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# Mixed-Integer Optimization for Ship Retrofitting in Green Logistics

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**Abstract:** Maritime transportation plays a pivotal role in global trade and international supply chains. However, the sector is also a significant source of emissions. One of the most promising technologies for reducing these emissions is air lubrication, which involves installing bubbles along the hull of a ship. Despite its potential, the design of cost-effective bubble-installation plans for ship fleets over the planning horizon remains unexplored in the literature. This paper addresses this gap by proposing a mathematical programming model designed to optimize the installation of bubble-based systems. We present several propositions concerning the model's properties, supported by rigorous proofs. To validate the model's effectiveness, we conduct a series of computational experiments. The findings demonstrate that our optimization model enables shipping companies to devise bubble-installation plans that are cost-effective. This contribution not only extends the current understanding of emission reduction technologies in maritime transportation, but also offers practical insights for their implementation.

**Keywords:** maritime logistics; bubble installation; mixed-integer programming; sensitivity analysis

**MSC:** 90C11



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## 1. Introduction

The shipping sector is of great importance for global trade; approximately 80% of international trade relies on marine-based support, and hence, it is quite significant to the world economy [1]. Due to the scale of the shipping sector, it leads to severe emissions of green house gas (GHG), representing about 3% of the total GHG emissions [2]. Thus, the International Maritime Organization (IMO) proposes strict regulations to limit air pollutants like $NO_x$, $SO_x$, and $CO_2$ [3].

In order to comply with the regulations proposed by IMO, some shipping companies choose to install different kinds of scrubbers on ships to reduce the emissions. Yang et al. [4] conducted a research to evaluate the actual performance of some $SO_x$ scrubber systems. The evaluation results show that though the emissions of $SO_x$ are reduced to some extent, particulate matter emissions are increased at the same time. Chen et al. [5] indicated that scrubber systems deployed on ships increase the consumption of fuel and the installation costs of them are relatively high. Hence, more advanced techniques to reduce the emissions have been introduced in recent years. Among them, air lubrication is one of the most effective ways to reduce shipping emissions by saving fuel consumption of ships [6].

Air lubrication [7] can reduce hull friction by using air as a lubricant. Specifically speaking, a layer of air is generated between the specially profiled underside and water surface, so that the ship effectively glides through the water, hence reducing the energy use during the voyage. One of the methods based on air lubrication technology is installing bubbles on ships [7], which has been proved to be effective to reduce the fuel consumption, thereby decreasing the emissions from ships. However, the optimization models focusing on designing cost-effective bubble-installation plans for the saving of fuel consumption are seldom introduced [8].

### 1.1. Literature Review

In recent years, the maritime shipping industry has played an increasingly important role in the global economy [9]. Internationalized consumerism has expanded the need for remote trading, and the intensive exchange of remote-manufacturing-site products has boosted delivery across continents through maritime routes [10]. Along with the rapid development of the marine industry, the control of emissions of air pollutants has become increasingly strict [11]. According to Malloupas et al. [12], existing ways of reducing emissions of ships can be generally divided into three different categories: using alternative fuels in shipping, using renewable resources in shipping, and adapting new technologies to produce less emissions during the voyage. Since this research focuses on how the new technology, specifically, installing the bubble based on the air lubrication technology, helps ships reduce the emission of air pollutants, we mainly review the literature within this category.

Reducing the emission of air pollutants by installing equipment has already become a practical way for shipping companies to modify their ships [13]. The exhaust gas recirculation (EGR) system [5] was firstly deployed to ships to reduce the $NO_x$ emissions. However, it increases particulate matter emissions and fuel consumption at the same time [14]. Thus, it was replaced by the selective catalytic reduction (SCR) system later [5]. The installation of SCR has been proved to be a reliable technical solution to satisfy the limits imposed on both $NO_x$ and $SO_x$ emissions, and the fuel consumption drops as well. Despite its great performance in reducing the emissions, its large size limits its actual usage, as it is not applicable to most existing ships [15]. With the development of technologies, more advanced techniques have been applied to reduce the emission from ships. Nowadays, one of the most state-of-art techniques to reduce the emissions of ships is air lubrication [7], which has been proved to be effective in reducing the energy consumption and pollution of ships. Specifically, bubble installation, which means reducing the fuel consumption by installing a series of air dispensers on the bottom of the ship to spray air bubbles that form an air carpet at the bottom of the ship to reduce frictional drag resistance, has already been applied by Mitsubishi Heavy Industries on a new built ship, the Till-Deymann [16], and proved to significantly reduce fuel consumption.

Despite reducing emissions by deploying new techniques to ships, it is also of great importance to provide accurate deployment plans for shipping companies to scale up their ships' optimization of reducing emissions by establishing a mathematical model. Although some studies have been conducted to maximize the fuel-saving performance of ships, such as Yan et al. [17] and Zaccone et al. [18], few studies have focused on designing an optimization model to evaluate the effectiveness of bubble installation. Thus, this paper proposes an optimization model to analyze the bubble-installation issue in detail. Subsequently, we analyze the characteristics of the model and conduct several sensitivity analysis experiments. The results show that the optimization model proposed in this paper is able to effectively assist the shipping companies in designing bubble installation plans.

