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2 **Three-dimensional Fatigue Crack Evaluation Using**
3 **Densely Connected Convolutional Network-assisted**
4 **Nonlinear Ultrasonics**

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27 **Abstract**

28 Linear and nonlinear ultrasonic guided wave (UGW)-based methods are available for detection
29 and evaluation of fatigue cracks and monitor the growth under cyclic loadings. However, linear
30 methods usually fail to detect early-stage fatigue cracks of undersized dimensions, while nonlinear
31 methods struggle to assess cracks once they have extended to a certain degree. The fusion of linear
32 and nonlinear UGW features offers new opportunity to enhance evaluation precision and
33 effectiveness, yet it necessitates highly complex theoretical modeling. Motivated by this,
34 densely connected convolutional network-based regression model using the spectra of UGW
35 signals is proposed in this study for the evaluation of three-dimensional (3D) fatigue cracks from
36 embryonic initiation, through progressive growth to the formation of macroscopic cracks. Via the
37 continuous wavelet transform (CWT), the spectra of UGW signals captured during fatigue crack
38 initiation and growth are obtained, which embrace both linear and nonlinear features of UGW.
39 Subsequently, spectra images are used as the input of the famed densely connected convolutional
40 network, DenseNet, to extract the implicit features of UGW. Finally, the last fully connected layer
41 of the DenseNet is modified as a regression layer to predict the length and depth of 3D fatigue
42 cracks. For validating the proposed model, a dataset with a total number of 400 UGW signals is
43 established based on the experiment and simulation of 3D corner crack growth. A comparison
44 study is conducted between the proposed DenseNet-based regression model and a previously
45 reported nonlinear UGW-based model for 3D corner crack evaluation. Results show that the
46 proposed model can characterize the length and depth of a corner crack in both initiation and
47 growth stages. The proposed model represents a promising proof-of-concept for deep learning-
48 based quantitative evaluation of 3D fatigue cracks.

49
50 **Keywords:** Deep learning-based regression; densely connected convolutional network; ultrasonic
51 guided wave; 3D fatigue crack, quantitative evaluation

52 **1. Introduction**

53 Fatigue cracks are widely found in critical components of engineering structures, which are liable
54 for numerous sudden catastrophic failures of entire structures, such as flight vehicles, rail tracks,
55 infrastructures, and so on. It is reported that fatigue is associated with over 90 % of all mechanical
56 failures [1]. Therefore, early awareness, continuous monitoring, and accurate evaluation of fatigue
57 cracks are of great significance for ensuring the integrity and safety of structures. For this purpose,
58 ultrasonic guided wave (UGW)-based methods have been developed to detect and characterize
59 both macroscopic and small-scale fatigue cracks [2-7]. The principle of UGW-based fatigue crack
60 identification and assessment rests on the idea that the propagation characteristics of a probing
61 UGW undergo a certain degree of modulation and alteration upon interaction with a fatigue crack.
62 This UGW-crack interaction gives rise to distinct wave scattering phenomena, including
63 reflection, transmission, mode conversion, resonant frequency shift, wave energy transfer, and the
64 generation of higher- and sub-order harmonics. Therefore, fatigue cracks can be inversely
65 identified and evaluated through careful examination of the alterations in UGW attributes by
66 employing suitable analytical models.

67
68 Linear UGW-based methods making use of linear wave features such as the reflection and
69 transmission coefficients [8-11], delay in time-of-flight [12], degree of wave energy dissipation
70 [13], and mode conversion [14], have been widely applied to identify fatigue cracks. These linear
71 methods show particular effectiveness in detecting and evaluating fatigue cracks with the
72 characteristic dimensions comparable to the wavelength of the probing UGW - a scale at which
73 the cracks may have already progressed to a macroscopic extent, resulting in a magnitude
74 substantial enough to induce fatal structural failure. To detect the small-scale fatigue cracks as
75 early as possible, nonlinear UGW-based methods have been proposed recently, making up for the
76 shortcomings of linear methods in identifying imperceptible fatigue cracks. Approaches in this
77 category can be broadly classified into harmonic generation [2, 15-18], resonant frequency shift

78 [19] and wave mixing [20-22]. Representatively, Xiang *et. al.* [23] developed a model to correlate
79 the fatigue crack location with the wavelet coefficients at the second harmonic frequency of Lamb
80 waves, which can be applied for the localization of barely visible fatigue cracks. Based on the
81 elastodynamic reciprocity theorem, Xu *et. al.* [24] proposed an analytical model to shed light on
82 the second harmonic generation arise from the contact acoustic nonlinearity (CAN) at ‘breathing’
83 cracks and derived a closed-form solution to the magnitude of crack-induced second harmonic,
84 which can be employed for detection and characterization of tiny fatigue cracks. Shen *et. al.* [25]
85 proposed a nonlinear ultrasonic resonance spectral correlation technique for fatigue crack
86 quantification and tailored a nonlinear reduced-order oscillator model to illuminate the CAN at
87 the fatigue cracks. Results demonstrated that the proposed technique is able to detect fatigue
88 cracks as short as 1.5 mm. Sohn *et. al.* [4, 26] designed the two-wave and three-wave mixing
89 techniques in which two or three UGWs with distinct central frequencies are excited in the
90 targeted structure simultaneously and the nonlinear sideband harmonics will appear at the sum
91 and difference of the input frequencies due to the existence of fatigue cracks. The crack-induced
92 sideband harmonics are inversely used to detect and localize early-stage fatigue cracks and
93 provide timely warning of imminent failure.

94
95 Although the use of linear features of UGW proves particularly efficient when the characteristic
96 dimensions of targeted fatigue cracks are comparable to the wavelength of the probing UGW and
97 the nonlinear UGW is adept at detecting incipient fatigue cracks, both methodologies fall short in
98 providing reliable identification and assessment of fatigue cracks throughout their early-stage
99 initiation and stable growth phases. The fusion of linear and nonlinear attributes of UGW presents
100 a viable solution to overcome this limitation, yet intricate modeling is imperative to establish a
101 correlation between these wave attributes and fatigue crack severity, due to the diverse UGW-
102 crack interaction mechanisms that vary across different stages of crack evolution. Amongst
103 limited effort, Kundu *et. al.* [27] proposed a holistic technique for sensing impact damage

104 initiation and progression in composite plates, by combining linear and nonlinear features of
105 UGW. In that technique, the evolution of impact damage is divided into early, intermediate, and
106 advanced stages. The linear and nonlinear UGW were applied to evaluate damage in the early and
107 advanced stages, respectively, and the joint use of both shown effectiveness in the intermediate
108 stage. There is an absence of foundational analysis correlating linear and nonlinear damage
109 indicators with damage severity, as well as the precise demarcation of the three stages of damage
110 evolution. Consequently, this shortfall poses challenges in practical applications of the holistic
111 technique for damage evaluation.

