A Lagrangian Relaxation Approach for the Electric Bus Charging Scheduling Optimization Problem

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Abstract

The planning and operational decision-making problems of electric transit systems have received significant attention recently in the process of transport electrification. Given an electrified electric transit system with constructed charging facilities, a coordinated bus charging schedule strategy can improve the system's operating efficiency by fully utilizing available charging resources. This paper proposes a novel optimization approach for the electric bus charging scheduling problem. To tackle the nonlinear relationship between the amount of energy and the time spent charging, this paper discretizes the decision variables for the charging schedule into time intervals in the first place. A linear integer program is formulated with the objective of minimizing the system's total charging time. A Lagrangian relaxation-based solution approach is proposed to decompose the model into subproblems with respect to individual vehicles. A tailored constraint generation scheme is developed to enhance computing efficiency. Numerical examples are conducted to verify the proposed models and solution algorithms. Our experiments confirm that the proposed coordinating charging strategy outperforms greedy strategies, such as first-in-first-out, by optimizing the charging order. The results also provide a number of insights that can help transit operators design cost-effective electric transit operational plans.

Keywords: electric bus; bus charging scheduling; nonlinear charging function; integer program; Lagrangian relaxation.

1. Introduction

Rapid urbanization and motorization have resulted in a sharp rise in vehicle ownership and created substantially environmental (e.g., noise and pollutant emissions) and social (e.g., energy consumption, traffic congestion) problems. Road transport is also considered as a severe threat to global warming due to greenhouse gas (GHG) emissions. It is reported that the transportation sector accounts for more than 25% of the worldwide energy consumption (Juan et al., 2016) and 27% of GHG emissions (Fan et al., 2018). However, because of the strong built-in inertia with physical and regulatory infrastructure in the urban transportation system, it is challenging to tackle urban environmental problems solely from transport aspects through enhancing the system mobility or restricting the use of private vehicles.

As an alternative to the fuel-powered vehicle, green transport fleets for mass mobilities have a promising potential to reduce negative climate impacts from urban transport and improve air quality conditions. Over the past decades, many cities worldwide, such as Shenzhen, Hong Kong, New York, Chicago, Curitiba, among many others, have been committed to providing considerable and continuous investment in the electrification of urban transportation (Wang et al., 2019; Tran et al., 2021). China has ranked first in terms of the market share among the world's electric transit market (Du et al., 2019). By the end of 2017, Shenzhen becomes the world's first city which replaces all conventional fuel-powered buses with battery-powered ones, where more than 16,000 vehicles are deployed, and 500 charging stations are built equipped with 8,000 chargers (Lin et al., 2019). About 48% of emissions (nearly 100% of particulate matter) are reduced compared to fossil fuel-powered buses.

For a newly planning electric transit system, an integrated planning scheme is needed, which should optimize the planning (e.g., charging facility location) and operational (e.g., itinerary, frequency, and schedule) decisions comprehensively (Liu et al., 2020). While for a fully electrified transit system, where the construction of electrification infrastructure has been completed, it is natural to optimize operational decisions with the restriction of available resources (including a limited number of chargers, the maximum permissible electrical peak load, etc.). Hence, it is well recognized that developing a cost-effective bus operational strategy by coordinating the existing bus operational schedule and the charging plan is of essential importance for the electric transit system (Li, 2016).

In practice, the electric bus (EB) charging scheduling strategies are distinct at different times of the day (Chen et al., 2018): at night, most of EBs are out of operation and can be fully charged for the next day's operation; in the daytime, EBs should be recharged according to the predetermined operational schedule. Houbbadi et al. (2019) define these two types of charging strategies as overnight charging and opportunity charging. At the planning level, the decisions of where and how to deploy the charging infrastructure only concern the tradeoff between construction cost and the maximum coverage of all recharging stations, which cannot guarantee that each bus can be recharged immediately when it arrives at the charging station. Hence, unnecessary delays would occur because of the overlapped charging times.

To mitigate the charging congestion at stations, this paper proposed an EB charging scheduling optimization model which aims to design a coordinated charging schedule for the opportunity charging in the daytime. The proposed model aims to minimize the total charging time of the entire fleet in the electric transit network with the restrictions of the existing bus operational schedule and charging facilities. Considering the nonlinearity of the battery charging function, a discretization technique is adopted to reformulate the relationship of the battery's state-of-charge (SOC) and charging time, aiming to ensure the computational tractability of the optimization model.

1.1 Literature review

Over the past decade, increasing awareness of the electrification of the urban transit system has led to a growing body of literature on the subject of optimization models for bus charging scheduling (see Table 1). The coordinated charging strategy is of great importance for the transit network with fixed routes and schedules (Wang et al., 2017), and any delay occurring in the charging process would cause the unreliability of further services. Compared with the generic electric vehicle (EV), the coordinated charging problem of EB is more complicated for the following two reasons: 1) due to the vehicle mass and long driving range, an EB may have a 55 kWh or larger battery capacity (30 kWh for generic EVs), resulting in a longer charging time and extended occupancy of charging facilities; 2) to maintain reasonable bus service frequency and reliability, the charging process must be completed within a relatively short time frame with respect to a predetermined schedule (De Filippo et al., 2014). Hence, the optimization model designed for the EV coordinated charging problem has been widely recognized to be unsuitable for electric transit systems (He et al., 2019).

Considering the time-dependent vehicle state, the EB charging strategies can be classified into dynamic and static cases. In the dynamic case, the EB charging process is controlled with respect to predictable operation or real-time feedback measurements from the system (Korkas et al., 2017). In the static case, the decision-making process of a bus charging schedule is designed from a long-term perspective. In this paper, we focus on the latter case which seeks to develop a stable bus charging schedule that determines when, where and how long a bus would charge.

Table 1. Review of existing works.

Literature	Model	Objective	Decision variable	Solution method
Li (2013)	ILP	Min. of the operation cost	VS	Exact algorithm; column generation
Paul et al. (2014)	-	Min. of charging energy	-	Greedy algorithm
You et al. (2015)	ILP	Min. of the total system cost	VS	Exact algorithm; dual decomposition
Wang et al. (2017)	MILP	Min. of the total annual cost	VS	Commercial solver
Chen et al. (2018)	MILP	Min. of the sum of electricity purchase costs	VS	Commercial solver
Rogge et al. (2018)	MILP	Min. of the total cost of ownership	VS	Heuristic algorithm
Wei et al. (2018)	MILP	Min. of the total cost	VS; VM	Commercial solver
Houbbadi et al. (2019)	NP	Min. of the battery aging cost	VS	Gradient-based optimization method
Abdelwahed et al. (2020)	MILP	-Min. of the impact on grid -Min. of the total charging cost	VS	Commercial solver
Bagherinezhad et al. (2020)	SOCP	Min. of the total operational cost	VS	Commercial solver
He et al. (2020)	NP	Min. of the total charging cost	VS	Commercial solver
Rinaldi et al. (2020)	MILP	Min. of the total operational cost	VS	Exact algorithm; decomposition
Zhou et al. (2020)	ILP	Min. of the peak-valley load	VS	Commercial solver
Bie et al. (2021)	ILP	-Min. of delays in departure time -Min. of energy consumption -Min. of bus procurement cost	VS	Heuristic algorithm
Liang et al. (2021)	ILP	Max of the operator’s welfare	VS; OD; VR	
Lu et al. (2021)	MILP	Min. of the total cost	VS	Heuristic algorithm
Zhang et al. (2021)	MILP	Min. of the total operational cost	VS	Exact algorithm
This paper	ILP	Min. of the total charging time	VS; VM	Lagrangian decomposition

Note: ILP: integer linear program; MILP: mixed-integer linear program; SOCP: second-order cone program; NP: nonlinear program; OD: order dispatching; VR: vehicle rebalancing; VS: vehicle charging schedule; VM: vehicle mileage.

There is a large body of research on optimization models of bus charging scheduling (see Table 1). Li (2013) defines the EB scheduling problem as a vehicle-scheduling problem with route constraints concerning the maximum route distance and maximum distance before battery renewal for EBs. Vehicle-scheduling models for both battery-swapping and fast charging are proposed. You et al. (2015) optimize the schedule of battery charging in the battery-swapping station to guarantee that each bus can find a fully charged battery to switch. Wei et al. (2018) investigate the unique spatio-temporal characteristics of the EB deployment. They indicate that: 1) to guarantee the long daily operation time, both the en-route charging at bus terminals and overnight charging at the garage are needed; and 2) to smooth the transition from traditional fuel-powered buses to EBs, the space-time trajectories of EBs should fit into the current route and schedule. In this regard, a spatio-temporal optimization model is proposed to minimize the cost of vehicle replacement. Houbbadi et al. (2019) mainly focus on the centralized overnight charging and first take the battery aging effects in the bus charging scheduling problem. With the help of advanced energy storage technology, Chen et al. (2018) are among the first to study the scheduling problem of the energy storage system charging and discharging in the energy storage system considering the time-of-use (TOU) electricity price.

