic and tactical decisions in a closed-loop supply chain network design problem considering price-sensitive demand. However, they did not directly investigate price and demand as variables and did not consider quantity discounts. Safaei et al. (2022) studied a multi-level, multi-period closed-loop SCND problem by using time series for demand forecasting. Their focus, however, was more on optimizing tactical-level decisions and did not address price-sensitive demand and quantity discounts. Table 1 offers a comprehensive overview of the key research efforts, shedding light on the primary contributions of this study.

Based on the literature review presented in Table 1, it is evident that in the majority of studies focusing on joint optimization at the strategic and tactical levels, the optimization at the strategic level primarily revolves around single-level location decision problems. The introduction of multi-level networks significantly increases complexity. Furthermore, at the tactical decision level, there is limited literature that simultaneously considers both price and demand as variables, with only two articles addressing this aspect. Additionally, there is only one paper that takes into account price-sensitive demand and quantity discounts, especially at the all-units quantity discount. Finally, in the context of SCND problems, most studies tend to lean towards heuristic algorithms due to the nonlinearity of the models and the complexity of solving them. Our proposed three-level network structure model encompasses considerations for price-sensitive demand from the consumer side, as well as quantity discounts from the supplier side. We extend this to multi-product logistics networks and make decisions regarding raw material procurement through incremental quantity discount.

Table 1
Tabular literature.

Articles	OBJ	Strategic decisions		Tactical decisions							Factors			Methods
		Manu-se	Ware-se	Quan-dis	Prod	Inven	Trans	Price	Dem	Others	Muti-prod	P-D	Quan-dis	
Tamjidzad & Mir-mohammadi (2018)	CM			√		√					√		All	Local search
Fattahi et al. (2015)	PM	√	√		√	√	√	√			√	√		SA
Yang et al. (2019)	CM			√			√						All	IA
Jackson & Munson (2019)	CM								√		√		All	CRCA-RP
Safaei et al. (2022)	CM				√	√	√			√	√			GA
Momenitabar et al. (2022)	CM	√	√		√	√	√			√	√	√		MCGP-UF
Wang et al. (2016)	CM				√	√				√		√		MMSE
Ahmadi-Javid & Hoseinpour (2015a)	PM		√			√	√	√		√		√		LRA
Ahmadi-Javid & Ghandali (2014)	PM		√					√		√		√		LRA
Ahmadi-Javid & Hoseinpour (2015b)	PM		√					√		√	√	√		LRA
Xiao & Qi (2016)	PM							√	√	√		√	All	CoM
Xiao et al. (2016)	PM							√	√	√		√		SG, NG
Our work	PM	√	√	√	√	√	√	√	√		√	√	INCR	SOCP, OA

Note 1: OBJ: Objective function, Manu-se: Manufacturer selection, Ware-se: Warehouse selection, Quan-dis: Quantity discount selection, Prod: Production, Inven: Inventory, Trans: Transportation, Dem: Demand, Muti-prod: Multiple products, P-D: Price-demand, Quan-dis: Quantity discount.
Note 2: All: All-units quantity discount, INCR: incremental quantity discounts, CM: Cost minimization, PM: Profit maximization.
Note 3: SA: Simulated annealing, IA: Iterative algorithm, CRCA-RP: Common replenishment cycle approach with refinement policy, GA: Genetic algorithm, MCGP-UF: Multi-Choice Goal Programming Approach with Utility Function, MMSE: MMSE forecasting technique, LRA: Lagrangian relaxation algorithm, CoM: Coordination mechanism, SG: Stackelberg game, NG: Nash game, SOCP: Second-order cone programming, OA: outer-approximation.

On this basis, we tackle the nonlinearity of the model by the SOCP method to obtain an accurate solution, and address large-scale instances by the outer-approximation method. In summary, the main novelties and contributions of this paper can be summarized as follows:

(1) Multi-level integrated decision-making: This paper establishes an MINLP model with the objective of maximizing the total supply chain profit to integrate location decisions at the strategic level and production, transportation, inventory, product prices, consumer demand, and discount selection decisions at the tactical level.

(2) Price-sensitive demand: This paper introduces the constraint of the price-demand function in the SCND problem, describes the correlation between demand and price in customer segments, and captures the elastic range of demand at different price points.

(3) Incremental quantity discount: Implemented in a stepwise manner, the discount gradually increases as the number of purchases increases, so that customers can enjoy greater price concessions when they purchase a larger number of products. As the number of purchases increases, the discount gradually increases.

(4) Solution algorithm: Due to the nonlinear nature of the model, this study proves the existence of the optimal solution of the problem and the feasibility of implementing the following two algorithms by analyzing the nature of the problem: the SOCP method and the outer-approximation method. The SOCP method can obtain optimal solutions for small- and medium-scale experiments, while the outer-approximation method can find approximate optimal solutions in a reasonable time for large-scale experiments.

3. SCND model considering pricing-demand relationships

3.1. Description of the problem

This paper studies a multi-period, three-echelon supply chain network structure, which consists of multiple manufacturers, multiple warehouses, and a single customer group. A single customer group is defined as a customer type with the same consumption concept and demand, and the demands of each member in the customer group are independent. The production, transportation, inventory, and sales of products over multiple periods are considered in this supply chain network, as shown in Figure 1.

Upstream in the supply chain network, manufacturers need to purchase raw materials when manufacturing products. To ensure sufficient stock of raw materials, this study assumes that these manufacturers choose the order-up-to-level, that is, the manufacturer n assesses the inventory at each period t to determine whether it is lower than the maximum inventory level H_n^{max}, and then determines the current purchase amount of raw materials q_{nt}. If the inventory is h_{nt} at the time of inspection, the order quantity is $q_{nt} = H_n^{max} - h_{nt}$. The unit purchase cost of raw materials per

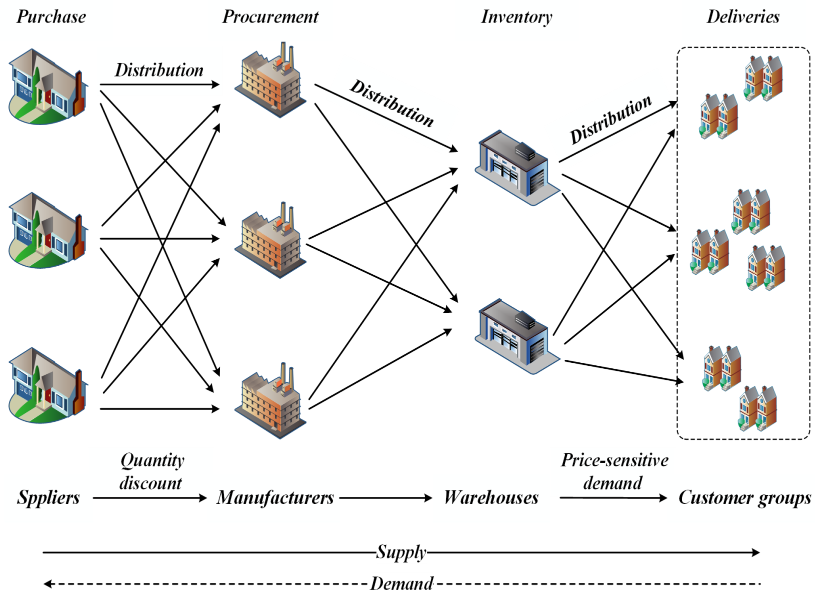

Fig. 1 Supply chain network structure

period is different. Meanwhile, when purchasing raw materials, manufacturers often face quantity discount provided by suppliers to reduce the procurement cost of raw materials. Figure 2 shows the cost of incremental quantity discounts corresponding to purchase quantities at different discount levels. The more the quantity purchased, the lower the price. The growth trend of the total cost will be slower and slower.

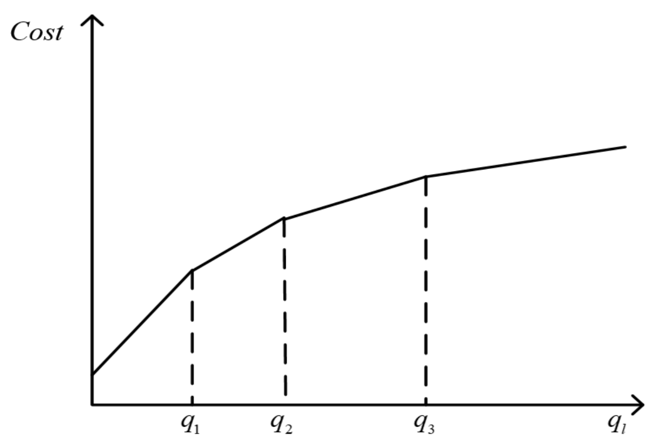

Fig. 2 Incremental quantity discount cost for order quantity q_l

Figure 3 shows the purchase price of the incremental quantity discount corresponding to the upper limit of different discount levels. With the increase of the purchase volume, the purchase price presents a step-by-step decrease, thereby stimulating the increase of the purchase volume.