### 1.2. Contributions and Organizations

The main contributions of this paper lie in the following aspects. A newly designed optimization model focusing on installing the bubble on the ship is established. The aim of the model is to help the shipping company to maximize its profits during the planning period by designing a cost-effective bubble-installation plan. Besides, several propositions originated from the model are put forward and their corresponding rigorous proofs are presented to enhance our methodological contribution.

The remainder of the paper is as follows. Section 2 illustrates the description of the problem and establishes the model. Section 3 analyzes the model by putting forward several propositions and proves them rigorously. Section 4 conducts sensitivity analysis experiments to analyze the effectiveness of the proposed model. Section 5 concludes the study and puts forward future research directions.

## 2. Problem Statement and Mathematical Model

This section describes the studied problem. We list needed notations in Table 1.

**Table 1.** Notations.

| Notations | Descriptions |
|---|---|
| Parameters | |
| $V$ | The number of ships possessed by the shipping company. |
| $T$ | The number of year from now on, i.e., the planning horizon. |
| $p_t$ | The fuel price of year $t$ ($t = 1, \ldots, T$). |
| $q_v$ | The amount of annual fuel consumption (ton) of ship $v$ ($v = 1, \ldots, V$). |
| $a_v$ | The percentage of fuel consumption saved by installing bubble for ship $v$ ($v = 1, \ldots, V$). |
| $c_v$ | The amount of money charged by the bank for installing bubble for ship $v$ ($v = 1, \ldots, V$) annually. |
| $\alpha$ | The annual interest rate of bank deposits. |
| Decision Variables | |
| $x_{vt}$ | Binary variable that equals 1 if ship $v$ ($v = 1, \ldots, V$) installs bubble at the start of year $t$ ($t = 1, \ldots, T$), or 0 otherwise. |
| $y_{vt}$ | Binary variable that equals 1 if ship $v$ ($v = 1, \ldots, V$) has bubble in year $t$ ($t = 1, \ldots, T$), or 0 otherwise. |
| $m_{vt}$ | Continuous variable that indicates the cost-saving of fuel consumption of ship $v$ ($v = 1, \ldots, V$) in year $t$ ($t = 1, \ldots, T$). |
| $n_t$ | Continuous variable that indicates the profits of company at the end of year $t$ ($t = 1, \ldots, T$). |

Consider a shipping company that possesses $V$ ships. For every individual ship $v$ ($v = 1, \ldots, V$), the annual fuel consumption is represented as $q_v$. In recent years, stricter regulations have been gradually imposed on the carbon emissions of ships. As a response, many ships decide to install the bubble to meet the requirements of the new regulations. The bubble is able to decrease the consumption of fuel, and thus, reduce the carbon emissions of the ship. Meanwhile, saving fuel consumption means that a ship can spend less money during its navigation, which brings more profits to the shipping company. Hence, shipping companies start to make the plan for installing bubbles for their ships.

We consider a planning horizon of a total of $T$ years. Assume that the bank charges $c_v$ for ship $v$ that completes the bubble installation at the end of every year, and the charging fee includes both the installation and maintenance costs. Furthermore, a ship equipped with bubble is able to save fuel consumption by a proportion of $a_v$. Besides, assume the fuel price in year $t$ in the planning horizon is $p_t$ ($t = 1, \ldots, T$). For the shipping company, the balance will be generated if the amount of cost saved by installing the bubbles for its ships exceeds the expense it spends on the bubble installation for those ships. Furthermore, the balance remaining at the end of each year can be deposited in the bank to make profits with the annual interest of $\alpha$. Furthermore, $t'$ refers to the index of the time period from time period 1 to time period $t$.

The objective function of this study is to maximize the profits of the shipping company for bubble-installation decisions for the given planning horizon. To this end, we need to decide whether to install the bubble for ship $v$ or not at each year; therefore, the decision variable is denoted as $x_{vt}$, which equals 1 if ship $v$ installs the bubble at the start of year $t$, or 0 otherwise. Consequently, we define $y_{vt}$ as a binary variable to determine whether ship

$v$ is equipped with the bubble in year $t$; here, $y_{vt}$ equals 1 if it is, or 0 otherwise. Then, the amount of fuel cost saved by ship $v$ in year $t$ can be denoted as $m_{vt}$, which is determined by its bubble-installation status $y_{vt}$, fuel-saving performance by installing bubble $a_v$, fuel consumption $q_v$, and the fuel price $p_t$ in year $t$.

In order to specify the profits gained by the shipping company due to the installation of bubbles, we consider the net present value, which indicates the profits of the shipping company at the end of year $T$ by summing the cost savings and the interests generated by the savings before the end of year $T$. Moreover, savings should be larger than or equal to the amount charged by the bank at the end of every year $t$ since the settlement is performed at that time.