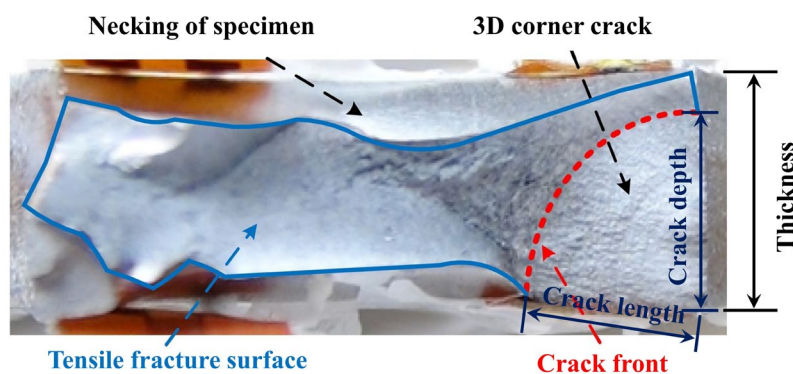
112

113 Recent years have witnessed the rapid development of deep learning (DL) and its successful
114 applications in various fields such as image recognition, natural language processing, speech
115 recognition, and no exception for crack detection in engineering structures. To deal with the
116 detection of cracks in concrete bridges, Deng *et. al.* [28] applied the You Only Look Once
117 (YOLO) model to automatically locate the concrete cracks with bounding box from raw real-
118 world images contaminated with handwriting scripts. Long *et. al.* [29] proposed a novel global
119 and local dual-scale faster region-based convolutional neural network (Faster R-CNN) to predict
120 the fatigue crack length based on the images captured by a camera. Results demonstrate that the
121 proposed method can effectively measure the propagating small cracks during cyclic loading and
122 can be applied for fatigue crack growth rate measurement. Based on the UGW method, Sohn *et.*
123 *al.* [30] designed a hybrid framework that incorporates higher-order spectral analysis of Lamb
124 waves with a long short-term memory (LSTM)-based DL model for early detection of fatigue
125 cracks. Results validated the robustness of the proposed framework for reliable fatigue crack
126 detection in noisy environments. Guo *et. al.* [31] used three pre-trained DL models (*e.g.*, VGG16,
127 ResNet50, and DenseNet161) to extract features from laser-generated ultrasonic waves and
128 developed a multiplicative LSTM (mLSTM)-based classifier for automatic quantification of
129 subsurface defects. Defects with the dimensions ranging from 0.2 mm to 6.0 mm were divided

130 into 30 categories and the trained mLSTM-based classifier can predict the defect size with an
131 overall accuracy of 98.26 %.

132
133 Previous studies have shown that DL possesses the advantage of establishing implicit connections
134 between physical quantities. This superb merit enables it to circumvent the intricate mathematical
135 modeling necessitated by the fusion of linear and nonlinear features of UGW for fatigue crack
136 evaluation from small-scale initiation to macroscopic evolution. On the other hand, although good
137 supply of research has been conducted based on DL-based classification for fatigue crack
138 detection and assessment, there are very few regression models for the continuous monitoring and
139 evaluation of embryonic three-dimensional (3D) fatigue cracks which do not penetrate the entire
140 thickness of the structures. Different from the two-dimensional (2D) fatigue cracks penetrating
141 the entire thickness of structures which have been investigated extensively, 3D, non-penetrating
142 cracks grow in both length and depth directions under cyclic loading, as schematically illustrated
143 in **Fig. 1**. 3D fatigue cracks, represented by surface cracks and corner cracks, play a vital role in
144 damage tolerance assessment and health monitoring of engineering structures, as the fatigue life
145 in the 3D, non-penetrating stage accounts for more than 80 % of the total fatigue life [32-34].
146 Therefore, it is of great significance to develop a regression model to continuously monitor and
147 accurately evaluate the 3D fatigue cracks from their initiation to the macroscopic progression.

148



149

150 **Fig. 1.** Schematic of a representative 3D corner crack and its progressive evolution under fatigue loading [35].

151

152 In recognition of the lack of DL-based regression able to monitor the growth and evaluate the
153 severity (*i.e.*, crack length and depth) of 3D fatigue cracks, a regression model based on the famed
154 densely connected convolutional network, DenseNet [36], is proposed in this study, using the
155 spectra of UGW which embrace both linear and nonlinear wave features. By performing a time-
156 frequency analysis via the continuous wavelet transform (CWT), the spectra of raw UGW signals
157 measured during the initiation and growth stages of a 3D fatigue crack are obtained. The DenseNet
158 is subsequently recalled for extracting time-frequency domain implicit features from the UGW
159 spectra images. The last fully connected layer of DenseNet is further modified as a regression
160 layer to predict the length and depth of the 3D fatigue crack. Both experiment and simulation are
161 performed to acquire the UGW signals during the initiation and growth stages of a 3D corner
162 crack emanating from a fastener hole, the most widely detected 3D fatigue crack in engineering
163 structures. Real-world and simulated UGW signals are collected to establish a dataset for the
164 training and validation of the proposed model. A comparison study between the proposed
165 DenseNet-based regression model and the previously reported model combining nonlinear UGW
166 and fatigue crack growth theory is conducted. Results demonstrate that the proposed model is not
167 only able to evaluate the corner cracks in the initiation and early progressive growth stage, but
168 can also predict the crack length and depth accurately when a macroscopic fatigue crack is formed.
169 This provides a new way for life-cycle monitoring and assessment of fatigue cracks.

170

171 **2. DenseNet-based Regression for 3D Fatigue Crack Evaluation**

172 The framework of the proposed DenseNet-based regression model for 3D fatigue crack evaluation
173 is detailed in this section. Firstly, the CWT is adopted to obtain the spectra of raw UGW signals
174 in the time-frequency domain. Subsequently, the spectra images are used as the input to train the
175 DenseNet with the last fully connected layer being modified to a regression layer for predicting
176 the length and depth of 3D fatigue cracks.

177

178 **2.1 Time-frequency Feature Extraction Via CWT**

179 The CWT stands as a pivotal mathematical technique extensively employed in signal processing
180 and analysis. Derived from the broader framework of the wavelet transform, the CWT offers a
181 means to scrutinize the time-frequency representation of a signal in a continuous manner, making
182 it particularly adept at capturing localized features in the time-frequency domain. Different from
183 the discrete wavelet transform (DWT) which dissects a signal into discrete segments of varying
184 scale, the CWT permits an adaptable window function that can be dynamically adjusted to capture
185 intricate signal components across various scales. This flexibility in window size facilitates a
186 nuanced exploration of signal characteristics, which is essential in applications spanning diverse
187 fields such as image processing, audio analysis, and biomedical signal interpretation.

188
189 In the proposed model, the CWT is adopted to extract the time-frequency domain features and
190 obtain the spectra of raw UGW signals captured throughout the fatigue crack initiation and growth
191 stages. For a given raw UGW signal $s(t)$, the CWT is performed via the following integral
192 manipulation, as

$$193 \quad S_{\omega}(a, b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} s(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt, \quad (1)$$

194 where $a \in R^+$ is the scale indicator and $b \in R^+$ is the translational value. The overline represents
195 the operation of complex conjugate and $\psi(\cdot)$ represents the mother wavelet which is a continuous
196 function in both time and frequency domains. In the proposed model, the morse wavelet is utilized
197 as the mother wavelet to acquire the spectra of measured UGW signals.

198 199 **2.2 DenseNet-based Regression for 3D Crack Evaluation**

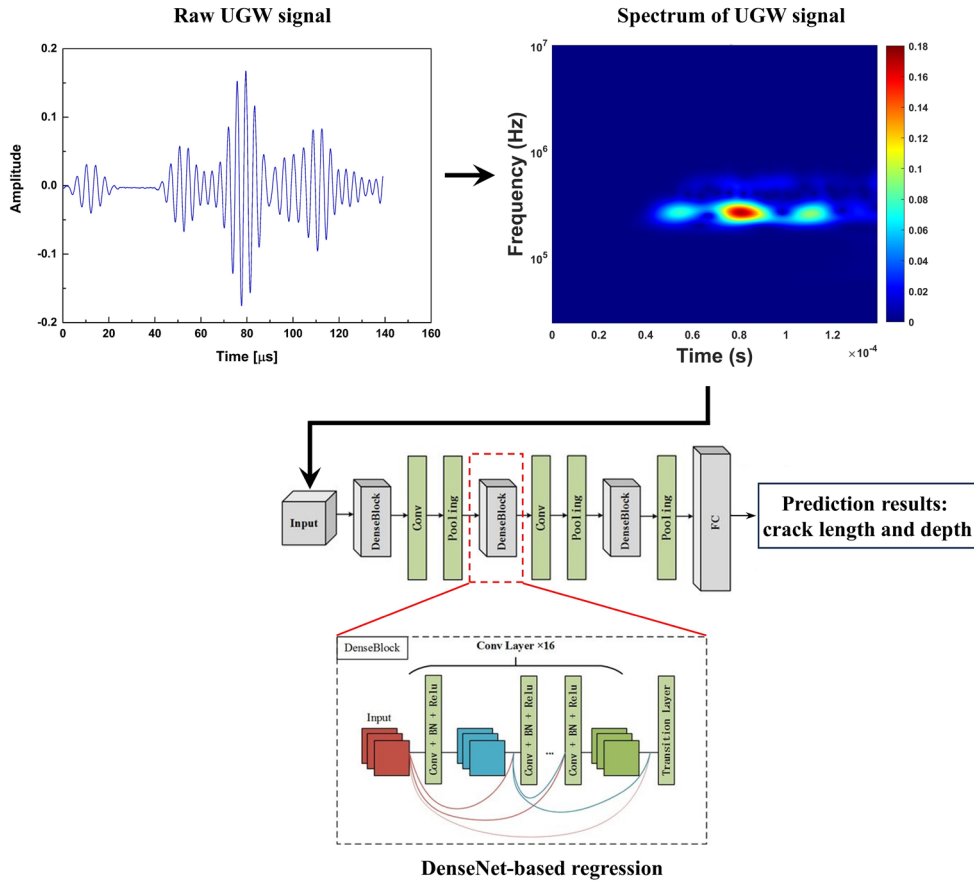
200 After obtaining the spectra of raw UGW signals via the CWT, a DL-based regression model is
201 employed to extract implicit features from the spectra images for the prediction of the length and
202 depth of 3D fatigue cracks. The DenseNet, a highly sophisticated densely connected convolutional