There are also several studies related to the integrated optimization of the electric transit network and the corresponding bus charging schedule. Wang et al. (2017) propose a concurrent modeling framework that optimizes the charging station locations, number of chargers, and bus recharging schedule. The result shows that the total number of recharging activities is influenced significantly by the EB maximum driving range. For a newly constructed electric transit system, the smooth transition from fuel-powered buses to EBs is a major concern. Rogge et al. (2018) focus on the charging scheduling problem of a mixed fleet of buses, namely diesel and electric buses. The proposed model aims to minimize the total cost of ownership of electric vehicle fleets in the entire transit system. The results show that a mixed fleet could be advantageous depending on the features of the bus routes.

The vehicle’s nonlinear charging function has been widely recognized as one of the critical concerns in the decision-making problems of the electric transportation system, which makes the optimization models to be highly nonlinear and nonconvex (Xu and Meng, 2019). Pelletier et al. (2017) have conducted a comprehensive analysis of the battery degradation and charging/discharging behavior of EV batteries. It is found that the battery in an EV is usually charged through two phases: linear phase (or the constant current phase where the SOC increases linearly) and nonlinear phase (or the constant voltage phase where the current decreases exponentially and the SOC increases concavely). Montoya et al. (2017) are among the first to investigate the electric vehicle routing problem with nonlinear charging functions. Each charging mode (e.g., slow, moderate, and fast) is approximated by a three-phase piecewise linear function. Similarly, Pelletier et al. (2018) also use the piecewise linear charging function which is divided into three phases with respect to the charging current. Due to the fact that the analytical expression of SOC in the constant voltage phase does not exist, Xu and Meng (2019) develop an implicit function of SOC with respect to the charging time based on the battery circuit model discussed by Pelletier et al. (2017). Froger et al. (2019) propose an arc-based formulation to track the time and SOC via piecewise linear approximations of nonlinear functions. Lee (2021) develops a novel extended charging stations network which explicitly considers the nonlinear charging function without any approximation. It is the first time that the global optimal is achieved with an exact nonlinear charging time function. Zhang et al. (2021) also adopt the piecewise linear charging function proposed by Pelletier et al. (2017) and further consider the battery degradation effect.

1.2 Objectives and contributions

According to the literature, the EB is characterized by three features: 1) longer charging time requirements, 2) strict operational schedule, and 3) enough charging power for the next trip. Though there have been some works in developing operational scheduling for EBs, the major limitation of existing studies lies in that the objectives of bus scheduling over-emphasize the reduction of operating costs, including electricity cost (considering the time-of-use electricity price), infrastructure investment cost (capacities of charging stations), and environmental cost (emission issues). The trade-off between the reduction of operation cost and the improvement of service reliability by reducing unnecessary delays has received little attention.

Moreover, from the operations research perspective, the traditional bus scheduling problem is to cover all bus trips in the timetable with known starting and ending times (Li and Head, 2009), which is usually formulated in the mixed-integer programming (MIP). Due to the nonlinearity of the vehicle charging function, the EB coordinated charging scheduling problem is usually nonlinear and nonconvex that cannot be solved efficiently by the existing solution algorithms. Although the shape of the charging function is well-known, deriving the analytical expression to describe it is intrinsically complex due to various environmental factors, such as current, voltage, battery's SOC, and temperature (Wang et al., 2013). Most of the existing studies simplify this issue by introducing linear function approximations (Felipe et al., 2014; Schiffer and Walther, 2017). Little attention has been paid to the discretized approach to model the nonlinear charging function.

To sum up, there still lacks efficient methods to calibrate the charging function of both EV and EB. Although some linearization techniques, such as the piecewise linearization (Montoya et al., 2017; Pelletier et al., 2018), have been widely applied to approximate the charging function, which would inevitably result in a relatively complex formulation and generate additional variables and constraints to the primal problem. Additionally, how to comprehensively embed the nonlinear charging function in the bus charging scheduling problem and to formulate a polynomially solvable programming model are still open questions in the literature due to its strong nonlinearity and NP-hardness. Moreover, the bus charging process is usually inconsistent in practice, where the bus with an urgent and scheduled trip is allowed to be charged in a prior order. Little attention has been paid to model the inconsistent charging process, which is of considerable significance during rush hours.

Hence, the contribution of this study is threefold. First, a novel bus charging scheduling optimization model from a network-level is formulated for the daytime opportunity charging mode. Second, a discretization technique is developed to track the time and SOC of the EB during charging, which intends to tackle the nonlinearity caused by the charging function. Third, a Lagrangian relaxation-based solution approach is developed based on the proposed optimization model, and a tailored constraint generation scheme is designed to enhance the solution algorithm's efficiency.

The remainder of this paper is organized as follows. Section 2 gives a generic description of the bus charging scheduling problem. The optimization model formulated for the network-based bus charging scheduling problems is presented in section 3. Section 4 develops a Lagrangian relaxation-based solution algorithm and a tailored constraint generation method, which could enhance the efficiency of the Lagrangian relaxation method. Section 5 gives numerical experiments to illustrate the model and solution method. Conclusions are presented in section 6.

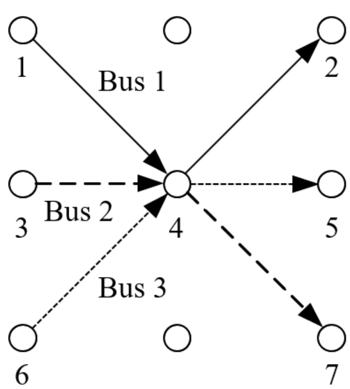
2 Problem description

An electric transit system is usually composed of three main components: charging stations, timetable, trip-assignment schedule (see Fig. 1). The timetable records the arrival and departure times of a bus at each station. A bus trip can then be defined by the departure time at a station and the arrival time at another station. The relationship between buses and trips is recorded by the trip-assignment schedule.

The deployment of charging stations (including their location and corresponding facility capacity) is usually determined in the planning stage (Li, 2016; He et al., 2019; Lin et al., 2019). This study focuses on the optimization of EB's daytime opportunity charging schedule at the operational level with given charging station deployment and trip-assignment schedule. The timetable of all buses is assumed to be known as a priori. This assumption is acceptable with the consideration of eliminating the negative impacts of adjusting the regular operating schedule of the transit system. Consequently, by optimizing the bus charging schedule for the transit operator, the proposed model determines when, where, and how long each bus should be charged.

Compared with overnight charging, daytime opportunity charging has the following characteristics: 1) The battery is not necessary to be fully charged at every charging station. To avoid redundant charging time and reduce the occupation time of chargers, an EB can only be charged to a target SOC that guarantees the energy consumption for the next trip; 2) Congestion issue may occur at the station when the charging demand exceeds the number of chargers; 3) The charging process of an EB can be interrupted in the case that another EB which arrives later but has limited time to be charged.

A feasible charging schedule of the illustrated electric transit network is presented in Fig. 2. Assume that station 4 has only one charger. Bus 3 arrives at station 4 at 8:30 and starts to charge immediately. Bus 1 arrives later but only has one hour for dwelling and charging. In such cases, the charging process of bus 1 can be interrupted and bus 1 would be charged first without waiting in the queue or violating the following scheduled trips.



Trip-assignment Schedule

	Station 1		Station 4		Station 2	
Bus 1	Arrival time	Departure time	Arrival time	Departure time	Arrival time	Departure time
	-	7:30	8:45	9:15	10:30	-
	Station 3		Station 4		Station 7	
Bus 2	Arrival time	Departure time	Arrival time	Departure time	Arrival time	Departure time
	-	7:45	9:00	10:00	10:45	-
	Station 6		Station 4		Station 5	
Bus 3	Arrival time	Departure time	Arrival time	Departure time	Arrival time	Departure time
	-	7:15	8:30	9:30	10:15	-

Figure 1. Example of an electric transit system.

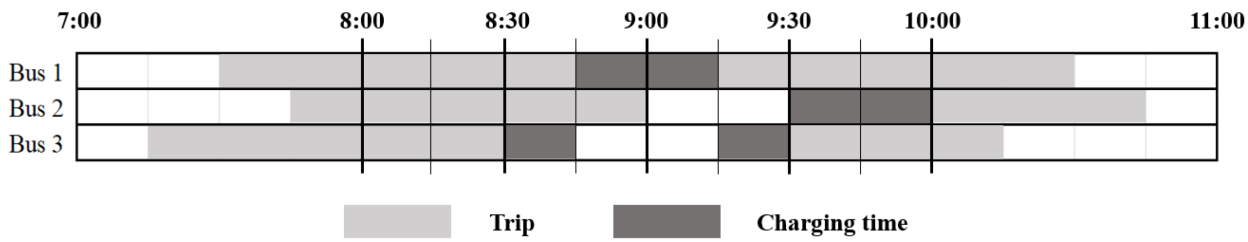


Figure 2. Trip and charging schedule.

The notations of the sets, parameters, and variables used in the following sections are listed in Table 2.