Now, we should determine $x_{vt}$, $y_{vt}$, and $m_{vt}$ to maximize the profits of the company at the end of the year $T$. The mathematical model [M0] can be presented as follows:

$$\max n_T \quad (1)$$

$$\text{s.t.} \sum_{t=1}^{T} x_{vt} \leq 1, \quad v = 1, \ldots, V \quad (2)$$

$$y_{vt} = \sum_{t'=1}^{t} x_{vt'} \quad v = 1, \ldots, V, t = 1, \ldots, T, \quad (3)$$

$$m_{vt} = p_t a_v q_v y_{vt} \quad v = 1, \ldots, V, t = 1, \ldots, T, \quad (4)$$

$$n_t = \sum_{v=1}^{V} \sum_{t'=1}^{t} (1+\alpha)^{t-t'} (m_{vt'} - c_v y_{vt'}) \quad t = 1, \ldots, T, \quad (5)$$

$$x_{vt} \in \{0, 1\} \quad v = 1, \ldots, V, t = 1, \ldots, T, \quad (6)$$

$$y_{vt} \in \{0, 1\} \quad v = 1, \ldots, V, t = 1, \ldots, T, \quad (7)$$

$$m_{vt} \geq 0 \quad v = 1, \ldots, V, t = 1, \ldots, T, \quad (8)$$

$$n_t \geq 0 \quad t = 1, \ldots, T. \quad (9)$$

Objective function (1) maximizes the profits of company at the end of year $T$, which is calculated by deducting the costs spent on bubble installation from the sum of fuel costs saved by installing the bubble and the interest earned from the bank. Constraints (2) indicate that ship $v$ is able to install the bubble at most once during the planning horizon. Constraints (3) indicate whether ship $v$ is equipped with bubble in year $t$. Constraints (4) indicate the amount of fuel costs saved by ship $v$ in year $t$. For example, if ship $v$ installs the bubble at or before the start of year $t$, then $m_{vt}$ will be the multiplication of the fuel price of year $p_t$, the amount of fuel consumption of ship $v$ annually $q_v$ and $a_v$, and the percentage of fuel consumption saved by installing bubble for ship $v$. Constraints (5) indicate the profits gained by the company at the end of every year. Constraints (6) and (7) illustrate that $x_{vt}$ and $y_{vt}$ are both binary variables. Constraints (8) and (9) indicate that $m_{vt}$ and $n_t$ are both non-negative variables.

## 3. Model Analysis

In this section, we present several properties that are related to our presented model and corresponding rigorous proofs.

**Proposition 1.** *The model [M0] has an optimal solution.*

**Proof.** Since the model in question adheres to the framework of a mixed-integer linear programming model. Consequently, to substantiate Proposition 1, two critical conditions must be verified: (1) it is imperative to demonstrate the existence of a feasible solution within the parameters of the model and (2) it is essential to establish that the objective function value of the model is constrained by an upper bound.

For the first criterion, we let $x_{vt} = 0$ $(v = 1, \ldots, V, t = 1, \ldots, T)$. Following this premise and given Constraints (3), it logically ensures that $y_{vt} = 0$ $(v = 1, \ldots, V, t = 1, \ldots, T)$. Then, $m_{vt} = 0$ $(v = 1, \ldots, V, t = 1, \ldots, T)$ due to Constraints (4), and $n_t = 0$ $(t = 1, \ldots, T)$ due to Constraints (5). This sequence of derivations facilitates the identification of solutions that adhere to all of the constraints stipulated by the model, thereby affirming the existence of a feasible solution within its framework.

To satisfy the second criterion, it is imperative to establish that the objective function possesses an upper bound. By examining Constraints (5), the objective function can be expressed as:

$$n_T = \sum_{v=1}^{V} \sum_{t'=1}^{T} (1+\alpha)^{T-t'} (m_{vt'} - c_v y_{vt'}). \tag{10}$$

If $y_{vt'} = 0$ $(v = 1, \ldots, V, t' = 1, \ldots, T)$ and according to Constraints (5), we can obtain an upper bound of the objective function shown as follows:

$$n_T = \sum_{v=1}^{V} \sum_{t'=1}^{T} (1+\alpha)^{T-t'} m_{vt'}. \tag{11}$$

Furthermore, according to the stipulations of Constraints (4), $m_{vt'}$ is constrained to be less than or equal to $p_{t'} a_v q_v$. Given that $p_{t'}$, $a_v$, and $q_v$ are constants—being parameters of the model—it follows that $p_{t'} a_v q_v$ represents a constant threshold. Consequently, the objective function $n_T$ is conclusively bounded by $\sum_{v=1}^{V} \sum_{t'=1}^{T} (1+\alpha)^{T-t'} p_{t'} a_v q_v$. This demonstrates that the objective function has an upper bound, fulfilling the second criterion.

In summary, the analysis substantiates the existence of a feasible solution within the model and confirms that the objective function is constrained by an upper bound. These findings collectively validate the assertion that the model is endowed with an optimal solution. □

**Proposition 2.** *Constraints (7) can be removed.*

**Proof.** To prove that the removal of Constraints (7) does not change the optimal solution to the model, we first analyze their interactions with other constraints of the model. According to Constraints (6), $x_{vt}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$ is a binary decision variable that indicates whether ship $v$ $(v = 1, \ldots, V)$ installs the bubble in year $t$ $(t = 1, \ldots, T)$. Thus, $x_{vt}$ can only be set to 0 or 1. Furthermore, according to Constraints (2), $\sum_{t=1}^{T} x_{vt} \leq 1$ $(v = 1, \ldots, V)$, which means that for ship $v$, it can install the bubble at most once from year 1 to year $T$. Besides, Constraints (3) show that $y_{vt} = \sum_{t'=1}^{t} x_{vt'}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$, which indicate that $y_{vt}$ is the cumulative sum of $x_{vt'}$ from $t' = 1$ to $t$. Since $x_{vt}$ has been restricted to be less than or equal to 1 and it can only be set to 0 or 1, $y_{vt}$ can only be set to 0 or 1 as well. Hence, Constraints (7) can be removed because it is implied by Constraints (2), (3), and (6). □