203 network with a number of compelling merits over other powerful convolutional neural network
204 (CNN) models such as VGG and ResNet, is utilized for the feature extraction and 3D fatigue crack
205 evaluation. The DenseNet comprises interconnected dense blocks which are chained sequentially,
206 interspersed with transition layers placed between contiguous dense blocks. Each dense block is
207 composed of multiple densely connected convolutional layers. The architecture of the DenseNet
208 and the details of a dense block are shown in **Fig. 2**.

209
210 The core principle underpinning the DenseNet is the dense connectivity pattern it introduces
211 among layers within a dense block. In contrast to traditional CNN architectures (*i.e.*, VGG and
212 ResNet) where information flows in a linear sequential manner, DenseNet establishes dense
213 interconnections between layers, enabling each layer to receive input not only from its very first
214 preceding layer but also from all preceding layers. This distinctive structural attribute engenders
215 a rich feature propagation mechanism, efficaciously promoting information exchange and
216 reusability throughout the entire network. By fostering these dense connections, DenseNet
217 combats the vanishing gradient issue, which exceedingly hampers the training of deep networks.
218 Additionally, the architecture's compactness and parameter efficiency render it conducive to
219 learning features from relatively small datasets. Therefore, the DenseNet is an appropriate model
220 to perform feature extraction and learning for 3D fatigue crack evaluation, considering the
221 difficulties in constructing a big dataset consisting of thousands of UGW signals.

222
223 With the manipulation of the dense blocks and transition layers, the implicit features carried by
224 the spectra of the raw UGW signals are extracted. Ultimately, the last fully connected layer of the
225 DenseNet is modified as a regression layer with two outputs, for the prediction of the two
226 characteristic indicators of 3D fatigue cracks, *i.e.*, crack length and depth. **Fig. 2** recaps the overall
227 framework of the proposed DenseNet-based regression model.

228



229

230

Fig. 2. Framework of the proposed DenseNet-based regression model for 3D fatigue crack evaluation.

231

232 3. Experiment and Simulation for Data Generation

233

For constructing a dataset for the training and validation of the proposed DenseNet-based regression model, fatigue crack growth and ultrasonic testing is performed, in which UGW signals are collected throughout the initiation and growth stages of 3D corner cracks emanating from a fastener hole. Considering the dataset based on pure experimental data is relatively small for the training of a DL model, numerical simulation is further conducted using the finite element method (FEM) to obtain synthetic UGW signals for dataset expansion.

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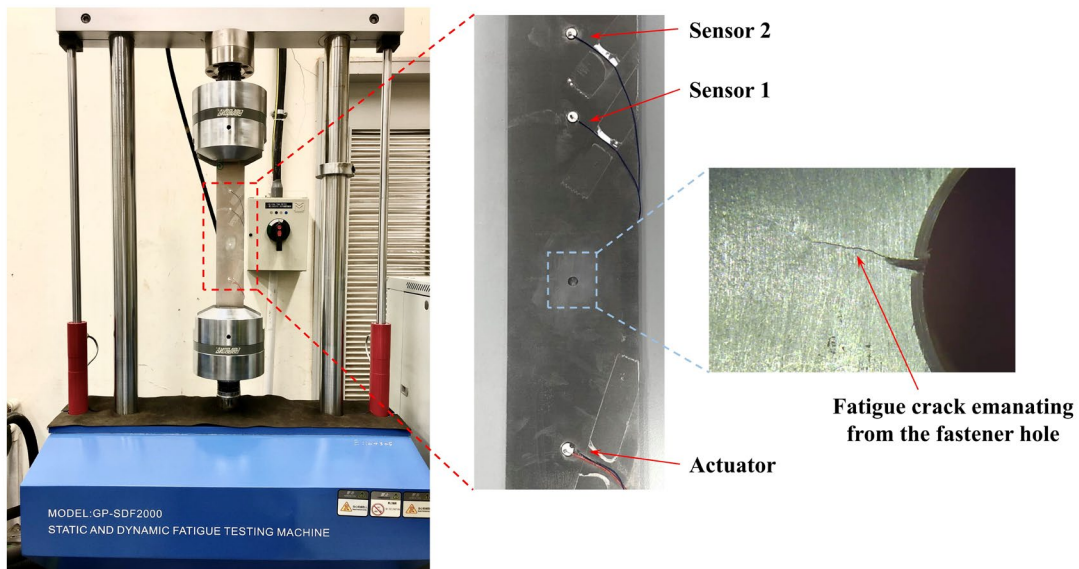
240 3.1 Data Generation based on Fatigue Crack Growth and Ultrasonic Testing

241

Two aluminum plate-like specimens (labeled as Specimen 1 and Specimen 2, respectively), measuring 500 mm in length, 80 mm in width, and 3 mm in thickness, are prepared for the fatigue

242

243 crack growth testing. A through-thickness fastener hole with a diameter of 6 mm is drilled in the
244 center of each specimen and a triangular artificial notch is inscribed at the fastener hole in the
245 plane perpendicular to the direction of fatigue loading, so that a corner crack can initiate and
246 progress under the cyclic loading. Three lead zirconate titanate (PZT) wafers are mounted on the
247 surface of the specimen, one acting as the actuator to excite probing UGW and the other two
248 severing as the UGW sensors to capture propagating UGW signals, as shown in **Fig. 3**. The
249 actuator is 100 mm from the center of the fastener hole and the two sensors are placed on the other
250 side of the hole with the distance of 100 mm and 150 mm from the hole center, respectively.
251



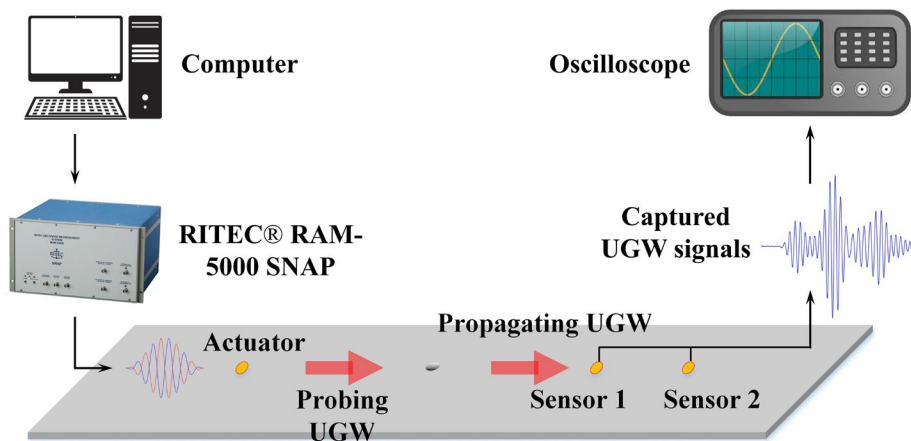
252
253 **Fig. 3.** Set-up of the fatigue crack growth testing and the tested specimen with a 3D corner crack emanating
254 from the fastener hole.
255