After the manufacturer purchases the raw materials, these purchased raw materials are used to produce the product. These produced products should be transferred to the warehouse. The supply chain network contains several warehouses. Furthermore, products are transported to different

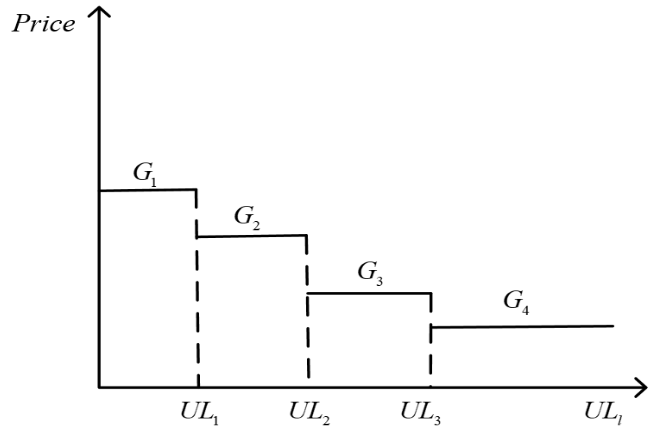

Fig. 3 Cost of purchasing raw materials for Upper limit of discount level UL_l

warehouses by the manufacturers' fleet, which generates transportation costs. Products shipped to the warehouses can be stored in warehouses or shipped to a customer base for sale, but products cannot be shipped between warehouses.

Downstream of the supply chain network, the consumer zone considers price-sensitive demand and describes the relationship between them by introducing price-demand functions. Figure 4 shows an example of the price-demand function used in this paper. When the price of product i is 0, the demand reaches the maximum D_i; when the price of product i exceeds the consumer's expectation b_i, the demand is 0, i.e., no one would buy it.

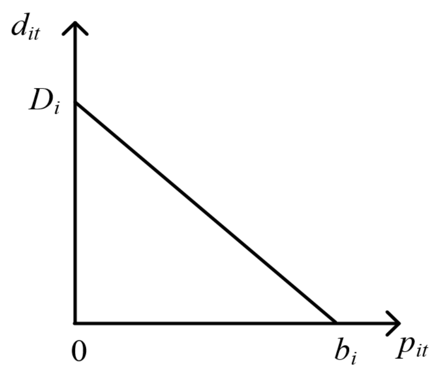

Fig. 4 Price-demand function

According to Figure 4, the functions of price and demand can be written as follows:

$$d_{it} = D_i - p_{it} \cdot D_i/b_i, 0 \leq p_{it} \leq b_i, \forall i \in I, t \in T. \tag{1}$$

3.2. Mathematical model

The underlying assumptions of the model are as follows:

(1) Manufacturers use the (t, H_n^{max}) strategy to replenish raw materials.

(2) The products produced by manufacturers can only be stored in warehouses.

The notations of this paper are as follows:

Sets

T	Set of periods, $t \in T$, $T = \{1, 2, ..., t, ..., T\}$;
I	Set of products, $i \in I$, $I = \{1, 2, ..., i, ..., I\}$;
W	Set of warehouses, $w \in W$, $W = \{1, 2, ..., w, ..., W\}$;
N	Set of manufacturers, $n \in N$, $N = \{1, 2, ..., n, ..., N\}$;
L	Set of discount levels, $l \in L$, $L = \{0, 1, 2, ..., l, ..., L\}$;

Parameters

UL_{nl}	Upper limit for discount level l for manufacturer n;
F_n	Fixed cost for the selection of manufacturer n;
R_w	Fixed cost for the opening of warehouse w;
G_{nlt}	Cost of purchasing one unit of raw materials by manufacturer n at discount level l during period t;
U_{nlt}	Cost of producing one unit of product i by manufacturer n during period t;
L_{nt}	Cost of holding one unit of raw materials by manufacturer n during period t;
L_{wit}	Cost of holding one unit of product i at warehouse w during period t;
V_{nwit}	Cost of transporting one unit of product i between manufacturer n and warehouse w during period t;
C_{wit}	Cost of transporting one unit of product i between warehouse w and the customer zone during period t;
A_i	Amount of raw materials required per unit of product i;
h_{n0}	Amount of raw materials held at manufacturer n at the beginning of the period;
h_{wi0}	Amount of product i held at warehouse w at the beginning of the period;
H_n^{max}	Maximum stock level of raw materials at the manufacturer n;
Ω_n^{max}	Maximum production capacity at manufacturer n;
K_n^{max}	Maximum transportation capacity amount of manufacturer n for forwarding products to all warehouses per period;
H_w^{max}	Maximum inventory capacity at warehouse w;
K_w^{max}	Maximum transportation capacity amount of warehouse w for forwarding products to all customers per period;
b_i	The parameter of the price-demand function; when the price of product i exceeds b_i, the demand is 0;
D_i	Upper bound value for the demand of product i;

Decision variables

p_{it}	Price of product i during period t;
d_{it}	Demand of product i after its price is determined during period t;

σ_{it} Actual amount of product i purchased during period t;

P_{it} Total revenue of product i during period t;

x_n 1 if manufacturer n is selected, and 0 otherwise;

y_w 1 if warehouse w is open, and 0 otherwise;

z_{nw} 1 if manufacturer n and warehouse w are selected and open, and 0 otherwise;

ω_{nlt} 1 if raw materials purchased by manufacturer n at discount level l during period t, and 0 otherwise;

q_{nlt} Amount of raw materials purchased by manufacturer n at discount level l during period t;

$\hat{q}_{nit}$ Amount of product i produced at manufacturer n during period t;

h_{nt} Amount of raw materials held at manufacturer n during period t;

h_{wit} Amount of product i held at warehouse w during period t;

λ_{nwit} Amount of product i transported from manufacturer n to warehouse w during period t;

s_{wit} Amount of product i transported from warehouse w to customers during period t.

The [MINLP] mathematical model is as follows:

[MINLP]

$$\begin{aligned}\text{maximize} \quad Z = & \sum_{i\in I}\sum_{t\in T} p_{it}\sigma_{it} - \sum_{n\in N} F_n x_n - \sum_{w\in W} R_w y_w - \sum_{n\in N}\sum_{t\in T}\sum_{l\in L} G_{nlt} q_{nlt} \\ & - \sum_{n\in N}\sum_{i\in I}\sum_{t\in T} U_{nit}\hat{q}_{nit} - \sum_{n\in N}\sum_{t\in T} L_{nt} h_{nt} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} L_{wit} h_{wit} \\ & - \sum_{n\in N}\sum_{w\in W}\sum_{i\in I}\sum_{t\in T} V_{nwit}\lambda_{nwit} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} C_{wit} s_{wit}\end{aligned} \tag{2}$$

subject to

$$\sum_{l\in L} q_{nlt} = H_n^{max} - h_{nt}, \forall\, n \in N,\ t \in T, \tag{3}$$

$$q_{nlt} \geq \left(UL_{nl} - UL_{n(l-1)}\right)\omega_{n(l+1)t}, \forall\, n \in N,\ l \in \{1,...,L-1\},\ t \in T, \tag{4}$$

$$q_{nlt} \leq \left(UL_{nl} - UL_{n(l-1)}\right)\omega_{nlt}, \forall\, n \in N,\ l \in \{1,...,L\},\ t \in T, \tag{5}$$

$$\omega_{n(l+1)t} \leq \omega_{nlt}, \forall\, n \in N,\ l \in \{0,...,L-1\},\ t \in T, \tag{6}$$

$$\sum_{w\in W} \lambda_{nwit} = \hat{q}_{nit}, \forall n \in N, i \in I, t \in T, \tag{7}$$

$$\sum_{i\in I} \hat{q}_{nit} \leq \Omega_n^{max} x_n, \forall n \in N, t \in T, \tag{8}$$

$$\sum_{i\in I} \lambda_{nwit} \leq K_n^{max} \cdot x_n \cdot y_w, \forall n \in N, w \in W, t \in T, \tag{9}$$

$$\sum_{i\in I} h_{wit} \leq H_w^{max} \cdot y_w, \forall w \in W, t \in T, \tag{10}$$

$$\sum_{i\in I} s_{wit} \leq K_w^{max} \cdot y_w, \forall w \in W, t \in T, \tag{11}$$

$$\sum_{w\in W} s_{wit} = \sigma_{it}, \forall i \in I, t \in T, \tag{12}$$

$$\sigma_{it} \leq d_{it}, \forall i \in I, t \in T, \tag{13}$$

$$d_{it} = D_i - p_{it} \cdot D_i/b_i, 0 \leq p_{it} \leq b_i, \forall i \in I, t \in T, \tag{14}$$

$$h_{n(t-1)} + \sum_{l\in L} q_{nlt} = h_{nt} + \sum_{i\in I} A_i \cdot \hat{q}_{nit}, \forall n \in N, t \in T\backslash\{1\}, \tag{15}$$

$$h_{n0} + \sum_{l\in L} q_{nl1} = h_{n1} + \sum_{i\in I} A_i \cdot \hat{q}_{ni1}, \forall n \in N, \tag{16}$$

$$h_{wi(t-1)} + \sum_{n\in N} \lambda_{nwit} = h_{wit} + s_{wit}, \forall w \in W, i \in I, t \in T\backslash\{1\}, \tag{17}$$

$$h_{wi0} + \sum_{n\in N} \lambda_{nwit} = h_{wit} + s_{wit}, \forall w \in W, i \in I, t = 1, \tag{18}$$

$$x_n, y_w, z_{nw}, \omega_{nlt} \in \{0,1\}, \forall n \in N, w \in W, t \in T, \tag{19}$$

$$p_{i\,t}, d_{it}, q_{nlt}, \hat{q}_{nit}, h_{nt}, h_{wit}, \lambda_{nwit}, s_{wit} \geq 0, \forall n \in N, w \in W, l \in L, i \in I, t \in T, \tag{20}$$