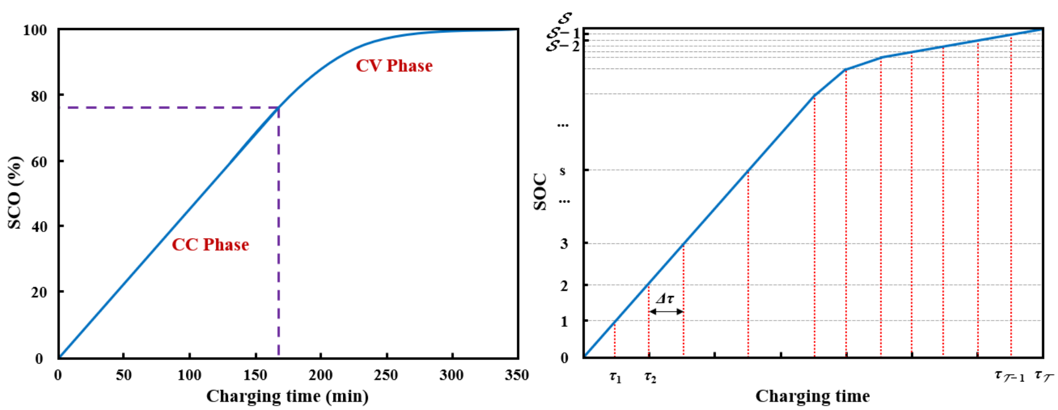
Table 2. List of notations.

Notation	Description
Set	
L	Set of bus lines.
N	Set of charging stations.
S	Set of states for the battery power.
T	Set of timestamps of the entire planning horizon.
$\boldsymbol{\tau}$	Set of timestamps in the charging function.
Parameter	
$a_{l,j}$	Scheduled arrival time of a bus in line l at the j-th station.
$d_{l,j}$	Scheduled departure time of a bus in line l at the j-th station.
$D_{l,j}$	Electricity consumption of a bus in l when traveling between the j-th station and the $(j+1)$-th station. $l \in L$, $j \in \{1,...,N_l - 1\}$ (%).
G_l	Minimum required SOC of a vehicle in bus line l, $l \in L$ (%).
H	Total charging time from the empty state (SOC=0%) to the fully charged state (SOC=100%) (hours).
M_n	Number of chargers at station n, $n \in N$.
N_l	Total number of stations contained in bus line l, $l \in L$.
$\mathcal{S}$	Total number of battery states.
$\mathcal{T}$	Total number of time intervals in the charging function.
U_l	Maximum number of periods of buses in line l is allowed to be charged at each station.
$\Delta\tau$	The length of a time interval.
Variable	
$u_{l,j}^{tt'}$	Binary variable for charging schedule, i.e., if $u_{l,j}^{tt'} = 1$ if a bus in line l is charged during time period $[t, t']$ at the j-th station, and $u_{l,j}^{tt'} = 0$, otherwise.
$v_{l,j,s}$	Binary variable for battery state, i.e., if $v_{l,j,s} = 1$ if the SOC of a bus in l is s when departing from the j-th station; and $v_{l,j,s} = 0$, otherwise.

3 Mathematical formulation

At the network level, the bus charging scheduling problem becomes a system-wide problem that requires the coordination between the charging and operational schedules. Due to the fact that the electricity price is usually flat in the daytime, the influence of TOU electricity price on the scheduling problem is not considered. Hence, the proposed scheduling model only aims to find an optimal charging strategy to improve the system operational efficiency, e.g., reducing the waiting time at a station, increasing the service reliability with the given bus line itinerary and schedule, among others.

Consider an electric transit network defined on a connected graph $G=(N,L)$, where N and L are sets of charging stations and bus lines, respectively. The itinerary of each bus line is divided into trips between charging stations included in the line. And all lines originate and terminate at the same bus terminal. Let N_l denote the total number of stations contained in bus line l, $l\in L$. Let $n(l,j):n(l,j)\in N_l$ denote the index of the j-th station of bus l in the station set N_l. Denote the scheduled arrival and departure times of bus line l at the j-th station as $a_{l,j}$ and $d_{l,j}$, respectively.



(a) The nonlinear charging function (b) The discretized charging function

Figure 3. Discretization of the nonlinear charging function.

As mentioned earlier, the battery's charging behavior in terms of the charging power and the SOC is usually described by a nonlinear function (see Fig. 3(a)). In this regard, the resulting bus charging scheduling problem is usually formulated in the nonlinear and nonconvex programs, which is difficult to be solved by conventional exact solution methods. To obtain the exact solution efficiently, linearization techniques, such as piecewise linearization, are widely used to approximate the nonlinear charging function (Montoya et al., 2017; Pelletier et al., 2019). But the number of variables and constraints would inevitably increase, resulting in a more complex formulation. To avoid the nonlinear components, the SOC of a vehicle can be discretized with respect to the nonlinear charging function. Note that batteries of all EBs are assumed to be homogeneous in terms of battery size, power consumption, and aging degree, and they have the same charging function.

Let $\Phi(\tau)$ denote the nonlinear charging function, and $\tau \in [0, H]$, where H (hours) is the battery charging time duration from empty state (SOC=0%) to the fully charged state (SOC=100%). As illustrated in Fig. 3(b), $\Phi(\tau)$ can be divided into a number of equal intervals. Assume that the length of each time interval is $\Delta\tau$. The total number of time intervals, $\mathcal{T}$, can be obtained by $\mathcal{T} = 60 \times H/\Delta\tau$. Accordingly, the battery charging duration can be represented by a set of discrete time intervals. Let $\boldsymbol{\tau}$ denote the set of timestamps at the end of each time interval, i.e., $\boldsymbol{\tau} = \{\tau_0, \tau_1, \tau_2, ..., \tau_{\mathcal{T}}\}$. In this regard, the battery charging function is then transformed into a discrete mapping between charging time and battery's SOC, that is, each timestamp corresponds to a battery state s. Let S denote the set of states for the battery power, $S = \{s \mid s = 0, 1, 2, ..., \mathcal{S}\}$, and $\mathcal{S} = \mathcal{T}$. In sum, each state represents the number of time interval a vehicle is charged from the state with zero battery power, i.e., $s = 0$ indicates that the vehicle's SOC is zero; $s = 1$ represents the amount of battery power charged by one time interval from $s = 0$; and $s = 2$ charged by two time intervals. Considering the nonlinearity of the charging function, the increment of the battery power between each state is inconsistent, i.e., the battery charged between s and $s+1$ is not necessarily equal to that between $s-1$ and s. With a little abuse of notation, we let $\varphi(\tau)$ represent the mapping between timestamps and battery states, and $s = \varphi(\tau)$. The inverse mapping is denoted as $\tau = \varphi^{-1}(s)$.

The number of chargers at station n is denoted by M_n. As presented in the example above, in the proposed coordinated bus charging scheme, when the scheduled charging periods of two buses are overlapped, the charging process of the bus which has already started charging could be interrupted. The bus with an urgent trip is allowed to be charged in a prior order. Hence, the charging process of a bus is allowed to be inconsistent and could be divided into disjunctive periods. In this regard, a parameter U_l is introduced to regulate the maximum number of periods of buses in line l is allowed to be charged at each station until the expected SOC is reached. Additionally, the entire planning horizon is also discretized equally with interval $\Delta\tau$. The set of timestamps with respect to the planning horizon is denoted by T.

The charging schedule of each bus is described by the binary variable $u_{l,j}^{tt'}$, where $u_{l,j}^{tt'}=1$ if the bus in line l is charged during the time period $[t,t']$ at the j-th station. Hence, the proposed network-based bus charging scheduling problem can thus be formulated as follows. For each $j=1,...,N_l$, define the binary variable $v_{l,j,s}=1$ if the battery state of the bus in l is s when departing from the j-th station; and $v_{l,j,s}=0$, otherwise.

[P]

$$\min \sum_{l\in L}\sum_{j=1}^{N_l}\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}(t'-t)u_{l,j}^{tt'} \tag{1}$$

subject to

$$\sum_{s\in S} v_{l,j,s}=1,\ l\in L, j\in\{1,...,N_l\}, \tag{2}$$

$$\sum_{s\in S} v_{l,j,s}-D_{l,j}\ge G_l,\ l\in L, j\in\{1,...,N_l-1\}, \tag{3}$$

$$v_{l,1,Q}=1,\ l\in L, \tag{4}$$

$$\sum_{s\in S} v_{l,j-1,s}\cdot s+\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}u_{l,j}^{tt'}\cdot\varphi\left(\varphi^{-1}\left(\sum_{s\in S}v_{l,j,s}\cdot s\right)+t'-t\right)-\sum_{s\in S}v_{l,j,s}\cdot s=D_{l,j-1},\ l\in L, j\in\{2,...,N_l\}, \tag{5}$$

$$\sum_{t'=a_{l,j}}^{t}\sum_{t''=t+1}^{d_{l,j}}u_{l,j}^{t't''}\le 1,\ l\in L, j\in\{2,...,N_l\}, t\in\{a_{l,j},...,d_{l,j}-1\}, \tag{6}$$

$$\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}u_{l,j}^{tt'}\le U_l,\ l\in L, j\in\{1,...,N_l\}, \tag{7}$$

$$\sum_{l\in L}\sum_{j=1,n(l,j)=n}^{N_l}\sum_{t'=a_{l,j}}^{t}\sum_{t''=t+1}^{d_{l,j}}u_{lj}^{t't''}\le M_n,\ n\in N, t\in T, \tag{8}$$

$$v_{l,j,s}\in\{0,1\},\ l\in L, j\in\{1,...,N_l\}, s\in S, \tag{9}$$

$$u_{l,j}^{tt'}\in\{0,1\},\ l\in L, j\in\{1,...,N_l\}, t\in\{a_{l,j},...,d_{l,j}-1\}, t'\in\{a_{l,j}+1,...,d_{l,j}\}, \tag{10}$$

The objective function (1) minimizes the total charging time of transit system. Constraint (2) assigns a battery state to each bus that leaves a station. Constraint (3) ensures that each bus has a certain battery state when arriving at a station. Note that by assigning non-negative values to G_l, constraint (3) ensures that each bus in line l will have sufficient battery power to arrive at the j-th station when it departs from $(j-1)$-th station. Constraint (4) ensures that the battery state of each bus is Q when the bus departs from the first station. Constraint (5) establishes the relationship between the battery state of a bus when arriving at a station and the battery state of the bus when departing from the station. Constraint (6) ensures that the charging periods of each bus at each station do not overlap. Constraint (7) imposes an upper limit on the number of charging periods of each bus at each station. Constraint (8) imposes an upper limit on the number of buses charged simultaneously at each station. Constraints (9) and (10) assign binary values to the decision variables.