256 A fatigue testing machine (model: GP-SDF2000) is utilized to perform the fatigue crack growth
257 testing. A pre-cracking fatigue loading with the maximum tensile load of 30 kN and the stress
258 ratio of 0.1 is first applied to introduce an embryonic 3D corner crack from the tip of the artificial
259 notch at the fastener hole. The length (dimension in the specimen surface) and depth (dimension
260 in the thickness direction of specimen) of the corner crack are continuously monitored by a
261 microscope and a slender mirror inserted into the fastener hole. Once the embryonic corner crack

262 grows to an extent of ~ 1 mm in length and depth, the pre-cracking stage is suspended and the
263 fatigue loading is regulated to 2~20 kN with a frequency of 10 Hz, to perform the growth testing
264 of the 3D corner crack. The PZT wafers are mounted onto the specimen after the pre-cracking
265 stage to minimize the risk of debonding between wafers and specimen. The fatigue crack growth
266 testing is paused every 1,000 cycles, to measure the length and depth of the corner crack and
267 conduct ultrasonic testing to acquire UGW signals. The test is terminated when the 3D corner
268 crack penetrates the specimen thickness and a 2D, through-thickness crack is formed.

269
270 During each pause of the fatigue crack growth testing, ultrasonic testing is performed to obtain
271 crack-modulated UGW signals. A high-power ultrasonic measurement system (RITEC® RAM-
272 5000 SNAP) is used to excite the actuator to generate a 5-cycle Hanning-windowed sinusoidal
273 toneburst at a central frequency of 250 kHz into the specimen. Upon modulated by the corner
274 crack, the propagating UGW is captured by the two sensors with an oscilloscope at a sampling
275 frequency of 200 MHz. The UGW signals are averaged 1,024 times to reduce the measurement
276 uncertainty from testing system and environment. **Fig. 4** schematically displays the set-up of the
277 ultrasonic testing.

278



279

280 **Fig. 4.** Schematic of the ultrasonic testing set-up for acquiring UGW signal data during the fatigue crack
281 initiation and growth stages.

282

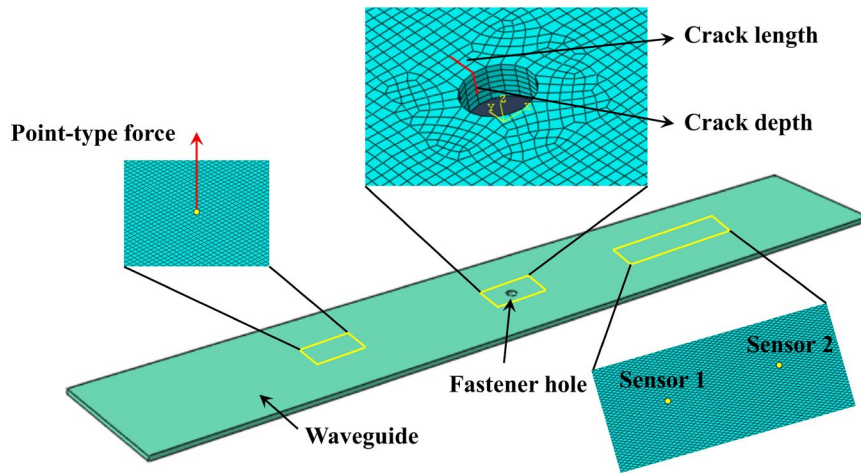
283 To acquire more UGW signals in the ultrasonic testing during the pause of fatigue testing, a pulse-
284 inversion technique [37], in which two identical probing UGWs with the same magnitude but in
285 opposite phase are excited, is adopted in the experiment. With this technique, 4 UGW signals can
286 be obtained by the two sensors in a single pause. The UGW signal data acquired from the entire
287 fatigue crack growth testing of Specimen 1 and Specimen 2 are collected to construct a dataset
288 for the training and validation of the proposed model.

289

290 **3.2 Dataset Expansion with FEM Simulation**

291 A sufficiently large dataset is of the utmost importance for training a DL model and enabling it to
292 learn and infer from new data, while the dataset constructed based on the UGW signals acquired
293 from the experiment in Section 3.1 is relatively small. Consequently, FEM simulation is
294 conducted using the commercial software ABAQUS®/EXPLICIT to generate synthetic UGW
295 signals for dataset expansion. A plate-like waveguide finite element (FE) model (density: 2700
296 kg/m³; Young's modulus: 73 GPa; Poisson's ratio: 0.33) with the same geometrical configuration
297 as the tested specimens is built, as shown in **Fig. 5**. A series of semi-elliptical corner cracks with
298 various severities (crack length and depth ranging from 1 to 3 mm) are simulated at the fastener
299 hole and in each simulated case, the crack surface is perpendicular to the length direction of the
300 waveguide. Material nonlinearity is not considered in the simulation due to its negligible intensity
301 compared with the CAN induced by fatigue cracks, according to the results of previous studies
302 [38, 39]. No special boundary layers (*e.g.*, perfect matching layer (PML) or absorbing layer with
303 increased damping (ALID)) are set around the waveguide model to simulate the actual boundary
304 reflection of probing UGW and to make the synthetic signals more approximate to real-world
305 UGW signals.

306



307

308 **Fig. 5.** FE model of the waveguide containing a semi-elliptical corner crack emanating from a fastener hole.

309

310 To generate a probing UGW in the waveguide, a toneburst with the same cycle and frequency as
 311 the excitation signal used in the experiment is excited by applying a point-type force at a node on
 312 the surface of the waveguide, as illustrated in **Fig. 5**. Two other nodes are further selected as the
 313 sensing points to capture propagating UGW signals. The locations of the excitation and sensing
 314 points are referred to the installation placement of the three PZT wafers in the experiment.
 315 Synthetic UGW signals obtained from the FEM simulation are added into the dataset consisting
 316 of experimental signals for dataset expansion for the training and validation of the proposed
 317 DenseNet-based regression model.

318

319 **4. Results and Discussion**

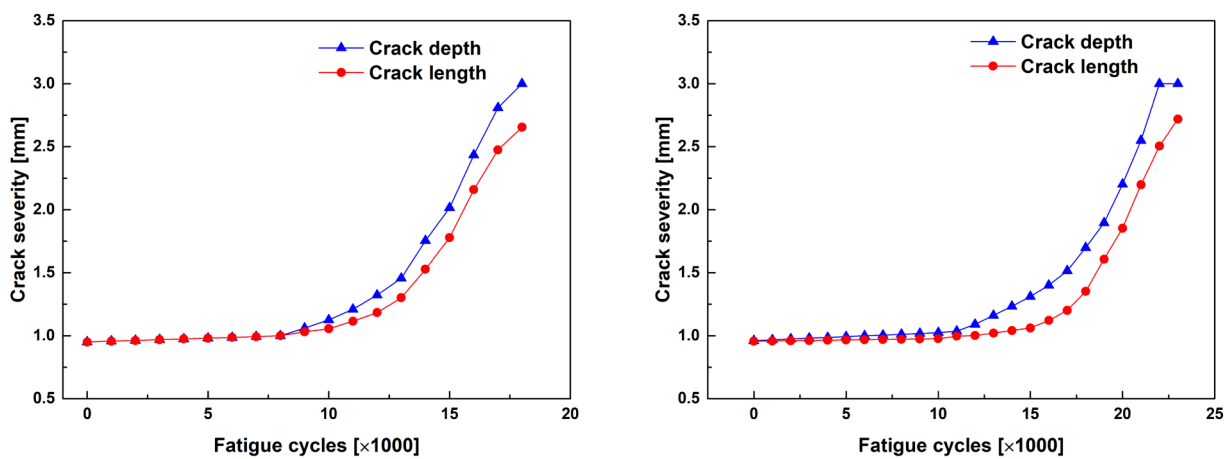
320 To validate the developed DenseNet-based regression model, the experimental and simulated
 321 UGW signals are converted to RGB images via the CWT, by which means an image dataset is
 322 built for training and verifying the proposed model. A comparison study is further conducted
 323 between the proposed model and a previously reported nonlinear UGW-based model, to reveal
 324 the superiority of the proposed DenseNet-based regression model in evaluating 3D fatigue cracks
 325 in both embryonic initiation and progressive growth stages.