**Proposition 3.** *Constraints (8) can be removed.*

**Proof.** According to Constraints (4), $m_{vt} = p_t a_v q_v y_{vt}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$, we can find that $m_{vt}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$ is composed of two parts: $p_t a_v q_v$ $(v = 1, \ldots, V, t = 1, \ldots, T)$ and $y_{vt}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$. For the first part, its components are all of the parameters that are larger than or equal to 0. Thus, the multiplication of these parameters are definitely larger than or equal to 0. For the second part, we have proved in Proposition 2 that $y_{vt}(v = 1, \ldots, V, t = 1, \ldots, T)$ can only be set to 0 or 1. Hence, $m_{vt}(v = 1, \ldots, V, t = 1, \ldots, T)$, which is the multiplication of the two parts, is absolutely larger than or equal to 0. So, Constraints (8) can be removed. □

**Proposition 4.** *In an optimal solution, it is possible that some ships install the bubble while others choose not to install it.*

**Proof.** We prove this proposition by showing an example in the first year. Assume that there are two ships, ship $v_1$ and ship $v_2$, with an annual bubble-installation cost of 1 million USD and 1.5 million USD, respectively. For ship $v_1$, its annual fuel consumption $q_1$ is 30,000 tons and the percentage of fuel consumption saved by installing the bubble is 1%. For ship $v_2$, its annual fuel consumption $q_1$ is 10,000 tons and the percentage of fuel consumption saved by installing the bubble is 2%. Furthermore, the fuel price $p_t(t = 1, \ldots, T)$ is 5000 USD per ton. According to Constraints (4) and (5), ship $v_1$ can save $30,000 \times 0.01 \times 5000 = 1.5$ million USD by installing the bubble. Furthermore, since the cost of installing the bubble for ship $v_1$ is 1 million USD, ship $v_1$ will absolutely choose to install the bubble because it can gain the profits of $1.5 - 1 = 0.5$ million USD. For ship $v_2$, it can save only $10,000 \times 0.02 \times 5000 =$ 1 million USD by installing the bubble according to Constraints (4) and (5). However, the installation cost of ship $v_2$ is 1.5 million USD, which exceeds the amount of money that ship $v_2$ can save by installing the bubble. Hence, ship $v_2$ will definitely refuse to proceed the bubble installation. Thus, ship $v_1$ chooses to install the bubble but ship $v_2$ will not proceed the installation. □

**Proposition 5.** *In an optimal solution, it is possible that two ships both install the bubble while the installing times are different.*

**Proof.** We prove this proposition by showing an example within two years. Assume there are two ships, ship $v_1$ and ship $v_2$, with an annual bubble-installation cost of 1 million USD and 1.5 million USD, respectively. For ship $v_1$, its annual fuel consumption $q_1$ is 30,000 tons and the percentage of fuel consumption saved by installing the bubble is 1%. For ship $v_2$, its annual fuel consumption $q_1$ is 10,000 tons and the percentage of fuel consumption saved by installing the bubble is 2%. Suppose that the fuel price at the first year $p_1$ is 5000 USD per ton and it increases to 10,000 USD per ton in the second year. Following the proof of Proposition 5, in the first year, ship $v_1$ chooses to install the bubble while ship $v_2$ rejects to do so. However, in the second year, ship $v_2$ would decide to install the bubble since the increase of the fuel price makes it profitable for ship $v_2$ to proceed the bubble installation. □

**Proposition 6.** *If the fuel price remains the same every year, an optimal solution exists whereby each ship either installs the bubble at the start of the first year or does not install it at all.*

**Proof.** According to Constraints (6), we can conclude that for ship $v$ $(v = 1, \ldots, V)$, the amount of money saved by installing the bubble $m_{vt}(v = 1, \ldots, V, t = 1, \ldots, T)$ does not change since the first year under the condition that the fuel price remains the same every year. Furthermore, from Constraints (7), they indicate that the profits gained by each ship $v$ $(v = 1, \ldots, V)$ are affected by the amount of money saved by installing the bubble $m_{vt}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$ and the bubble-installation cost $c_v$ $(v = 1, \ldots, V)$. Thus, for a ship $v$ $(v = 1, \ldots, V)$, if the installation of the bubble is profitable, then it will install the bubble at the beginning of the first year since it will gain more profits if it completes the installation as early as possible. On the other hand, for a ship $v$ $(v = 1, \ldots, V)$, if the amount of money $m_{vt}$ $(v = 1, \ldots, V, t = 1, \ldots, T)$ saved by installing the bubble is unable to cover the installation cost $c_v$ $(v = 1, \ldots, V)$, it will not install the bubble at the beginning of the first year, and it will be impossible to install in the following years, obviously. □

**Proposition 7.** *If the fuel price strictly decreases every year, all of the optimal solutions entail that each ship either installs the bubble at the start of the first year or does not install it thereafter.*