4 Solution method

It has been well discussed that the classical multiple-station vehicle scheduling problem is NP-hard, which is computationally challenging to be solved to optimality (Li and Head, 2009). This study develops a Lagrangian relaxation-based solution algorithm (Niu et al., 2018; Huang et al., 2021). To reduce the searching space for large-scale problems, the primal model ($\mathbf{P}$) is decomposed into subproblems with respect to individual buses. An enhancement to the regular Lagrangian relaxation algorithm is proposed to accelerate the convergence of the algorithm.

4.1 Lagrangian relaxation

Observe that the primal problem $\mathbf{P}$ has two types of decisions: 1) the charging schedule of a bus and 2) the optimal battery state when a bus departs from the station. One important point of this observation is that constraints in $\mathbf{P}$ can also be classified into categories with respect to decision variables, namely, constraints associated with the bus schedule, battery state, and coupling constraints. It can be seen that the objective function of $\mathbf{P}$ only contains the variables concerning the bus's charging schedule. Hence, the constraints containing only the bus schedule variables, i.e., constraints (7) and (8), can be dualized into the objective function, by introducing multipliers $\mu_{l,j}$ and $\lambda_{k,t}$, and $\mu_{l,j} \geq 0$, $\lambda_{k,t} \geq 0$. Then, the Lagrangian relaxation problem can be decomposed into $|L|$ subproblems, each of which involves only the decision of charging planning for one bus.

Therefore, the Lagrangian relaxation problem can be constructed as follows.

[$\mathbf{P}_{\text{LR}}(\boldsymbol{\mu},\boldsymbol{\lambda})$]

$$\begin{aligned}\min \; & \sum_{l\in L}\sum_{j=1}^{N_l}\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}(t'-t)u_{l,j}^{tt'}+\sum_{l\in L}\sum_{j=1}^{N_l}\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}\mu_{l,j}u_{l,j}^{tt'}-\sum_{l\in L}\sum_{j=1}^{N_l}\mu_{l,j}U_l \\ & +\sum_{l\in L}\sum_{n\in N}\sum_{j=1,n(l,j)=n}^{N_l}\sum_{t\in T}\sum_{t'=a_{l,j}}^{t}\sum_{t''=t+1}^{d_{l,j}}\lambda_{n,t}u_{l,n}^{t't''}-\sum_{n\in N}\sum_{t\in T}\lambda_{n,t}M_k\end{aligned} \tag{11}$$

subject to constraints (2)-(6), (9), (10).

In the above model, $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are vectors of Lagrangian multipliers. Let $L(\boldsymbol{\mu},\boldsymbol{\lambda})$ denote the optimal objective value of problem $\mathbf{P}_{\text{LR}}(\boldsymbol{\mu},\boldsymbol{\lambda})$. The goal of the Lagrangian relaxation algorithm is to solve the following Lagrangian dual problem:

[$\mathbf{P}_{\text{LD}}$]

$$\max_{\boldsymbol{\mu},\boldsymbol{\lambda}} \; L(\boldsymbol{\mu},\boldsymbol{\lambda}) \tag{12}$$

The optimal values of $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ can be searched and updated using a subgradient optimization procedure (Fisher, 2004), which would be presented in detail in the next section.

After removing the constant term $-\sum_{l\in L}\sum_{j=1}^{N_l}\mu_{l,j}U_l-\sum_{n\in N}\sum_{t\in T}\lambda_{n,t}M_k$ from the objective function (11), the relaxed problem will decompose into $|L|$ subproblems. The mathematical formulation of the l-th subproblem is presented below:

[$\mathbf{P}'_l$]

$$\min \; \sum_{j=1}^{N_l}\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}(t'-t)u_{l,j}^{tt'}+\sum_{j=1}^{N_l}\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}\mu_{l,j}u_{l,j}^{tt'}+\sum_{n\in N}\sum_{j=1,n(l,j)=n}^{N_l}\sum_{t\in T}\sum_{t'=a_{l,j}}^{t}\sum_{t''=t+1}^{d_{l,j}}\lambda_{n,t}u_{l,n}^{t't''} \tag{13}$$

subject to

$$\sum_{s\in S}v_{l,j,s}=1,\; j\in\{1,\ldots,N_l\}, \tag{14}$$

$$\sum_{s\in S}v_{l,j,s}-D_{l,j}\geq G_l,\; j\in\{1,\ldots,N_l-1\}, \tag{15}$$

$$v_{l,1,s}=1, \tag{16}$$

$$\sum_{s\in S}v_{l,j-1,s}\cdot s+\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}u_{l,n_{l,j}}^{tt'}\cdot\varphi\left(\varphi^{-1}\left(\sum_{s\in S}v_{l,j,s}\cdot s\right)+t'-t\right)-\sum_{s\in S}v_{l,j,s}\cdot s=D_{l,j-1},\; j\in\{2,\ldots,N_l\}, \tag{17}$$

$$\sum_{t'=a_{l,j}}^{t} \sum_{t''=t+1}^{d_{l,j}} u_{l,j}^{t't''} \le 1,\ j \in \{2,...,N_l\}, t \in \{a_{l,j},...,d_{l,j}-1\}, \tag{18}$$

$$v_{l,j,s} \in \{0,1\},\ j \in \{1,...,N_l\}, s \in S, \tag{19}$$

$$u_{l,j}^{tt'} \in \{0,1\}, j \in \{1,...,N_l\}, t \in \{a_{l,j},...,d_{l,j}-1\}, t' \in \{a_{l,j}+1,...,d_{l,j}\}. \tag{20}$$

It can be observed that the subproblem $\mathbf{P}_l'$ is a 0-1 integer program with $N_l|S| + \sum_{n\in N_l}(d_{l,n}-a_{l,n})^2$ binary variables and $3N_l - 1 + \sum_{n\in N_l}(d_{l,n}-a_{l,n})$ constraints. When $(d_{l,n}-a_{l,n})$ is large for each $n \in N_l$, solving the problem by commercial integer program solvers would be impossible due to the enormous number of variables. In the following, we will reduce the problem size by transforming problem $\mathbf{P}_l'$ into a compact form. We introduce new decision variables as follows. For each $j = 1,...,N_l$ and $t \in \{a_{l,j},...,d_{l,j}\}$, let $\tilde{u}_{l,j,t} = 1$ if the bus in line l is being charged a the j-th station during time period $[t, t+1]$; and $\tilde{u}_{l,j,t} = 0$, otherwise. For each $j = 1,...,N_l$, let $\tilde{v}_{l,j}$ denote the battery state of bus l when departing from the j-th station. Consider the integer program:

[$\mathbf{P}_l''$]

$$\min \sum_{j=1}^{N_l} \sum_{t=a_{l,j}}^{d_{l,j}} (1+\mu)\tilde{u}_{l,j,t} + \sum_{n\in N} \sum_{j=1,n(l,j)=n}^{N_l} \sum_{t=a_{l,j}}^{d_{l,j}} \lambda_{n,t}\tilde{u}_{l,j,t} \tag{21}$$

subject to

$$\tilde{v}_{l,1} = \mathcal{S}, \tag{22}$$

$$\tilde{v}_{l,j-1} + \sum_{t=a_{l,j-1}}^{d_{l,j}-1} \tilde{u}_{l,j-1,t} - \tilde{v}_{l,j} = D_{l,j-1},\ j \in \{2,...,N_l\}, \tag{23}$$

$$\tilde{u}_{l,j,t} \in \{0,1\},\ j \in \{1,...,K_l\}, t \in \{a_{lj},...,d_{lj}\}, \tag{24}$$

$$\tilde{v}_{l,j} \in \{G_l,...,S\},\ j \in \{1,...,K_l\}. \tag{25}$$

The following proposition states that the optimal objective value of problem $\mathbf{P}_l'$ can be obtained by solving problem $\mathbf{P}_l''$:

Proposition 1: Solving problem $\mathbf{P}_l''$ yields the optimal objective value of problem $\mathbf{P}_l'$.