326

327 4.1 Experimental Results and Verification of FEM Simulation

328 The length and depth of the corner crack emanating from the fastener hole in Specimen 1 and
329 Specimen 2 are measured by the microscope continuously at every pause of the fatigue crack
330 growth testing, as shown in **Fig. 6**. It can be observed that under the cyclic fatigue loading, the
331 3D corner crack initiates in the first a few kilocycles, then progressively grows in both length and
332 depth directions, and eventually penetrates the entire thickness of the specimen and forms a 2D
333 crack. By way of illustration using Specimen 1, the corner crack initiates before 9, 000 fatigue
334 cycles and the length and depth of the corner crack remain largely unchanged during this period.
335 After 8, 000 cycles, the crack length and depth increase with respect to fatigue cycles. The crack
336 depth reaches 3 mm at 18, 000 cycles, indicating that the corner crack penetrates the whole
337 thickness of the specimen and the fatigue crack growth testing is terminated accordingly. Typical
338 photographs of the corner crack in different stages are displayed in **Fig. 7**, showing the evolution
339 of the corner crack in both length and depth directions under the cyclic loading.

340



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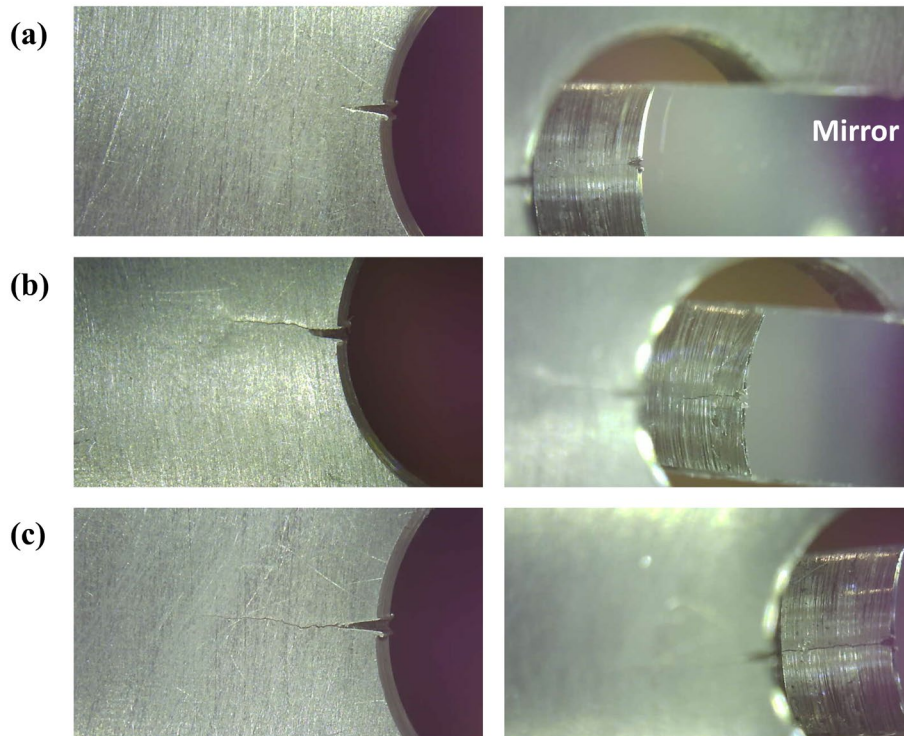
(a)

(b)

342

343 **Fig. 6.** Measured length and depth of the corner crack emanating from the fastener hole in (a) Specimen 1;
344 and (b) Specimen 2.

345



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Fig. 7. Photographs of the corner crack in Specimen 1 in (a) crack initiation stage; (b) progressive growth stage; and (c) when the corner crack penetrates the whole thickness of the specimen. Left column: crack length, right column: crack depth.

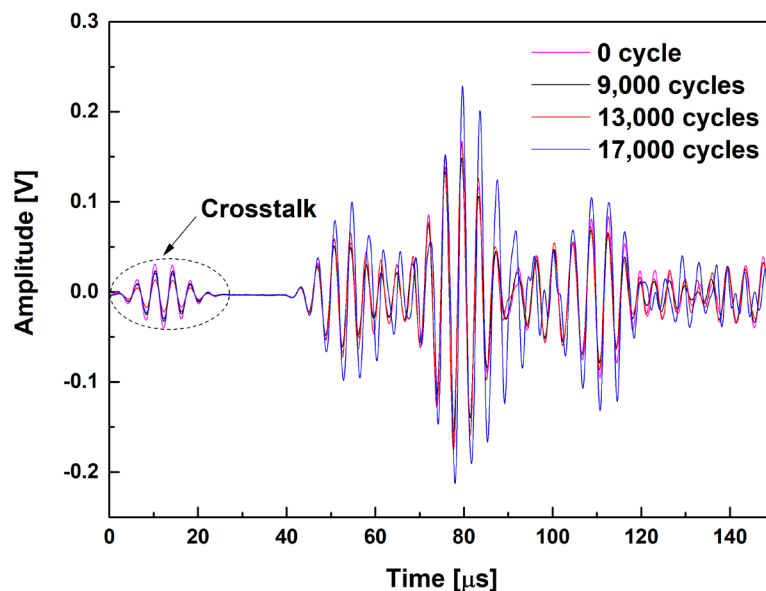
351 As depicted in **Fig. 6**, the fatigue testing for Specimen 1 is terminated at 18,000 cycles when the
352 corner crack penetrates the whole thickness of the specimen, while for Specimen 2, it concludes
353 at 23,000 cycles. The fatigue testing for the two specimens is paused every 1,000 cycles for
354 conducting ultrasonic testing to acquire UGW signals, indicating that the fatigue testing of the
355 two specimens is paused 19 and 24 times (ultrasonic testing is also performed at 0 cycle) for
356 ultrasonic measurement, respectively. By taking advantage of the pulse-inversion technique, 4
357 UGW signals can be obtained by the two sensors mounted on the specimen in a single pause,
358 resulting in a total of 172 UGW signals acquired in the experiment.

359

360 Representative UGW signals captured by Sensor 1 mounted on Specimen 1 at different fatigue
361 cycles are shown in **Fig. 8 (a)**. It can be observed from the raw signals that the magnitude of the
362 signals, a typical linear feature of UGW, varies against the fatigue cycles, while the phase and

363 waveform remain largely unchanged. By performing CWT, the time-domain UGW signals
364 displayed in **Fig. 8 (a)** are converted to RGB images representing the time-frequency spectra of
365 the original signals, as shown in **Fig. 8 (b)**. A rectangular window function is utilized before the
366 implementation of the CWT, to eliminate the effect of the crosstalk interference in the raw signals
367 induced by the measurement instruments. It can be found that during the crack initiation period
368 (*i.e.*, 0 cycle and 9,000 cycles), the spectra of UGW signals only contain linear components at the
369 fundamental frequency (250 kHz). On the contrary, apart from linear components, nonlinear
370 components at higher frequency range can be found in the spectra of UGW signals captured during
371 the crack growth stage (*i.e.*, 13,000 cycles and 17,000 cycles). This can be attributed to the CAN
372 induced by the ‘breathing’ behavior of fatigue cracks, which has been extensively scrutinized in
373 previous studies [3, 40-42]. The personified ‘breathing’ describes the periodic opening and closing
374 of a fatigue crack during the propagation of a probing UGW, causing the nonlinear modulation of
375 the crack on the probing UGW and the resultant higher-order harmonic generation which can be
376 clearly seen in **Fig. 8 (b)**.