**Proof.** At the beginning of the first year, if a ship $v$ $(v = 1, \ldots, V)$ is unable to make profits by installing the bubble, it will not choose to proceed the bubble installation. Since the fuel price strictly decreases along the time goes by, the amount of the money saved by the bubble installation also decreases year by year. So, it is obviously unnecessary for this ship to install the bubble in the following years. On the other hand, if a ship $v$ $(v = 1, \ldots, V)$ is able to make profits by installing the bubble, we can analyze this situation from its contradiction. We have known that the amount of money saved by the bubble installation decreases with the decline of the fuel price. Thus, for ship $v$ $(v = 1, \ldots, V)$, which can make profits in the first year by installing the bubble, it will install the bubble as early as possible to maximize its profits. For example, if ship $v$ $(v = 1, \ldots, V)$ installs the bubble in the second year, it will lose the opportunity to gain these profits in the first year. It is easy to find that for ship $v$ $(v = 1, \ldots, V)$, the later it installs the bubble, the fewer profits it will gain. As a result, it is undeniable that the ship will choose to install the bubble at the beginning of the first year. In conclusion, for ship $v$ $(v = 1, \ldots, V)$, it will either install the bubble at the beginning of the first year or not install it at all. □

## 4. Computational Experiments

This section conducts computational experiments to verify the effectiveness of our proposed model. We adopt Gurobi to solve our model and conduct experiments based on Python. The experiments are conducted on Intel(R) Core(TM) i5-10200H CPU @ 2.40 GHz with 16 GB RAM powered by Windows 10. We first set initial values for parameters to obtain basic results. Furthermore, sensitivity analyses are conducted to examine the impact of these parameters.

The settings of fuel price and interest rate are based on Loennechen et al. [19] and Cochrane et al. [20], respectively. Specifically, the fuel price is set to be 500 USD per ton initially according to Loennechen et al. [19], and the annual interest rate of the bank is set to be 4.5% per year according to Cochrane et al. [20]. We set the number of ships possessed by the shipping company, $V$, to be 10. For $T$, we set it to be 20 years, which means that our model intends to deal with the bubble-installation decisions within the following 20 years. For the fuel price of year $t$, $p_t$ $(t = 1, \ldots, T)$, referring to Kim et al. [21], we set it to be a fixed value as 500 USD per ton. For the amount of fuel consumption of ship $v$ $(v = 1, \ldots, V)$ annually, $q_v$ $(v = 1, \ldots, V)$, referring to Kim et al. [21], we set that all of the ships' fuel consumption to be 5000 tons per year. For the percentage of fuel consumption saved by installing the bubble, $a_v$ $(v = 1, \ldots, V)$, referring to Kim et al. [21], we set it to be 15%. For the amount of money charged by the bank for installing the bubble for ship $v$ $(v = 1, \ldots, V)$ every year, we set it to be 0.5 million USD. Furthermore, for the annual interest rate of bank deposits, referring to Kim et al. [21], we set it to be 4.5% per year.

### *4.1. Basic Results*

The experimental results given our initial parameter settings are shown in Table 2. According to Table 2, it is easy to find that all of the ships decide to install the bubbles in the first year, since they can gain profits by proceeding the bubble installation. According to Constraints (2), all of the ships are able to install the bubble at most once during the planning horizon. Thus, no ship installs the bubble in the following years because they have all installed the bubble in the first year. The experiment results verify the property shown in Proposition 6, which states that if the fuel price remains the same every year, there exists an optimal solution whereby each ship either installs the bubble at the start of the first year or does not install it at all. Furthermore, based on our parameter settings, all of the ships are recommended to install the bubble in the first year.

**Table 2.** Installation schedule for bubbles on ships over 20 years.

| **Ship/Year** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ship 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 6 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 8 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 9 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ship 10 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

### *4.2. Sensitivity Analyses*

In this section, several experiments are conducted for the sensitivity analyses. Within each experiment, only one parameter is changed and the others remain the same as our initial settings. This is to analyze the impact of the changed parameter on the optimal value of the objective function.

#### 4.2.1. Impact of Fuel Price on Net Savings

In the first experiment of sensitivity analyses, we select the fuel price $p_t (t = 1, \ldots, T)$ as the parameter to be changed and remain other parameters unchanged. We set the initial value of fuel price of the first scenario to be 500 USD per ton and increase 25 USD to the fuel price per ton in the second scenario. Then, the fuel price per ton of the current scenario is always 25 USD higher than that of the previous scenario. The experiment results are shown in Figure 1.