Proof. It is clear that the optimal solution of $\mathbf{P}_l''$, $\tilde{u}_{l,j,t}$, gives the exact time periods that a bus is being charged. Let $U_{l,j}$ denote the total charging time of the bus in line l at its j-th station. The number of buses that are charging simultaneously at station n during time period t is denoted by $m_{n,t}$. Given the optimal charging schedule of bus l at its j-th station, $\tilde{u}_{l,j,t}$, $U_{l,j}$ and $m_{n,t}$ can be obtained as follows: 1) let $U_{l,j}=0$ and $m_{n,t}=0$; 2) for t in $\left[a_{l,j}+1, d_{l,j}\right]$, if $\tilde{u}_{l,j,t-1}=1$ and $\tilde{u}_{l,j,t}=0$, then $U_{l,j}=U_{l,j}+1$; 3) for t in $\left[a_{l,j}, d_{l,j}\right]$, then $m_{n,t}=m_{n,t}+\tilde{u}_{l,j,t}$. Thus, $\sum_{l\in L}\sum_{j=1}^{N_l}\sum_{t=a_{l,j}}^{d_{l,j}-1}\sum_{t'=t+1}^{d_{l,j}}\left(t'-t\right)\cdot u_{l,j}^{tt'}=\sum_{l\in L}\sum_{j=1}^{N_l}U_{l,j}$ and $\sum_{l\in L}\sum_{j=1,n(l,j)=n}^{N_l}\sum_{t'=a_{l,j}}^{t}\sum_{t''=t+1}^{d_{l,j}}v_{lj}^{t't''}=m_{n,t}$, $n\in N$, $t\in T$. □

Instead of solving the 0-1 integer program of $\mathbf{P}_l'$ directly, it suffices to solve the compact formulation of $\mathbf{P}_l''$, which has much fewer variables and constraints.

4.2 Solving the Lagrangian decomposition problem

For any value of the Lagrangian multipliers $\mu_{l,j}$ and $\lambda_{n,t}$, the sum of the optimal value of each subproblem $\mathbf{P}_l'$ gives a lower bound for the primal problem $\mathbf{P}$. Note that the optimal solution of the Lagrangian dual problem $\mathbf{P}_{\mathrm{LD}}$ may or may not be feasible for the primal problem $\mathbf{P}$. If not, a heuristic algorithm is usually applied to find a feasible solution and to obtain an upper bound for the primal problem. In this subsection, a constructive greedy heuristic approach to the problem is developed, which is embedded in an iterative algorithm based on the Lagrangian decomposition. A subgradient updating scheme is used to determine the Lagrangian multipliers at each iteration.

4.2.1 *k*-greedy heuristic algorithm

The *k*-greedy heuristic algorithm is a kind of constructive heuristic algorithm that utilizes the *k*-nearest neighbor search technique when selecting the feasible charging period between EB's arrival and departure times. Different from Paul and Yamada (2014), which is among the first to apply the *k*-greedy-based approach to obtain proper EB charging schedules, vehicle deployment among different lines is not allowed in our study. The operation diagram of an illustrated bus line is shown in Fig. 4. The feasible charging periods are generated and selected according to the itinerary of a bus line. At each station, *k* candidate charging plans that guarantee the next trip's energy are generated, and then the feasibility conditions would be checked. Note that if the available charging period of a station cannot guarantee the next trip's energy consumption, the last station is revisited, the current charging plan of which is then adjusted by generating new candidate charging plans which could guarantee the energy consumption of the next two trips. The detailed algorithm is presented using a flow chart illustrated in Fig. 5.

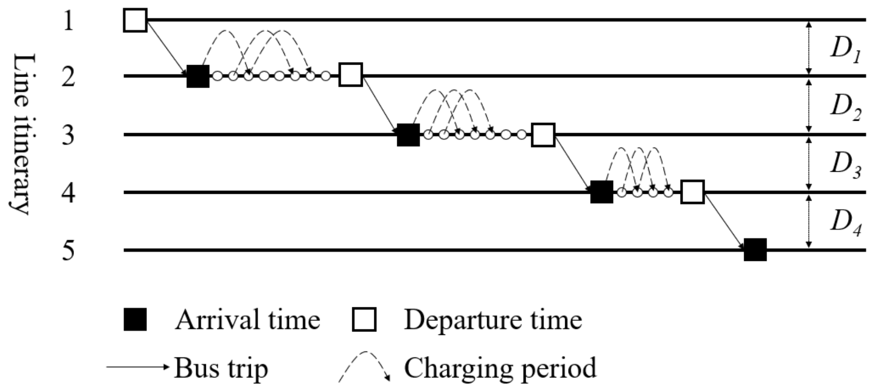


Figure 4. Illustration of the selection feasible charging process.

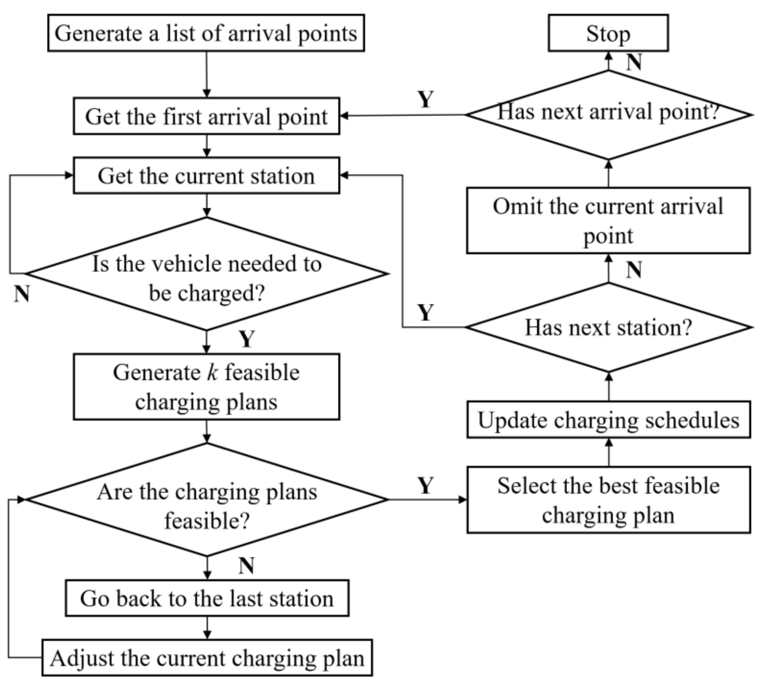


Figure 5. Flow chart of the *k*-greedy algorithm.

4.2.2 Subgradient Lagrangian search algorithm

In literature, there exists a wide variety of search methods which at least approximately find the best possible lower bound of the primal problem **P** (Rardin, 2017). In this section, we apply the most popular search method, i.e., the subgradient search, which utilizes a generalized steepest gradient search mechanism. The full statement of the subgradient search over Lagrangian duals is provided in **Algorithm 1** as follows.

Algorithm 1: Subgradient Lagrangian search algorithm

Step 0: **Initialization**

Set iteration $i \leftarrow 0$.

Initialize the set of Lagrangian multipliers $\mu_{l,j}^{(0)} \leftarrow 0$, $\lambda_{n,t}^{(0)} \leftarrow 0$.

Set the best lower bound $lb^* \leftarrow -\infty$, the best upper bound $ub^* \leftarrow +\infty$.

Step 1: **Solve the decomposed subproblems**

Solve the decomposed subproblems $\mathbf{P}_l''$, and obtain the optimal solution $\tilde{u}_{l,j,t}^{(i)}$ and $\tilde{v}_{l,j}^{(i)}$.

Solve the Lagrangian relaxation problem $\mathbf{P}_l'$.

Based on **Proposition 1**, obtain $U_{l,j}^{(i)}$ and $m_{n,t}^{(i)}$.

Step 2: **Generate the lower bound**

Calculate the values of the objective functions in $\mathbf{P}$ and $\mathbf{P}_l''$, $obj^{(i)}$ and $obj_{LR}^{(i)}$.

If all the relaxed constraints are satisfied, i.e., $U_{l,j}^{(i)} \le U_l$ for every station in each bus line and $m_{n,t}^{(i)} \le M_n$ for each station during each period, update the lower bound $lb^* = \max\left\{lb^*, obj_{LR}^{(i)}\right\}$.

Step 3: **Generate the upper bound**

Use the *k*-greedy heuristic algorithm to generate a feasible charging plan using the current $U_{l,j}^{(i)}$ and $m_{n,t}^{(i)}$, and obtain the value of objective function, $obj_{k-greedy}^{(i)}$.

Update the upper bound $ub^* = \max\left\{ub^*, obj_{k-greedy}^{(i)}\right\}$.

Step 4: **Compute the optimal gap**

$gap = \left(ub^* - lb^*\right) \big/ ub^*$.

Step 5: **Update Lagrangian relaxation multipliers using subgradient search**

Determine the step size δ, $\delta = 1/2(n+1)$.