$$\sigma_{it} \geq 0, \forall i \in I, t \in T. \tag{21}$$

The objective function (2) describes the total profit of the supply chain network. The first item in the objective function represents the total revenue of the products sold; the second item represents the construction cost of warehouses; the third item represents the construction cost of manufacturers; the fourth item represents the procurement cost of raw materials; the fifth item represents the production cost at manufacturers; the sixth item represents the inventory cost of the raw materials at manufacturers; the seventh item represents the inventory cost of products at the warehouses; and the last two items represent the transportation cost of the products. Constraints (3) represent the purchase amount of raw materials at the beginning of period t. Constraints (4) and Constraints (5) ensure that the purchase quantity of raw materials at each discount level is between the lower and upper limits according to the incremental quantity discount of each supplier in each period. Constraints (6) help determine the discount level for raw materials purchased from each supplier in each period. Constraints (7) indicate that all products produced by manufacturer n are shipped to warehouse w. Constraints (8) are capacity constraints, indicating that the manufacturers' production levels cannot exceed their maximum production capacity. Constraints (9) indicate that the transportation volume of the products from the manufacturers to the warehouses cannot exceed the maximum transportation capacity of manufacturers. Constraints (10) indicate that the inventory of the products in warehouse w cannot exceed its maximum inventory capacity. Constraints (11) indicate that the transportation volume of the products from warehouse w to the consumer cannot exceed its maximum transportation capacity. Constraints (12) indicate that the quantity of products transported from the warehouse must meet the demands of consumers. Constraints (13) indicate that the actual purchase amount of consumers is less than or equal to their demand after the price is determined. Constraints (14) represent the price-demand relationships. Constraints (15) and (16) represent the flow balance of the products production. Constraints (17) and (18) represent the flow balance of the products transportation. Constraints (19) clarify the numerical definition of 0-1 variables. Constraints (20) and (21) clarify the numerical definition of the non-negative variables.

In the model [MINLP], it is obvious that two nonlinearities exist. One is in the first item of the objective function and the other is in constraints (9). Here, the decision variable z_{nw} is introduced to convert constraints (9) to the following constraints:

$$\sum_{i \in I} \lambda_{nwit} \leq K_n^{max} \cdot z_{nw}, \forall n \in N, w \in W, t \in T, \tag{22}$$

$$z_{nw} \leq x_n, \forall n \in N, w \in W, \tag{23}$$

$$z_{nw} \leq y_w, \forall n \in N, w \in W, \tag{24}$$

$$z_{nw} \in \{0, 1\}, \forall n \in N, w \in W. \tag{25}$$

4. Solution for the SCND model considering pricing-demand relationships

Due to the nonlinearity of the first item in the objective function (2), solving the [MINLP] mathematical model is particularly difficult. Discretizing the price is the most common way to deal with the nonlinear part of the model. Fattahi et al. (2015) discretized the pricing level by introducing the price-demand functions and solved it by using a linear relaxation-based simulated annealing algorithm. However, the proposed algorithm introduces additional integer decision variables, which affect the quality of the solution. Therefore, the properties of the MINLP model are first analyzed, and then, two different algorithms are proposed accordingly in this research.

4.1. Analysis of the SCND model considering pricing-demand relationships

The model [MINLP] has the following characteristics and can derive the following relevant theorems:

Theorem 1: The model [MINLP] has an optimal solution.

Proof. All constraints in [MINLP] are linear and are connected by $\leq$, $=$, and $\geq$. There are no $<$ and $>$ constraints. Therefore, the feasible region of [MINLP] is closed. The feasible region of [MINLP] is bounded because of the constraints (3), (8)-(11), (13), (14), (20), and (21). Hence, the feasible region is a compact set. The feasible region is not empty, because there is a feasible solution where all decision variables are zero. Therefore, the model [MINLP] has an optimal solution.

Theorem 2: The decision variable $(\sigma_{it}, d_{it}, \forall i \in I, t \in T)$ of [MINLP] has an optimal solution $(\sigma^*_{it}, d^*_{it}, \forall i \in I,\, t \in T)$. For $\forall i \in I, t \in T$, if $\sigma^*_{it} > 0$, then $\sigma^*_{it} = d^*_{it}$.

Proof. The theorem is proved by contradiction. Theorem 1 illustrates the existence of an optimal solution for [MINLP]. Suppose that an optimal solution $(\hat{\sigma}_{it},\ \hat{d}_{it}, \forall i \in I, t \in T)$ exists and the solution satisfies $0 < \hat{\sigma}_{\bar{i}\,\bar{t}} < \hat{d}_{\bar{i}\,\bar{t}}$ for $\bar{i} \in I, \bar{t} \in T$.

Since $p_{\bar{i}\,\bar{t}}(d_{\bar{i}\,\bar{t}})$ is strictly decreasing on $[0, D_i]$ and $0 < \hat{\sigma}_{\bar{i}\,\bar{t}} < \hat{d}_{\bar{i}\,\bar{t}} \leq D_i$, then $p_{\bar{i}\bar{t}}(\sigma_{\bar{i}\bar{t}}) > p_{\bar{i}\bar{t}}(d_{\bar{i}\bar{t}})$. We have $p_{\bar{i}\,\bar{t}}(\sigma_{\bar{i}\,\bar{t}})\sigma_{\bar{i}\,\bar{t}} > p_{\bar{i}\,\bar{t}}(d_{\bar{i}\,\bar{t}})\sigma_{\bar{i}\,\bar{t}}$ because $\sigma_{\bar{i}\,\bar{t}} > 0$. Therefore, if the price $p_{\bar{i}\,\bar{t}}(d_{\bar{i}\,\bar{t}})$ increases to $p_{\bar{i}\,\bar{t}}(\sigma_{\bar{i}\,\bar{t}})$, the profit (2) will also increase. Then, the new solution $(\sigma_{i\,t} = \hat{\sigma}_{i\,t}, \forall i \in I, t \in T\,; d_{i\,t} = \hat{d}_{i\,t}, \forall i \in I \backslash \{\,\bar{i}\}, t \in T \backslash \{\,\bar{t}\}; d_{\bar{i}\,\bar{t}} = \hat{\sigma}_{\bar{i}\,\bar{t}})$ is feasible and better than the original solution $(\hat{\sigma}_{i\,t},\ \hat{d}_{i\,t}, \forall i \in I, t \in T)$, which contradicts the assumption that the solution $(\hat{\sigma}_{i\,t},\ \hat{d}_{i\,t}, \forall i \in I, t \in T)$ is optimal. Theorem 2 is proved.

Therefore, constraints (12), (13), and (14) can be transferred to:

$$\sum_{w \in W} s_{wit} = D_i - p_{it} \cdot D_i / b_i, 0 \leq p_{it} \leq b, \forall i \in I, t \in T. \tag{26}$$

By proving the properties of the optimal solution, actual purchases σ_{it} can then be replaced by demand d_{it} in the objective function (2).

4.2. Second-order cone programming

The nonlinearity in the first term of the objective function cannot be solved directly using the mathematical programming software CPLEX. Therefore, the problem was converted into an SOCP problem to obtain the exact optimal solution in this section.

Theorem 3 can be obtained from the introduction of the price-demand functions, see constraints (1) and Figure 3.

Theorem 3: The function $d_{it}(p_{it})$ is strictly decreasing on [0, b_i] and conforms to the Lipschitz continuity.

The function $d_{it}(p_{it})$ has an inverse function due to strict monotonicity, and the inverse function can be expressed as:

$$p_{it} = b_i - d_{it} \cdot b_i/D_i, 0 \le d_{it} \le D_i, \forall i \in I, t \in T. \tag{27}$$

According to constraints (27), the function $p_{it}(d_{it})$ is also a strictly decreasing function as shown in Figure 4.

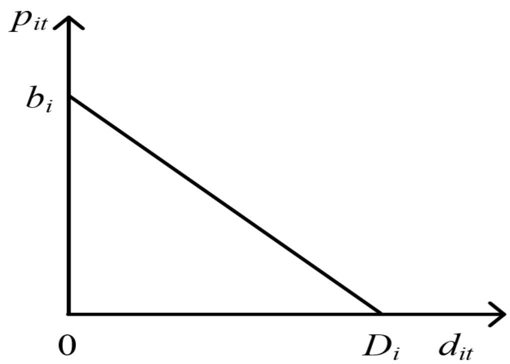

Fig. 5 Demand-price function

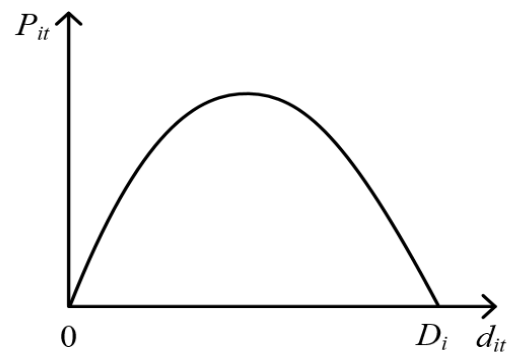

Fig. 6 Demand-revenue function

The rationality of the Lipschitz continuity is that demands originate from multiple consumers. When the price changes a little, the demand does not fluctuate significantly.

P_{it} is defined as the revenue of product i during period t, i.e., $P_{it} = p_{it} \cdot d_{it}$. According to constraints (27), we can get:

$$P_{it} = (-{d_{it}}^2 + D_i \cdot d_{it}) \cdot b_i/D_i, 0 \le d_{it} \le D_i, \forall i \in I, t \in T. \tag{28}$$

Theorem 4: P_{it} is strictly concave on $[0, D_i]$ and continuously differentiable about d_{it}.

The demand-revenue function is shown in Figure 5.