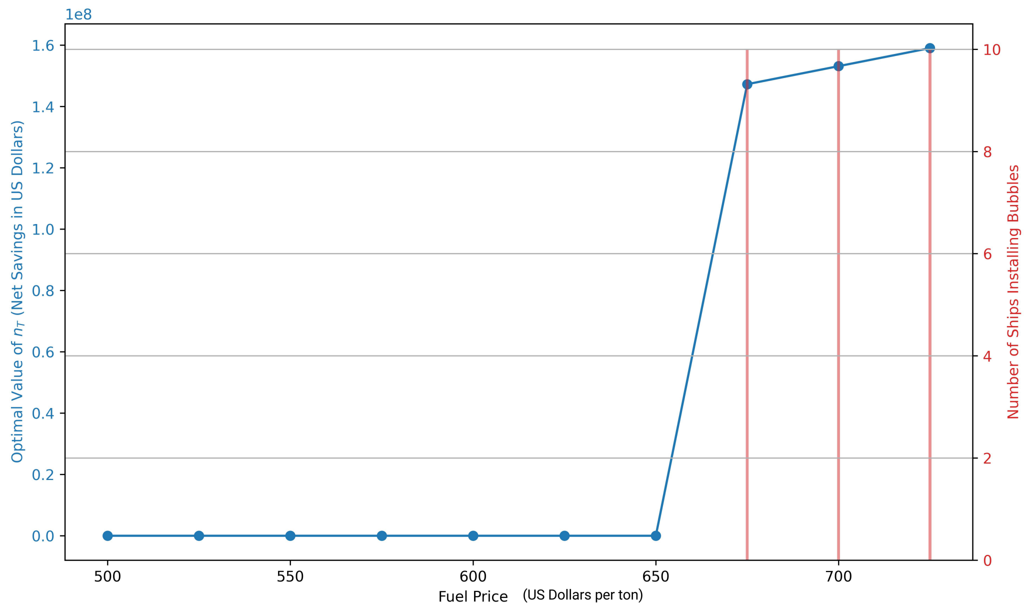


**Figure 1.** Impact of fuel price on net savings.

According to Figure 1, we can find that the optimal value of objective function remains 0 until the fuel price increases to 650 USD per ton. The reason for this can be referred to $p_t$ $(t = 1, \ldots, T)$ and Constraints (5). Specifically, if the fuel price is not high enough, then the amount of fuel costs saved by ships is not able to cover the costs for installing the bubble. Thus, the optimal value of objective function is 0 due to the situation in which no actions are taken, which means that the bubble is not installed and no cost is saved. However, the optimal value of the objective function sharply increases when the fuel price is increased to

675 USD per ton, since at this point, installing the bubble begins to be profitable and all of the ships decide to proceed the bubble installation. Furthermore, when the fuel price begins to increase to a higher level, we can find that the optimal solution of the objective function increases at the same time since the money saved by installing the bubble increases along with the increase of the fuel price. Furthermore, the higher the fuel price is, the more money ships will save.

#### 4.2.2. Impact of Annual Fuel Consumption per Ship on Net Savings

In the second experiment of sensitivity analysis, we adjust the fuel price $p_t$ $(t = 1, \ldots, T)$ to 1000 USD per ton and keep it fixed in the following experiments. In this experiment, we select the annual fuel consumption per ship as the variable across different scenarios. It increases from 5000 tons per ship in the first scenario, and 250 tons per ship is added in every new scenario until the last scenario. The experiment results are shown in Figure 2.

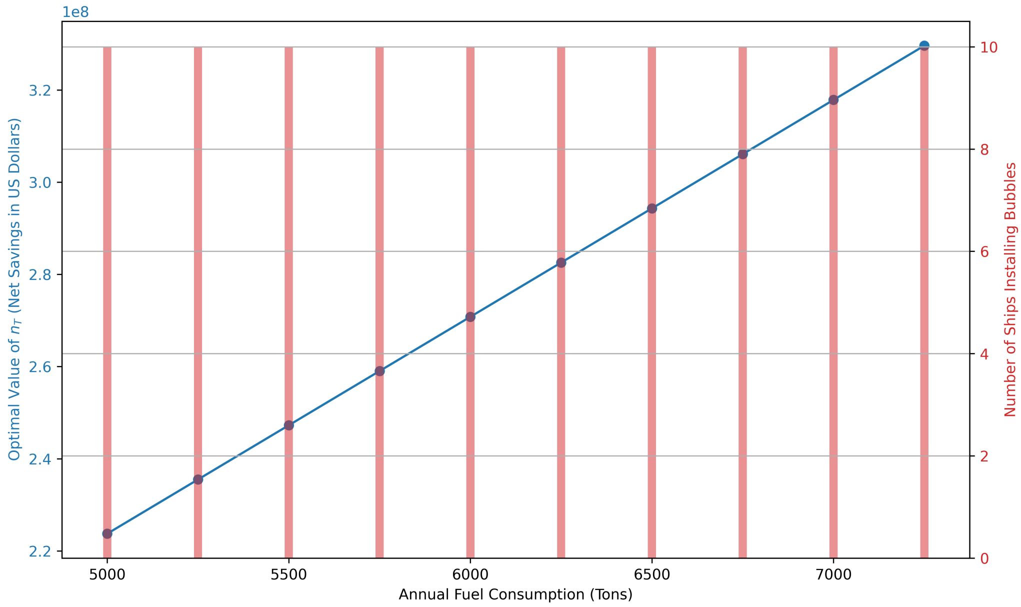


**Figure 2.** Impact of annual fuel consumption per ship on net savings.

According to Figure 2, we can find that all of the ships decide to install bubbles to make profits in every scenario. This is because in the first scenario, where the annual fuel consumption per ship is 5000 tons, the fuel costs saved by installing the cost exceed the amount of money paid to the bank. Besides, the annual fuel consumption per ship is increased linearly across scenarios. Thus, the fuel costs saved by installing bubbles will increase at the same time. As a result, all of the ships decide to install bubbles to gain more profits from the first scenario to the last one. Furthermore, the net savings increase linearly as well with the growth of the annual fuel consumption per ship.

#### 4.2.3. Impact of Annual Money Charged by Banks for Installing the Bubble per Ship on Net Savings

In the third experiment of sensitivity analyses, we select the money charged by the bank by installing bubbles for ships as the variable. The initial cost is 500,000 USD in the first scenario, and we increase this by 25,000 USD in the next scenario until the last scenario. The results of the experiment are shown in Figure 3.