Update Lagrangian multipliers: $\mu_{l,j}^{(i+1)} = \max\left\{\mu_{l,j}^{(i)} + \delta \cdot \Delta\mu_{l,j}^{(i)},\quad 0\right\}$,

$\lambda_{n,t}^{(i+1)} = \max\left\{\lambda_{n,t}^{(i)} + \delta \cdot \Delta\lambda_{n,t}^{(i)},\quad 0\right\}$, where $\Delta\mu_{lj}^{(i)} = \left(U_{l,j}^{(i)} - U_l\right) \Big/ \left(\sum_{l \in L} \sum_{j=1}^{N_l} \mu_{l,j} \left(U_{l,j}^{(i)} - U_l\right)^2\right)^{1/2}$,

and $\Delta\lambda_{n,t}^{(i)} = \left(m_{n,t}^{(i)} - M_n\right) \Big/ \left(\sum_{n \in N} \sum_{t \in T} \left(m_{n,t}^{(i)} - M_n\right)^2\right)^{1/2}$.

Step 6: **Termination condition test**

If optimal gap is less than the pre-determined value ε, i.e., $gap \le \varepsilon$, then **stop**;

else if the iteration number i is larger than the maximum iteration number $I_{\max}$, then **stop**;

else, let $i = i + 1$, and go to ***Step 2***.

4.3 Algorithm enhancements

In this section, we present an algorithmic enhancement to accelerate the convergence of the Lagrangian relaxation algorithm.

4.3.1 Constraint generation in Lagrangian relaxation

It can be observed that when the number of the Lagrangian multipliers is large, the computation burden of solving the Lagrangian dual problem $\mathbf{P}_{\text{LD}}$ increases because of the high occurrence of degeneracy, i.e., different values of $(\boldsymbol{\mu}, \boldsymbol{\lambda})$ can lead to the same value of the objective $L(\boldsymbol{\mu}, \boldsymbol{\lambda})$. The convergence speed of the subgradient optimization method would inevitably slow down because of this degeneracy effect. For instance, it can be observed that constraint (8) is imposed for each $n \in N$, $t \in T$. It is obvious that the charging function can be approximated closely if it is divided into more time intervals, resulting in a large set of T and a large number of the Lagrangian multipliers $\lambda_{n,t}$ which decreases the convergence speed of the subgradient optimization method.

To overcome this deficiency, a constraint generation strategy is proposed to reduce the number of Lagrangian multipliers. The basic idea of this strategy is to eliminate constraint (8) from the primal problem $\mathbf{P}$, and then add this constraint iteratively back as needed. Specifically, initialize all the Lagrangian multipliers $\lambda_{n,t}$ to zero. At the i-th iteration in **Algorithm 1**, consider the solution generated by solving the Lagrangian relaxation $\mathbf{P}_{\text{LD}}$. If this solution violates constraint (8) for certain $n \in N$ or $t \in T$, the violated constraint can then be added to $\mathbf{P}$ (if this constraint has not been imposed yet). Note that the violations of constraint (8) can be easily identified by a complete enumeration.

Let $\Omega^{(i)}$ be the set of (n,t) pairs for which constraint (8) is added to $\mathbf{P}$. Instead of updating all the Lagrangian multipliers $\lambda_{n,t}$ in the i-th iteration of the i-th iteration, it is sufficient to update the multipliers $\lambda_{n,t}$ for each $(n,t) \in \Omega^{(i)}$. The constraint generation is performed iteratively until the subgradient updating scheme is terminated.

4.3.2 An efficient algorithm for solving $\mathbf{P}_l''$

In each iteration in **Algorithm 1** for solving the Lagrangian decomposition problem, problem $\mathbf{P}_l''$ for each $l \in L$ is solvable to commercial solvers. The computational performance of

Algorithm 1 depends largely on the efficiency of solving $\mathbf{P}_l''$. Instead of using the commercial solver directly, an efficient algorithm for solving $\mathbf{P}_l''$ is proposed in this subsection.

For each $l \in L$, $j \in \{1,.,,,N_l\}$, and $t \in a_{l,j},...,d_{l,j}$, define

$$\lambda'_{l,j,t} = \begin{cases} \lambda_{n,t}, & if \ \exists n \in N, n(l,j) = n \\ 0, & otherwise. \end{cases} \tag{26}$$

Furthermore, define

$$C_{l,j,t} = \lambda'_{l,j,t} + \mu_{l,j} + 1. \tag{27}$$

Then, for each $l \in L$, $\mathbf{P}_l''$ can be reformulated as follows:

[$\mathbf{P}_l'''$]

$$\min \sum_{j=1}^{N_l} \sum_{t=a_{l,j}}^{d_{l,j}} C_{l,j,t} \tilde{u}_{l,j,t} \,. \tag{28}$$

subject to constraints (22)-(25)

In $\mathbf{P}_l''$, $C_{l,j,t}$ can be interpreted as the charging cost incurred by buses in l during time period t at the j-th station. Problem $\mathbf{P}_l'''$ assigns time periods to buses in l for charging to ensure that the vehicle's battery state is within $[G_l, S]$ when arriving at each station, while the total charging cost is thus minimized. Consider the moment at which a bus in line l departs from the j-th ($1 < j < N_l$) station with battery state $\tilde{v}_{l,j}$. For each $j' \in \{1,..., j\}$, let $\hat{T}_{l,j'}$ denote the set of time periods during which a bus in line l is charged at the j'-th station. If the battery state of this bus is insufficient to arrive at the $(j+1)$-th station (i.e., $\tilde{v}_{l,j} - L_{l,j} < G_l$), additional time periods should be assigned to this bus for charging before departing from the j-th station, which can be realized by assigning $G_l + L_{l,j} - \tilde{v}_{l,j}$ additional time periods that incur the minimum charging cost to the bus from the set $\bigcup_{j'=1}^{j} \left(\{a_{l,j'},..., d_{l,j'}\} \setminus \hat{T}_{l,j'} \right)$. Based on this observation, a tailored iterative algorithm is proposed to solve problem $\mathbf{P}_l''$ instead of using the commercial solver.

Specifically, if the battery power of a bus is not sufficient to travel to the next station, additional charging time periods would be assigned to the bus. The pseudo-code of this algorithm is presented in **Algorithm 2**. Note that in this algorithm, $j(t)$ is the index of the station such that $t \in \{a_{l,j(t)},...,d_{l,j(t)}\}$, and each $t \in \bigcup_{j=1}^{N_l} \{a_{l,j},...,d_{l,j}\}$ corresponds to a unique $j(t)$.

Algorithm 2: The improved algorithm for solving $\mathbf{P}_l''$

Set $\tilde{u}_{l,j,t} \leftarrow 0$, $\forall j \in \{1,...,N_l\}$, $t \in \{a_{l,j},...,d_{l,j}\}$,
    $\tilde{v}_{l,j} \leftarrow 0$, $\forall j \in \{2,...,N_l\}$, $\tilde{v}_{l,1} \leftarrow S$ and $i \leftarrow 1$
while $i < N_l$ **do**
    Set $T_{l,i} \leftarrow \bigcup_{j=1}^{i} \{a_{l,j},...,d_{l,j}\}$
    while $\tilde{v}_{l,j} - L_{l,\mathrm{i}} < G_l$ **do**
        Set $t^* \leftarrow \arg\min_{t \in T_{l,i} : \tilde{u}_{l,j(t),t}=0} \{C_{l,j(t),t}\}$
        if $\tilde{v}_{l,j} + 1 \le S$ for all $j \in \{j(t^*),...,i\}$ **then**
            Set $\tilde{v}_{l,j} \leftarrow \tilde{v}_{l,j} + 1$ for each $j \in \{j(t),...,i\}$
            Set $\tilde{u}_{l,j(t^*),t^*} \leftarrow 1$
        end if
        Set $T_{l,i} \leftarrow T_{l,i} \setminus \{t^*\}$
    end while
    Set $i \leftarrow i+1$
end while

The aim of **Algorithm 2** is to assign time periods to the bus in line l for charging to guarantee that the battery state of bus l is at least G_l when it arrives at each station, which incurs the minimum charging cost. It can be observed that **Algorithm 2** is a polynomial algorithm with a complexity of $O(H_l)$, and $H_l = \left|\bigcup_{j=1}^{N_l} \{a_{l,j},...,d_{l,j}\}\right|$.