Therefore, constraints (28) can be transferred into constraints (29):

$$P_{it} \le (-{d_{it}}^2 + D_i \cdot d_{it}) \cdot b_i/D_i, 0 \le d_{it} \le D_i, \forall i \in I, t \in T. \tag{29}$$

Furthermore, constraints (29) can be re-written as:

$$-{d_{it}}^2 + D_i \cdot d_{it} \ge D_i \cdot P_{it}/b_i, 0 \le d_{it} \le D_i, \forall i \in I, t \in T, \tag{30}$$

$${d_{it}}^2 - D_i \cdot d_{it} \le -D_i \cdot P_{it}/b_i, 0 \le d_{it} \le D_i, \forall i \in I, t \in T. \tag{31}$$

Then, constraints (31) can be transformed to a set of hyperbolic inequalities, as shown in the following:

$$(d_{it} - D_i/2)^2 \le -D_i \cdot P_{it}/b_i + D_i^2/4, 0 \le d_{it} \le D, \forall i \in I, t \in T. \tag{32}$$

The above hyperbolic inequalities can be rewritten as a set of SOCP constraints:

$$\left\|(2d_{it} - D_i, -D_i \cdot P_{it}/b_i + {D_i}^2/4 - 1)\right\|_2 \le -D_i \cdot P_{it}/b_i + {D_i}^2/4 + 1, 0 \le d_{it} \le D_i, \forall i \in I, t \in T, \tag{33}$$

where $\|\cdot\|_2$ denotes the Euclidean norm.

Therefore, the model [MINLP] can be changed to model [MINLP-SOCP]:

[MINLP-SOCP]

$$\begin{aligned} \text{maximize } Z = &\sum_{i\in I}\sum_{t\in T} P_{it} - \sum_{n\in N} F_n x_n - \sum_{w\in W} R_w y_w - \sum_{n\in N}\sum_{l\in L}\sum_{t\in T} G_{nlt} q_{nlt} \\ &- \sum_{n\in N}\sum_{i\in I}\sum_{t\in T} U_{nit}\hat{q}_{nit} - \sum_{n\in N}\sum_{t\in T} L_{nt} h_{nt} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} L_{wit} h_{wit} \\ &- \sum_{n\in N}\sum_{w\in W}\sum_{i\in I}\sum_{t\in T} V_{nwit}\lambda_{nwit} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} C_{wit} s_{wit} \end{aligned} \tag{34}$$

subject to constraints (3)-(8), (10), (11), (15)-(20), (22)-(26), and (33).

4.3. Outer-approximation

Although the SOCP model can obtain the exact optimal solution, it can only deal with small and medium-sized problems in a reasonable time. Therefore, the outer-approximation method is proposed in this section to find the approximate optimal solution of the problem and improve the efficiency of solving large-scale problems.

As mentioned above, define P_{it} as the revenue of product i during period t, and the revenue funtion changes from $P_{it} = p_{it} \cdot \sigma_{it}$ to $P_{it} = p_{it} \cdot d_{it}$ by the above theorems. According to constraints (14), the following constraints can be obtained:

$$P_{it} = D_i \cdot p_{it} - {p_{it}}^2 \cdot D_i/b_i, 0 \le p_{it} \le b_i, \forall i \in I, t \in T. \tag{35}$$

According to constraints (35), the maximum value of P_t on [0, b_i] is obtained at $b_i/2$ (symmetry axis) and the maximum value is $D_i \cdot b_i/4$. The graph of the function $P_{it}(p_{it})$ on [0, b_i] is shown in Figure 6.

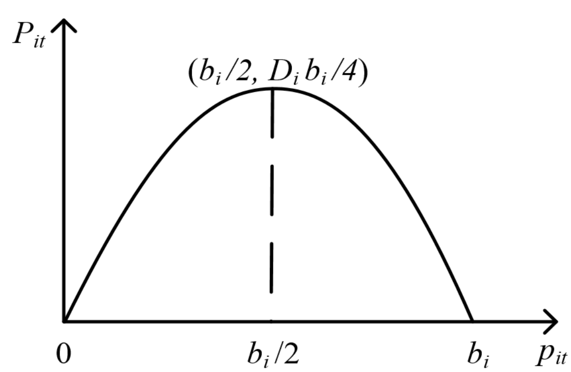

Fig. 7 Price-revenue function

The model [MINLP] can be transferred to the [MINLP-OA1] model (where OA stands for outer-approximation).

[MINLP-OA1]

$$\begin{aligned}\text{maximize } Z = &\sum_{i\in I}\sum_{t\in T} P_{it} - \sum_{n\in N} F_n x_n - \sum_{w\in W} R_w y_w - \sum_{n\in N}\sum_{l\in L}\sum_{t\in T} G_{nlt} q_{nlt} \\ &- \sum_{n\in N}\sum_{i\in I}\sum_{t\in T} U_{nit}\hat{q}_{nit} - \sum_{n\in N}\sum_{t\in T} L_{nt}h_{nt} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} L_{wit}h_{wit} \\ &- \sum_{n\in N}\sum_{w\in W}\sum_{i\in I}\sum_{t\in T} V_{nwit}\lambda_{nwit} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} C_{wit}s_{wit}\end{aligned} \tag{36}$$

subject to constraints (3)-(8), (10), (11), (15)- (20), (22)-(27), and (35).

Since the function $P_{it}(p_{it})$ is concave on [0, b_i], the outer-approximation method is used to approximate it.

For $\forall p_{it} \in [0, b_i]$, P_{it} is approximated by using the sum of many piecewise-linear functions and its approximation error e can be controlled within a predeterminged tolerance level. Define $\Psi = \{1, 2, ..., |\Psi|\}$ as the set of tangents and the tangent's expression is: $r_{it}^{\psi} \times p_{it} + sk_{it}^{\psi}$, where r_{it}^{ψ} and sk_{it}^{ψ} represent the slope and intercept of tangent ψ, respectively. Then, the piecewise-linear approximation functions can be written as:

$$\bar{P}_{it} = \min\left\{r_{it}^{\psi} \times p_{it} + sk_{it}^{\psi}, \forall i \in I, t \in T, \psi \in \Psi\right\}. \tag{37}$$

The characteristics of the piecewise-linear function $\bar{P}_{it}(p_{it})$ and the concave function $P_{it}(p_{it})$ indicate:

$$\bar{P}_{it} \geq P_{it}, \forall i \in I, t \in T. \tag{38}$$

The following constraints can be obtained by constraints (37) and (38):

$$P_{it} \leq r_{it}^{\psi} \times p_{it} + sk_{it}^{\psi}, \forall i \in I, t \in T, \psi \in \Psi. \tag{39}$$

Therefore, the model [MINLP-OA1] can be modified as model [MINLP-OA2]:

[MINLP-OA2]

$$\begin{aligned}\text{maximize } Z = &\sum_{i\in I}\sum_{t\in T} P_{it} - \sum_{n\in N} F_n x_n - \sum_{w\in W} R_w y_w - \sum_{n\in N}\sum_{t\in T}\sum_{l\in L} G_{nlt} q_{nlt} \\ &- \sum_{n\in N}\sum_{i\in I}\sum_{t\in T} U_{nit}\hat{q}_{nit} - \sum_{n\in N}\sum_{t\in T} L_{nt}h_{nt} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} L_{wit}h_{wit} \\ &- \sum_{n\in N}\sum_{w\in W}\sum_{i\in I}\sum_{t\in T} V_{nwit}\lambda_{nwit} - \sum_{w\in W}\sum_{i\in I}\sum_{t\in T} C_{wit}s_{wit}\end{aligned} \tag{40}$$

subject to constraints (3)-(8), (10), (11), (15)-(20), (22)-(26), and (39).

To make it easier to understand, the tangent generating steps of the piecewise-linear approximation functions are described in the following. Figure 7 illustrates the algorithm (taking five tangent lines of period t as an example).

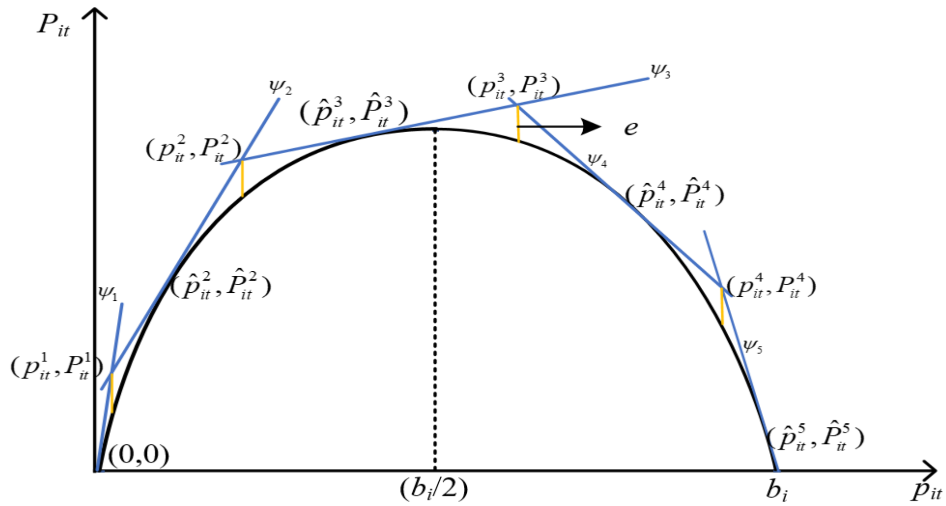

Fig. 8 Generation of the piecewise-linear approximation functions

Step 0: Set $\psi = 1$ and the tangent point $(\hat{p}_{it}^1, \hat{P}_{it}^1) = (0, 0)$. Pass the point $(\hat{p}_{it}^1, \hat{P}_{it}^1)$ and draw the tangent ψ of the function $P_{it}(p_{it})$.