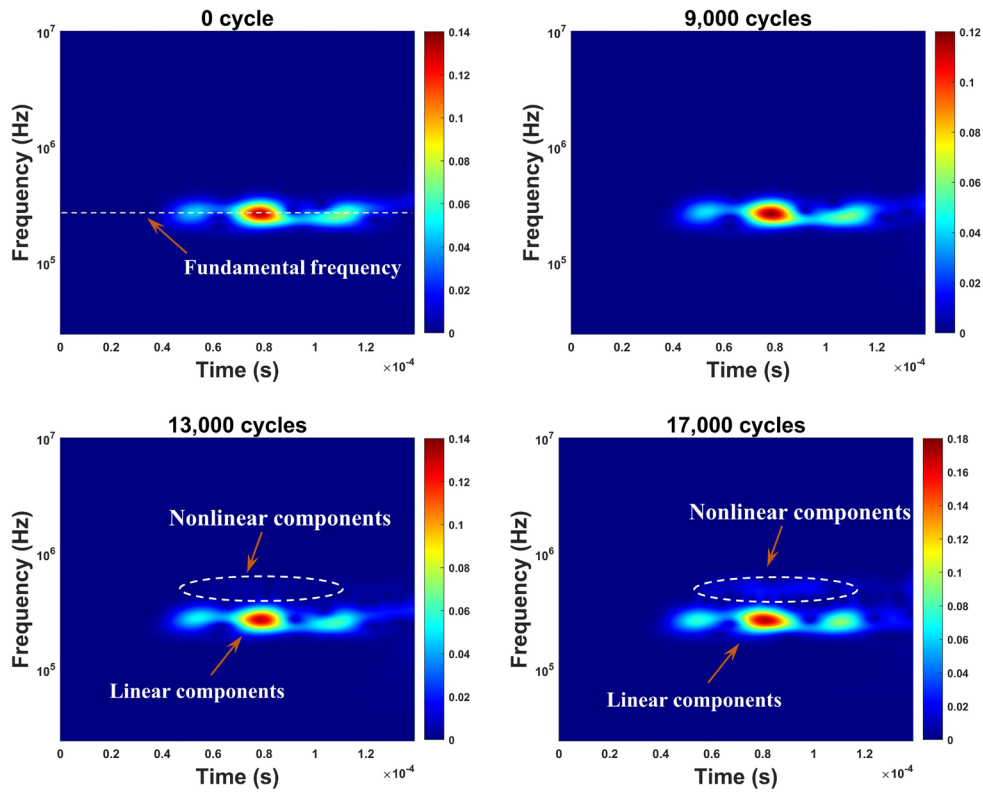
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378

379

(a)



(b)

Fig. 8. (a) Typical UGW signals captured from Specimen 1 at different fatigue cycles; and (b) the corresponding spectra of the selected GUV signals obtained via the CWT.

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385 The RGB images of the spectra of the experimental UGW signals acquired from Specimen 1 and
386 Specimen 2, embracing both linear and nonlinear features of the captured UGWs, are collected to
387 construct an image dataset for subsequent training and validation of the proposed DenseNet-based
388 regression model. All the images are scaled to 224×224 before training and validation, to match
389 the input unit of the DenseNet.

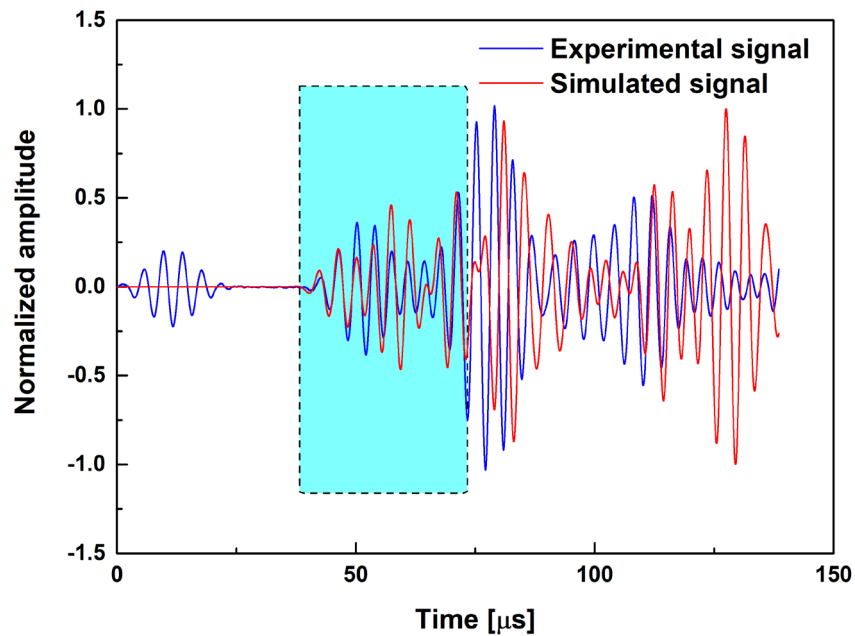
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391 Due to the experimental data is insufficient to train a DL model, FEM simulation is conducted for
392 the purpose of dataset expansion. 57 FE models are built to obtain synthetic UGW signals from
393 waveguides with corner cracks of different severities (crack length and depth ranging from 1 to 3
394 mm) emanating from a fastener hole. In each simulation case, two sensing points with the same
395 locations of the two sensors used in experiment are selected to acquire UGW signals. The pulse-

396 inversion technique adopted in the experiment is also utilized in the FEM simulation, leading to
397 a total of 228 artificial UGW signals obtained from the simulation.

398
399 Reliable and verified FEM simulation results are crucial for training the proposed model. A
400 representative simulated UGW signal captured from the waveguide containing a corner crack
401 with the length of 1.2 mm and depth of 1.5 mm is illustrated in **Fig. 9**, and further compared with
402 its experimental counterpart which is obtained from Specimen 2 when the corner crack measuring
403 1.201 mm long and 1.514 mm deep at 17,000 cycles. For better comparison, the simulated and
404 experimental UGW signals are normalized by their peak values, respectively. It can be observed
405 that the simulated signal is in good agreement with the experimental signal, especially in the first-
406 arrival wave packet as highlighted in the dashed region, which contains the majority of the crack-
407 related UGW information. It is therefore concluded that the FEM simulation is able to provide
408 reliable synthetic UGW signals to expand dataset for training the model.

409



410

411 **Fig. 9.** The simulated UGW signals obtained from the waveguide containing a corner crack with the length of
412 1.2 mm and depth of 1.5 mm and the comparison with its experimental counterpart.

413

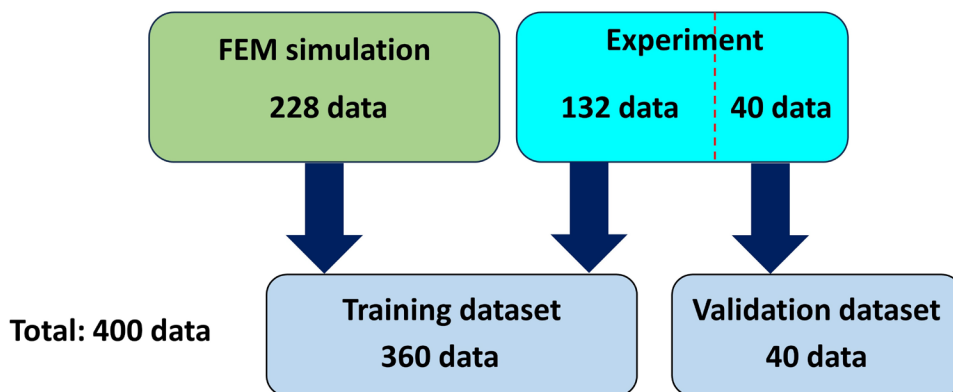
414 The CWT is subsequently recalled to convert the synthetic UGW signals obtained from FEM
415 simulation into RGB images for dataset expansion for a better training performance. Like the
416 images obtained from experimental signals, the spectral images of simulated signals are also re-
417 sized to 224×224 to fulfill the requirement of DenseNet.