4.3.3 The enhanced Lagrangian relaxation algorithm

Combining the constraint generation technique and the tailored iterative algorithm, a novel solution algorithm for the Lagrangian relaxation problem is constructed in **Algorithm 3**. The pseudo-code of **Algorithm 3** is presented as follows:

Algorithm 3: The enhanced Lagrangian relaxation algorithm

Set $\underline{Z} \leftarrow 0$, $\overline{Z} \leftarrow +\infty$, $u_{l.j} \leftarrow 0$, $\forall l \in L$, $j \in \{1,...,N_l\}$,
    $\lambda_{n,t} \leftarrow 0$, $\forall n \in N$, $t \in T$, $\Omega \leftarrow \varnothing$
while termination conditions are not satisfied **do**
    Solve the problem P_l'' for each $l \in L$ using **Algorithm 2** to obtain the solution

$\{\tilde{\mathbf{u}}, \tilde{\mathbf{v}}\}$ and the lower bound $\underline{Z}(\boldsymbol{\mu}, \boldsymbol{\lambda})$

5: **if** $\underline{Z}(\boldsymbol{\mu}, \boldsymbol{\lambda}) > \underline{Z}$ **then**

6: Set $\underline{Z} \leftarrow \underline{Z}(\boldsymbol{\mu}, \boldsymbol{\lambda})$

7: **end if**

8: Based on Proposition 1, obtain the solution of P_l', $\{\mathbf{u}, \mathbf{v}\}$

9: Identify the set of $\{n,t\}$ pairs for which constraint (20) is violated by u

10: Let Ω' denote the identified set of $\{n,t\}$ pairs

11: Set $\Omega \leftarrow \Omega \cup \{(n,t) \in \Omega' | (n,t) \notin \Omega\}$

12: Generate the upper bound of problem P, $\overline{Z}(\boldsymbol{\mu}, \boldsymbol{\lambda})$

13: **if** $\overline{Z}(\boldsymbol{\mu}, \boldsymbol{\lambda}) < \overline{Z}$ **then**

14: Set $\overline{Z} \leftarrow \overline{Z}(\boldsymbol{\mu}, \boldsymbol{\lambda})$

15: **end if**

16: **for** $l \in L$, $j \in \{1, ..., N_l\}$ **do**

17: Update $\lambda_{n,t}$

18: **end for**

19: **End while**

Note that the updating scheme of $u_{l,j}$ and $\lambda_{n,t}$ in lines 17 and 20 of **Algorithm 3** is same as ***Step 4*** of **Algorithm 1**.

5 Numerical experiments

In this section, we conducted numerical experiments to evaluate the proposed model and algorithm in this study. The algorithm was coded in python, and $\mathbf{P}_l''$ was solved by Gurobi 9.0. All computational experiments were conducted on an Intel Core i7-9750H CPU at 2.60 GHz with 16 GB RAM.

5.1 Experimental setup

We tested the proposed model and algorithms with random instances including 40-300 stations. The original data, including the location of stations and corresponding trip schedule, can be retrieved from the online dataset (NEO, 2013). The planning horizon is divided into 600-time intervals, each of which represents one minute. The minimum required SOC, G_l, is set to 5%. The real battery charging profile is adopted from the work of Zündorf (2014), which is then discretized by the method presented in Section 3.

5.2 Optimal results

To better evaluate the performances of the proposed algorithms, the following four solution strategies are applied and compared: 1) using Gurobi to solve the primal problem **P** directly without any reformulation procedures; 2) using **Algorithm 1** to solve **P** based on the Lagrangian decomposition; 3) using Gurobi to solve the Lagrangian relaxation problem and updating the multipliers by **Algorithm 2**; and 4) using the enhanced Lagrangian relaxation method, i.e., **Algorithm 3**. Table 3 summarizes the computational results for random instances.

Table 3 Performances of the proposed model and algorithms.

Instances	**Gurobi**		**Algorithm 1**		**Gurobi + Algorithm 2**		**Algorithm 3**	
	Obj	Running time	Obj	Running time	Obj	Running time	Obj	Running time
40/8/3/3	124	1.65	124	0.20	124	0.18	124	0.07
40/10/3/3	279	1.99	279	0.22	279	0.19	279	0.07
40/12/3/3	294	2.13	294	0.25	294	0.24	294	0.11
50/10/3/3	302	2.22	302	0.21	302	0.22	302	0.16
50/12/3/3	339	2.55	339	0.25	339	0.23	339	0.17
50/15/3/3	550	3.46	550	0.39	550	0.28	550	0.27
80/15/3/3	482	5.39	482	0.35	482	0.86	482	0.51
80/30/3/3	721	9.73	721	0.48	721	5.28	721	1.14
80/40/3/3	996	9.43	996	0.62	996	1.54	996	1.08
150/30/3/3	1,019	17.58	1,019	0.69	1,019	0.62	1,019	0.89
150/50/3/3	1,189	28.49	1,189	1.10	1,189	1.06	1,189	0.93
150/70/3/3	1,498	54.96	1,498	2.89	1,498	2.64	1,498	2.08
300/70/3/3	1,373	118.84	1,373	3.47	1,373	4.75	1,373	1.69
300/70/4/3	1,373	118.84	1,373	1.62	1,373	1.57	1,373	1.61
300/70/5/3	1,373	118.84	1,373	1.58	1,373	1.59	1,373	1.56
300/150/3/3	-	-	3,481	9.67	3,481	7.48	3,481	17.82
300/150/4/3	-	-	3,481	7.09	3,481	6.76	3,481	3.61
300/150/5/3	-	-	3,481	6.17	3,481	6.42	3,481	3.31

Each instance is named after $N/L/M_k/U_l$. Table 3 presents the average of 20 runs for all instances of each data set of each algorithm. Column "Obj" reports the average solution values of our implemented algorithms. Column "Running time" presents the computational time measured in CPU seconds. The results indicate that our proposed enhanced Lagrangian relaxation algorithm outperforms the solution strategies of using the solver directly or the standard subgradient method. When the scale of instances gets larger in terms of the number of stations and lines, our enhanced algorithms are more stable and more efficient than the standard algorithms.

The dominant influencing factor on the computing efficiency can be attributed to the operation on the "tight" constraint. Comparing the results of solution strategies of 2) and 3), it can be observed that reducing the number of updating multipliers can indeed speed up the convergence speed of the algorithm if the constraint (16) is relaxed, which is considered as a tight one in the original problem.

5.3 Comparison between charging strategies

In practice, if no bus charging scheduling strategy is applied, the EBs' charging processes are usually disorganized and difficult to describe mathematically. For tractability reasons, we further assume that, without any coordinated measurements, EBs are charged in a first-in-first-out (FIFO) order (the bus which arrives earlier will be charged first, and the later buses are charged immediately afterward) (Qin et al., 2016). Moreover, each bus is allowed to charge at a single station for only one time, and the charging process would not be interrupted. In line with the proposed coordinated charging (CC) scheme, the charging process is completed if the current SOC is enough for the next trip.

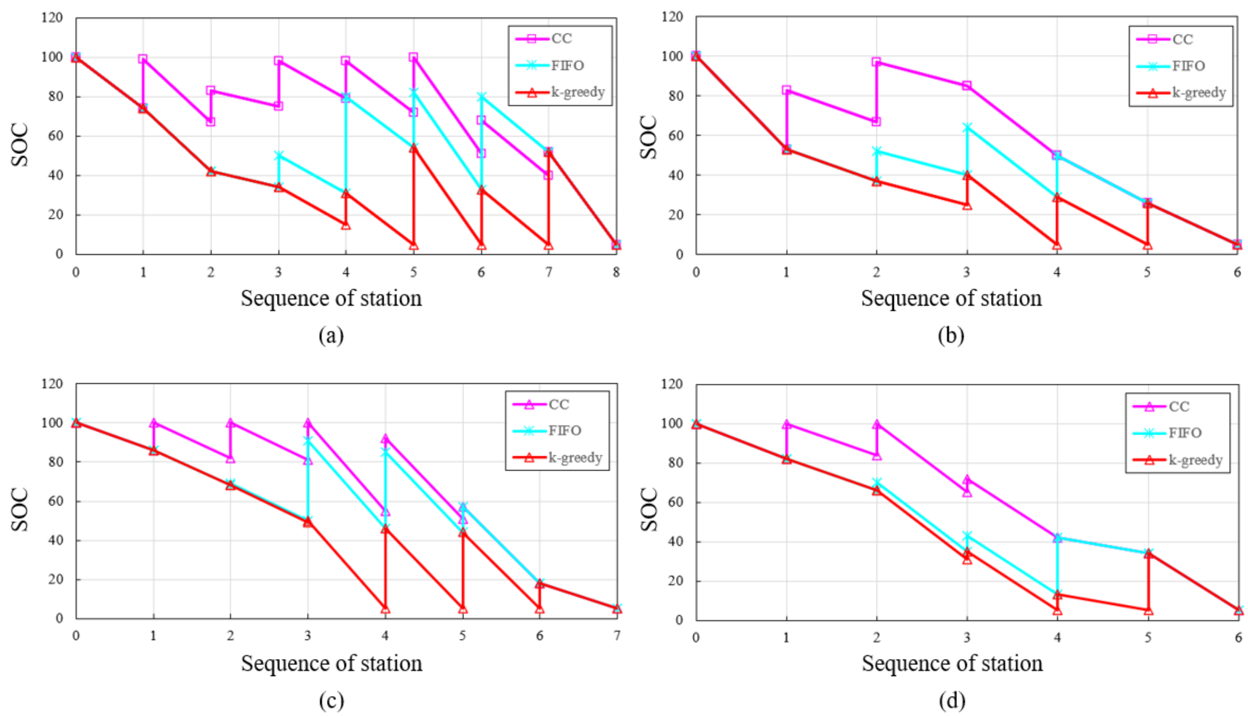


Figure 6. Charging files of four typical lines.

The charging files of four bus lines in the group of instances 40/12/3/3 in three different charging schemes (CC, FIFO, and *k*-greedy) are shown in Fig. 6, from which the following observations can be made: 1) In the CC scheme, most of the charging processes are completed at the previous stations of bus itineraries, which indicates that vehicles are likely to charge additional energy at the first several stations and reduce the charging time at latter stations; 2) In the *k*-greedy scheme, which has an opposite result, vehicles are more likely to charge at latter stations rather the previous stations of their itineraries. It can be attributed to the fact that the *k*-greedy heuristic algorithm tends to minimize the total charging time by decreasing the number of charging processes, where vehicles only need to be charged when the remaining SOC is not enough for the next trip; and 3) In the FIFO scheme, the distribution of charging processes seems to be even among bus's itinerary.