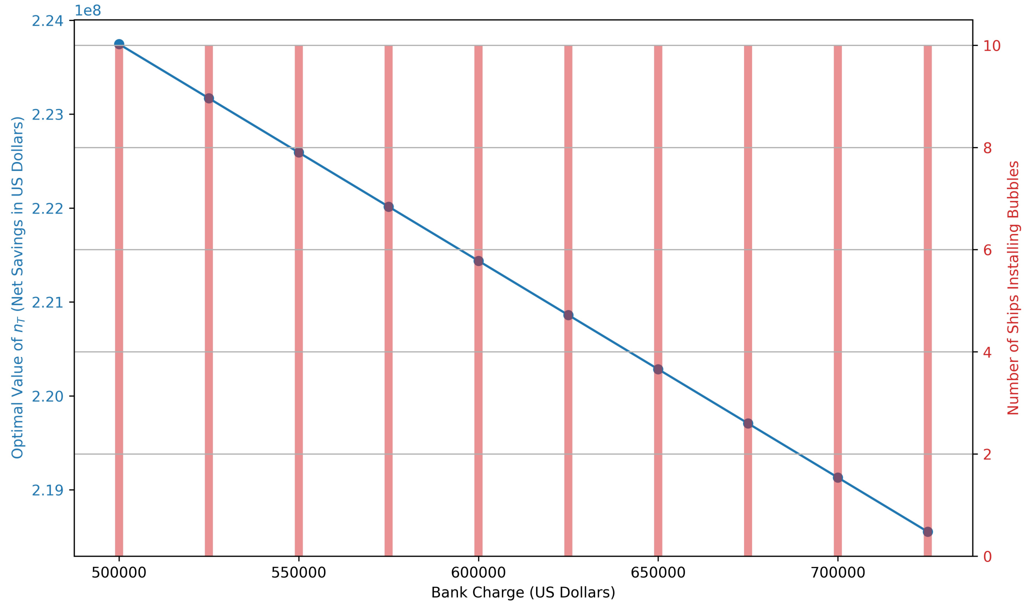


**Figure 3.** Impact of annual money charged by banks for installing the bubble per ship on net savings.

According to Figure 3, it is not hard to find that all of the ships decide to install the bubble in every scenario. This is due to the driven force of profits. In this case, all of the ships can gain profits when the money paid to the bank annually is 500,000 USD. Though the money charged by the bank increases in the following scenarios, which makes the net savings decrease linearly across scenarios, the profits can still be gained by the installation of bubbles among all of the scenarios. Hence, all of the ships still decide to install the bubbles to guarantee that they can earn as much money as possible.

#### 4.2.4. Impact of Annual Interest Rate on Net Savings

In the fourth experiment of sensitivity analyses, the annual interest rate is selected as the variable. We set it to be 0.045 initially in the first scenario, and increase it by 0.005 in every new scenario until the last scenario. The experiment results are shown in Figure 4.

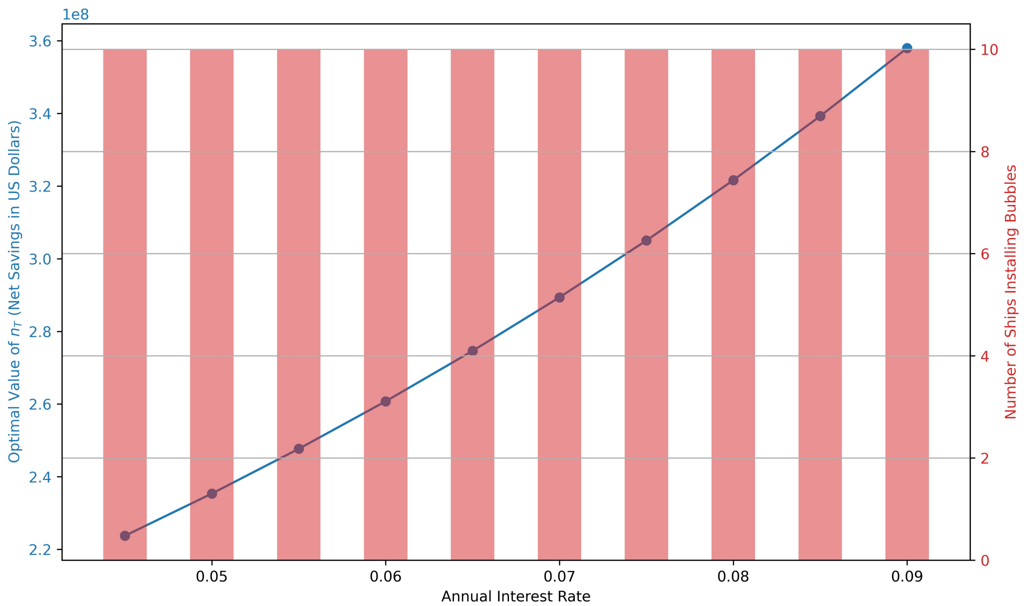


**Figure 4.** Impact of annual interest rate on net savings.

According to Figure 4, we can find that the change of net savings is a bit different from previous experiments; specifically, it increases in a non-linear way though the annual interest rate of the bank is increased linearly. The reason for this is because that the net savings in the whole 20 years is calculated year by year, and the money saved within an exact year is composed of two parts: the money saved in that year and the interest earned from the bank generated by the total money saved in previous years. It is obvious that the latter part increases non-linearly along with the linear change of the annual interest rate. Apart from this, the number of ships that decide to install the bubble are always 10 in every scenario. This is because that profits can be gained by installing the bubble and the increase of annual interest rate of the bank makes the profits keep going up across scenarios.