418

419 **4.2 Dataset for Model Training and Validation**

420 Based on the experiment and FEM simulation, a total of 400 data samples, including 178
421 experimental data and 228 simulation data, are collected and split to build the training and
422 validation datasets, for the training and validation of the proposed DenseNet-based model. As
423 illustrated in **Fig. 10**, the training dataset (360 data) accounts for 90 % of the total acquired data,
424 consisting of all simulation data and part of the experimental data (132 data). The remaining 10
425 % of the total data is utilized to construct the validation dataset comprising pure experimental
426 data, to demonstrate the effectiveness of the trained model in predicting the length and depth of
427 the corner crack using real-world UGW signals.

428



429

430 **Fig. 10.** Allocation of simulation and experimental data for model training and validation.

431

432 **4.3 Training and Validation Results**

433 With the training dataset containing both simulated and experimental data, the training of the
434 proposed DenseNet-based regression model is implemented on a computer with 64 Gigabytes

435 RAM, an Intel(R) Core(TM) i9-11900K CPU @ 3.50 GHz and a NVIDIA GeForce RTX 3080
436 GPU. The batch size and the number of training epoch are set to be 18 and 100, respectively,
437 resulting in a total of 2,000 iterations (the parameters of the model are updated in every single
438 iteration) throughout the entire training process. Adam optimizer is utilized to optimize the
439 training efficiency, considering its superb merits in terms of convergence speed, memory
440 requirement and generalization to new data. The learning rate is initially set to be 0.001 and then
441 dynamically adjusted as the training progresses according to a pre-defined schedule, to prevent
442 the chaotical bounce around the local minimum value.

443
444 The mean squared error (MSE) is chosen as the loss function as it is appropriate to update the
445 model parameters in regression tasks, which is defined as

$$446 \quad MSE = \frac{1}{N} \sum_{i=1}^N (x_i - \hat{x}_i)^2, \quad (2)$$

447 where N is the batch size, x_i and \hat{x}_i represent the true and predicted values, respectively. **Fig. 11**
448 displays the variation of the training loss within the 100 epochs of the training process, from which
449 it can be observed that the loss drastically drops in the first few epochs, then gradually decreases
450 and finally stabilizes at around 0.006. The minimum value of the model loss during the whole
451 training process is 0.00562 at the 92nd epoch. The rapid convergence of the loss value verifies the
452 effectiveness of the training of the proposed model using the combination of simulated and
453 experimental data.

454

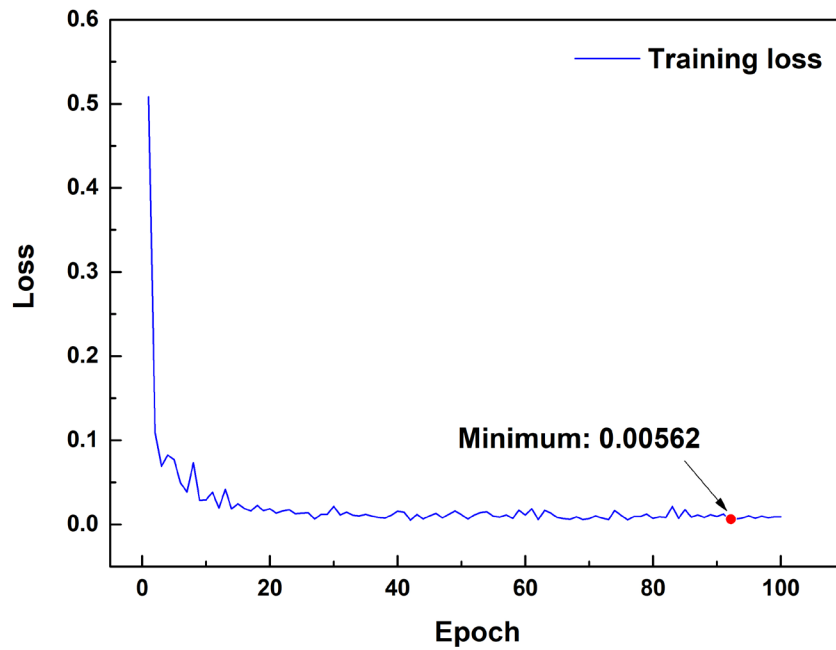


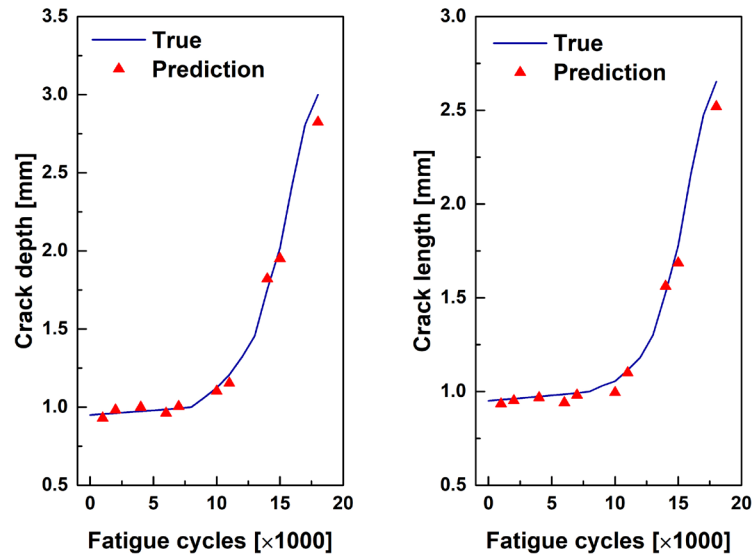
Fig. 11. The loss of the proposed regression model during the training process.

455
 456
 457
 458 The validity of the proposed DenseNet-based regression model in evaluating the severity of 3D
 459 fatigue cracks is testified by applying the trained model to predict the length and depth of the
 460 corner crack in the two specimens using the validation dataset. **Fig. 12** illustrates the predicted
 461 results as well as the comparison with true values measured in the experiment. Good agreement
 462 between the predicted and measured results throughout the whole fatigue crack growth testing has
 463 demonstrated that the proposed regression model is capable of evaluating 3D fatigue cracks not
 464 only in their embryonic stage, also in the stable growth stage, in a continuous and quantitative
 465 manner. The relative errors of the crack length and depth predicted by the trained model are
 466 summarized in **Table 1**, from which it can be found that the maximum relative error in the two
 467 specimens is 5.97 %, showing the high accuracy of the model prediction.

Table 1. Summary of the relative errors of model prediction.