Table 4 presents the comparison of the system performance between the proposed CC and FIFO charging scheme in the case of $N = 50$, $L = 20$. Note that the number of charging times of a bus in the same station U_l in CC is set to 1 arbitrarily in line with FIFO. It can be observed that the total charging times of both schemes are the same because both these two schemes aim to minimize the total charging time under the constraint of guaranteeing the service trips in the first place. In the case that each bus station only has one charger, the FIFO outperforms the CC in queueing time but causes more schedule delay. That is because, in the CC scheme, buses could be charged with additional battery power instead of just satisfying the battery requirement of the next trip (see Fig. 7). While the FIFO is a greedy heuristic which only takes the required batter power of the next trip instead of optimizing its charging behavior at a system level. Another observation is that, when U_l is determined, increasing the number of chargers at each station would not improve the system performances, i.e., decreasing the queueing time or the schedule delay.

Table 4 The comparison between the proposed CC and FIFO charging scheme.

No. of chargers	Queueing time		Charging time		Schedule delay	
	FIFO	CC	FIFO	CC	FIFO	CC
1	14	72	443	443	38	0
2	10	17	443	443	32	0
3	10	17	443	443	32	0
4	10	17	443	443	32	0
5	10	17	443	443	32	0

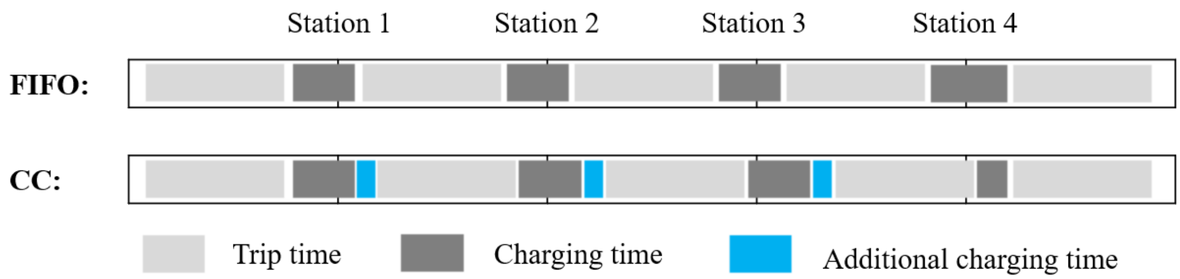
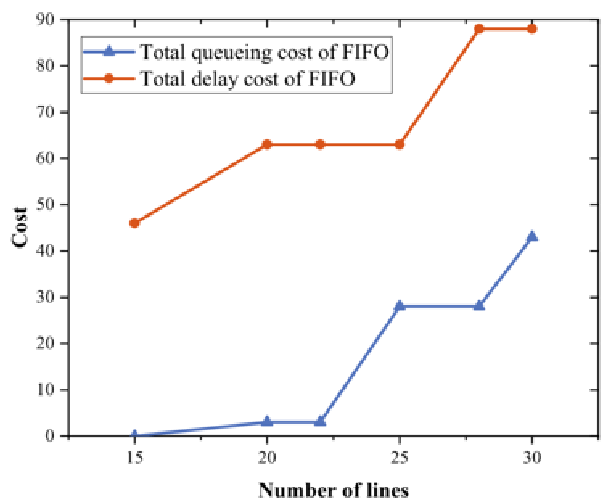
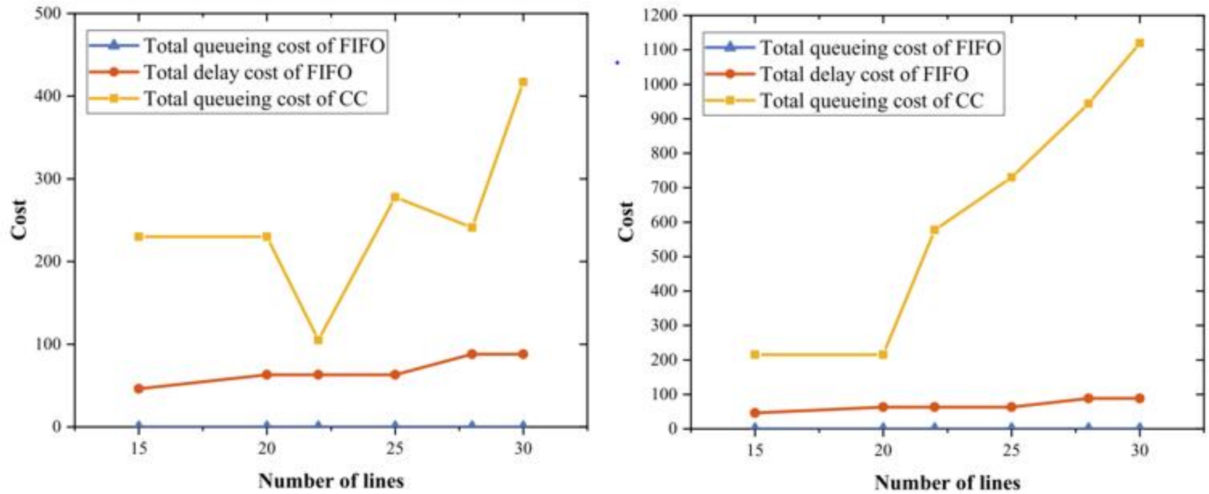


Figure 7. The illustration of the additional charging time.

In Fig. 8, three indicators are used to evaluate the impacts of the number of chargers on the system performance with respect to the different number of bus lines in the transit system. The experiments are conducted on the instance with $N = 50$ and $U_l = 3$. Note that in Fig. 8(a), the total queueing cost is not presented because no feasible solution is found with only one charger at each station. The results show that, in the FIFO scheme, both queueing and delay costs are positively correlated with the number of lines. When the number of chargers is given, the increase of bus lines will result in an increase of queueing and delay costs due to the high level of schedule overlap among different bus lines. In the CC scheme, the delay cost is eliminated because of the strict constraint of schedule adherence. However, the queueing cost increases dramatically because some vehicles must wait in lines even though they arrive earlier, which is in accordance with the basic assumption of the CC scheme that the vehicle with a tighter schedule has a higher priority to charge first compared with those which arrives earlier but with loose schedule constraint.



(a) Instance: 50 nodes, $U_l = 3$, $M_n = 1$.



(b) Instance: 50 nodes, $U_l = 3$, $M_n = 2$. (c) Instance: 50 nodes, $U_l = 3$, $M_n = 3$.

Figure 8. The impacts of the number of chargers on the system performance.

6 Conclusions

In this paper, a Lagrangian relaxation approach for the electric bus charging scheduling optimization problem has been proposed. Due to the limited resources concerning the charging facility, it is essential to design a cost-effective and coordinated charging scheduling strategy. An ILP is proposed to design the optimal EB charging schedule to minimize the total charging time of the entire electric transit network. To tackle the nonlinearity caused by the charging function, this paper discretizes the decision variables for the charging schedule into time intervals. Both two models are formulated in linear integer programs. A Lagrangian relaxation-based solution algorithm. To reduce the searching space for large-scale problems, the network-based model is decomposed into subproblems with respect to one bus. An enhancement to the basic Lagrangian relaxation algorithm is proposed to accelerate the convergence of the algorithm. The results show that: 1) our proposed enhanced Lagrangian relaxation algorithm outperforms the solution strategies of using the solver directly or the standard subgradient method; 2) the limited capacity constraint is considered as the “tight” constraint in the proposed model.

The optimal result of the charging strategy provides guidance for the transit operator to design a charging schedule and improve the system’s operational efficiency accordingly. Two key insights that we can deduce from these results are as follows. First, the optimal distribution of charging location varies in different charging schemes. In the proposed CC, EBs are more likely to charge for a short period in the previous stations of itineraries; while, in greedy strategies (including both *k*-greedy and FIFO), EBs only charge when the remaining battery energy is not enough for the next trip. The strategy of CC outperforms *k*-greedy and FIFO especially in the case of limited charging capacity. For instance, EBs in the CC scheme intend to charge for additional energy once there exists available chargers and would not violate the predetermined schedule as well. However, in greedy schemes, EBs have to charge at a certain station otherwise the remaining battery power cannot guarantee the service of the next trip. Hence, they have to wait for charging even though the schedule is violated. Second, there is a large body of research on optimization models of charging station deployment including location and capacity. The expansion of the existing charging facility is inevitable when the charging demand increases. However, the unbalance between charging demand and supply can first be addressed from the operational level in a more cost-effective way by coordinating the EBs' charging schedule, e.g., allowing the EB with an urgent and scheduled trip to be charged in a prior order.

Several potential enhancements could be considered in future works: 1) integrate the coordinated charging scheme in the charging facility location and EB network design problem to develop a systematic EB planning and operation approach; 2) take into account more external factors concerning the city's power grid; and 3) investigate the bus charging scheme in a complex and realistic environment, such as the stochastic trip time, battery aging, schedule reliability, among many others.

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