Step 1: Calculate the slope r_{it}^{ψ} and intercept sk_{it}^{ψ} of the tangent ψ:

$$r_{it}^{\psi}{=}D_i \cdot (b_i - 2\hat{p}_{it}^{\psi})/b_i, \tag{41}$$

$$sk_{it}^{\psi}{=}\hat{P}_{it}^{\psi} - r_{it}^{\psi} \cdot \hat{p}_{it}^{\psi}. \tag{42}$$

Step 2: Set the error e between the tangent ψ and the function $P_{it}(p_{it})$. At the error value point, there is $P_{it}(p_{it}){=}\psi{-}\varepsilon$. We can then obtain the error value point $(p_{it}^{\psi}, P_{it}^{\psi})$:

$$P_{it}^{\psi}{=}r_{it}^{\psi} \cdot p_{it}^{\psi} + sk_{it}^{\psi}. \tag{43}$$

Step 3: Determine whether $p_{it}^{\psi} \leq b_i$. If yes, let $\psi = \psi + 1$ and continue at Step 4. Otherwise, the calculations stay ends and go to Step 5.

Step 4: Pass the point $(p_{it}^{\psi-1}, P_{it}^{\psi-1})$ and draw the tangent ψ. Go to Step 1.

Step 5: Obtain all the tangent lines.

Based on the outer-approximation of the revenue function $P_{it}(p_{it})$, the model can be directly solved by adding linear constraints (39).

Theorem 5: The lower bound for the number of tangent lines to the revenue function $P_{it}(p_{it})$ is $2\pi / \left\{ \arccos \left[\sqrt{\left(1 + {D_i}^2\right)^3} \Big/ \left(\sqrt{\left(1 + {D_i}^2\right)^3} - 2D_i e / b_i \right) \right] \right\}$, and the upper bound for the number is $2\pi / \arccos\left[-b_i/(2D_i e)\right]$.

Proof. The method for calculating the number of tangent lines to the parabola function $P_{it}(p_{it})$ is:

At first, we take a circle with circumference l and radius r (as shown in Figure 8) as an example. The circle can be approximated by a group of tangent lines. Furthermore, the number of the tangent lines is decided by the the error value e. The curvature of a circle is fixed, whereas the curvature of a parabola constantly changes. Therefore, the number of tangent lines can be calculated by the parabolic curvature.

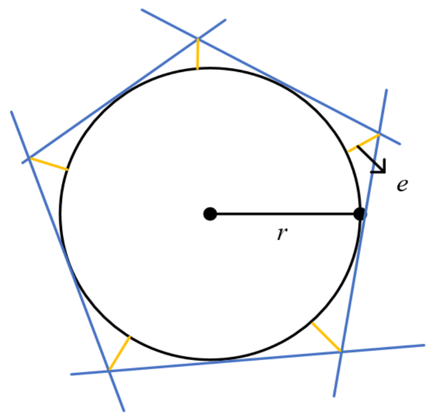

Fig. 9 Schematic diagram of the number of required tangent lines

Furthermore, it is obviously that the greater the curvature is, the more the tangents are required. So the upper bound of the number of tangents can be obtained at the maximum curvature. Similarly, the lower bound of the number of tangents can be obtained at the minimum curvature.

Then, we can calculate the number of tangent lines of the revenue function $P_{it}(p_{it})$ as follows: The curvature radius of the parabola $y = ax^2 + bx + c(a \neq 0)$ is $\rho = (1+y')^3/y''$. The curvature radius at the point $\left(-b/2a, (4ac-b^2)/4a\right)$ is $1/2a$, and the curvature radius at $(-b/a, 0)$ is $\sqrt{(1+b^2)^3}/2a$.

(1) When the curvature radius is $1/2a$, the maximum curvature is obtained and the circumference of a circle is π/a. Then the upper bound for the number of tangent lines can be calculated. When the error is e, the length of the circumference that a tangent can cut is $\arccos{(2ae)}^{-1}/2a$, and the number of required tangent lines is $2\pi/\arccos{(2ae)}^{-1}$.

(2) When the curvature radius is $\sqrt{(1+b^2)^3}/2a$, the minimum curvature is obtained and the circumference of a circle is $\pi\sqrt{(1+b^2)^3}/a$. Then the lower bound for the number of tangent lines can be calculated. When the error is e, the length of the circumference that can be cut by a tangent is $\sqrt{(1+b^2)^3}\cdot\arccos\left[\sqrt{(1+b^2)^3}\Big/\left(\sqrt{(1+b^2)^3}+2ae\right)\right]/2a$, and the number of required tangent lines is $2\pi/\left\{\arccos\left[\sqrt{(1+b^2)^3}\Big/\left(\sqrt{(1+b^2)^3}-2ae\right)\right]\right\}$.

By putting the parameters of constraints (35) into the above conclusions, both the maximum and minimum curvature radii of the revenue function $P_{it}(p_{it})$ can be thus obtained. Then, the lower bound (ψ_{LB}) and upper bound (ψ_{UB}) for the numbers of tangent lines to the function $P_{it}(p_{it})$ can be calculated: $\psi_{LB} = 2\pi/\left\{\arccos\left[\sqrt{(1+{D_i}^2)^3}\Big/\left(\sqrt{(1+{D_i}^2)^3}-2D_ie/b_i\right)\right]\right\}$ and $\psi_{UB} = 2\pi/\arccos{[-b_i/(2D_ie)]}$.

Theorem 3 is proved.

5. Numerical experiments

To assess the performance of the model and the algorithms proposed in this paper, the mathematical programming software CPLEX (12.5.1) was used to compare solutions solved by different algorithms at the same scale. These numerical experiments were conducted on a personal computer with Intel Core i7-4790 CPU (3.60 GHz), with 16.00 GB of RAM.

5.1. Results for three different scale problems

The relevant parameters involved in the experiments are uniformly distributed within the range shown in Table 2. For example, the values of F_n (Fixed cost for the selection of manufacturer n) are randomly distributed in [7000, 8000]. The experiment uses 3 discount levels. The parameters related to the discount level are uniformly distributed in the ranges given in Table 3.

Table 2
Parameters setting for the experiments.

Parameters	Value	Parameters	Value	Parameters	Value
F_n	[7000, 8000]	L_{wit}	[0.5, 1.5]	U_{nit}	[5, 6.5]
R_w	[3000, 4000]	L_{nt}	[0.3, 1.3]	C_{wit}	[1.2, 2.2]
D_i	[4000, 5000]	V_{nwit}	[1, 2]	h_{wi0}	[40, 60]
b_i	[50, 55]	h_{n0}	[100, 150]	Ω_n^{max}	[1500, 2200]
A_i	[3, 5]	H_n^{max}	[2000, 2200]	K_w^{max}	[1000, 1200]
K_n^{max}	[2200, 2500]	K_w^{max}	[800, 1000]		

Table 3
Parameter settings related to different discount levels.

The discount level	Parameters	Value	Parameters	Value
1	G_{n1t}	[10, 13]	UL_{n1}	[800, 1000]
2	G_{n2t}	[9, 10]	UL_{n2}	[1800, 2000]
3	G_{n3t}	[8, 9]	UL_{n3}	[2800, 3000]

First, a sensitivity analysis experiment of the error value e was conducted to select the appropriate parameter for the outer-approximation algorithm. Table 4 shows the effects of different error values e on the solution results for $N = 3$, $W = 9$, $I = 3$, and $T = 10$. Table 4 shows that the gaps of the objective values between the outer-approximation algorithm and the SOCP algorithm decrease with gradually decreasing error value e. However, the computing time increases with decreasing error value e. The gap between both algorithms is relatively small and the computing time is relatively short at an error value of 5. Therefore, an error value of 5 ($e = 5$) was chosen for the outer-approximation algorithm to compare the solution quality and CPU time between both algorithms.

Table 4
Error value sensitivity analysis.

Error Values	Time (s)		OBJ		GAP$_{\mathrm{OBJ}}$ (%)
	OA	SOCP	OA	SOCP	
50	2.265		185897		0.794
40	2.706		185569		0.616
30	2.808		185246		0.441
20	2.859		184921		0.265
10	3.941	8.775	184818	184433	0.209
5	4.592		184654		0.120
1	10.352		184597		0.089
0.1	60.288		184456		0.012
0.01	90.067		184443		0.005

Notes: (1) Time is the computation time (s). (2) OBJ shows objective values. (3) GAP$_{\mathrm{OBJ}}$ is calculated by: (OBJ$_{\mathrm{OA}}$ - OBJ$_{\mathrm{SOCP}}$)/OBJ$_{\mathrm{SOCP}}$.

To ensure the validity of the comparison between both algorithms, this study sets experimental parameters for three sets of different scales, including small-, medium-, and large-sized experiments. Tables 5, 6, and 7 show the numerical experimental results of both the outer-approximation algorithm and the SOCP algorithm in three scales.

Table 5
Comparison of experimental results of both developed algorithms for small scales.

Instances	T	N	W	I	Time (s)		OBJ		GAP$_{OBJ}$ (%)
					OA	SOCP	OA	SOCP	
1	5	1	5	2	0.651	0.865	123174	123110	0.052
2	5	1	5	3	0.605	0.648	191598	191432	0.087
3	5	1	7	3	0.596	0.714	198030	197873	0.080
4	5	1	7	4	0.828	1.171	265531	265381	0.057
5	5	2	7	4	0.863	1.190	275589	275448	0.051
6	5	2	7	5	0.688	1.050	345659	345498	0.047
7	5	2	9	5	0.940	1.696	377950	377815	0.036
8	5	2	9	6	1.155	3.534	437017	436757	0.060

Notes: (1) Time represents the computation time (s). (2) OBJ represents the objective values. (3) GAP$_{OBJ}$ is calculated by: (OBJ_{OA} - OBJ_{SOCP})/OBJ_{SOCP}.