	Relative error of crack length (%)		Relative error of crack depth (%)	
	Maximum	Mean	Maximum	Mean
Specimen 1	5.69	2.30	5.83	2.69
Specimen 2	3.95	1.14	5.97	1.70

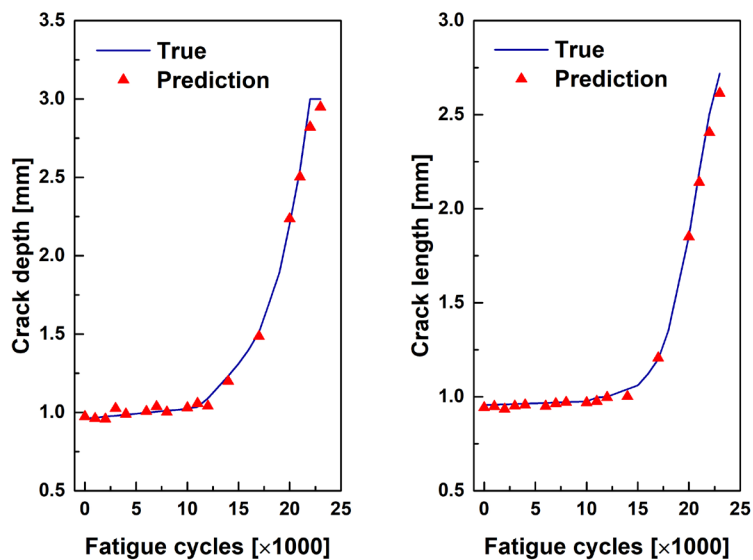
470



471

472

(a)



473

474

(b)

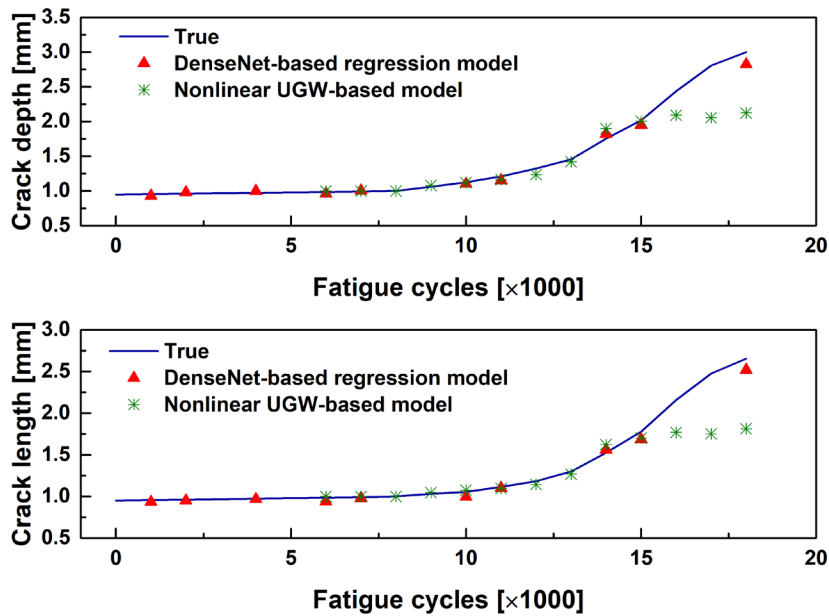
475 **Fig. 12.** Predicted length and depth of the corner crack in (a) Specimen 1 and (b) Specimen 2 using the trained
476 DenseNet-based regression model, as well as the comparison with experimental results.

477

478 4.4 Comparison Study

479 A comparison study is conducted to highlight the superb merit of the proposed DenseNet-based
480 regression model over previously reported nonlinear UGW-based model in the assessment of 3D
481 fatigue cracks, spanning from crack initiation, through stable growth, to macroscopic crack
482 formation. Based on the elastodynamic analysis, the nonlinear interaction between a fatigue crack

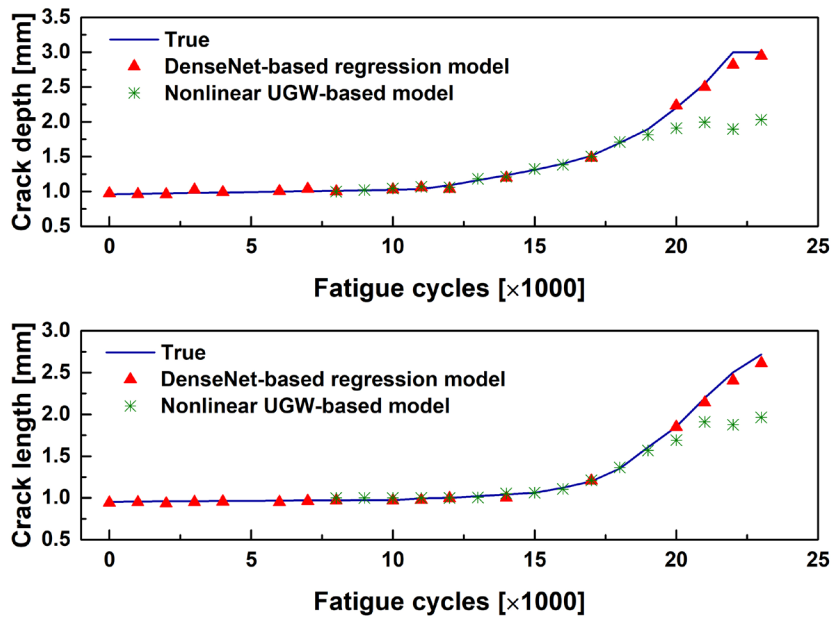
483 and probing UGW was scrutinized and a crack-surface-area-dependent nonlinear damage index
484 was proposed for characterizing the severity of fatigue cracks [16]. To take a step further, a two-
485 step, 3D fatigue crack assessment model capable of evaluating the crack length and depth was
486 developed by combining the nonlinear damage index and the correlation between crack length
487 and depth arising from a fatigue crack growth model [24]. In this section, the nonlinear UGW-
488 based model is applied to evaluate the length and depth of the corner crack in Specimen 1 and
489 Specimen 2 using the experimentally captured UGW signals. **Fig. 13.** compares the predicted
490 results of the proposed DenseNet-based regression and the nonlinear UGW-based model.
491



492

493

(a)



(b)

494
 495
 496 **Fig. 13.** Length and depth of the corner crack in (a) Specimen 1 and (b) Specimen 2 predicted by the proposed
 497 DenseNet-based regression model and the nonlinear UGW-based model [24], as well as the comparison with
 498 experimental results.
 499

500 It can be observed from **Fig. 13** that both models can predict the crack length and depth with high
 501 accuracy during the crack initiation and the early stage of crack growth when the crack size is less
 502 than 2 mm, for example, the first 15,000 fatigue cycles of Specimen 1 and the first 19,000 fatigue
 503 cycles of Specimen 2. However, the nonlinear UGW-based model fails to assess the crack severity
 504 when a macroscopic crack is formed (*i.e.*, crack grows beyond 2 mm in this study). In this stage,
 505 dimensions of the crack are relatively large, and the large crack opening displacement imposes
 506 restrictions on the nonlinear modulation of the crack on the probing UGW, leading to the
 507 inapplicability of the nonlinear damage index. On the contrary, by learning both linear and
 508 nonlinear features of the captured UGW signals, the proposed DenseNet-based regression model
 509 can still evaluate the severity of the crack even when the corner crack has already penetrated the
 510 whole thickness of the specimen, demonstrating its usability throughout the initiation and growth
 511 stages of 3D fatigue cracks.
 512

513 **5. Concluding Remarks**

514 A DL-based regression model making use of the DenseNet framework is developed in this study,
515 targeting quantitative evaluation of 3D, non-penetrating fatigue cracks from their embryonic
516 initiation, through progressive growth to formation of macroscopic cracks. CWT is adopted to
517 convert the UGW signals into RGB images representing their time-frequency domain spectra,
518 which includes both linear and nonlinear features of UGW signals. The spectra images are
519 subsequently used as the input of the DenseNet to extract implicit features. The last fully
520 connected layer of DenseNet is modified as a regression layer to predict the length and depth of
521 the 3D fatigue crack. To validate the effectiveness of the proposed DenseNet-based regression
522 model, fatigue crack growth and ultrasonic testing is performed to acquire UGW signals during
523 the initiation and growth stages of a corner crack emanating from a fastener hole. FE simulation
524 is further conducted for dataset expansion. The collected experimental and simulated data are split
525 into training and validation datasets, for training and verifying the proposed model, respectively.
526 A comparison study is undertaken and the results demonstrate that compared with previously
527 reported nonlinear UGW-based model, the proposed DenseNet-based regression model can
528 evaluate the length and depth of 3D fatigue cracks with high accuracy not only in the crack
529 initiation stage, but also in the stable growth stage, showing its versatility in early awareness and
530 long-term assessment of fatigue cracks.

531

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