### *4.3. Impact of Saving Rate of Fuel by Installing Bubble per Ship on Net Savings*

In the fifth experiment of sensitivity analyses, we take the saving rate of fuel by installing bubble per ship as the variable and increases it by 0.015 from 0.15 across ten scenarios. Furthermore, the experiment results are shown in Figure 5.

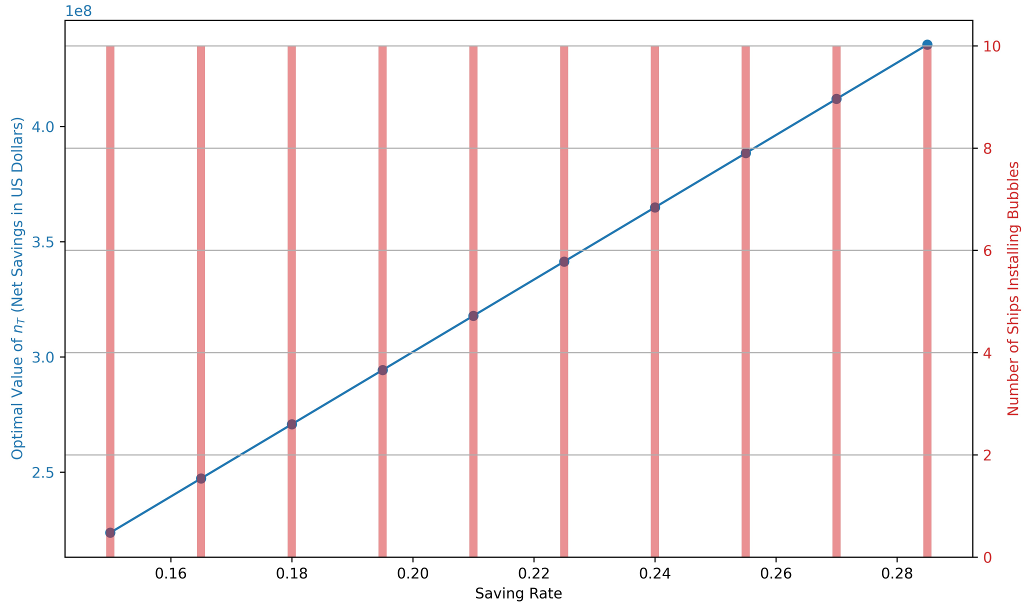


**Figure 5.** Impact of saving rate of fuel by installing bubble per ship on net savings.

According to Figure 5, the net savings increase linearly across scenarios with the increase of saving rate of fuel by installing the bubble per ship due to Constraints (4) and (5). Furthermore, all of the ships are determined to install bubbles since the profits have already been generated when the saving rate is 0.15, and they can absolutely gain more profits with the increase of the saving rate.

## 5. Conclusions

To achieve substantial reductions in maritime emissions, air lubrication technology emerges as a preferred solution. Among various innovations in this field, bubble installation has proven to be one of the most effective methods for decreasing emissions, thereby helping shipping companies meet the international standards. Despite its effectiveness, there is a scarcity of optimization models in existing research that focus on the installation of this technology. To address this gap, this paper introduces a mixed-integer programming model with the aim of maximizing the profits of shipping companies by devising a cost-effective bubble-installation strategy for a fleet during the planning horizon. Specifically, our model is designed to optimize the installation schedule for bubbles. We present several propositions that delineate the model's characteristics, each supported by detailed explanations. Additionally, we perform sensitivity analyses to demonstrate its responsiveness to changes in key parameters. Furthermore, according to our experimental results, we do suggest that shipping companies should take the bubble installation issues into detailed considerations. It is quite clear that the installation of the bubble can remarkably reduce the cost and improve the revenue at the same time. Besides, since earlier installations help ships to save more fuel, shipping companies are advised to install bubbles for their ships as soon as possible in order to gain more profits. Furthermore, we suggest that related subsidy policies should be put forward by regulatory authorities so that more shipping companies will install bubbles on their ships. Specifically speaking, regulatory authorities may charge less taxes or fees for ships with bubbles.

Nevertheless, this study has certain limitations. First, the parameter variations in our sensitivity analyses are relatively simplistic, often changing one at a time. Future research could benefit from more complex, dynamic changes, including simultaneous adjustments to multiple parameters, which could yield more robust results. Besides, since we consider the static issue in the proposed model, we may further take dynamic issues into considerations in the future study. Moreover, our model assumes uniformity across all ships in the fleet, a simplification that might limit the generalizability of our findings. Future efforts might consider varying ship characteristics to provide a more nuanced understanding of the model's applicability across different ship types. Furthermore, environmental impact analysis to quantify the reduction in emissions and its benefits to the environment and public health may also be conducted in the future study. Most importantly, more complicated experiments with more data from the real world should be conducted in the future research, which will make our model become more practical to be adopted in real scenarios.

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