Since the SOCP algorithm can obtain an optimal solution, the gap and CPU time were compared between the outer-approximation algorithm and the SOCP algorithm. Table 5 shows that the outer-approximation algorithm and the SOCP algorithm can obtain the solution in a shorter computation time for small-scale problems, and the gap of results between both algorithms is very small. The average gap value is about 0.0588% for the outer-approximation algorithm and the SOCP algorithm.

The gaps of results are also not evident for medium-scale instances in Table 6, and the average gap value is about 0.1144% for the outer-approximation algorithm and the SOCP algorithm.

Table 6
Comparison of experimental results of both developed algorithms for middle scales.

Instances	T	N	W	I	Time (s)		OBJ		GAP$_{OBJ}$ (%)
					OA	SOCP	OA	SOCP	
9	10	3	9	3	55.365	53.903	234684	233512	0.502
10	10	3	9	4	50.758	120.534	336636	336352	0.084
11	10	3	11	4	35.934	30.206	350508	350206	0.086
12	10	3	11	5	16.085	19.333	487126	486835	0.060
13	10	4	11	5	26.832	217.654	489126	488848	0.057
14	10	4	11	6	45.431	1131.526	569665	569189	0.084
15	10	4	13	6	51.661	1117.336	589043	587256	0.304
16	10	4	13	7	176.904	3296.290	674985	674122	0.128

Notes: (1) Time represents the computation time (s). (2) OBJ represents the objective values. (3) GAP$_{OBJ}$ is calculated by: (OBJ_{OA} - OBJ_{SOCP})/OBJ_{SOCP}.

However, the CPU time of the SOCP algorithm increased noticeably compared to the outer-approximation algorithm.

Table 7 also provides a comparison between the outer-approximation algorithm and the SOCP algorithm for large-scale instances. For the SOCP algorithm, the optimal solution cannot been obtained for more than 4 hours. Experiments indicate that the SOCP algorithm cannot solve instances where the scale of the case exceeds 20 periods, 5 manufacturers, 13 warehouses, and 4 products; however, the outer-approximation algorithm can solve these cases within reasonable computing time.

Table 7
Comparison of experimental results of both developed algorithms for large scales.

Instances	T	N	W	I	Time (h)		OBJ		GAP$_{OBJ}$ (%)
					OA	SOCP	OA	SOCP(*)	
17	20	5	13	4	0.9	≥ 4	315875	298542	-
18	20	5	13	5	1.1	≥ 4	594788	575853	-
19	20	5	15	5	1.5	≥ 4	680302	659475	-
20	20	5	15	6	1.8	≥ 4	751012	721217	-
21	20	6	15	6	1.8	≥ 4	708496	669917	-
22	20	6	15	7	1.9	≥ 4	852931	819432	-
23	20	6	17	7	2.1	≥ 4	899782	850263	-
24	20	6	17	8	2.6	≥ 4	1102087	1089521	-

Notes: (1) Time represents the computation time (h). (2) OBJ represents the objective values. (3) GAP$_{OBJ}$ is calculated by: (OBJ_{OA} - OBJ_{SOCP})/OBJ_{SOCP}. (4) ≥ 4 indicates that the computation time is more than 4 h and the optimal solution cannot be found. (5) (*) indicates that the value is the result of the experimental calculation of 4 h. (6) (-) represents that the value cannot be calculated.

Based on the numerical results, the following conclusions can be drawn:

(1) For small and medium-sized experiments, SOCP algorithm can obtain the optimal solution in a short time. the outer-approximation algorithm can also find the approximate optimal solution in an effective time. In terms of solving accuracy, the average gap between the outer-approximation

method and SOCP algorithm is 0.0588%.

(2) For large-sized experiments, the SOCP algorithm cannot solve the model, while the outer-approximation algorithm can still obtain approximate solutions within a reasonable time. This implies that the outer-approximation algorithm is an effective method to solve the proposed MINLP model on large-sized experiments.

5.2. Comparison with the results of the problems without discount levels

Tables 8, 9, and 10 show the experimental results of the two algorithms at three different scales without considering the discount level. The advantage of the outer-approximation algorithm in solving medium- and large-scale problems is reflected by the results of comparing the quality and efficiency of the algorithms.

Table 8
Experimental results of both developed algorithms for small scales without discount levels.

Instances	T	N	W	I	Time (s)		OBJ		GAP$_{OBJ}$ (%)
					OA	SOCP	OA	SOCP	
1	5	1	5	2	0.192	0.151	112475	112248	0.00202
2	5	1	5	3	0.195	0.194	181536	180897	0.00353
3	5	1	7	3	0.294	0.161	192591	192353	0.00124
4	5	1	7	4	0.280	0.170	252403	251959	0.00176
5	5	2	7	4	1.244	1.186	247303	247040	0.00106
6	5	2	7	5	0.397	0.197	324215	323884	0.00102
7	5	2	9	5	0.348	0.208	357686	357149	0.00150
8	5	2	9	6	0.685	0.589	413947	413488	0.00111

Notes: (1) Time represents the computation time (s). (2) OBJ represents the objective values. (3) GAP$_{OBJ}$ is calculated by: (OBJ_{OA} - OBJ_{SOCP})/OBJ_{SOCP}.

Table 9
Experimental results of both developed algorithms for middle scales without discount levels.

Instances	T	N	W	I	Time (s)		OBJ		GAP$_{OBJ}$ (%)
					OA	SOCP	OA	SOCP	
9	10	3	9	3	10.226	7.175	185105	184433	0.00364
10	10	3	9	4	11.963	30.549	265182	264502	0.00257
11	10	3	11	4	3.967	0.447	263976	263300	0.00257
12	10	3	11	5	3.560	20.001	393326	392635	0.00176
13	10	4	11	5	28.458	0.598	377904	377219	0.00182
14	10	4	11	6	21.113	78.755	461111	460631	0.00104
15	10	4	13	6	8.423	211.062	474663	473702	0.00203
16	10	4	13	7	13.587	2418.346	568995	568252	0.00131

Notes: (1) Time represents the computation time (s). (2) OBJ represents the objective values. (3) GAP$_{OBJ}$ is calculated by: (OBJ_{OA} - OBJ_{SOCP})/OBJ_{SOCP}.

Table 11 compares the results of the two models obtained by the SOCP and the outer-approximation method at different scales. We can see that considering the incremental quantity

Table 10
Experimental results of both developed algorithms for large scales without discount levels.

Instances	T	N	W	I	Time (s)		OBJ		GAP$_{OBJ}$ (%)
					OA	SOCP	OA	SOCP(*)	
17	20	5	13	4	394.585	$\geq 4h$	238495	229156	-
18	20	5	13	5	644.393	$\geq 4h$	444029	435056	-
19	20	5	15	5	965.325	$\geq 4h$	503654	499863	-
20	20	5	15	6	1940.635	$\geq 4h$	557763	549765	-
21	20	6	15	6	2399.359	$\geq 4h$	519251	513265	-
22	20	6	15	7	2965.354	$\geq 4h$	649691	642651	-
23	20	6	17	7	2657.422	$\geq 4h$	664633	656210	-
24	20	6	17	8	3448.729	$\geq 4h$	798300	790280	-

Notes: (1) Time represents the computation time (s). (2) OBJ represents the objective values. (3) GAP$_{OBJ}$ is calculated by: (OBJ_{OA} - OBJ_{SOCP})/OBJ_{SOCP}. (4) $\geq 4h$ indicates that the computation time is more than 4 h and the optimal solution cannot be found. (5) (*) indicates that the value is the result of the experimental calculation of 4 h. (6) (-) represents that the value cannot be calculated.

discount can greatly improve the profits of the entire supply chain network. As manufacturers receive incremental quantity discount when purchasing raw materials, there is an impact on the purchase volume per purchase. On the small scale problems, it can increase profits by about 3%-11%. On the medium scale problems, the profit can be increased by about 19%-33%. On the large scale problems, it can increase profits by about 31%-38%. Therefore, for logistics network activities with a long planning period, price discounts provided by suppliers can greatly improve the profits of the entire supply chain network.

In addition to the impact of discount level on network profits in Table 11, we can also see the solution efficiency of the two algorithms at different scales. The outer-approximation method uses piecewise linear functions to approximate the calculation, and the solution obtained is not accurate. However, because smaller error values are chosen, for small-scale instances, the difference between the solution obtained using the outer-approximation method and the exact solution derived from SOCP is minimal. In small and medium-sized experiments, the solution error value under the two schemes using the outer-approximation method and the SOCP is within 1%. However, for large-scale instances, SOCP cannot provide a solution within a reasonable time. For instances 17-24, SOCP results are limited by a 4-hour compute time limit. In this case, the error value of the two algorithms is controlled within 7% under the two schemes. However, due to the long solution time of SOCP, the outer-approximation method is preferred to solve large-scale problem instances. Overall, as the size of the comparison instances increase, the differences between the two algorithms become more and more significant. Due to the NP-hard of problem (see, e.g., Ahmadi-Javid & Hoseinpour, 2015b; Fattahi et al., 2015), the computational efficiency of the outer-approximation method is more advantageous for larger instances, especially when SOCP is difficult to provide a solution within a reasonable time.

Table11

Comparison of experimental results of two mathematical models for different scales.

Scales	Instances	Discount level		Non-Discount level		$\mathrm{GAP}_{\mathrm{OA}}$ (%)	$\mathrm{GAP}_{\mathrm{SOCP}}$ (%)
		$OBJ_{\text{OA-1}}$	$OBJ_{\text{SOCP-1}}$	$OBJ_{\text{OA-2}}$	$OBJ_{\text{SOCP-2}}$		
Small scales	1	123174	123110	112475	112248	9.512	9.677
	2	191598	191432	181536	180897	5.543	5.824
	3	198030	197873	192591	192353	2.824	2.87
	4	265531	265381	252403	251959	5.201	5.327
	5	275589	275448	247303	247040	11.438	11.499
	6	345659	345498	324215	323884	6.614	6.673
	7	377950	377815	357686	357149	5.665	5.786
	8	437017	436757	413947	413488	5.573	5.627
Middle scales	9	234684	233512	185105	184433	26.784	26.611
	10	336636	336352	265182	264502	26.945	27.164
	11	350508	350206	263976	263300	32.78	33.006
	12	487126	486835	393326	392635	23.848	23.992
	13	489126	488848	377904	377219	29.431	29.593
	14	569665	569189	461111	460631	23.542	23.567
	15	589043	587256	474663	473702	24.097	23.972
	16	674985	674122	568995	568252	18.628	18.631
Large scales	17	315875	298542	238495	229156	32.445	30.279
	18	594788	575853	444029	435056	33.953	32.363
	19	680302	659475	503654	499863	35.073	31.931
	20	751012	721217	557763	549765	34.647	31.186
	21	708496	669917	519251	513265	36.446	30.521
	22	852931	819432	649691	642651	31.283	27.508
	23	899782	850263	664633	656210	35.38	29.572
	24	1102087	1089521	798300	790280	38.054	37.865

Notes: (1) $OBJ_{\text{OA-1}}$ represents the objective values for our model considering the discount level using outer-approximation algorithm. (2) $OBJ_{\text{SOCP-1}}$ represents the objective values for our model considering the discount level using SOCP method. (3) $OBJ_{\text{OA-2}}$ represents the objective values for our model without considering the discount level using outer-approximation algorithm. (4) $OBJ_{\text{SOCP-2}}$ represents the objective values for our model without considering the discount level using SOCP method. (5) $\mathrm{GAP}_{\mathrm{OA}}$ is calculated by: ($OBJ_{\text{OA-1}}$ – $OBJ_{\text{OA-2}}$) / $OBJ_{\text{OA-2}}$. (6) $\mathrm{GAP}_{\mathrm{SOCP}}$ is calculated by: ($OBJ_{\text{SOCP-1}}$ – $OBJ_{\text{SOCP-2}}$) / $OBJ_{\text{SOCP-2}}$.

From the perspective of model results and algorithm performance, in highly competitive environments, considering price-sensitive demand contributes to a better understanding of the market and consumers by enterprises, enabling them to formulate more suitable pricing and marketing strategies. This, in turn, increases sales revenue and profits, allowing them to gain a sustainable competitive advantage in fiercely competitive markets. This underscores the necessity of our study in treating price and demand as decision variables.

In the context of research on price-sensitive demand, Fattahi et al. (2015) and Ahmadi-Javid & Hoseinpour (2015a) have done some research. However, they encountered limitations in directly obtaining decision solutions for both price and demand variables. On the other hand, while Mogale et al. (2022) also treated demand and price as decision variables, their pricing scheme was not a multi-period dynamic decision. Building upon this, we further consider incremental quantity discount at the upstream manufacturer's procurement stage. Based on the results, it is evident that considering incremental quantity discount creates opportunities for profit growth in the supply chain network. In terms of algorithmic solutions, the SOCP and outer-approximation method have contributed to filling the research gap in handling such NP-hard problems.

6. Conclusion and Outlook

This paper studies a three-echelon multi-period SCND problem that considers both price and demand. At the same time, the impact of incremental quantity discount offered by the suppliers on the profitability of the supply chain network is considered. This study also assumes prices as decision variables and establishes an MINLP model with the goal to maximize the profit of the supply chain network. Due to the nonlinearity of the objective function, first, the proposed MINLP model is analyzed and the conclusion is obtained that the model [MINLP] has an optimal solution. We further get an important property of this optimal solution that the actual amount of purchased product is equal the demand. This allows us to use SOCP to solve the problem directly through commercially available software such as CPLEX and obtain an exact solution. Meanwhile, these conclusions are vital to solve the problem by outer-approximation method. In addition, both the upper and lower bounds for the number of tangent lines to the revenue function are calculated. Finally, numerical experiments have been conducted to validate the significance of the model and the efficacy of the developed algorithms.

The most important findings and management insights are the following:

(1) The research results provide direct decision-making solutions for product pricing and customer demand. This enables managers to make more accurate pricing decisions by responding to changes in demand while determining location and inventory decisions.

(2) Considering incremental quantity discount can greatly improve the profit of the whole supply chain network. Especially for logistics network activities with a long planning period, the profit

improvement is more obvious.

(3) The SOCP algorithm can find the optimal solutions for small-scale and medium-scale problems. However, it cannot solve large-scale problems.

(4) The solution of the outer-approximation algorithm is close to the optimal solution for small-scale and medium-scale problems. Moreover, it can still obtain approximate optimal solutions within a reasonable computing time for large-scale problems.

Furthermore, there were certain limitations in conducting this study, which can serve as directions for future research efforts. As a primary suggestion, uncertainty methods, such as stochastic optimal control (Gökgöz & Öktem, 2021; Savku & Weber, 2022), robust optimization (Özmen et al., 2017; Khalilpourazari & Hashemi Doulabi, 2022; Goli et al., 2023; Lotfi et al., 2023; Tirkolaee et al., 2023), and grey systems (Gökgöz & Öktem, 2021; Masoomi et al., 2022), can be applied to models to improve the robustness. From the perspective of computation, the effectiveness of linear approximation (Taylan et al., 2018) and the more effective accurate algorithms can be further considered to provide the optimal solutions to large-scale instances within a reasonable time.

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Data Availability Statemen

The data that support the findings of this study are available from the authors, upon reasonable request.

References

Ahmadi, A., & Ghasemi, P. (2022). Pricing strategies for online hotel searching: a fuzzy inference system procedure. *Kybernetes*, (pp. –).

Ahmadi-Javid, A., & Ghandali, R. (2014). An efficient optimization procedure for designing a capacitated distribution network with price-sensitive demand. *Optimization and Engineering*, *15(3)*, 801–817.

Ahmadi-Javid, A., & Hoseinpour, P. (2015a). Incorporating location, inventory and price decisions into a supply chain distribution network design problem. *Computers* & *Operations Research*, *56*, 110–119.

Ahmadi-Javid, A., & Hoseinpour, P. (2015b). A location-inventory-pricing model in a supply chain distribution network with price-sensitive demands and inventory-capacity constraints. *Transportation Research Part E: Logistics and Transportation Review*, *82*, 238–255.

Ardjmand, E., Weckman, G. R., Young, W. A., Sanei Bajgiran, O., & Aminipour, B. (2016). A robust optimisation model for production planning and pricing under demand uncertainty. *International Journal of Production Research*, *54(13)*, 3885–3905.

Azevedo, N., Pinheiro, D., Xanthopoulos, S. Z., & Yannacopoulos, A. N. (2018). Contingent claim pricing through a continuous time variational bargaining scheme. *Annals of Operations Research*, *260*, 95–112.

Babazadeh, R., Razmi, J., Pishvaee, M. S., & Rabbani, M. (2017). A sustainable second-generation biodiesel supply chain network design problem under risk. *Omega*, *66*, 258–277.

Badri, H., Bashiri, M., & Hejazi, T. H. (2013). Integrated strategic and tactical planning in a supply chain network design with a heuristic solution method. *Computers* & *Operations Research*, *40(4)*, 1143–1154.

Baud-Lavigne, B., Agard, B., & Penz, B. (2016). Simultaneous product family and supply chain design: An optimization approach. *International Journal of Production Economics*, *174*, 111–118.

Behzadi, G., O'Sullivan, M. J., Olsen, T. L., Scrimgeour, F., & Zhang, A. (2017). Robust and resilient strategies for managing supply disruptions in an agribusiness supply chain. *International Journal of Production Economics*, *191*, 207–220.

Burke, G. J., Carrillo, J., & Vakharia, A. J. (2008). Heuristics for sourcing from multiple suppliers with alternative quantity discounts. *European Journal of Operational Research*, *186(1)*, 317–329.

Cachon, G. P., & Harker, P. T. (2002). Competition and outsourcing with scale economies. *Management Science*, *48(10)*, 1314–1333.

Chen, X., He, C., Chen, Y., & Xie, Z. (2023). Internet of Things (IoT)—blockchain-enabled pharmaceutical supply chain resilience in the post-pandemic era. *Frontiers of Engineering Management*, *10(1)*, 82–95.

Cheng, C., & Tang, L. (2018). Robust policies for a multi-stage production/inventory problem with switching costs and uncertain demand. *International Journal of Production Research*, *56(12)*, 4264–4282.

De Matta, R., & Miller, T. (2015). Formation of a strategic manufacturing and distribution network with transfer prices. *European Journal of Operational Research*, *241(2)*, 435–448.

Fattahi, M., Govindan, K., & Keyvanshokooh, E. (2017). Responsive and resilient supply chain network design under operational and disruption risks with delivery lead-time sensitive customers. *Transportation Research Part E: Logistics and Transportation Review*, *101*, 176–200.

Fattahi, M., Mahootchi, M., Govindan, K., & Husseini, S. M. M. (2015). Dynamic supply chain network design with capacity planning and multi-period pricing. *Transportation Research Part E: Logistics and Transportation Review*, *81*, 169–202.

Georgiadis, M. C., Tsiakis, P., Longinidis, P., & Sofioglou, M. K. (2011). Optimal design of supply chain networks under uncertain transient demand variations. *Omega*, *39(3)*, 254–272.

Ghoreishi, M., Weber, G. W., & Mirzazadeh, A. (2015). An inventory model for non-instantaneous deteriorating items with partial backlogging, permissible delay in payments, inflation-and selling price-dependent demand and customer returns. *Annals of Operations Research*, *226*, 221–238.

Goli, A., Ala, A., & Mirjalili, S. (2023). A robust possibilistic programming framework for designing an organ transplant supply chain under uncertainty. *Annals of Operations Research*, *328*, 493–530.

Guo, C., & Li, X. (2014). A multi-echelon inventory system with supplier selection and order allocation under stochastic demand. *International Journal of Production Economics*, *151*, 37–47.

Gupta, M., Kaur, H., & Singh, S. P. (2021). Multi-echelon agri-food supply chain network design integrating operational and strategic objectives: a case of public distribution system in india. *Annals of Operations Research*, (pp. 1–58).

Gökgöz, N., & Öktem, H. (2021). Modeling of tumor-immune system interaction with stochastic hybrid systems with memory: a piecewise linear approach. *Advances in the Theory of Nonlinear Analysis and its Application*, *5(1)*, 25–38.

Hanh, N. T. M., Chen, J. M., & Van Hop, N. (2022). Pricing strategy and order quantity allocation with price-sensitive demand in three-echelon supply chain. *Expert Systems with Applications*, *206*, 117873.

Hanjoul, P., Hansen, P., Peeters, D., & Thisse, J. F. (1990). Uncapacitated plant location under alternative spatial price policies. *Management Science*, *36(1)*, 41–57.

Hsieh, C. C., Liu, Y. T., & Wang, W. M. (2010). Coordinating ordering and pricing decisions in a two-stage distribution system with price-sensitive demand through short-term discounting. *European Journal of Operational Research*, *207(1)*, 142–151.

Huang, H., He, Y., & Li, D. (2018). Coordination of pricing, inventory, and production reliability decisions in deteriorating product supply chains. *International Journal of Production Research*, *56(18)*, 6201–6224.

Jackson, J. E., & Munson, C. L. (2019). Common replenishment cycle order policies for multiple products with capacity expansion opportunities and quantity discounts. *International Journal of Production Economics*, *218*, 170–184.

Ke, G. Y., & Bookbinder, J. H. (2018). Coordinating the discount policies for retailer, wholesaler, and less-than-truckload carrier under price-sensitive demand: A tri-level optimization approach. *International Journal of Production Economics*, *196*, 82–100.

Keyvanshokooh, E., Ryan, S. M., & Kabir, E. (2016). Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated benders decomposition. *European Journal of Operational Research*, *249(1)*, 76–92.

Khalilpourazari, S., & Hashemi Doulabi, H. (2022). A flexible robust model for blood supply chain network design problem. *Annals of Operations Research*, *328*, 701–726.

Kılıç, D. K., & Uğur, Ö. (2023). Hybrid wavelet-neural network models for time series. *Applied Soft Computing*, *144*, 110469.

Li, F., Chen, Z. L., & Tang, L. (2017). Integrated production, inventory and delivery problems: Complexity and algorithms. *INFORMS Journal on Computing*, *29(2)*, 232–250.

Lotfi, R., Haqiqat, E., Rajabi, M. S., & Hematyar, A. (2023). Robust and resilience budget allocation for projects with a risk-averse approach: A case study in healthcare projects. *Computers & Industrial Engineering*, *176*, 108948.

Margolis, J. T., Sullivan, K. M., Mason, S. J., & Magagnotti, M. (2018). A multi-objective optimization model for designing resilient supply chain networks. *International Journal of Production Economics*, *204*, 174–185.

Masoomi, B., Sahebi, I. G., Fathi, M., Yıldırım, F., & Ghorbani, S. (2022). Strategic supplier selection for renewable energy supply chain under green capabilities (fuzzy bwm-waspas-copras approach). *Energy Strategy Reviews*, *40*, 100815.

Mogale, D. G., De, A., Ghadge, A., & Aktas, E. (2022). Multi-objective modelling of sustainable closed-loop supply chain network with price-sensitive demand and consumer's incentives. *Computers & Industrial Engineering*, *168*, 108105.

Mohammed, A., & Wang, Q. (2017). The fuzzy multi-objective distribution planner for a green meat supply chain. *International Journal of Production Economics*, *184*, 47–58.

Momenitabar, M., Dehdari Ebrahimi, Z., Arani, M., Mattson, J., & Ghasemi, P. (2022). Designing a sustainable closed-loop supply chain network considering lateral resupply and backup suppliers using fuzzy inference system. *Environment, Development and Sustainability*, (pp. 1–34).

Mönch, L., Uzsoy, R., & Fowler, J. W. (2018). A survey of semiconductor supply chain models part i: semiconductor supply chains, strategic network design, and supply chain simulation. *International Journal of Production Research*, *56(13)*, 4524–4545.

Nickel, S., Saldanha-da Gama, F., & Ziegler, H. P. (2012). A multi-stage stochastic supply network design problem with financial decisions and risk management. *Omega*, *40*, 511–524.

Özceylan, E., & Paksoy, T. (2013). A mixed integer programming model for a closed-loop supply-chain network. *International Journal of Production Research*, *51(3)*, 718–734.

Özmen, A., Kropat, E., & Weber, G. W. (2017). Robust optimization in spline regression models for multi-model regulatory networks under polyhedral uncertainty. *Optimization*, *66(12)*, 2135–2155.

Pan, F., & Nagi, R. (2013). Multi-echelon supply chain network design in agile manufacturing. *Omega*, *41(6)*, 969–983.

Peng, P., Snyder, L. V., Lim, A., & Liu, Z. (2011). Reliable logistics networks design with facility disruptions. *Transportation Research Part B: Methodological*, *45(8)*, 1190–1211.

Pervin, M., Roy, S. K., & Weber, G. W. (2018). Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration. *Annals of Operations Research*, *260*, 437–460.

Safaei, S., Ghasemi, P., Goodarzian, F., & Momenitabar, M. (2022). Designing a new multi-echelon multi-period closed-loop supply chain network by forecasting demand using time series model: a genetic algorithm. *Environmental Science and Pollution Research*, *29(53)*, 79754–79768.

Sagaert, Y. R., Kourentzes, N., De Vuyst, S., Aghezzaf, E. H., & Desmet, B. (2019). Incorporating macroeconomic leading indicators in tactical capacity planning. *International Journal of Production Economics*, *209*, 12–19.

Sahling, F., & Kayser, A. (2016). Strategic supply network planning with vendor selection under consideration of risk and demand uncertainty. *Omega*, *59*, 201–214.

Savku, E., & Weber, G. W. (2022). Stochastic differential games for optimal investment problems in a markov regime-switching jump-diffusion market. *Annals of Operations Research*, *312(2)*, 1171–1196.

Song, M., Yuan, S., Bo, H., Song, J., Pan, X., & Jin, K. (2022). Robust optimization model of anti-epidemic supply chain under technological innovation: learning from COVID-19. *Annals of Operations Research*, (pp. 1–31).

Tamjidzad, S., & Mirmohammadi, S. H. (2018). A two-stage heuristic approach for a multi-item inventory system with limited budgetary resource and all-units discount. *Computers & Industrial Engineering*, *124*, 293–303.

Tang, L., Li, F., & Chen, Z. L. (2019). Integrated scheduling of production and two-stage delivery of make-to-order products: Offline and online algorithms. *INFORMS Journal on Computing*, *31(3)*, 493–514.

Taylan, P., Uysal, E., & Tez, M. (2018). On curvature measurements of the nonlinear errors in variable models by application study. *Journal of Statistics and Management Systems*, *21(5)*, 741–765.

Temocin, B. Z., Korn, R., & Selcuk-Kestel, A. S. (2018). Constant proportion portfolio insurance in defined contribution pension plan management. *Annals of Operations Research*, *266(1-2)*, 329–348.

Tirkolaee, E. B., Aydin, N. S., & Mahdavi, I. (2023). A bi-level decision-making system to optimize a robust-resilient-sustainable aggregate production planning problem. *Expert Systems with Applications*, *228*, 120476.

Umpfenbach, E. L., Dalkiran, E., Chinnam, R. B., & Murat, A. E. (2018). Promoting sustainability of automotive products through strategic assortment planning. *European Journal of Operational Research*, *269(1)*, 272–285.

Wang, N., Lu, J., Feng, G., Ma, Y., & Liang, H. (2016). The bullwhip effect on inventory under different information sharing settings based on price-sensitive demand. *International Journal of Production Research*, *54(13)*, 4043–4064.

Yang, F., Huang, Y. H., & Li, J. (2019). Alternative solution algorithm for winner determination problem with quantity discount of transportation service procurement. *Physica A: Statistical Mechanics and its Applications*, *535*, 122286.

Zhen, L., & Li, H. (2022). A literature review of smart warehouse operations management. *Frontiers of Engineering Management*, *9(1)*, 31–55.

Zokaee, S., Jabbarzadeh, A., Fahimnia, B., & Sadjadi, S. J. (2017). Robust supply chain network design: an optimization model with real world application. *Annals of Operations Research*, *257(1)*, 15–44.